Executive Summary of Finance 430

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Finance 430-62/63/64, Winter 2011

Weekly Topics:

1. Present and Future Values, Annuities and Perpetuities
2. More on NPV
3. Capital Budgeting
4. Stock Valuation
5. Bond Valuation and the Yield Curve
6. Introduction to Risk – Understanding Diversification
7. Optimal Portfolio Choice
8. The CAPM
9. Capital Budgeting Under Uncertainty
10. Market Efficiency

Summary of Week 1: Present Value Formulas

1. **Present Value:** Present value of a single cash flow received n periods from now:
$$\text{PV} = C \times \frac{1}{(1+r)^n}.$$
2. **Future Value:** Future value of a single cash flow invested for n periods:
$$\text{FV} = C \times (1+r)^n.$$
3. **Perpetuity:** Present value of a perpetuity, $\boxed{\text{PV} = \frac{C}{r}}$.
4. **Growing Perpetuity:** Present value of a constant growth perpetuity, $\boxed{\text{PV} = \frac{C}{r-g}}$.
5. **Annuity:**

   Present value of an annuity paying C at the end of each of n periods
$$\boxed{\text{PV} = \frac{C}{r}\left[1 - \frac{1}{(1+r)^n}\right]}.$$
   Future value of an annuity paying C at the end of each of n periods
$$\text{FV} = (1+r)^n \times PV = (1+r)^n \frac{C}{r}\left[1 - \frac{1}{(1+r)^n}\right] = C \times \frac{(1+r)^n - 1}{r}.$$

6. **Growing Annuity:** Present value of a growing annuity

$$\boxed{\text{PV} = C \times \begin{cases} \frac{1}{r-g}\left[1-\left(\frac{1+g}{1+r}\right)^n\right] & \text{if } r \neq g \\ n/(1+r) & \text{if } \ r = g. \end{cases}}$$

Future value of growing annuity

$$\text{FV} = (1+r)^n \times PV = C \times \begin{cases} \frac{1}{r-g}[(1+r)^n - (1+g)^n] & \text{if } r \neq g \\ n(1+r)^{n-1} & \text{if } r = g. \end{cases}$$

7. **Excel Functions for Annuities:** PV, PMT, FV, NPER
   Other Excel functions: SOLVER.
   Excel spreadsheet with examples from Week 1 handout posted on course page.
   Additional Excel examples in Excel handout posted on Blackboard.

8. **Remember:** There are no Excel functions for perpetuities, growing perpetuities or growing annuities. Just type the formula directly into a cell in Excel.

Summary of Week 2: More on NPV

1. **Interest rate per compounding period:** $\frac{r_{APR}}{k}$

   Effective annual rate (EAR): If you invest \$1 at an APR of r_{APR} with compounding k times per year, then after one year you have

$$(1 + EAR) = \left(1 + \frac{r_{APR}}{k}\right)^k$$

   and after t years you have $(1 + EAR)^t$

   Changing interest rates over time or below market rate financing:

   - Calculate cash flows using contract interest rate
   - Calculate principal still owed using contract interest rate
   - Calculate present value (market value) using going market rate.

2. **Covered interest rate parity:**

$$FS_{0,n}\,(\$/£) = S_0\,(\$/£) \times \frac{(1 + r_\$)^n}{(1 + r_£)^n}.$$

   $S_0\,(\$/£)$: Current (spot) exchange rate. $FS_{0,n}$: Forward exchange rate set today (time 0) for exchanging \$ and £ at time n. $r_\$$: \$ interest rate. $r_£$: £ interest rate.

3. **Nominal vs. real cash flows:** (i is the expected inflation rate)

$$C_t^{\text{real}} = \frac{C_t^{\text{nominal}}}{(1+i)^t}$$

Growth rates of real and nominal cash flows:

$$1 + g^{\text{real}} = \frac{1 + g^{\text{nominal}}}{1+i}$$

Nominal vs. real interest rates:

$$1 + r_{\text{real}} = \frac{1 + r_{\text{nominal}}}{1+i}.$$

Discounting real cash flows at the real interest rate or nominal cash flows at the nominal interest rate gives the **same present value**:

$$PV = \frac{C_t^{\text{nominal}}}{\left(1 + r_{\text{nominal}}\right)^t} = \frac{C_t^{\text{nominal}}}{\left(\left(1 + r_{\text{real}}\right) \times (1+i)\right)^t} = \frac{C_t^{\text{nominal}} / (1+i)^t}{\left(1 + r_{\text{real}}\right)^t} = \frac{C_t^{\text{real}}}{\left(1 + r_{\text{real}}\right)^t}.$$

Relating nominal and real cash flows and interest rates:

$$\begin{tikzcd}
\text{\$ today} \arrow[d, "\times 1 \text{ (equal)}"'] & & \text{\$ in } t \text{ years} \arrow[ll, "\times \frac{1}{(1+r_{nominal})^t}"'] \arrow[d, "\times \frac{1}{(1+i)^t}"] \\
\text{Purchasing power today} & & \text{Purchasing power in } t \text{ years} \arrow[ll, "\times \frac{1}{(1+r_{real})^t}"]
\end{tikzcd}$$

4. **After tax cash flows and interest rates at tax rate τ:**

$$\begin{aligned}
C_t^{\text{After-tax}} &= (1-\tau) \times C_t^{\text{Before-tax}} \\
r^{\text{After-tax}} &= (1-\tau) \times r^{\text{Before-tax}}.
\end{aligned}$$

Summary of Week 3: Capital Budgeting

1. **Cash Flow Formulas**

$$
\begin{aligned}
C_t &= \text{Project Cash Inflows}_t - \text{Project Cash Outflows}_t \\
C_t &= (1-\tau)(\text{OPREV - OPEXP - DEPR})_t + \text{DEPR}_t - \text{CAPEX}_t - (\text{WC}_t - \text{WC}_{t-1}) \\
C_t &= (1-\tau)(\text{OPREV - OPEXP})_t + \tau\text{DEPR}_t - \text{CAPEX}_t - (\text{WC}_t - \text{WC}_{t-1})
\end{aligned}
$$

   - Only **changes** in working capital have cash flow implications, so we include $(\text{WC}_t - \text{WC}_{t-1})$ in the cash flow formula. Working capital is defined as:

$$\text{WC}_t = \text{Inventory}_t + \text{Accounts Receivable}_t - \text{Accounts Payable}_t$$

   - Sale of capital enters as negative CAPEX.

$$\text{After-tax cash flow from asset sale} = \text{Sale price} - \tau \times (\text{Sale price} - \text{Book value})$$

   where the book value is the amount of the purchase price that has not yet been depreciated.

2. The **net present value (NPV)** of a project's cash flows is:

$$\text{NPV} = C_0 + \frac{C_1}{1+r} + \cdots + \frac{C_n}{(1+r)^n}$$

   where C_t is the cash flow at time t.

3. A project's **internal rate of return (IRR)** is the interest rate that sets the net present value of the project cash flows equal to zero:

$$0 = C_0 + \frac{C_1}{(1+\text{IRR})} + \frac{C_2}{(1+\text{IRR})^2} + \cdots + \frac{C_n}{(1+\text{IRR})^n}.$$

4. A project's **profitability index** is the ratio of the net present value to the constrained resource consumed:

$$\text{PI} = \frac{\text{NPV}}{\text{Constrained resource consumed}}$$

Summary of Week 4: Stock Valuation

1. **Realized return on a stock:**

$$\frac{P_{t+1} - P_t}{P_t} + \frac{DIV_{t+1}}{P_t}$$

2. **General stock price formula:** The price of a stock is the PV of its future dividends

$$P_0 = \Sigma_{t=1}^{\infty} \frac{DIV_t}{(1+r)^t}.$$

3. **Gordon growth model, one-stage version: Constant growth forever**

   Stock price:

$$P_t = \frac{DIV_{t+1}}{r-g} = \frac{POR \times EPS_{t+1}}{r-g} = \frac{POR \times ROE \times BE_t}{r - PBR \times ROE}$$

   Price-earnings ratio:

$$\frac{P_t}{EPS_{t+1}} = \frac{POR}{r-g}$$

   Present value of growth opportunities: Calculate the price twice, with and without growth, and compare. Or calculate it in one step as:

$$\text{PVGO}_t = P_t - \frac{\text{EPS}_{t+1}}{r}$$

   which could also be calculated as

$$\text{PVGO}_t = \frac{\left(\frac{\text{ROE}}{r} - 1\right) \times \text{PBR} \times \text{EPS}_{t+1}}{r-g}.$$

4. **Gordon growth model, two-stage version: Stable growth from time T on**

   Stock price:

$$P_0 = \sum_{t=1}^{T} \frac{DIV_t}{(1+r)^t} + \frac{P_T}{(1+r)^T}$$

   - P_T is the usual expression from the standard Gordon growth model

$$P_T = \frac{\text{DIV}_{T+1}}{r - g_{late}}$$

     with $g_{late} = PBR_{late} \times ROE_{late}$,
     and $DIV_{T+1} = POR_{late} \times EPS_{T+1} = POR_{late} \times ROE_{late} \times BE_T$.

   - If growth is constant and equal to g_{early} up to time T, then dividends for the first T years are a growing annuity with $g_{early} = PBR_{early} \times ROE_{early}$. In that case:

$$P_0 = \frac{DIV_1}{r - g_{early}} \left[1 - \left(\frac{1+g_{early}}{1+r}\right)^T\right] + \frac{1}{(1+r)^T} \left[\frac{DIV_{T+1}}{r - g_{late}}\right]$$

     where $DIV_1 = POR_{early} \times EPS_1$.

Price-earnings ratio: Calculate the price, P_t. Calculate the earnings, EPS_{t+1}. Then just divide to get the (forward) PE ratio, P_t/EPS_{t+1}.

Present value of growth opportunities: Calculate the price twice, with and without growth, and compare. No easy formula.

5. **Stock pricing allowing for stock issues or repurchases. Still no debt. Constant growth.**

   - We can adjust the (constant growth) Gordon growth model formula to fit this case:

     $$P_0 = \frac{DIV_1}{r - g_{\text{dividends/share}}}, \quad \text{with} \quad g_{\text{dividends/share}} = \frac{PBR \times ROE \times POR + RPR \times r}{POR + RPR}$$

     where

     – DIV_1 (dividends per share) is total dividends paid at time 1 divided by the number of shares at time 0 (i.e. at time 1 before the repurchase).

     – RPR (the repurchase rate) is the fraction of earnings used to repurchase shares, so $POR + PBR + RPR = 1$. A negative value of RPR means that the firm is raising external capital by issuing more shares.

   - We can get the same answer using the **total payout model**. It states that

     $$P_0 = \frac{\text{PV(Future total dividends and repurchases)}}{\text{Shares outstanding at time 0}}$$

     where

     $$\text{PV(Future total dividends and repurchases)} = \frac{\text{Total dividends and repurchases at time 1}}{r - g}$$

     and $g = PBR \times ROE$. Again, negative repurchases means that new shares are issued.

6. **Expected versus realized returns:** In both versions of the Gordon growth model r denotes the return investors expect to earn (investors' "required return").

   - If no news arrives during the period, they will earn r.
   - If news about cash flows or about investors' required return (going forward) arrive during the period, then the realized return may be higher or lower than what was expected.

Summary of Week 5: Bond Valuation

1. **Price of a j-period zero coupon bond** (=discount bond=STRIPS) with face value \$100:

$$B_j = \frac{100}{(1+y_j)^j}.$$

This formula also defines the yield to maturity on the j-period zero coupon bond, y_j, as the (constant) discount rate that equates the discounted cash flow to the price (i.e. the IRR to buying the bond). Reorganizing the formula to get y_j gives

$$y_j = \left(\frac{100}{B_j}\right)^{1/j} - 1.$$

2. **Price of a j-year coupon bond** with face value F and annual coupon payments C

$$P_j = \frac{C}{(1+y_1)} + \frac{C}{(1+y_2)^2} + ... + \frac{C}{(1+y_{j-1})^{j-1}} + \frac{C+F}{(1+y_j)^j}$$

or expressed directly in terms of the zero-coupon bond prices

$$P_j = C \times \frac{B_1}{100} + C \times \frac{B_2}{100} + ... + C \times \frac{B_{j-1}}{100} + (C+F) \times \frac{B_j}{100}.$$

where P_j is the price of the coupon bond, B_t, t=1,...,j, is the prices of the zero coupon bonds, C is the dollar coupon payment on the coupon bond, and F is the face value of the coupon bond.

Calculating the yield to maturity y of a coupon bond: The yield to maturity, y, is defined as the internal rate of return to buying the bond. It is given by the equation:

$$P_j = \frac{C}{(1+y)} + \frac{C}{(1+y)^2} + ... + \frac{C}{(1+y)^{j-1}} + \frac{C+F}{(1+y)^j}$$

(and can be solved for using the IRR function in Excel).

3. **Calculating PVs (when the term structure is not necessarily flat):**

Method I (**use prices of zero coupon bonds**):

$$PV = C_1 \times \frac{B_1}{100} + C_2 \times \frac{B_2}{100} + ... + C_n \times \frac{B_n}{100}$$

Method II (**use yields to maturity of zero coupon bonds**):

$$PV = \frac{C_1}{1+y_1} + \frac{C_2}{(1+y_2)^2} + ... + \frac{C_n}{(1+y_n)^n}$$

Method III (**use forward interest rates**):

$$PV = \frac{C_1}{1+y_1} + \frac{C_2}{(1+y_1)(1+f_2)} + ... + \frac{C_n}{(1+y_1)(1+f_2)...(1+f_n)}$$

(note that $f_1 = y_1$).

4. **Duration** (D) of a security that pays cash flows C_1, C_2, ..., C_T (when the yield curve is flat at y):

$$D = 1 \times \frac{PV(C_1)}{PV(C_1, C_2, ..., C_T)} + 2 \times \frac{PV(C_2)}{PV(C_1, C_2, ..., C_T)} + \cdots + T \times \frac{PV(C_T)}{PV(C_1, C_2, ..., C_T)}.$$

Note that the PV of all the cash flows, $PV(C_1, C_2, ..., C_T)$, is just the price of the security.

5. **Sensitivity of the price of a security to changes in the yield curve** is approximately given by

$$\text{\% Change in value of security} \;\approx\; -\frac{D}{1+y} \times \text{Change in yield (in \% points)}$$

where y is the level of the yield curve **before** any changes. $D/(1+y)$ is sometime called "modified duration" and denoted D^*.

6. **Duration of a portfolio of securities (when the yield curve is flat at y):**

$$D_{\text{Portfolio}} = D_{\text{Security 1}} \times \left(\frac{\text{Value(Security}_1)}{\text{Portfolio Value}}\right) + D_{\text{Security 2}} \times \left(\frac{\text{Value(Security}_2)}{\text{Portfolio Value}}\right) + \cdots$$

7. Relationship between **prices of zero coupon bonds and forward interest rates**:

$$f_j = \frac{B_{j-1}}{B_j} - 1.$$

8. Relationships between **yields on zero coupon bonds and forward interest rates**:

$$(1 + y_j)^j = (1 + y_1)(1 + f_2)(1 + f_3) \ldots (1 + f_{j-1})(1 + f_j)$$

and also

$$(1 + y_j)^j = (1 + y_{j-1})^{j-1}(1 + f_j).$$

Week 6: Introduction to Risk

1. **A single asset**

   Expected return: $E(r) = \Sigma_{j=1}^{n} p_j \times r_j$

   Variance of return: $\sigma^2(r) = \Sigma_{j=1}^{n} p_j \times [r_j - E(r)]^2$

   Standard deviation of return: $\sigma(r) = \sqrt{\sigma^2(r)}$.

2. **Portfolios of two risky assets, A and B**

   Return (realized return): $r_p = x_A r_A + x_B r_B$,
   where $x_B = 1 - x_A$.

   Expected return: $E(r_p) = x_A E(r_A) + x_B E(r_B)$

   Variance of return:

$$\begin{aligned} \sigma_p^2 &= x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2 x_A x_B \sigma_{A,B} \\ &= x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2 x_A x_B \rho_{A,B} \sigma_A \sigma_B. \end{aligned}$$

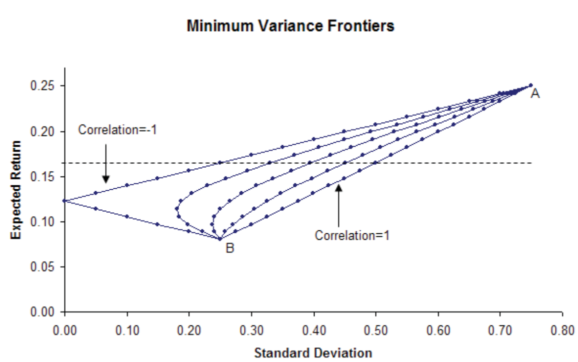
   Standard deviation of return: $\sigma_p = \sqrt{\sigma_p^2}$.

- For $\rho_{A,B} = 1$ there is is **no diversification benefit**. Then

$$\sigma_p^2 = x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2 x_A x_B \sigma_A \sigma_B = (x_A \sigma_A + x_B \sigma_B)^2$$

  and $\sigma_p = x_A \sigma_A + x_B \sigma_B$ so the portfolio standard deviation is simply the weighted average of the standard deviations of the two assets.

- For $\rho_{A,B} < 1$ there is a **diversification benefit**. The portfolio standard deviation is now less than the weighted average of the standard deviations of the two assets. Thus by diversifying you can get less risk for the same expected return or higher expected return for the same risk.



3. **Portfolios of many risky assets:** For a portfolio with n risky assets. x_i =proportion of wealth invested in asset i.

Realized return:

$$r_p = x_1 r_1 + x_2 r_2 + \cdots + x_n r_n = \sum_{i=1}^{n} x_i r_i$$

Expected return:

$$E(r_p) \;=\; x_1 E(r_1) + x_2 E(r_2) + \cdots + x_n E(r_n) = \sum_{i=1}^{n} x_i E(r_i)$$

Variance:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{i,j} = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{\substack{j=1 \\ i\neq j}}^{n} x_i x_j \sigma_{i,j}$$

For equal-weighted portfolio of n risky assets, each portfolio share is $1/n$ and the portfolio variance is

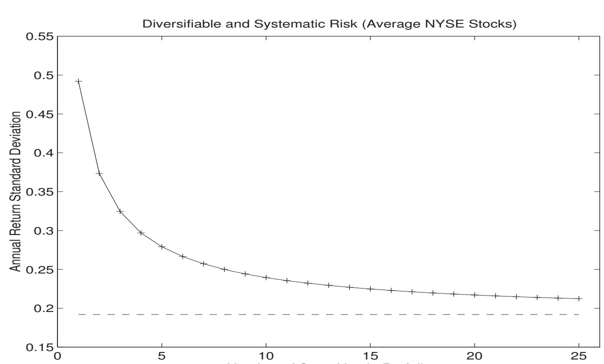
$$\sigma_p^2 = \left(\frac{1}{n}\right)\overline{\sigma^2} + \left(\frac{n-1}{n}\right)\overline{cov}$$

where $\overline{\sigma^2}$ is the average variance of the n assets and $\overline{cov}$ is the average covariance of the n assets. As n becomes large

$$\left(\frac{1}{n}\right) \to 0,\; \left(\frac{n-1}{n}\right) \to 1,\; \text{so } \sigma_p^2 \to \overline{cov}.$$

Point: For very well diversified portfolios of risky assets, the portfolio risk is determined only by the average covariance of the assets in the portfolio, not by the variances of the assets.

This plot shows how the standard deviation of a portfolio of average NYSE stocks changes as we change the number of stocks in the portfolio.



Summary of Week 7: Optimal Portfolio Choice

1. Portfolios of a risky asset (A) and a riskless asset (f).

Return (realized return): $r_p = xr_A + (1-x)\,r_f$

Expected return: $E(r_p) = xE(r_A) + (1-x)\,r_f = r_f + x\,(E(r_A) - r_f)$

Variance of return: $\sigma_p^2 = x^2\sigma_A^2$

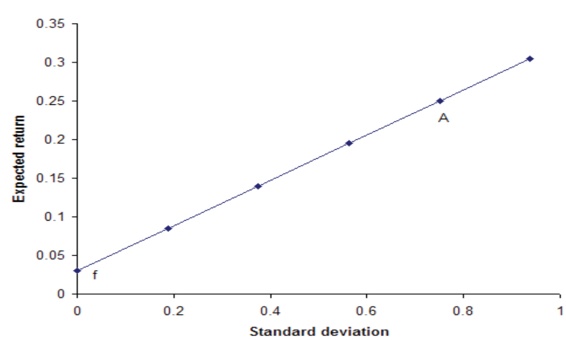
Standard deviation of return: $\sigma_p = x\sigma_A$.

If you are willing to tolerate portfolio risk of σ_p, choose a portfolio share for the risky asset of $x_A = \frac{\sigma_p}{\sigma_A}$. If you do this, your portfolio's expected return will be

$$\begin{aligned} E(r_p) &= r_f + \frac{\sigma_p}{\sigma_A}\,(E(r_A) - r_f) \\ &= r_f + \frac{E(r_A) - r_f}{\sigma_A}\sigma_p : \ \textbf{Capital Allocation Line (CAL)}. \end{aligned}$$

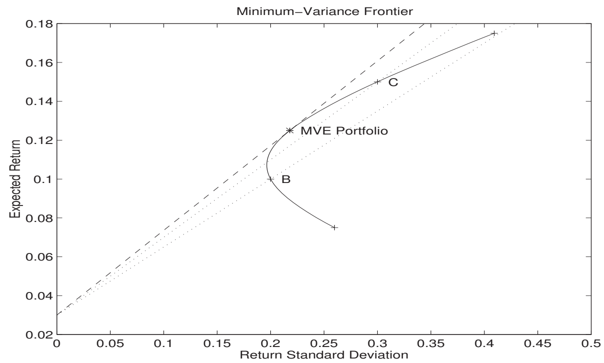
Intercept: The riskless interest rate. Slope: Sharpe ratio of the risky asset.

The CAL gives the expected return on a portfolio invested in a combination of a riskless asset and a risky asset, for each possible chosen value of the portfolio standard deviation.



2. Optimal portfolio choice with a riskless asset and two or more risky assets

- For any choice of risky asset portfolio we can draw the CAL resulting from combining this risky asset portfolio with holdings of the riskless asset.
- The best portfolio of risky assets with which to combine the riskless asset is the one which leads to the steepest CAL (less risk for same expected return/higher expected return for same risk).
- The best portfolio of risky assets is called the **mean-variance efficient (MVE)** portfolio of risky assets, or the tangency portfolio.



- The resulting CAL is the "**optimal CAL**". The expression for the optimal CAL is

$$E\left(r_p\right) = r_f + \frac{E\left(r_{MVE}\right) - r_f}{\sigma_{MVE}} \sigma_p.$$

- **Efficient portfolios** lie on the optimal CAL and combine f and MVE.
- **Punch line: Two funds are enough** - All (smart) investors should choose an efficient portfolio, i.e. a combination of the riskless asset and the MVE portfolio of risky assets. The exact point chosen depends on how **risk averse** the investor is. Investors should control the risk of their portfolio not by reallocating among risky assets, but through the split between risky and riskless assets.

- **Formulas for efficient portfolios:**

  Return (realized return): $r_p = x_{MVE} r_{MVE} + x_f r_f$
  Expected return: $E(r_p) = x_{MVE} \cdot E(r_{MVE}) + x_f \cdot r_f = r_f + x_{MVE} \cdot (E(r_{MVE}) - r_f)$
  Variance: $\sigma_p^2 = x_{MVE}^2 \sigma_{MVE}^2$
  Standard deviation: $\sigma_p = x_{MVE} \sigma_{MVE}$.
- In the case of two risky assets, you can find the MVE portfolio of risky assets using Solver in Excel (see spreadsheet posted for example).
- Adding more risky assets improves the minimum variance frontier of risky assets. We do not solve for the MVE in cases with more than two risky assets, but once you have the MVE portfolio of risky assets, everything works as in the case with two risky assets (and a riskless asset).

3. Equilibrium: Asset prices must adjust to ensure that

MVE Portfolio of Risky Assets=Market Portfolio of Risky Assets.

Week 8: The CAPM

- **Capital market line (CML):**

  When investors invest in a combination of the riskless asset and the market portfolio, their portfolio will be on the **capital market line (CML)**.

  The CML is the capital allocation line CAL for which the risky asset portfolio is the market portfolio. Therefore, on the CML:

$$\begin{aligned} r_p &= r_f + x_m (r_m - r_f) \ , \\ E(r_p) &= r_f + x_m (E(r_m) - r_f) \, , \\ \sigma_p &= x_m \sigma_m \end{aligned}$$

  so, combining these

$$E(r_p) = r_f + \frac{E(r_m) - r_f}{\sigma_m} \sigma_p .$$

- **Capital asset pricing model (CAPM):**

  A model of what the expected return on a security i will be in equilibrium. According to the CAPM

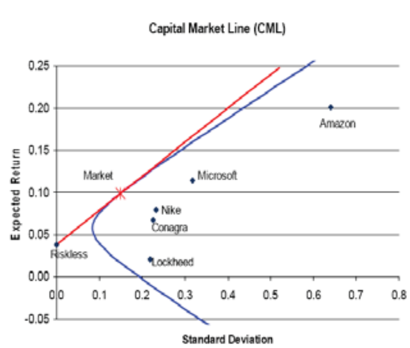
$$E(r_i) = r_f + \beta_i [E(r_m) - r_f] \, , \text{ where } \beta_i = \frac{cov(r_{i,t}, r_{m,t})}{\sigma_m^2} .$$

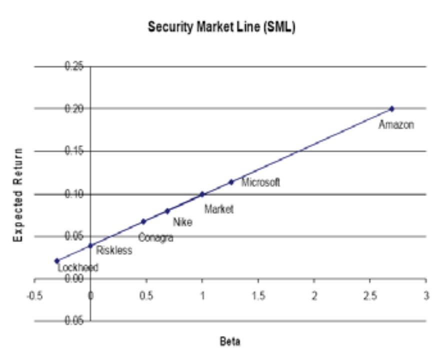
  If the expected return on any asset i differed from the above, the market portfolio would not be mean-variance efficient and the supply of risky assets would not equal the demand for risky assets.

- **Security market line (SML):**

  Illustrates the CAPM by plotting the formula $E(r_i) = r_f + \beta_i [E(r_m) - r_f]$ to show that a security's expected return depends linearly on its β.

- **Comparing the CML and SML:**





- **Interpreting β as a regression coefficient:**

  β is the regression coefficient in the regression

  $$r_{i,t} - r_{f,t} = \alpha_i + \beta_i \left(r_{m,t} - r_{f,t}\right) + \varepsilon_{i,t}$$

  where

  $\beta_i \left(r_{m,t} - r_{f,t}\right)$: **Undiversifiable**=market =systematic component. Has variance $\beta_i^2 \sigma_m^2$

  $\varepsilon_{i,t}$: **Diversifiable**=firm-specific=unsystematic=idiosyncratic component. Has variance $\sigma_{\varepsilon_i}^2$.

  The total return variance for asset i is thus:

  $$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\epsilon_i}^2.$$

  According to the CAPM, $E\left(r_i\right) - r_f = \beta_i \left[E\left(r_m\right) - r_f\right]$, so only the systematic risk driven by β_i affects the expected return ($\sigma_{\varepsilon_i}^2$ doesn't enter the CAPM formula at all). **You don't get paid to take on unsystematic risk.**

  Furthermore, according to the CAPM the regression intercept α_i should be close to zero.

  The **regression** R^2 measures the fraction of total firm i risk σ_i^2 which is due to market risk, i.e. the fraction you get paid to hold:

  $$R^2 = \frac{\text{systematic risk}}{\text{total risk}} = \frac{\beta_i^2 \sigma_m^2}{\sigma_i^2} = \frac{\beta_i^2 \sigma_m^2}{\beta_i^2 \sigma_m^2 + \sigma_{\varepsilon_i}^2}.$$

- **Alpha (α):** The difference between an asset's actual expected return and what something with its beta should return according to the CAPM is called the asset's **alpha**:

  $$\alpha_i = E(r_i) - \left[r_f + \beta_i \left(E(r_m) - r_f\right)\right]$$

  So for an asset with α, the expected return is

  $$E(r_i) = \alpha_i + r_f + \beta_i \left(E(r_m) - r_f\right).$$

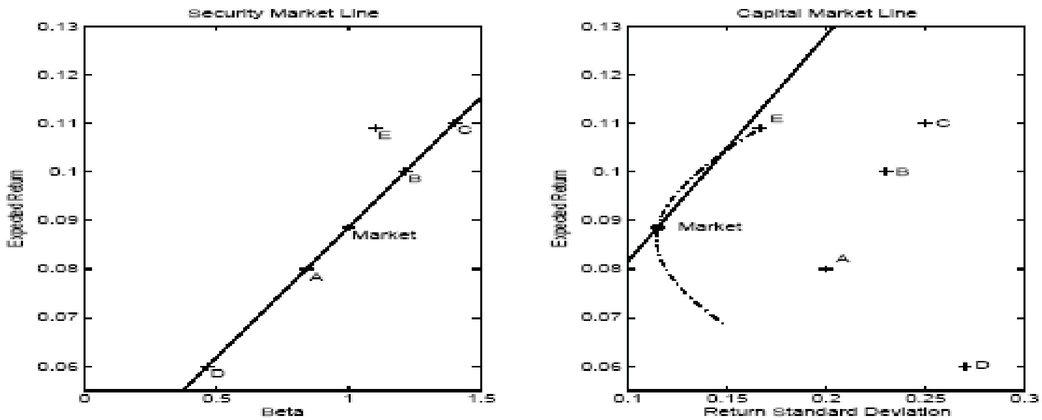
  You estimate the α as:

  $$\alpha_i = \overline{r}_i - (\overline{r_f} + \beta_i [\overline{r}_m - \overline{r_f}]).$$

- **Graph practice:** What do the SML and CML graphs look like if a security has an expected return below or above what the CAPM predicts? (i.e. has negative or positive alpha)

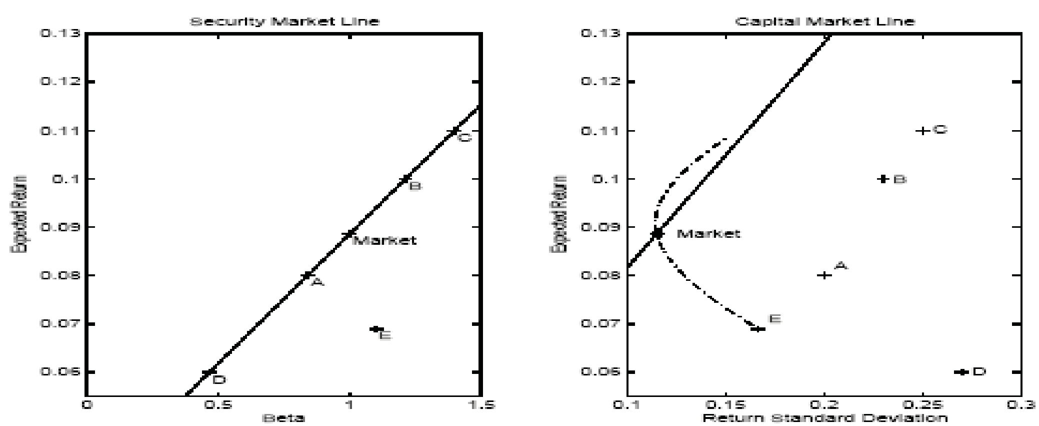
  If $E\left(r_E\right) > r_f + \beta_E \left[E\left(r_m\right) - r_f\right]$ then:

  - $\alpha_E = E\left(r_E\right) - \left(r_f + \beta_E \left[E\left(r_m\right) - r_f\right]\right) > 0.$
  - Asset E plots above the SML.
  - The market portfolio of risky assets is no longer mean-variance efficient. A combination of M and E provides a higher Sharpe ratio. This combination involves a positive portfolio share for E.



If $E(r_E) < r_f + \beta_E [E(r_m) - r_f]$ then:

- $\alpha_E = E(r_E) - (r_f + \beta_E [E(r_m) - r_f]) < 0$.
- Asset E plots below the SML.
- The market portfolio of risky assets is no longer mean-variance efficient. A combination of M and E provides a higher Sharpe ratio. This combination involves a negative portfolio share for E.



- **β of a portfolio of n assets:**

$$\beta_p = x_1\beta_1 + x_2\beta_2 + ... + x_n\beta_n$$

- **Multi factor models:** Add more measures of systematic risk to the regression. The Fama French Carhart 4-factor model is the leading multi-factor model currently being used. Estimate betas in this regression:

$$\begin{aligned} r_{i,t} - r_{f,t} &= \alpha_i + \beta_{i,1}\left(r_{m,t} - r_{f,t}\right) + \beta_{i,2}\left(r_{small,t} - r_{big,t}\right) \\ &\quad + \beta_{i,3}\left(r_{high,t} - r_{low,t}\right) + \beta_{i,4}\left(r_{winners,t} - r_{losers,t}\right) + \varepsilon_{i,t}. \end{aligned}$$

  Then calculate expected returns from:

$$\begin{aligned} E\left(r_i\right) - r_f &= \beta_{i,1}\left[E\left(r_m\right) - r_f\right] + \beta_{i,2}\left[E\left(r_{small}\right) - E\left(r_{big}\right)\right] \\ &\quad + \beta_{i,3}\left[E\left(r_{high}\right) - E\left(r_{low}\right)\right] + \beta_{i,4}\left[E\left(r_{winners}\right) - E\left(r_{losers}\right)\right]. \end{aligned}$$

Week 9: Capital Budgeting Under Uncertainty

The capital budgeting process involves the following steps:

1. Using the **project** beta and the CAPM, calculate the discount rate (cost of capital) for the project.
2. Discount the **expected** cash flows, both inflows and outflows, from the project at the project discount rate, to get the NPV of the project.
3. Accept all positive NPV projects, reject all negative NPV projects.

To get the project beta you will generally:

1. Find a **comparable firm** (or collection of firms) whose business consists of projects similar to yours, and which are publicly traded (include your own firm if the project is similar to your existing business and your firm is publicly traded).
2. Estimate the **equity betas** of these comparable firms using regressions. Also estimate **debt betas** if the firm has enough debt that it is risky.
3. For each firm, calculate the **asset beta** or firm beta, using the equity betas, debt betas, and capital structure:

$$\beta_A = \frac{E}{V}\beta_E + \frac{D}{V}\beta_D.$$

where
$$V = E + D, \quad E/V = \frac{1}{1+D/E}, \quad D/V = \frac{D/E}{1+D/E}.$$

4. Use an **average** of the comparable firms' asset betas as an estimate of your project beta.
5. **Plug it into the CAPM** to get the cost of capital for the project
$$E(r_A) = r_f + \beta_A \left(E(r_m) - r_f\right).$$

Notice also that the cost of capital for the project (or firm) ($= E(r_A)$) can be expressed as a weighted average of the expected return on equity and on debt:
$$E(r_A) = \frac{E}{V}E(r_E) + \frac{D}{V}E(r_D).$$

The beta of a portfolio of assets can come in handy when working with companies that have multiple divisions (or cash holdings):
$$\beta_A = \frac{V_{Div.1}}{V_{Div.1} + V_{Div.2}} \times \beta_{A,Div.1} + \frac{V_{Div.2}}{V_{Div.1} + V_{Div.2}} \times \beta_{A,Div.2}.$$
A similar equation holds for equity:
$$\beta_E = \frac{E_{Div.1}}{E_{Div.1} + E_{Div.2}} \times \beta_{E,Div.1} + \frac{E_{Div.2}}{E_{Div.1} + E_{Div.2}} \times \beta_{E,Div.2}$$
(and for debt).

Week 10: Market Efficiency

Markets are efficient if securities are always properly priced, meaning that
$$P_0 = \Sigma_{t=1}^{\infty} \frac{E\left(C_t\right)}{\left(1+E(r)\right)^t}$$
where r_t is the appropriate discount rate (expected return) for the riskiness of the cash flows.

- Market are thus efficient **if prices reflect information** about cash flows (and cash flow risk). To consider **how** efficient markets are we look at how much information seems to be reflected in prices.
  - **Weak form efficiency:** You cannot make abnormal (risk-adjusted) returns by trading based on past prices.
  - **Semi-strong form efficiency:** You cannot make abnormal (risk-adjusted) returns by trading based on public information.
  - **Strong form efficiency:** You cannot make abnormal (risk-adjusted) returns by trading based on any type of information you could possibly dig up about the company.
- The **evidence** generally supports weak form efficiency and semi-strong form efficiency, but not strong form efficiency.

Tools used in testing market efficiency:

- Calculating a **mutual fund manager's** α: For manager i (or fund i):

$$\alpha_i = \overline{r}_i - (\overline{r_f} + \beta_i[\overline{r}_m - \overline{r_f}]).$$

  α_i should be zero according to the CAPM, so α_i measures how much the manager outperformed the CAPM. A positive, statistically significant α_i is interpreted as investment skill.

- **Event studies:** Can be used to determine if markets react in an efficient way to public information announcements. The steps involved are as follows.

  1. Identify the event dates (the announcement dates).
  2. Label the event date by date "$t = 0$". Collect data from before and after the event date.
  3. Run a regression using data from before the event to estimate the β and α for each stock i in the sample, β_i and α_i
  4. Now calculate each stock's daily **abnormal returns ($\epsilon_{i,t}$)** by subtracting from its daily returns $r_{i,t}$ the return predicted by the regression:

$$\epsilon_{i,t} = r_{i,t} - (\alpha_i + r_{f,t} + \beta_i \times (r_{m,t} - r_{f,t}))$$

  5. Next, construct the **average abnormal return** by averaging across the N firms in the sample:

$$\bar{\epsilon}_t = \frac{\epsilon_{1,t} + \epsilon_{2,t} + \cdots + \epsilon_{N,t}}{N}$$

  6. The **Cumulative Abnormal Return (CAR)** is then defined as the sum of the average abnormal returns up to that point:

$$\text{CAR}_{-6,t} = \bar{\epsilon}_{-6} + \bar{\epsilon}_{-5} + \cdots + \bar{\epsilon}_t$$

  7. **Plot the CAR against event time.**
     - A flat CAR line *after* $t = 0$ is evidence in favor of semi-strong form efficient markets: All news is reflected in the price right after the announcement.
     - A jump in the CAR on the announcement date is evidence against strong form efficient markets: The information announced was not already reflected in the price prior to the announcement.
     - If markets are not strong-form efficient, but are semi-strong form efficient, then we should see a jump in the CAR line on the announcement day as the new information is incorporated into the price.
     - The slope of the CAR line *before* the announcement is not informative about semi-strong form efficiency.