Property Rights and the Efficiency of Bargaining*

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1 Introduction

Property rights specify an initial default position from which agents may subsequently bargain to determine their ultimate allocation. Following the seminal article of Grossman and Hart (1986), the economics literature discussing the optimal allocation of property rights has largely focused on how they affect ex ante investments, under the assumption that bargaining always results in ex post efficient outcomes. In this paper, we instead examine how property rights affect the efficiency of bargaining and the final allocations that result.[1]

According to the "Coase Theorem" (Coase 1960), in the absence of "transaction costs," parties will reach Pareto efficient agreements regardless of initial property rights. We instead examine settings in which this may not happen due to transaction costs associated with asymmetric information. That property rights may matter for the efficiency of bargaining can be seen by comparing Myerson and Satterthwaite's (1983) conclusion that private

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[1]Matouschek (2004) studies this question as well; we discuss the relation to our paper below.

information must generate inefficiency in bargaining between a buyer and a seller, with Cramton, Gibbons, and Klemperer's (1987) demonstration that efficient bargaining mechanisms do exist for more evenly distributed (or randomized) property rights. Here we examine more broadly the nature of optimal property rights in such settings.

In addition to simple property rights of the sort considered by Myerson and Satterthwaite (1983) and Cramton et al. (1987), the legal literature has considered other forms of property rights. Calebresi and Melamud (1972) first highlighted the distinction between "property rules," which correspond to the simple property rights of the economics literature, and "liability rules," in which an agent may harm another agent (e.g., by polluting) but must make a damage payment to the victim. These liability rules may therefore be thought of as an option-to-own.[2] Calabresi and Melamud (1972) considered such liability rules to be desirable only when bargaining is impractical (in which case they can make the final allocation responsive to values), but subsequent work [Ayres and Talley (1994), Kaplow and Shavell (1995-6, 1996), Ayres (2005)] has suggested the possibility that liability rules may also be desirable when bargaining is possible but imperfect. Ayres (2005) also considered in a two-agent setting "dual chooser" rules in which both agents can exercise options. In general, liability rules and dual-chooser rules may both be viewed as particular forms of *property rights mechanisms*, in which the default outcome depends on messages sent by the various agents.

This paper advances the literature in two ways. First, we establish a wide class of economic settings and property rights (including both simple property rights and liability rules) in which efficient bargaining is impossible. Second, for these environments in which inefficiency is unavoidable, we examine the optimal allocation of property rights, including simple property rights, liability rules, and dual-chooser rules.[3] In addition to its implications for the allocation of formal property rights, our analysis can also be applied to the allocation of decision rights within firms.

Our inefficiency result unifies a number of results in the earlier literature (Myerson and Satterthwaite (1983), Mailath and Postlewaite (1990), Williams (1999), Figueroa and Skreta (2008), Che (2006)). In contrast to the earlier literature, our approach to establishing inefficiency does not require

[2]Demski and Sappington (1991) and Noldeke and Schmidt (1995) consider the use of option-to-own contracts to induce efficient ex ante investments.

[3]In Segal and Whinston (2011) we establish sufficient conditions for first-best efficiency to be achieved.

performing any computations. Instead, it requires only the verification of two simple conditions: (i) existence of "adverse opt-out types" and (ii) non-emptiness of the core (actually, non-emptiness of a larger set that we call the "marginal core") and its multi-valuedness with a positive probability.[4]

We define an "opt-out type" as a type whose non-participation is consistent with efficiency (for any types of the other agents). In addition, for settings that involve externalities, such as liability rules, we define an "adverse type" as a type who, when he does not participate and behaves noncooperatively (e.g., chooses optimally whether to damage others under a liability rule), minimizes the total expected surplus available to the other agents. (In settings with simple property rights, in which externalities are absent, any type is trivially an adverse type.) Our inefficiency result applies when each agent has a type that is simultaneously an opt-out type and an adverse type. This assumption is clearly restrictive – for example, it is not satisfied in the presence of intermediate (or randomized) property rights of the kind considered by Cramton et al. (1987) and Segal and Whinston (2011). Nevertheless, we show that this assumption is satisfied in a number of settings involving simple property rights, liability rules, and dual-chooser rules. (We also allow this assumption to hold in an asymptotic form: e.g., a type may become an "almost" adverse opt-out type as the type goes to $+\infty$.)

In contrast, the non-emptiness and multi-valuedness of the core is a typical feature of economic settings. For example, if, under an appropriate definition of "goods," a price equilibrium exists (e.g., a Walrasian equilibrium, or a Lindahl equilibrium), then it will be in the core, and "generically" the core will be multi-valued (except for some limiting "competitive" cases with a large number of agents, where the core may converge to a unique Walrasian equilibrium).

Having identified a class of settings in which achieving efficiency is impossible, we then turn to an analysis of the optimal allocation of property rights in those cases. In doing so, we take a mechanism design approach to bargaining, asking what property rights would be optimal if bargaining takes as efficient a form as possible given the allocation of property rights.

We use two different measures of effiiciency to identify optimal property rights. In the first, we assume that there is an outside agency who will subsidize the bargaining process in order to achieve efficiency and we examine

[4]Precursors to this approach can be found in Makowski and Ostroy (1989) and Segal and Whinston (2012).

the effect of property rights on the expected subsidy that is required. One corollary of our impossibility analysis is a simple formula for this expected subsidy. The formula allows us to compare the subsidies required by the various property rights that satisfy (i) and (ii). Among such property rights, we can identify those that minimize the intermediary's expected subsidy.

One interesting benchmark for comparison is the property rights that would maximize the expected surplus were bargaining impossible. With two agents and simple property rights that induce opt-out types, we show that the intermediary's expected first-best subsidy equals the expected bargaining surplus, and therefore minimizing this expected subsidy is equivalent to maximizing the expected status quo surplus. For example, in the buyer-seller model of Myerson and Satterthwaite (1983), if we can choose who should initially own the object, it is optimal to give it to the agent with the higher expected value for it. We also identify the optimal option-to-own (liability rule) in this same setting, and show that it is exactly the same as the optimal option-to-own when bargaining is impossible, involving an option price (damage payment) that equals the expected value (harm) of the non-choosing agent (victim). As in the case without bargaining, the optimal option-to-own is strictly better than the best simple property right.

However, the equivalence between what is best for minimizing the expected first-best bargaining subsidy and what is best absent bargaining generally breaks down when we have more than two agents: in such cases, we instead want to raise the values of coalitions including all but one agent (reducing the "hold-out power" of individual agents). We illustrate the difference in two examples, one concerning the optimal ownership of spectrum, and the other examining the optimal liability rule when there ar emany victims.

Evaluating property rights by their effects on the expected subsidy required for first-best bargaining may not be the right thing to do, since in most cases a benevolent mediator willing to subsidize bargaining is not available. Our second efficiency measure is instead the maximal ("second-best") expected surplus that can be achieved in budget-balanced bargaining. Analysis of the second-best problem is complicated by the fact that the optimal allocation rule depends on the identity of the agents least willing to participate (the "critical types"), which in turn depends on the allocation rule. Unfortunately, we are unable to solve for the second-best bargaining procedure at a comparable level of generality to our first-best subsidy calculation. For this reason, we focus on the simple case of just two agents whose values are drawn from the interval $[0, 1]$. For this case, we characterize the second-best bargaining mechanism for a given liability rule. We then illustrate the solution and the dependence of the maximal expected surplus on the option price for the case in which both agents' values are distributed uniformly. We find that the second-best expected surplus is maximized by setting the option price equal to the expected value of the affected party, which is $1/2$ under our normalization. Thus, the optimal option price under second-best bargaining proves to be the same as the price that minimizes the expected first-best subsidy, which is in turn the same as the optimal price in the absence of bargaining. (We conjecture that this equivalence between the optimal option price with and without bargaining extends to all caes in which the value of the party exercising the option is drawn uniformly.)

However, we also find a significant difference in how the second-best expected surplus and the expected first-best subsidy vary with the option price. Namely, while in the uniform example the expected first-best subsidy is always lower the closer the price is to $1/2$, the second-best expected surplus does not increase monotonically with such changes. Instead, we find that setting the price close to 0 or to 1 yields a lower expected second-best surplus than setting it at exactly 0 or 1 (which corresponds to giving one of the agents a simple property right to the object). In fact, we show that the same conclusion extends to *all* distributions of the two agents' valuations (not just uniform). Thus, contrary to the intuition one might take from the results of Cramton et. al (1987), less extreme property rights (in the form of a option price that sometimes leads to exercise of the option) may be worse than extreme ones.

In addition to Myerson and Satterhwaite (1983) and Cramton, Gibbons, and Klemperer (1987), a number of other papers examine the effect of property rights on bargaining efficiency. Most, like our previous paper [Segal and Whinston (2011)] establish conditions under which the first best is achievable [see our (2011) paper or Segal and Whinston (2013) for additional references]. Matouschek (2004) was the first paper to consider second-best optimal property rights under asymmetric information bargaining. He studied a model in which asset ownership x is set irrevocably ex ante, and bargaining over other decisions q occurs ex post after agents' types are determined. In contrast to much of our analysis, bargaining is not allowed to redistribute the initial property rights. He finds that, depending on the parameters, the optimal property rights x will either maximize the total surplus at the disagreement point (as if no renegotiation were possible) or minimize it (as if renegotiation were possible over both x and q). Mylovanov and Troger (2012) analyzes a two-agent setting like ours, but instead uses a specific bargaining protocol in which one agent has the power to make a take-it-or-leave-it offer to the other agent. Finally, in unpublished notes, Che (2006) examines the optimal option-to-own for minimizing the expected first-best subsidy.

The paper is organized as follows: In Section 2 we describe our basic model. Section 3 derives our inefficiency result. In Section 4, we analyze the optimal property rights for minimizing the first-best subsidy. Section 5 analyzes optimal second-best property rights. Section 6 extends our analysis to consider dual-chooser rules. Finally, Section 7 concludes.

2 Set-Up

We consider a general model with N agents, indexed by $i = 1, ..., N$, who bargain over a nonmonetary decision $x \in X$, as well as a vector $t \in \mathbb{R}^N$ of monetary transfers. Each agent i privately observes a type $\theta_i \in \Theta_i$, and his resulting payoff is $v_i(x, \theta_i) + t_i$. We assume that the types $(\tilde{\theta}_1, \ldots, \tilde{\theta}_N) \in \Theta_1 \times \ldots \times \Theta_N$ are independent random variables.

We will be interested in examing what is achievable given some initial property rights when the agents engage in the best possible bargaining procedure after their types are realized. To this end, we take a mechanim design approach to bargaining. Appealing to the Revelation Principle, we focus on direct revelation mechanisms $\langle \chi, \tau \rangle$, where $\chi : \Theta \to X$ is the decision rule, and $\tau : \Theta \to \mathbb{R}^N$ is the transfer rule. In particular, we will be interested in implementing an *efficient* decision rule χ^*, which solves:

$$\chi^*(\theta) \in \underset{x \in X}{\arg\max} \sum_i v_i(x, \theta_i) \text{ for all } \theta \in \Theta.$$

We let $V(\theta) \equiv \sum_i v_i(\chi^*(\theta), \theta_i)$ be the maximum total surplus achievable in state θ.

When considering direct revelation mechanisms that correspond to bargaining mechanisms, we restrict them to satisfy *budget balance*:

$$\sum_i \tau_i(\theta) = 0 \text{ for all } \theta \in \Theta.$$

and *(Bayesian) Incentive Compatibility*:

$$\begin{aligned} & \mathbb{E}[v_i(\chi(\theta_i, \widetilde{\theta}_{-i}), \theta_i) + \tau_i(\theta_i, \widetilde{\theta}_{-i})] \\ \geq \ & \mathbb{E}[v_i(\chi(\theta'_i, \widetilde{\theta}_{-i}), \theta_i) + \tau_i(\theta'_i, \widetilde{\theta}_{-i})] \text{ for all } i, \theta_i, \theta'_i \in \Theta_i \end{aligned}$$

Next we consider participation constraints. For this purpose, we need to describe what outcome each agent i expects when he refuses to participate in the bargaining mechanism. In general, this outcome will depend on the types of the other agents. For example, the other agents may make some noncooperative choices under a liability rule, and these choices may depend on their types. Alternatively, the other agents may be able to bargain with each other over some parts of the outcome without the participation of agent i, and this bargaining may have externalities on agent i. It is also possible that if agent i refuses to participate, the default will involve a noncooperative game among agents, and the outcome of this game will depend on all the agents' types.

To incorporate all these possibilities, we assume that if agent i refuses to participate and the state of the world is θ, the nonmonetary decision is $\hat{x}_i(\theta)$, and agent i receives a transfer $\widehat{t}_i(\theta)$. The resulting reservation utility of agent i is therefore

$$\widehat{V}_i(\theta) \equiv v_i(\hat{x}_i(\theta), \theta_i) + \widehat{t}_i(\theta).$$

For example, in the simple special case of a fixed status quo $\left(\hat{x}, \widehat{t}_1, ..., \widehat{t}_N\right)$ that either cannot be renegotiated at all without all agents' participation or whose renegotiation by a subset of agents does not affect nonparticipating agents (e.g., because renegotiation can only involve exchange of private goods), the reservation utility would take the form $\widehat{V}_i(\theta) = v_i(\hat{x}, \theta_i) + \widehat{t}_i$. In general, the functions $\hat{x}_i(\theta)$ and $\widehat{t}_i(\theta)$ depend on both the property rights and assumptions about bargaining, but for most of the analysis we will take these functions as given.

Given these functions and the resulting reservation utility, the (interim) individual rationality constraints of agent i can be written as

$$\mathbb{E}[v_i(\chi(\theta_i, \tilde{\theta}_{-i}), \theta_i) + \tau_i(\theta_i, \tilde{\theta}_{-i})] \geq \mathbb{E}[\widehat{V}_i(\theta_i, \tilde{\theta}_{-i})] \text{ for all } \theta_i. \qquad (1)$$

We will say that property rights *permit efficient bargaining* if they induce functions $\{\hat{x}_i(\cdot), \widehat{t}_i(\cdot)\}_{i=1}^N$ such that there exists a budget-balanced, incentive-compatible, and individually rational mechanism implementing an efficient decision rule $\chi^*(\cdot)$.

3 An Inefficiency Theorem

In this Section, we provide a set of sufficient conditions ensuring that efficient bargaining is impossible given a set of initial property rights. Our result will have Myerson and Satterthwaite's (1983) result, and several others as special cases.

3.1 Characterization of Intermediary Profits

It will prove convenient to focus on mechanisms in which, for some vector of types $(\widehat{\theta}_1, ..., \widehat{\theta}_N)$, payments take the following form:

$$\tau_i(\theta|\hat{\theta}_i) = \sum_{j\neq i} v_j\left(\chi^*\left(\theta\right), \theta_j\right) - K_i(\hat{\theta}_i) \tag{2}$$

$$\text{where } K_i(\hat{\theta}_i) = \mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i})]. \tag{3}$$

Note that these payments describe a Vickey-Clarke-Groves ("VCG") mechanism [see Mas-Colell, Whinston, and Green (1995), Chapter 23]. The portion of the payment that depends the agents' announcements, $\sum_{j\neq i} v_j\left(\chi^*\left(\theta\right), \theta_j\right)$, causes each agent i to fully internalize his effect on aggregate surplus, thereby inducing him to announce his true type and implementing the efficient allocation rule $\chi^*(\cdot)$. The fixed participation fee $K_i(\hat{\theta}_i)$, on the other hand, equals type $\hat{\theta}_i$'s expected gain from participating in the mechanism absent the fixed charge, so it causes that type's IR constraint to hold with equality. If we imagine that there is an intermediary in charge of this trading process, its expected profit with this mechanism, assuming all agents participate, is given by

$$\begin{aligned}\pi(\hat{\theta}) &= \mathbb{E}\left[\sum_i \tau_i(\tilde{\theta}|\hat{\theta}_i)\right] \\ &= \left(\sum_i \mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i})]\right) - (N-1)\,\mathbb{E}[V(\tilde{\theta})]. \end{aligned} \tag{4}$$

To ensure that all types participate, the participation fee for each agent i can be at most $\inf_{\hat{\theta}_i\in\Theta} K_i(\hat{\theta}_i)$, resulting in an expected profit for the intermediary of

$$\overline{\pi} \equiv \inf_{\hat{\theta}\in\Theta} \pi(\hat{\theta}). \tag{5}$$

If there exists a type $\hat{\theta}_i$ achieving the infimum, i.e.,

$$\hat{\theta}_i \in \arg\min_{\theta_i \in \Theta_i} \mathbb{E}\left[V(\theta_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\theta_i, \tilde{\theta}_{-i})\right],$$

it will be called agent i's *critical type*. This is a type that has the lowest net expected participation surplus in the mechanism.

The sign of the expected profit (5) determines whether property rights permit efficient bargaining:[5]

Lemma 1 *(i) Any property rights at which $\overline{\pi} \geq 0$ permit efficient bargaining. (ii) If, moreover, for each agent i, Θ_i is a smoothly connected subset of a Euclidean space, and $v_i(x, \theta_i)$ is differentiable in θ_i with a bounded gradient on $X \times \Theta$, then property rights permit efficient bargaining only if $\overline{\pi} \geq 0$.*

3.1.1 Adverse Opt-Out Types

For each agent i, let

$$\widehat{V}_{-i}(\theta) \equiv \sum_{j \neq i} v_j(\hat{x}_i(\theta), \theta_j) - \widehat{t}_i(\theta)$$

denote the joint payoff of agents other than i when agent i chooses not to participate in the bargaining mechanism. (Observe that we assume there is budget balance in the event of nonparticipation, so that the collective transfer to agents other than i when agent i opts out is $-\widehat{t}_i(\theta)$.) Since $V(\theta)$ is the maximal achevable surplus in state θ, we have:

$$\widehat{V}_i(\theta) + \widehat{V}_{-i}(\theta) = \sum_j v_j(\hat{x}_i(\theta), \theta_j) \leq \sum_j v_j(\chi^*(\theta), \theta_j) = V(\theta) \text{ for all } \theta \in \Theta. \tag{6}$$

Definition 2 *Given property rights, type θ_i of agent i is an **opt-out type** if $\hat{x}_i(\theta_i, \theta_{-i}) = \chi^*(\theta_i, \theta_{-i})$ for all θ_{-i}.*

[5] Versions of this result appear, for example, in Makowski and Mezzetti (1994), Krishna and Perry (1998), Neeman (1999), Williams (1999), Che (2006), Schweizer (2006), Figueroa and Skreta (2008), Segal and Whinston (2011), and Segal and Whinston (2012). Part (i) of the Lemma can be proven by building a budget-balanced mechanism as suggested by Arrow (1979) and d'Aspremont and Gérard-Varet (1979), and satisfying all agents' participation constraints with appropriate lump-sum transfers. Part (ii) follows from the classical Revenue Equivalence Theorem.

Note that if θ_i is an opt-out type, then $V(\theta_i, \theta_{-i}) = \widehat{V}_i(\theta_i, \theta_{-i}) + \widehat{V}_{-i}(\theta_i, \theta_{-i})$ for all θ_{-i}. That is, there are never any gains from trade between type θ_i and the other agents, regardless of their types.

Definition 3 *Given property rights, type θ_i of agent i is an **adverse type** if it minimizes $\mathbb{E}[\widehat{V}_{-i}(\theta_i, \tilde{\theta}_{-i})]$.*

Type θ_i is an adverse type if, conditional on agent i opting out, agents other than i are worst off collectively when agent i's type is θ_i. Note in particular that any type θ_i is trivially an adverse type when agent i imposes no externalities on the other agents, so $\widehat{V}_{-i}(\theta)$ does not depend on θ_i. This is the case, for example, with simple property rights.

Example 4 *Suppose that each of two agents $i = 1, 2$ has a value θ_i for an object, where θ_i is drawn from distribution F_i Agent 1 faces a liability rule with price p. In this case, $\widehat{V}_1(\theta_1, \theta_2) = \max\{\theta_1 - p, 0\}$ and $\widehat{V}_2(\theta_1, \theta_2) = \theta_2 F_1(p) + (1 - F_1(p))p$. Then agent 2's type $\theta_2 = p$ is an opt-out type, since when he has this value the outcome of agent 1's exercise decision is efficient regardless of agent 1's type. That type of agent 2 is trivially an adverse type because agent 1"s payoff when exercising the option does not depend on agent 2's type.*

On the other hand, both $\theta_1 = 0$ and $\theta_1 = 1$ are opt-out types for agent 1. Of these two types, $\theta_1 = 0$ is an adverse type for agent 1 when $\mathbb{E}[\theta_2] < p$, since then agent 2 prefers for agent 1 to exercise the option and agent 1 never does, while $\theta_1 = 1$ is an adverse type for agent 1 when $\mathbb{E}[\theta_2] > p$.

The significance of these definitions for our results stems from the following observation:

Lemma 5 *When agent i has a type θ_i° that is both an adverse type and an opt-out type, it is a critical type.*

Proof. We can then write for all $\theta_i \in \Theta_i$,

$$\begin{aligned}
\mathbb{E}\left[V(\theta_i^\circ, \tilde{\theta}_{-i}) - \widehat{V}_i(\theta_i^\circ, \tilde{\theta}_{-i})\right] &= \mathbb{E}\left[\widehat{V}_{-i}(\theta_i^\circ, \tilde{\theta}_{-i})\right] \\
&\leq \mathbb{E}\left[\widehat{V}_{-i}(\theta_i, \tilde{\theta}_{-i})\right] \\
&\leq \mathbb{E}\left[V(\theta_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\theta_i, \tilde{\theta}_{-i})\right]
\end{aligned}$$

where the equality is because θ_i° is an opt-out type, the first inequality is because θ_i° is an adverse type, and the second inequality is by (6). ■

Our results will apply not only to settings in which adverse opt-out types exist, but also to settings in which their existence is only of the following asymptotic form:

Definition 6 *The **adverse opt-out property** holds for agent i if there exists a sequence $\{\theta_i^k\}_{k=1}^\infty$ in Θ_i such that as $k \to \infty$,*

$$\begin{aligned} \mathbb{E}\left[V(\theta_i^k, \tilde{\theta}_{-i}) - \widehat{V}_i(\theta_i^k, \tilde{\theta}_{-i}) - \widehat{V}_{-i}(\theta_i^k, \tilde{\theta}_{-i})\right] &\to 0, \textit{ and} \\ \mathbb{E}\left[\widehat{V}_{-i}(\theta_i^k, \tilde{\theta}_{-i})\right] &\to \inf_{\hat{\theta}_i} \mathbb{E}\left[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})\right]. \end{aligned}$$

Note that this property holds whenever agent i has an adverse opt-out type θ_i° (in which case we can let $\theta_i^k = \theta_i^\circ$ for all k), but it may also hold in other cases – e.g., sometimes we may need to take a sequence with $\theta_i^k \to +\infty$ (in which case we may say informally that $\theta_i = +\infty$ is an adverse opt-out type). This property allows us to express the intermediary's expected profits as follows:

Lemma 7 *If the the adverse opt-out property holds for each agent, the intermediary's profits (5) can be written as follows:*

$$\bar{\pi} = \sum_i \inf_{\hat{\theta}_i \in \Theta_i} \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] - (N-1)\,\mathbb{E}[V(\tilde{\theta})]. \tag{7}$$

Proof.

$$\begin{aligned} \bar{\pi} &= \sum_i \inf_{\hat{\theta}_i \in \Theta_i} \mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i})] - (N-1)\,\mathbb{E}[V(\tilde{\theta})] \\ &= \sum_i \inf_{\hat{\theta}_i \in \Theta_i} \left\{\mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] + \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})]\right\} - (N-1)\,\mathbb{E}[V(\tilde{\theta})]. \end{aligned}$$

On the one hand, (6) guarantees that this expression is greater or equal to the right-hand side of (7). On the other hand, the adverse opt-out property for agent i ensures that

$$\inf_{\hat{\theta}_i \in \Theta_i} \left\{\mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] + \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})]\right\} \le \inf_{\hat{\theta}_i \in \Theta_i} \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})].$$

Hence, if this holds for all agents, we obtain (7). ■

3.2 Inefficiency Result

Proof. The adverse opt-out property is a restrictive property, but it will hold in a number of settings of interest. On the other hand, the second property we require in Proposition 9 below is usually satisfied. It makes use of the following notion: ■

Definition 8 *$w \in \mathbb{R}^N$ is a **marginal core payoff vector** in state θ if*

(i) $\sum_{j \neq i} w_j \geq \widehat{V}_{-i}(\theta)$ *for all i, and*

(ii) $\sum_i w_i = V(\theta)$.

Compared to the usual notion of the core, the marginal core considers only coalitions that include N-1 agents. Condition (i) simply says that the coalition consisting of all agents except agent i does not block (assuming "blocking" yields the coalition the same collective payoff it receives when agent i opts out), while condition (ii) says that the maximal total surplus is achieved. Using (ii), condition (i) can be rewritten as $w_i \leq V(\theta) - \widehat{V}_{-i}(\theta)$, i.e., no agent i can receive more than his marginal contribution to the total surplus.

Proposition 9 *Suppose that the assumptions of Lemma 1(ii) hold, the adverse opt-out property holds for each agent, and the set of marginal core payoff vectors is non-empty in all states and multi-valued with a positive probability. Then efficient bargaining is impossible.*

Proof. (7) implies that

$$\begin{aligned}\bar{\pi} &\leq \sum_i \mathbb{E}[\widehat{V}_{-i}(\tilde{\theta})] - (N-1)\,\mathbb{E}[V(\tilde{\theta})] \\ &= \mathbb{E}\left[V(\tilde{\theta}) - \sum_i [V(\tilde{\theta}) - \widehat{V}_{-i}(\tilde{\theta})]\right] \qquad (8)\end{aligned}$$

Now, for a marginal core payoff vector w, for each i we have

$$w_i \leq V(\theta) - \widehat{V}_{-i}(\theta).$$

If the marginal core is multi-valued, then there exists such a w with at least one inequality strict, and so

$$V(\theta) = \sum_i w_i < \sum_i \Big(V(\theta) - \widehat{V}_{-i}(\theta) \Big) .$$

If this inequality holds with positive probability, (8) implies that $\overline{\pi} < 0$, and so the impossibility of efficient bargaining is implied by Lemma 1(ii). ■

3.3 Some Applications

The assumptions of Proposition 9 cover many classical economic settings. For one example, consider the double-auction setting of Williams (1999), in which there are N_s sellers with values drawn from a distribution on $[\underline{\theta}_s, \overline{\theta}_s]$ and N_b buyers with values drawn from a distribution on $[\underline{\theta}_b, \overline{\theta}_b]$ with $(\underline{\theta}_b, \overline{\theta}_b) \cap (\underline{\theta}_s, \overline{\theta}_s) \neq \varnothing$. Since this is a setting without externalities, all types are trivially adverse types. Moreover, since the IR constraints (1) imply that the functions $V_{-i}(\cdot)$ have no effect on whether efficient bargaining can be achieved, in this setting we can without loss assume that agents $-i$ trade efficiently among themselves in the event that agent i opts out. If so, then (i) a buyer of type $\underline{\theta}_b$ is an opt-out type if either $\underline{\theta}_b \leq \underline{\theta}_s$ or $N_b > N_s$, and (ii) a seller of type $\overline{\theta}_s$ is an opt-out type if either $\overline{\theta}_s \geq \overline{\theta}_b$ or $N_s > N_b$. Moreover, a competitive equilibrium exists in every state and is not unique with a positive probability. Since a competitive equilibrium is always in the core (and, hence, in the marginal core), Proposition 9 applies when both (i) and (ii) hold.[6]

Proposition 9 also applies to the public good setting of Mailaith and Postlewaite (1990), in which each of N consumers' values is drawn from a distribution on $[0, \overline{\theta}]$, and the cost of provision is $c > 0$ (whose allocation could be assumed to be equal split among participating agents in the default outcome). Assume (without loss of generality) that if an agent i opts out, the other agents choose the level of the public good to maximize their joint payoff,

[6] The argument can also be extended to show impossibility whenever $N_b = N_s$. In this case, note that in an efficient allocation any agent of type below $\underline{\theta} \equiv \max\{\underline{\theta}_s, \underline{\theta}_b\}$ receives an object with probability zero, so is therefore indistinguishable from type $\underline{\theta}$, and any agent of type above $\overline{\theta} \equiv \min\{\overline{\theta}_s, \overline{\theta}_b\}$ receives an object with probability one, so is therefore indistinguishable from type $\overline{\theta}$. Therefore, the profit in the mechanism must be the same as if all agents' types were instead distributed on the same interval $[\underline{\theta}, \overline{\theta}]$ (with possible atoms at its endpoints), in which case efficient bargaining is impossible by the argument in the text.

without any payment to or from agent i. In this case, each agent's type 0 is both an opt-out type and an adverse type (minimizing the probability of provision). Since a Lindahl equilibrium exists in every state and is not unique with a positive probability, and a Lindahl equilibrium is in the core (and the marginal core), Proposition 9 applies.

In the double-auction setting, Gresik and Satterthwaite (1989) find that the inefficiency in an ex ante optimal mechanism shrinks to zero as $N_b, N_s \to \infty$. Intuitively, this relates to the fact that the core converges (in probability) to the unique competitive equilibrium of the continuous limit economy, hence in the limit the agents can fully appropriate their marginal contributions [as in Makowski and Ostroy (1989, 1995, 2001)]. .Speciifcally, a buyer's marginal contribution, in the limit, equals his value minus the equilibrium price, while a seller's marginal contribution equals the equilibrium price minus his cost. So the intermediaries profit converges to zero. In contrast, in the public good setting of Mailaith and Postlewaite (1990), the core grows in relative size as $N \to \infty$, and inefficiency is exacerbated: each agent's marginal coontribution when the good is provided and he is non-pivotal equals his value. As the number of agents grows large, each agent becomes non-pivotal with probability one, and the intermediaries deficit approaches the probability of provision times the project's cost. (In second-best, the probability of providing the public good in any mechanism goes to zero)

4 Optimal Property Rights for Minimizing the Expected First-Best Subsidy

Recall from Lemma 7 that the expected subsidy needed to achieve the first best when the adverse opt-out property holds for all agents:

$$\overline{\pi} = \inf_{\hat{\theta}_1,\ldots,\hat{\theta}_N} \left[V(\tilde{\theta}) - \sum_i [V(\tilde{\theta}) - \widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] \right]. \tag{9}$$

Using this formula we rank property rights possessing this property in terms of this criterion, which in general amounts to maximizing the sum $\sum_i \inf_{\hat{\theta}_1} \widehat{V}_{-i}(\theta_i^\circ, \tilde{\theta}_{-i})$. For example, when adverse opt-out types $\theta_1^\circ, ..., \theta_N^\circ$ exist for all agents, the property rights should be set to maximize the sum of payoffs received by the adverse opt-out types (and, hence, critical types) of the remaining agents when a single agent opts out. In the remainder of this section we explore the implications of this prescription.

4.1 Two Agents

We first consider situations with two agents and analyze optimal property rights for an indivisible good. Specifically, as in Myerson and Satterthwaite (1983), each agent i's value θ_i for the good is drawn from a full support distribution F_i on $[0, 1]$. We first consider which of the two agents should own the good if the goal is to minimize the expected first-best bargaining subsidy. We then investigate whether options to own can improve on simple ownership. This corresponds to the legal literature's question of whether property rules or liability rules are better.[7]

4.1.1 Who Should Own?

Consider any situation with two agents in which the property rights induce a fixed status quo $(\widehat{x}, \widehat{\tau}_1, ..., \widehat{\tau}_N)$, and the two agents both have adverse opt-out types. We then have

$$\begin{aligned}\widehat{V}_{-1}(\theta_1^\circ, \theta_2) &= v_2(\widehat{x}, \theta_2) + \widehat{\tau}_2, \\ \widehat{V}_{-2}(\theta_1, \theta_2^\circ) &= v_1(\widehat{x}, \theta_1) + \widehat{\tau}_1,\end{aligned}$$

and, using (9), the expected first-best subsidy is

$$\begin{aligned}\overline{\pi}(\hat{x}) &= \mathbb{E}\{V(\tilde{\theta}) - [V(\tilde{\theta}) - \widehat{V}_{-1}(\theta_1^\circ, \widetilde{\theta}_2)] - [V(\tilde{\theta}) - \widehat{V}_{-2}(\widetilde{\theta}_1, \theta_2^\circ)]\} \\ &= \mathbb{E}\{v_2(\widehat{x}, \tilde{\theta}_2) + v_1(\widehat{x}, \tilde{\theta}_1) + (\widehat{\tau}_1 + \widehat{\tau}_2) - V(\tilde{\theta})\} \\ &= \mathbb{E}\{v_2(\widehat{x}, \tilde{\theta}_2) + v_1(\widehat{x}, \tilde{\theta}_1) - V(\tilde{\theta})\} < 0.\end{aligned}$$

In words, a mediator who implements the first best must subsidize the entire renegotiation surplus. Thus, the status quo $\hat{x}$ that minimizes the expected subsidy (within a class of those that have opt-out types) must maximize the expected status quo surplus $\mathbb{E}[v_1(\widehat{x}, \widetilde{\theta}_1) + v_2(\widehat{x}, \widetilde{\theta}_1)]$. Thus, we have:

[7] In Section 6 we also discuss "dual chooser" rules in settings with two agents.

Proposition 10 *Suppose that the assumptions of Lemma 1(ii) hold and there are two agents. Then the among the property rights that cause both agents to have adverse opt-out types the one that minimizes the first-best subsidy is the one that maximizes the two agent's' joint payoff in the absence of bargaining.*

Since, as we saw in Section 3, both agents have opt-out types in the setting of Myerson and Satterthwaite, we have the following corollary:

Corollary 11 *Consider the Myerson-Satterthwaite setting in which each of two agents $i = 1, 2$ has value $\theta_i \in [0, 1]$ drawn from a full support distribution F_i. Then assigning wonership to the agent with the higher expected value minimizes the first-best subsidy.*

Thus,to minimize the first-bestbargaining subsidy, ownership is best assigned exactly as if bargaining were impossible.

4.1.2 Property Rules vs. Liability Rules

We now consider the possibility that instead of a simple property right, one agent may be given an option to own. Specifically, imagine that agent 1 can choose to acquire the good from agent 2 at a price p. This arrangement may be thought of as a liability rule in which agent 1 can take the good from agent 2, but must then make damage payment p to agent 2.

As we saw in Example 4, both agents will have adverse opt-out types in this case. For agent 2 it is his type $\hat{\theta}_2 = p$, while for agent 1 it is his type $\hat{\theta}_1 = 1$ if $\mathbb{E}[\theta_2] < p$, amd type $\hat{\theta}_1 = 0$ if $\mathbb{E}[\theta_2] > p$. In that case, the mediator profit can be written as

$$\begin{aligned}
\bar{\pi} &= \mathbb{E}\left[\widehat{V}_1(\widetilde{\theta}_1) + \widehat{V}_2(\hat{\theta}_1, \widetilde{\theta}_2) - V(\tilde{\theta})\right] \\
&= \underbrace{\mathbb{E}\left[\widehat{V}_1(\widetilde{\theta}_1) + \widehat{V}_2(\widetilde{\theta}_1, \widetilde{\theta}_2) - V(\tilde{\theta})\right]}_{<0 \text{ when option exercise is not first-best}} + \underbrace{\mathbb{E}\left[\widehat{V}_2(\hat{\theta}_1, \widetilde{\theta}_2) - \widehat{V}_2(\widetilde{\theta}_1, \widetilde{\theta}_2)\right]}_{\leq 0 \text{ since } \hat{\theta}_1 \text{ is an adverse type}}
\end{aligned}$$

Observe that the first term is the expected welfare loss from agent 1's optimal exercise of the option. It is negative since agent 1's exercise decision is not always ex post optimal. However, the inefficiency is minimized when $p = \mathbb{E}[\theta_2]$, which is the optimal exercis eprice in the absence of bargaining.

The second term, on the other hand, is non-positive, and is strictly negative unless agent 2 is indifferent about agent 1's exercise decision. That occurs when $p = \mathbb{E}[\theta_2]$. Thus, we see that when agent 1 has the option, the option price that minimizes the first-best subsidy has $p = \mathbb{E}[\theta_2]$ and it results in a positive expected subsidy; achieving the first best is impossible.[8] This option price corresponds exactly to the traditional legal liability rule in which the damage payment equals the victim's expected damage.

Next, consider which agent should have the option. When agent i gets the option and $p = \mathbb{E}[\theta_{-i}]$, the first-best subsidy exactly equals the welfare loss from agent i's optimal exercise of the option in the absence of bargaining. Hence, the agent i should be give the option if and only if he is best assigned the option when bargaining is impossible.

Finally, since the case of a simple property right corresponds to setting $p = 0$ or $p = 1$, the optimal liability rule is strictly better than the best simple property right, exactly as in the case without bargaining.

In summary:

Proposition 12 *In the Myerson and Satterthwaite setting, the option-to-own (i.e., liability rule) that minimizes the expected first-best bargaining subsidy sets the option price equal to the non-chooser's ("victim's") expected value ("harm") and assigns the option to the agent whose optimal exercise in the absence of bargaining results in the greatest expected surplus. The resulting expected subsidy is lower than with assignment of any simple property right.*

4.2 More than Two Agents

When there are more than two agents, choosing the subsidy-minimizing property rights requires that we consider the consider coalitional, rather than individual, values. For example, shifting the property right to a private good (generating no externalities) from one agent to another is efficiency enhancing in the absence of bargaining if it increasing the joint payoff of the two agents. In contrast, this change increases efficiency when bargaining is possible (in the sense of reducing the expected first-best subsidy) if it raises the sum of the two values that these two agents can each individually achieve when in a coalition with all of the other agents.

[8] Che (2006) also derives this result and notes the impossibility of two agents achieving efficiency under a liability rule.

We illustrate the new effects through two examples.

4.2.1 Application: Spectrum Licenses

Consider the following example: Simple property rights to two spectrum licenses, L_1 and L_2, are to be allocated among three firms. Firms 1 and 2 are specialists and each firm $i = 1, 2$ has a value $\theta_i \in \mathbb{R}_+$ for license L_i, drawn from distribution F with mean μ, and no value for license L_{-i}. Firm 0 is a generalist firm, and has a value $\theta_0 \in \mathbb{R}_+$ for both licenses, and value $\lambda\theta_0$ for just one of the licenses, where θ_0 is drawn from distribution G, with mean μ_0, and $\lambda \in (0, 1)$. The values θ_1, θ_2, and θ_0 are independent random variables. When $\lambda < 1/2$, the licenses are complements for G; when $\lambda > 1/2$, they are substitutes. For example, the licenses might be in two different regions, with firms 1 and 2 being regional firms and firm G being a national firm. In that case, G is likely to find the two licenses complements ($\lambda > 1/2$). Alternatively, the licenses might be to different frequencies, with firms 1 and 2 each having a product that can use just one of the frequencies effectively, while firm G may have two products, each of which can use either frequency, and one of which is more profitable than the other. In that case, the frequencies are substitutes ($\lambda > 1/2$).

We will compare an allocation of **BOTH** licenses to G with an allocation of **NONE** of the licenses to G. Absent bargaining, the expected surplus is larger at BOTH than at ONE if

$$\mu_0 - 2\mu > 0 \tag{10}$$

Note that the best choice between these two allocations of property rights in the absence of bargaining is independent of λ.

Now consider the subsidy-minimizing property rights when there is bargaining. Under both of these property rights allocations, the adverse opt-out property holds. The following table summarizes the coalitional values, under the assumption that each two-agent coalition maximizes its joint payoff:

Property Rights Allocation:	$\hat{V}_{-G}$	$\hat{V}_{-1}$	$\hat{V}_{-2}$
BOTH	0	$\max\{\theta_0, \lambda\theta_0 + \theta_2\}$	$\max\{\theta_0, \lambda\theta_0 + \theta_1\}$
NONE	$\theta_1 + \theta_2$	$\max\{\lambda\theta_0, \theta_2\}$	$\max\{\lambda\theta_0, \theta_1\}$

So, BOTH is better than NONE if

$$\sum_{i=1,2} \mathbb{E}[\max\{\theta_0, \lambda\theta_0 + \theta_i\}] > 2\mu + \sum_{i=1,2} \mathbb{E}[\max\{\lambda\theta_0, \theta_i\}],$$

which can be rewritten as

$$\mu_0 - 2\mu > (2\lambda - 1)\mu_0 + \sum_{i=1,2} (\mathbb{E}[\max\{0, \theta_i - \lambda\theta_0\} - \mathbb{E}[\max\{0, \theta_i - (1-\lambda)\theta_0\}]) \tag{11}$$

Thus, bargaining changes the optimal property rights according to the sign of the term on the right-hand side of (11). This term equals zero when $\lambda = 1/2$, so that the licenses are neither substitutes nor complements for firm G. In that cae, the best property rights allocation is the same as in the absence of bargaining. The derivative of the right-hand side with respect to λ is

$$\mu_0 \left(2 - \sum_{i=1,2} [\Pr(\theta_i - \lambda\theta_0 \geq 0) + \Pr(\theta_i - (1-\lambda)\theta_0 \geq 0)] \right) > 0$$

Thus, when $\lambda > 1/2$ (substitutes), bargaining pushes the optimal property rights toward NONE, and when $\lambda < 1/2$ (complements) bargaining pushes the optimal property rights toward BOTH.

4.2.2 Application: Liability Rule for Pollution

Consider a setting in which agent 0 (the "firm") chooses whether to pollute, labeled by $x \in \{0, 1\}$. The firm's utility is $v_0(x, \theta_0) = \theta_0 x$, where $\theta_0 \in \mathbb{R}_+$. denotes its value for polluting. Agents $i = 1, ..., N$ are consumers, whose utilities are given by $v_i(x, \theta_i) = (1-x)\theta_i$ with $\theta_i \in \mathbb{R}_+$. Efficient pollution is therefore given by

$$\chi^*(\theta) = 1 \text{ if and only if } \theta_0 \geq \sum_{i \geq 1} \theta_i.$$

We assume that, for all i, $\tilde{\theta}_i$ has a full-support absolutely continuous distribution on $\mathbb{R}_+$.

The property rights are given by a liability rule: the firm can choose to pollute, in which case it must pay pre-specified "damages" $p_i \geq 0$ to each consumer $i \geq 1$. Thus, if the firm does not participate in bargaining, it optimally chooses $\hat{x}_0(\theta) = \chi^*(\theta_0, p)$, and its transfer is given by $\hat{\tau}_0(\theta) = -\left(\sum_i p_i\right)\hat{x}_0(\theta)$.

We must also specify what happens if agent $i \geq 1$ does not participate. To obtain the results in the simplest possible way, we assume for now that all the other agents then bargain efficiently among each other, given that agent i must be paid compensation p_i if pollution is chosen. Thus, they optimally choose pollution level $\hat{x}_i(\theta) = \chi^*(p_i, \theta_{-i})$, and agent i's compensation is $\hat{\tau}_i(\theta) = p_i \hat{x}_i(\theta)$. (We will discuss the role of this assumption later.)

Given these assumptions, it is easy to see that each agent $i \geq 1$ has an opt-out type $\theta_i^\circ = p_i$. This type is also trivially adverse, since the agent imposes no externalities on the others. Hence, by Lemma 5, it is agent i's critical type.

The firm, on the other hand, has two opt-out types: $\theta_0 = 0$ (which never pollutes in the first best and does not pollute when it does not participate) and $\theta_0 = +\infty$ (which always pollutes in the first best and pollutes when it does not participate). Furthermore, $\theta_0 = 0$ is an adverse type if $\sum_{i\geq 1} p_i \geq \mathbb{E}[\sum_{i\geq 1} \tilde{\theta}_i]$ while $\theta_0 = +\infty$ is an adverse type if the inequality is reversed. (Of course, formally speaking $\theta_0 = +\infty$ is not a "type," but taking a sequence $\theta_0^k \to +\infty$ shows that the firm does satisfy the adverse opt-out property.)

Finally, it is easy to see that the core is nonempty-valued and multi-valued with a positive probability. Hence, Theorem **??** implies that efficient bargaining is impossible.

Remark 13 *How would this conclusion be affected if agent $i \geq 1$ expected a different outcome $\hat{x}_i(\theta)$ from non-participation, while still expecting compensation $\hat{\tau}_i(\theta) = p_i \hat{x}_{-i}(\theta)$ according to the liability rule? Observe that the reservation utility of type $\theta_i = p_i$ is independent of $\hat{x}_i(\theta)$. Since type $\theta_i = p_i$ was agent i"s critical type above, where efficient bargaining was impossible, the participation constraints of this type continue to imply that efficient bargaining is impossible.*

Furthermore, we can argue that if an intermediary can choose $\hat{x}_i(\theta)$ following nonparticipation of agent $i \geq 1$ to minimize the expected first-best subsidy, then it can do no better than setting $\hat{x}_i(\theta) = \chi^*(p_i, \theta_{-i})$, as we assumed above. Indeed, since the intermediary has to satisfy the participation constraint of type $\theta_i = p_i$ regardless of $\hat{x}_i(\theta)$, formula (7) bounds below the intermediary's expected subsidy. On the other hand, by choosing $\hat{x}_i(\theta) = \chi^*(p_i, \theta_{-i})$ the intermediary ensures that type $\theta_i = p_i$ is a critical type, and therefore its participation constraints imply all the other types' par-

ticipation constraints, so the lower bound on the expected subsidy is actually achieved. Therefore, the following analysis of optimal damages p applies to the situation where the intermediary can choose $\hat{x}_i(\theta)$ optimally following nonparticipation by individual agents.

Now we identify the vector of damages $p = (p_1, .., p_N)$ that minimizes the expected first-best subsidy. Using (7), the maximization problem can be written as

$$\max_{p_1,..,p_N \geq 0} \sum_{i=0}^{N} \mathbb{E}\left[V_{-i}(\theta_i^\circ, \tilde{\theta}_{-i})\right]$$

where

$$\mathbb{E}\left[V_{-0}(\theta_0^\circ, \tilde{\theta}_{-0})\right] = \min\left\{\sum_{i\geq 1} \mathbb{E}[\tilde{\theta}_i], \sum_{i\geq 1} p_i\right\}$$

and

$$\mathbb{E}\left[V_{-i}(\theta_i^\circ, \tilde{\theta}_{-i})\right] = \mathbb{E}\left[\max\left\{\tilde{\theta}_0 - p_i, \sum_{j\neq i,\, j\geq 1} \tilde{\theta}_j\right\}\right] \text{ for } i \geq 1.$$

Note that using the Envelope Theorem, for $i \geq 1$,

$$\frac{\partial \mathbb{E}\left[V_{-i}(\theta_i^\circ, \tilde{\theta}_{-i})\right]}{\partial p_i} = -\Pr\left\{\tilde{\theta}_0 - p_i > \sum_{j\neq i,\, j\geq 1} \tilde{\theta}_j\right\} \in (-1, 0)\,, \qquad (12)$$

while

$$\frac{\partial \mathbb{E}\left[V_{-0}(\theta_0^\circ, \tilde{\theta}_{-0})\right]}{\partial p_i} = \left\{\begin{array}{c} 1 \text{ if } \sum_{i\geq 1} \mathbb{E}[\tilde{\theta}_i] > \sum_{i\geq 1} p_i \\ \textit{otherwise.} \end{array}\right\}.$$

Therefore, at the optimum we must have $\sum_{i\geq 1} p_i = \sum_{i\geq 1} \mathbb{E}[\tilde{\theta}_i]$, i.e., the total damages paid by the firm should be equal to the total expectation damages for the affected parties. This would also be optimal in a setting where bargaining is impossible.

However, in contrast to the setting without bargaining, it now matters how the damages are allocated among consumers. The problem of optimal allocation of damages can be formulated as

$$\max_{p_1,..,p_N \geq 0} \sum_{i\geq 1} \mathbb{E}[V_{-i}\left(p_i, \tilde{\theta}_{-i}\right)] \text{ s.t. } \sum_{i\geq 1} p_i = \sum_{i\geq 1} \mathbb{E}[\tilde{\theta}_i].$$

Note by (12), $\partial \mathbb{E}[V_{-i}(p_i, \tilde{\theta}_{-i})]/\partial p_i$ is nondecreasing in p_i, so the objective function is convex, and is therefore maximized at a vertex of the feasible set, i.e., a point p such that $p_{i^*} = \sum_{i \geq 1} \mathbb{E}[\tilde{\theta}_i]$ for

$$i^* \in \arg\max_{j \geq 1} \left\{ \mathbb{E}\left[V_{-j}\left(\sum_{i \geq 1} \mathbb{E}[\tilde{\theta}_i], \tilde{\theta}_{-j} \right) \right] + \sum_{i \geq 1, i \neq j} \mathbb{E}\left[V_{-i}(0, \tilde{\theta}_{-i}) \right] \right\}$$

and $p_i = 0$ for all $i \neq i^*$. Thus, all of the damages should be paid to a single consumer, with the consumer selected to maximize the total expected surplus of coalitions excluding that single consumer.

5 Optimal Property Rights with Second-Best Bargaining

In many circumstances, there isn't a planner available to subsidize trade. In that case, a more appropriate approach to determining optimal property rights involves looking at second-best mechanisms that maximize expected surplus subject to a budget balance constraint. Analyzing that problem, however, is complicated by the interplay between the mechanism chosen and the agents' "critical types" (i.e., those types whose participation constraints bind). Those critical types depend on the mechanism being employed, but the best mechanism depends on the agent's critical types (because they determine which IR constraints bind). In this section, we analyze this problem. As this is a much harder problem than the first-best problem studied earlier, we restrict attention to a case with two agents trading a single indivisible good.

Myerson and Satterthwaite (1983) characterized the optimal second-best mechanism for the case of simple property rights, where one agent is a seller (the initial owner) and the other agent is the buyer. The optimal mechanism is shown in Figure 1, which leads to a surplus loss of 7/64 (from the first-best surplus of 3/4).[9] It involves a trading "gap" $l = 1/4$, which represents the amount that the buyer's value must exceed the seller's value for trade to occur.

Consider, first, how the second-best expected surplus varies as we change which of the two agents is the owner in the Myerson-Satterthwaite setting. To

[9] Cramton, Gibbons, and Klemperer (1987) showed that the first best is achievable for the convex set of intermediate property rights if randomized property rights are possible [see also Segal and Whinston (2011)].

do this continuously, we allow for randomized ownership, where, say, agent 1 gets the good with probability $x \in [0, 1]$. We have the following result:

Proposition 14 *If payoffs are linear in x, then the maximal second-best welfare is convex in the status quo level of x, $\hat{x}$.*

Proof. Let $\langle \chi, \tau \rangle$ be a second-best optimal mechanism for status quo $\hat{x}$ and $\langle \chi', \tau' \rangle$ be a second-best optimal mechanism for property rights $\hat{x}'$. For $\alpha \in [0, 1]$, consider the mechanism $\langle \chi'', \tau'' \rangle = \alpha \langle \chi, \tau \rangle + (1 - \alpha) \langle \chi', \tau' \rangle$. (Given linearity of payoffs, it is equivalent to the randomized mechanism that implements $\langle \chi, \tau \rangle$ with probability α and $\langle \chi', \tau' \rangle$ with probability $1-\alpha$.) Mechanism $\langle \chi'', \tau'' \rangle$ inherits IC and BB from mechanisms $\langle \chi, \tau \rangle$ and $\langle \chi', \tau' \rangle$, and satisfies IR for status quo $\hat{x}'' = \alpha \hat{x} + (1 - \alpha) \hat{x}'$. ■

The proposition implies, in particular, the set of status quos for which first-best is achieved is convex (the set is nonempty by Segal-Whinston 2011), and that a move in the direction of this set raises the second-best expected surplus.

We next investigate what can be achieved with a liability rule in which one agent (or firm) is given the option to own the good (or pollute) in return for paying p to the other agent. Without loss of generality, we will take the agent who has this option to be agent 1. Note that if $p = 0$ then agent 1 will always exercise his option in the default, so it is equivalent to agent 1 being the owner with a simple property right. If, instead, $p = 1$, then agent 1 will never exercise his option, so it is equivalent to agent 2 being the owner with a simple property right. Hence, the *optimal* liability rule cannot be worse than the optimal simple property right. However, we will see that there are always some liability rules that are worse than the best simple property right.

Our analysis hinges on identifying critical types. For the passive agent 2, we can observe any type whose probability of trade in the mechanism is p is a critical type. Indeed, any other type can guarantee the same participation surplus by pretending to be such a type. In general, there will be not a single critical type but an interval $\left[\underline{\theta}_2, \bar{\theta}_2\right]$ of them which we will identify. As for the active agent 1, we observe that this agent's critical types always include either $\widehat{\theta}_1 = 0$, or $\widehat{\theta}_1 = 1$, or both. To see this, observe that in the default outcome this agent's payoff is $V_1(\theta_1) = \max\{\theta_1 - p, 0\}$, which is a convex function whose derivative is 0 below p and 1 above p. This agent's expected payoff $U_1(\theta_1)$ in any mechanism, on the other hand, has a derivative $U_1'(\theta_1)$ at each θ_1 that equals that type's expected probability of receiving the good in the mechanism, so $U_1'(\theta_1) \in [0, 1]$ for all θ_1.

Recall that $\theta_i \in [0,1]$ for $i = 1,2$ and that agent 1 is the active chooser, possessing the default call option with price p. We consider a general case in which the c.d.f.'s of the two agents' types are F_1, F_2 respectively, with strictly positive densities f_1, f_2.

For $i = 1,2$, and for $\lambda \in [0,1]$ let

$$\underline{\omega}_i(\theta_i|\lambda) \equiv \theta_i - \lambda \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \text{ and } \bar{\omega}_i(\theta_i|\lambda) \equiv \theta_i + \lambda \frac{F_i(\theta_i)}{f_i(\theta_i)}$$

denote agent i's virtual values when his downward/upward ICs bind. We assume that both $\underline{\omega}_i(\cdot|1)$ and $\bar{\omega}_i(\cdot|1)$ are strictly increasing and continuous functions in θ_i (this implies the same properties for any $\lambda \in [0,1]$). Note that

$$\underline{\omega}_i(\theta_i|\lambda) \leq \theta_i \leq \bar{\omega}_i(\theta_i|\lambda),$$

and that the inequalities are strict for $\theta_i \in (0,1)$, provided that $\lambda > 0$.

Also, for $\gamma \in [0,1]$, let

$$\omega_1(\theta_1|\lambda,\gamma) \equiv (1-\gamma)\underline{\omega}_1(\theta_1|\lambda) + \gamma\bar{\omega}_1(\theta_1|\lambda).$$

Finally, for $\lambda \in [0,1]$ and $\gamma \in [0,1]$, define $\bar{\theta}_1(\theta_2|\lambda,\gamma) \equiv \omega_1^{-1}(\bar{\omega}_2(\theta_2|\lambda)|\lambda,\gamma)$ and $\underline{\theta}_1(\theta_2|\lambda,\gamma) \equiv \omega_1^{-1}(\underline{\omega}_2(\theta_2|\lambda)|\lambda,\gamma)$. Under our assumptions they are both continuous increasing functions and $\bar{\theta}_1(\theta_2|\lambda,\gamma) \geq \underline{\theta}_1(\theta_2|\lambda,\gamma)$ for all θ_2.

We begin with the following characaterization result (all proofs for results in this Section are in the Appendix):

Lemma 15 *The second-best solution takes the following form (with probability 1): For some fixed $\hat{\lambda}$ and γ*

$$x_1(\theta_1,\theta_2) = \begin{cases} 1 & \textit{for} \quad \theta_1 > \hat{\theta}_1(\theta_2), \\ 0 & \textit{for} \quad \theta_1 < \hat{\theta}_1(\theta_2), \end{cases} \tag{13}$$

where

$$\hat{\theta}_1(\theta_2) \equiv \max\left\{\underline{\theta}_1(\theta_2|\lambda,\gamma), \min\left\{p, \bar{\theta}_1(\theta_2|\lambda,\gamma)\right\}\right\}. \tag{14}$$

Furthermore,

$$\gamma E[\hat{\theta}_1(\tilde{\theta}_2) - p] \quad \geq \quad 0 \textit{ and } (1-\gamma) E[\hat{\theta}_1(\tilde{\theta}_2) - p] \leq 0. \tag{15}$$

For the specific case in which both agents' types are drawn from the uniform distribution, this characterization implies that the second-best mechanism takes the following form:

Proposition 16 *When both agents' types are drawn from the uniform distribution and there is a liability rule in which agent 1 has the option to own in return for a payment of $p \in [0,1]$, the optimal second-best allocation rule takes the following forms, for some function $l(p)$:*

- ***For** $p < 3/8$**:** $x_1(\theta_1, \theta_2) = 1$ if and only if (i) $\min\{\theta_1, p\} \geq \theta_2$, (ii) $\theta_1 \geq p$ and $\theta_2 \in [p, p + l(p)]$, or (iii) $\theta_1 \geq \theta_2 - l(p)$ and $\theta_2 > p + l(p)$;*

- ***For** $p \in [3/8, 5/8]$**:** $x_1(\theta_1, \theta_2) = 1$ if and only if (i) $\theta_1 \geq \theta_2 + l(p)$ and $\theta_2 < 3/8$, (ii) $\theta_1 \geq p$ and $\theta_2 \in [3/8, 5/8]$, or (iii) $\theta_1 \geq \theta_2 - l(p)$ and $\theta_2 > 5/8$;*

- ***For** $p > 5/8$**:** $x_1(\theta_1, \theta_2) = 1$ if and only if (i) $\theta_1 \geq \theta_2 \geq p$, (ii) $\theta_1 \geq p$ and $\theta_2 \in [p, p - l(p)]$, or (iii) $\theta_1 \geq \theta_2 + l(p)$ and $\theta_2 < p - l(p)$.*

Figures 2-4 show the sets of types for which agent 1 receives the good for the three cases identified in Proposition 16. The three cases correspond to situations in which agent 1's critical type is $\widehat{\theta}_1 = 1$ (for $p < 3/8$), $\widehat{\theta}_1 = 0$ (for $p > 5/8$), and both types 0 and 1 are critical types (for $p \in [3/8, 5/8]$). [Note that the critical type is 1 (resp. 0) for low (resp. high) p, which are cases where the property right is relatively close to agent 1 (resp. 2) having a simple ownership right.] The function $l(p)$ is similar to the Myerson-Satterthwaite gap seen in Figure 1, and like that gap its size is set to achieve budget balance.

The resulting second-best inefficiency is derived in the Appendix. Figure 5 graphs the resulting inefficiency, the loss in expected surplus from the first best level, as a function of p. For comparison, the figure also shows the inefficiency with no bargaining and the deficit for a planner who would subsidize trade to achieve the first best. As can be seen in the figure, the optimal property right has $p = 1/2$ – equal to the expected value of the non-choosing agent – in all three cases. This is not generally true (we have counter-examples), although we conjecture that the optimal option price has this feature whenever the active agent's type is drawn from the uniform distribution. (In Lemma 19 we show that this is true whenever the IR constraints of the lowest and highest types of the active agent bind.)

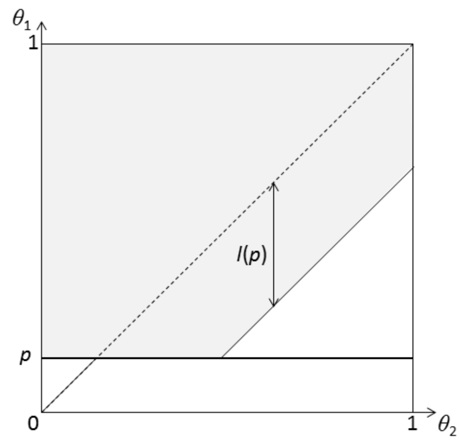


Figure 1: Second-Best for p < 3/8

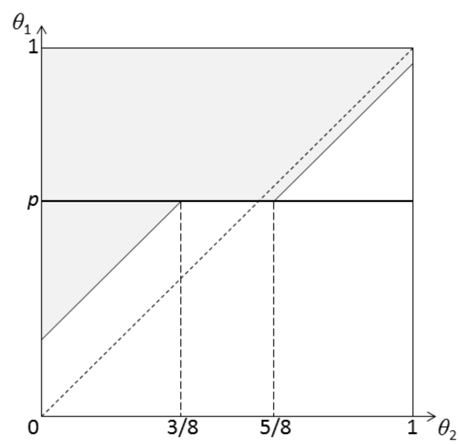


Figure 2: Second-Best for 3/8 < p < 5/8

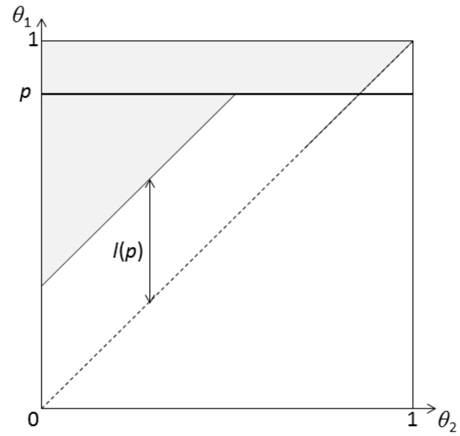
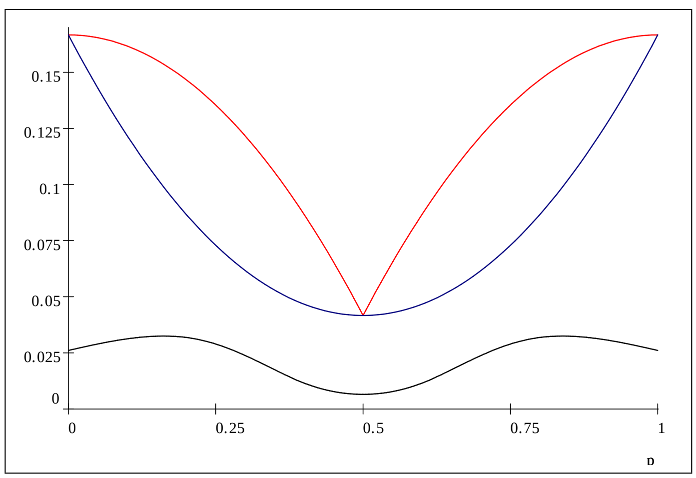


Figure 3: Second-Best for p > 5/8



Expected Subsidy Needed for First-Best
Expected Inefficiency if there were No Bargaining
Expected Inefficiency in Second-Best Bargaining

Referring again to Figure 5, we see that, perhaps surprisingly, the surplus achievable with a liability rule is not monotone increasing as p moves toward $1/2$, and is in fact lower for p close to 0 (resp. 1) than at $p = 0$ (resp. 1).

That is, a slightly interior p is worse than the simple property right it is near.The fact that a liability rules which induce default allocations close to but different from a simple property right are worse than that simple property right does not depend on our assumption of a uniform distribution. As the following Proposition shows, it is true for *any* distributions of values for the two agents:

Proposition 17 *There exists a $\delta > 0$ such that any liability rule with $p \in [1, 1-\delta]$ (resp, $p \in [0, \delta]$) has a lower second-best expected surplus than $p = 1$ (resp. $p = 0$), which is equivalent to simple ownership by agent 2 (resp. agent 1).*

Intuitively, starting at $p = 1$ a small reduction in p increases the expected payoff in the default to essentially all types of the passive agent 2 (whose default payoff when $p = 1$ simply equals his type), since he then gains $(p - \theta_2)[1 - F_1(p)]$.[10]

6 Dual-Chooser Rule

For another application, we consider a "dual-chooser" rule with two agents as described by Ayres (2005): agent 2 is the initial owner of the good, but agent 1 can get it if both agents agree to this at a pre-specified price p. We assume that both agents' values for the good are drawn from the same interval, which we normalize to be $[0, 1]$. Our first observation is that with this rule, agent 2's type $\hat{\theta}_2 = 1$ is an adverse opt-out type, while agent 1's type $\hat{\theta}_1 = 0$ is an adverse opt-out type (these types never trade, either in the default mechanism or in the efficient mechanism). Since these types have the same reservation utilities as in the standard Myerson-Satterthwaite setting in which agent 2 is the owner, we obtain that the expected first-best subsidy is the same as in the Myerson-Satterthwaite setting, regardless of p.

As for the second-best expected surplus, we note that for any p, we observe that it cannot exceed that in the Myerson-Satterthwaite setting where agent 2 has a simple property right. Indeed, the participation constraints of types $\hat{\theta}_2 = 1$ and $\hat{\theta}_1 = 0$ must still be satisfied, and the reservation utilities of these types are the same as in the Myerson-Satterthwaite setting, regardless of p.

[10] The complication in the proof is that this is not true for type $\theta_2 = 1$ (or types near it for $p < 1$).

On the other hand, the second-best expected surplus can be *strictly lower* than in the Myerson-Satterthwaite setting. For example,

Proposition 18 *If $\tilde{\theta}_1, \tilde{\theta}_2 \sim U[0,1]$, then the M-S mechanism fails IR for the dual-chooser rule with posted price $p \in \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, 1\right)$.*

Proof. Consider first agent 1. His expected utility in the dual-chooser rule is $\hat{U}_1(\theta_1) = \max\{0, (\theta_1 - p)\, p\}$, while his expected utility in the M-S mechanism is $U_1(\theta_1) = (\theta_1 - 1/4)_+^2 /2$ (this can be calculated either from the dominant-strategy "pricing" implementation of this allocation rule, or by the integral formula from the allocation rule). So clearly for $p < 1/4$, the IR of $\theta_1 = 1/4$ will fail. For $1/4 \leq p < 1/2$, consider type $\theta_1 = p + 1/4$ (this will actually be the "critical type"), for him the participation surplus is $p^2/2 - p/4 = p\,(p - 1/2)/2 < 0$.

Now consider agent 2. His expected utility in the dual-chooser rule is $\hat{U}_2(\theta_2) = \theta_2 + \max\{0, (1-p)(p - \theta_2)\}$. while his expected utility in the M-S mechanism is $U_2(\theta_2) = \theta_2 + (3/4 - \theta_2)_+^2 /2$. So clearly for $p > 3/4$, the IR of $\theta_2 = 3/4$ will fail. For $1/2 < p \leq 3/4$, consider type $\theta_2 = p - 1/4$ (this will actually be the "critical type"), for him the participation surplus is $(1-p)^2/2 - (1-p)/4 = (1-p)(1/2 - p)/2 < 0$. ■

So, while we didn't solve for 2nd-best, we know it looks something like this?? This can be contrasted with the expected surplus achieved if there is no bargaining, which for any $p \in (0,1)$ is *strictly higher* than under a simple property right.

7 Conclusion

Our results have implications for several literatures. In organizational economics, losses due to ex post bargaining inefficiencies were a central theme of Williamson's Transaction Cost Economics approach to the firm. One can view our analysis, in which we study how property rights can affect those losses, as taking the Grossman-Hart-Moore [Grossman and Hart (1986), Hart and Moore (1990), Hart (1995)] Property Rights Theory approach of asking how asset ownership affects efficiency, but doing so focusing instead on Williamson's costs of haggling, rather than on inefficiencies in ex ante investments. Like the Property Rights Theory, our approach has implications not only for asset ownership, but also for allocation of decision rights within firms.

In the legal literature, ever since Calebresi and Malemud (1972), scholars have been interested in the performance of different property right regimes. Our results shed new light on this issue when bargaining is imperfect due to the presence of asymmetric information. In particular, we have highlighted the effect that bargaining has on the choice among property rights regimes, relative to the case in which bargaining is impossible.

Finally, from the perspective of mechanism design, we provide a new set of sufficient conditions characterizing when efficiency through bargaining is impossible, which applies not only to the traditional case of simple property rights, but also to more general property rights mechanisms.

8 Appendix: Proofs

8.1 Proof of Lemma 15

Proof. Note than in any Bayesian incentive compatible mechanism, agent 2's expected consumption $1-E[x_1(\tilde{\theta}_1,\theta_2)]$ must be nondecreasing in θ_2, therefore there will be a type $\hat{\theta}_2$ such that

$$\left\{1-F_1(p)-E[x_1(\tilde{\theta}_1,\theta_2)]\right\} sign(\theta_2-\hat{\theta}_2)\geq 0 \text{ for all } \theta_2 \tag{16}$$

Consider the designer's "relaxed problem" in which she chooses $\hat{\theta}_2$, the allocation rule $x(\cdot)$, and interim expected utilities $U_1(\cdot)$, $U_2(\cdot)$ to maximize expected surplus subject to (16), expected budget balance, first-order incentive compatibility (ICFOC), agent 1's participation constraints $\mathrm{IR}_1(0)$ and $\mathrm{IR}_1(1)$, and agent 2's participation constraint $\mathrm{IR}_2(\hat{\theta}_2)$.[11] The Lagrangian for this problem (leaving ICFOC as constraints) is

$$\begin{aligned}
&E\left[\tilde{\theta}_1x_1(\tilde{\theta})+\tilde{\theta}_2(1-x_1(\tilde{\theta}))\right]+E\left[\delta(\tilde{\theta}_2)(1-F_1(p)-x_1(\tilde{\theta}))sign(\widetilde{\theta}_2-\hat{\theta}_2)\right]\\
&+\lambda\left\{E[\tilde{\theta}_1x_1(\tilde{\theta})+\tilde{\theta}_2(1-x_1(\tilde{\theta}))]-E[U_1(\tilde{\theta}_1)]-E[U_2(\tilde{\theta}_2)]\right\}\\
&+\mu_0U_1(0)+\mu_1(U_1(1)-(1-p))+\nu[U_2(\hat{\theta}_2)-p[1-F_1(p)]-\hat{\theta}_2F_1(p)]\\
&\text{s.t. ICFOC}
\end{aligned}$$

[11] By standard arguments that adjust the transfer rule [e.g., Lemma 1 in Segal and Whinston (2011)], ex ante budget balance can be strengthened to ex post budget balance without affecting expected surplus or any of the other constraints. Moreover, provided that the allocation rule is monotone, all IC and IR constraints will be satisfied for the reasons stated in text.

It is easy to see that $\lambda > 0$ (since the first-best is impossible), while $\mu_0, \mu_1 \geq 0$ must satisfy the complementary slackness conditions

$$\mu_0 U_1(0) = 0 \text{ and } \mu_1 [U_1(1) - (1-p)] = 0 \tag{17}$$

and $\delta(\cdot) \geq 0$ must satisfy the complementary slackness conditions

$$\delta(\theta_2) \left\{ E[x_1(\tilde{\theta}_1, \theta_2)] - (1 - F_1(p)) \right\} = 0 \text{ for all } \theta_2 \neq \hat{\theta}_2. \tag{18}$$

Note that the solution must have $\nu = \lambda$ (otherwise we could raise the Lagrangian by adding a constant to $U_2(\cdot)$ without affecting ICFOC), and $\mu_0 + \mu_1 = \lambda$ (otherwise we could raise the Lagrangian by adding a constant to $U_1(\cdot)$ without affecting ICFOC). Hence, we can rewrite the Lagrangian as

$$\begin{aligned}
&(1+\lambda) E\left[\tilde{\theta}_1 x_1(\tilde{\theta}) + \tilde{\theta}_2(1 - x_1(\tilde{\theta}))\right] + E\left[\delta(\tilde{\theta}_2)(1 - F_1(p) - x_1(\tilde{\theta})) sign(\widetilde{\theta}_2 - \hat{\theta}_2)\right] \\
&-\lambda\left(\frac{\lambda - \mu_1}{\lambda}\right)\left\{E[U_1(\tilde{\theta}_1)] - U_1(0)\right\} - \lambda\left(\frac{\mu_1}{\lambda}\right)\left\{E[U_1(\tilde{\theta}_1)] - U_1(1) + 1 - p\right\} \\
&-\lambda\left\{E[U_2(\tilde{\theta}_2)] - U_2(\hat{\theta}_2)\right\} - \lambda\left\{p[1 - F_1(p)] + \hat{\theta}_2 F_1(p)\right\}
\end{aligned}$$

Note also that we can always satisfy one of the complementary slackness conditions (17) by adding a constant to $U_1(\cdot)$. To be able to satisfy both of them at the same time while satisfying $\text{IR}_1(1)$ and $\text{IR}_1(0)$, we need to be in one of the following three cases: (i) $\mu_0, \mu_1 > 0$ implies $U_1(1) - U_1(0) = 1 - p$, (ii) $\mu_1 = 0$ implies $\mu_0 = \lambda > 0$ hence $U_1(0) = 0$ and $U_1(1) - U_1(0) = U_1(1) \geq 1 - p$, or (iii) $\mu_0 = 0$ implies $\mu_1 = \lambda$ hence $U_1(1) = 1 - p$ and $U_1(1) - U_1(0) = 1 - p - U_1(0) \leq 1 - p$.

For now, fix $\hat{\theta}_2$ and consider maximizing with respect to the allocation rule $x_1(\cdot)$ the simplified Lagrangian (which drops all terms not involving the allocation rule)

$$\begin{aligned}
&(1+\lambda) E\left[\tilde{\theta}_1 x_1(\tilde{\theta}) + \tilde{\theta}_2(1 - x_1(\tilde{\theta}))\right] - E\left[\delta(\tilde{\theta}_2) x_1(\tilde{\theta}) sign(\widetilde{\theta}_2 - \hat{\theta}_2)\right] \\
&-\lambda\left(\frac{\lambda - \mu_1}{\lambda}\right)\left\{E[U_1(\tilde{\theta}_1)] - U_1(0)\right\} - \lambda\left(\frac{\mu_1}{\lambda}\right)\left\{E[U_1(\tilde{\theta}_1)] - U_1(1) + 1 - p\right\} \\
&-\lambda\left\{E[U_2(\tilde{\theta}_2)] - U_2(\hat{\theta}_2)\right\}
\end{aligned}$$

Now using ICFOC and integration by parts, dividing by $1+\lambda$, and letting $\hat{\lambda} \equiv \lambda/(1+\lambda)$, $\hat{\delta}(\theta_2) \equiv \delta(\theta_2)/(1+\lambda)$ and $\gamma \equiv \mu_1/\lambda$, this simplified Lagrangian can be rewritten as

$$E\left[\begin{array}{c} \omega_1(\tilde{\theta}_1|\hat{\lambda},\gamma)x_1(\tilde{\theta}) + \left\{1\{\tilde{\theta}_2 \geq \hat{\theta}_2\}\underline{\omega}_2(\tilde{\theta}_2|\hat{\lambda}) + 1\{\theta_2 < \hat{\theta}_2\}\bar{\omega}_2(\tilde{\theta}_2|\hat{\lambda})\right\}(1-x_1(\tilde{\theta})) \\ -sign(\theta_2 - \hat{\theta}_2)\delta(\tilde{\theta}_2)x_1(\tilde{\theta}) \end{array}\right].$$

It is maximized pointwise by a solution of the form (13), where

$$\hat{\theta}_1(\theta_2) = \begin{cases} \omega_1^{-1}(\bar{\omega}_2(\theta_2|\hat{\lambda}) - \hat{\delta}(\theta_2)\,|\hat{\lambda},\gamma) & \text{for} \quad \theta_2 < \hat{\theta}_2, \\ \omega_1^{-1}(\underline{\omega}_2(\theta_2|\hat{\lambda}) + \hat{\delta}(\theta_2)\,|\hat{\lambda},\gamma) & \text{for} \quad \theta_2 > \hat{\theta}_2. \end{cases} \tag{19}$$

Let $\bar{\theta}_2(\theta_1|\lambda,\gamma) \equiv \{\theta_2 : \underline{\theta}_1(\theta_2|\lambda,\gamma) = \theta_1\}$ and $\underline{\theta}_2(\theta_1|\lambda,\gamma) \equiv \{\theta_2 : \bar{\theta}_1(\theta_2|\lambda,\gamma) = \theta_1\}$, and define $\bar{\theta}_2 \equiv \bar{\theta}_2(p|\hat{\lambda},\gamma)$ and $\underline{\theta}_2 \equiv \underline{\theta}_2(p|\hat{\lambda},\gamma)$. Note that under our assumptions we have $\underline{\theta}_2 \leq \bar{\theta}_2$. Observe that by (16) and the definition of $\underline{\theta}_2, \bar{\theta}_2$, we have $\hat{\theta}_1(\theta_2) \leq p < \bar{\theta}_1(\theta_2|\lambda,\gamma)$ for all $\theta_2 \in (\underline{\theta}_2, \hat{\theta}_2)$ and $\hat{\theta}_1(\theta_2) \geq p > \underline{\theta}_1(\theta_2|\lambda,\gamma)$ for all $\theta_2 \in (\hat{\theta}_2, \bar{\theta}_2)$. This in turn implies, using (19), that for all $\theta_2 \in (\min\{\underline{\theta}_2, \hat{\theta}_2\}, \max\{\hat{\theta}_2, \bar{\theta}_2\})$ we have $\hat{\delta}(\theta_2) > 0$ and therefore for all such θ_2, by (18), $E[x_1(\tilde{\theta}_1,\theta_2)] = 1 - F_1(p)$, which implies $\hat{\theta}_1(\theta_2) = p$.

Next, for $\theta_2 < \min\{\underline{\theta}_2, \hat{\theta}_2\}$, $\hat{\delta}(\theta_2) \geq 0$ implies that $\hat{\theta}_1(\theta_2) \leq \bar{\theta}_1(\theta_2|\lambda,\gamma) < p$, therefore $E[x_1(\tilde{\theta}_1,\theta_2)] > 1 - F_1(p)$, and so by (18) $\hat{\delta}(\theta_2) = 0$, implying $\hat{\theta}_1(\theta_2) = \bar{\theta}_1(\theta_2|\lambda,\gamma)$. Similarly for $\theta_2 > \max\{\hat{\theta}_2, \bar{\theta}_2\}$, $\hat{\delta}(\theta_2) \geq 0$ implies that $\hat{\theta}_1(\theta_2) \geq \underline{\theta}_1(\theta_2|\lambda,\gamma) > p$, therefore $E[x_1(\tilde{\theta}_1,\theta_2)] < 1 - F_1(p)$, and so by (18) $\hat{\delta}(\theta_2) = 0$, implying $\hat{\theta}_1(\theta_2) = \underline{\theta}_1(\theta_2|\lambda,\gamma)$. Thus, the solution takes the form

$$\hat{\theta}_1(\theta_2) = \begin{cases} \bar{\theta}_1(\theta_2|\lambda,\gamma) & \text{for} \quad \theta_2 < \min\{\underline{\theta}_2, \hat{\theta}_2\}, \\ p & \text{for} \quad \theta_2 \in (\min\{\underline{\theta}_2, \hat{\theta}_2\}, \max\{\hat{\theta}_2, \bar{\theta}_2\}), \\ \underline{\theta}_1(\theta_2|\lambda,\gamma) & \text{for} \quad \theta_2 > \max\{\hat{\theta}_2, \bar{\theta}_2\}. \end{cases} \tag{20}$$

Now consider optimal choice of $\hat{\theta}_2$: since the solution for $\hat{\theta}_2 < \underline{\theta}_2$ or $\hat{\theta}_2 > \bar{\theta}_2$ is feasible in the problem for $\hat{\theta}_2 \in [\underline{\theta}_2, \bar{\theta}_2]$ (while the converse is not true), it is optimal to choose $\hat{\theta}_2 \in [\underline{\theta}_2, \bar{\theta}_2]$, in which case the function $\hat{\theta}_1(\theta_2)$ is given by (14).

The complementary slackness conditions (17) are given by (15), since by ICFOC and Fubini's Theorem

$$U_1(1) - U_1(0) = \int_0^1 E[x_1(\theta_1,\tilde{\theta}_2)]d\theta_1 = E\left[\int_0^1 x_1(\theta_1,\tilde{\theta}_2)d\theta_1\right] = 1 - E[\hat{\theta}_1(\tilde{\theta}_2)]. \tag{21}$$

Finally, note that the constructed solution actually satisfies all the incentive constraints (since for each i, $E_{\tilde{\theta}_{-i}} x_i[(\theta_i, \tilde{\theta}_{-i})]$ is nondecreasing in θ_i) and all of the participation constraints (by the argument in the text before the proposition). ■

We now describe a transfer rule that implements the allocation rule above in a dominant strategy IC mechanism that has the right participation constraints binding. When $\gamma < 1$, i.e., $IR_1(0)$ binds, we let $t_1(\theta_1, \theta_2) = -\hat{\theta}_1(\theta_2) x_1(\theta_1, \theta_2)$ – i.e., agent 1 pays $\hat{\theta}_1(\theta_2)$ when he consumes the object.[12] When $\gamma > 0$, i.e., $IR_1(1)$ binds, we let $t_1(\theta_1, \theta_2) = -p + (1 - x_1(\theta_1, \theta_2)) \hat{\theta}_1(\theta_2)$ – i.e., agent 1 first takes the object at p and then is paid $\hat{\theta}_1(\theta_2)$ when he gives it up.[13] For $\gamma \in (0, 1)$, by (15) the two payments have the same expectation over θ_2 for every θ_2. In particular, in that case we can elect the first option for t_1 when $\theta_1 < p$ and the second option when $\theta_1 > p$, yielding transfer rule

$$t_2(\theta_1, \theta_2) = \left\{ \begin{array}{cl} -\hat{\theta}_1(\theta_2) x_1(\theta_1, \theta_2) & \text{if } \theta_1 < p, \\ -p + (1 - x_1(\theta_1, \theta_2)) \hat{\theta}_1(\theta_2) & \text{if } \theta_1 > p, \end{array} \right\} \tag{22}$$

As for agent 2, we let

$$t_2(\theta_1, \theta_2) = \left\{ \begin{array}{cl} \underline{\theta}_2\left(\theta_1 | \hat{\lambda}, \gamma\right) x_1(\theta_1, \theta_2) & \text{if } \theta_1 < p, \\ p - \bar{\theta}_2\left(\theta_1 | \hat{\lambda}, \gamma\right) (1 - x_1(\theta_1, \theta_2)) & \text{if } \theta_1 > p, \end{array} \right\} \tag{23}$$

That is, if agent 1 would not exercise his option at the default, then agent 2 receives $\underline{\theta}_2\left(\theta_1 | \hat{\lambda}, \gamma\right)$ whenever he sells the object, while if agent 1 would exercise his option at the default, then agent 2 receives p but pays back $\bar{\theta}_2\left(\theta_1 | \hat{\lambda}, \gamma\right)$ whenever he ends up keeping the object.[14]

Adding the two transfer rules (22) and (23) yields a budget deficit of

$$\begin{aligned} \left[\underline{\theta}_2\left(\theta_1 | \hat{\lambda}, \gamma\right) - \hat{\theta}_1(\theta_2)\right] x_1(\theta_1, \theta_2) \text{ when } \theta_1 &< p, \\ [\hat{\theta}_1(\theta_2) - \bar{\theta}_2\left(\theta_1 | \hat{\lambda}, \gamma\right)] (1 - x_1(\theta_1, \theta_2)) \text{ when } \theta_1 &> p. \end{aligned} \tag{24}$$

[12] Since $x_1(0, \theta_2) = 1$ for all θ_2 when $\gamma < 1$, type 0 of agent 1 has an expected payoff of 0.

[13] Since $x_1(0, \theta_2) = 0$ for all θ_2 when $\gamma > 1$, type 1 of agent 1 has an expected payoff of $1 - p$.

[14] Observe that with this payment rule, type $\hat{\theta}_2$ of agent 2 has expected payoff $\hat{\theta}_2 F_1(p)$ $+ p[1 - F_1(p)]$, so $IR_2(\hat{\theta}_2)$ holds with equality.

8.2 Proof of Lemma 19

Lemma 19 *Suppose that agent 1's type θ_1 is uniformly distributed and let $\hat{\theta}_1(\theta_2|p)$ describe the second-best allocation rule given p [as specified in (15)]. Then, if for any p and p' both $IR_1(0)$ and $IR_1(1)$ bind in the optimal second-best mechanism, then $\hat{\theta}_1(\theta_2|p') = \hat{\theta}_1(\theta_2|p) + (p' - p)$. Among such p, expected surplus is maximized by setting $p = E(\theta_2)$.*

Proof. Define the two regions $A_p \equiv \{\theta : \theta_1 \in (\hat{\theta}_1(\theta_2|p), p)\}$ and $B_p \equiv \{\theta : (p, \hat{\theta}_1(\theta_2|p))\}$. By (15), the probabilities of these two regions must be equal (these are the two regions in which the final allocation differs from what would happen if agent 1 simply exercised his option optimally). Observe that with a constant shift in $\hat{\theta}_1(\theta_2)$ budget balance is preserved: For every state θ in region A_p, the deficit is now exactly δ smaller, while for every state θ in region B_p, the deficit is now exactly δ larger. Given the uniform distribution of θ_1 and the fact that we integrate in each case over the same sets of θ_2, this change has no effect on the expected deficit. Thus, if $\hat{\theta}_1(\theta_2|p)$ maximizes expected surplus with budget balance under default p, then $\hat{\theta}_1(\theta_2|p') = \hat{\theta}_1(\theta_2|p) + (p' - p)$ must do so under default p'.

Next, observe that the optimal mechanism in the region in which $\text{IR}_1(0)$ and $\text{IR}_1(1)$ both bind therefore [again, given the uniform distribution of θ_1] has a constant improvement in expected surplus over the expected surplus arising when agent 1 simply exercises his option optimally. Since the latter expected surplus is maximized at $p = 1/2$, so is the former. ∎

8.3 Proof of Proposition 16

Consider the special case of F_1, F_2 being uniform on $[0, 1]$, in which

$$\underline{\omega}_i(\theta_i|\lambda) = (1+\lambda)\,\theta_i - \lambda \text{ and } \bar{\omega}_i(\theta_i|\lambda) = (1+\lambda)\,\theta_i. \tag{25}$$

Then we have

$$\begin{aligned}
\underline{\theta}_2(\theta_1) &= \theta_1 - \underline{l} \\
\bar{\theta}_2(\theta_1) &= \theta_1 + \bar{l} \\
\bar{\theta}_1(\theta_2) &= \theta_2 + \underline{l} \\
\underline{\theta}_1(\theta_2) &= \theta_2 - \bar{l}
\end{aligned}$$

and

$$\hat{\theta}_1(\theta_2) = \min\left\{\max\left\{\theta_2 - \bar{l}, p\right\}, \theta_2 + \underline{l}\right\},$$

where $\underline{l} = \lambda(1-\gamma)/(1+\lambda)$ and $\bar{l} = \lambda\gamma/(1+\lambda)$.

8.3.1 Only $\mathbf{IR}_1(0)$ binds

Now consider the relaxed problem in which we ignore $\mathrm{IR}_1(1)$. The solution to that problem corresponds to the case in which $\gamma = 0$, hence $\bar{l} = 0$. Let $\underline{l} = l = \lambda/(1+\lambda)$. We can use the following transfer for agent 1:

$$\begin{aligned} t_1(\theta_1, \theta_2) &= -\hat{\theta}_1(\theta_2)\, x_1(\theta_1, \theta_2) \text{ when } \theta_1 < p, \\ t_1(\theta_1, \theta_2) &= -E[\hat{\theta}_1(\tilde{\theta}_2)] + (1 - x_1(\theta_1, \theta_2))\, \hat{\theta}_1(\theta_2) \text{ when } \theta_1 > p. \end{aligned}$$

since in both cases $E[t_1(\theta_1, \tilde{\theta}_2)] = E[\hat{\theta}_1(\tilde{\theta}_2)x_1(\theta_1, \tilde{\theta}_2)]$. Given these transfers, the budget deficit is then

$$\begin{aligned} \left[\underline{\theta}_2(\theta_1) - \hat{\theta}_1(\theta_2)\right] x_1(\theta_1, \theta_2) \text{ when } \theta_1 &< p, \\ \left[\hat{\theta}_1(\theta_2) - \bar{\theta}_2(\theta_1)\right](1 - x_1(\theta_1, \theta_2)) + p - E[\hat{\theta}_1(\tilde{\theta}_2)] \text{ when } \theta_1 &> p. \end{aligned}$$

Focusing first on the region where $p < \theta_1 < \theta_2$, the subsidy there is the whole gains from trade $\theta_2 - \theta_1$. Since the efficient expected gains from trade in the M-S model with $U[0,1]^2$ distribution is 1/6, the expected gains from trade on the region $p < \theta_1 < \theta_2$ is $(1-p)^3/6$ (the probabilities and the gains themselves are scaled by $1-p$).[15] Now, in the region $\theta_2 + l < \theta_1 < p$, the subsidy $\theta_1 - \theta_2 - 2l$ can be interpreted as (a) paying the gains from trade as if agent 1's value were $\theta_1 - l$ and trade were efficient for that value, and then (b) getting back l on every trade that happened. In expectation, (a) yields $(p-l)^3/6$, and (b) yields $l(p-l)^2/2$.[16] Finally, we have the term $p - E[\hat{\theta}_1(\tilde{\theta}_2)]$, which has to be paid when $\theta_1 > p$, which in expectation costs $(1-p)\left[(p-l)^2/2 - (1-p)^2/2\right]$. Adding all the terms yields

[15] In general, the Myerson-Satterthwaite deficit with uniform distributions on $[0,1]$ and a "gap" equal to l is $(1-4l)(1-l)^2/6$ [see Myerson and Satterhwaite (1983, p. 277)]. So when $l = 0$, the deficit is 1/6. We get $(1-p)^3/6$ because the probability of being in the region $p < \theta_1 < \theta_2$ is $(1-p)^2$ and the region is $[0,1]^2$ scaled down by $(1-p)$.

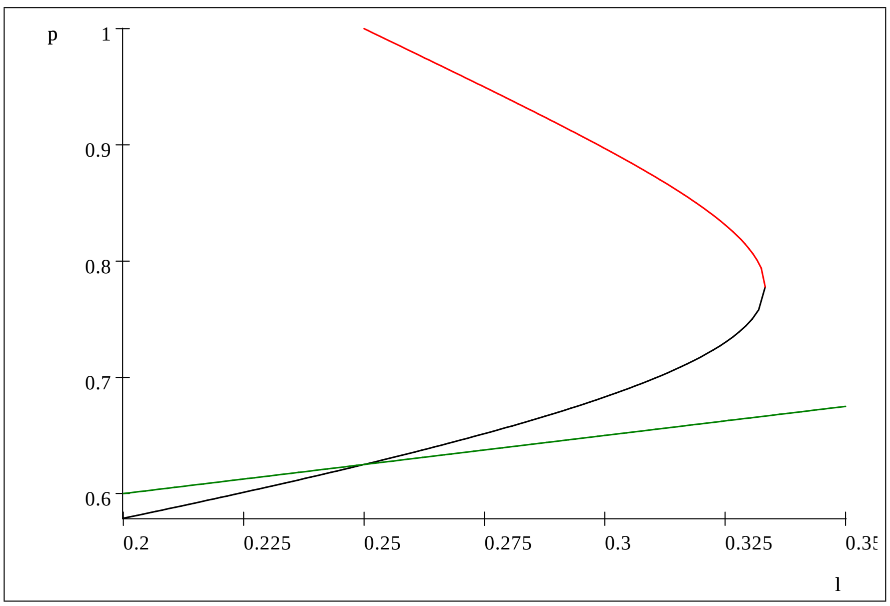
[16] Alternatively, the region $\max\{\theta_1, \theta_2\} < p$ is a scaled-down version of a Myerson-Satterthwaite $[0,1]^2$ trading box with a "gap" equal to l/p. Using the formula in footnote XX, the Myerson-Satterthwaite deficit would be $(1-4\frac{l}{p})(1-\frac{l}{p})/6$. The probability of an outcome in this region is p^2 and the deficit is scaled down by p, so this region contributes $(p-4l)(p-l)/6$ to the expected deficit.

$$\begin{aligned}
& (1-p)^3/6 + (p-l)^3/6 - l(p-l)^2/2 + \left((p-l)^2/2 - (1-p)^2/2\right)(1-p) \\
= \ & p - lp + \frac{1}{2}l^2 - \frac{2}{3}l^3 - \frac{1}{2}p^2 + l^2p - \frac{1}{3}
\end{aligned}$$

Requiring ex ante budget balance sets this expression to 0. We want to express l through p but it's easier to do the reverse. The solutions are:

$$p = 1 - l(1-l) \pm \frac{1}{3}\sqrt{3(1-l)^3(1-3l)}$$

Putting both solutions on the same graph as red and black curves we get



Combining the red and black curves yields $l(p)$, which we see is inverse U-shaped.

The green line is the boundary of when $\text{IR}_1(1)$ is satisfied, which is when $E[\hat{\theta}_1(\theta_2)] \leq p$; i.e., $p - l \geq 1 - p$, or $p \geq (1+l)/2$. Intersecting the line with the black curve we solve

$$1 - l(1-l) - \frac{1}{3}\sqrt{3(1-l)^3(1-3l)} = (1+l)/2$$

The solution is: $l = \frac{1}{4}$, and teh corresponding level of p is $p = 1 - l(1-l) - \frac{1}{3}\sqrt{3(1-l)^3(1-3l)} = 5/8 = 0.625$

Thus,since the solution to the relaxed problem satisfies $\text{IR}_1(1)$ when $p \geq 5/8$, it is the actual solution in those cases. The case where $p \leq 3/8$ and only $\text{IR}_1(1)$ binds is symmetric.

Expected Welfare Loss Calculation Now, calculate the expected welfare loss when $p \geq 5/8$. It is

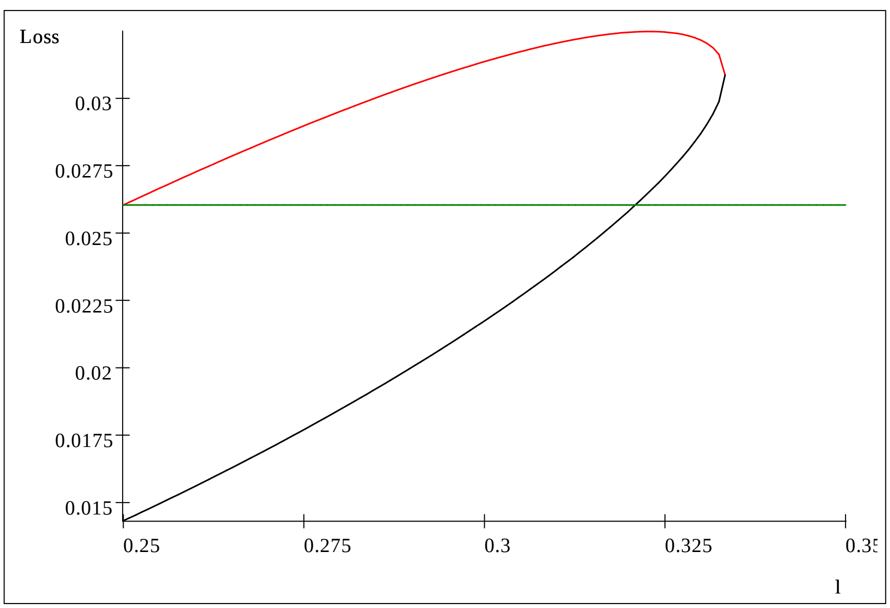
$$p^3/6 - (p-l)^3/6 - l(p-l)^2/2 = \frac{1}{6}l^2(3p-2l).$$

(The first term is if there were no trade at all for values below p, the second term is expected gains from trade on the triangle below p assuming that l is wasted each time, and the third term accounts for l not being wasted.)

Substituting p from the red and black curves into this expression yields the welfare loss

$$\frac{1}{6}l^2\left(3l^2 - 5l + 3 \pm \sqrt{3(1-l)^3(1-3l)}\right),$$

which can be plotted as follows:



Reducing p from 1 corresponds to moving along the curve clockwise, starting on the red curve and then shifting to the black curve. Note that the loss is increasing p as we reduce p from 1 for most of the red curve. The green line here is the M-S welfare loss (i.e., when $p = 1$ and $l = 1/4$) which is $\frac{5}{192}$. The loss exceeds the M-S loss for $l > \hat{l} \approx 0.321$, which corresponds to $\widehat{p} \geq p \approx 0.720$.

The point where welfare loss is maximized is given by solving

$$\begin{aligned} 0 &= \frac{d}{dl}\left(l^2\left(3l^2 - 5l + 3 + \sqrt{3(1-l)^3(1-3l)}\right)\right) \\ l &\in [1/4, 1/2] \end{aligned}$$

The solution is $l \approx 0.323$, which corresponds to $p \approx 0.839$.

8.3.2 Only $\mathbf{IR}_1(1)$ binds

By symmetry, the solution has only $\text{IR}_1(1)$ bind when $p \leq 3/8$ and results, for each such p, in identical welfare losses to the case with $p + 1/2$.

8.3.3 Both $\mathbf{IR}_1(0)$ and $\mathbf{IR}_1(1)$ bind

When $p \in (3/8, 5/8)$ both $\text{IR}_1(0)$ and $\text{IR}_1(1)$ must bind. Applying (24), the budget deficit in this case is

$$\begin{aligned} \theta_1 - \theta_2 - 2\underline{l} \text{ when } \theta_2 + \underline{l} &< \theta_1 < p, \\ \theta_2 - \theta_1 - 2\bar{l} \text{ when } p &< \theta_1 < \theta_2 - \bar{l}, \\ & \quad 0 \text{ otherwise.} \end{aligned}$$

By (15), $E[\hat{\theta}_1(\tilde{\theta}_2)] = p$. This implies that the probabilities of the two regions $A \equiv \{\theta : \theta_2 + \underline{l} < \theta_1 < p\}$ and $B \equiv \{\theta : p < \theta_1 < \theta_2 - \bar{l}\}$ are equal (these are the two regions in which the final allocation differs from what would happen if agent 1 simply exercised his option optimally). This involves having $\bar{l} = \underline{l} + (1 - 2p)$.

Consider first the case of $p = 1/2$. In this case, regions A and B have equal probability when $\underline{l} = \bar{l}$. The optimal allocation in this case can be interpreted as separate Myerson-Satterthwaite mechanisms for the cases $\theta_1, \theta_2 < 1/2$ (in which agent 1 is the buyer) and $\theta_1, \theta_2 > 1/2$ (in which agent 1 is the seller), with no cross-subsidization. The unique gap that achieves budget balance and maximizes expected surplus is half of the Myerson-Satterthwaite gap: $l_{1/2} = 1/2 \cdot 1/4 = 1/8$. For other $p \in (3/8/, 5/8)$, we apply Lemma BOTH IRs BIND.

8.3.4 Proof of Proposition 17

Consider the Myerson-Satterthwaite solution, which corresponds to the case of $p = 1$ and $\gamma = 0$ of the above Lemma. Let λ_1 denote the Lagrange multiplier on expected budget balance in this solution, and let $\widehat{\lambda}_1 \equiv \lambda_1/(1+\lambda_1)$.

Now, fix the option price $\overline{p}$ and a type $\hat{\theta}_2$ and consider the program $R(\overline{p}, \hat{\theta}_2)$ of choosing the allocation rule, the utility mappings $U_1(\cdot)$ and $U_2(\cdot)$, and the "ironing point p" [which affects the solution through constraint (16)] to maximize expected surplus plus λ_1 times expected revenue subject to only $\text{IR}_2(\hat{\theta}_2)$, $\text{IR}_1\,(0)$, ICFOC, and constraint (16), with optimization being over the allocation rule $x_1(\cdot)$ and $p \in \left[\hat{\theta}_2, 1\right]$. We will first show that there is a type $\hat{\theta}_2 \in (\underline{\theta}_2\,(1)\,, 1)$ such that the solution has $p = 1$ and the same allocation rule as in the Myerson-Satterthwaite solution. Thus, this program achieves the Myerson-Satterthwaite expected surplus,and, if $\overline{p} = 1$, satisfies xpected budget balance. Moreover, it also satisfies $\text{IR}_2(\theta_2)$ for all θ_2 as well as monotonicity (and thus, global IC). Thus, the value of program $R(1, \hat{\theta}_2)$ is exactly the second-best (Myerson-Satterthwaite) expected surplus.

To see this, observe that by arguments in the proof of Lemma 15), the solution takes the form described by (13) and (20) [note that we are in the case $\underline{\theta}_2\,(p) \leq \underline{\theta}_2\,(1) < \hat{\theta}_2$]. Now maximization of the Lagrangian over p takes the form

$$p \in \arg \max_{p' \in \left[\hat{\theta}_2, 1\right]} \left(1 - F_1\left(p'\right)\right) E\left[\hat{\delta}(\tilde{\theta}_2) sign(\tilde{\theta}_2 - \hat{\theta}_2)\right]. \tag{26}$$

By (19) and the fact that $\underline{\omega}_i(\theta_i|\hat{\lambda}) \leq \theta_i \leq 1$ for each $i = 1, 2$, $\theta_i \in [0, 1]$, we must have

$$\begin{aligned} \hat{\delta}\,(\theta_2) &= \max\{\bar{\omega}_2(\theta_2|\hat{\lambda}_1) - \underline{\omega}_1(p|\hat{\lambda}_1, 0)\} \geq \max\{\bar{\omega}_2(\theta_2|\hat{\lambda}_1) - 1, 0\} \text{ for } \theta_2 < \hat{\theta}_2, \\ \hat{\delta}\,(\theta_2) &= \max\{\underline{\omega}_1(p|\hat{\lambda}_1) - \underline{\omega}_2(\theta_2|\hat{\lambda}_1, 0)\} \leq 1 - \underline{\omega}_2(\theta_2|\hat{\lambda}_1) \text{ for } \theta_2 > \hat{\theta}_2, \end{aligned}$$

and therefore

$$
\begin{aligned}
& E[\hat{\delta}(\tilde{\theta}_2) sign(\tilde{\theta}_2 - \hat{\theta}_2)] \\
\leq & \int_{\underline{\theta}_2(p|\hat{\lambda}_1,0)}^{\underline{\theta}_2(1|\hat{\lambda}_1,0)} [1-\bar{\omega}_2(\theta_2|\hat{\lambda})] dF_2(\theta_2) + \int_{\underline{\theta}_2 1|\hat{\lambda}_1,0)}^{\hat{\theta}_2} [1-\bar{\omega}_2(\theta_2|\hat{\lambda})] dF_2(\theta_2) \\
& + \int_{\hat{\theta}_2}^{1} [1-\underline{\omega}_2(\theta_2|\hat{\lambda})] dF_2(\theta_2) \\
= & \int_{\underline{\theta}_2(p|\hat{\lambda}_1,0)}^{\underline{\theta}_2(1|\hat{\lambda}_1,0)} [1-\bar{\omega}_2(\theta_2|\hat{\lambda})] dF_2(\theta_2) + \int_{\underline{\theta}_2(1|\hat{\lambda}_1,0)}^{1} [1-\bar{\omega}_2(\theta_2|\hat{\lambda})] dF_2(\theta_2) \\
& + \int_{\hat{\theta}_2}^{1} [\bar{\omega}_2(\theta_2|\hat{\lambda}) - \underline{\omega}_2(\theta_2|\hat{\lambda})] dF_2(\theta_2) .
\end{aligned}
$$

The second integral is strictly negative [since $\bar{\omega}_2(\theta_2|\hat{\lambda}) > \bar{\omega}_2(\underline{\theta}_2(1)|\hat{\lambda}) = 1$ for all $\theta_2 > \underline{\theta}_2(1)$], while the first and third approach zero as $\hat{\theta}_2 \to 1$ [the third integral equals $\hat{\lambda}\left(1-\hat{\theta}_2\right)$, while $\underline{\theta}_2(p|\hat{\lambda}_1, 0)) \to \underline{\theta}_2(1|\hat{\lambda}_1, 0))$ as $\hat{\theta}_2$, and hence p, approaches 1]. Hence, their sum is negative for $\hat{\theta}_2 \in (\underline{\theta}_2(1), 1)$ close enough to 1. Then (26) implies that the program for such values of $\hat{\theta}_2$ is solved by setting $p = 1$.

Now fix $\hat{\theta}_2^* \in (\underline{\theta}_2(1), 1)$ observe the following:

- The second-best expected surplus for $p = 1$ equals the value of program $R(1, \hat{\theta}_2^*)$.
- The value of program $R(1, \hat{\theta}_2^*)$ exceeds the value of program $R(\overline{p}', \hat{\theta}_2^*)$ for any $\overline{p}' \in (\hat{\theta}_2^*, 1)$. This follows because a change from $\overline{p} = 1$ to $\overline{p}' \in (\hat{\theta}_2^*, 1)$ tightens the constraint $\text{IR}_2(\hat{\theta}_2)$ in the relaxed program by $[1 - F_1(p)](p - \hat{\theta}_2)$ and does not affect any other constraints.
- The value of program $R(\overline{p}', \hat{\theta}_2^*)$ for any $\overline{p}' \in (\hat{\theta}_2^*, 1)$ exceeds the value achieved if instead the ironing point p is set equal to $\overline{p}'$ (by the argument above), which in turn exceeds the value that is achieved when we maximize expected surplus plus $\hat{\lambda}_1$ times expected revenue subject to all of the IR and IC constraints.
- Since the Myerson-Satterthwaite surplus exceeds the value that is achieved when we maximize expected surplus plus $\hat{\lambda}_1$ times expected revenue

subject to all of the IR and IC constraints, it must exceed the second-best surplus that is achievable with option price $\overline{p}'$ (otherwise the second-best solution would have at least as large a value of expected surplus plus $\hat{\lambda}_1$ times expected revenue as the Myerson-Satterthwaite expected surplus).

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