**Econ** 401

**Mathematical Methods** 

**FALL 2015** 

Instructor: Wojciech Olszewski, wo@northwestern.edu

Time and Location: On days other than Weds, Lectures: 9 A.M.-12 P.M.; Problem Sessions: 1-2 P.M.; Andersen Hall, Room 3245; On Weds, TA Sessions: 9 A.M.-12 P.M. and 1-2 P.M. (The time and location

of the last two classes, which overlap with the fall quarter, will be announced later.) Teaching Assistants:

John Farragut (JohnFarragut2016@u.northwestern.edu) and Abdoulaye Ndiaye (laaye.ndiaye@gmail.com)

Course Overview

The purpose of this course is to refresh incoming PhD students with essential mathematical tools for their programs. The course covers topics in topology, real analysis, optimization, linear algebra and probability theory. Aside from providing mathematical tools, this course aims to contribute to developing the level of mathematical sophistication necessary to conduct research in many areas of economics. The course will therefore emphasize logical clarity and mathematical rigor, along with the ability to follow and construct proofs. Some part of each class will be devoted to writing rigorous proofs. Except Wednesdays, there will be daily lectures, followed by problem sessions and daily problem sets. On Wednesdays, there will be extra long TA sessions solving questions and reviewing issues with which the students say they had problems. On one of the last days of class, there will be a mandatory exam. I will be available to discuss final exams during the first weeks of the fall quarter. For those who feel they may benefit from some additional mathematical training, I would also be happy to discuss courses offered by the mathematics department at that time, or potential other readings. I will also be available for a limited number of extra classes during the fall to cover math material that students are having trouble with.

Textbooks

textbook designed for this kind of class is Simon and Blume (1994): Mathematics for Economists. As an introduction to topology and real analysis, I would recommend Rudin (1976): Principles of Mathematical Analysis, or Ok (2005): Real Analysis with Economic Applications. The former book is a classic math textbook for this material. The latter book contains some more advanced material, and its advantage is that its author is an active-in-reaserch economist. Oxtoby (1980): Measure and Category is an excellent introduction to probability theory and topology, but its focus is somewhat different from a typical math refresher course.

The primary reference for this course will be a set of lecture notes, which will distributed in class. A

Students should also consider studying popular probability theory textbooks such as Billingsley or Shiryaev.

The mathematical appendix of Mas-Colell, Winston and Green (1995): Microeconomic theory contains a

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condensed overview of mathematical tools which are used in microeconomic theory. Finally, Aliprantis and Border (2006): In finite Dimensional Analysis: A Hitchhiker's Guide is a great reference book containing many results from various areas of mathematics, which turned out useful in economics.

## "Prerequisite" readings

Some students ask for readings, for the vacation before entering the graduate school. I would recommend to look first at the appendix of *Mas-Colell, Whinston and Green*. If you know, and can prove on your own, most theorems from this appendix, you will not need any in advance readings. You may use your time to study more advanced probability textbooks, or review *Aliprantis and Border*. Otherwise, I would recommend to read *Rudin* if you have no, or little, experience with writing proofs; and to read *Simon and Blume* if you see large parts of the material for the first time, or have seen it a long time ago. From Rudin, I would recommend Chapters: 2, the first five sections of 3, 4-5, the first three sections of 6, 7, and 9. Particularly relevant chapters from *Simon and Blume* are 7-19, 23, and 29-30.

## Course Outline (Tentative)

The following is a tentative course outline.

- Date Lecture
- 8/27 **Topology:** Metrics, Open Sets, Sequences
- 8/28 Compactness, and other properties of topological spaces
- 8/31 Mappings, Correspondences, (Hemi-) Continuity, Fixed Points, Min-Max Theorem
- 9/1 Linear Algebra: Systems of Equations, Elementary (and other basic) Operations
- 9/3 Determinant, and other basic concepts
- 9/4 Eigenvalues, Eigenvectors, Jordan Theorem
- 9/8 **Real Analysis:** Differentiation, and basics of Integration
- 9/10 Differentiation, and basics of Integration
- 9/11 **Optimization:** Unconstraint Optimization, Equality-Constrained Optimization
- 9/14 Inequality-Constrained Optimization
- 9/15 Convexity, and basic concepts and results in Convex Analysis
- 9/17 **Probability Theory:** Basic Concepts in Discrete Spaces
- 9/18 Lebesgue Measure, and more general objects and concepts
- 9/21 Examination
- 9/22 Linear and Dynamic Programming