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"The Leveling Axiom"

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The Leveling Axiom

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Abstract

We characterize general constraints under which rational choices are characterized by asymmetric revealed preferences. A key feature of our main characterization result is expressed by the leveling axiom. We also consider the special case of a law-abiding decision maker who chooses optimally among legal options. We show that the law does not necessarily satisfy the leveling axiom and, therefore, transitivity adds empirical content to law-abiding choices.

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1 Introduction

This paper examines the question of whether transitivity adds empirical content to asymmetry in revealed preferences in a general model of constrained optimization. We deliver a full characterization of the constraints for which asymmetric revealed preferences fully characterizes rationalizable choice functions. In addition, a simplification of our main condition produces a novel axiom on economic constraints that we call the leveling axiom. Finally, we consider a specific economic constraint in decision-making: the law. That is, a law-abiding citizen chooses rationally and freely among the options that are both feasible and legal. We explore particular features of the law and show legal doctrines that violate the levelling axiom. Hence, transitivity adds empirical content to asymmetry in revealed preferences of a law-abiding citizen.

This paper is organized as follows: The introduction discusses informally our general model of constrained optimization and the problem of whether transitivity adds empirical content to asymmetry in revealed preferences. The introduction also introduces the leveling axiom and the special case of a law-abiding citizen. In section 2, we present a formal model of constrained optimal choice under general constraints. In section 3, we present the $L-$WARP theorem. This result provides a general characterization of constraints for which asymmetric revealed preferences fully characterizes rationalizable choice functions. Section 4 considers the special case of a law-abiding citizen. Section 5 presents a brief literature review. Section 6 concludes. Proofs are in the appendix.

1.1 Revealed Preferences in Constrained Optimization Models

Paul Samuelson (1938) introduced the weak axiom of revealed preference (WARP). Ever since, there has been interest in whether transitivity adds empirical content to asymmetry in revealed preferences. This question is sometimes referred to as the equivalence between WARP and the strong axiom of revealed preference (SARP), see Chambers and Echenique (2016) for a review.

We also examine the question of whether transitivity adds empirical content to asymmetry in revealed preferences in the following general constrained optimization setting: a decision maker maximizes a utility function u subject to a consideration set $L(B)$. The consideration set $L(B)$ is a subset of the set B of feasible options. Thus, a consideration function L takes, as input, a non-empty set B and returns, as output, a non-empty subset of B. The constrained optimal choice $C(B)$ maximizes the decision maker's utility among the options in the consideration set $L(B)$. That is,

$$C(B) = \text{argmax } u(x) \text{ subject to } x \in L(B). \tag{1}$$

Given L, a $L-$rationalizable choice function C is such that for some utility function u, (1) holds. For simplicity, we restrict attention to the case without indifferences and, hence, $C(B)$ is a single option. By definition, $C(B) \in L(B)$ in any $L-$rationalizable choice function C. This is assumed throughout the paper and, hence, when we refer to a characterization of $L-$rationalizable choice functions we either implicitly or explicitly assume $C(B) \in L(B)$.

In this setting, a simple axiom captures asymmetric revealed preferences. This axiom can be traced back at least to Richter (1966). We refer to this axiom as the $L-$WARP axiom. It requires that if two different options x and y belong to two consideration sets $L(B)$ and $L(B')$, then x cannot be chosen in one of these consideration sets while y is chosen in the other consideration set. This follows because if two options belong to the same consideration set, then chosen options are revealed preferred to rejected options.

For some, but not all, considerations function L, asymmetric revealed preferences fully characterizes $L-$rationalizable choice functions. This paper fully characterizes the consideration functions L for which asymmetric revealed preferences characterize $L-$rationalizable choice functions C. We now consider motivating examples.

1.2 Abstract Examples and the Leveling Axiom

Take the consideration function $L^u(B) = B$ for every set B, i.e., L^u is the identity function and does not produce any additional restriction on B. Then, L^u-WARP is equivalent to WARP, and asymmetric revealed preferences fully characterizes rationalizable choice functions. On the other extreme, take a consideration function L^r such that $L^r(B)$ is equal to a single option. Then, L^r completely restricts choice and $C(B) \in L^r(B)$ suffices to characterize rationalizable choice functions. Thus, both in the fully unrestricted case ($L = L^u$) and in the fully restricted case ($L = L^r$), asymmetric revealed preferences (plus $C(B) \in L(B)$) fully characterizes $L-$rationalizable choice functions.

Let's now restrict attention to three alternatives x, y and z. For several intermediary choice functions L (i.e., neither L^r nor L^u), asymmetric revealed preferences still fully characterizes $L-$rationalizable choice functions. For example, let L^1 be such that

$$L^1(x, y) = \{x, y\};\ L^1(y, z) = \{y, z\};\ L^1(x, z) = \{z\};\ L^1(x, y, z) = \{y, z\}. \qquad (2)$$

Given a choice function C, let u^1 be an utility function such that $u^1(x) > u^1(y)$ if $C(x, y) = x$ and $u^1(y) > u^1(x)$ if $C(x, y) = y$; $u^1(y) > u^1(z)$ if $C(y, z) = y$ and $u^1(z) > u^1(y)$ if $C(y, z) = z$. It is straightforward to see that if C satisfies L^1-WARP (and $C(B) \in L(B)$), then u^1 rationalizes C. Now let us make L^1 a bit less restrictive so that in the choice between x and z, both options are allowed. Let L^2 be a consideration function such that

$$L^2(x, y) = \{x, y\};\ L^2(y, z) = \{y, z\};\ L^2(x, z) = \{x, z\};\ L^2(x, y, z) = \{y, z\}.$$

The key distinction between L^1 and L^2 is that, L^2 imposes no restrictions on the binary choices, while L^1 restricts the choice between x and z to just z. This will prove significant in the determination of why L^1 satisfies the Leveling Axiom and L^2 does not satisfy the Leveling Axiom (see Definition 2.1 below).

Under L^2, asymmetric revealed preferences do not fully characterize L^2-rationalizable choice functions. Consider a cyclic choice function $\bar{C}$ such that

$$\bar{C}(x, y) = x; \bar{C}(y, z) = y; \bar{C}(x, z) = z; \bar{C}(x, y, z) = y.$$

Given L^2, no utility function u can rationalize $\bar{C}$ because L^2 makes no restrictions on the binary choices and, hence, cyclic choices in the binary sets would imply cyclic preferences. However, $\bar{C}$ satisfies asymmetric revealed preferences because the only pair of options that belong to two different L^2- consideration sets are y and z. Moreover, in these two consideration sets, the same choice y is made.

Moving from L^1 to L^2, delivers more information about the decision-maker's preferences because now a choice between x and z is no longer restricted. Moreover, now (i.e., in L^2 instead of L^1) transitivity adds empirical content to asymmetry in revealed preferences. However, if L^2 is made even less restrictive, and there is even more information about the decision-maker's preferences, now transitivity is back to not adding empirical content to asymmetry in revealed preferences. This follows because once the choice between x, y, and z becomes unrestricted, then L^2 becomes L^u. It is, therefore, not the amount of information on the decision-maker choices that determines whether transitivity adds empirical content to asymmetry in revealed preferences. Instead, in the case of three options, it is whether the consideration function satisfies the leveling axiom.

Definition 1 *A consideration function L satisfies* ***the leveling axiom*** *if for any three different options x, y and z*

$$L(x, y) = \{x, y\},\ L(y, z) = \{y, z\},\ L(x, z) = \{x, z\} \Longrightarrow L(x, y, z) = \{x, y, z\}.$$

The leveling axiom says that if, among three options, in all binary choices the consideration function does not restrict choice, then in the issue of all three options, the consideration function also does not restrict choice. Under a consideration function L that satisfies the leveling axiom, $L-$rationalizable choice functions are characterized by $L-$WARP. Under a consideration function L that does not satisfy the leveling axiom, $L-$rationalizable choice functions are not characterized by $L-$WARP. For example, L^1 satisfies the leveling axiom and under L^1, rationalizable choice functions are characterized by L^1-WARP. L^2 does not satisfy the leveling axiom and under L^2, L^2-rationalizable choice functions are not characterized by L^2-WARP.

The role of the leveling axiom is easy to understand intuitively. If the binary choices are restricted, then the leveling axiom holds and transitivity does not add empirical content to asymmetry in revealed preferences. This follows because it is only when binary choices are unrestricted that cyclic choices imply cyclic preferences. However, once binary choices are unrestricted, then the choice between all three options must also be unrestricted. Otherwise, transitivity adds empirical content to asymmetry in revealed preferences, as in the case of consideration function L^2. All of this holds in the case of three options. This paper extends this characterization to the general case of any finite number of options.

We introduce an axiom on consideration functions, called a basic structure, and show that if the consideration function L has a basic structure then, $L-$rationalizable choice functions C are characterized by $L-$WARP. If the consideration function L does not have a basic structure, then $L-$rationalizable choice functions C are not characterized by $L-$WARP. Thus, a basic structure demarcates the constraints for which transitivity adds empirical content to asymmetric revealed preferences. Naturally, in the special case of three options, a basic structure reduces to the leveling axiom.

1.3 Law-Abiding Citizen

The economic constraints L on the decision-maker could be a budget constraint or could be something of an entirely different nature. We now consider an economic constraint that is not usual in formal decision-theoretical models: the law. In this interpretation, the decision-maker is a law-abiding citizen. That is, the decision-maker maximizes an utility function u over the set of options $L(B)$ that are legal when B is the set of feasible options. This model was used by Katz and Sandroni (2017, 2023) and is based on the idea that the formal properties of the law will directly affect the logical properties of law-abiding choices.

Consider three options x, y and z. If any of these options is illegal in a binary choice, then the leveling axiom holds. In this case, asymmetric revealed preferences characterize law-abiding choices. For example, consider a decision-maker who is asked to participate in a bank robbery. Consider the options x (to participate in a bank robbery), y (to suffer severe burns), and z (to lose a precious manuscript). In this case, x is legal when the alternative is y; and x is illegal when the alternative is z. This follows because the defense of duress is available to a defendant who was pressured into committing a crime with the threat of serious pain or injury, but not with the threat of losing a manuscript (even if the defendant cares more about losing the manuscript than protecting his body). Thus, the simple fact that it is not legal to participate in a bank robbery when the alternative is to lose a precious manuscript implies that, in this example, the leveling axiom holds and asymmetric revealed preferences characterize law-abiding choices.

In the duress example above, law-abiding choices can be cyclic for some preference orders (even though they are characterized by asymmetric revealed preferences). First, in this example, the law is given by L^1. This is fairly straightforward and the details can be found in Katz and Sandroni (2017). Now consider a law-abiding citizen who prefers x over y over z. Then, the law-abiding citizen chooses x over y (because x is preferred to y and both options are legal); chooses y over z (because y is preferred to z and both options are legal); and chooses z over x (because only z is legal when the alternative is x). This example shows how the logical structure on the law can directly impact the properties of law-abiding choices. The key question here is whether law-abiding choices can *fail* to be characterized by asymmetric revealed preferences. In the case of three options, this question reduces to the question of whether the law can violate the leveling axiom. The central difficulty in answering this question is that very little is known about the logical properties of the law as formal economic constraints. That is, the translation of legal doctrines into a formal decision-theoretic setting is still in its infancy.

We show two legal principles that can violate the leveling axiom. The first is the principle that allows someone to legally infringe of someone else's rights in emergencies, provided that this infringement is put to the most socially beneficial use. We show that this doctrine violates the leveling axiom. A second legal principle that can lead to violations of the leveling axiom relates to the determination of how much risk it is legal to impose on others. The law usually allows the decision-maker to impose higher risks on others when the decision-maker cannot eliminate these risks entirely and the only choice available is to impose higher or lower risks on others. The translations of these legal principles into formal economic constraints can be found in Section 4 of this paper.

2 Decision-Theoretic Model

Let A be a finite set of alternatives. An *issue* B is a non-empty subset of A. Let $\mathcal{B}$ be the set of all issues. A *consideration function* is a mapping $L : \mathcal{B} \longrightarrow \mathcal{B}$ that takes an issue B, as input, and returns, as output, a non-empty subset, $L(B)$, of B. A *choice function* C is a mapping $C : \mathcal{B} \longrightarrow A$ such that $C(B) \in B$. A *utility function* u is a mapping $u : A \longrightarrow \Re$, where $\Re$ is the set of real numbers. [To simplify the language we do not make a distinction between functions and correspondences]. An utility function u is *without indifferences* if $u(x) \neq u(y)$ for all $x \in A$ and $y \in A$, $x \neq y$.

Definition 2 *Given a consideration function L, a choice function C is **in**$-L$ if $C(B) \in L(B)$ for any $B \in \mathcal{B}$.*

That is, C is in$-L$ if for any issue B, the choice $C(B)$ is in the consideration set $L(B)$.

Definition 3 *C is an $L-$**rationalizable choice function** if C is in$-L$ and there exists an utility function u without indifferences such that for any $B \in \mathcal{B}$,*

$$u(C(B)) > u(y) \text{ for all } y \in L(B),\ y \neq C(B).$$

A $L-$rationalizable choice function is such that the choice $C(B)$ is the best alternative in the consideration set $L(B)$, for some utility function u.

2.1 Transitivity and Asymmetry

Given a consideration function L, and a choice function C, let R $(= R^{C,L})$ be a binary relation such that given any two options $x \in A$ and $y \in A$, $x \neq y$,

$$x\ R\ y \text{ if and only if } \{x, y\} \subseteq L(B) \text{ and } x = C(B) \text{ for some } B \in \mathcal{B}.$$

So, $x\ R\ y$ indicates that x is revealed to be preferred to y. The next result (Proposition 1) is known and follows Richter (1966).

Proposition 1 Given a consideration function L, C is a $L-$rationalizable choice function if and only if C is in$-L$ and R is an asymmetric and acyclic binary relation.

In Proposition 1, revealed preferences are required to be acyclic. That is, for an arbitrary consideration function L, asymmetric and acyclic revealed preferences fully characterize rationalizable choices. We now turn to the basic question: for which consideration functions, asymmetric revealed preferences suffices to fully characterize rationalizable choices?

3 The L-WARP Theorem

Definition 4 *Given a consideration function L, a choice function C satisfies* **L$-$*WARP*** *if for any two issues B_1 and B_2*

$$C(B_1) \in L(B_2) \text{ and } C(B_2) \in L(B_1) \Longrightarrow C(B_1) = C(B_2).$$

$L-$WARP tracks the same idea as in WARP (see Samuelson (1938)). If $C(B_2) \in L(B_1)$ then $C(B_1)$ is revealed preferred to $C(B_2)$, because $C(B_1)$ and $C(B_2)$ belong to $L(B_1)$ and $C(B_1)$ is chosen. Analogously, $C(B_1) \in L(B_2)$ implies that $C(B_2)$ is revealed preferred to $C(B_1)$. Hence, $L-$WARP requires asymmetric revealed preferences, but it does not *require* transitive or acyclic revealed preferences.

We now ask what is the weakest condition on L that assures that $L-$rationalizable choice functions are fully characterized by in$-L$ choice functions C that satisfy $L-$WARP. That is, we ask for which consideration functions, asymmetric revealed preferences characterize constrained optimal choice.

Definition 5 *A consideration function L has* ***a basic structure*** *if for any set of distinct options x_1, x_2,...., x_n, $n \geq 3$, and issues B_1, B_2, ...B_n, such that*

$$\{x_i, x_{i+1}\} \subseteq L(B_i),\ i \in \{1,...,n-1\},\ \{x_n, x_1\} \subseteq L(B_n),$$

then when $n = 3$ there is an issue B such that

$$\{x_1, x_2, x_3\} \subseteq L(B) \subseteq \bigcup_{i=1,2,3} L(B_i); \tag{3}$$

and when $n > 3$ there is an issue B, and a pair of distinct options x_k and x_l such that $\{x_k, x_l\} \subseteq \{x_1,\ x_2,....,\ x_n\}$, $\{x_k, x_l\} \neq \{x_i, x_{i+1}\}$, $i \in \{1,...,n-1\}$, $\{x_k, x_l\} \neq \{x_n, x_1\}$, and

$$\{x_k, x_l\} \subseteq L(B) \subseteq \bigcup_{i=1,...,n} L(B_i). \tag{4}$$

The definition of a basic structure has two parts, (3) and (4). Condition (3) is essentially the leveling axiom. It ensures that if there is a revealed preference cycle of length 3, then there is a consideration set with all three options to help produce a $L-$WARP violation.

Remark 1 If a consideration function has a basic structure, it satisfies the leveling axiom. In the special case that A has three options (i.e., $\#(A) = 3$), a consideration function has a basic structure if and only if it satisfies the leveling axiom.

Remark 1 follows directly from (3). Thus, if attention is restricted to three options, then a basic structure reduces to the simpler leveling axiom.

Condition (4) ensures that if there is a revealed preference cycle of length greater than 3, then there will be a smaller revealed preference cycle. Assume that x_i is chosen over x_{i+1}, $i \in \{1,...,n-1\}$, and for each $i \in \{1,...,n-1\}$, both options x_i and x_{i+1} are in a consideration set. Also assume that x_n is chosen over x_1, and both options x_n and x_1 are in a consideration set. If $n > 3$, then there is a revealed preference cycle of length greater than 3. However, assume that there is a pair of options $\{x_k, x_l\}$, with non-consecutive indices and different from $\{x_n, x_1\}$, who also belong to a consideration set. Then, we can construct a new cycle bypassing the options with indices between k and l. The shortening of the revealed preference cycle can be made until we have a cycle of length 3 in which case, by (3), a violation of $L-$WARP can be produced.

The L-WARP Theorem Consider any consideration function L that has a basic structure. Then, C is a $L-$rationalizable choice function if and only if C is in$-L$ and satisfies $L-$WARP. Moreover, consider any consideration function L that does not have a basic structure. Then, there are choice functions that are in$-L$ and satisfy $L-$WARP, but are not $L-$rationalizable choice functions.

The $L-$WARP Theorem shows that a basic structure characterizes the consideration functions L for which $L-$WARP characterizes rationalizable choice functions. In particular, (only) with consideration functions that have a basic structure, asymmetric revealed preferences characterize constrained optimal choice. Equivalently, a basic structure demarcates the consideration functions for which transitivity adds empirical content to asymmetric revealed preferences.

4 The Law and the Leveling Axiom

We now focus on a particular economic constraint: the law. In this interpretation, the consideration set $L(B)$ is the set of legal options when the feasible options is B [we may refer to L as the *law*]. Hence, the choice function C in (1) delivers optimal decisions of a law-abiding citizen (see Katz and Sandroni (2017, 2023)).

We can now ask whether optimal legal choices are characterized by asymmetric revealed preferences. We argue that the law does not necessarily satisfy the leveling axiom and, hence, transitivity may add empirical content to legal decision making.

Example 1 Providing help The law does not satisfy the leveling axiom.

Two victims find themselves in urgent need of medicine that belongs to a third party. The medicine is sufficient to save two lives. The decision-maker is not this party, but instead is a potential rescuer who has no formal obligations toward the victims. To take the medication from the third party is justified by the doctrine of necessity.[1] There are three possible outcomes: (y) no one gets the medicine, (x) only one person gets the medicine, (z) both persons get the medicine.

If the feasible options are x and y, then both are legal. It is legal to fail to provide help, there being no general duty to aid.[2] But it is also legal to provide help. Thus $L(x, y) = \{x, y\}$. If the feasible options are y and z, or x and z, then both options are legal for the same reason. The choice between y and z could occur when the medicine, if provided, would be seized by both victims and so both persons can be helped or none of them can be helped. The choice between x and z could occur if the medicine has been obtained by one of the victims and the decision maker can either ensure that the second victim also gets the medicine or not. That is, in a choice between x and z, y (no one gets the medicine) is excluded. Hence, in

[1] Model Penal Code 3.02: "Conduct which the actor believes to be necessary to avoid a harm or evil to himself or to another is justifiable, provided that ... the harm or evil sought to be avoided by such conduct is greater than that sought to be prevented by the law defining the offense charged."

[2] Model Penal Code 2.01: "Liability for the commission of an offense may not be based on an omission unaccompanied by an action unless... a duty to perform the omitted act is otherwise imposed by law."

the choice between x and z it is necessary that one of the victims already has the medicine or will obtain it, regardless of what the potential rescuer does. In this case, the choice of the potential rescuer is just one of "nonfeasance" or "feasance" with respect to the other victim. Thus, $L(y,z) = \{y,z\}$ and $L(x,z) = \{x,z\}$.

Finally, suppose the choice is between x, y, and z. Whereas y and z remain legal, x is no longer legal because it is not legal to infringe on someone else's rights and then fail to provide maximum assistance. To some extent, this is implicit in the requirement that all available benefits from infringement of rights must be obtained, if an action is to be justified by necessity. That is, the doctrine of necessity allows for the infringement of individual rights under special circumstances. However, in cases where infringement of individual rights are justified by social benefits, the compensating social benefits cannot be reduced. For example, if someone's rights is infringed so that other people lives are saved, then, legally, one must save as many lives as can be saved from this infringement of rights**.** This principle has been endorsed independently in the scholarly literature as well (see Alexander et al. (2009)). Thus, $L = L^2$ and the law is as in L^2, where

$$L^2(x,\, y) = \{x,y\},\ L^2(y,z) = \{y,z\},\ L^2(x,z) = \{x,z\},\text{ and } L^2(x,y,z) = \{y,z\}. \tag{5}$$

Example 2: Imposing costs The law does not satisfy the leveling axiom.

A manufacturer producing widgets emits pollutants in small enough quantities so as to be not guilty of negligence. The decision-maker is this manufacturer. A typical definition of negligence, Model Penal Code 2.02 provides that "a person acts negligently with respect to a material element of an offense when he should be aware of a substantial and unjustifiable risk that the material element exists or will result from his conduct." So, it is legal to not manufacture widgets (y) and it is also legal to manufacture widgets that pollute at small enough levels (x). That is, $L(x,y) = \{x,y\}$. Now consider a new process (z) that costs a bit more, but pollutes even less. Clearly, $L(y,z) = \{y,z\}$ as the pollution levels in z are smaller than the pollution levels in x. However, if the decision-maker has the option to shut down the factory, he is probably legally obligated to switch to that new process z (or shut down the factory), as it would be negligent to run the factory and pollute at a higher level, when there is an alternative technology of similar costs that pollutes less. That is, $L(x,y,z) = \{y,z\}$. On the other hand, if the decision-maker does not have the authority or the capability to shut down the factory, then the law may not require him to switch to that new process z. That is, $L(x,z) = \{x,z\}$. To see this last point from a legal perspective, assume that the manufacturer belongs to a larger corporation who ordered the manufacturer to produce widgets at the corresponding level of pollution in x. There is no option y where widgets are not produced, presumably because the widgets will be produced by another manufacturer, if the decision-maker refuses to produce them. However, the decision-maker can produce the widgets at a lower level of pollution in z, either by disobeying orders to do x or by convincing the larger corporation to produce the widgets at the lower level of pollution in z. In this case, the manufacturer chooses z either by not disobeying orders or by not convincing the corporation to switch to the new process z. Either way, the law interprets the failure of disobeying these orders of production or the failure in persuading the corporation to switch processes as an omission and, hence, it is legal (Model Penal Code 2.01). As in Example 1, the law is like in (5). That is, $L = L^2$ and, hence, the law violates the leveling axiom.

It follows from examples 1 and 2, and the $L-$WARP theorem, that transitivity may add empirical content to asymmetric revealed preferences of law-abiding choices.

4.1 The Law from a Revealed Preference Perspective

At first glance, it may seem natural to model legality as follows: An option x is illegal if there is a feasible alternative y that is sufficiently better than x. Formally, there is an asymmetric binary relation $\succ$ ("the legal relation") such that x is illegal in B if $y \succ x$, for some $y \in B$. In particular, in a binary choice between x and y, if x is illegal and y is legal, then y is revealed to be better than x, by the legal relation $\succ$. As our examples show, the law is context-dependent and, hence, legality should not be modelled by an asymmetric binary relation $\succ$. The reasons and implications for the context-dependent nature of the law are discussed in greater detail in Katz and Sandroni (2017, 2023). Here, we point out that our examples showing that the law is context-dependent are based on time-honored, and normatively appealing principles. Moreover, the context-dependent nature of the law affects what can be revealed about the preferences of a law-abiding citizen.

Consider Example 1. It is based on well-established legal principles and yet, these principles may lead to a law as in L^2 that violates the levelling axiom. Consider now the law as in L^2 and a law-abiding citizen who has cyclic preferences. Unlike say a law such as in L^1, these cyclic preferences are revealed by choice because all options in binary choices in L^2 are legal and, hence, a choice implies a preference for the chosen option over the rejected option. Moreover, in L^2, the non-ordered nature of the decision-maker preferences can only be revealed by cyclic choices. This follows because, in L^2, the consideration sets are the same in the issue of all three options and in the issue y, z. Hence, the choice in the issue x, y, z does not deliver additional (i.e., beyond choices in binary issues) information about law-abiding citizen's preferences.

5 Literature Review

5.1 Revealed Preference Theory

The literature on revealed preferences is too large to be reviewed here (see, among many contributions, Aguiar et al. (2020), Azrieli et al. (2018) Beatty and Crawford (2011), Border and Segal (1994), Clippel and Rozen (2014), Evren et al. (2019), Forges and Minelli (2009), Gorno (2019), Heufer (2014), Matzkin and Richter (1991), Mariotti (2008), Nishimura et al. (2017), and Peters and Wakker (1994)). Even the literature on the specific question of the equivalence between WARP and SARP, is too large to be review here. In the context of budget constraints, Rose (1958) showed this equivalence for two goods. With three or more goods, Gale (1960) showed that WARP and SARP may differ. These results have been qualified in different ways (see, for example, Blundell et al. (2015), Bossert (1993), Heufer (2014), Peters and Wakker (1994), Quah (2006), Reny (2015)). Cherchye et al. (2018) delivered a characterization of the equivalence between WARP and SARP for multiple goods.

The closest paper to ours is a working paper by Caradonna (2020). Like this paper, Caradonna (2020) considers an abstract decision-theoretic model and shows the class of constrained optimal choice functions that are characterized by asymmetric revealed preferences. The key feature in Caradonna (2020) is that only some choices of the decision-maker are observed by an outsider. Formally, in the case that a choice is not observed and in the case of a single option consideration set, no inferences about preferences can be made.

Our results and the results in Caradonna (2020) were developed independently and simultaneously. Moreover, there are fundamental differences between our approach and the one in Caradonna (2020). First, the proofs and the entire formalization differ. In particular, Caradonna (2020) does not define economic constraints given by a consideration function L. Hence, there is no explicit axiomatization of consideration functions in Caradonna (2020). Specifically, Caradonna (2020) does not define a basic structure and does not define the leveling axiom either. These axioms are a focus of this paper. They deliver explicit characterizations of economic constraints that can be used to address questions such as, for example, whether transitivity adds empirical content to the choices of a rational, law-abiding citizen. Finally, our use of formal decision-theoretic concepts to law is novel.

Another paper related to ours is Tyson (2013). Tyson (2013) identifies preferences in a general constrained optimization setting, as we do in this paper. However, unlike this paper, the consideration function is not assumed to be observable in Tyson (2013). Instead, Tyson (2013) takes only the choices to be observable and characterized the choice correspondence that are induced by consideration functions L that are assumed to satisfy some conditions. In contrast, the focus of our characterization is not on the choice function, but on the economic constraints given by consideration functions. Another difference is that, unlike this paper, Tyson (2013) is not explicitly interested in the question of whether transitivity adds empirical content to revealed preferences. Instead, Tyson (2013) uses a SARP-like acyclicity condition as a way to characterize revealed preferences.

5.2 Two-Stage Choice Models

There are several two-stage decision-theoretic models in the literature. In these models, as in this paper, the decision-maker chooses the highest utility option, among those in a consideration set. Among two-stage decision-theoretic models, we formally address the shortlisting theory of Manzini and Mariotti (2007), the inattention theory of Lleras et al. (2017) and Masatlioglu, Nakajima and Ozbay (2012), and the rationalization model of Cherepanov et al. (2013). This list is highly incomplete and there are many other related models in the literature. For example, Salant and Rubinstein (2006, 2012) develop a model of choices from lists. Au and Kawai (2011) produce a special case of the Manzini and Mariotti (2007) shortlisting model with transitive rationales and axiomatized it with a SARP-like condition. Horan (2016) characterizes the model of Au and Kawai (2011) in terms of context effects. Houy (2007) characterizes sequentially rational choice in a model related to the shortlisting theory of Manzini and Mariotti (2007). Masatlioglu and Suleymanov (2019) consider constrained optimization in a product network. There are also several multi-self decision theoretic models such as Clippel and Eliaz (2012) and Heller (2012) and several bounded rationality models such as Ok et al. (2015), Eliaz and Spiegler (2011), Eliaz et al. (2006) and Bernheim and Rangel (2009).

The two-stage choice models that we formally address differ from this paper in the same way that the results in Tyson (2013) differ from the results in this paper. First, unlike this paper, the consideration functions in these two-stage choice models are not assumed to be observable. In addition, the focus of these papers is on characterizing constrained optimal choice based on consideration functions with assumed properties. In contrast, the focus of this paper is on characterizing consideration functions for which asymmetric revealed preferences characterize optimal choice.

The consideration function L in the shortlisting theory of Manzini and Mariotti (2007) is such that for some asymmetric binary relation $\succ$

$$L(B) = \{x \in B \mid \nexists\ y \in B \text{ for which } y \succ x\}. \tag{6}$$

The consideration function L in the inattention theory of Lleras et al. (2017), called *consideration filters*, and in the Cherepanov et al. (2013) rationalization model is such that

$$\text{if } B \subseteq B^* \text{ then } L(B^*) \bigcap B \subseteq L(B). \tag{7}$$

The consideration function L in the inattention theory of Masatlioglu, Nakajima and Ozbay (2012), called *attention filters*, is such that

$$L(B) = L(B \setminus x) \text{ whenever } x \notin L(B). \tag{8}$$

Consideration filters and attention filters violate the levelling axiom. This follows because L^2 (5) is a consideration filter and an attention filter, but it violates the levelling axiom. The consideration functions in the shortlisting theory of Manzini and Mariotti (2007) satisfies the levelling axiom. This follows because if all options are in the consideration sets in all binary choices, then the asymmetric binary relation $\succ$ cannot rank one option higher than another. Hence, in the issue of all three options, the consideration set is also the set of all three options. However, the consideration function in the shortlisting theory of Manzini and Mariotti (2007) does not have a basic structure. To see this, consider four options x_1, x_2, x_3, x_4 and a consideration function in (6) based on the asymmetric binary relation $\succ$ such that $x_1 \succ x_3$ and $x_2 \succ x_4$. The only two pairs of distinct options x_k and x_l such that $\{x_k, x_l\} \subseteq \{x_1,\ x_2, \ldots,\ x_4\}$, $\{x_k, x_l\} \neq \{x_i, x_{i+1}\}$ $i = 1, 2, 3$, and $\{x_k, x_l\} \neq \{x_1, x_4\}$ are the pairs $\{x_1, x_3\}$ and $\{x_2, x_4\}$. However, in any issue that contains x_1 and x_3, x_3 is not in the consideration set and in any issue that contains x_2 and x_4, x_4 is not in the consideration set. It follows that transitivity adds empirical content to optimal choices in the shortlisting theory of Manzini and Mariotti (2007), the inattention theory of Lleras et al. (2017) and Masatlioglu, Nakajima and Ozbay (2012), and in the rationalization model of Cherepanov et al. (2013). In addition, while the consideration functions of several models do not satisfy the leveling axiom, there are models where the consideration functions lack a basic structure, but the leveling axiom cannot rule them out.

The levelling axiom holds for any consideration function L that satisfies Sen's (1977) property β. Sen defines property β as follows: If x and y both belong to $L(B)$, and B is a subset of B^*, then x must belong to $L(B^*)$ if y does. Note that some option, say option y, must be in the consideration set $L(x, y, z)$. It follows that under property β, if $L(x, y) = \{x, y\}$, $L(y, z) = \{y, z\}$ and $y \in L(x, y, z)$, then $L(x, y, z) = \{x, y, z\}$. Hence, the levelling axiom holds under property β. The law, however, violates the levelling axiom, and it also violates Sen's properties α and β (see Katz and Sandroni (2023)). Finally, the levelling axiom also holds if the consideration function L satisfies Sen's (1977) property γ. Sen defines property γ as follows: For any class M of sets, if x belongs to $L(S)$ for all S in M, then belongs to $L(\bigcup M)$. Now assume that L does not make restrictions on binary choices (otherwise the levelling axiom holds). Consider any option in $\{x, y, z\}$. This option is in two binary subsets of $\{x, y, z\}$ where the union of these two subsets is $\{x, y, z\}$. In addition, this option is also in L of any of binary subset. Hence, $L(x, y, z) = \{x, y, z\}$ if property γ holds. It follows that the levelling axiom is satisfied if property γ holds.

5.3 Decision Theory and the Law

While there are formal models of social norms (see, for example, Richter and Rubinstein (2020)), the use of formal decision-theoretic models to understand the law is still in its infancy. Katz and Sandroni (2017, 2023) used the same decision-theoretic model as in this paper to show that legality is context-dependent (positively), that legality must be context-dependent (normatively). For example, Katz and Sandroni (2023) show that the law is and must be openly (i.e., without deception) manipulable because the law is and must be quite context-dependent. However, the main examples in Katz and Sandroni (2017, 2023) do not violate the levelling axiom. In Katz and Sandroni (2017) main examples, the law is as in L^1 (see (2)). In L^1, law-abiding choices can be cyclic, but preferences are not be revealed to be cyclic. In contrast, in some examples in this paper, the law is as in L^2 (see (5)). Then, cyclic preferences are revealed by cyclic choice.

Several generalizations of our basic model of law are possible. The decision-maker could take illegal actions (i.e., options in $B \backslash L(B)$), at some cost. There could be restrictions on the domain of issues over which choices are observed, in addition to restrictions on consideration functions imposed by the law. Finally, legality could also depend on the choice of issue. For example, consider a decision-maker under attack who has two options: (x) kill his attacker or (y) be killed. In the issue $\{x, y\}$, x can be self-defense and, hence, legal. However, assume that previously the decision maker had the option (z) to escape, but decided to close the escape route (i.e., chose the issue $\{x, y\}$) and afterwards choose x in $\{x, y\}$. Under some, by not all, circumstances, the self-defense argument can be denied, and x is no longer legal. These generalizations should be pursued in future work.

6 Conclusion

In models of constrained optimization, the properties of constraints directly affect the properties of choices made under these constraints. We characterize the constraints for which optimal choice functions are characterized by asymmetric revealed preferences. We refer to the leveling axiom as a key feature of constraints that lead to optimal choices that can be characterized by asymmetric revealed preferences. In a special case of our model, the decision maker is a rational law-abiding citizen who chooses optimally among legal and feasible options. We show that the law may violate the leveling axiom. Thus, even though the law may lead a rational law-abiding citizen to make cyclic choices, transitivity may add empirical content to rational, law-abiding choices.

7 Appendix

Proof of Proposition 1: Let C be an in$-L$ choice function. Assume that R is an asymmetric and acyclic binary relation. By topological ordering, R may be extended to an asymmetric order P (see Cormen et al. (2001)). Let u be the associated (with P) utility function without indifferences. Consider any issue $B \in \mathcal{B}$. If $y \in L(B)$ and $C(B) \in L(B)$, $y \neq C(B)$, then $C(B)\ R\ y \Longrightarrow C(B)\ P\ y$. Thus, C is a $L-$rationalizable choice function.

Assume that C is a $L-$rationalizable choice function. Let P be the asymmetric preference order associated with the utility function u. Assume, by contradiction, that $x\ R\ y$ and $y\ R\ x$, $x \neq y$. Then, for some $B \in \mathcal{B}$, $\{x, y\} \subseteq L(B)$ and $x = C(B)$ and for some $B' \in \mathcal{B}$, $\{x, y\} \subseteq L(B')$ and $y = C(B')$. So, $x\ P\ y$ and $y\ P\ x$. This contradicts the asymmetry of P. Also assume, by contradiction, that $x\ R\ y$ and $y\ R\ z$, and $z\ R\ x$, $x \neq y \neq z$. Then, for some $B \in \mathcal{B}$, $\{x, y\} \subseteq L(B)$ and $x = C(B)$; for some $B' \in \mathcal{B}$, $\{y, z\} \subseteq L(B')$ and $y = C(B')$; for some $B'' \in \mathcal{B}$, $\{x, z\} \subseteq L(B'')$ and $z = C(B'')$. Thus, $x\ P\ y$, $y\ P\ z$, and $z\ P\ x$. This contradicts the transitivity of P.$\blacksquare$

Proof of the L$-$WARP Theorem: Assume that L has a basic structure, C is in$-L$ and satisfies $L-$WARP. Assume, by contradiction, that R is cyclical. So, there are distinct options x_1, x_2,...., x_n, and issues B_1, B_2, ...B_n, such that

$$(x_i,\, x_{i+1}) \subseteq L(B_i),\, i \in \{1,...,n-1\};\, (x_n,\, x_1) \subseteq L(B_n),\, x_i = C(B_i),\, i \in \{1,...,n\}. \tag{9}$$

If $n > 3$ there are options x_k and x_l and issue B such that (4) holds.

Step 1. If $C(B) = x_j \in \{x_1,\, x_2,....,\, x_n\}$ then there exists $x_f \in \{x_1,\, x_2,....,\, x_n\}$, such that $x_f \neq x_j$, $x_f \in L(B)$ and $\{x_f, x_j\} \neq \{x_i, x_{i+1}\}$, $i \in \{1,...,n-1\}$, $\{x_f, x_j\} \neq \{x_n, x_1\}$.

If $x_j \in \{x_k, x_l\}$ then, by (4), $x_f \in \{x_k, x_l\}$, $x_f \neq x_j$. If $x_j \notin \{x_k, x_l\}$ then $x_f = x_k$ if $\{x_j, x_k\} \neq \{x_i, x_{i+1}\}$, $i \in \{1,...,n-1\}$, $\{x_j, x_k\} \neq \{x_n, x_1\}$ or $x_f = x_l$ if $\{x_j, x_l\} \neq \{x_i, x_{i+1}\}$, $i \in \{1,...,n-1\}$, $\{x_j, x_l\} \neq \{x_n, x_1\}$. If (I) $\{x_j, x_k\} = \{x_i, x_{i+1}\}$ for some $i \in \{1,...,n-1\}$ or $\{x_j, x_k\} = \{x_n, x_1\}$ and (II) $\{x_j, x_l\} = \{x_i, x_{i+1}\}$ for some $i' \in \{1,...,n-1\}$ or $\{x_j, x_l\} = \{x_n, x_1\}$ then either (1) $x_j = x_1$ and either x_k or x_l is x_n or (2) $x_j = x_{i+1}$, $i \in \{1,...,n-1\}$, and either x_k or x_l is x_i. Either (1) or (2) contradicts $L-$WARP. In (1), either x_k or x_l is $C(B_n)$, $x_j \in L(B_n)$, $x_j = C(B)$, $\{x_k, x_l\} \subseteq L(B)$. In (2), either x_k or x_l is $C(B_i)$, $x_j \in L(B_i)$, $\{x_k, x_l\} \subseteq L(B)$.

Step 2. If $n > 3$ then there are distinct options y_1, y_2,...., y_m, and issues B'_1, B'_2, ...B'_m, such that $m < n$ and (9) holds.

If $C(B) = x_j \in \{x_1,\, x_2,....,\, x_n\}$ then x_j and x_f is given by step 1. If $j < f$ then $m = j + n - f + 1$, $y_1 = x_1$, ... , $y_j = x_j$, $y_{j+1} = x_f$, $y_{j+2} = x_{f+1}$, ..., $y_m = x_n$; and $B'_1 = B_1$,.... , $B'_j = B$, $B'_{j+1} = B_f$, ..., $B'_m = B_n$. Thus, $m < n$ (because $j < f \Longrightarrow j < f - 1$). If $f < j$ then $y_1 = x_f$, $y_2 = x_{f+1}$,....,$y_m = x_j$; and $B'_1 = B_f$, $B'_2 = B_{f+1}$,.....,

$B'_{m-1} = B_{j-1}$, $B'_m = B$. Thus, $m = j - f + 1 < n$ (because $j \leq n$ and $f > 1$ if $j = n$).

If $C(B) = y \notin \{x_1, x_2, ..., x_n\}$. Then, $y \in L(B_{\bar{\imath}})$ for some $\bar{\imath} \in \{1,...,n\}$. If $\bar{\imath} = k$ then $C(B) = y \in L(B_k)$ and $C(B_k) = x_k \in L(B)$. So, $y = x_k$. An analogous contradiction with $L-$WARP holds if $\bar{\imath} = l$. So, $\bar{\imath} \neq k$ and $\bar{\imath} \neq l$. Without loss of generality, assume $k < l$. If $k < \bar{\imath} < l$ then $m = \bar{\imath} - k + 2$, $y_1 = x_k$, ... , $y_{\bar{\imath}-k+1} = x_{\bar{\imath}}$, $y_m = y_{\bar{\imath}-k+2} = y$; $B'_1 = B_k$,.... , $B'_{\bar{\imath}-k+1} = B_{\bar{\imath}}$, $B'_m = B$. So, $m < n$ because $\bar{\imath} \leq n-1$ and $\bar{\imath} = n-1 \Rightarrow x_l = x_n \Rightarrow k > 1$. If $\bar{\imath} < k$ then $m = n - l + \bar{\imath} + 2$, $y_1 = x_l$, ...,$y_{n-l+1} = x_n$, $y_{n-l+2} = x_1$,...., $y_{n-l+\bar{\imath}+1} = x_{\bar{\imath}}$, $y_m = y$; $B'_1 = B_l$,.... , $B'_{n-l+1} = B_n$, $B'_{n-l+2} = B_1$, ..., $B'_{n-l+\bar{\imath}+1} = B_{\bar{\imath}}$, $B'_m = B$. So, $m < n$ because $\bar{\imath} < l - 2$. If $\bar{\imath} > l$ then $m = \bar{\imath} - l + 2$, $y_1 = x_l$, ..., $y_{\bar{\imath}-l+1} = x_{\bar{\imath}}$, $y_m = y$; $B'_1 = B_l$,.... , $B'_{\bar{\imath}-l+1} = B_{\bar{\imath}}$, $B'_m = B$. So, $m < n$ because $l \geq 3$.

By Step 2, there are distinct options x_1, x_2, x_3 and issue B such that (9) and (3) hold (the case $n = 2$ contradicts $L-$WARP directly). So, $C(B) \in L(B_{\bar{\imath}})$ for some $\bar{\imath} \in \{1, 2, 3\}$, $x_{\bar{\imath}}$ $= C(B_{\bar{\imath}})$ and $x_{\bar{\imath}} \in L(B)$. By $L-$WARP, $C(B) = x_{\bar{\imath}}$. So, $C(B) \in L(B_k)$ for some $k \in \{1, 2, 3\}$, $k \neq \bar{\imath}$, and $C(B_k) \in L(B)$. This contradicts $L-$WARP. Thus, R is acyclic and, by Proposition 1, C is a $L-$rationalizable choice function. The proof that any $L-$rationalizable choice functions is in$-L$ and satisfies $L-$WARP is straightforward and holds for any consideration function L.

Assume that L does not have a basic structure. Then, there are distinct options x_1, x_2,...., x_n, and issues B_1, B_2, ...B_n, such that

$$\{x_i, x_{i+1}\} \subseteq L(B_i),\ i \in \{1,...,n-1\};\text{ and } \{x_n, x_1\} \subseteq L(B_n)$$

and for all issues B, (3) does not hold when $n = 3$ and (4) does not hold when $n > 3$.

Consider any asymmetric preference order P that ranks x_i above x_{i+1}, $i = 1, ..., n-1$. Any option in $\bar{A} \equiv \bigcup_{i=1,...,n} L(B_i)$, but not in $\{x_1, ..., x_{n-1}\}$, is ranked below x_n. Any option that is not in $\bar{A}$ is ranked above x_1.

Let C be the in$-L$ choice function such that for any issue B, $C(B) \in L(B)$ and

$$\begin{array}{lll} C(B) = x_n & \text{if} & \{x_1, x_n\} \subseteq L(B) \subseteq \bar{A}; \\ C(B)\ P\ w \text{ for all } w \in L(B), w \neq C(B) & \text{if} & \{x_1, x_n\} \nsubseteq L(B) \text{ or } L(B) \nsubseteq \bar{A}. \end{array}$$

We show that C satisfies $L-$WARP. Consider two issues $\tilde{B}$ and $\bar{B}$ and assume that

$$C(\tilde{B}) \in L(\bar{B}) \text{ and } C(\bar{B}) \in L(\tilde{B}).$$

If $\{x_1, x_n\} \subseteq L(B) \subseteq \bar{A}$ for $B = \tilde{B}$ and $B = \bar{B}$ then $C(\tilde{B}) = C(\bar{B}) = x_n$. If $\{x_1, x_n\} \nsubseteq L(B)$ or $L(B) \nsubseteq \bar{A}$ for $B = \tilde{B}$ and $B = \bar{B}$ then $C(\tilde{B})\ P\ C(\bar{B})$ and $C(\bar{B})\ P\ C(\tilde{B})$. So, $C(\tilde{B}) = C(\bar{B})$.

Assume $\{x_1, x_n\} \subseteq L(\tilde{B}) \subseteq \bar{A}$ and $\{x_1, x_n\} \nsubseteq L(\bar{B})$ or $L(\bar{B}) \nsubseteq \bar{A}$. The case where $\tilde{B}$ and $\bar{B}$ are exchanged is analogous. So, $C(\tilde{B}) = x_n$. If $L(\bar{B}) \nsubseteq \bar{A}$ then $C(\bar{B}) \notin \bar{A}$ (because options outside $\bar{A}$ are ranked, by P, higher than those inside $\bar{A}$). This contradicts $C(\bar{B}) \in L(\tilde{B}) \subseteq \bar{A}$. Thus, $L(\bar{B}) \subseteq \bar{A}$ and $\{x_1, x_n\} \nsubseteq L(\bar{B})$. Now $C(\tilde{B}) = x_n \in L(\bar{B}) \Longrightarrow x_1 \notin L(\bar{B})$.

If $n > 3$ then $\{x_1, x_n\} \subseteq L(\tilde{B}) \Longrightarrow x_i \notin L(\bar{B})$, $i \in \{2, ..., n-1\}$ (because (4) does not hold). If $n = 3$ then $C(\bar{B}) \neq x_2$ otherwise $\{x_1, x_2, x_3\} \subseteq L(\tilde{B}) \subseteq \bar{A}$. Thus, $C(\bar{B}) = x_n$ (because $x_n \; P \; w$ for all $w \in \bar{A} \bigcap \{x_1, ..., x_{n-1}\}^c$).

By definition, $x_i = C(B_i)$, $i \in \{1, ..., n\}$. Thus, x_i is revealed preferred to x_{i+1}, $i = 1, ..., n-1$ and x_n is revealed preferred to x_1. Hence, R is cyclical. By Proposition 1, C is not a $L-$rationalizable choice function.■

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