Abstract
When a buying firm delegates the bidding for an asset to a manager whose incentives are partially based on accounting income, the manager’s bid is affected by how accounting values are calculated. If the manager is privately informed about firm-specific synergies, then the accounting regime also affects the seller’s choice of reserve price. Hence the probability that the asset is sold to the buyer with the highest valuation depends on the choice of accounting regime.

This paper studies the informational properties, efficiency and wealth distribution effects of three prominent valuation methods in the context of corporate acquisitions.

The purchase price method (PP) requires the acquirer to book the acquisition at historical cost, thereby underestimating the asset value by the amount of the acquirer’s surplus in the transaction.

The exit-value method (EV) is downward biased and forces the acquirer to book an accounting loss on the date of acquisition. Given that bidders’ incentives may depend on the accounting report, EV connects otherwise independent valuations and induces a reporting-based winner’s curse. Because EV iteratively links a firm’s willingness to pay to the next best buyer’s willingness to pay, it can lead to firesale-like asset values. Relative to PP, EV implies lower asset prices, a decrease (increase) in the surplus of the seller (acquirer) and an increase in the probability that the asset attains its best use when the market is sufficiently competitive.

Finally I examine an alternative method in which the asset is valued at its perceived value-in-use (VU). Under this method, the acquirer bids more aggressively to increase perceived value-in-use, leading to a shift of surplus from the acquirer to the seller. Yet, VU is unbiased and always maximizes the probability that the asset attains its best use.

1 INTRODUCTION
Since Coase first articulated a formal theory of transaction costs, researchers have explored what constitutes the perimeter of the firm, and how transactions occurring within its boundaries may not be well-represented by market arm’s length transactions.

When an asset is acquired by a firm, it ceases to be part of a market price system. An accountant attempting to value the asset therefore faces a paradox, as there are (at least) three natural and
well-defined competing concepts of economic value: (i) the purchase price (i.e., historical cost), (ii) the value at which the firm could potentially resell the asset (i.e., the exit value), or (iii) the value that the firm can extract from the asset (i.e., the value in use). In standard Arrow-Debreu perfectly competitive markets, the law of one price implies that these three values should be identical. However, as the asset crosses the boundaries of the firm, at the time of purchase, the value-in-use should be greater than the price paid for the asset — or else the firm should not have purchased the asset. Likewise, the price paid should be greater than the price the seller would pay to purchase the asset back; otherwise, the seller should not have sold the asset.

In the early twentieth century, the accounting profession adopted the purchase price method, which values an acquisition at the outlay of resources incurred as of the acquisition date. This method has been criticized as backward-looking, even when considering the instant post-acquisition, because it reflects neither the benefits that can be derived from using the asset, nor the price at which the asset could be resold.\(^1\) Despite this criticism, conceptual statements in the 1930’s emphasized the ideas of verifiability and objectivity, and historical cost has been widely used since then.

The exit value method, by contrast, is more forward-looking for it prescribes a valuation based on the net realizable value that the asset would bring in the market if it were sold on the reporting date. Jacob Viner, an early member of the Chicago School of Economics, considered current exit values as the only proper basis for accounting. Skeptics of the exit-value method have, in contrast, emphasized its non-observable nature and implied lack of objectivity, claiming that while the exit value is a meaningful metric, it is also difficult to audit. Recently, both GAAP and IFRS have begun to depart from the long-standing tradition of historical cost, emphasizing, instead, the use of exit values (FAS-157 and IAS-39). Although the notion of exit values is considered modern, the use of exit values was not unusual a century ago (see Matheson, 1884).

Finally, the value-in-use method requires valuing an asset at the discounted value of the asset’s future cash flows. The idea that the appropriate value for an asset is the discounted stream of the future net revenues was strongly advocated by Irving Fisher (1897, 1930) who argued that this criterion correctly measures the creation of value, i.e., the economic definition of income. This

\(^1\)“It is, of course, unlikely that balance sheets will be drawn up in the indicated manner; this is a matter for the future. But it is clear that present balance sheets already contain an element of expectation and speculation.” Morgenstern (1963).
criterion was later modernized by Gerard Debreu (Theory of Value, 1959) who argued that firms maximize the value-in-use of their assets. However, some accountants have opposed the adoption of this criterion, arguing that the future stream of cash flows is uncertain and its estimation necessarily unreliable.\(^2\)

This paper proposes an analytical framework to understand the economic effects of alternative accounting methods in the context of corporate acquisitions. In the model, a seller (target) offers an asset for sale to a pool of potential buyers. The seller sells the asset in a first-price sealed bid auction, publicly announcing a minimum starting price and then collecting sealed bids from prospective buyers.\(^3\) A buyer does not directly bid for the asset but delegates the bidding to an agent (the manager). The manager, being privately informed about buyer-specific synergies, faces a tension between maximizing the buyer’s surplus and reporting high accounting income when deciding how much to bid. Yet, the manager must report any transaction in compliance with the prevailing accounting standard. In this context, I study three accounting standards: the purchase price method (PP), the exit-value method (EV) and the perceived value-in-use method (VU). I examine the effects of these methods on bidders’ incentives to purchase an asset and the seller’s incentive to sell the asset. Then, I derive which accounting method leads to the most efficient allocation of the asset, and I explore how accounting methods may affect the allocation of the trading surplus between buyers, managers and sellers.

The analysis of each method yields the following conclusions. The purchase price method underestimates the acquisition value to the buyer by the amount of the acquirer’s surplus in the transaction because this method prescribes reporting the acquisition value at its historical cost. This underestimation implies that the manager must report no income on the acquisition and, as a result, that the bidding incentives of buyers and managers get perfectly aligned.

The exit-value method forces the acquirer to book an accounting loss on the date of acquisition, because the expected resale price is below its purchase price. Since the resale price depends on rival-bidders’ valuations, and particularly on the valuation of unsuccessful bidders, this method con-

\(^2\)"The non-availability of the future series of data, except for certain fragmentary items attaching to the near future, not only prevents the systematic development of realized income statistics to the point of large usefulness but prevents also a full development of capital valuation. For without reliable estimates of all future series to be discounted, reliable present valuations are impossible." Canning (1929).

\(^3\)In a study of 400 takeovers during the 1990s, Boone and Mulherin (2007) find that half of the target firms are sold in auctions with multiple-bidders and the rest are sold in one-on-one negotiations.
nects otherwise independent valuations and induces a reporting-based winner’s curse. Furthermore, accounting reports reflect fire-sale values because this method iteratively links a firm’s willingness to pay to the next best buyer’s willingness to pay.

I finally examine an alternative method in which the asset value is reported at its expected value-in-use conditional on the purchase price. Under VU, the manager reports the perceived surplus on the acquisition as an accounting gain. Managers overbid in a failed attempt to affect the “market perception” of the acquisition’s surplus.

When these three methods are compared, the paper demonstrates that VU is the most efficient regime as measured by the probability that the asset reaches its best use. An accounting regime affects this probability in two manners: (1) it alters the way bidders’ perceive the benefits from acquiring the asset, sometimes discouraging efficient transactions because bidders anticipate the transaction would force them to report large accounting losses. (2) An accounting regime also affects how effective is the seller’s monopolistic tendency to charge a high reserve price as a means to induce more aggressive bidding behavior. In these two respects, VU is the most appealing method. In equilibrium, VU aligns bidder’s perceptions of the benefits of acquiring the asset with the asset’s economic value, thereby inducing the right incentives for bidders to participate in the auction. Second, since under VU the accounting implications from the acquisition are in equilibrium independent of the seller’s reserve price, bidding behavior becomes less sensitive to the reserve price thereby reducing seller’s incentives to resort on the reserve price.

We also show that PP is the least efficient method when the seller’s own valuation is low and/or the market is relatively competitive, because under this method the seller has generally the greatest incentives to set the highest reserve price and inefficiently retain the asset. Since under PP the accounting valuation of the acquisition is given by the purchase price, and since the purchase price normally depends on the reserve price, the seller can affect bidder’s willingness to bid also through the impact that the reserve price has on the accounting report.

Second, the paper demonstrates that VU (EV) maximizes (minimizes) expected seller revenues. VU induces the highest revenues because managers tend to overbid under VU, driven by the accounting gains they are allowed to report. By contrast, EV induces the lowest expected revenues because managers tend to underbid in response to the accounting loss they must report in equilibrium. Moreover, the sensitivity of expected revenues with respect to accounting incentives
drastically vary across regimes. In fact, I show that the relative accounting power of the manager’s incentives (a) does not affect expected revenues under PP, (b) increases expected revenues under VU, and (c) decreases expected revenues under EV.

Third, the analysis of wealth distribution effects shows that, unlike sellers and buyers, managers’ preferences for accounting regimes perfectly align with social efficiency: a manager always prefers the regime that maximizes the probability of trade. This is surprising, given that managers’ reporting concerns drive the differences of efficiency among regimes. Ex-ante, the manager is exclusively concerned by efficiency because all his reporting incentives are ultimately absorbed by the seller, either by higher prices when reporting concerns induce overbidding, or by lower prices when the opposite is true. In contrast, both the seller and the buyer face a tension between the expected prices and the probability of trade induced by an accounting regime.

Fourth, the paper shows that, although EV and VU prescribe reporting estimates rather than observations, the three regimes are equally informative. In fact, the acquirer’s private information can always be perfectly inferred from the accounting report irrespective of which regime prevails. Thus, the choice of accounting method does not pose a “relevance-reliability” trade-off. Yet, both EV and PP underestimate the asset value. Although this type of bias is usually part of standard-setters’ concerns, this paper shows that a greater underestimation may be beneficial for social efficiency when it corrects the sellers’ monopolistic tendency to withhold the asset.

1.1 INSTITUTIONAL BACKGROUND

This paper studies valuation standards at a given point in time, in particular at initial recognition. In practice, accounting valuations are time-dependent. For instance, the U.S. accounting standard on business acquisitions, FAS-141(R), prescribes a valuation method at initial recognition and a different method to be applied afterwards. In particular, FAS-141(R) requires the buyer to use the purchase price method to report the acquisition at initial recognition (except in the case of a bargain purchase). At this date, the buyer must identify the target’s net assets and then allocate the excess of the price paid over the “fair value” of these assets to an intangible asset called goodwill. While the target’s identifiable net assets are, in general, subject to fair value revaluations, goodwill

\[4\] The accounting treatment is asymmetric for a “bargain purchase,” namely, when the purchase price is lower than the fair value of the target’s identifiable net assets. In this case, the buyer reports a gain on this price differential and books the acquisition at the fair value of its identifiable net assets.
is, by contrast, subject to periodic impairment tests. So the revaluation method is not only time-dependent but also contingent on whether or not the asset is tangible.

Under GAAP, the revaluation method also depends on the availability of public information and on market conditions. In fact, FAS-157 defines fair value essentially as an exit price and prescribes revaluations using (i) quoted prices for identical assets in liquid markets if they are available, otherwise (ii) an estimate based on public information and finally, only in the absence of public “observables,” (iii) an estimate based on firm-specific information.

1.2 ACQUISITIONS: SOME STYLIZED FACTS

Acquisitions are perhaps the most important means by which firms grow. From January 1980 to January 2000, 70,000 mergers and acquisitions were completed worldwide, for a value close to $9 trillion. While the 1980s accounted for about one-fifth of the total number and value, the 1990s accounted for four-fifths (Bradley and Sundaran, 2006).

The most detailed study on how firms are sold is Boone and Mulherin (2007). In their sample, including 400 acquisitions undertaken by U.S. public firms during the 1990s, and representing over $1 trillion, the authors find that 50% of the targets choose an auction as the selling mechanism and the rest choose a one-on-one negotiation. The authors describe the acquisition as a five step process: (1) a selling firm hires an investment banker and considers the number of potential bidders to contact, (2) the contacted bidders sign confidentiality/standstill agreements, whereby the bidders receive non-public information but agree not to make an unsolicited bid, (3) those signing agreements submit preliminary indications of interest, (4) a subset of the bidders indicating preliminary interest submit binding sealed offers, and (5) the winning bidder is determined.

However, corporate auctions are often informal mechanisms without clearly specified rules, unlike the textbook version of an auction. Yet, Boone and Mulherin (2009) show that sellers tend to specify some commitment rules. For example, the seller signs a no-shop provision that prohibits the seller from further soliciting offers from other potential bidders. Using information from SEC filings, the authors find that 91% of the deals have termination fees and/or lock-up stock options that compensate original bidders if the target is acquired by other companies. In such cases, the average size of the fees as a percentage of deal value is close to 3%. These commitment devices raise the probability of success of the auction, as bidders elicit their valuations without the fear of
being held-up ex-post by the seller.

The present paper studies determinants of the failure rate of acquisition attempts. The empirical evidence on the magnitude of this rate is sparse. First, the actual rate of failed sales is much higher than reported publicly. Some firms attempt a sale, but do not find a buyer or do not get a high enough price. Unfortunately, the only way to typically know about these failed sales is if the firm is taken over in a subsequent attempt, as the merger background might discuss prior attempts at selling the firm. Second, the rate varies drastically across samples and studies. Boone and Mulherin (2007) for instance report a rate of 6% in a sample relatively dominated by large takeover targets. At the other extreme, Bradley et al. (1983), using a sample of 371 acquisitions from 1958-1980 report a failure rate close to 35%. Perhaps the most precise estimate is given by Coates and Subramanian (2000): they consider 3,254 acquisition events and find that the probability of failure approaches 15% of the attempts.

The present paper also studies wealth distribution effects, which are perhaps the main concern of the empirical literature. In fact, a vast part of the empirical literature centers on the allocation of wealth between buyers and sellers. Some results are the following. Bradley et al. (1987), studying a sample of 236 acquisitions, from 1963-1984, find that acquisitions increase the combined wealth of the stockholders of both the target and acquiring firm by 7.4%. Conventional wisdom, nevertheless, believes that the allocation of these gains between the target and acquirer is uneven: on average, target shareholders would realize significant gains, while acquirer shareholders would suffer slight losses. The most recent and comprehensive study challenges this view: Bradley and Sundaran (2006) study a sample of 12,476 acquisitions, undertaken by 4,116 U.S. firms publicly listed on the NYSE, ASE, or NASDAQ during the years 1990-2000. They find that, in the aggregate, the excess market value change in the acquirers’ stock price during the five days surrounding acquisition announcements is $110 billion, or about 4% of the value of all targets acquired. However, these gains depend on the organizational form of the target: acquirers lose 5.7% of the value of public targets, but they gain 26.1% of the aggregate value of all non-public targets acquired. The reason for this difference is an open question.

\footnote{I thank Audra Boone for this insight.}
1.3 LITERATURE REVIEW

This paper relates to the literature on the economic consequences of accounting standards that originated with Demski (1973) who demonstrated the general impossibility of a complete normative ranking of accounting standards. A subsequent strand of the literature, particularly starting with Kanodia (1980), studies the efficiency effects of accounting standards on firms’ investment decisions (Kanodia, Sapra and Venugopalan 2004; Dye and Sridhar 2008). A parallel strand studies the effects of accounting standards on the efficiency of contracting outcomes (Magee 1978; Gigler, Kanodia, Sapra and Venugopalan 2009). A third strand emphasizes the trade-off between information quality and cost that is implicit in the choice of accounting standards (Bachar et al. 1997; Kirschenheiter 1997; Demski et al. 2009). All three strands show, in a variety of settings, that more informative accounting standards may be detrimental to economic efficiency. Like Plantin, Sapra and Shin (2008), the present paper centers on the effects of standards on the efficiency of trade. While their paper studies the role of market revaluations in inducing inefficient sales of assets by a group of homogenous sellers, my model examines the role of three accounting standards in facilitating the acquisition of a single asset among heterogenous buyers. The main reason for studying standards in the context of acquisitions stems from the relevance of acquisitions as a source of economic growth. Arguably, acquisitions are firms’ main source of growth (Bradley, Desai and Kim 1988; Bradley and Sundaram 2006).

The extant literature views accounting standards as prescribing the disclosure of a noisy version of the information that managers privately observe. For instance, Demski, Lin and Sappington (2009) examine lower-tail revaluation policies. Others, such as Dye (2002), consider standards that mandate a dichotomic classification of the information, e.g., “good” or “bad news”. Part of the literature considers the case in which the noise of the reporting signal arises endogenously either from managerial manipulation or from some form of managerial discretion in the choice of reporting procedure. For instance, Fishman and Hagerty (1990) study a setting in which the manager observes a vector of signals but must choose to disclose only one among a subset of the original signals. They show that limiting discretion, namely the number of signals among which the manager can choose, may increase the informativeness of a disclosure. I adopt a somewhat different approach. I study standards as being general valuation principles that must be based on verifiable information. These principles may rely on direct observation or prescribe an estimate.
When the principle prescribes an estimate, the equilibrium determines the accounting valuation.

The present paper is the first using auction theory to evaluate accounting standards. Auctions provide a general framework in which a perfectly competitive market is only a particular (limit) case. With few exceptions [Dye and Sridhar 2008; Plantin, Sapra and Shin 2008] the literature on accounting standards relies on the assumption of perfectly competitive capital markets. The present paper relaxes this assumption in order to understand how the performance of actual accounting principles is affected by the organization of the capital market. Insofar as the properties of an accounting standard may depend on how competitive the market is, allowing for the possibility of imperfectly competitive capital markets is important.

The paper proceeds as follows. The model and the three accounting regimes are presented in section 2. Section 3 solves the model for each of these regimes and section 4 compares their properties. Finally, section 5 summarizes the central findings and discusses potential extensions. An appendix contains proofs of the main results.

2 THE MODEL

An overview of the sequence of events of the model is this: an individual who owns an asset ("the seller") offers the asset for sale in an auction to N prospective buyers. The highest bidder wins the asset for the price he bids (i.e., this is a first price auction ("FPA")) provided that bid exceeds the seller's publicly announced reservation price; if not, the seller retains ownership of the asset. Each bidder bids without knowing other bidders' bids (i.e., this is a sealed bid auction). These buyers have distinct uses for the asset and, as a consequence, have different (private) assessments of the value of owning the asset.

The buyers are indexed by \( i \in \{1, 2, \ldots, N\} \). Buyer \( i 's \) value-in-use from purchasing the asset is \( x_i \), which is the realization of the random variable \( X_i \). The set of random variables \( \{X_i\}_{i=1}^N \) is identically and independently distributed (iid), with (common) distribution \( F(\cdot) \), density \( f(x) \), support \([0, w]\), and finite mean. \( x_i \) is private information of buyer \( i \), but the distribution \( F(\cdot) \) is common knowledge. The seller's value to retaining the asset is \( x_0 \in (0, w) \).

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7While we take the form of the auction (first price and sealed bid) as exogenous, there are many reasons to study such auctions: for example, they are commonly used in practice, they are simple, they are insensitive to manipulation (e.g., "bids from the "chandelier"), etc.
If a bidder with value-in-use $x_i$ wins the auction with the bid/price $p$, the bidder’s surplus from the purchase is $x_i - p$; losing bidders obtain no surplus from participating in the auction. In the analysis that follows, we assume that the winning bidder’s utility from purchasing the asset is not determined just by the surplus he derives from the purchase, but also depends on whether the accounting rules in place require the buyer to recognize a gain or loss on the purchase. Specifically, I assume there is a constant $\delta \in [0, 1]$ such that when bidder $i$ has value-in-use for the asset $x_i$ and he purchases the asset at price $p$ but the prevailing accounting rule prescribes to book the acquisition at $R^A(p)$ then the bidder’s utility is given by:

$$\delta (R^A(p) - p) + (1 - \delta) (x_i - p).$$

The amount $R^A(p) - p$ is the accounting gain/loss the buyer recognizes upon the purchase when accounting rule $A$ is in effect. This accounting loss/gain may not necessarily be recognized at the time of the acquisition but perhaps in a subsequent period.

With historical cost accounting, $R^A(p) = p$, in which case the buyer’s utility is determined just by his surplus from the purchase. If, alternatively, accounting allowed estimates of the buyer’s gain to be recognized at the time of acquisition of the asset, then in this representation both the booked gain and the buyer’s actual surplus from the purchase would affect the buyer’s utility from the purchase. If accounting rules required the buyer to book a loss at the time of acquisition of the asset, then - consistent with this representation - both this loss and the "real" surplus from purchasing the asset would presumably affect the bidder’s utility from purchasing the asset.

This representation of the bidder’s utility admits several alternative interpretations. For instance, if the buyer is a firm, and the asset being purchased is one of the firm’s principal assets, then were the owner of the firm to sell the firm at a later date, the amount the firm sells for at this later date may depend in part on how much accounting income the firm reported in the past. Under this interpretation, the weights $\delta$ and $1 - \delta$ in the bidder’s utility function reflect the owner’s relative assessments of the contributions of the booked gain to the firm’s subsequent selling price and the value of the surplus to the firm of the asset’s purchase up to the point of sale.

Alternatively, if the buyer is a firm, with the owners of the firm delegating the purchase decision

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8Note also that in our setting there is no ex-post uncertainty, so that goodwill impairments are not an issue. In a companion paper we are addressing the issue of ex-post uncertainty and goodwill impairments.
to the firm’s manager, the manager’s compensation may depend in part on whatever accounting
income is recorded at the time of purchase. Under this interpretation, the weights \( \delta \) and \( 1 - \delta \)
could reflect how the accounting gain and the firm’s surplus from the purchase differentially affect
the present value of the manager’s compensation. (In the following, to add context to our results,
we often describe results using this second interpretation of the model.) As a third (and related)
interpretation, a firm with accounting-based debt covenants or other accounting-based contracts
could find that an asset acquisition has value not simply because it generates surplus for the firm
but also because it slackens debt covenants or otherwise favorably affects the firm’s accounting-
based contracts. In this interpretation, the weights \( \delta \) and \( 1 - \delta \) reflect the relative value to the firm
of these accounting and "real" effects of the purchase.

Before leaving this discussion of the utility function of bidders, we wish to also note that this
utility function implicitly takes all bidders to be risk-neutral, and it focuses on the influence of rules
that affect the accounting for the asset at the time the asset is acquired, rather than accounting
rules, such as depreciation policies, that affect the accounting for the asset over time. (In future
work, we intend to study dynamic issues related to accounting valuations too.)

2.1 THREE ACCOUNTING REGIMES

We study three accounting regimes, each founded on a different valuation principle, that differentially
affect the “accounting portion” of the buyer’s/reporting entity’s utility from purchasing the asset.
Switching between these regimes obviously has the direct effect of changing the winning bidder’s utility from acquiring the asset at a given purchased price. But, as we show below, it also
has the indirect effect of altering each bidder’s bidding behavior and hence affects the equilibrium
distribution of the winning bid/asset’s purchase price.

The accounting principles are:

a) **Purchase Price (PP):** under this principle, the acquisition of an asset must be reported at
its purchase price. The accounting value of the acquired asset, as of the acquisition date, is simply
the price paid by the acquirer:

\[
R^{PP}(p) \equiv p.
\]

GAAP requires this method for valuing nonfinancial assets and also, under some circumstances,
for valuing a business acquisition. In fact, the accounting standard governing business acquisitions
in the U.S., FAS-141(R), requires the buyer to use the purchase-price method to report the acquired firm’s acquisition value (except in the case of a bargain purchase). The requirement of using the purchase-price method is, indeed, implicit in the way goodwill has to be calculated under FAS-141 (R):

The acquirer shall recognize goodwill as of the acquisition date, measured as the excess of the consideration transferred [...] over the net of the acquisition-date amounts of the identifiable assets acquired and the liabilities assumed (FAS-141 (R), page 11).

One important feature of purchase price accounting this regime is its simplicity and auditability: this rule mandates reporting the transaction price. Clearly, it does not require the use of any estimates for its implementation.

b) Exit Value (EV): under his principle, the acquisition of an asset must be reported value at the price he would obtain were he to resell the asset on the reporting date. This principle is consistent with what the accounting standard on fair value measurements, FAS-157, defines as the fair value of a transaction:

Fair value is the price that would be received to sell an asset [...] in an orderly transaction between market participants at the measurement date (FAS-157, pp. 6).

This valuation standard applies especially to financial assets, but also to a business acquisition in the case of a bargain purchase, namely, when the purchase price of the acquisition is lower than the fair value of the target’s identifiable net assets.

In the context we are studying, where the asset is initially sold to one of a group of N bidders in an auction, it is natural to consider the asset’s exit value to be the hypothetical price the winning bidder would receive were he turn around and sell the asset to one of the N – 1 bidders who failed to win the asset in the original auction. If we let \( V_{N-1} (p) \) denote the expected resale price obtained from reselling the asset (after the initial auction took place) to the N – 1 loosing bidders in the initial auction when it is common knowledge that the winning bidder in the original auction paid

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9 As is well known, at the acquisition date the buyer must, first, identify the target’s net assets and then allocate the excess of the price paid over the fair value of these assets to goodwill. Thus, the acquisition is reported at is purchase price, although decomposed into tangible and intangible assets. This distinction is not a concern of the present paper.
For the asset, we define the exit value price by:

\[ R^{EV}(p) \equiv V_{N-1}(p). \]

One contrast between the PP method and the EV method is that the latter requires the use of an estimate, because the hypothetical resale price is not directly observable.

c) Value in Use (VU): under this principle, the asset’s reported value is given by an assessment of the asset’s discounted cash flows to the purchaser of the asset. We take this “assessment” to be an appraisal of the asset’s value based on the winning bid:

\[ R^{VU}(p) \equiv \mathbb{E}(X|p). \]

For its implementation, this rule requires that auditors (or whoever verifies the asset’s value) formulate a model (in this case, model the bidders’ bidding rules) in order to construct the assessment \( \mathbb{E}(X|p) \) from knowledge of the winning bid \( p \). In the absence of a liquid market for the asset, FAS-157 implicitly accepts this method as a possible valuation approach:

The income approach uses valuation techniques to convert future amounts (for example, cash flows or earnings) to a single present amount (discounted). The measurement is based on the value indicated by current market expectations about those future amounts. Those valuation techniques include present value techniques; option-pricing models, such as the Black- Scholes-Merton formula (a closed-form model) and a binomial model (a lattice model), which incorporate present value techniques; and the multiperiod excess earnings method, which is used to measure the fair value of certain intangible assets (FAS-157, page 9).

3 ANALYSIS

3.1 PRELIMINARIES

In this section, I define and study the equilibrium of the first-price auction (FPA) with a public reserve price under a generic accounting rule \( R^A(\cdot) \). I restrict attention to regular accounting rules, which I define as follows.

Assumption 1 (Regularity) An accounting rule \( R^A(b) \) is regular if it satisfies:
(i) $R^A(b)$ is differentiable a.e.

(ii) $R^A(b)$ is weakly increasing in $b$.

(iii) \( \frac{\partial [\delta R^A(b) - b]}{\partial b} \geq 0 \)

Condition (i) excludes discontinuous accounting rules that establish partitions or prescribe classifications such as “going concern versus not going concern” or “capital versus operating leases” etc. (see Dye, 2002). Condition (ii) is natural and says that the recorded value of the acquired asset must be non decreasing in the amount paid. Condition (iii) imposes a restriction on the speed at which accounting values can increase as a function of the purchase price. This restriction rules out situations in which bidders’ payoffs increase in the amount they pay due to their accounting incentives.

Before presenting the definition of equilibrium, we make a few preliminary observations and notational conventions. When accounting rule $R^A(\cdot)$ is in effect, we denote the seller’s reserve price as $r^A$. Bids below $r^A$ will not be accepted by the seller. When bidder $i$ has value-in-use $x_i$ for the asset and accounting rule $R^A(\cdot)$ is in effect, bidder $i$’s bid is denoted by $b_i = \beta^A_i(x_i, r^A)$, and bidder $i$’s bidding function is denoted by $\beta^A_i(\cdot)$. In the following, we confine the study to symmetric bidding strategies, i.e., where $\beta^A_i(\cdot)$ is the same for all $i$. This restriction is natural, since the bidders’ values-in-use $X_i$ are all iid. By confining attention to the symmetric cases, we can sometimes simplify the notation and write a bidder’s bidding strategy as $\beta^A(\cdot)$.

We consider the bidder’s search for an optimal bidding strategy as a two-step process. First, after the seller’s announcement of the auction’s reserve price $r^A$, and after the bidder’s observation of his own value in use for the asset, the bidder calculates his optimal bid to submit were he to participate in the auction, taking the seller’s reserve price and his expectation of what bidding strategies other bidders will adopt as given. Second, the bidder can calculate his expected utility from submitting that bid. If his expected utility is lower than what he would receive by staying out of the auction, he submits no bid; otherwise, he bids his optimal bid.

The bidder’s participation decision is determined by whether he gets non negative expected utility from participating in the auction by submitting a bid equal to the reserve price $r^A$. The threshold value $x^A$ for a bidder’s value in use which determines whether the bidder participates in the auction is given by:

\[
(1 - \delta) x^A + \delta R(r^A) - r^A = 0
\]
We refer to the bidder with private value \(x^A\) as the marginal bidder; this bidder, by definition, would exactly break even if he acquired the asset by paying the reserve price. Furthermore, the reserve price is the optimal bid for the marginal bidder when the accounting rule \(R(\cdot)\) is regular (and satisfies \(\frac{\partial[R(b) - b]}{\partial b} \leq 0\)).

The second step in a bidder’s decision problem is to select how much to bid. If bidder, say \(i\), submits a sealed bid of \(b\) when his private value is \(x_i\), then his payoff is

\[
\begin{cases}
\delta (R^A(b) - b) + (1 - \delta)(x_i - b) & \text{if } b > \max_{j \neq i} b_j \\
0 & \text{if } b < \max_j b_j
\end{cases}
\]

under accounting regime \(R^A(\cdot)\). Suppose that all bidders \(j \neq i\) use bidding strategy \(A(\cdot)\), where \(A(\cdot)\) is increasing. Denote by \(Y_i^{N-1} = \max_{j \neq i} X_j\) the maximum private value among bidder \(i\)’s \(N - 1\) opponents. Then, bidder \(i\) wins the auction with bid \(b\) if and only if \(b \geq A(Y_i^{N-1}, r^A)\) or equivalently whenever \(Y_i^{N-1} \leq A^{-1}(b, r^A)\). Note that bidder \(i\) anticipates that his opponents’ bids may depend on the \(r^A\). In the following I may sometimes omit the dependence of \(A(\cdot)\) on \(r^A\) or I may alternatively write this dependence in terms of \(x^A\) as \(A(Y_i^{N-1}, x^A)\).

Let \(G(\cdot)\) and \(g(\cdot)\) be the distribution and density of \(Y_i^{N-1}\). Then \(A(\cdot)\) is an equilibrium bidding strategy if for each bidder \(i = 1, \ldots, N\) and each \(x_i \geq x^A\), \(A(\cdot)\) satisfies

\[
A(x_i, r^A) \in \arg \max_{b \geq r^A} \left[\delta R^A(b) + (1 - \delta)x_i - b\right] G(Y_i^{N-1} < A^{-1}(b, r^A)).
\]

(3)

In formulating their bids, bidders must trade off the prospect of getting the asset at a low price against the probability of increasing their chances of winning the asset. Condition (3) requires that if, say, bidder \(i\) believes that all bidders other than bidder \(i\) adopt the bidding strategy \(A(\cdot)\) – and so, in particular if bidder \(j \neq i\) bids \(A(x_j, r^A)\) for the asset when he would get value in use \(x_j\) for it– then, it is optimal for bidder \(i\) also to use bidding strategy \(A(\cdot)\). In other words, \(A(\cdot)\) has to be a fixed point of the reaction function describing each bidder’s optimal bidding function taking as given other bidders’ (common) bidding functions.

To complete the specification of an equilibrium we must describe, for both the EV and VU regimes, how the accounting rule is operationalized in equilibrium. No such description is necessary.
for the PP rule, as it does not depend on any inferences for its implementation. The following definition of equilibrium explicitly embeds the accounting rules’ implementation in its specification (in part (iii)).

**Definition 1** A symmetric equilibrium when accounting rule \( R^A(\cdot) \) is in effect, consists of a bidding function \( \beta^A(\cdot) \) and a reserve price \( r^A \) such that:

(i) Given \( r^A \) and \( R^A(\cdot) \), for each \( i = 1, \ldots, N \) and each \( x \in [0, w] \) the bidding function \( \beta^A(\cdot) \) satisfies

\[
\beta^A(x_i, r^A) \in \arg \max_{b \geq r^A} u(x_i, b) \equiv [(1 - \delta) x_i + \delta R^A(b) - b] \Pr \left[ \beta^A \left( Y_i^{N-1}, r^A \right) \leq b \right]
\]

(4)

if \( u(x_i, \beta^A(x_i, r^A)) \) \( \geq 0 \), otherwise \( \beta^A(x_i, r^A) = 0 \), where \( Y_i^{N-1} \equiv \max \{ X_1, X_2, \ldots, X_{i-1}, X_{i+1}, \ldots, X_N \} \).

(ii) Given \( \beta^A(\cdot) \), and \( R^A(\cdot) \) then the optimal reserve price \( r^A \) maximizes

\[
r^A \in \arg \max_r \mathbb{E} \left[ \left( \beta^A (Y_i^{N}, r) - x_0 \right) \cdot \Pr \{ \beta^A (Y_i^{N}, r) \geq r \} \right]
\]

where \( Y_i^{N} \equiv \max \{ X_1, \ldots, X_N \} \).

(iii) Given \( \beta^A(\cdot) \), and \( r^A \), the accounting rule \( R^A(\cdot) \) is consistent with regime \( A \). In particular, for all \( b \):

(a) if \( A = PP \), then

\[
R^{PP}(b) = b;
\]

(b) if \( A = EV \), then

\[
R^{EV}(b) = V_{N-1}(b);
\]

where \( V_{N-1}(b) \) is an estimate of the resale price (to be defined later).

(c) if \( A = VU \), then

\[
R^{VU}(b) = \mathbb{E} \left[ X_i | \beta^{VU}(X_i, r^A) = b \right].
\]

The definition’s principal features are the following. Part (i) describes the requirements for the bidding function \( \beta^A(\cdot) \) to be part of the equilibrium. Part (ii) stipulates that the seller maximizes expected revenues when choosing the auction’s reserve price \( r^A \). As was noted previously, the

---

\(^{11}\) In practice some accounting standards implicitly refer to an equilibrium notion: for instance the Black and Scholes formula which has been adopted by some accounting standards (FAS-157) is based on the equilibrium principle of no-arbitrage opportunities.

\(^{12}\) To simplify the statement of the definition I will subsume the participation strategy into the bidding strategy, and represent the decision to stay out of the auction by a bid of zero.
choice of reserve price $r^A$ determines the participation threshold $x^A$, consisting of the minimum value-in-use a bidder has to realize in order to be willing to participate in the auction. Part (iii) requires that the inference used to implement the accounting rule is consistent with the bidders’ equilibrium bidding strategies. In both the VU and EV accounting regimes, the purchase price contains information that has to be “unpacked” by understanding the player’s equilibrium strategies under these regimes. More precisely, for EV and VU, the rule is defined as the appropriate Bayesian inference based on the asset’s purchase price.

In the following, I start the analysis by describing the equilibrium bidding strategy $\beta^A(\cdot)$ for a generic and regular accounting rule $R^A(\cdot)$ and an arbitrary reserve price $r^A$. For $\beta^A(\cdot)$ to be a Nash equilibrium, it is necessary that $b = \beta^A(x)$ be a best response for bidder $i$ when bidders $j \neq i$ adopt strategy $\beta^A(\cdot)$. Thus if $b = \beta^A(x)$ maximizes the bidder’s expected utility, as described in (3), it must satisfy the following first order condition:

$$\left[\delta \left( R^A(b) - 1 \right) - (1 - \delta) \right] G (\beta^{-1}_A(b)) + \frac{g \left( \beta^{-1}_A(b) \right)}{\beta'_A (\beta^{-1}_A(b))} \left[ \delta \left( R^A(b) - b \right) + (1 - \delta) (x - b) \right] = 0. \tag{5}$$

After some algebraic manipulations the first order condition (5) yields the following differential equation:

$$\frac{d \left[ G (x) \beta^A(x) \right]}{dx} = (1 - \delta) x g(x) + \delta \frac{d \left[ R^A (\beta^A(x)) \right] G (x)}{dx} \tag{6}$$

Further, since there is a strictly increasing relation between $r^A$ and $x^A$, we can think of the seller choosing the participation threshold, rather than choosing the reserve price. In solving the problem, we will exploit the equivalence between these two choices.

We can differentiate bidder $i$'s utility function,

$$u(b, x) = \left[ \delta \left( R^A(b) - b \right) + (1 - \delta) (x - b) \right] G (\beta^{-1}_A(b)),$$

with respect to $b$ to obtain the following F.O.C:

$$\left[\delta \left( R^A(b) - 1 \right) - (1 - \delta) \right] G (\beta^{-1}_A(b)) + \frac{g \left( \beta^{-1}_A(b) \right)}{\beta'_A (\beta^{-1}_A(b))} \left[ \delta \left( R^A(b) - b \right) + (1 - \delta) (x - b) \right] = 0$$

where I have used the inverse function theorem. Note that, for convenience, I switched the placement of the superscript $A$, writing $\beta^A(x)$ but $\beta^{-1}_A(b)$ to denote the inverse of $\beta^A(x)$. I also write $\beta'_A(x)$ to denote the partial derivative of $\beta_A(x)$ with respect to $x$. In a symmetric equilibrium, $b = \beta^A(x)$, and thus the above equation yields the following differential equation

$$\left[\delta \left( R'_A (\beta^A(x)) - 1 \right) - (1 - \delta) \right] G(x) + \frac{g(x)}{\beta'_A(x)} \left[ \delta \left( R^A (\beta^A(x)) - \beta^A(x) \right) + (1 - \delta) (x - \beta^A(x)) \right] = 0$$

which can be expressed as

$$\frac{dG(x) \beta^A(x)}{dx} = (1 - \delta) x g(x) + \delta R^A \left( \beta^A(x) \right) g(x) + \delta R'_A \left( \beta^A(x) \right) \beta'_A(x) G(x)$$
Besides equation (6), we know from previous discussions, that the marginal bidder’s optimal bid is \( r^A \) so that \( \beta^A (x^A, r^A) \) must satisfy:

\[
\beta^A (x^A, r^A) = r^A. \tag{7}
\]

Solving equation (6) and (7) yields the equilibrium bidding function for any given participation threshold the seller may choose when \( A \) is the prevailing regime. Once this equilibrium bidding function \( \beta^A (\cdot) \) has been determined, the seller’s expected revenue from setting the reserve price at an arbitrary value \( r \) is given by:

\[
\Pi^A_N (r) = \int_{\frac{r - \delta R^A (x)}{1 - \delta}}^{r} \left( \beta^A (x, r) - x_0 \right) dP_N (x). \tag{8}
\]

where \( \frac{dP_N (x)}{dx} \) is the probability density of \( N^{th} \) order statistic of the sample of bidders’ private values. \( \int_{\frac{r - \delta R^A (x)}{1 - \delta}}^{r} \beta^A (x, r) dP_N (x) \) is the expected value of the highest bid when the reserve price is \( r \).

The following proposition characterizes the equilibrium for a given generic and regular accounting rule \( R^A (\cdot) \).

or more compactly

\[
\frac{d}{dx} \left[ G (x) \beta^A (x) \right] = (1 - \delta) x g (x) + \delta \frac{d}{dx} \left[ R^A (\beta^A (x)) G (x) \right]. \tag{ODE}
\]

Moreover, for the marginal bidder with value \( x^A \) it must be true

\[
\beta^A (x^A) = (1 - \delta) x^A + \delta R^A \left( \beta^A \left( x^A \right) \right). \]

With this condition we can solve the above differential equation for \( \beta^A (x) \). In fact, integrating both sides of \( \text{ODE} \) yields

\[
G (x) \beta^A (x) = \int_{x^A}^{x} (1 - \delta) y g (y) dy + \delta R^A \left( \beta^A \left( x^A \right) \right) G (x) + G \left( x^A \right) \left( \beta^A \left( x^A \right) - \delta R^A \left( \beta^A \left( x^A \right) \right) \right)
\]

which can be rewritten as

\[
G (x) \beta^A (x) = \int_{x^A}^{x} (1 - \delta) y g (y) dy + \delta R^A \left( \beta^A \left( x^A \right) \right) G (x) + (1 - \delta) G \left( x^A \right) x^A \tag{BID}
\]

Now, integrating \( \int_{x^A}^{x} y g (y) dy \) by parts gives \( \int_{x^A}^{x} y g (y) dy = y G (y) \bigg|_{x^A}^{x} - \int_{x^A}^{x} G (y) dy \). Substituting this into \( \text{BID} \) and rearranging, results in

\[
\beta^A (x) = (1 - \delta) \left( x - \int_{x^A}^{x} \frac{G (y)}{G (x)} dy \right) + \delta R^A \left( \beta^A \left( x \right) \right).
\]
Lemma 1 For any regular accounting rule $R^A(\cdot)$, there exists a unique symmetric equilibrium, defined by each bidder adopting the same strictly increasing bidding function $\beta^A(\cdot)$ along with the seller setting the minimum bid $r^A$ (leading to value in use threshold $x^A = \frac{r^A-\delta R^A(x^A)}{1-\delta}$) defined as follows:

$$\beta^A(x, r^A) = (1 - \delta) \left(x - \int_{x^A}^{x} \frac{G(y)}{G(x)} dy\right) + \delta R^A(\beta^A(x, r^A))$$ (9)

and

$$r^A \in \arg \max_r \int_{x^A}^{w} \left[(1 - \delta) \psi(x) + \delta R^A(\beta^A(x, r)) - x_0\right] dP_N(x)$$ (10)

where $\psi(x) = x - \frac{1-F(x)}{f(x)}$.\footnote{15}{The function $\psi(p)$ is known, in the auction literature, as the virtual valuation of a bidder with private value $p$. Bulow and Klemperer (1989) show that this function can be interpreted as the marginal revenue of a monopolist with demand function $1-F(p)$.

Lemma (1) gives the equilibrium bidding strategy and the reserve price for a given accounting rule $R^A(\cdot)$. It is already apparent from (9) and (10) that both the reserve price and the bidding strategy are affected by how the accounting rule is operationalized. Bidders choose how much to bid weighing the surplus from purchasing the asset and the accounting implications from the transaction. If we fix a reserve price $r^A$ and the bidder’s private value $x$, then a greater $R^A(\beta^A(x, r^A))$ must necessarily result in a higher bid.

On the other hand, the integrand in (10) reveals that the amount the seller can extract from bidders and his choice of reserve price is affected by how the accounting rule is implemented in equilibrium. We will see below that regardless of which regime is in place, the seller announces a reserve price above his own valuation $x_0$. By doing so the seller reduces the probability that the asset is sold to any bidder but he also raises the average price conditional on the asset being sold, because a higher reserve price induces more aggressive bidding behavior. The reserve price not only determines which bidders are excluded from the auction but also determines how aggressive each bidder’s bid is. Of course, a reserve price above $x_0$ results in a social-efficiency loss because the seller inefficiently retains the asset with positive probability. As Section 4.1 discusses, the magnitude of this inefficiency crucially depends on the accounting regime.

To complete the specification of the equilibrium we need to specify an accounting regime and obtain the corresponding accounting rule. This paper only considers three regimes, but Lemma 1
can readily be applied to any regular accounting rule the reader may consider relevant. Before considering our first regime, we make the following assumption about $\psi(x)$ which plays an important role in guaranteeing the monotonicity of bidding strategies.

**Assumption 2** $\psi(x)$ is strictly increasing in $x$.

This condition is satisfied by many standard probability distributions including the Normal, Exponential and Uniform distributions.

### 3.2 A BENCHMARK: THE PURCHASE PRICE METHOD

PP is a natural benchmark because it is equivalent to a standard auction where bidders do not have accounting incentives. In fact, recalling that the accounting rule associated with this regime is $R^{PP}(p) \equiv p$, we see that the manager’s payoff $u(b, x)$ from bidding $b$ when his private value is $x$ is given by

$$u(b, x) = [(1 - \delta)(x - b) + \delta \cdot 0] \Pr(\text{win}|b). \quad (11)$$

where $\Pr(\text{win}|b)$ denotes the probability of winning the auction by bidding $b$. From (11) we see that the incentives of managers and shareholders get perfectly aligned at the time of the auction. In choosing his bid, the manager maximizes the firm’s expected surplus from acquiring the asset $(x - b)$ because, even if the manager faces no opponents in the auction (and as a result pays a relatively low price relative to the asset’s economic value) he cannot report any income upon the purchase. Under PP, a change in $\delta$ is equivalent to rescaling the manager’s payoff but has no effect on the manager’s optimal bid.

Upon substituting $R^{PP}(p) = p$ into Lemma 1 we automatically obtain the equilibrium of the purchase price regime, i.e., when $A = PP$. Table 1 summarizes the equilibrium, both in its general form as well as assuming $X_i \sim \text{Uniform}[0, 1]$.

<table>
<thead>
<tr>
<th>General</th>
<th>Uniform</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^{PP}(x)$</td>
<td>$x - \int_{x^{PP}}^{x} \frac{G(y)}{G(x)} dy$</td>
</tr>
<tr>
<td>$x^{PP}$</td>
<td>$\psi(x^{PP}) - x_0 = 0$</td>
</tr>
<tr>
<td>$R^{PP}(b)$</td>
<td>$b$</td>
</tr>
<tr>
<td>$\Pi_N^{PP}(w)$</td>
<td>$\int_{x^{PP}}^{w} \left[ \psi \left( x^{PP} \right) - x_0 \right] dP_N(x)$</td>
</tr>
</tbody>
</table>

Table 1: The Purchase Price Method
Examining the bidding function $\beta^{PP}(x)$, we see that bids are always below bidders’ private values $x$ because in an FPA bidders “shade” their values when choosing how much to bid (as $\int_{x}^{\infty} \frac{G(y)}{G(x)} dy > 0$). Moreover, since $\frac{G(y)}{G(x)} = \left[ \frac{F(y)}{F(x)} \right]^{N-1} < 1$, then the degree of shading depends on the number of bidders: higher competition among bidders reduces the intensity of shading. Consider the extreme cases: as $N \to \infty$ the bidding function becomes simply $\beta^{PP}(x) = x$. In contrast when only two bidders are competing, $N = 2$, then $\beta^{PP}(x) = x - \int_{x}^{\infty} \frac{F(y)}{F(x)} dy$. Generally, we can conclude that (i) the acquisition price understates the acquisition value and (ii) this understatement is amplified when there is little competition among bidders, i.e., $N$ is small.

Consider the Uniform distribution example. Assume $N = 4$ and $x_0 = 0$, then we get that the auction fails with probability 6.5%; bidders bid 82% of their values (equivalently, the accounting system underestimates by 18% the asset value); and the seller makes 77% of the expected revenues he would make if he knew bidders’ values.

### 3.3 THE EXIT VALUE METHOD

The EV method mandates reporting the acquisition value at the asset’s resale price, i.e., what the winning bidder could sell the asset for if that bidder had to turn around and sell the asset (immediately after its acquisition). Often the resale price is unobservable. In liquid markets the resale price of an asset can be approximated by the selling prices of similar assets on the date the asset was sold. However, the observation of similar transactions is rarely available for a business acquisition; an estimate is then necessary.

Defining the expected resale price poses some conceptual difficulties. This estimate depends on variables such as which mechanism is used to resell the asset or the number of bidders that would participate in a resale auction. Moreover, the expected resale price may also depend on the acquisition price even when bidders’ values are independent.

In the presence of reporting concerns the resale price of a buyer depends on the resale price of the next buyer, which in turns depends on the resale price of the third buyer and so on. Thus, calculating the auctions resale price may require to solve a sequence of interconnected auctions. The interested reader is referred to the Appendix D for the solution to this problem. However, to ease the exposition, in this section I adopt a simplified definition of the exit value whose properties resemble those of the expected resale price. I define the exit value as a Bayesian estimate of the
expected revenue in a resale first price auction, among the N-1 bidders who lost in the original auction, given that only that only the winning bid (the purchase price) in the original auction is public. However, I assume that active bidders in the resale auction do not have reporting concerns so their valuations are just the asset’s use value $x_i$. I denote the expected revenues in this resale auction by $V_{N-1}(p)$ and thus define the accounting rule under EV as:

$$R^{EV}(p) = V_{N-1}(p)$$ (12)

This definition entails a Bayesian inference about the average resale price based on the purchase price. Note that although bidder’s valuations are independent of one another, the purchase price $p$ is correlated with the resale price. Specifically, if the original auction’s bidding strategies are strictly increasing, then the auction’s purchase price can be used to obtain an upper bound for the private values of active bidders in the resale auction (which is obtained by inverting the bidding function $\beta^{EV-1}(p)$). Knowledge of this upper bound allows us to update the distribution of private values of active bidders in a very simple manner: the posterior pdf of the private value of a typical bidder in the resale auction, when the purchase price was $p$, is given by $f(x; p) = \frac{f(x)}{F(\beta^{EV-1}(p))}$.

Denoting by $P_{N-1}(x; p)$ the cdf associated with the highest private value among the active bidders in the resale auction when the purchase price in the original auction was $p$, then from (Myerson, 1981) we know that the expected revenue in the (first price) resale auction is given by:

$$V_{N-1}(p) = \max_y \int_y^{\beta^{EV-1}(p)} \left[ x - \frac{F(\beta^{EV-1}(p)) - F(x)}{f(x)} \right] dP_{N-1}(x; p).$$

Since $V_{N-1}(p)$ assumes that the bidders in the resale auction do not have reporting concerns, the number $V_{N-1}(p)$ overestimates the actual expected resale price, because bids are naturally lower when bidders valuations are linked to resale options. Yet $V_{N-1}(p)$ is a good approximation to the expected resale price if the number of competing bidders $N$ is high. This is true because as $N \to \infty$ the expected resale price to an acquirer with private value $x$, converges to $x$ regardless of whether $\delta = 0$ or not. Furthermore, the overestimation incurred by $V_{N-1}(p)$ does not qualitatively affect the number $V_{N-1}(p)$ overestimates the actual expected resale price, because bids are naturally lower when bidders valuations are linked to resale options. Yet $V_{N-1}(p)$ is a good approximation to the expected resale price if the number of competing bidders $N$ is high. This is true because as $N \to \infty$ the expected resale price to an acquirer with private value $x$, converges to $x$ regardless of whether $\delta = 0$ or not. Furthermore, the overestimation incurred by $V_{N-1}(p)$ does not qualitatively affect

\[\text{This is all the information that can be extracted from the purchase price in an independent value setting.}\]

\[\text{Condition 2 also ensures that the updated virtual valuation } \psi(x; p) = x - \frac{F(\beta^{EV-1}(p)) - F(x)}{f(x)} \text{ is strictly increasing.}\]

\[\text{As the number of competing bidders increase, bidders’ tendency to shade their values both in the original and the resale auction vanishes. Second, the expected gap between the private value of the winner in the original auction and the private value of the winner in the resale auction also shrinks as } N \text{ grows. These arguments explain why } \lim_{N \to \infty} V_{N-1}(x) = x.\]

22
the results. All the results presented in Section 4 hold if instead we used the definition provided in Appendix D.

The next proposition presents the solution of the EV regime.

**Proposition 1** There exists a unique equilibrium under the EV regime that can be characterized as follows:

<table>
<thead>
<tr>
<th></th>
<th>General</th>
<th>Uniform</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^{EV}(x)$</td>
<td>$(1 - \delta) \left( x - \int_{x}^{E^{EV}} \frac{G(y)}{G(x)} dy \right) + \delta V_{N-1}(x)$</td>
<td>$\left[ (1 - \delta) \left( \frac{N-1+(\beta^{EV})^N}{N} \right) \right] + \delta \gamma_N$</td>
</tr>
<tr>
<td>$R^{EV}(p)$</td>
<td>$V_{N-1}(p)$</td>
<td>$\gamma_N \cdot \beta^{EV-1}(p)$ where $\gamma_N = \frac{N-2+(\frac{1}{2})^{N-1}}{N}$</td>
</tr>
<tr>
<td>$x^{EV}$</td>
<td>$(1 - \delta) \psi \left( x^{EV} \right) + \delta V_{N-1}(x) - x_0 = 0$</td>
<td></td>
</tr>
<tr>
<td>$\Pi^{EV}_N$</td>
<td>$\int_{x^{EV}}^{w} [(1 - \delta) \psi \left( x \right) + \delta V_{N-1}(x) - x_0] dP_N(x)$</td>
<td>$\left( 2 \left( 1 - \delta \right) + \gamma_n \right) \frac{N}{N+1} \left( 1 - \left( x^{EV} \right)^{N+1} \right)$</td>
</tr>
</tbody>
</table>

Table 2: The Exit Value Method

To get a better sense of the results let us consider the Uniform distribution example for the particular case of $N = 4$, $x_0 = 0$ and $\delta = \frac{1}{2}$. With these assumptions we find that the probability that the auction fails is 2.4% (as opposed to 6.5% under PP); bidders bid, on average, 66% of their values (as opposed to 82% under PP), and sellers make 64% of the revenues they would make if they knew bidders’ values (as opposed to 77% under PP).

The equilibrium bidding function $\beta^{EV}(\cdot)$ incorporates not only the benefits from using the asset but also its resale value, not because buyers are planning to resell the asset but because the accounting regime uses the resale price as the proper valuation for the asset. Notice also that the exit value is always below the actual purchase price, which implies this method forces the acquirer to report a loss in equilibrium, regardless of how competitive the market is. This is perhaps surprising; one would think that if only a few bidders are competing, then a very strong shading would drive the equilibrium price even below the exit value. But notice that the exit value is also depressed by a dry market: the number $V^{N-1}(p)$ clearly decreases as $N$ goes down. In equilibrium, a bidder always submits a bid above his average resale price.\(^{19}\)

\(^{19}\)This can be seen by considering that the first term of $\beta^{EV}(x) \geq E \left( Y^{N-1} | Y^{N-1} < x \right) = \left( x - \int_{x_0}^{x} \frac{G(y)}{G(x)} dy \right)$. Note also that $E \left( Y^{N-1} | Y^{N-1} < x \right)$ is the expected private value of the winner in the resale auction. This number has to be greater than $R^{EV}(\beta^{EV}(x))$ because in the resale auction bidders would not bid exactly their values but an amount strictly smaller.
Under the accounting regime EV, bidders’ endogenous valuations become non independent, which results in the auction displaying a sort of winner’s curse. A manager, to some extent, receives bad news upon winning the auction: he realizes that rival bidders should have lower private values, which means that the expected resale price should be below the purchase price. As a result the manager must report an accounting loss on the acquisition date. In anticipation of this loss, the manager reduces the amount of his bid.

EV’s valuations emulate a “fire-sale”, namely, when the owner sells the asset at a price lower than the asset’s value-in-use. This undervaluation is magnified both by a low market liquidity (represented by a low number of bidders) and by a high level of accounting incentives (reflected in a high \( \delta \)). In the extreme case, if there is only one buyer, the exit value method would require full expensing of the asset.

If instead of (12) we used the expected resale price as the definition of exit value (see Appendix D), then the accounting values are iteratively connected to resale markets characterized by progressively fewer and lower valuation bidders. This connection is strengthened by a high \( \delta \). In the limit as \( \delta \) approaches 1, EV prescribes a valuation that approaches zero, for any given level of market liquidity. Paradoxically, the more managers are eager to report high values the lower the values they can report in equilibrium, when EV prevails.

3.4 THE PERCEIVED VALUE-IN-USE METHOD

Recall that under the value-in-use regime, the book value of the acquired asset is calculated by estimating the asset’s cash flows conditional on the asset’s purchase price or \( R^{VU} (p) \equiv \mathbb{E} (X|p) \). Under this regime bids become, to some extent, a manager’s device to signal the profitability of his transactions. This method is operationalized by inferring the asset value from its purchase price. Since the equilibrium strategies are strictly increasing, the asset value to the acquirer can be perfectly inferred from the purchase price as

\[
\mathbb{E} (X|p) = \beta^{VU^{-1}} (p),
\]

which in turn implies that the accounting report always reflects exactly the economic value of

\footnote{The winner’s curse traditionally applies to common-value auctions. However, EV makes values “common”: a bidder’s valuation is tied to the resale price and, through the resale price, is tied to other bidders’ valuations.}
the asset:

\[ R^{VU}(\beta^{VU}(x)) = x. \]  \(14\)

VU has the appealing properties of both being unbiased and fully revealing the acquisition value. Upon substituting \(14\) into Proposition 1, we obtain the equilibrium under VU, which Proposition 2 describes.

**Proposition 2** There exists an equilibrium in the VU regime that can be characterized as follows:

<table>
<thead>
<tr>
<th></th>
<th>General</th>
<th>Uniform</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta^{VU}(x))</td>
<td>((1 - \delta)\left(x - \int_{x}^{VU} G(y) \frac{G(y)}{G(x)} dy\right) + \delta x)</td>
<td>(x \left(\frac{N(1+\delta)-1+\left(\frac{x^{VU}}{N}\right)^N}{N}\right))</td>
</tr>
<tr>
<td>(R^{VU}(p))</td>
<td>(\beta^{VU^{-1}}(p))</td>
<td>(\beta^{VU^{-1}}(p))</td>
</tr>
<tr>
<td>(x^{VU})</td>
<td>(\psi\left(x^{VU}\right) + \delta \frac{1-F\left(x^{VU}\right)}{f\left(x^{VU}\right)} = x_0)</td>
<td>(\frac{1 - \delta + x_0}{2 - \delta})</td>
</tr>
<tr>
<td>(\Pi^{VU}_N(w))</td>
<td>(\int_{x}^{w} \left(\psi\left(x\right) + \delta \frac{1-F\left(x\right)}{f\left(x\right)} - x_0\right) dP_N\left(x\right))</td>
<td>(\frac{N - 1 + \delta + (x^{VU})^N}{N + 1})</td>
</tr>
</tbody>
</table>

Table 3: The Value in Use Method

If we consider the Uniform distribution example with \(N = 4\), \(\delta = 50\%\) and \(x_0 = 0\), then we find that the auction would fail with probability 1.23% (as opposed to 6.5% under PP); bidders bid 90% of their values (as opposed to 82% under PP); and the seller makes on average 88% of the revenues he would make if he knew bidders values (as opposed to 77% under PP).

The VU method induces overbidding relative to the benchmark (PP). Consider \(\beta^{VU}(x)\). The first term that \(\beta^{VU}(x)\) weighs, \(\left(x - \int_{x}^{VU} \frac{G(y)}{G(x)} dy\right)\), resembles the bidding function of a standard auction (see Krishna, 2002, page 24), and is lower than a bidder’s private value \(x\), again, because of the shading effect. However, the second term of this bidding function, is exactly the bidder’s private value \(x\), as this is the value the bidder ends up reporting in equilibrium. Bidders decide how much to bid while in part attempting to inflate the accounting-report value of the asset. Although they do not fool these expectations they nevertheless overpay.

This method sometimes may seem difficult to implement because it requires the calculation of the inference \(I3\). But note that in takeover contexts, this inference need not be made by the preparer of the accounting report or the standard setter: if capital markets are efficient, then the share price of the acquirer reflects the economic value of the acquisition; the value-in-use could then be recovered from the acquirer’s stock price reaction to the acquisition announcement.
The next section examines this method further, so here I just observe that although the reserve price is independent of market's liquidity, it nevertheless depends on the relative power of accounting incentives as reflected in $\delta$. In the limit, as $\delta$ approaches 1 the reserve price approaches $x_0$ which implies that trade takes place with probability one. Intuitively as $\delta$ approaches 1, managers tend to bid exactly their private values. As a consequence, the seller need not rely on the reserve price to elicit bidders values.

4 DISCUSSION

This section compares the three regimes along the following dimensions: efficiency, informational properties and wealth distribution. While efficiency and information seem to be the main concerns of relevance to standard setters in the US, wealth distribution effects are also likely to influence standard setters' priorities.\(^{21}\)

4.1 THE RELATIVE EFFICIENCY OF ACCOUNTING REGIMES

As explained earlier, the seller announces a reserve price $r^A$ above his own valuation $x_0$. Therefore, if all bidders have valuations that fall short of this price, then the auction fails and the seller retains the asset, in some cases, inefficiently.

The participation threshold $x^A$ associated with the seller’s reserve price $r^A$ varies across accounting regimes and is an index of the regime’s efficiency because the probability the asset fails to reach its best use and is inefficiently retained by the seller, increases monotonically in the participation threshold. In the following we refer to $\Pr(x_0 \leq Y^N \leq x^A)$ as the efficiency of an accounting regime $A$.

When bidders have accounting incentives, the way accounting values are calculated affect the efficiency of the market in two different ways. First, accounting rules affect bidders’s valuations, thereby affecting their decision to participate in the auction. In some cases, profitable transactions may be discouraged by the accounting implications of a transaction, e.g., when the transaction forces bidders to declare large accounting losses. The second way accounting rules may affect the

\(^{21}\)"The mission of the U.S. Securities and Exchange Commission is to protect investors, maintain fair, orderly, and efficient markets, and facilitate capital formation" S.E.C website (http://www.sec.gov/about/whatwedo.shtml).

"The objective of general purpose external financial reporting is to provide information that is useful to present and potential investors and creditors and others in making investment, credit, and similar resource allocation decisions" FASB (2006).
efficiency of the market has to do with the seller’s monopolistic tendency to restrict the amount of trade via the reserve price. Some accounting regimes may affect this tendency by altering the effectiveness of the reserve price at inflating bidding strategies. The interplay of these two effects determines the level of participation in the auction and thus the ranking of efficiency of accounting regimes.

Proposition 3  The efficiency of accounting regimes can be characterized as follows: there exists a value $\hat{x}_0$ such that if $x_0 \leq \hat{x}_0$, then, $x^{\text{PP}} \geq x^{\text{EV}} \geq x^{\text{VU}}$ and if $x_0 > \hat{x}_0$, then, $x^{\text{EV}} \geq x^{\text{PP}} \geq x^{\text{VU}}$. The value $\hat{x}_0$ is defined by $\psi(\hat{x}_0) = V_{N-1}(\hat{x}_0)$.

This result contains two parts. First, it shows that VU is always the most efficient regime. That is, VU is the accounting regime that ensures the highest probability the asset reaches its most productive use. This result holds generally: for any level of competition $N$, any level of accounting incentives $\delta$ and any $x_0$. Second, Proposition 3 shows that the relative efficiency of EV depends on the size of the seller’s valuation $x_0$ and establishes that EV is more efficient than PP if and only the seller’s valuation is relatively low. In the following I explain Proposition 3. I may sometimes make statements like "regime X reduces bidder’s valuations" which should be understood as "regime X reduces bidder’s valuations relative to regimes Y and Z".

VU is the most efficient regime because (1) it gives the seller the weakest incentives to rely on a high reserve price to induce aggressive bidding behavior. This occurs because in equilibrium the accounting rule is independent of the reserve price therefore bidding functions tend to be less sensitive to the reserve price. (2) VU does not depress bidders’ valuations. Given that the accounting rule is unbiased, bidders’ equilibrium valuations coincide with the asset’s use value which means that bidders have in principle the right incentives to participate in the auction. In particular, no bidder would stay out of the auction because of the accounting implications that would follow the transaction which are in fact favorable because the acquirer reports gains on the transaction.

Consider now the efficiency of EV. Recall that EV forces bidders to report losses upon acquiring the asset. These losses depress bidders’ valuations discouraging some bidders from participating in the auction thus preventing some potentially efficient trades. This effect is clearly amplified if the seller’s own valuation is high; in that case the seller becomes more reluctant to trade, he sets
a higher reserve price and, in the extreme, when no bidder can afford the seller’s own valuation, trade breaks down. Clearly in this extreme case EV would be the least efficient regime. Yet, there are also circumstances under which EV is relatively efficient. Much like VU, the EV regime discourages the seller from relying –as much as PP– in the monopolistic practice of a high reserve price since bidding functions are relatively insensitive to the reserve price. This effect compensates EV’s undervaluation and explains why EV is more efficient than PP when the seller’s valuation is low.

To understand the superiority of EV vis-a-vis PP when \( x_0 \) is sufficiently low some observations are required. (1) EV’s efficiency is lower the stronger is EV’s undervaluation of the asset (2) when EV’s undervaluation is extremely strong, i.e., when EV requires full expensing of the acquisition or \( R^{EV}(\cdot) = 0 \), bidders behave like in a standard auction but with their valuations scaled down from \( x \) to \( (1 - \delta) x \). That is, bidders behave like under PP but with their private values scaled down by \( (1 - \delta) \) (3) Scaling down bidders’ valuations clearly affects the reserve price (as the seller has to adjust to these new (lower) valuations) but it does not affect the participation threshold unless \( x_0 > 0 \). These observations allows us to conclude that when \( x_0 = 0 \), the efficiency of EV and PP are the same only under EV’s worst case scenario (\( R^{EV}(\cdot) = 0 \)). Hence, EV’s efficiency has to be greater than PP’s when \( R^{EV}(\cdot) > 0 \).

The above arguments show that the efficiency of EV relative to PP switches at some point depending on the seller’s valuation \( x_0 \). A technical argument shows that there is only one such point \( \hat{x}_0 \) that depends on \( N \) and \( \delta \). Whenever the seller’s valuation is below \( \hat{x}_0 \) the efficiency of EV will be greater than that of PP. We now turn to the comparative statics.

**Corollary 1** (a) \( x^{VU} \) and \( x^{PP} \) are independent of \( N \). However, \( x^{EV} \) decreases in \( N \) and \( x^{EV} \) converges to \( x^{VU} \) when \( N \to \infty \). (b) As \( \delta \to 0 \) all three methods are equally efficient. If \( \delta \to 1 \), \( VU \) becomes fully efficient or \( x^{VU} \to x_0 \). In general \( \frac{\partial x^{VU}}{\partial s} \leq 0 \); \( \frac{\partial x^{PP}}{\partial s} = 0 \) and \( \frac{\partial x^{PP}}{\partial s} = \left\{ \begin{array}{ll} \leq 0 & \text{if } x_0 \leq \hat{x}_0(N, \delta) \\ \geq 0 & \text{if } x_0 > \hat{x}_0(N, \delta) \end{array} \right. \).

Corollary 1 (a) studies the effect of market liquidity \( N \) on the efficiency of accounting regimes. It

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22 An analogy may help to understand this. When \( x_0=0 \), the seller behaves like a monopoly with zero marginal cost. Think of a monopoly who suffers a negative shock in his demand so that his inverse demand function is rescaled from \( p(q) \) to \( (1 - \delta) p(q) \). Clearly the monopoly would adjust the price, however the quantity would remain the same because although the marginal revenue would be lower after the shock, the sign of the marginal revenue would not be altered.
establishes that only the efficiency of EV is affected by a low level of liquidity. It is well known that in a standard auction, the equilibrium reserve price (and participation threshold) is independent of the number of bidders \( N \). This result is confirmed only for VU and PP. Under EV, the participation reserve price/participation threshold decreases in \( N \) so that the efficiency of EV is greater in more liquid markets. This occurs because the undervaluation that EV induces, tends to vanish as the market becomes more competitive. In fact, EV converges to VU in perfectly competitive markets (as \( N \to \infty \))\(^{23}\).

Corollary 1 (b) shows that VU’s efficiency increases in the size of accounting incentives \( \delta \). Moreover, as \( \delta \to 1 \), VU becomes fully efficient (or \( x^{VU} = x_0 \)) so that the asset always achieves its best use. This occurs because as \( \delta \) increases, bidding strategies become more aggressive reducing seller’s need/incentives to rely on the reserve price to elicit bidders’ private values. In the limit as \( \delta \to 1 \), bidders bid exactly their private values \( \beta^{VU} (x) = x \) and the seller has no need to use the reserve price to elicit those values.

By now it should be clear why the efficiency of PP does not depend on \( \delta \), so we skip the discussion of that case. Instead we consider the case of EV. Under EV, an increase in \( \delta \) involves two countervailing effects: it decreases bidders’ valuations in equilibrium and it decreases the effectiveness of the reserve price in eliciting bidders’ private values. While the first effect goes against the efficiency of EV, the second effect improves it, so that in principle it is not clear whether a higher \( \delta \) is beneficial for EV’s efficiency or not. Yet, our analysis shows that the first effect is more relevant when \( x_0 \) is high. If the seller’s valuation is relatively high, an increase in \( \delta \) aggravates the bidder’s undervaluation, which would expel more bidders out of the auction. If the seller’s valuation is relatively low, this effect is less relevant because the seller has flexibility to adjust to the new circumstances by reducing the reserve price. Instead an increase in \( \delta \) would induce a relatively strong decrease in the effectiveness of the reserve price as a means to affect bidder’s behavior.

\(^{23}\) This result is apparent from consideration of the bidding function \( \beta^{EV} (x,k) \) in Table ??, \( \beta^{EV} (\cdot) \) clearly converges to \( \beta^{VU} (x,k) \) as \( N \to \infty \), because, as was noted before, the resale price of a bidder who acquires the asset with a private value \( x \) converges also to \( x \) as \( N \to \infty \) or \( \lim_{N \to \infty} V_{N-1} (x) = x \).
4.2 IS THERE A RELEVANCE-RELIABILITY TRADE-OFF?

Many valuation methods are biased. In a frictionless world this type of bias is nominal, reversible, and thus innocuous. However, in practice, nominal distortions may produce real effects for a variety of reasons that include market incompleteness, contract stickiness, and bounded rationality. This is perhaps a reason why standard setters are concerned about the representational-faithfulness (or simply the bias) of financial statements. Proposition 4 studies the bias associated with the accounting regimes.

**Proposition 4** The three regimes are equally informative, namely for all \( A \in \{PP, EV, VU\} \), and all \( x \), \( \mathbb{E} (X|R^A (\beta^A (x))) = x \). However,

(a) \( PP \) and \( EV \) understate the underlying economic value;
(b) \( VU \) is the only unbiased method;
(c) \( EV \)'s bias is uniformly greater than \( PP \)'s bias. In fact, for all \( x \geq x^{PP} \)

\[ R^{EV} (\beta^{EV} (x)) \leq R^{PP} (\beta^{PP} (x)) \leq R^{VU} (\beta^{VU} (x)) = x; \]

(d) The three methods are consistent (i.e., the bias vanishes as the number of bidders \( N \) grow);
(e) If the definition of \( EV \) is that given in Appendix D, then \( EV \)'s bias increases in \( \delta \).

Proposition 4 demonstrates that all three methods convey the same information, i.e., they all fully reveal the asset value. This is true because no matter which accounting method is in place, the equilibrium associated with that method induces a monotonic mapping between bidders’ bids and the accounting report. This result by itself implies that no relevance-reliability trade-off exists. Even though both \( EV \) and \( VU \) prescribe the report of an estimate, these estimates are all just as informative as the asset’s purchase price. Contrary to conventional wisdom, in this model, estimates are not less informative than are observations.

\(^{24}\) There is a vast literature in accounting and macroeconomics documenting the real effects of nominal shocks. This is, in fact, the basic assumption of both Keynesian economics and the Positive Theory of Accounting (see Watts and Zimmerman, 1978).

\(^{25}\) "The objective of this Statement is to improve the relevance, representational faithfulness, and comparability of the information that a reporting entity provides in its financial reports about a business combination and its effects" FAS-141(R).

Reliability is "the quality of information that assures that information is reasonably free from error and bias and faithfully represents what it purports to represent". SFAC No.2, FASB, 1980.
However, only $VU$ is unbiased. In contrast both $PP$ and, especially, $EV$ understate the asset’s value.\textsuperscript{26} Yet, the three methods are statistically consistent in the sense that they converge to the asset’s underlying economic value as the number of bidders approaches infinity. In the case of $PP$, this convergence occurs because more competition reduces the extent of bidders’ shading, thereby reducing the gap between values and bids. In the case of $EV$, the convergence happens because greater competition in the auction increases the expected resale price through two different channels: on the one hand, resale markets inherit this greater level of competition which increases the expected resale price; on the other hand, as the number of bidders grows, the expected gap between the highest and the second highest private value shrinks.\textsuperscript{27}

Furthermore, $EV$’s bias is exacerbated by the relative importance of accounting incentives: an increase in $\delta$ results in bidders becoming more concerned about exit values (which are biased) as opposed to the asset’s fundamental economic value.

The next section discusses wealth distribution effects of the three accounting standards. These effects naturally follow from the valuation bias induced by each accounting regime. We will see, for example, that a downward bias depresses bidding leading, on average, to a reduction in seller revenues.

4.3 WEALTH DISTRIBUTION EFFECTS

The choice of accounting regime differentially affects the welfare of managers, sellers and buyers. In this section I consider how the allocation of surplus among these three groups is determined by the choice of accounting regime and to what extent the preference of each group is aligned with social efficiency.

Unlike sellers and buyers, managers’ incentives are aligned with social efficiency. In fact, a manager always prefers the regime with the lowest reserve price. Proposition\textsuperscript{5} justifies this assertion.

[Proposition 5] The manager’s expected surplus is maximized (minimized) by the accounting regime with the lowest (highest) participation threshold.

\textsuperscript{26}Prior empirical studies Bradley and Sundaram (2006) implicitly find that the purchase price method has, on average, underestimated the market value of acquisitions in the US during the 1990s, in nearly 4% of the aggregate value of the targets acquired or a remarkable 26.1% if the sample is restricted to all non-public targets acquired.

\textsuperscript{27}If the market is more densely populated one should expect the Nth and N-1th order statistic to be closer to each other.
Proof. The manager’s ex ante expected surplus from regime $A$ can be expressed as
\[
E[u(\beta^A(X), X)] = \int_{x^A}^{w} [(1 - \delta) x + \delta R^A(x)] G(x) f(x) dx - \frac{\Pi^A}{N}.
\] (15)
where I used the shortcut $R^A(x)$ for $R^A(\beta^A(x))$. Seller’s expected revenues are given by
\[
\Pi^A = \int_{x^A}^{w} [(1 - \delta) \psi(x) + \delta R^A(x) - x_0] dP_N(x)
\]
(16)
Substituting (16) into (15) one obtains the expected utility of a manager
\[
E[u(\beta^A(X), X)] = \int_{x^A}^{w} [(1 - \delta) \frac{1 - F(x)}{f(x)} + x_0] \frac{dP_N(x)}{N}.
\] (17)
which is obviously decreasing in $x^A$. ■

Managers concern for social efficiency is somewhat striking; after all, managers’ distorted incentives drive the differences in the efficiency across regimes. Had managers no reporting incentives, all three regimes would be efficiency equivalent. The reason managers’ unique concern is social efficiency is that, ex-ante, the implications of manager’s reporting incentives are fully absorbed by the seller: the seller enjoys higher prices when the manager reports gains and bears depressed prices when the manager reports losses. Note that Proposition 5 is general: it applies to any set of regular accounting regime.

Buyers’ preferences, in contrast, are not aligned with social efficiency but are perhaps particularly opposed to it. In fact, the most efficient regime $VU$ is always the least preferred by buyers as $VU$ induces on average significantly higher expected prices as well as a higher managerial compensation than the other two regimes.

**Proposition 6** If $A \in \{PP, EV, VU\}$, then $VU$ always minimizes buyers’ welfare. If $x_0 \leq \hat{x}_0$ buyers’ preferred regime is EV.

Proof. Let $EU^A$ denote buyer’s ex ante expected utility under regime $A$. Then,
\[
EU^A = \int_{x^A}^{w} xG(x) dF(x) - Eu^A - \frac{\Pi^A}{N}.
\] It is easy to check that
\[
EU^A = \delta \int_{x^A}^{w} (x - R^A(x)) \frac{dP_N(x)}{N}, \text{ thus } EU^{VU} = 0.
\] ■

Buyers inclination for EV is intuitive when $x_0 \leq \hat{x}_0$. Under these circumstances, EV provides buyers with the lowest expected prices, the lowest expected managerial compensation and still a relatively high chance of trading the asset, at least higher than the chance induced by PP’s. By
contrast, the benefits of EV are not so clear when we approach the market’s breakdown point, i.e.,
when the seller himself has a high valuation for the asset. Note, however, that buyers may very
well prefer EV in circumstances in which EV is the least efficient method, just in view of the high
surplus they derive from this method.

The choice of accounting regime also creates a tension between revenue maximization and
social efficiency. In particular, a seller ranks the accounting regimes based on the magnitude of the
expected income induced by each regime, because the expected level of reported income determines
how aggressive bidders’ strategies are. While expected accounting income induces overbidding
under VU, expected accounting losses induce underbidding under EV. Proposition 7 compares
expected revenues across regimes.

**Proposition 7** If $A \in \{PP, EV, VU\}$, then VU (EV) maximizes (minimizes) expected revenues.
In fact:

1. $\Pi^E_V \leq \Pi^P_P \leq \Pi^V_U$
2. $\Pi^A_N$ is increasing in $N$ for all $A = EV, PP$ and $VU$
3. $\frac{\partial \Pi^E_V}{\partial \delta} \leq 0$, $\frac{\partial \Pi^P_P}{\partial \delta} = 0$, $\frac{\partial \Pi^V_U}{\partial \delta} \geq 0$
4. $\lim_{\delta \to 0} \Pi^E_V = \lim_{\delta \to 0} \Pi^V_U = \Pi^P_P$

Proposition 7 reveals that the seller may sometimes prefer inefficient regimes. For instance,
the seller always prefers regime PP to regime EV in spite EV’s is sometimes more efficient. Yet,
unlike buyer’s preference for accounting regimes, the seller’s preference is more aligned with social
efficiency: in fact, the seller’s preferred regime coincides with the socially most efficient regime
(VU). Proposition 7 also studies comparative statics results. Not surprisingly, the seller benefits
from a larger number of bidders in the auction as more competition always increases revenues. By
contrast, the effect of $\delta$ on seller’s revenues depends on the accounting regime: an increase in $\delta$
increases revenues under VU but it decreases them under EV.

5 CONCLUDING REMARKS

This paper studies alternative methods for acquisition accounting when the seller optimally chooses
the selling mechanism and the buyer delegates the bidding to an agent who (i) is privately informed
about buyer-specific synergies, (ii) has incentives to report high accounting income, but (iii) must comply with the prevailing accounting standard in reporting the value of the acquired asset.

The exit-value method requires an acquirer to book an accounting loss on the date of acquisition, measured as the difference between the price paid and the expected price that would have been paid by the second best buyer. By connecting otherwise independent valuations, this method induces an accounting based winners’ curse. Given that the exit value method iteratively links a firm’s willingness to pay to the willingness to pay of the next best buyer, it can lead to depressed asset values, in particular when accounting incentives are stronger.

The purchase price method measures the fair value of the asset at the purchase price and is required by GAAP. This method implies higher asset prices, increases (decreases) in the surplus of the acquirer (seller) and a decrease in allocative efficiency.

Under the value-in-use method, the acquirer bids more aggressively to increase perceived value in use, leading to a shift of surplus from the acquirer to the seller. Yet, the value in use method is unbiased and maximizes efficiency as measured by the probability that the asset is sold.

The analysis is restrictive in at least two respects. First, managers’ contracts are exogenous and are thus independent of the accounting regime. A perhaps fruitful extension would endogenize the agency problem that justify managers’ reporting concerns. Second, although this paper studies standards that prescribe the disclosure of estimates, these estimates are always verifiable because all parties’ beliefs about the acquisition coincide after observing the purchase price. This general agreement based on public information is not realistic in practice. In the model, it is the consequence of assuming that agents have common prior beliefs. Departures from this assumption may be important when analyzing standards because managerial manipulation may particularly affect the development and performance of estimates.

A possible extension is to allow for cross sectional variation in the importance of accounting incentives in bidders’ preferences. When this is the case, the winning bidder might sometimes be a bidder with relatively strong accounting incentives instead of relatively large synergies. Another natural extension of the analysis may consider revaluations in subsequent periods, once the auction has taken place. For instance, one can explore the relative benefits of impairments vis-a-vis symmetric revaluations when combined with an accounting standard’s initial recognition principles.

Another extension may consider the case in which buyers enter the auction having an initial
stake in the target, and the accounting standard mandates revaluations of this stake based on
the price realized in the auction. One would expect some distortions in the bidding behavior of
managers, even under PP, because managers would seek to revaluate upwards their balance sheets
by overbidding in the auction. As in “toehold” bidding (see, e.g., Burkart 1995) managers would
play two roles, that of bidders of the target firm and that of hypothetical sellers of their initial
assets to rival bidders.
A PROOFS

A.1 PROOF OF PROPOSITION 1

Assuming that $R^A (b)$ is regular we can differentiate bidder i’s utility function,

$$ u (b, x) = \left[ \delta (R^A (b) - b) + (1 - \delta) (x - b) \right] G \left( \beta^{-1}_A (b) \right), $$

with respect to $b$ to obtain the following FOC:

$$ \left[ \delta (R^A (b) - 1) - (1 - \delta) \right] G \left( \beta^{-1}_A (b) \right) + \frac{g (\beta^{-1}_A (b))}{\beta'_A (\beta^{-1}_A (b))} \left[ \delta (R^A (b) - b) + (1 - \delta) (x - b) \right] = 0 \quad (19) $$

where I used the inverse function theorem. Note that, for convenience, I moved the superscript $A$, and wrote $\beta^A (x)$ but $\beta^{-1}_A (b)$ to denote the inverse of $\beta^A (x)$. I also wrote $\beta'_A (x)$ to denote the partial derivative of $\beta_A (x)$ with respect to $x$.

In a symmetric equilibrium, $b = \beta^A (x)$, and thus (19) yields the following differential equation

$$ \left[ \delta (R'_A (\beta^A (x)) - 1) - (1 - \delta) \right] G (x) + \frac{g (x)}{\beta'_A (x)} \left[ \delta (R^A (\beta^A (x)) - \beta^A (x)) + (1 - \delta) (x - \beta^A (x)) \right] = 0 $$

which can be expressed as

$$ \frac{dG (x) \beta^A (x)}{dx} = (1 - \delta) x g (x) + \delta R^A (\beta^A (x)) g (x) + \delta R'_A (\beta^A (x)) \beta'_A (x) G (x) $$

or more compactly

$$ \frac{d \left[ G (x) \beta^A (x) \right]}{dx} = (1 - \delta) x g (x) + \delta \frac{d \left[ R^A (\beta^A (x)) G (x) \right]}{dx}. \quad (20) $$

Moreover, for the marginal bidder, the following condition must hold:

$$ \beta^A (x^A) = (1 - \delta) x^A + \delta R^A (\beta^A (x^A)) $$

With this condition we can solve (20) for $\beta^A (x)$. In fact, integrating both sides of (20) yields

$$ G (x) \beta^A (x) = \int_{x^A}^{x} (1 - \delta) y g (y) dy + \delta R^A (\beta^A (x^A)) G (x) + G (x^A) \left( \beta^A (x^A) - \delta R^A (\beta^A (x^A)) \right) $$

The inverse function theorem says $\frac{\partial \beta^{-1}_A (b)}{\partial b} = \frac{1}{\beta'_A (x)}$. 

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which can be rewritten as

\[ G(x) \beta^A(x) = \int_{x^A}^x (1 - \delta) yg(y) \, dy + \delta R^A(\beta^A(x^A)) G(x) + (1 - \delta) G(x^A) x^A \]  

(21)

Now, integrating \( \int_{x^A}^x yg(y) \, dy \) by parts gives

\[ \int_{x^A}^x yg(y) \, dy = yG(y) \bigg|_{x^A}^x - \int_{x^A}^x G(y) \, dy. \]

Substituting this into (21) and rearranging, results in

\[ \beta^A(x) = (1 - \delta) \left( x - \int_{x^A}^x G(y) \, dy \right) + \delta R^A(\beta^A(x^A)). \]  

(22)

The bidding function is then \( \beta^A(x) \) implicitly defined by (22). A solution for \( \beta^A(x) \) exists and is unique for all three methods. Having obtained the equilibrium bidding function we can calculate what the seller revenues are expected to be. In a FPA the winner pays what he bids and thus the net expected payment \( m_A(x) \) by a bidder with value \( x \) is

\[ m_A(x) \equiv \Pr(\text{Win}) \times (\text{AmountBid-x}_0) \]

\[ \equiv G(x) (\beta^A(x) - x_0) \]

\[ = (1 - \delta) \left( G(x)x - \int_{x^A}^x G(y) \, dy \right) + \delta R^A(\beta^A(x)) G(x) - x_0G(x) \]

The ex-ante net expected payment of a bidder is therefore

\[ \mathbb{E}[m_A(X)] = \int_{x^A}^w m_A(x) f(x) \, dx \]

and the seller’s expected revenues are

\[ \Pi^A_N = \sum_{i=1}^N \mathbb{E}[m_A(X)] \]

which, in virtue of buyers’ symmetry, becomes simply

\[ \Pi^A_N = N \cdot \mathbb{E}[m_A(X)] \]  

(23)

Equation (23) can be written more explicitly as

\[ \Pi^A_N = N (1 - \delta) \int_{x^A}^w \left( G(x)x - \int_{x^A}^x G(y) \, dy \right) dF(x) \]

\[ + N \int_{x^A}^w \delta R^A(\beta^A(x)) G(x) \, dF(x) - \int_{x^A}^w NG(x) \, dF(x) x_0 \]  

(24)

but changing the order of integration one can see that
\[
\int_{x^A}^{w} \int_{x^A}^{x} G(y) \, dy \, dF(x) = \int_{x^A}^{w} G(x) \, (1 - F(x)) \, dx
\]

Substituting this into (24) yields:

\[
\Pi_N^A (w, x^A) = \int_{x^A}^{w} \left[ (1 - \delta) \psi(x) + \delta R^A (\beta^A(x)) - x_0 \right] \, dP_N(x) \tag{25}
\]

where \( \psi(x) = x - \frac{1 - F(x)}{f(x)} \) and \( \frac{dP_N(x)}{dx} \equiv p_N(x) = NG(x) \, f(x) \) is the density of the highest among \( N \) values.

Now,

\[
x^A \equiv \arg \sup_{z \in [0, w]} \Pi_N^A (w, z)
\]

Moreover, by the Theorem of the Maximum, there exists \( x^A \in [0, w] \) such that

\[
x^A \in \arg \max_{z \in [0, w]} \int_{x^A}^{w} \left[ (1 - \delta) \psi(x) + \delta R^A (\beta^A(x)) - x_0 \right] \, dP_N(x)
\]

in fact \( x^A > x_0 \). Moreover, a sufficient condition for \( x^A \) to be unique is that \( R^A (\beta^A(x)) \) is non-decreasing. So far, we have only proved that \( \beta^A(x) \) as defined by (22) is a candidate solution that satisfies the first order necessary conditions. It is not difficult to show that \( \beta^A(x) \) is indeed an equilibrium. In fact assume bidder one bids \( \beta^A(z) \) when his value is \( x \) but all bidders \( j \neq 1 \) conform to the equilibrium strategy \( \beta^A(x) \). Then bidder 1 payoffs are

\[
u(\beta(z), x) = \left[ (1 - \delta) x + \delta R (\beta^A(z)) - \beta^A(z) \right] G(z)
\]

Substituting (22) into the above equation gives

\[
u(\beta(z), x) = (1 - \delta) \left[ (x - z) + \int_{x^A}^{z} \frac{G(y)}{G(z)} \, dy \right] G(z)
\]

But

\[
u(\beta(x), x) - u(\beta(z), x) = (1 - \delta) \left[ G(z) (z - x) - \int_{x}^{z} G(y) \, dy \right] \geq 0
\]

regardless of whether \( z \geq x \) or \( z \leq x \).

\[\text{Note that both } \psi(x) \text{ and } P(x) \text{ are functions of } w.\]
A.2 THE PP REGIME

Recall that $PP$ is defined by

$$R^{PP}(b) \equiv b,$$  \hfill (26)

The boundary condition for this method is defined by

$$\beta^{PP}(x^{PP}) = (1 - \delta) x^{PP} + \delta R^{PP}(\beta^{PP}(x^{PP}))$$

which by (26) becomes

$$\beta^{PP}(x^{PP}) = x^{PP}. \hfill (27)$$

In general, by Proposition 1, the equilibrium bidding strategy is

$$\beta^{PP}(x) = x^{PP} \frac{G(x^{PP})}{G(x)} + \int_{x^{PP}}^{x} \frac{g(y)}{G(x)} dy$$

$$= x - \int_{x^{PP}}^{x} \frac{G(y)}{G(x)} dy,$$

and the seller revenues, for a given reserve threshold $z$, are determined by

$$\Pi_{N}^{PP}(z) = \int_{z}^{\psi(x) - x_0}NG(x)f(x)dx $$ \hfill (28)

so the seller’s optimization problem is to maximize $\Pi_{N}^{PP}(z)$ or

$$\Pi_{N}^{PP} = \max_{x} N \int_{z}^{\psi(x) - x_0}G(x)f(x)dx$$

By condition 2, the solution to this problem exists, and is uniquely determined by,

$$x^{PP} = \psi^{-1}(x_0)$$

Not surprisingly we see that $PP$ replicates the outcome when $\delta = 0$.

\textbf{Example 1} Assuming $X \sim \text{Uniform}[0, 1]$, then:

$$x^{PP} = \frac{1 + x_0}{2}$$

$$\beta^{PP}(x) = x \frac{N - 1 + \left(\frac{x^{PP}}{x}\right)^N}{N}$$

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\[ \Pi^V_N = \int_{x_{PP}}^{1} (2x - 1 - x_0) N x^{N-1} dx \]

A.3 PROOF OF PROPOSITION 1

The EV method mandates \( R^{EV} (b) \equiv V_{N-1} (\beta^{EV-1} (b)) \) where \( \beta^{EV-1} (\cdot) \) is the equilibrium symmetric and strictly increasing bidding strategy that we need to find. By proposition (1) it is easy to verify:

\[ \beta^{EV} (x) = (1 - \delta) \left( x - \int_{x_{EV}}^{x} \frac{G (y)}{G (x)} dy \right) + \delta V_{N-1} (x) \]

which is indeed strictly increasing function of \( x \). Recall \( V_{N-1} (x) = \max_z \int_z^x \psi (t, x) dP_{N-1} (t; x) \), then

\[ \Pi^{EV} = \max_{x_{EV}} \int_{x_{EV}}^{w} [(1 - \delta) \psi (x) + \delta V_{N-1} (x) - x_0] dP_N (x) \]

which is clearly maximized at \( x^{EV} \), defined as

\[ (1 - \delta) \psi (x^{EV}) + \delta V_{N-1} (x^{EV}) = x_0. \]

If \( x^{EV} \geq w \) the seller retains the asset.

Example 2 Assuming \( X_i \sim \text{Uniform}[0,1] \), then:

The expected resale price for a bidder with value \( x \) is given by

\[ V_{N-1} (x) = \max \int_x^1 \left( t - \frac{1 - \frac{t}{x}}{1 - \frac{1}{x}} \right) (N - 1) \left( \frac{t}{x} \right)^{N-2} \frac{1}{x} dt \]

\[ = x \frac{N - 2 + 2^{-N+1}}{N} \cdot \]

\[ x^{EV} = N \frac{1 - \delta + x_0}{2N - N\delta - 2\delta + \delta 2^{-N+1}} \]

\[ \beta^{EV} (x) = (1 - \delta) x \frac{N - 1 + \left( \frac{x^{EV}}{x} \right)^N}{N} + \delta V_{N-1} (x) \]

\[ \Pi^V_N = \int_{x^{EV}}^{1} [(1 - \delta) (2x - 1) + \delta V_{N-1} (x) - x_0] N x^{N-1} dx \]
A.4 PROOF OF PROPOSITION 2

The VU method mandates

\[ R^{VU}(b) \equiv \mathbb{E}(X|b) \]

which in a symmetric and increasing equilibrium with bidding strategies denoted by \( \beta^{VU}(x) \) means

\[ R^{VU}(b) = \beta^{-1}_{VU}(b) = \beta^{-1}_{VU}(\beta^{VU}(x)) = x \]  \hspace{1cm} (29)

By proposition [1] it is easy to verify

\[ \beta^{VU}(x) = (1 - \delta) \left( x - \int_{x^{VU}}^{x} \frac{G(y)}{G(x)} dy \right) + \delta x \]

which is indeed strictly increasing. Now,

\[ \Pi^{VU}_{N}(z) = N \int_{z}^{w} \left( x - (1 - \delta) \frac{1 - F(x)}{f(x)} - x_0 \right) G(x) f(x) dx \]

So that \( \Pi^{VU}_{N} \) is

\[ \Pi^{VU}_{N} \equiv \max_{z} \Pi^{VU}_{N}(z) \] \hspace{1cm} (30)

The solution to (30) exists and is unique. In fact, the optimal reserve price is determined by the next equation

\[ x^{VU} = (1 - \delta) \frac{1 - F(x^{VU})}{f(x^{VU})} + x_0 \]

Example 3 Assuming \( X_i \sim \text{Uniform}[0, 1] \), then:

\[ x^{VU} = \frac{1 - \delta + x_0}{2 - \delta} \]

\[ \beta^{VU}(x) = x \frac{N - 1 + \delta + (1 - \delta)^N \left( \frac{x^{VU}}{x} \right)^N}{N} \]

\[ \Pi^{VU}_{N} = \max_{z} N \int_{z}^{1} (x - (1 - \delta)(1 - x) - x_0) x^{N-1} dx \]

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A.5 PROOF OF PROPOSITION 3

a) The fact that \( x^{VU} \) is always lower than \( x^{EV} \) and \( x^{PP} \) is clear, and follows because \( \Phi^{VU} (x) \) is uniformly greater than both \( \Phi^{EV} (x) \) and \( \Phi^{PP} (x) \). By comparing the virtual valuations \( \Phi^{EV} (x) \) and \( \Phi^{PP} (x) \) we see that there exists a cutoff \( \hat{x} \) where \( \Phi^{PP} (\hat{x}) = \Phi^{EV} (\hat{x}) = \hat{\Phi} \) defined by \( \psi^{EV} (\hat{x}) = V_{N-1} (\hat{x}) \). If \( x_0 \leq \hat{\Phi} \), then \( x^{EV} = \Phi^{EV-1} (x_0) \leq \Phi^{PP-1} (x_0) \), so that the ranking is \( x^{VU} \leq x^{EV} \leq x^{PP} \). By contrast, if \( x_0 \geq \hat{\Phi} \) then the ranking

\[
x^{VU} \leq x^{PP} \leq x^{EV}.
\]

b) When \( \delta = 1 \), clearly \( x^{VU} = x_0 \) whereas \( x^{EV} \) is given by \( V_{N-1} (x^{EV}) = x_0 \). I only prove the comparative statics \( \frac{\partial x^A}{\partial \delta} \) for \( EV \). The participation threshold satisfies the following equation when \( EV \) prevails:

\[
\Phi^{EV} (x^{EV}, \delta) - x_0 = (1 - \delta) \psi (x^{EV}) + \delta R^{EV} (\beta^{EV} (x^{EV})) - x_0 = 0.
\]

Notice that although \( \beta^{EV} (x^{EV}) \) depends on \( \delta \), \( R^{EV} (x^{EV}) \) in equilibrium does not. By the implicit function theorem

\[
\frac{\partial x^{EV}}{\partial \delta} = - \frac{\Phi^{EV}_x (x^{EV}, \delta)}{\Phi^{EV} (x^{EV}, \delta)} = \frac{\psi (x^{EV}) - R^{EV} (x^{EV})}{\Phi_x (x^{EV}, \delta)}.
\]

so that

\[
\frac{\partial x^{EV}}{\partial \delta} = \frac{\psi (x^{EV}) - R^{EV} (x^{EV})}{\Phi_x (x^{EV}, \delta)}
\]

Since \( \Phi_x (x, \delta) \geq 0 \), the sign of \( \frac{\partial x^{EV}}{\partial \delta} \) depends on \( \psi (x^{EV}) - R^{EV} (x^{EV}) \). But \( \psi (x^{EV}) - R^{EV} (x^{EV}) \geq 0 \) if and only if \( x_0 \geq \hat{\Phi} \) as was shown above.

c) Since \( \lim_{N \to \infty} V_{N-1} (x) = x \) it is clear that \( \lim_{N \to \infty} x^{EV} = x^{VU} \).

B PROOF OF PROPOSITION 4

The fact that all three methods are equally informative follows from \( \beta^A (x) > 0 \) for all \( A \in \{EV, PP, VU\} \) and all \( x \).

a) Straightforward.

b) The unbiasedness of VU is true by definition.
c) The fact that EV’s bias is greater than that of PP’s comes from the fact that $\beta^{PP} (x) \geq E \left[ Y^N \mid Y^N \leq x \right]$. But clearly $E \left[ Y^N \mid Y^N \leq x \right] \geq V_{N-1} (x) = R^{EV} (\beta^{EV} (x))$.

d) The consistency of PP was shown in the text. The consistency of EV comes from the fact that

$$\lim_{N \to \infty} \max_z \int_z^x \psi (y;x) dP_{N-1} (y;x) = x$$

### C PROOF OF PROPOSITION 7

a) Fix a participation threshold $z$. Then bidding strategies are:

$$\beta^{VU} (x,z) = (1 - \delta) \left[ x - \int_z^w \frac{G(y)}{G(x)} dy \right] + \delta x$$

$$\beta^{PP} (x,z) = x - \int_z^w \frac{G(y)}{G(x)} dy$$

$$\beta^{EV} (x,z) = (1 - \delta) \left[ x - \int_z^w \frac{G(y)}{G(x)} dy \right] + \delta R^{EV} (x)$$

I argue that $\beta^{VU} (x,z) \geq \beta^{PP} (x,z) \geq \beta^{EV} (x,z)$. The first inequality is clear since $x \geq x - \int_z^w \frac{G(y)}{G(x)} dy$. The last inequality follows because $x - \int_z^w \frac{G(y)}{G(x)} dy \geq E \left[ Y^N \mid Y^N < x \right] \geq R^{EV} (x)$. Therefore, since $\Pi^A = \max_z \int_z^w \beta^A (x,z) dP_N (x)$ by revealed preferences it must be the case that

$$\Pi^{EV} \leq \Pi^{PP} \leq \Pi^{VU}.$$  

b) To prove that $\Pi^A_N$ is increasing in $N$ follows by induction and revealed preferences.

c) The result follows from observing that $\frac{\partial \beta^{EV} (x)}{\partial \delta} \leq 0, \frac{\partial \beta^{PP} (x)}{\delta} = 0, \frac{\partial \beta^{VU} (x)}{\partial \delta} \geq 0$.

d) Straightforward.

e) Straightforward.

### D EXIT VALUES AS THE EXPECTED RESALE PRICE

In order to better understand this method I use a two-step process. First I simplify the problem and consider a FPA without reserve price. Then I solve the complete version in which the seller optimally chooses the reserve price.
D.1 NO RESERVE PRICE

The EV method mandates reporting the acquisition value at the exit price. Defining this price poses some conceptual difficulties. The exit price may depend on things such as which mechanism is used to resell the asset or the number of bidders that would participate in the auction in subsequent rounds. Moreover, the resale price may also depend on the acquisition price even when bidders’ values are independent. In fact, as my analysis below demonstrates, EV induces an artificial linkage among bidders’ payoffs which results in fire-sale like accounting values particularly when $\delta$ is high.

In fact, the accounting report would be always partially tied to the most illiquid possible market, but particularly when accounting based incentives $\delta$ are high.

Recall that this method dictates

$$R_{N}^{EV}(b) \equiv V_{N-1}(b)$$

where $V_{N-1}(b)$ is the expected resale price given that the asset was acquired by paying $b$ and $V_{N-1}(b)$ must be estimated as the expected successful bid among the remaining $N-1$ bidders when method $EV$ is in place. Of course, this resale auction is just a thought experiment only required to estimate the exit price.

Now, if $\Pi_{N-1}^{EV}(w)$ represents expected revenues in a FPA with $N-1$ bidders whose values are known to belong to $[0, w_{N-1}]$, then $V_{N-1}(b)$ could be defined by

$$V_{N-1}(b) = \mathbb{E} [\Pi_{N-1}^{EV}(w_{N-1}) | b].$$

This definition of the exit price is general. It acknowledges the possibility that the resale auction would potentially suffer from greater uncertainty about remaining bidders’ private values than the actual auction. While the seller in the actual auction knows the upper bound of bidders’ values, a reseller would possibly have to estimate this bound.

An interesting feature of this analysis lies in the recursive nature of $V_{N-1}(b)$. The acquirer’s expected revenues from reselling the asset among the remaining $N-1$ potential buyers, depends on what the next buyer would earn if he were also to resell the asset among the remaining $N-2$ bidders and so on until only one potential buyer remains.\(^{30}\) Of course, if the seller does not set a

\(^{30}\)I am implicitly assuming that in this hypothetical sequence of auctions, only loosing bidders remain active in subsequent auctions. In other words succesful bidders cannot re-purchase the asset in subsequent rounds, but they
minimum price in the auction, this recursion would stop at $V_1^{EV} (\cdot) = 0$, because when there is a single bidder, this bidder would optimally bid zero. Consequently, to estimate the expected resale price one has to solve a sequence of hypothetical auctions indexed by the number of bidders that would participate in each round. For convenience, I denote by $t_n = N - n$ the round in which only $n$ (out of the $N$) bidders remain active.

One may wonder why $V_{N-1} (b)$ depends on $b$; after all, buyers’ valuations are independent. The reason that $V_{N-1} (b)$ depends on $b$ is because, in general, the winning bid is a signal of the private values of active bidders so that $b$ indirectly determines the expected resale price.

Let us examine the solution of this problem. In order to obtain the rule that $EV$ would require, we need to solve for the sequence of equilibrium expected resale prices. I denote this sequence by $\{V_{n-1} (b_n)\}_{n=2}^N$, where the number $V_{n-1} (b_n)$ represents the expected resale price when $(n - 1)$ bidders are active and the current and prior winning bids were $b_n = \{b_N, ..., b_{n+1}, b_n\}$.

To find this sequence we need to apply $EV$ recursively in each round. I proceed by defining $EV$ at round $t_n$ analogously as I did before:

$$R_n^{EV} (b_n) = V_{n-1} (b_n)$$

Now, if the equilibrium in each round is such that the asset is allocated to the highest value, then, at round $t_n$, the upper bound of the distribution of values of active bidders denoted by $w_n$, will be updated in a very simple manner: active bidders must have lower private values than the last winner’s inferred value.

From convenience, throughout the following I omit the superscript that identifies $EV$ in the bidding function. So, if $\beta_{n+1} (x)$ denotes the equilibrium bidding strategy at round $t_{n+1}$ relative to regime $EV$, then:

$$w_n = \beta_{n+1}^{EV^{-1}} (b_{n+1}) \text{ for all } n = 1, .. N - 1.$$  

The next lemma formalizes this idea.

**Lemma 2** If in each round, bidding strategies are strictly increasing, then the random variable that describes the upper bound of the distribution of values of active bidders at round $n$ is degenerate, actually drop out of the game. This seems like the most natural way to define the $EV$ standard, but one could certainly consider alternative definitions where a seller could become again a buyer in a later round. If this is the case, the exit price would arise from some bargaining between the first buyer and the second buyer.
and is given by

\[ w_n = \beta_n^{EV-1} (b_{n+1}) \text{ for all } n = 1, \ldots, N - 1. \]

If strategies are increasing, the history of prices will be progressively decreasing because in each round the object will be allocated to the bidder with the highest value. The winning bid becomes thus a signal of the upper bound of active bidders’ private values; moreover, the last winning bid becomes a sufficient statistic for this bound. Furthermore, if strategies are strictly monotonic, then the expected exit price is a Markov chain that depends only on the last period price, thus \( V_n (b_{n+1}) \) boils down simply to \( V_n (b_{n+1}) \). Moreover the monotonicity of the equilibrium implies that the exit price would be \( \Pi_n^{EV} (w_n) \) namely the expected revenues from running an auction among \( n \) bidders whose valuations are known to belong to \([0, w_n]\). The next lemma formalizes these ideas.

**Lemma 3** If in each round, bidding strategies are strictly increasing then:

i) \[ V_{n-1} (b_n) = V_{n-1} (b_{n+1}) \]

Moreover

ii) \[ V_{n-1} (b_n) = \Pi_n^{EV} (w_{n-1} (b_{n+1})) \]

Bidders’ values are known to belong to \([0, w_n (b_{n+1})]\) at \( t_n \). However, what is the posterior distribution of an active bidder value at \( t_n \)? Further, how is this distribution updated in the Bayesian sense? The monotonicity of the equilibrium in each round again simplifies the problem substantially: in each period the distribution is truncated from above. In particular, if \( F_n (x) \) denotes the updated distribution of the value of an active bidder at \( t_n \), then this distribution is given by:

\[ F_n (y) = \frac{F (y)}{F (w_n)} \]

Hence the distribution and density of the maximum among the rivals of an active bidder at \( t_n \) are:

\[ G_n (y) = \left[ \frac{F (y)}{F (w_n)} \right]^{n-1}; \text{ and} \]

\[ g_n (y) = G_n' (y), \]
respectively. These insights reveal that if the problem is regular, the equilibrium is given by a sequence of auctions in which the Bayesian updating about private values follow a very simple structure: active bidders values must be distributed over an interval whose upper bound is inferred from the prior round winner’s bid.

In solving this sequence of auctions we need to keep track of two state variables: the upper bound of the distribution of values $w_n$ and the number of active bidders that participate in the current round $n$. I can now define the equilibrium of this problem.

**Definition 2** An Increasing Symmetric Nash Equilibrium (ISNE) relative to regime EV consists of a sequence of auctions each of which is described by four functions: a bidding function $\beta_n(x)$, an accounting rule $R_n^{EV}(b_n)$, a revenue function $\Pi_n^{EV}(w_n)$ and a Bayesian updating rule $w_n(b_{n+1})$, such that:

a) Given $R_n^{EV}(\cdot)$ and $x$, the bidding strategy $\beta_n(x)$ satisfies

$$\beta_n(x) \in \arg \max_{b_n} \{(1-\delta)x + \delta R_n^{EV}(b_n) - b_n\} G_n(\beta_n^{-1}(b_n))$$

for all $x \in [0, w_n(b_{n+1})]$ and for all $n = 2, \ldots N$.

And $\beta_1(x) = 0$ for all $x \in [0, w_1(\cdot)]$.

b) Given $w_n(\cdot)$ and $\beta_n(x)$, then the revenue function satisfies

$$\Pi_n^{EV}(w_n(\cdot)) = \int_0^{w_n(\cdot)} \beta_n(x) p_n(x) \, dx$$

for $n = 1, \ldots N^{31}$

c) Given $b_n$, $\beta_{n-1}(\cdot)$ and $w_{n-1}(\cdot)$ the equilibrium accounting rule satisfies

$$R_n^{EV}(b_n) = \Pi_{n-1}^{EV}(w_{n-1}(\cdot))$$

for all $n = 1, \ldots N$.

d) Bayesian updating

$$w_{n-1}(b_n) = \beta_n^{-1}(b_n) \text{ for all } n = 2, \ldots N.$$  

e) And boundary conditions:

$$w_N = w$$

$$\Pi_0^{EV} = 0$$

---

31 $p_n(x) = nG_n(x)f_n(x)$ is the density of the highest value among $n$ active bidders. Although not explicit, this density is clearly a function of $w_n$. 

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Notice that bidders’ utility functions are determined as part of the equilibrium, in so far as the accounting rule is defined by the equilibrium. Part a) describes the symmetric equilibrium bidding functions in each round as maximizing bidders’ expected payoffs from participating in the auction. Part b) defines seller’s expected revenues as the expected payment of the winner. Part c) defines the accounting rule recursively as the expected revenues in the resale auction conditional on the bids of prior rounds. Finally, part d) defines how beliefs about active bidders’ valuations evolve along this sequence of auctions. Since I solve a more general problem in the next section, here I content myself with describing the equilibrium by means of an example. In fact, it is instructive to consider an example with a closed form solution.

**Example 4** Assume \( X_i \sim Uniform[0, 1] \).

Then an equilibrium relative to the EV regime is characterized by:

\[
\beta_n(x) = \left[ 1 - \frac{1}{n} \left( \frac{1 - \delta^n}{1 - \delta} \right) \right] x
\]

\[
\Pi_{n}^{EV}(w_n(\cdot)) = \frac{n - \frac{1 - \delta^n}{1 - \delta}}{n + 1} w_n(\cdot)
\]

\[
R_{n}^{EV}(b_n) = \frac{N - 1 - \frac{1 - \delta^{n-1}}{1 - \delta}}{N} b_n
\]

\[
w_n(b_{n+1}) = \frac{b_{n+1}}{1 - \frac{1}{n+1} \left( \frac{1 - \delta^{n+1}}{1 - \delta} \right)}.
\]

For the derivation of this example refer to the appendix.

Figure 1 illustrates expected revenues (or the exit-price for a fixed \( w = 1 \)) and their sensitivity to \( \delta \). There we see that expected revenues are increasing in the number of bidders \( n \) and converge to the upper bound \( w = 1 \) as \( n \) approaches infinity (namely when the market is perfectly competitive). The figure also shows that expected revenues are decreasing in \( \delta \).

### D.2 OPTIMAL RESERVE PRICE

The previous analysis assumed that the selling mechanism was a FPA without reserve price and thus omitted the fact that sellers can optimally choose the way to sell the asset. Here I allow (re)sellers to choose the optimal reserve price within the class of FPA.\(^{32}\) As in the prior section I

\(^{32}\)Since a FPA implements the optimal mechanism there is no loss of generality in restricting the analysis to this particular type of auction.
define EV recursively as

\[ R_{n}^{EV}(b_{n}) \equiv V_{n-1}(b_{n}) \]  

(31)

but now \( V_{n-1}(b_{n}) \) represents the exit price that would arise from an optimal FPA with \( n - 1 \) participants when the re-seller himself acquired the asset by bidding (and paying) \( b_{n} \).

As previously, the equilibrium resale price \( V_{n-1}(b_{n}) \) could, in principle, depend on the whole hypothetical history of prices \( b_{n} \). However if in each round symmetric buyers use an increasing strategy \( \beta_{n}(x) \), then the winner’s value is revealed as the highest among active bidders. Therefore \( b_{n+1} \) is a sufficient statistic for \( V_{n-1}(b_{n}) \) with respect to \( b_{n} \) so that, mutatis mutandis, Lemma 2 and 3 would also hold here. Consequently, we can define a symmetric, strictly increasing and differentiable equilibrium as follows:

**Definition 3** A Bayes-Nash Equilibrium relative to regime EV consists of a sequence of \( N \) optimal auctions, \( \{\Gamma_{n}\}_{n=1}^{N} \). Each auction \( \Gamma_{n} \) is described by five functions: a bidding function \( \beta_{n}(x) \), an accounting rule \( R_{n}^{EV}(b_{n}) \), a revenue function \( \Pi_{n}^{EV}(w_{n}) \), a reserve price function \( x_{n}^{EV}(w_{n}) \), and a Bayesian updating rule \( w_{n}(b_{n+1}) \), such that:

a) Given \( x, R_{n}^{EV}(\cdot), x_{n}^{EV}(\cdot), w_{n}(\cdot) \) and \( b_{n+1} \) the bidding strategy \( \beta_{n}(x) \) satisfies

\[ \beta_{n}^{EV}(x) = \left\{ \begin{array}{ll} \arg \max_{b_{n}} [(1 - \delta) x + \delta R_{n}^{EV}(b_{n}) - b_{n}] G_{n} \left( \beta_{n}^{-1}(b_{n}) \right) & \text{if } n = 2, \ldots, N \\ x_{1}^{EV}(w_{1}) & \text{if } n = 1 \end{array} \right. \]
for all $x \in [x_{n}^{EV}(w_{n}), w_{n}(b_{n+1})]$ otherwise the bidding function is zero.

b) Given $x_{n}^{EV}(\cdot), w_{n}(\cdot)$, and $\beta_{n}(x)$ the revenue function and the reserve price are defined by

$$
\Pi_{n}^{EV}(w_{n}(\cdot)) = \max_{x_{n}^{EV}} \int_{x_{n}^{EV}}^{w_{n}(\cdot)} \beta_{n}(x) p_{n}(x) \, dx
$$

for $n = 1, \ldots, N$.

c) Given $b_{n}, \beta_{n-1}(\cdot)$ and $w_{n-1}(\cdot)$ the equilibrium accounting method satisfies

$$
R_{n}^{EV}(b_{n}) = \Pi_{n-1}^{EV}(w_{n-1}(b_{n}))
$$

for $n = 2, \ldots, N$.

d) Bayesian updating

$$
w_{n-1}(b_{n}) = \beta_{n-1}^{-1}(b_{n})
$$

for $n = 2, \ldots, N$; and

e) boundary conditions:

$$
\Pi_{0} = 0
$$

$$
w_{N} = w
$$

Implicit in the definition of the bidding functions is the fact that bids may depend on the reserve price, not only for the marginal bidder but in general. Part b) defines seller’s expected revenues as the expected payment of the successful bidder. Part c) defines the accounting rule as expected revenues in a resale auction conditional on prior round bids. Finally, part d) defines how beliefs about the valuations of active bidders evolve along the sequence of rounds. Theorem 1 establishes the existence of this equilibrium.

**Theorem 1** There exists a Bayes-Nash Equilibrium under the EV regime given by:

a) Equilibrium bidding strategies:

$$
\beta_{n}^{EV}(x) = \begin{cases} 
(1 - \delta) \left(x - \int_{x_{n}^{EV}} x_{n}^{EV} \frac{G_{n}(y)}{G_{n}(x)} \, dy \right) + \delta \Pi_{n-1}^{EV}(x) & \text{if } n = 2, \ldots, N, \\
\beta_{1}^{EV}(x) & \text{if } n = 1
\end{cases}
$$

for all $x \in [x_{n}^{EV}, w_{n}(b_{n+1})]$, otherwise bid zero.

b) Equilibrium revenues are:

$$
\Pi_{n}^{EV}(w_{n}) = \max_{x_{n}^{EV}} \int_{x_{n}^{EV}}^{w_{n}} \left( \delta \Pi_{n-1}^{EV}(x) + (1 - \delta) \psi_{n}(x) \right) p_{n}(x) \, dx
$$
for \( n = 1, \ldots, N \), where \( \psi_n(x) = x - \frac{1-F_n(x)}{f_n(x)} \).

The reserve price solves

\[
\delta \Pi_{n-1}^{EV}(x_n^{EV}) + (1 - \delta) \psi_n(x_n^{EV}) = 0
\]

c) Equilibrium accounting rule

\[
P_n^{EV}(b_n) = \Pi_{n-1}^{EV}(w_{n-1}(b_n))
\]

for \( n = 1, \ldots, N \).

d) Bayesian updating

\[
w_{n-1}(b_n) = \beta_n^{-1}(b_n)
\]

for \( n = 2, \ldots, N \); and
e) boundary conditions are:

\[
\Pi_0^{EV} = 0
\]

\[
w_N = w
\]

Part a) shows that bidding strategies weigh both the benefits from acquiring the asset as perceived by current shareholders and the asset resale value. Although in equilibrium the asset would never be resold, the combination of bidders’ reporting incentives along with the accounting regime induces the connection between payoffs and resale prices. This linkage also drives the recursive nature of seller’s expected revenues that we see in part b). Seller revenues can only be defined iteratively in reference to the next seller revenues. Furthermore, solving this recursion yields, at the same time, the accounting rule. In fact, the solution to the functional equation of part b) defines the accounting method as stated in part c). Although a closed form solution is not guaranteed, this equation can always be solved recursively.\(^{33}\)

Inspection of this equilibrium reveals that the properties of EV are sensitive to market conditions and asset characteristics. In particular, this method’s undervaluation is amplified by market’s illiquidity and when accounting based incentives are important.\(^{34}\) In fact, the effect of \( \delta \) is particularly strong: as \( \delta \) approaches one, EV would prescribe reporting the acquisition at zero. In a sense

\(^{33}\)Furthermore this equation has a stationary point as \( n \to \infty \). This can easily be established appealing to Banach fixed point theorem.

\(^{34}\)As Plantin, Sapra and Shin (2008) point out, these considerations may underlie banks reluctance to the use of fair values.
this method is paradoxical: the stronger are bidders’ reporting concerns the lower the values they would be allowed to report are. Proposition 8 describes this and other features of the EV regime.

**Proposition 8** The EV method can be characterized as follows:

a) \( \Pi_n^{\text{EV}} (w_n) \) decreases in \( \delta \) and \( \lim_{\delta \to 1} \Pi_n^{\text{EV}} (w_n) = 0 \).

b) \( \Pi_n^{\text{EV}} (w_n) \) is increasing in \( n \), \( \lim_{n \to \infty} \Pi_n (w_n) = w_n \).

c) \( \Pi_n^{\text{EV}} (w_n) \) is increasing in \( w_n \) and \( \Pi_n (0) = 0 \).

d) \( x_n^{\text{EV}} \) is decreasing in \( n \).

e) \( \frac{\partial x_n^{\text{EV}}}{\partial p} \leq 0 \)

f) \( \Pi_n^{\text{EV}} (w_n) \) is differentiable almost everywhere.

Also the efficiency of EV is sensitive to market conditions and asset characteristics. Greater liquidity improves efficiency because it strengthens competition among bidder, thereby allowing the seller to relax the reserve price. Efficiency is also enhanced by accounting incentives \( \delta \). The seller response to an increase in \( \delta \) would be to reduce the reserve price in anticipation of bidders having more depressed payoffs and more lower bidding strategies.

This method illustrates the difference between a principle, like the exit price that is general, and its specific implementation through a rule like \( R_N^{\text{EV}} (\cdot) \). This estimate, being predicated on a notion of equilibrium, is sensitive to market and asset details.

It is instructive to derive the equilibrium using the Uniform distribution example.

**Example 5** In the Uniform distribution example, the revenue function is a linear function of the upper bound \( w_n \):

\[
\Pi_n (w_n) = \gamma_n w_n.
\]

where the coefficient \( \gamma_n \) solves the following difference equation:

\[
\gamma_n = (1 - \delta) \frac{n - 1}{n + 1} + \frac{\delta \gamma_{n-1} n + \left( \frac{1}{2 + \frac{1}{1 - \delta} \gamma_{n-1}} \right)^n (1 - \delta)}{(n + 1)} \tag{32}
\]

\[
\gamma_1 = \frac{1 - \delta}{4}
\]

\[
w_N = 1
\]

and the optimal reserve price is given by

\[
x_n = \frac{w_n}{2 + \frac{\delta}{1 - \delta} \gamma_{n-1}}
\]
Unfortunately (32) does not have a closed form solution, yet its characterization is relatively easy. In fact, one can verify the following properties:

a) \( \Pi_{n}^{EV} \in \left[ \frac{1-\delta}{4}, 1 \right] \).
b) \( \Pi_{n}^{EV} - \Pi_{n-1}^{EV} > 0 \) and \( \lim_{n \to \infty} \Pi_{n}^{EV} = 1 \)
c) \( \frac{\partial \Pi_{n}^{EV}}{\partial \pi} < 0 \) and \( \lim_{p \to 1} \Pi_{n}^{EV} = 0 \) and \( \lim_{\delta \to 0} \Pi_{n}^{EV} = \frac{n-1+2^{-n}}{n+1} \)
d) \( x_{n}^{EV} - x_{n-1}^{EV} < 0 \) and \( \lim_{n \to \infty} x_{n}^{EV} = \frac{1-\delta}{2-\delta} \)
e) \( \frac{\partial x_{n}^{EV}}{\partial \pi} \leq 0 \)

Figure ?? shows the reserve price of EV as a function of \( \delta \). There we see how the reserve price goes down as the number of bidders (and accounting incentives) increase. Interestingly the reserve price is bounded below by a positive number, for all \( \delta > 0 \).

![Reserve prices graph](image.png)

Reserve prices. The graph assumes \( X_i \sim \text{Uniform}[0, 1] \).

### D.3 PROOF OF THEOREM 1

The equilibrium resale price \( R_{n-1}(b_n) \) may generally depend on the whole "history" of hypothetical prices \( \{b_N, b_{N-1}, \ldots, b_{n+1}\} \). If in each round symmetric buyers use an increasing strategy \( \beta_{n}^{EV}(x) \) then the winner’s value is revealed as the highest among active bidders. Therefore \( b_{n+1} \) is a sufficient statistic for \( R_n(\cdot) \) with respect to the history of prices \( \{b_N, b_{N-1}, \ldots, b_{n+1}\} \). So that Lemma 2 and Lemma 3 also hold here.
Knowing that one can adapt the first order conditions (6), together with the definition of EV as stated by (31), in order to obtain the $t_n$ equilibrium bidding strategy which we next present:

$$
\frac{d}{dx} \left[ G_n(x) \beta_n^{EV}(x) \right] = (1 - \delta) x g_n(x) + \delta \frac{d}{dx} \left[ \Pi_{n-1}^{EV} \left( \beta_n^{EV}(x) \right) \right] G_n(x)
$$

(33)

This equation is now solved using the boundary condition defined by equation (??).

To gain further intuition about EV, here I derive the solution in the intuitive order: first I obtain $\beta_n^{EV}$ and then $\Pi_{n}^{EV}(\cdot)$.

The solution to equation (33) yields buyer’s expected payment in the auction at $t_n$ given that the accounting rule is $\Pi_{n-1}^{EV}(\cdot)$. If the reserve price, in $t_n$, is denoted by $x_n$, one can solve equation (33) to obtain the equilibrium bidding strategy:

$$
\beta_n^{EV}(x, x_n) = (1 - \delta) \left[ x_n \frac{G_n(x_n)}{G_n(x)} + \int_{x_n}^{x} \frac{g_n(y)}{G_n(x)} dy \right] + \delta \Pi_{n-1}^{EV}(x)
$$

Although we have not made it explicit here, it is important to recall that $G_n(x)$, much like in Section D.1, is a function of $w_n$. Adapting (10) one can write the seller’s revenue function as:

$$
\Pi_n(w_n) = \max_{x_n^{EV}} \left[ \int_{x_n^{EV}}^{w_n} \left( \delta \Pi_{n-1}^{EV}(x) + (1 - \delta) \psi_n(x) \right) dP_N(x) \right]
$$

(34)

where $\frac{dP_n(x)}{dx} = nG_n(x) f_n(x)$ describes the density of the highest value among $n$, at $t_n$. This equation holds for any given $n$. In particular, since $w_N = w$ then the seller’s expected revenue at round zero is:

$$
\Pi_N^{EV}(w) = \max_{x_N^{EV}} \left[ \int_{x_N^{EV}}^{w} \left( \delta \Pi_{N-1}^{EV}(x) + (1 - \delta) \psi(x; w) \right) dP_N(x; w) \right]
$$

(35)

The solution to equation (35) yields both the sequence of reserve prices and the sequence of revenue functions. The latter in turn defines the accounting rule under EV.

To actually solve equation (34), we need to determine the boundary condition $\Pi_1^{EV}(w_1)$\footnote{Intuitively one could simply use $\Pi_0^{EV} = 0$ as a boundary condition. However solving for $\Pi_1^{EV}$ is probably more insightful.}. It is easy to see that $\Pi_1^{EV} R(w_1)$ is the outcome of the optimal take-it-or-leave it offer that the seller can make to a single bidder whose valuation belongs to $[0, w_1]$, when this bidder expects a resale price of zero. This bidder accepts the offer only if his value $x$ is above the threshold $x^*$ defined by

$$
-\delta p + (1 - \delta) (x^* - p) = 0
$$
or

\[ x^* = \frac{p}{1 - \delta}. \]

Consequently, \( \Pi_1^{EV}(w_1) \) is defined by

\[
\Pi_1^{EV}(w_1) = \max_p \left( 1 - F_1 \left( \frac{p}{1 - \delta} \right) \right) p. 
\] (36)

Although, in general, equation (34) does not have an explicit solution for \( \Pi_n^{EV}(w) \).

D.4 DERIVATION OF EXAMPLE 4

We conjecture that there is an equilibrium where \( \Pi_n(w_n) = \gamma_n w_n \). Then, one can write equation (35) as

\[
\Pi_n(w_n) = \frac{n}{w_n} \int_0^{w_n} \left[ (w_n - y) (1 - \delta) (n - 1) y^{n-1} + \delta \gamma_{n-1} y^n \right] dy
\]

which by a suitable change in variables becomes

\[
\Pi_n(w_n) = nw_n \int_0^1 \left( (1 - \delta) (n - 1) (u^{n-1} - u^n) + \delta \gamma_{n-1} u^n \right) du
\] (37)

hence our conjecture is right and

\[
\Pi_n(w_n) = \gamma_n w_n. 
\] (38)

Moreover, to obtain \( \gamma_n \) we need to solve the following difference equation,

\[
\gamma_n = \frac{(1 - \delta) (n - 1) + n \delta \gamma_{n-1}}{n + 1}
\] (39)

and because in a first price auction \( \Pi(w_1, 1) = 0 \), then \( \gamma_1 = 0 \).

Consequently, the solution to (39) is

\[
\gamma_n = \Pi_n^{EV} = \frac{n}{n + 1} - \frac{1 - \delta^n}{(n + 1) (1 - \delta)}.
\]

Note that \( \gamma_n \) represents the fraction of the true economic value of the asset that can be reported, therefore \( 1 - \gamma_n \) represents the bias of the EV.
D.5 DERIVATION OF EXAMPLE 5

As a first step notice that \( f_n(x) = \frac{1}{w_n}, F_n(x) = \frac{x}{w_n}, G_n(x) = \left( \frac{x}{w_n} \right)^{n-1} \) and \( xg_n(x) = (n-1) \left( \frac{x}{w_n} \right)^{n-1} \).

Then using equation (35), we obtain

\[
\Pi_n(w_n) = \max \left\{ \frac{n}{x_n} \left( \frac{x_n}{w_n} \right)^{n-1} \left( 1 - \frac{x_n}{w_n} \right) + \int_{x_n}^{w_n} \left( 1 - \frac{y}{w_n} \right) (n-1) \frac{y^{n-1}}{w_n} dy + \delta \int_{x_n}^{w_n} \gamma_{n-1} \left( \frac{y}{w_n} \right)^{n-1} \frac{y}{w_n} dy \right\}
\]

Let us conjecture, \( \Pi_{n-1}(x) = \gamma_{n-1}x \). Then

\[
\Pi_n(w_n) = \max \left\{ \frac{n}{x_n} \left( \frac{x_n}{w_n} \right)^{n-1} \left( 1 - \frac{x_n}{w_n} \right) + \int_{x_n}^{w_n} \left( 1 - \frac{y}{w_n} \right) (n-1) \frac{y^{n-1}}{w_n} dy + \delta \int_{x_n}^{w_n} \gamma_{n-1} \left( \frac{y}{w_n} \right)^{n-1} \frac{y}{w_n} dy \right\}
\]

Define \( u = \frac{y}{w_n} \). Then by a change in variables,

\[
\Pi_n(w_n) = n (1 - \delta) \max \left\{ w_n \left( \frac{w_n}{w_n} \right)^{n-1} \left( 1 - \frac{w_n}{w_n} \right) + (n-1) w_n \int_{x_n}^{w_n} (1 - u) u^{n-1} du + n\delta w_n \int_{x_n}^{w_n} \gamma_{\alpha n-1} u^{n-1} du \right\}
\]

If we define a “normalized” reserve price at \( t_n \),

by \( A_n = \frac{x_n}{w_n} \). Then,

\[
\Pi_n(w_n) = n (1 - \delta) \max \left\{ A_nA_n^{n-1} (1 - A) + (n-1) w_n \int_{A_n}^{1} (1 - u) u^{n-1} du + n\delta w_n \int_{A_n}^{1} \gamma_{\alpha n-1} u^{n-1} du \right\}
\]

Hence,

\[
\Pi_n(w_n) = w_n n \max \left\{ \left( 1 - \delta \right) A_n^{n-1} (1 - A) + (n-1) \int_{A}^{1} (1 - u) u^{n-1} du + \delta \int_{A}^{1} \gamma_{\alpha n-1} u^{n-1} du \right\}
\]

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which means that our conjecture \( \Pi_n (w_n) = \gamma_n w_n \) was correct.

Optimizing with respect to \( A \), yields

\[
\Pi_n = \frac{\delta \gamma_{n-1} n + (n - 1 + A_n^n) (1 - \delta)}{(n + 1)} w_n
\]

at

\[
A_n = \frac{1}{2 + \frac{\delta}{1 - \delta} \gamma_{n-1}}
\]

Now, in order to obtain the boundary condition \( \Pi_1 \) we need to solve

\[
\Pi_1 = \max_p \left( 1 - \frac{p}{w_1} \right)
\]

Which implies \( \gamma_1 = \frac{1 - \delta}{4} \). Hence \( \gamma_n \) comes from the solution to

\[
\gamma_n = (1 - \delta) \frac{n - 1}{n + 1} + \frac{\delta \gamma_{n-1} n + \left( \frac{1}{2 + \frac{\delta}{1 - \delta} \gamma_{n-1}} \right)^n (1 - \delta)}{(n + 1)}
\]

\[
(40)
\]

Unfortunately this non linear difference equation does not have a closed form solution, yet its characterization is relatively easy. In fact, one can verify the following properties for this uniform example,

a) \( \Pi_n \in \left( \frac{1 - \delta}{4}, 1 \right) \).

b) \( \Pi_n - \Pi_{n-1} > 0 \) and \( \lim_{n \to \infty} \Pi_n = 1 \)

c) \( \frac{\partial \Pi_n}{\partial p} < 0 \) and \( \lim_{p \to -1} \Pi_n = 0 \) and \( \lim_{\delta \to 0} \Pi_n = \frac{n-1+2^{-n}}{n+1} \)

d) \( x_{EV}^n - x_{EV}^{n-1} < 0 \) and \( \lim_{n \to \infty} x_{EV}^n = \frac{1 - \delta}{2 - \delta} \)

e) \( \frac{\partial x_{EV}^n}{\partial p} \leq 0 \)
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