INTERNATIONAL ECONOMIC REVIEW
Vol. 22, No. 1, February, 1981

ON THE SCOPE OF THE STOCKHOLDER UNANIMITY THEOREMS*

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1. INTRODUCTION

If a firm is owned by several stockholders each of whom has different risk preferences, then whose risk preferences determine whether the firm accepts or rejects a risky, proposed change in its production plan? It might appear that this is a political problem whose resolution is necessarily through political means such as proxy fights. This appearance, however, is only partially accurate. Arrow [1964], Ekern [1973], Ekern and Wilson [1974], Leland [1973, 1978], Baron [1977], and others have developed a theory showing that if a sufficient variety of securities are traded on the stock market, then conflict among stockholders does not occur. The reason is that if an economy has enough different securities, then in equilibrium each stockholder's preferences towards risk is aligned with every other stockholder's preferences. Consequently stockholders are unanimous in their evaluations of risky investment projects.

The purpose of this paper is to inquire if any reason exists why one should expect a sufficient variety of securities to be traded on the stockmarket to achieve stockholder unanimity. The answer I propose is of a mixed character. Within an economy that has no transaction costs an incentive does exist to introduce a variety of securities onto the market that is sufficient to assure stockholder unanimity for certain classes of firms' decisions. A decision, for example, to increase production capacity for a currently marketed, successful product falls into this class. But it is impossible for a variety of securities to exist within an economy that is sufficient to assure that stockholder unanimity exists for all classes of firms' decisions. A decision, for example, to be the first firm to invest in a radically different production technology is not and cannot be assured of unanimity.

The first, affirmative part of my answer appears to be well known by those investigators who have been most active in this area of research, though no one, to my knowledge, has explicitly stated it in a paper.[2] The second, negative part

* Manuscript received August 22, 1977; revised March 4, 1980.

[1] The comments of David Baron, Robert Forsythe, and an anonymous referee each contributed substantially to the development of this paper. Research support form the J.L. Kellogg Graduate School of Management through an IBM Research Professorship and the Center for Advanced Study in Managerial Economics and Decision Sciences is gratefully acknowledged.

[2] Ross [1976], for example, comes close to explicitly stating this result on two occasions. First, he states (p. 76):

> Furthermore, in general, it is less costly to market a derived asset generated by a primitive than to issue a new primitive, and there is at least some reason to believe that options will be created until the gains are outweighed by the set-up costs.

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of my answer appears to be original. It is because the two parts of my answer taken as a whole are fundamental to assessing the importance of the developing theory of stockholder unanimity that I derive both parts here explicitly.

This paper is organized as follows. Section 2 describes the model that is used throughout. It is a standard, frictionless, two-period, one good model of an economy with firms and consumers. The one added feature that it contains is a distinction between the observable component of the state of nature and the unobservable component of the state of nature. The next three sections contain the formal analysis of the model. In Section 3 I introduce the idea of spanning the observable component of the state space. In Section 4 I show that an incentive exists for entrepreneurs to introduce new securities into the economy as long as the observable component is not spanned. That such an incentive exists implies that the economy is not in full, long run equilibrium whenever the observable component is not spanned.

In Section 5 I derive, based in particular on Leland's [1978] work, two conditions that assure stockholder unanimity whenever, as is likely according to Section 4's result, the state space's observable component is spanned. In Section 6 I interpret these conditions on the assumption that the observable component is in fact spanned and arrive at the following conclusion. Stockholders are likely to be unanimous towards a proposed investment project if the returns it yields are only a function of the state space's observable component and risks that are objectively assessable. If, however, the proposed project's returns depend on the unobservable component with probabilities that are primarily subjective, then stockholder unanimity is no longer assured. Such an eventuality is possible because a proposed investment project may, in effect, be a proposal for that firm to conduct an experiment that will make observable an aspect of the state of nature that was formerly unobservable under that and all other firms' original production plans. For example, if initially no firm has a plan to implement a new and very different production technology, then the economic feasibility (or infeasibility) of that technology is an unobservable aspect of the state of nature because no firm intends to do the experiment of trying that technology out and actually observing its economic feasibility. Moreover, if the technology is different enough, then no objective assessment of the risks involved may be possible, i.e., equally qualified engineers may disagree vehemently on the probability of success. Thus it is for the truly innovative investment projects that stockholder unanimity probably breaks down, not for the routine investment projects that firms most commonly consider. Section 7 concludes the paper by posing some related questions that are beyond the scope of this paper.

(*Continued*)

and, second, he states (p. 78):

> ...we are neglecting the consideration that the creation of markets in new assets will be costly... If costs are sufficiently high, it will be inefficient to open all the markets even if it does permit all the states to be spanned. (If costs are low, however, unless markets have significant public goods aspects, it is not clear why they will not be open in competition.)

2. THE MODEL

Consider a one consumption good, two period economy with $\mathscr{K}=\{1,\ldots,K\}$ states of nature, $\mathscr{F}=\{1,\ldots,F\}$ firms, and $\mathscr{I}=\{1,\ldots,I\}$ consumers.[3] Let firms, for the moment, have fixed production plans that determine how their returns will vary with the state of nature. In the first period consumers and firms are ignorant of the state of nature. At the beginning of the second period they learn the identity of the subset of $\mathscr{K}$ within which the true state is contained. Where convenient and where confusion is unlikely the first period is referred to as now and the second period is then. Transaction costs are assumed to be zero.

Let each state of nature $k\in\mathscr{K}$ be represented as an ordered pair $k=(st)$ where $s\in\mathscr{S}=\{1,\ldots,S\}$, $t\in\mathscr{T}=\{1,\ldots,T\}$, and $k=(s-1)T+t$. Given the firms' fixed production plans, the first component is called the observable component of the state of nature and the second component is called the unobservable component of the state of nature. An element s of $\mathscr{S}$ is called the observable event s and, similarly, an element t of $\mathscr{T}$ is called the unobservable event t. The observable component identifies the state in sufficient detail to determine how much each firm, given its current production plan, will earn in period two. That component becomes common knowledge of both firms and consumers at the beginning of period two. The unobservable component, as its name implies, is not observable by any firm or agent in either period and therefore no firm's returns can depend on its identity.

The information that the observable component contains includes the values of macro variables such as the money supply, OPEC's pricing policies, and Congressional action on regulatory policy. It also includes the values of micro variables such as consumer demand for a new product that some firm f will introduce in period two as part of its fixed production plan. This last item is observable since at the end of period two consumers will have revealed their preferences towards the product, which in turn affect the firm's earnings.[4] Therefore, given firm f's current production plan, investors nnow its returns will be conditional upon the observable component s. Unobservable states contain information about contingencies that would be observable if the appropriate experiments were done. Nevertheless the several firms' production plans are such that those experiments are not to be done. For example, the unobservable component contains information about consumer demand for different products that no firm is offering in period one or, given their fixed production plans, is planning to offer in period two. It also contains information about the feasibility

[3] All the conclusions of this paper remain valid in a multiperiod, many good, rational expectations model of the type that, for example, Hart [1975] used.

[4] The example of a new product is formally inconsistent with the model's assumption of a single consumption good except for the fact, pointed out in footnote 3, that the assumption of a single consumption good is inessential and is present only to simplify the notation.

and cost of those production technologies that no firm is currently using or planning to use.

In the first period (now) consumers receive initial endowments of the single consumption good and shares of each firm's stock. Trade takes place in period one as each consumer, given the vector of market clearing prices, adjusts his initial endowment of consumption good and stockholdings to that portfolio of consumption and stockholding that is maximal according to his preferences and budget constraint. In the second period (then) no trade takes place: each consumer consumes the additional consumption good endowment he receives then and the returns on his stockholdings. Let the endowment of consumption good consumer i receives now be $\omega_1^i > 0$ and the endowment he receives then if state $k \in \mathscr{K}$ occurs is $\omega_2^i(k) > 0$. Since the t component of $k = (st)$ is defined to be unobservable, necessarily $\omega_2^i(k) = \omega_2^i(st) = \omega_2^i(s1) = \cdots = \omega_2^i(sT) \equiv \omega_2^i(s)$ where $\omega_2^i(s)$ is introduced as a convenient notation. Let his consumption of the consumption good be x_1^i now and $x_2^i(s1) = \cdots = x_2^i(sT) \equiv x_2^i(s)$ then if state $k = (st)$ occurs. The consumption good can not be carried over from now to then. Let the endowment of firm f's stock that consumer i receives now be $\bar{z}_f^i$. Adopt the convention that, for all $f \in \mathscr{F}$, $\sum_{\mathscr{F}} \bar{z}_f^i = 1$. Let consumer i's holding of firm f's stock now after all trades are complete be z_f^i. A negative z_f^i is permissible and corresponds to a short sale of the stock. An individual i is called an initial stockholder of firm f if $\bar{z}_f^i > 0$ and a final stockholder if $z_f^i > 0$.

Assume that each firm is under the direction of a manager who owns stock in his firm and acts in the interests of stockholders (himself included) whenever those interests are well defined.[5] As alluded to above, assume each firm f has a production plan that specifies what actions it plans to take both now and, contingent on the state of nature, then. Given its production plan, firm f pays a return then of $a_f(s) \equiv a_f(s1) = \cdots = a_f(sT)$ units of consumption good if state $k = (st)$ occurs. Thus at the beginning of the second period, a final stockholder i receives $z_f^i a_f(s)$ of consumption good if state $k = (st)$ occurs. He pays $z_f^i a_f(s)$ of consumption good if he is a short seller of firm f.

Let the price now of firm f's stock be p_f. Every individual i is a price taker and picks his vector $z^i = [z_1^i, \ldots, z_F^i]$ of stockholdings and vector $x^i = [x_1^i, x_2^i(11), \ldots, x_2^i(ST)]$ of consumption so as to maximize his utility

$$U^i[x_1^i, x_2^i(11), \ldots, x_2^i(1T), \ldots, x_2^i(ST)] \tag{1}$$

subject to budget constraints

$$x_1^i + \textstyle\sum_{\mathscr{F}} z_f^i p_f \leq \omega_1^i + \textstyle\sum_{\mathscr{F}} \bar{z}_f^i p_f; \tag{2}$$

$$x_2^i(k) \leq \omega_2^i(k) + \textstyle\sum_{\mathscr{F}} z_f^i a_f(k), \qquad k = 1, \ldots, K \tag{3}$$

and the observability constraints

[5] Thus I assume that the moral hazard problem of providing managers with incentives to act in the interest of stockholders does not exist. For a discussion of this problem see, for example, Jensen and Meckling [1976].

(4) $$x_2^i(s1) = x_2^i(s2) = \cdots = x_2^i(sT) \equiv x_2^i(s) \qquad s = 1,\dots, S;$$

(5) $$a_f(s1) = a_f(s2) = \cdots = a_f(sT) \equiv a_f(s) \qquad s = 1,\dots, S,\ f = 1,\dots, F.$$

Assume that U^i is i's strictly monotonic, continuously differentiable von Neuman-Morgenstern utility function with the form:

(6) $$U^i[x_1^i,\dots, x_2^i(ST)] = \sum_{s=1}^{S} \sum_{t=1}^{T} \eta^i(st)u^i[x_1^i, x_2^i(st)]$$

where u^i is i's state independent utility for consumption now and then and $\eta^i(st)$ is i's subjective probability of state (st) being realized.[6] Assume that for any set of strictly positive prices $p=\{p_1,\dots, p_F\}$ the maximizing consumption bundle $x^i=\{x_1^i,\dots, x_2^i(K)\}$ has strictly positive components.

Let the only securities traded be the stocks of the F firms. A vector of prices $p=(p_1,\dots, p_F)$, a vector of consumption plans $x=(x^1,\dots, x^I)$, and a vector of stockholdings $z=(z^1,\dots, z^I)$ is a $\mathscr{F}$-equilibrium for a given set of production plans if (a) for all $i \in \mathscr{I}$, the plan (x^i, z^i) maximizes i's utility given the constraints (2), (3), (4), and (5) and (b) the market for each firm's stock clears, i.e., $\sum z_f^i=1$ for all $f \in \mathscr{F}$. I defer for the moment defining equilibrium when the set of securities traded is not fixed to be the set $\mathscr{F}$.

3. EQUILIBRIUM, IMPLICIT PRICES, AND SPANNING

Let $\mathscr{L}$ be the LaGrangian expression formed from i's maximization problem (1). Given a set of prices $p=(p_1,\dots, p_F)$, the first order conditions for i's stockholdings $z^i=(z_1^i,\dots, z_F^i)$ and consumption plan $x^i=\{x_1^i, x_2^i(11),\dots, x_2^i(ST)\}$, which is constrained to satisfy the observability requirement (4), to be maximal are:

(7) $$\frac{\partial \mathscr{L}}{\partial x_1^i} = \frac{\partial U^i[x^i]}{\partial x_1^i} - \lambda^i = 0;$$

(8) $$\frac{\partial \mathscr{L}}{\partial x_2^i(s)} = \sum_{\mathscr{T}} \frac{\partial U^i[x^i]}{\partial x_2^i(s)} - \delta_s^i = 0, \qquad s \in \mathscr{S} = \{1, \dots, S\};$$

(9) $$\frac{\partial \mathscr{L}}{\partial z_f^i} = -\lambda^i p_f + \sum_{\mathscr{S}} \delta_s^i a_f(s) = 0, \qquad f \in \mathscr{F} = \{1, \dots, F\}.$$

The $S+1$ LaGrange multipliers λ^i and δ_s^i have the usual interpretations: λ^i is the marginal utility for i of a unit of consumption good now and δ_s^i is the marginal utility for i of a unit of consumption good then if state s occurs. Let $\delta_{st}^i \equiv \partial U^i[x^i]/\partial x_2^i(st)$ be the marginal utility for i of consumption good then if state st occurs. Define $\psi_{st}^i=\delta_{st}^i/\lambda^i$ to be i's implicit price now for consumption good then if state st occurs. It is how much consumption good now i is willing

[6] Consumers may generally be expected to have subjective probabilities concerning the likelihood of unobservable states being realized. For example, I have a subjective probability concerning the existence of intelligent life on a planet somewhere else in the universe.

to pay for one unit of consumption good then if state st occurs.

Using (7) and (8) and the definitions of δ^i_{st} and ψ^i_{st}, the F equations of (9) may be rewritten for each consumer i as:

$$\begin{aligned}
&\textstyle\sum_{\mathcal{T}} \psi^i_{1t} a_1(1) + \sum_{\mathcal{T}} \psi^i_{2t} a_1(2) + \cdots + \sum_{\mathcal{T}} \psi^i_{st} a_1(S) = p_1 \\
&\textstyle\sum_{\mathcal{T}} \psi^i_{1t} a_2(1) + \sum_{\mathcal{T}} \psi^i_{2t} a_2(2) + \cdots + \sum_{\mathcal{T}} \psi^i_{st} a_2(S) = p_2 \\
&\qquad\vdots \\
&\textstyle\sum_{\mathcal{T}} \psi^i_{1t} a_F(1) + \sum_{\mathcal{T}} \psi^i_{2t} a_F(2) + \cdots + \sum_{\mathcal{T}} \psi^i_{si} a_F(S) = p_F .
\end{aligned} \tag{10}$$

Define $\psi^i_s = \sum_{\mathcal{T}} \psi^i_{st}$ to be i's implicit price now for consumption then if observable event s is realized. *It*, analogous to ψ^i_{st}, is how much i is willing to pay now for one unit of consumption then if observable event s occurs. System (10) may be rewritten in matrix form:

$$A\psi^i_{\mathcal{S}} = p \tag{11}$$

where

$$A = \begin{bmatrix} a_1(1) & a_1(2)\cdots a_1(S) \\ a_2(1) & a_2(2)\cdots a_2(S) \\ \vdots & \vdots \qquad \vdots \\ a_F(1) & a_F(2)\cdots a_F(S) \end{bmatrix}, \ \psi^i_{\mathcal{S}} = \begin{bmatrix} \psi^i_1 \\ \psi^i_2 \\ \vdots \\ \psi^i_S \end{bmatrix}, \ p = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_F \end{bmatrix}.$$

The observable component of the state space is *spanned* if rank $A = S$. In other words, the observable component is spanned if there are at least as many firms with independent return vectors a_f as observable events $s \in \mathcal{S} = \{1,\ldots, S\}$.

Given the returns matrix A and the price vector p, equation (11) may be interpreted as an equilibrium restriction on each consumer's implicit prices for observable events. The assumption that each consumer's utility function is strictly monotonic implies that the implicit prices ψ^i_{st} and ψ^i_s are strictly positive. Therefore the set of column vectors y that satisfy (11) in an admissible manner is $\Omega^{S-F} = \{y \mid y \in R^S_+ \text{ and } Ay = p\}$ where R^S_+ is the S-dimensional nonnegative orthant. This is a convex subset of R^S_+ with dimensionality ($S - \text{rank}\, A$) that consists of a unique point only if rank $A = S$, i.e., all individuals necessarily have identical implicit prices for each observable event s only if the observable component is spanned. If the observable component is not spanned, then two individuals i and j who are in equilibrium may have different vectors of implicit prices for the observable events, i.e., perhaps $\psi^i_{\mathcal{S}} \neq \psi^j_{\mathcal{S}}$.

Note, however, that even if the observable component is spanned, the implicit prices ψ^i_{st} for unobservable events st may not be identical. This is because, for any $s \in \mathcal{S}$,

$$\textstyle\sum_{\mathcal{T}} \psi^i_{st} \equiv \psi^i_s = \psi^j_s \equiv \sum_{\mathcal{T}} \psi^j_{st} \tag{12}$$

does not imply that $\psi^i_{st} = \psi^j_{st}$ necessarily, i.e., $\psi^i_{st} \neq \psi^j_{st}$ quite possibly. Market completeness, not spanning, guarantees that implicit prices for unobservable events as well as observable events are identical across individuals. Completeness requires that the number of securities with linearly independent return vectors equal ST, the number of states of the world. Existence of ST linearly independent securities would require that the returns of securities vary not only with respect to the observable component, but also with respect to the unobservable component, which is impossible by definition.

4. THE INCENTIVE TO ACHIEVE SPANNING OF THE OBSERVABLE COMPONENT

This section shows that an economy is not in full equilibrium unless the set of observable components is spanned. Specifically if the observable component is not spanned, then entrepreneurs can make a riskless profit by introducing new securities. Therefore, if one is willing to assume that the economy does tend towards equilibrium, then spanning of the observable component is a consequence of the market process, not an assumption that may be arbitrarily made about the market structure.[7]

Permit any consumer i to issue a new security, labeled g, subject to the requirements that its returns (a) be nonnegative and (b) be a function only of the state space's observable component and not of its unobservable component.[8] Thus, exactly as for each firm's stock, $a_g(s) \equiv a_g(s1) = a_g(s2) = \cdots = a_g(sT) \geq 0$ for all $s \in \mathscr{S}$. Issuance of such a security is feasible because the observable component is observable and thus contracts can be made contingent on it. An $\mathscr{F}$-equilibrium is a full equilibrium only if no consumer can make a riskless profit by introducing a new security g onto the market. Thus if a $\mathscr{F}$-equilibrium is a full equilibrium, then each consumer i has exhausted his opportunities for maximization with respect to his consumption plan x^i, his trading plan z^i, and the possibilities of introducing new securities. In a $\mathscr{F}$-equilibrium each consumer takes the set of securities as given and maximizes only with respect to his plans x^i and z^i.

The paper's first proposition is: in the absence of transaction costs, a necessary condition for a $\mathscr{F}$-equilibrium to be a full equilibrium is that the stocks of the F firms span the observable component. The only exception to this occurs when the market states are not spanned and, at some $\mathscr{F}$-equilibrium, all consumers by chance have the same implicit prices. This is an unlikely occurrence if consumers' utility functions, subjective probability assessments, and endowment streams are heterogeneous. Demonstration of the proposition is as follows.

[7] Transaction costs prevent spanning from being perfectly achieved in real financial markets. The practical importance of this imperfection is, however, difficult to gauge because transaction costs for the creation of new financial securities (options, etc.) are low.

[8] The creation of new securities by entrepreneurs is a frequent occurrence in United States' security markets. Two examples from recent history are the creation of options markets for certain common stocks and the creation of new futures markets for some commodities. See Ross [1976] for an analysis of how the creation of an options market increases the number of linearly independent securities even though options are based on existing securities. See Sandor [1973] for a historical account of how the Chicago Board of Trade and the professional traders who compose it established the market in plywood futures.

Assume, contrary to the result, that the economy is in full equilibrium without the observable component being spanned. Therefore Ω^{S-F} contains a multiplicity of points and, unless consumers have identical utility functions and endowment streams, almost certainly a pair of consumers i, j exist who have unequal implicit prices over the observable component: $\psi^i_{\mathscr{S}} \neq \psi^j_{\mathscr{S}}$. Since the vectors $\psi^i_{\mathscr{S}}$ and $\psi^j_{\mathscr{S}}$ are distinct points in R^S_+, they are also disjoint convex sets. Therefore a hyperplane exists that separates them, i.e., a vector $a_g = [a_g(1) \cdots a_g(S)] > 0$ and scalars p^+_g and p^-_g exist such that either

$$(13) \qquad 0 < \textstyle\sum_{\mathscr{S}} a_g(s)\psi^i_s < p^-_g < p^+_g < \sum_{\mathscr{S}} a_g(s)\psi^j_s$$

or

$$(14) \qquad 0 < \textstyle\sum_{\mathscr{S}} a_g(s)\psi^j_s < p^-_g < p^+_g < \sum_{\mathscr{S}} a_g(s)\psi^i_s .$$

The components of a_g may be chosen to be nonnegative because $\psi^i_{\mathscr{S}}$ and $\psi^j_{\mathscr{S}}$ are single points within R^S_+. Assume without loss of generality that (13) is satisfied. Suppose a third consumer $e \in \mathscr{I}$ — the entrepreneur — offers to sell at price p^+_g and buy at price p^-_g the vector of returns $a_g(s)$. Let this return vector be called security g. Given the offer price of p^+_g individual j wants to buy from individual e some quantity of security g because the utility he attaches to the purchase of one unit of g at that price is:

$$\begin{aligned} \Delta U^j &= \textstyle\sum_{\mathscr{S}} \sum_{\mathscr{T}} \frac{\partial U^j(x^j)}{\partial x^j_2(st)} a_g(s) - p^+_g \frac{\partial U^j(x^j)}{\partial x^j_1} \\ &= \textstyle\sum_{\mathscr{S}} \sum_{\mathscr{T}} \delta^j_{st} a_g(s) - p^+_g \lambda_j \\ (15) \qquad &= \lambda_j \left[\textstyle\sum_{\mathscr{S}} \sum_{\mathscr{T}} \psi^j_{st} a_g(s) - p^+_g\right] \\ &= \lambda_j \left[\textstyle\sum_{\mathscr{S}} \psi^j_s a_g(s) - p^+_g\right]. \\ &> 0 \end{aligned}$$

The first line is a first degree Taylor series approximation of the utility consequences of purchasing the unit of stock, which is accurate provided all components of a_g are small. The second line follows from equations (7) and (8), the third and fourth lines follow from the definitions of ψ^j_{st} and ψ^j_s, and the inequality follows from (13) because $\lambda^j > 0$. Similarly, individual i wants to sell to individual e some quantity of g at price p^-_g. If k astutely selects the prices p^+_g and p^-_g, then the quantities that i wants to sell and j wants to buy will be equal and e can make a riskless profit of $y(p^+_g - p^-_g) > 0$ where y is the positive quantity traded. Therefore, the economy is not in full equilibrium because e has an incentive to introduce a new security. This contradicts the original assumption that the market is in equilibrium and establishes the proposition.

After some entrepreneur has introduced security g, then a new $\mathscr{F}$-equilibrium may be achieved with individuals trading the F stocks plus the new security g.[9]

[9] Since security g is purely a set of transfers from one individual to another and not a claim on the real returns of a firm, the market clearing condition for security g is $\sum_{\mathscr{I}} z^i_g = 0$, not $\sum_{\mathscr{I}} z^i_g = 1$.

Exactly as before, a necessary condition for this new $\mathscr{F}$-equilibrium to be a full equilibrium is that the $F+1$ securities being traded span the set of observable components. If they do not span the set of market states, then entrepreneurs have an incentive to introduce another security g'. Clearly, in a world of no transactions costs, this process continues until the variety of securities traded are sufficient to span the observable component of the state space.

5. SPANNING AND STOCKHOLDER UNANIMITY[10]

The preceding section showed that the market process does tend to insure that the observable component is spanned. This section derives two conditions under which a firm's stockholders will be unanimous towards an investment project that constitutes a change in the firm's production plan and, if adopted, will change the firm's vector of state contingent returns. The first of these conditions is well known and the second was recently developed by Leland [1978]. They are both derived here because in Section 6 they are given a specific interpretation that follows directly from the particular derivations that are presented here.

Throughout this section suppose that the state space's observable component is spanned. Take the endowment streams of consumption goods for consumers $[\omega_1^i, \omega_2^i(11),..., \omega_2^i(ST)]$ as fixed. Suppose additionally that consumers' initial endowments of stock $[\bar{z}_1^i,..., \bar{z}_F^i]$ together with their consumption good endowments are such that they constitute an equilibrium allocation, i.e., when the market opens no trades take place. Let $\hat{p}=(\hat{p}_1,..., \hat{p}_F)$ be the equilibrium price vector for this initial situation.[11] Because the observable component is spanned and the economy is in equilibrium, all consumers have identical implicit prices for the observable events, i.e., $\psi_{\mathscr{S}}^i=\psi_{\mathscr{S}}^j$ for all $i, j \in \mathscr{S}$.

Now, turning to consideration of the first condition that guarantees unanimity, suppose firm f proposes a project such that its vector of returns changes from $a_f \equiv [a_f(11),..., a_f(st),..., a_f(ST)]$ where $a_f(s1)=\cdots=a_f(st)=\cdots=a_f(sT)$ to $a_f + b \equiv [a_f(11)+b(11),..., a_f(ST)+b(ST)]$ where b is a ST-dimensional vector. Let b have two characteristics. First, let b, as does a_f, vary only with the observable component of the state, i.e., $b(s)\equiv b(s1)=b(s2)=\cdots=b(sT)$. Second, let the magnitude of b be small enough compared to the economy as a whole to justify price taking behavior. In particular, assume that every consumer takes his equilibrium implicit prices $\psi_{\mathscr{S}}^i$ as invariant with respect to a small change in firm f's returns' vector a_f. Given these assumptions, all stockholders of the firm are unanimous in approving or disapproving the project.

[10] The case I treat here is called *ex post* stockholder unanimity in the literature. If one is willing to make the strong assumption of perfect foresight with respect to equilibrium stock prices and implicit prices (as is the case in rational expectations equilibria), then *ex ante* unanimity can also be shown. Baron [1977] provides a clear exposition and discussion of the intricacies of *ex ante* unanimity versus *ex post* unanimity.

[11] This equilibrium initial situation is the result of previous trading activity that is not explicitly included within this model.

This is seen by picking an arbitrary stockholder i and calculating the effect the change b in the firm's returns has on his utility. Since b is small and since $\bar{z}^i_f b$ is how much his consumption in period two changes, adoption of b causes i's utility to change by the quantity:

$$
\begin{aligned}
\Delta U^i &= \bar{z}^i_f \textstyle\sum_{\mathscr{S}} \sum_{\mathscr{T}} \dfrac{\partial U^i(x^i)}{\partial x^i_2(st)}\, b(st) \\
&= \bar{z}^i_f \textstyle\sum_{\mathscr{S}} \sum_{\mathscr{T}} \delta^i_{st} b(st) \\
&= \bar{z}^i_f \lambda^i \textstyle\sum_{\mathscr{S}} \sum_{\mathscr{T}} \psi^i_{st} b(st) \\
&= \bar{z}^i_f \lambda \textstyle\sum_{\mathscr{S}} \psi^i_s b(s)
\end{aligned} \tag{16}
$$

where the algebra parallels the algebra of equation (15). Inspection of the last line shows that ΔU^i necessarily has the same sign for all stockholders because (a) by definition $\bar{z}^i_f > 0$ for all stockholders, (b) $\lambda^i = \partial U^i / \partial x^i_1 > 0$ since consumers are assumed to have strictly increasing utility functions, and (c) the sum $\sum_{\mathscr{S}} \psi^i_s b(s)$ is invariant across individuals since spanning of the observable component guarantees $\psi^i_{\mathscr{S}} = \psi^j_{\mathscr{S}}$ for all pairs (i, j) of consumers. Therefore the firm's stockholders unanimously approve or disapprove the project. If, however, the state space's observable component had not been spanned, then unanimity would not have been assured because $\psi^j_{\mathscr{S}}$ would not necessarily have equaled $\psi^i_{\mathscr{S}}$ for every pair of consumers.

Crucial to the above derivation of stockholder unanimity under spanning of the state space's observable component is the requirement that the components of b only vary with respect to observable events s. This may be seen by supposing that the firm proposes a project such that $b(st') \neq b(st'')$ for two states $st', st'' \in \mathscr{K}$. Spanning of the observable component only guarantees $\sum_{\mathscr{T}} \psi^i_{st} \equiv \psi^i_s = \psi^j_s \equiv \sum_{\mathscr{T}} \psi^j_{st}$ for all $s \in \mathscr{S}$ and all $i, j \in \mathscr{I}$, which does not imply $\psi^i_{st} = \psi^j_{st}$. Therefore, for this case, the fourth line of (16) does not follow from the third line. Consequently $\sum_{\mathscr{S}} \sum_{\mathscr{T}} \psi^i_{st} b(st)$ and $\sum_{\mathscr{S}} \sum_{\mathscr{T}} \psi^j_{st} b(st)$ may be of different signs, which is to say that stockholder unanimity may fail.

Leland [1978] in a significant and very interesting paper has developed a condition that is sufficient for stockholder unanimity for this case where the state space's observable component is spanned and the proposed project's returns' vector b varies with the unobservable events t. It is: if all consumers place identical conditional probabilities $\eta^i(t \mid s) \equiv \eta^i(st) / \sum_{\mathscr{T}} \eta^i(st)$ on the possibility of the unobservable event t being realized given that the observable event s is realized, then unanimity is preserved. Moreover, Leland shows, it is not really necessary that all consumers agree on these conditional probabilities; rather all that is necessary is that they would all agree if they had the same, better information as the firm's management.

Leland's result may be derived as follows. Recall that by definition $\psi^i_{st} = \delta^i_{st} / \lambda^i$ and $\psi^i_s = \sum_{\mathscr{T}} \psi^i_{st}$. Substitution from equations (8) and (6) into these definitions and differentiation of U^i explicitly gives:

$$\psi^i_{st} = \frac{1}{\lambda^i} \eta^i(st) \frac{\partial u^i[x^i_1, x^i_2(st)]}{\partial x^i_2(st)} \tag{17}$$

and

$$\psi^i_s = \frac{1}{\lambda^i} \sum_{\mathscr{T}} \eta^i(st) \frac{\partial u^i[x^i_1, x^i_2(st)]}{\partial x^i_2\,(st)}\,. \tag{18}$$

The change in consumer i's expected utility resulting from adoption of a project whose returns vector b varies with the unobservable component is therefore:

$$\begin{aligned} \Delta U^i &= \bar{z}^i_f \sum_{\mathscr{S}} \sum_{\mathscr{T}} \frac{\partial U^i[x^i]}{\partial x^i_2\,(st)} b(st) \\ &= \bar{z}^i_f \sum_{\mathscr{S}} \sum_{\mathscr{T}} \eta^i(st) \frac{\partial u^i[x^i_1, x^i_2(st)]}{\partial x^i_2(st)} b(st) \\ &= \bar{z}^i_f \sum_{\mathscr{S}} \frac{\partial u^i[x^i_1, x^i_2(s)]}{\partial x^i_2(s)} \sum_{\mathscr{T}} \eta^i(st) b(st). \end{aligned} \tag{19}$$

Line three follows from line two because the observability constraints (4) guarantee that $x^i_2(st') = x^i_2(st'') = x^i_2(s)$ for all $s \in \mathscr{S}$ and all $t', t'' \in \mathscr{T}$, which implies that $\partial u^i[x^i_1, x^i_2(st)]/\partial x^i_2(st) = \partial u^i[x^i_1, x^i_2(s)]/\partial x^i_2(s)$. Moreover this latter fact implies that (18) can be rewritten as

$$\frac{\partial u^i[x^i_1, x^i_2(s)]}{\partial x^i_2(s)} = \frac{\lambda^i \psi^i_s}{\sum_{\mathscr{T}} \eta^i(st)}\,. \tag{20}$$

Substitution of (20) into (19) gives

$$\begin{aligned} \Delta U^i &= \lambda^i \bar{z}^i_f \sum_{\mathscr{S}} \psi^i_s \sum_{\mathscr{T}} \left(\frac{\eta^i(st)}{\sum_{\mathscr{T}} \eta^i(st)} \right) b(st) \\ &= \lambda^i \bar{z}^i_f \sum_{\mathscr{S}} \psi^i_s \sum_{\mathscr{T}} \eta^i(t|s) b(st). \end{aligned} \tag{21}$$

This expression implies unanimity whenever the observable component is spanned and consumers have identical subjective probabilities $\eta(t \mid s) \equiv \eta^1(t \mid s) = \cdots = \eta^i(t \mid s) = \cdots = \eta^I(t \mid s)$ for all $t \in \mathscr{T}$ and $s \in \mathscr{S}$. This is seen by defining $b'(s) \equiv \sum_{\mathscr{T}} \eta(t \mid s) \cdot b(st)$, which is the expected return of the project if s is realized, and noting that (21) with $b'(s)$ substituted is identical to (16). Leland's further conclusion that if stockholders had the better information that the firm's manager has, then they would be unanimous for or against the project follows from the assumption that the manager is a stockholder who acts solely in accordance with his interests as a stockholder.

6. CONCLUSIONS

Formally I have shown three results in the preceding sections. First, if initially the observable component of the state space is not spanned, then in a world of no transaction costs an incentive exists for individual entrepreneurs to introduce new securities in sufficient variety to span the observable component of the state space. Second, given that the observable component is spanned, stockholders of a firm are unanimous concerning the acceptance or rejection of any proposed project whose returns are a function only of the observable component. Stockholders, however, may disagree concerning production plan changes whose returns vary with the unobservable component as well as with the observable component. Third, as Leland [1978] originally showed, unanimity remains assured even if the project's returns vary with the unobservable component provided that (a) the observable component is spanned and (b) all consumers place (or would place if they had the same, better information that the firm's manager has) identical conditional probabilities on the unobservable events $t \in \mathscr{T}$.

The interpretation that I give these results is, as indicated in the paper's introduction, that unanimity is likely to exist for proposed projects that are routine and is unlikely to exist for proposed projects that are truly innovative and represent a significant new experiment within the economy. Examples of routine projects are (a) the expansion or contraction of capacity for existing product lines and (b) the introduction of a new product that is only marginally different from already marketed products. Examples of innovative projects are (a) major investment in a new, radically different production technology and (b) introduction of consumer product that is genuinely different in concept and is not a variation on established themes.

Construction of a definition that (a) distinguishes between routine and innovative projects and (b) demonstrates that that definition is consistent with both the earlier section's formal results and this section's interpretation is most efficiently accomplished by discussing an example of both a routine project and an innovative project. Suppose, for the routine project example, firm f is considering increasing its manufacturing capacity by constructing a major plant addition. The paper's first formal result indicates that the observable component of the state space is certainly spanned for all practical purposes because, if it were not, financial entrepreneurs would introduce new securities until spanning were achieved. This, however, does not guarantee unanimity because the returns from construction of the addition necessarily depend on the state space's unobservable component.

Specifically, the cost of the addition depends on, among other factors, the stability of the soil on which the addition is to be built. If the soil is unexpectedly unstable, then the cost of constructing the foundations will jump by an order of magnitude and correspondingly decrease the project's returns. Clearly the soil conditions, unless the addition is actually built, is an aspect of the state space that is unobservable. Consequently, for the purposes of this example, the state space may be described as $\mathscr{S} = \{1,\ldots, S\}$, the observable component, and $\mathscr{T} = \{1, 2\}$, the unobservable component where $t=1$ denotes stable soil and $t=2$ denotes unstable soil. Moreover the soil stability on which the addition will be built is a characteristic of nature; it does not change with other contingencies within the economy. Therefore every consumer will regard the state space's two components as statistically independent, i.e., $\eta^i(st)=\eta^i(s)\eta^i(t)$ and $\eta^i(t\,|\,s)=\eta^i(t)$ for each $i \in \mathscr{I}$.

Before firm f's manager makes his decision on whether to build the plant addition, he will secure an engineering report on the soil conditions at the proposed construction site. This report will allow him to assess quite objectively the risk that soil conditions will turn out to be unfavorable, i.e., based on the report he will revise his prior probability judgments $\eta^m_t(1)$ and $\eta^m_2(2)$ concerning soil conditions at the plant site. Moreover, because soil engineering is a well developed profession, his judgment will be objective in the sense that if all consumers read the engineering report and other information used by the manager in making his probability judgment, then they would agree very closely with his judgment. This means that Leland's condition for unanimity is met: the state space's observable component is spanned and all consumers would agree with the manager's probability assessment concerning the unobservable component if they had access to the better information on which he bases his judgment. Thus routine projects may be defined to be those projects for which generally accepted techniques exist for objectively assessing the probabilities of the relevant unobservable events.

Consider, as an example of an innovative project, firm f's decision concerning investment into a radical new technology for the smelting of iron ore. Suppose that this technology has been tested in a pilot plant, but never implemented on a commercial scale, an endeavor that involves scaling up the pilot by a factor of fifty. Moreover suppose that real technical controversy exists as to whether the process can be successfully scaled up by such a factor, i.e., equally qualified and informed engineers disagree substantially on what the probabilities of success are.[12]

Exactly parallel to the first example, the state space's observable component may be expected to be spanned, the state space may be described by $\mathscr{S}=\{1,\dots, S\}$ and $\mathscr{T}=\{1, 2\}$ where $t=1$ denotes feasibility of the technology and $t=2$ denotes infeasibility, and consumers probability judgments $\{\eta^i_t(1), \eta^i_t(2)\}$ concerning the unobservable component are independent of their probability judgments $\{\eta^i_s(1), \dots, \eta^i_s(S)\}$ concerning the observable component. The difference with the first example is that even if all consumers had access to the same, better information that firm f's manager is using in making his decision concerning the proposed technology, then consumers would still seriously disagree among themselves on what the probabilities of success are. This is because no generally accepted technical methodology exists for evaluating the probabilities. Consequently

[12] Currently such differences in opinion appear to be prevalent in regard to the commercial promise of fusion as an energy source. Harsanyi [1968, pp. 498–500] has argued that such differences in probability judgments generally stem from differences in information. I disagree because for the case of fusion it seems evident that two equally qualified engineers might fail to agree on the chances of success even if they were given unlimited time to exchange information. In other words, the source of such disagreements is, at least in part, differences in fundamental beliefs, not differences in information. The only way to get agreement in such cases is to do the experiment of trying to develop fusion as a commercial energy source and to observe the outcome.

Leland's conditions are not met and stockholder unanimity is not guaranteed. Thus innovative projects may be defined to be those projects for which generally accepted techniques do not exist for objectively assessing the probabilities of the relevant unobservable events.

In summary, this theoretical discussion suggests two conclusions regarding stockholder unanimity. First, stockholder unanimity is likely for investment decisions such as plant expansion that essentially involve more of the same. Second, decisions that involve substantial innovation such as the implementation of a radical technology are likely to create division among stockholders. This latter conclusion places absolute limits on the extent that the market can mediate among stockholders' diverse risk preferences and subjective probabilities. Within a dynamic economy decisions of this latter, nonroutine type periodically face firms as new technologies are discovered and new products are conceived. Stockholders inevitably will disagree over which of these ideas are worth substantial investment. It is their very newness that makes it impossible for the market to have established a set of implicit prices by which managers can evaluate their appropriateness for investment.

7. SOME UNANSWERED QUESTIONS

The discussion above is incomplete in that it implicitly raises a number of important and interesting questions that I have not yet addressed. Three of these questions follow. First, and of obvious significance, the paper contains no discussion of welfare effects. In particular, what are the welfare implications of the conclusion that the security market is necessarily incomplete because securities can not be written that depend on the success or failure of innovations that no firm is trying. The work of Hart [1975] and Grossman [1977] on optimality within incomplete markets indicates that no simple answer is likely to exist to this question. Second, this paper has taken the division of the state space into observable and unobservable components as exogenous. This is clearly inappropriate, however, because the past choices of firms as to which innovations to adopt has determined this division. Therefore an important question is under what conditions a firm is likely to decide to go ahead with an innovative project for which stockholder unanimity does not exist. Third, and dependent on the answer to the second question, what do these limits on the scope of stockholder unanimity imply for public policy towards innovation.

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