A measure of partisan advantage in redistricting
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Extended Abstract

We propose a measure of artificial partisan advantage in drawing electoral districts. This measure is simple to calculate. It provides an “easily administrable standard” to determine whether a redistricting map is a partisan gerrymander. In Vieth vs Jubelier (2004), League of United Latin American Citizens vs Perry (2006), and Gill v Whitford (2016) cases, the Supreme Court sought -and did not find- such a measure.

The idea is to compare the results obtained under a given map, with the results that parties would have obtained, had seats been allocated according to the division of the state into counties. County lines are neutral, largely fixed, and not subject to political contests. For a given state $S$, and a given party $P$, sum up the fraction of the population of state $S$ in all counties in which party $P$ received most votes. This fraction is the fraction of seats that party $P$ would win if each district would correspond to exactly one county, and each district/county received a number of seats proportional to the population of the county. We stress that we do not propose that we create seats according to county lines. Rather, we propose that we compare the seat allocation according to a given redistricting map, to the neutral seat allocation that would attain according to this allocation based on who wins each county. The difference between the two seat allocations is the artificial partisan advantage that the party obtained by virtue of the specific drawing of electoral maps.

This measure has the virtues of being very easy to compute, and that it accounts for the geographical sorting of the population. Consider a hypothetical a state in which half the population votes for the Democratic Party and the other half votes for the GOP. If the
population in this state sorts so that there is one urban county with 1/3 of the population in which everyone votes Democratic, and in all other (rural) counties 2/3 of the votes are for GOP and 1/3 for the Democratic Party, then 2/3 of the population lives in GOP counties, so the GOP would earn 2/3 of the seats according to county lines. To the extent that a redistricting map allows the GOP to earn more than 2/3 of the seats (or the Democratic Party to earn more than 1/3), we call the surplus an artificial partisan advantage, obtained by virtue of drawing an specific map that departs from county lines.

We compute this artificial partisan advantage for each state with at least two seats in the US House of Representatives districts in the 2016 election, and in the 2018 election. We find that in most states, the artificial advantage is small, and it can be rounded down to zero. However, in some states, the advantage is large. North Carolina is the worst offender: out of a delegation of 13 representatives, the GOP-drawn map provided an artificial partisan advantage of more than three seats. In particular, in the 2016 elections to the US House of Representatives in NC, the GOP obtained 53% of the total two-party vote, enough to win counties representing 54% of the population of the state, which would translate to 7 seats. So according to county maps, the NC delegation GOP-D split should have been 7-6. Instead, it was 10-3, for a net partisan gain for the GOP of 3 seats, or about a quarter of the state’s delegation.

The partisan advantage depends on the exact electoral result. We can compute it for the actual election result, and for other possible election results: the most focal is for a counterfactual in which the national popular vote is tied: the GOP and the Democratic party receive the same votes, across all states. Ex-ante, this is arguably the most interesting case, as it is the case in which the majority in the House of Representatives should be most in doubt.

In the actual 2016 election, the GOP obtained 62.7 million votes, and the Democratic party 61.4 million, so their shares of the two-party vote where 50.6% and 49.4%. The GOP won 241 seats, and the Democratic party won 194 seats. According to county lines, the GOP
would have won 233 seats, and the Democratic party would have won 202, so the artificial partisan advantage for the GOP was 8 seats.

In the counterfactual in which one in every hundred GOP votes in each state flipped to the Democrats, the national popular vote would have been tied. In this scenario, despite the overall tie in votes, the GOP would win in counties that comprise a slight majority of the population. In particular, according to county lines, the GOP would earn 220 seats, for 215 for the Democratic party, or a majority of 5. This slim majority is due to the sorting effect, which in the aggregate, benefits the GOP. This is our benchmark: in a tied election, neutral maps do not generate equal seats, nor equal wasted votes; in a nationally tied election, with neutral maps the GOP would obtain a slim majority of seats, due to sorting. However, with the actual maps, and in the hypothetical with a tied election results, the GOP would have won 238 seats, not 220. The difference of 18 seats is the artificial partisan advantage, cumulative and in addition to the sorting advantage, that the GOP derives from GOP-drawn maps in GOP-controlled legislatures, including 3 seats in North Carolina, 2.5 in Michigan, and 2.2 in Ohio, to name the three states with the greatest advantage.

Because counties can be of any size, their proportional share of the population can be any fractional number, and hence adding up the total share of seats deserved by a party according to counties results in a fractional number. An actual districting map must result on exact number of seats, so typically no map can yield exactly zero advantage to any party. Rather, we suggest that a map should yield a small artificial partisan advantage, small both in absolute terms, and in relative terms, given the size of the state delegation.

Define the following “**one plus ten percent**” rule: redistricting maps must be such that the number of seats of artificial partisan advantage must be no greater than one plus 10% of the state’s delegation size. Since the artificial partisan advantage depends on the actual election result, we can compute it, and we can check whether any given redistricting map violates the one plus 10% for any hypothetical election outcome, not just for the realized

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1To be precise, the popular vote would have been tied if 1.2% of the GOP votes in each county had switched to the Democrats.
outcome. A counterfactual of particular interest is the one in which the popular vote is tied nationally. We can also compute the artificial partisan advantage for election results in which control of the House would be in doubt, such as outcomes in which one party wins the popular vote by 1%, 2%, 3%, 4% or 5%. If a map violates the one plus ten percent test for any such close electoral outcome, then we suggest that it be presumed a partisan gerrymander.

The artificial partisan advantage for the actual electoral result is very easy to compute: the only necessary input is results by county, which are publicly available for every state, and the only computation is by addition across counties, division by total population, multiplication by size of the delegation, and subtraction to obtain the difference between the actual number of seats obtained with the given map, and the deserved number of seats, according to county lines. The artificial partisan advantage for any other hypothetical result is also easy to compute, but it requires an extra step: to compute the advantage for a counterfactual in which a given party’s vote share was \( x \) percentage points lower than in the actual result, we first transfer, in each county and district, a fraction \( \frac{x}{100} \) of the vote of the party to the other party, and then we use these revised vote tallies to check if the party would still win any given county or district.

For the 2016 election, North Carolina and Utah are the only two states that violate the one plus 10% rule both for the actual electoral outcome, and for the tied popular vote counterfactual. Both of these states used maps drawn by GOP legislatures, and both result in an excessive artificial partisan advantage of the GOP. In addition, Michigan (also GOP drawn) violates the one plus ten percent rule for the tied election counterfactual, but not for the actual election result, and Maryland (a Democratic gerrymander) violates it for the actual election result, but not for the tied election counterfactual.

Ohio (GOP drawn) and Arizona (commission drawn) violate a more stringent “one plus five percent” rule for both election results; Ohio favoring the GOP, and Arizona favoring the Democratic party. See the Figure 1, drawn for the tied election counterfactual: maps in
Figure 1: Artificial partisan advantage, in seats, 2016, under a tied popular vote.

the area sandwiched by the black arrows meets the lax “one plus ten percent” rule. Maps outside the area would provide an unfair artificial partisan advantage to one party.

We also checked the North Carolina’s violation was not specific to 2016. Now using data from the 2018 election, Figure 2 shows that the maps were set to give the GOP a large artificial partisan advantage of over three seats (out of a delegation of thirteen) for any plausible election outcome.

The figure shows the seats that the GOP would get in NC for any election result between the 2016 result (R +1.6%) and the 2018 result (D +8.5%), and the seats the GOP would deserve according to county lines. For any of these outcomes, the GOP wins the popular vote in North Carolina, and it wins counties representing a small majority of the population, so it deserves around seven seats, out of thirteen. But the exact set of counties it wins, and with it the number of seats it deserves, declines as the Democratic vote share increases.
In contrast, for any election outcome ranging from a close election, to a moderately large Democratic win, the number of seats the GOP wins in North Carolina under its district maps does not decline as the Democratic vote share increases: the GOP gets 10, or around three more than it deserves, regardless of the electoral outcome. This is a clear partisan gerrymander.

In the aggregate across all states, for the 2016 election to the US House of Representatives, the GOP won 50.7% of the two-party vote nationwide. It won a plurality of the vote in counties with 53.7% of the US population, so if seats were assigned in proportion to counties won weighed by population, the GOP would have deserved 53.7% of seats in the House. That is, the House would seat 233 Republicans and 202 Democrats. The actual seat outcome was 241 Republicans to 194 Democrats, or an aggregate artificial partisan advantage of 8 additional seats for the GOP.

If we consider counterfactual electoral results, shifting a fixed proportion of the vote in each county from one party to the other, and we look at the deserved seats according to counties, the total seats given the redistricting maps in 2016, and the GOP’s aggregate
artificial partisan advantage, we find that as the Democratic vote share increases from the actual loss by 1.7% to a Democratic 4% win, a large number of counties flip, so the population living in counties with a Democratic majority rises from 46.3% to 52.1% and the number of seats that the Democratic party deserves rises accordingly from 202 to 227, enough to control the chamber. However, while counties flip, districts do not: according to the maps in place in 2016, only 5 districts flip as the Democrats’ vote rises from losing by 1.7% to winning by 4%. The Democratic party number of seats climbs only to 199. The artificial partisan advantage shoots up to 28 seats.

The collective effect of the redistricting maps in all states is that for any electoral outcome in which the Democrats win by no more than five points, the US House of Representatives looks as if the GOP had won the election by two percentage points.

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<thead>
<tr>
<th>Vote D-R</th>
<th>GOP maj</th>
<th>Deserved</th>
<th>Partisan Advantage</th>
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<tbody>
<tr>
<td>48%-52%</td>
<td>57</td>
<td>45</td>
<td>6</td>
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<tr>
<td>49%-51%</td>
<td>51</td>
<td>37</td>
<td>7</td>
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<td>41</td>
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<td>51%-49%</td>
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<tr>
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<tr>
<td>53%-47%</td>
<td>37</td>
<td>-19</td>
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**Formal Definition**

Consider a given assembly, a given election to this assembly, and a given electoral unit that needs to be partitioned into \( s \) districts. Let \( L \) and \( R \) denote the two main parties running in the election for this assembly. In our application of interest, \( L \) and \( R \) for “left” and “right”. In our application of interest, party \( L \) is the Democratic Party, party \( R \) party is the GOP or Republican party, the electoral unit is one of the 50 states in the United States, and the assembly is either the House of Representatives (for any state with a delegation of at least two seats), or the lower or upper chamber of the legislative branch of the state’s government.
Figure 3:

Then for this election, for this assembly and for this state, let \( D \) be the set of electoral districts drawn according to the state’s district map, and let \( C \) be the set of counties. For each county \( j \in C \), let \( p_j \) be the population of county \( j \). Recall that \( s \) denotes the total number of seats alloted to the delegation of the state under consideration in the assembly under consideration. This number is equal to the total number of districts.

For any district \( i \in D \) and any county \( j \in C \), let \( v^L_{i,j} \) be the number of votes cast for the top candidate of party \( L \) party in county \( j \) in the race for the seat representing district \( i \), and similarly let \( v^R_{i,j} \) be the number of total votes cast for the top party \( R \) candidate in county \( j \) in the race for the seat that represents district \( i \).

For any \( i \in D \), Party \( L \) wins seat \( i \) if, summing across counties in the district \( \sum_{j \in C} v^L_{i,j} > \sum_{j \in C} v^R_{i,j} \) and party \( R \) wins the seat if \( \sum_{j \in C} v^L_{i,j} < \sum_{j \in C} v^R_{i,j} \). Define \( s^L_i \) and \( s^R_i \) by \( s^L_i = 1 \) and \( s^R_i = 0 \) if \( L \) wins seat \( i \), and \( s^L_i = 0 \) and \( s^R_i = 1 \) if \( R \) wins the seat.

Then \( s^L_T = \sum_{i \in D} s^L_i \) and \( s^R_T = \sum_{i \in R} s^R_i \) are the total seats won in the state by each party.

Similarly, for each county \( j \in C \), party \( L \) wins most votes in the county if \( \sum_{i \in D} v^L_{i,j} > \sum_{i \in D} v^R_{i,j} \).
and party $R$ wins most votes in the county if $\sum_{i \in D} v_{i,j}^L < \sum_{i \in D} v_{i,j}^R$, where we are now summing across all districts that overlap the county. Define $c_j^L$ and $c_j^R$ by $c_j^L = 1$ and $c_j^R = 0$ if $L$ wins most votes in county $j$, and $c_j^L = 0$ and $c_j^R = 1$ if $R$ wins most votes in county $J$.

Then $s_L = s \sum_{j \in C} p_j c_j^L$ is the number of seats deserved by party $L$, and $s_R = s \sum_{j \in C} p_j c_j^R$ is the number of seats deserved by party $R$, according to the neutral county lines.

**Definition 1** The artificial partisan advantage for party $L$ and for party $R$ are, respectively $s_T^L - s_L$, and $s_T^R - s_R$.

For any $x \in (0, 1)$, if we wish to compute the artificial partisan advantage for the counterfactual electoral result in which fraction $x$ of votes for party $L$ switched to party $R$, so that the two-party vote share of party $R$ increased by $200x$ percentage points, we first compute the counterfactual vote totals $\hat{v}_{i,j}^L = (1 - x)v_{i,j}^L$ and $\hat{v}_{i,j}^R = v_{i,j}^R + xv_{i,j}^L$ for each county $i$ and district $j$, and then we compute the artificial partisan advantage using the revised vote shares $\hat{v}_{i,j}^L$ and $\hat{v}_{i,j}^R$. Similarly, for the counterfactual in which fraction $x$ of the vote of party $R$ switched to $L$, we use $\hat{v}_{i,j}^L = v_{i,j}^L + xv_{i,j}^R$ and $\hat{v}_{i,j}^R = (1 - x)v_{i,j}^R$.

**Relevant Court Cases and literature**


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2 Note that for any $i$ and $j$ such that county $i$ and district $j$ do not overlap, these numbers are necessarily zero.