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“Electoral College and Election Fraud”

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# Electoral College and Election Fraud<sup>\*</sup>

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## Abstract

One frequently overlooked aspect of the U.S.-style electoral college system is that it discourages election fraud. In a presidential election based on the popular vote, competing political parties are motivated to manipulate votes in areas where they hold the most significant influence, such as states where they control local executive offices, legislatures, and the judiciary. However, with the electoral college system in place, the incentives for fraud shift to swing states where the local government is politically divided, and fraud is therefore more difficult and costly. Our theoretical model elucidates why the electoral college system provides more effective protection against election fraud compared to the popular vote system. While polarization makes fraud more likely, it does not affect the superiority of the electoral college system.

**Keywords:** presidential elections, electoral college, election fraud.

**JEL Classification:** P16, D74.

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# Introduction

The fact that the United States, a country known for its long history of uninterrupted elections under the same fundamental rules, does not elect its president through a popular vote is a constant source of public amusement and intense debate. According to a survey conducted by the Pew Research Center in January 2021, 55 percent of Americans favor electing the president based on the popular vote, while only 43 percent support the current system of the electoral college.<sup>1</sup> Even prior to the controversial 2016 and 2020 elections, headlines such as “How to Get Rid of the Electoral College” have been pervasive.<sup>2</sup> Since the adoption of the U.S. Constitution in 1787, there have been more than 700 congressional proposals aimed at reforming or repealing it (Strömberg, 2008). Notably, the issue of election fraud rarely features prominently in these discussions.

In this paper, we argue that the current system creates strong disincentives for national parties to engage in election fraud, which in turn might explain why discussions about the merits of the electoral college often neglect the issue. Under the electoral college, presidential candidates compete for votes on a state-by-state basis. The winner of each state obtains all the state’s electoral votes, the number of which is equal to the size of the state’s congressional delegation, which, in turn, is roughly aligned with the state’s population.<sup>3</sup> This system often results in very close elections in a select few states. In the 2020 presidential election, for instance, the incumbent President Trump faced losses of 10 thousand votes (0.3 percent) in Arizona, 12 thousand votes (0.2 percent) in Georgia, and 20 thousand votes (0.6 percent) in Wisconsin. Similarly, in 2016, Hillary Clinton experienced a combined margin of defeat of 76 thousand votes across three swing states.

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<sup>1</sup>“Majority of Americans continue to favor moving away from Electoral College”, *Pew Research Center*.

<sup>2</sup>“How to get rid of the Electoral College”, *Brookings Institute*. See also Schumaker and Loomis (2002) for a variety of perspectives on the contentious outcome of the 2000 presidential election between George W. Bush and Albert Gore.

<sup>3</sup>This is a simplification of the actual process in the U.S. As the number of electoral votes for each state is equal to the number of representatives of this state in the House, which is formed basing on proportional representation, plus two (the number of the state’s Senators), the less populous states are over-represented. Two states, Maine and Nevada, do not use the winner-take-all system to allocate their electoral votes.

A superficial analysis would suggest that the relatively small degree of fraud required to overturn the election in either case could present a significant opportunity, if not a temptation, to engage in such activities.

The contribution of this paper is to consider the critical point: under the electoral college, the states where fraudulent votes would be most valuable are also the states where it would be most difficult to obtain them. Consider, hypothetically, what President Trump would need to do in order to change the outcome of the election he lost in 2020. In the states that he lost closely – Arizona, Wisconsin, and Georgia – his opponents had a significant representation at all levels of government such as the state Supreme Courts, the lower and upper chambers of the state legislatures, and the states’ Congressional delegations. Thus, organizing fraud sufficient to swing the outcome in these states with the expectation that election supervisors, prosecutors, and judges would turn a blind eye would be extremely challenging. At the same time, it would be comparatively easier to obtain a substantial number of fraudulent votes in Republican-dominated states such as Tennessee, Texas, Alabama, or Oklahoma, where President Trump won by significant margins and enjoyed widespread support at all levels of government. However, while it may be easier to steal votes in any of former states, it would be pointless under the electoral college system.<sup>4</sup> In contrast, under the popular vote system, a vote in Tennessee would hold the same value as a vote in Wisconsin, and a party inclined to steal an election would have the opportunity to conduct larger-scale fraud more easily.

We elaborate on the intuition of this argument in a simple example in Section 2 and then present a general model. Our model features a finite number of states, a finite population that is symmetrically distributed across these states and that is subject to sym-

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<sup>4</sup>In 2020, Trump lost by 10,000 votes (0.3 percent) in Arizona, 12,000 votes (0.2 percent) in Georgia, and 20,000 votes (0.6 percent) in Wisconsin. Meanwhile, Trump won by 708,000 votes in Tennessee, 631,000 votes in Texas, 591,000 votes in Alabama. The “excess” votes from each of these states would have been sufficient to win, if were “transferred” to states that Trump lost. In 2016, Hillary Clinton lost by combined margin 76,000 votes in the three swing states; her popular vote margin was 2,865,000. In fact, in most of modern elections the votes won by the losing candidate in “their” states would have been sufficient to overturn the result if they were cast in pivotal states instead.

metric tastes shocks. These assumptions make sure that an equilibrium exists and, more critically, that the popular vote and electoral college systems yield identical outcomes in the absence of fraud. While restrictive, these assumptions allow us to focus on the issue of fraud rather than other well-studied consequences of electoral college such as pivotality and minority representation (Shapley and Shubik, 1954; Uslander, 1976; Strömberg, 2008; Wright, 2009). The main question we ask is which electoral system provides stronger deterrence to engaging in fraud that could change the outcome of the election.

It is not surprising that in extremely close elections the temptation to engage in some fraud is high. This fact should not be disheartening: after all, close elections are quite random anyway (Eggers et al., 2015), with the outcome affected by, e.g., weather on the election day (Shachar and Nalebuff, 1999; Gomez, Hansford and Krause, 2007; Fowler, 2015). At the other extreme, landslide elections such as 1964 or 1984 presidential races in the U.S. are next to impossible to defraud. For these reasons, our measure of resilience of the electoral system to fraud is how far apart the fair vote tallies of the two parties have to be in order to deter vote fraud. In other words, we ask the following question: How big should the difference in votes be to make *not* committing fraud a Nash equilibrium, and how does this threshold depend on the electoral system?

Our main result is formulated in these terms: under realistic assumptions, *the threshold that prevents fraud under the electoral college system is higher than under the popular vote*. These thresholds are determined from two constraints that each party that seeks to change the outcome of the election by the means of fraud needs to overcome. First, there should be enough votes to steal and people willing to engage in such a process in each state that the party targets; this is what we call the *feasibility constraint*. Second, the overall cost of the process, real or reputational, should not exceed the party's willingness to pay (or, equivalently, the budget available), which we call the *incentive constraint*. We show that on the margin, that is, when the society is just indifferent between the electoral college and the popular vote system, the two active constraints are the feasibility con-

straint of the electoral college system and the incentive constraint of the popular vote system. This observation drives the main result. It also allows us to study potentially complex questions, such as the consequences of polarization, in a straightforward way. Perhaps surprisingly, we show that an increase in polarization (within or between states) does not change which electoral system deters fraud more reliably, nor does it hurt the optimal system’s ability to deter fraud – but, interestingly, it may make whichever system is the worst even worse.

To make the case in the most transparent way, we assume that the political space is symmetric, and the number of jurisdictions and individuals living in those jurisdictions is large but finite. Thus, the Nash theorem guarantees existence of an equilibrium. Furthermore, we demonstrate that, in general, if there is an equilibrium that involves no fraud, it is a unique equilibrium. Armed with these insights, we investigate the conditions under which a Nash equilibrium that involves no fraud exists under either of the electoral systems.<sup>5</sup> To simplify formulas even more, when making comparisons we take the limit and make use of a continuous approximation; the advantage of this approach is also that it makes the problem of unequal sizes of jurisdictions moot. This results in a “minimalist” model that nevertheless allows us to make robust predictions.

One critical assumption that we make is that under both systems, the instances of fraud are checked by local authorities, regulators, and courts. This is natural given the way the electoral system works in the United States at present, and also that the most likely replacement, the so-called National Popular Vote Interstate Compact, would retain local counting and enforcement even while implementing the popular vote (see a discussion in Section 5). In other words, we are comparing apples to apples. What our

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<sup>5</sup>Importantly, we study the question of *deterrence of fraud* rather than that of competition and performance of fraud strategies. Doing the latter would, at least in the case of the electoral college, amount to *colonel Blotto*-style games of allocating scarce resources over many battlefields (Borel, 1953; Blackett, 1958; Konrad and Kovenock, 2009; Roberson and Kvasov, 2012; Kovenock and Roberson, 2012). Analyzing such games would be important and relevant for studying *legal* means of competition for the victory in the electoral college, including allocation of advertising campaigns, get-out-the-vote operations, or candidate visits. Given our question of electoral fraud, it is natural to focus instead on understanding the conditions under which fraud is deterred completely.

analysis is missing is the possibility of conducting the election, counting, and enforcement by a national election commissions, similar to ones that exist in other countries. We do not model such a commission, and note the possibility that, like any institution, it may be captured and corrupted. Our model, however, suggests that implementing the popular vote system without such a national authority and continuing to count votes and enforce integrity locally is inferior to the present system.

The literature on electoral college is both vast and limited. Primary criticisms of the electoral college include claims that it produces results that significantly deviate from that of the popular vote (Hinich and Ordeshook, 1974; Abbott and Levine, 1991; Strömberg, 2008; Hummel, 2011), reduces or skews voters' power (Banzhaf III, 1965; Mann and Shapley, 1964; Gelman and Katz, 2001; Gelman, Silver and Edlin, 2012), and unfairly empowers certain groups over others (Uslaner, 1976; Nelson, 1974). In one of the first positive models of campaign resource allocation, Brams and Davis (1974) predicted that candidates should allocate resources in proportion to about  $3/2$ 's of the power of the electoral votes in each state. Strömberg (2008), another positive model of campaign resource allocation, finds similar results by comparing the model's predictions to the number of campaign visits by state.

In the political science literature, different aspects of electoral college were extensively discussed, including the ones that we investigate. The issue of the election fraud in the context of electoral college has popped up before, e.g., in Adkison and Elliott (1997) or, in a legal context, in Florey (2017)). However, in the absence of an appropriate model, an argument might be lacking in logical consistency. For example, Best (1975) argued that while the electoral college restricts the effects of fraud to individual states, in which the fraud may be too insignificant to matter, the popular vote may see the temptation to commit fraud in a close race "blanketing the entire country". Our model shows that while the conclusion about the magnitude of fraud in the electoral college vs. the popular vote is correct, election fraud *spreading* under the popular vote would look different.

More plausibly, it will be concentrated in states with a relative domination of one party. Similarly, [Grofman and Feld \(2005\)](#) argue that the electoral college creates additional incentives for election fraud *at the state level* as the naive logic would predict. As our model demonstrates, the electoral college disincentivizes election fraud, dividing the states into two sets: those in which stealing votes is too expensive, and those in which stealing votes does not increase chances to win. [Swaim \(2020\)](#) correctly states that the electoral college deters fraud, yet considers the main mechanism to be the parties' inability to predict which states are going to be close. Our model shows that it does not matter: even if the whole fraud game is played *ex post*, the deterrence effect of the electoral college system is still in place.

In Section 5, we discuss election fraud in the U.S. and around the world (see [Lehoucq, 2003](#), and [Simpser, 2013](#), for a comprehensive overview of systematic causes of election fraud around the world). The formal literature on election fraud is limited, yet distinctive. One important strand of the literature is concerned with election fraud in authoritarian regimes and its consequences (see a recent review in [Egorov and Sonin, 2022](#)). In [Rozenas \(2010, 2016\)](#); [Little \(2012, 2015\)](#); [Egorov and Sonin \(2021\)](#) dictators manipulate elections to signal their popularity. In [Magaloni \(2010\)](#) and [Fearon \(2011\)](#), the threat of mass protests limits the incumbent's willingness to resort to fraud. In our model, while we microfound parties' opportunities to steal votes, the main argument does not depend on a specific type of fraud.

The rest of the paper is organized as follows. Section 2 provides a simple example that captures the intuition of the model. Section 3 introduces our main model. Section 4 contains all main results, including comparison between the electoral college and popular vote. Section 5 discusses practical implications of our results and their robustness, while Section 6 concludes.



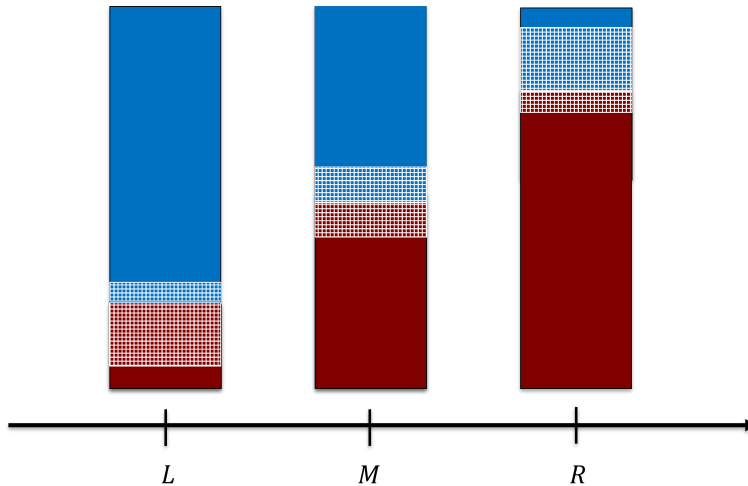


Figure 1: Expected vote shares in three states,  $L$ ,  $M$ , and  $R$ . The shaded area denotes votes that can be “stolen” by the other party.

## 2 A simple example

The following example illustrates the logic of our argument.

There are two parties,  $L$  and  $R$ , and three states of equal size, left-wing  $L$ , middle  $M$ , and right-wing  $R$ . Party  $L$  enjoys 70% expected support in state  $L$ , 50% support in state  $M$ , and 30% support in state  $R$ . The expected support for party  $R$  in these states is 30%, 50%, and 70%, respectively. Consider a scenario where a common valence shock increases the support of party  $L$  by an additional fraction  $x$ ,  $0 < x < 0.1$  in each of the states. Then party  $L$  wins the election under both the electoral college and the popular vote rules.

Suppose, however, that party  $R$  considers overturning the election by the means of electoral fraud, specifically by miscounting the votes for party  $L$  as votes for party  $R$ . Our question is which electoral system would be more vulnerable to such fraudulent activities.

Under the electoral college system, party  $R$  wins state  $R$ , loses state  $L$  by a large margin, and falls short of winning state  $M$  by a margin of  $2x$  votes. To win the election, party  $R$  would need to flip a fraction of  $x$  votes in state  $M$ , or slightly more than that. However, this would not be sufficient to secure victory under the popular vote system. Indeed, in

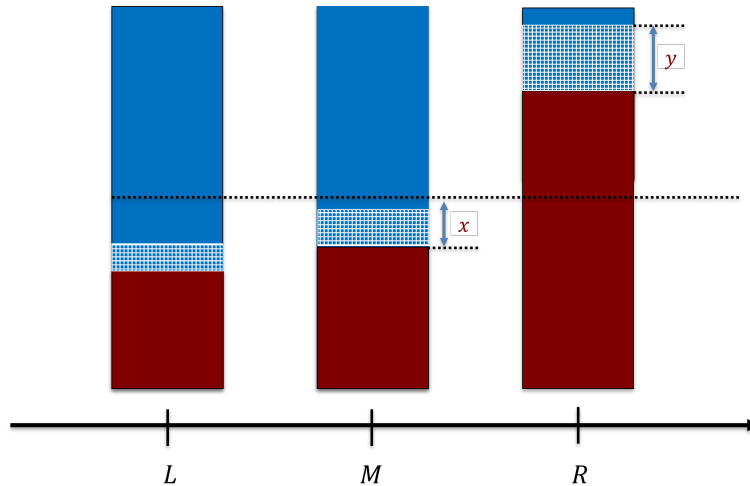


Figure 2: The vote share that party  $R$ , the loser in this particular outcome, could steal in state  $R$  would have been sufficient to overturn elections under the popular vote, but not under the electoral college as  $x < y$ .

the popular vote scenario, party  $R$  is winning a fraction  $0.5 - x$  of the total popular vote. To overcome this disadvantage, party  $R$  would need to flip a fraction  $x$  of the overall votes, for example, by flipping a fraction of  $x$  votes in each state. This implies a larger-scale fraud operation, three times as significant. This observation lends support to the notion that the popular vote system may be more resilient to electoral fraud compared to the electoral college system.

However, this reasoning overlooks the possibility that some votes may be easier to manipulate than others. There are factors that suggest conducting fraud in “friendly” jurisdictions could be simpler and less risky, as there may be a greater willingness among individuals to participate in fraudulent activities and provide cover for such actions. If so, instead of flipping a proportional share of votes in all states, party  $R$  may prioritize its efforts in the favorable jurisdiction  $\mathbf{R}$ . In this example, if party  $R$  successfully flips three times as many votes, namely  $3x$ , in state  $\mathbf{R}$  alone, it would be sufficient to secure a popular vote victory without resorting to fraud in the other two states. And, in line with this logic, flipping  $3x$  votes in state  $\mathbf{R}$  might indeed be easier than flipping  $x$  votes in state  $\mathbf{M}$ .

To be more specific, consider the case where  $x = 0.05$ . Suppose that flipping a vote

in any state requires the support of three randomly chosen people in that state who support party  $R$  in this election (for instance, the individuals responsible for counting that specific vote), and, even then, the success rate of flipping the vote is only 75%. In state  $\mathbf{M}$ , where 40% of the population supports party  $R$  (because  $50 - x = 40$ ), the share of votes that can be flipped would be  $0.75 \times 0.4^3 = 0.048$ , which is less than the 5% needed to flip  $\mathbf{M}$  and win the election under the electoral college system. On the other hand, in state  $\mathbf{R}$ , where 60% of the population supports party  $R$ , the share of votes that can be flipped would be  $0.75 \times 0.6^3 = 0.162$ , which is higher than the 15% needed in state  $\mathbf{R}$  to add 5% to the popular vote for party  $R$  and therefore win the popular vote. Thus, under these conditions, the electoral college system would impede efforts to overturn the election, whereas the popular vote system would not. (Figure 2 illustrates the same point.)

### 3 Formal model

**Elections.** Consider a country composed of an odd number of jurisdictions or states  $j \in \{0, \frac{1}{J}, \dots, \frac{I}{J}\}$ , where  $J$  is even. Each state consists of an odd number of similar precincts  $P$  each of which is populated by an odd number of individuals  $i \in \{0, \frac{1}{I}, \dots, \frac{I}{I}\}$  where  $I$  is even, so the number of individuals in each state is odd:  $N = P(I + 1)$ . Assuming odd numbers of states and individuals in each state rules out draws and simplifies the notation and analysis, and states (and precincts) of equal sizes allow us to highlight the main point without extra notation, but the results hold more generally than that. Precincts would be irrelevant in the case of fair elections, but they are important for modeling election fraud.

There are two political parties, left  $L$  and right  $R$ . One candidate from each party runs for president; abusing notation, we denote the candidates by  $L$  and  $R$  as well. We normalize all individuals' utilities by setting them equal to zero if the president is from party  $L$ , and denote the utility of individual  $i$  from state  $j$  if party  $R$  is in power by  $u_{ij}$ .

These net preferences for party  $R$  over party  $L$  are a combination of individual-level preferences and state-level preferences. More precisely, we assume that

$$u_{ij} = y(i) + z(j),$$

where  $y(\cdot)$  and  $z(\cdot)$  are monotonically increasing functions satisfying symmetry around  $1/2$ :  $y(1/2-x) = -y(1/2+x)$  and  $z(1/2-x) = -z(1/2+x)$  and  $z(1) > y(1)$ . The first assumption guarantees ex ante symmetry of preferences for both parties and across states, and the latter assumption ensures that in each state there are individuals supporting each party, which is both realistic and allows us to avoid corner cases. We will denote the c.d.f. of  $y(i)$  by  $\Psi(\cdot)$ . and the c.d.f. of  $z(j)$  by  $\Omega(\cdot)$ . For simplicity, we focus on the case where individual- and state-specific preferences are linear:

$$\begin{aligned} y(i) &= \beta \left( i - \frac{1}{2} \right), \\ z(j) &= \gamma \left( j - \frac{1}{2} \right), \end{aligned}$$

with  $\beta > \gamma$ . With this notation, as the number of individuals in each precinct (and thus in each state),  $I$ , goes up, the share of supporters of party  $L$  in state  $j$  tends to

$$\frac{1}{2} - \frac{\delta}{\beta} - \frac{\gamma}{\beta} \left( j - \frac{1}{2} \right).^6 \tag{1}$$

**Example 1.** Take  $\beta = 1$  and  $\gamma = 1/2$ . Then  $y(x) = x - 1/2$  and  $z(x) = (x - 1/2)/2$ . Then preferences of individuals in states are equally spaced, ranging from  $-3/4$  to  $1/4$  in the leftmost state 0 and between  $-1/4$  and  $3/4$  in the rightmost state 1. The share of individuals who prefer party  $R$  ranges from about  $1/4$  (for  $I$  sufficiently large) of support in the leftmost state to about  $3/4$  of support in the rightmost one.

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<sup>6</sup>Here and elsewhere, whenever we take a limit with respect to  $I$  with the sequence members defined for even numbers only, we assume that the limit is defined via a filter base that includes all sequences numbered by even numbers (Bourbaki, 2013). Limits with respect to  $J$  and  $P$  are defined in a similar way.

The model so far is completely symmetric. Assume that prior to voting, all individuals get a common shock  $\delta$  that affects the willingness to have the candidate from party  $R$  elected. Specifically, we define  $w_{ipj} = u_{ipj} + \delta$  and assume that each individual votes for party  $R$  candidate ( $v_{ipj} = R$ ) if  $w_{ipj} > 0$  and for party  $L$  candidate ( $v_{ipj} = L$ ) otherwise (note that sincere voting is indeed an equilibrium in the voting game). To ensure that both parties get a positive share of votes in all states, we assume that  $|\delta| < \beta - \gamma$ . Lastly, we assume that voters who are, given the shock, indifferent between voting for either candidate, will vote for candidate  $L$ ; since this can happen only for a finite number of values of  $\delta$ , this assumption does not have material consequences but simplifies notation.

**Election Fraud.** There are countless ways to conduct fraud. These include allowing ineligible (e.g., deceased) people to vote, voting multiple times, failing to count votes that were legitimately cast, putting ballots in the wrong pile when counting, falsification of protocols, etc. To keep matters simple, as well as to avoid dealing with turnout and the total number of votes, we focus on the fraud where votes cast for one candidate are counted for their opponent.

To be specific, we focus on fraud that takes place during counting the ballots. We assume that to commit fraud, party  $R$  (or  $L$ ) needs to find a sufficient number of individuals (or groups of individuals) that are willing to do so. One natural way to model this is as follows. Suppose that committing fraud in each precinct in any state requires conspiracy of all individuals involved in counting, which are assumed to be  $k \geq 2$  individuals randomly drawn from that precinct. For example, these could be those who count, supervise counting, perhaps other electoral officials or a local prosecutor who would look the other way or write off any inconsistencies as human errors. These individuals will conspire with probability  $\sigma$  (the opposite of honesty) if they all support the candidate who would benefit from the fraud, otherwise they conspire with probability zero.

We will further assume that the scope of fraud has limits on both the extensive and

intensive margins. On the intensive margin, the conspirators in each precinct are able to change at most share  $\lambda$  of votes; changing more votes could be challenged, detected on recounting and impossible to write off as an innocuous human error. On the extensive margin, the total share of votes that any party may change across all states is constrained by its incentives: we assume that winning the election brings each party benefit  $B$  but switching votes is costly (due to logistical or reputational considerations) and we capture this with a linear cost: changing share  $\alpha$  of all votes in the country costs party  $\alpha C$ . Denoting  $\mu = B/C$ , we get that a party would never switch more than share  $\mu$  of votes, even if it means moving from losing the election for sure to winning the election. In other words, fraud is assumed to be limited, to some extent, both locally (at the precinct level) and globally (for the whole election).

**Electoral systems.** Each individual casts their vote  $v_{ipj} \in \{L, R\}$ , but in the process of committing fraud these votes may be changed to some  $z_{ipj} \in \{L, R\}$ , which are what matters for the outcome of the election. We consider two electoral systems. Under popular vote (PV), candidate  $R$  is elected if and only if she gets a majority of votes:

$$\sum_{j=0}^J \sum_{p=1}^P \sum_{i=0}^I \mathbb{1}\{z_{ipj} = R\} > \sum_{j=0}^J \sum_{p=1}^P \sum_{i=0}^I \mathbb{1}\{z_{ipj} = L\}.$$

Under electoral college (EC), candidate  $R$  is elected if and only if she wins in a majority of states:

$$\sum_{j=0}^J \mathbb{1} \left\{ \sum_{p=1}^P \sum_{i=0}^I \mathbb{1}\{z_{ipj} = R\} > \sum_{p=1}^P \sum_{i=0}^I \mathbb{1}\{z_{ipj} = L\} \right\} > \frac{J+1}{2}.$$

Note that in the model up to now, the two electoral systems produce identical results with probability 1: candidate  $R$  wins under either system if  $\delta > 0$  and loses if  $\delta < 0$ .

**Timing.** The sequence of moves in the game is as follows.

1. In each precinct in each state, a random set of individuals  $C_{jp}$  with  $|C_{jp}| = k$  is picked

to become the electoral commission.

2. Nature picks parameter  $\delta$  from some distribution with full support on the real line and everyone observes it.
3. Each individual  $i$  in precinct  $p$  in state  $j$  casts a vote  $v_{ipj} \in \{L, R\}$ , with  $v_{ipj} = R$  if and only if  $w_{ipj} > 0$ .
4. Each party,  $L$  and  $R$ , observe which electoral commissions would be agreeable to conduct fraud, and then they decide, simultaneously, how many votes they want flipped in each such precinct.
5. Denoting the flipped votes in each precinct  $z_{ipj}$ , the winner of the election is determined according to the electoral system used (PV or EC).
6. Everyone gets payoffs. For voters, it is  $z_{ipj}$  if the candidate from party  $R$  is elected and 0 otherwise. For party  $R$ , it is  $B\mathbb{I}\{R \text{ won}\} - \alpha_R C$ , where  $\mathbb{I}$  is indicator and  $\alpha_R$  is the share of votes that party  $R$  flipped. For party  $L$  the payoff is analogous.

## 4 Analysis

We start by establishing existence and some general properties of equilibria with any finite number of states, electoral precincts, and citizens in each precinct. Then, to compare two systems, we will consider the limit case where the numbers of states, precincts in each state and individuals in each precinct are large.

**Existence and Uniqueness of Equilibria.** We first show existence of Nash equilibrium in the game between two parties. Given any distribution of actual votes  $\{v_{ipj}\}$ , each party's strategy set involves flipping a finite number of votes across states and precincts. This is a finite game, which therefore has at least one Nash equilibrium in, generally speaking, mixed strategies.

**Proposition 1.** *For each realization of  $\delta$ , there is a Nash equilibrium, possibly in mixed strategies.*

We are particularly interested in distinguishing situations where there is fraud in equilibrium from those where there is no fraud in equilibrium. Here, multiplicity of equilibria could present a problem. However, as it turns out, the uniqueness of the relevant equilibrium depends on the fundamentals of the model and not on equilibrium selection. Specifically, we show that generically, if there is fraud with a positive probability in one equilibrium, then there is fraud in every equilibrium. Denote the strategy where no fraud is committed by  $NF$  for each player; our next result shows that if  $(NF, NF)$  is a strict Nash equilibrium, then this equilibrium is unique. It remains to note that any such equilibrium is strict as long as  $\frac{B}{C}$  is not rational, which holds for a generic set of parameters  $B$  and  $C$ . Formally, we prove the following proposition.

**Proposition 2.** *Under either electoral system, if  $(NF, NF)$  is a strict Nash equilibrium, then it is a unique Nash equilibrium.*

**Proof of Proposition 2.** Indeed, suppose that in addition to  $(NF, NF)$ , there is another equilibrium,  $(\hat{s}_L, \hat{s}_R)$ , possibly in mixed strategies. Suppose without loss of generality that player  $L$  would lose, absent fraud. Then  $U_L(NF, NF) = 0$ , and furthermore  $U_L(NF, \hat{s}_R) = 0$  (because if  $L$  loses without fraud, then it loses with fraud by party  $R$  as well).

Now observe that  $U_L(\hat{s}_L, \hat{s}_R) \geq U_L(NF, \hat{s}_R) = 0$  since  $(\hat{s}_L, \hat{s}_R)$  is an equilibrium. This implies that  $U_L(\hat{s}_L, NF) \geq U_L(\hat{s}_L, \hat{s}_R) \geq 0$ , with the latter inequality holding since if party  $R$  switches from  $\hat{s}_R$  to  $NF$ , this cannot decrease party  $L$ 's payoff. Consequently,  $U_L(\hat{s}_L, NF) \geq 0 = U_L(NF, NF)$ , which contradicts the assumption that  $(NF, NF)$  is a strict Nash equilibrium. ■

**The No Fraud Conditions.** In light of Proposition 2, we are interested in characterizing the conditions under which  $(NF, NF)$  is an equilibrium, i.e., there exists an equilibrium that does not involve fraud. Proposition 2 guarantees that if there is no fraud in



one equilibrium, then there is no fraud in any equilibrium for generic parameter values. Naturally, fraud is less likely if the election is not too close. However, Proposition 3 below establishes a much stronger result.

We prove that for each of the two electoral systems, there exists a threshold such that any election outcome with vote difference exceeding this threshold is safe from fraud. This happens when the loser finds it infeasible or too costly to overturn the election, in which case the winner does not want to engage in fraud either. For large elections, these thresholds are easy to characterize using explicit formulas, which will enable to compare the two electoral systems.

More precisely, the pair of strategies  $(NF, NF)$  is an equilibrium if the losing party at least weakly prefers  $NF$  to committing fraud, because the winning party prefers no fraud as long as the losing party commits none. For the losing party to commit fraud, two constraints need to be satisfied. First, fraud must be feasible: the losing party should be able to flip enough votes to overturn the election. Second, committing fraud must be incentive-compatible, in the sense that the benefits of winning elections should be higher than the aggregate cost of fraud. Under either electoral system, both of these constraints are more likely to be satisfied when  $|\delta|$  is small; this is when the election is close and thus fewer votes need to be flipped to change the results. When the number of states, precincts and individuals are large, we are able to derive sharp conditions for when each of the constraint is satisfied, for each electoral system, and obtain complete characterization.

Let us start with the electoral college. Define  $\hat{\delta}_{EC}$  and  $\tilde{\delta}_{EC}$  as the unique positive solutions to the following equations:

$$\lambda\sigma \times \left( \frac{1}{2} - \frac{\hat{\delta}_{EC}}{\beta} \right)^k = \frac{\hat{\delta}_{EC}}{\beta}, \quad (2)$$

$$C \times H(\tilde{\delta}_{EC}) = B, \quad (3)$$

where

$$H(\delta) = \begin{cases} \frac{1}{2} \frac{\delta^2}{\beta\gamma} & \text{if } \delta \leq \frac{\gamma}{2}; \\ \frac{\delta}{2\beta} - \frac{\gamma}{8\beta} & \text{if } \delta > \frac{\gamma}{2}. \end{cases}$$

Define  $\delta_{EC} = \min \{ \hat{\delta}_{EC}, \tilde{\delta}_{EC} \}$ .

Equation (2) guarantees that fraud is feasible under the electoral college; if  $\delta > \hat{\delta}_{EC}$ , then there are not enough votes in the appropriate states for the losing party to overturn the outcome. Equation (3) reflects incentive compatibility; if  $\delta > \tilde{\delta}_{EC}$ , then the losing party does not have sufficient resources to engage in fraud. The function  $H(\delta)$  integrates the votes that the losing party needs to recover in the states that it lost, with the first formula corresponding to the case where the losing party won at least some states (the more typical scenario) and the second taking care of the case where it lost all states and needs to overturn the election in a half of the country.

First take any  $\delta \in (-\delta_{EC}, \delta_{EC})$ ; without loss of generality assume that it's nonnegative:  $\delta \geq 0$ , so absent fraud party  $L$  loses. Consider any state  $j$ . Recall that the share of individuals in this state supporting party  $L$  tends to (1) as  $I, P \rightarrow \infty$ . Notice that this equals  $\frac{1}{2} - \frac{\delta}{\beta}$  for state  $j = \frac{1}{2}$  and is higher than that in any state  $j < \frac{1}{2}$ . Thus, the number of votes that need to be switched in state  $j = \frac{1}{2}$  for party  $L$  to win tends to  $\frac{\delta}{\beta}$  under the same limit  $I, P \rightarrow \infty$ , and this limit is lower than that in  $j < \frac{1}{2}$ . By the law of large numbers, as  $I, P \rightarrow \infty$ , the share of votes that can be stolen in state  $j = \frac{1}{2}$  is  $\lambda\sigma \left( \frac{1}{2} - \frac{\delta}{\beta} \right)^k$ , and since we assumed  $\delta < \hat{\delta}_{EC}$ , stealing this number of votes is sufficient to change the election outcome in this state. By the argument above, in other states  $j < \frac{1}{2}$  that party  $L$  lost, the number of votes needed to be switched is lower and the share of votes that may be switched is higher, so overturning elections in those states is also feasible with probability arbitrarily close to 1. Thus, overturning elections is feasible for party  $L$ .

Let us now consider whether overturning elections is incentive-compatible for party  $L$ . The benefit of overturning elections is  $B$ . The cost is  $C$  times the share of votes that need to be changed. For large  $I$  and  $P$ , the share of votes that need to be changed in

state  $j$  is  $\frac{\gamma}{\beta} \left( j - \frac{1}{2} \right) + \frac{\delta}{\beta}$ , as follows from **1**, and this change is necessary only in states that party  $L$  lost. These are states  $j \in \left[ \max \left\{ \frac{1}{2} - \frac{\delta}{\gamma}, 0 \right\}, \frac{1}{2} \right]$ . As the number of states  $j \rightarrow \infty$ , the aggregate share of votes that need to be changed equals to

$$\int_{\max\{\frac{1}{2}-\frac{\delta}{\gamma},0\}}^{\frac{1}{2}} \left( \frac{\gamma}{\beta} \left( j - \frac{1}{2} \right) + \frac{\delta}{\beta} \right) dj = \min \left\{ \frac{\delta^2}{2\beta\gamma}, \frac{\delta}{2\beta} - \frac{\gamma}{8\beta} \right\}.$$

Note that since  $\delta < \tilde{\delta}_{EC}$ , party  $L$  strictly prefers to pay the cost to convert this share of votes as the benefit outweighs it. Consequently, if party  $R$  plays  $NF$ , doing the same is not a best response for party  $L$ , which proves that for  $|\delta| < \delta_{EC}$ ,  $(NF, NF)$  is not a Nash equilibrium; so, provided that  $I, P, J$  are large enough, any equilibrium involves fraud.

Consider the alternative case where  $|\delta| > \delta_{EC}$ ; without loss of generality assume that  $\delta$  is positive. In this case, either  $\delta > \hat{\delta}_{EC}$  or  $\delta > \tilde{\delta}_{EC}$ . In the first case, overturning elections in state  $j = \frac{1}{2}$  is possible with probability arbitrarily close to 0 for large  $I, P$ . In the latter case, the share of votes needed to be overturned is higher than  $\frac{B}{C}$  with an arbitrarily high probability as  $I, P, J \rightarrow \infty$ . Consequently, with a probability approaching 1, playing  $NF$  is a best response for party  $L$ . As for party  $R$ , if party  $L$  does not commit fraud and loses the election, then committing fraud does not make sense for party  $R$  either. As a result, if  $\delta > \delta_{EC}$ , the probability that  $(NF, NF)$  is equilibrium is arbitrarily close to 1 in the limit. This completes the proof for the electoral college case.

We now need to do a similar exercise for the case of popular vote. Define  $\hat{\delta}_{PV}$  and  $\tilde{\delta}_{PV}$  as unique positive solutions to the following equations:

$$\lambda\sigma \times \int_0^1 \left( \frac{1}{2} - \frac{\hat{\delta}_{PV}}{\beta} - \frac{\gamma}{\beta} \left( j - \frac{1}{2} \right) \right)^k dj = \frac{\hat{\delta}_{PV}}{\beta}, \quad (4)$$

$$C \times \frac{\tilde{\delta}_{PV}}{\beta} = B, \quad (5)$$

and let  $\delta_{PV} = \min \{ \hat{\delta}_{PV}, \tilde{\delta}_{PV} \}$ . Similarly to the case of electoral college, equation (4) guarantees that fraud is feasible under the popular vote. That is, if  $\delta > \hat{\delta}_{PV}$ , then there are

not enough votes for the losing party to overturn the outcome. Similarly, equation (5) reflects incentive compatibility: if  $\delta > \tilde{\delta}_{PV}$ , then the losing party would not engage in fraud because of the lack of resources.

Consider  $|\delta| < \delta_{PV}$ , and without loss of generality assume that it is positive. Now party  $L$  is losing absent fraud and the total share of votes that it needs overturned tends to  $\frac{\delta}{\beta}$  provided that  $I, P, J$  are large. Since  $\delta < \tilde{\delta}_{PV}$ , the benefits of winning outweigh the cost of fraud. As far as feasibility is concerned, for  $I, P$  sufficiently high, the share of votes that party  $L$  can switch in state  $j$  is  $\lambda\sigma\left(\frac{1}{2} - \frac{\delta}{\beta} - \frac{\gamma}{\beta}\left(j - \frac{1}{2}\right)\right)^k$ , and as the number of states goes up, the total share of votes that may be switched tends to the left-hand side of (4). Consequently, as  $I, P, J \rightarrow \infty$ ,  $NF$  is not a best response for party  $L$  with an arbitrarily high probability, and as a result there is fraud in equilibrium with probability tending to 1.

Conversely, if  $|\delta| > \delta_{PV}$ , and we again assume that  $\delta$  is positive, then for  $I, P, J$  sufficiently high, either  $\delta > \hat{\delta}_{PV}$  or  $\delta > \tilde{\delta}_{PV}$  is violated with an arbitrarily high probability. If so, as in the case of electoral college,  $(NF, NF)$  is an equilibrium with a probability tending to 1 as  $I, P, J \rightarrow \infty$ .

The following Proposition 3 summarizes formally the above discussion.

**Proposition 3.** *For  $\delta_{EC}$  and  $\delta_{PV}$  defined above:*

(i) *Under the electoral college system, if the absolute value of shock  $\delta$  satisfies  $|\delta| < \delta_{EC}$ , then  $\lim_{I,P,J \rightarrow \infty} \Pr_{EC}(NF, NF) = 0$ , and if the absolute value of shock  $\delta$  satisfies  $|\delta| > \delta_{EC}$ , then  $\lim_{I,P,J \rightarrow \infty} \Pr_{EC}(NF, NF) = 1$ ;*

(ii) *Under the popular vote system, if the absolute value of shock  $\delta$  satisfies  $|\delta| < \delta_{PV}$ , then  $\lim_{I,P,J \rightarrow \infty} \Pr_{PV}(NF, NF) = 0$ , and if the absolute value of shock  $\delta$  satisfies  $|\delta| > \delta_{PV}$ , then  $\lim_{I,P,J \rightarrow \infty} \Pr_{PV}(NF, NF) = 1$ .*

As Proposition 3 shows, for small shocks and therefore close vote tallies, large elections will feature fraud with probability 1. This is also not particularly surprising. Indeed, if the election is so close that only one vote would change the result, it is not reasonable

to believe that among thousands of people involved in counting, not a single one would make a deliberate mistake. At the same time, one could also argue that in extremely close elections, fraud is not a major societal problem: from a utilitarian perspective, in close elections, any outcome reflects the will of about half of the electorate.

At the same time, in a landslide election, fraud is next to impossible, because it is either not feasible, too costly, or both. Naturally, our results lead us to the following question: How far apart should the vote for the two parties be in order to deter fraud, and how does it depend on the electoral system? Within each electoral system, for large elections, the answer to the first question is already given by Proposition 3 already: there is no fraud for  $|\delta| > \delta_{PV}$  under the popular vote and for  $|\delta| > \delta_{EC}$  under the electoral college. We now turn to comparative statics results that will enable us to compare the two systems.

**Electoral College vs. Popular Vote.** The next proposition addresses the question of which electoral system deters fraud for a wider range of parameter values. We prove the following result.

**Proposition 4.** *Higher values of  $B$  and  $k$  or lower values of  $\lambda$ ,  $\sigma$ , or  $C$  make it more likely that  $\delta_{PV} > \delta_{EC}$ . In other words, electoral college deters fraud for a wider range of values of  $\delta$  than popular vote.*

**Proof of Proposition 4.** Let us first prove that the thresholds for each of the electoral systems defined in (2) and (4) (feasibility) and in (3) and (5) (incentive compatibility) satisfy the following inequalities:  $\hat{\delta}_{EC} < \hat{\delta}_{PV}$  and  $\tilde{\delta}_{EC} > \tilde{\delta}_{PV}$ .

To show that  $\hat{\delta}_{EC} < \hat{\delta}_{PV}$ , suppose, to obtain a contradiction, that  $\hat{\delta}_{EC} \geq \hat{\delta}_{PV}$ . We then

have, using (2), (4), and Jensen's inequality:

$$\begin{aligned}
\frac{\hat{\delta}_{PV}}{\beta} &= \lambda\sigma \times \int_0^1 \left( \frac{1}{2} - \frac{\hat{\delta}_{PV}}{\beta} - \frac{\gamma}{\beta} \left( j - \frac{1}{2} \right) \right)^k dj \\
&> \lambda\sigma \times \left( \frac{1}{2} - \frac{\hat{\delta}_{PV}}{\beta} \right)^k \geq \lambda\sigma \times \left( \frac{1}{2} - \frac{\hat{\delta}_{EC}}{\beta} \right)^k \\
&= \frac{\hat{\delta}_{EC}}{\beta} \geq \frac{\hat{\delta}_{PV}}{\beta}.
\end{aligned}$$

Since one inequality is strict, this is a contradiction, which proves that  $\hat{\delta}_{EC} < \hat{\delta}_{PV}$ .

To show that  $\tilde{\delta}_{EC} > \tilde{\delta}_{PV}$ , suppose, to obtain a contradiction, that  $\tilde{\delta}_{EC} \leq \tilde{\delta}_{PV}$  and consider two cases. If  $\tilde{\delta}_{EC} \leq \frac{\gamma}{2}$ , then by (3),  $\tilde{\delta}_{EC}$  satisfies  $\frac{1}{2} \frac{\tilde{\delta}_{EC}^2}{\beta\gamma} = \frac{B}{C}$ , and from (5) we have  $\frac{\tilde{\delta}_{PV}}{\beta} = \frac{B}{C}$ . We have

$$\frac{B}{C} = \frac{\tilde{\delta}_{PV}}{\beta} \geq \frac{\tilde{\delta}_{EC}}{\beta} \geq \frac{\tilde{\delta}_{EC}}{\beta} \times \frac{2\tilde{\delta}_{EC}}{\gamma} > \frac{1}{2} \frac{\tilde{\delta}_{EC}^2}{\beta\gamma} = \frac{B}{C},$$

which is a contradiction. Likewise, if  $\tilde{\delta}_{EC} > \frac{\gamma}{2}$ , then by (3) we have  $\frac{\tilde{\delta}_{EC}}{2\beta} - \frac{\gamma}{8\beta} = \frac{B}{C}$ , and therefore

$$\frac{B}{C} = \frac{\tilde{\delta}_{PV}}{\beta} \geq \frac{\tilde{\delta}_{EC}}{\beta} > \frac{\tilde{\delta}_{EC}}{2\beta} > \frac{\tilde{\delta}_{EC}}{2\beta} - \frac{\gamma}{8\beta} = \frac{B}{C},$$

which is again a contradiction. Therefore, we have proved that  $\tilde{\delta}_{EC} > \tilde{\delta}_{PV}$ .

Since  $\hat{\delta}_{EC} < \hat{\delta}_{PV}$  and  $\tilde{\delta}_{EC} > \tilde{\delta}_{PV}$ , of these four values, the minimum is equal to either  $\hat{\delta}_{EC}$  or  $\tilde{\delta}_{PV}$ :  $\min\{\delta_{EC}, \delta_{PV}\} = \min\{\hat{\delta}_{EC}, \tilde{\delta}_{PV}\}$ . Thus,  $\delta_{EC} < \delta_{PV}$  if and only if  $\hat{\delta}_{EC} < \tilde{\delta}_{PV}$ . Notice that  $\hat{\delta}_{EC}$  satisfies (2) and is therefore increasing in  $\lambda$  and  $\sigma$  and is decreasing in  $k$  (and does not depend on  $B$  or  $C$ ), whereas  $\tilde{\delta}_{PV}$  satisfies (5) and is therefore increasing in  $B$  and decreasing in  $C$  (and does not depend on  $\lambda$ ,  $\sigma$ , or  $k$ ). The comparative statics result immediately follows. ■

The intuition underlying Proposition 4 is as follows. In either system, fraud is possible if both the budget and the feasibility constraints are lax; in other words, if the losing party is both willing and able to commit fraud. Let us compare the corresponding constraints under the two electoral systems. First, the incentive constraint is tighter under popular

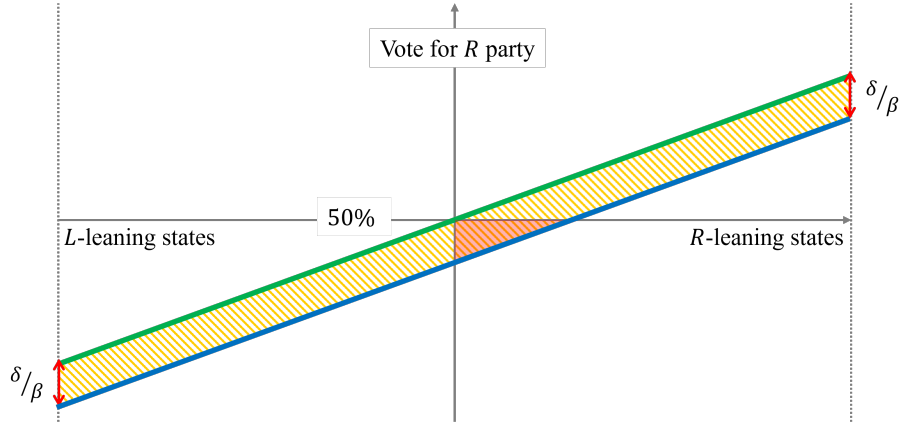


Figure 3: **When the feasibility constraint is *not* binding:** The yellow (shaded) area is the amount of vote party  $R$  needs to “steal” after a shock of size  $\delta$  to win the popular vote. Under the electoral college, the needed vote is a much smaller red (solid) area.

vote: it is possible that it is satisfied under electoral college and that it is violated under popular vote, but not vice versa. This is intuitive: to overturn elections under the electoral college, the losing party only needs to change a fraction of votes it would need to change under popular vote, as it only needs to undo some votes in the states that it would have won absent the shock (see Figure 3).

The situation is opposite when it comes to the feasibility constraint. Under the electoral college, the losing party is able to overturn the election if and only if it is able to flip the median state, which requires changing a share of  $|\delta|/\beta$  votes in that state. Indeed, in all other states where the party is losing but where it would win absent the shock, it is both easier to commit fraud (the party is more popular) and fewer votes need to be changed (for the same reason). Under the popular vote system, the losing party needs to find the same share of votes over the entire country. Since finding enough people willing to conspire is much easier in states where the party is very popular (or, in other words, it is sufficiently difficult to find many votes in the median state), the feasibility constraint is tighter under the electoral college system than under the popular vote (see Figure 4).

Proposition 4 is stated in terms of which parameters give the electoral college the advantage in preventing fraud. Of course, the converse of these statements is also true.

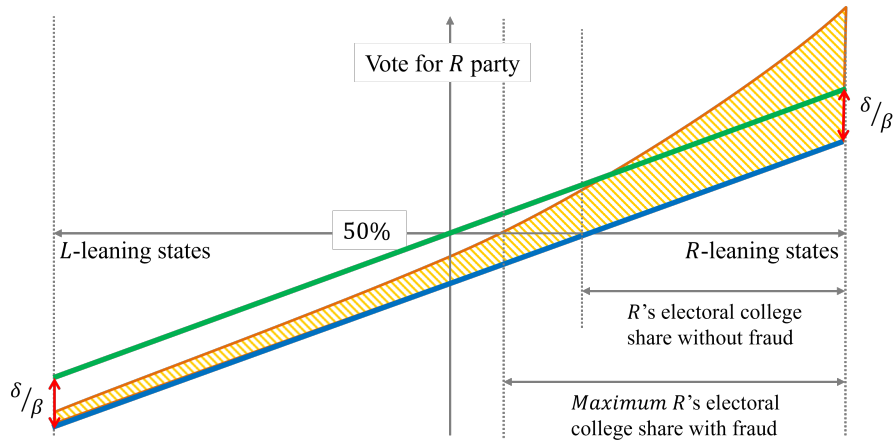


Figure 4: **When the feasibility constraint is binding:** In each state, the yellow (shaded) interval is the amount of vote party  $R$  can “steal” in this state. Though the total yellow (shaded) area might be sufficient to cover the popular vote deficit,  $R$  cannot overcome  $L$ ’s electoral college advantage after a shock of size  $\delta$ .

Lower values of  $B$  and  $k$  or higher values of  $\lambda$ ,  $\sigma$ , or  $C$  increase the likelihood of fraud under the electoral college relative to that of the popular vote.

The comparison between the two systems reduces to the following question: Is it the incentive constraint under the popular vote or the feasibility constraint under the electoral college that does a better job deterring fraud? With only two “active” constraints, the comparison is straightforward. A lower  $B$  or higher  $C$  tightens the incentive constraint, thus making the popular vote more attractive. Conversely, lower  $\lambda$  or  $\sigma$  or higher  $k$  make make conspiracies more difficult to assemble or less effective, which tightens the incentive constraint and makes the electoral college more effective.

**The Effect of Polarization.** We now turn to the effects of polarization on deterrence of fraud provided by the two electoral systems. In our framework, there are two parameters that capture political polarization. Parameter  $\beta$ , which enters in individual-specific preferences, is higher if individuals have more polarized preferences within any state – yet, changes in  $\beta$  do not affect the within-state average. An increase in  $\beta$  will decrease the share of persuadable people in any state, but will not affect the average alignment



of the state. In contrast, parameter  $\gamma$  increases polarization between states while leaving within-state polarization constant. In other words, a higher  $\gamma$  would make red states redder and blue states bluer, while reducing the number of purple states.

With respect to these polarization parameters we have the following comparative statics. First, consider an increase in state-level polarization, so  $\gamma$  is higher. This change has two consequences. Under the electoral college, this relaxes the incentive constraints, as the losing party loses fewer votes in fewer states, thereby making fraud cheaper. Under the popular vote, however, polarization affects the feasibility constraint, as the increased difficulty to commit fraud in hostile states is more than outweighed by reduced difficulty to commit fraud in friendly states. Remarkably, however, this increase in  $\gamma$  does not affect the feasibility constraint under the electoral college system (as it depends on the median state only) or the incentive constraint under the popular vote system (because the total number of votes that need to be changed is the same). Since the comparison between the two electoral systems hinges on these two latter constraints, an increase in between-state polarization does not change the relative ranking of the systems, nor does it effect the effectiveness of the better system at deterring fraud. At the same time, it can make the worse of the two systems even worse.

The situation is different with respect to individual-level polarization  $\beta$ . An increase in  $\beta$  makes all individuals less sensitive to shock  $\delta$ , and the losing party would now need to change fewer votes. This relaxes all constraints, and in particular it makes both the electoral college and the popular vote systems more prone to fraud. However, a more careful examination of the constraints reveals that the ranking of the two systems would not be affected in this case either. To sum up, we have the following result.

**Proposition 5.** *An increase in individual-level polarization  $\beta$  increases  $\delta_{EC}$  and  $\delta_{PV}$ , making fraud more likely in both the electoral college and the popular vote systems, yet preserves the relative ranking of the two electoral systems. An increase in state-level polarization  $\gamma$  does not affect  $\delta_{EC}$  or  $\delta_{PV}$  and thus does not affect the effectiveness of either system*

at deterring fraud or the ranking between the two systems. However, it makes the worse system weakly worse.

**Proof of Proposition 5.** Notice that the feasibility constraint (2) depends on  $\hat{\delta}_{EC}$  and  $\beta$  only through the ratio  $\frac{\hat{\delta}_{EC}}{\beta}$ , and similarly, the incentive constraint (5) depend on  $\tilde{\delta}_{PV}$  and  $\beta$  through the ratio  $\frac{\tilde{\delta}_{PV}}{\beta}$  only. Let  $\beta' > \beta$ . The values that solve (2) and (5) now satisfy  $\frac{\hat{\delta}'_{EC}}{\beta'} = \frac{\hat{\delta}_{EC}}{\beta}$  and  $\frac{\tilde{\delta}'_{PV}}{\beta'} = \frac{\tilde{\delta}_{PV}}{\beta}$ . Thus,  $\hat{\delta}'_{EC} > \hat{\delta}_{EC}$  and  $\tilde{\delta}'_{PV} > \tilde{\delta}_{PV}$ . In addition, it is straightforward to see from (3) and (4) that  $\tilde{\delta}'_{EC} > \tilde{\delta}_{EC}$  and  $\hat{\delta}'_{PV} > \hat{\delta}_{PV}$ , which implies that  $\delta'_{EC} > \delta_{EC}$  and  $\delta'_{PV} > \delta_{PV}$ . Thus, both electoral systems are more prone to fraud under  $\beta'$  than under  $\beta$ . Finally, the sign of  $\hat{\delta}'_{EC} - \tilde{\delta}'_{PV}$  is the same as that of  $\hat{\delta}_{EC} - \tilde{\delta}_{PV}$ , because

$$\hat{\delta}'_{EC} - \tilde{\delta}'_{PV} = \frac{\beta'}{\beta} \left( \hat{\delta}_{EC} - \tilde{\delta}_{PV} \right).$$

This shows that if  $\delta_{EC} < \delta_{PV}$ , then  $\hat{\delta}_{EC} < \tilde{\delta}_{PV}$  and hence  $\hat{\delta}'_{EC} < \tilde{\delta}'_{PV}$  and thus  $\delta'_{EC} < \delta'_{PV}$ , and vice versa. This shows that the increase in  $\beta$  does not change which electoral system's results are more preferable.

Finally, notice that neither (2) nor (5) depends on  $\gamma$ , and therefore if  $\gamma$  increases to  $\gamma'' > \gamma$ , we would still have  $\hat{\delta}''_{EC} = \hat{\delta}_{EC}$  and  $\tilde{\delta}''_{PV} = \tilde{\delta}_{PV}$ . For the last result, notice that  $\tilde{\delta}_{EC}$  and  $\hat{\delta}_{PV}$  are increasing in  $\gamma$ , as follows from (3) and (4), respectively. Therefore, the worse system may be affected by this change in  $\gamma$ , provided that  $\tilde{\delta}_{EC} < \hat{\delta}_{EC}$  for the electoral college system and that  $\hat{\delta}_{PV} < \tilde{\delta}_{PV}$  for the popular vote system, and if so, it becomes more prone to fraud. ■

## 5 Discussion

One inherent challenge with our theory is the absence of a readily available experiment to test it. While an experiment falsifying our theory could be easily constructed in principle, in practice, very few countries around the world elect a strong executive by anything

but popular vote. The countries that have a strong presidency are often the same countries where election fraud is rampant or, even worse, where incumbents retain power through wholesale election theft (Egorov and Sonin, 2021). While this is in line with our theory, it is a problem that we have no genuine counterfactual scenario and are unable to distinguish our explanation of rarity of fraud in the United States election from mere correlation or chance.

The relative absence of fraud in presidential elections in the United States is indeed remarkable, especially given the controversies, polarization, and the narrow margins of some recent elections. Notably, Donald Trump has claimed the possibility of massive vote fraud in both the 2016 and 2020 presidential elections, which he won and lost, respectively. These claims prompted extensive investigations, yet no compelling evidence of systematic election fraud has been found for either election (for 2016, see Cottrell, Herron and Westwood, 2018; for 2020, see Eggers, Garro and Grimmer, 2021). In 2020, judges, including those appointed by Republicans, rejected 100 percent of lawsuits alleging election fraud (over 30 in total) that were filed on behalf of the defeated candidate.

It is important, however, that the occurrence of election fraud that can alter election outcomes is not outside the realm of possibility in the United States, which, if this were the case, would make our main point moot. In fact, incidences of such fraud have been documented in the sub-national-level elections. For instance, Caro (2011a) provides detailed historical evidence showing that the 1949 Senate race in Texas was stolen by the eventual winner, future U.S. President Lyndon Johnson. The first volume of the Johnson's biography, Caro (2011b), contains evidence that the 1941 Senate race was stolen as well, this time from Johnson. When such fraud happens, the collected evidence points to a mechanism of fraud that is fully consistent with our theory: candidates steal votes in those counties where they possess substantial advantage over their opponents (for example, Parr county in the 1949 Texas election).

In the remainder of this Section, we discuss the implications of our model, both nor-

mative and positive, its relevance to the current discourse surrounding the National Popular Vote Interstate Compact, and the robustness of the results.

**Normative implications.** The idea of replacing the electoral college with a system that reflects the popular vote is more than a theoretical possibility. While completely eliminating the electoral college would likely necessitate a constitutional amendment, which is unlikely to happen in the foreseeable future, alternative proposals that would effectively implement the outcome of the popular vote have been put forward. One notable example is the National Popular Vote Interstate Compact, which entails an agreement among states (and the District of Columbia, which also has electoral votes) to allocate their electors to the candidate who wins the popular vote. The Interstate Compact is designed to come into effect once the number of electoral votes possessed by participating states reaches 270, which is the minimum required to enforce the outcome of the election; as of June 2023, this number equals 205. The constitutionality of the Interstate Compact remains unsettled (Brody, 2013), yet this idea is arguably the most viable approach to emulate the popular vote system (Keyssar, 2020).

This paper is applicable to analyzing the trade-off between the existing electoral college system and the proposed Interstate Compact. Notably, under both systems, each state remains responsible for determining its delegation to the electoral college, whereas certain additional changes that could be implemented as part of the switch to the popular vote system, such as a national election commission or federal enforcement of electoral laws, are not part of consideration under the Interstate Compact. Our theory, therefore, suggests that the Interstate Compact system would likely be more prone to election fraud than the existing system. While one would expect current swing states to lose significance as sources of fraudulent votes, widespread fraud in states where one party enjoys overwhelming support, such as California and Illinois for Democrats or Texas and Tennessee for Republicans, could become a real possibility. One implication of this pa-

per is that implementing a popular vote system while entrusting the counting of votes to local authorities may create a system that is particularly vulnerable to fraud.

Our model also speaks to the effects of polarization, and the predictions are largely “negative,” suggesting that polarization does not change the preferred electoral system. In other words, if the objective is to deter electoral fraud and if there are indications that the current electoral college system has been effective thus far, the increasing polarization of the recent years, both within states and between states, does not provide a compelling reason to switch to a popular vote system, as follows from our theory.

**Positive implications.** Our motivating example in the Introduction suggests that in 2016, the electoral college might have been instrumental in protecting the election against fraud, because it essentially compelled the losing candidate, Donald Trump, to seek fraudulent votes in states like Arizona and Georgia where obtaining such votes would be significantly challenging. Interestingly, Donald Trump and some of his associates, while failing to “find the votes,” propagated the opposite narrative. They argued that the election outcome was actually fraudulent, alleging that Democrats had committed fraud in several states, Arizona and Georgia in particular. This narrative has been proven to be false, and while some errors and irregularities are probably unavoidable, the scale could not be large enough to affect the election in this particular case (Eggers, Garro and Grimmer, 2021).

The logic of our paper suggests why such fraud would be highly implausible *even from a theoretical standpoint*. For individuals willing to commit fraud in favor of Democrats in Arizona and Georgia, it would be natural to expect that such attempts would be put in check by numerous Republicans holding statewide and local offices in these states. And indeed, investigations into alleged fraud were indeed conducted, and Republican officials, apart from being motivated by their own interests, were under intense pressure from Donald Trump. The fact of these investigations underscores the point that stealing

votes or other forms of electoral fraud is exceptionally difficult and risky in swing states. And even if one believes in all-powerful machines that deliver the vote in places like New York or Illinois, the electoral college system makes these machines irrelevant for presidential elections.

**Extensions.** Our model is presented in perhaps the simplest possible way with numerous simplifying assumptions, in order to convey the point in the simplest way possible. Nevertheless, the main implications appear to be robust to a number of realistic possibilities that are outside of the present model.

So far, our analysis has solely focused on aggregate shocks that impact all states uniformly. Of course, shocks state-specific shocks  $\delta_j$  that affect individuals in state  $j$  only are also possible. In the simplest case where such shocks are i.i.d. across states (and, to make things even simpler, each shock is uniformly distributed with mean 0), this is mathematically equivalent to an increase in polarization between states, and therefore is covered by Proposition 5. Specifically, this would not change the comparison between the two electoral systems we consider. Even when such shocks are not formally covered under the Proposition 5, the intuition that these shocks should not affect the feasibility constraint under the electoral college system and the incentive constraint under the popular vote system still holds, and we know that the relative “tightness” of these two constraints determines the superiority of one electoral system over the other.

One could object to our assumption that the party attempting to steal an election already knows the “fair” vote tally, enabling them to strategize accordingly. We believe that this is a good first approximation, as parties have access to local representatives, internal polls conducted prior to the election day, turnout data and, exit polls. Nevertheless, it is true that such data are imperfect, requiring potential fraudsters to allocate their efforts based on incomplete information. This would also be applicable if the party has to allocate resources and persuade individuals to commit fraud ahead of the election date.

Regardless of how one conceptualizes or models the information available to parties, imperfect information makes it relatively more difficult to commit fraud under the electoral college system than under the popular vote system. Indeed, under the former, lack of information requires committing fraud in more states than necessary as a precautionary measure. Under the popular vote system, a stolen vote holds value no matter where it is stolen. Consequently, if anything, imperfect information at the time of committing fraud is likely to make the electoral college system relatively more effective at deterring fraud compared to the popular vote system.

Finally, there exist numerous potential methods of fraud apart from miscounting. These include ballot stuffing, having people vote multiple times, reducing the vote for the opponent through the physical disposal of ballots, failure to deliver ballots from precincts with high expected share of votes for the opponent, voter intimidation, and so forth. There are likely other methods, unknown to the authors of this paper. These methods are different in terms of costs, coordination efforts required, the likelihood of being caught red-handed or detection using statistical methods. Nevertheless, as long as the main assumption of our paper holds, specifically that organizing fraud is easier in a favorable jurisdiction, as in [Caro \(2011a\)](#), we should expect that the key findings of this paper hold true, and the important implications of this paper remain applicable.

## **6 Conclusion**

From time to time, and certainly in the wake of close presidential elections such as those in 1960 and 2000, the possibility of vote fraud in the United States elections is discussed. The recent elections of 2016 and 2020 have been no exception, with a major candidate making the possibility of fraud a prominent talking point before and after the elections. The drawbacks of the electoral college system as a whole are also frequently raised, and given the current state of polarization in the United States, opinions on the electoral col-

lege versus the popular vote systems appear to be deeply divided as well.

In this paper, we study the two systems from the standpoint of their effectiveness in deterring fraud. While the answer is generally ambiguous and depends on parameter values, we emphasize the role of the electoral college system as an institution that disincentivizes electoral fraud. Under the electoral college system, a party seeking to overturn the election outcome must concentrate their fraudulent activities in swing states where the election was close, and the electorate is therefore evenly divided. In such circumstances, each fraudulent vote is very costly and difficult to obtain, as the opposing party is well-represented in elected and administrative bodies of the state and will devote efforts to combat fraud. In contrast, under the popular vote system, each party's efforts would be concentrated in states where they possess near-complete dominance, such as Alabama for Republicans or Massachusetts for Democrats recently, and even if a larger number of votes is required to overturn the election result, they may be relatively easier to obtain. Our model compares these two electoral systems in a unified setting and demonstrates that the electoral college system offers superior protection against election fraud under reasonable assumptions, and increasing polarization is unlikely to alter this conclusion.



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