Adverse Selection, Slow Moving Capital and Misallocation*

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PRELIMINARY AND INCOMPLETE

Abstract

Adverse selection is commonly used to explain inefficiencies in specific markets. In this paper,we incorporate an informational asymmetry into a decentralized dynamic economy and study its implications for aggregate and sector level dynamics. We show that it leads to slow moving capital, lagged investment and persistent misallocation of resources. The mechanism can help explain why economies recover slowly, even when the shock does not affect the overall productivity or potential output. The model generates a rich set of dynamics and provides a micro-foundation for convex adjustment costs.

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1 Introduction

An important factor in determining aggregate productivity in an economy is the allocative efficiency of its resources. For example, there is growing consensus among economists that misallocation is large enough to explain a significant part of the TFP gap across rich and poor countries.[1] Part of this persistence in misallocation has been linked to the failure of markets.[2] Productive resources are naturally heterogenous in their quality and informational asymmetries between market participants are commonly used to explain market failures. We incorporate these considerations into a dynamic model of capital reallocation and show that adverse selection can lead to slow moving capital, persistent misallocation of resources and provides a micro-foundation for convex adjustment cost models.

We consider a decentralized two-sector dynamic economy in which sectoral productivity shifts arrive randomly and create a reason for reallocating capital from one sector to the other. Capital reallocation takes place in a competitive market; the 'sellers' are firms in the less productive sector who own capital and the 'buyers' are firms in the more productive sector who demand capital. In the absence of any frictions, capital is immediately reallocated to the more productive sector following a productivity shift.

We introduce an information asymmetry by allowing capital to vary in quality and firms to privately observe the quality of the capital they own and operate. Buyers are at an information disadvantage and face an adverse selection problem (Akerlof, 1970) . In a static environment, this friction can lead to a complete breakdown in the market for capital. Within our (dynamic) economy, the adverse selection problem translates into a slow moving reallocation process; capital moves gradually from the less productive sector to the more productive one. In particular, we show there is a unique separating equilibrium in which the capital quality is distinguished by the time and price at which it trades. Because lower quality capital is less productive, firms with low quality capital are more anxious to sell their capital than firms with high quality capital. As a result, lower quality capital is reallocated more quickly, but at a lower price. Delays in reallocation generate real economic costs because capital, especially high quality capital, continues to operate in the less productive sector following a productivity shift.

We explore the implications of this mechanism for reallocation dynamics. Whether the productivity shift is permanent or transitory plays an important qualitative role for our results. We first look at the permanent shift case and examine the transition process from the

[1] For instance, Hsieh and Klenow (2009) compare the dispersion in productivity between the US, India and China and find substantially lower dispersion in the US. Using a fairly general model, they argue that if the dispersion in TFP in India and China were equal to US levels, TFP would be 30-60% higher.

[2] See, for example, Banerjee and Duflo (2005).

less productive to the more productive sector. We derive a direct link between the production technology and the rate at which capital reallocation takes place. With this link, the model yields qualitative predictions about the reallocation dynamics. For example, when the production technology has a constant elasticity of substitution (CES) in sector productivity and capital quality, the rate of reallocation increases over time when the factor inputs are complimentary, and decreases over time when quality and productivity are substitutes.

When shocks are transitory, the possibility of future productivity shifts affects the market value of capital today. Specifically, capital prices in the market depend not only on productivity, but also on how rate at which capital is reallocated. A firm that invests capital today accounts for the delays associated with reallocating the capital in the future. The anticipated possibility of misallocation leads to an information-driven illiquidity discount in capital prices, which captures the efficiency costs of misallocation. The discount varies with the severity of the adverse selection problem; as the dispersion of capital quality increases, the discount increases. Since higher quality capital takes longer to be reallocated, higher quality capital is associated with a larger discount.

In equilibrium, the illiquidity discount and the rate of reallocation are jointly determined. Perhaps surprisingly, this illiquidity discount serves to speed up the reallocation process. Specifically, the presence of the illiquidity discount implies that capital values are less sensitive to quality relative to the case when the productivity shift is permanent. This decreased sensitivity of price to quality serves to ameliorate the adverse selection problem and increase the rate of reallocation for at least some of the low-quality sellers.

We extend the model to study new investment. We introduce entrepreneurs who have the ability to create new units of capital – projects or firms – upon the arrival of an investment opportunity. Entrepreneurs are heterogenous in ability: highly skilled entrepreneurs create projects of higher quality. Entrepreneurs have limited capacity, so in order to start a new firm they must first sell their existing projects, about which they are privately informed. This model generates delay in the response of the economy to investment opportunities (e.g., technological innovations) and gradual increases in the measured productivity of the new sector. When entrepreneurs ability is sufficiently persistent across investment opportunities, aggregate measured productivity drops in response to innovations. This obtains because the first adopters of the new technology are the lower-ability entrepreneurs.

The dynamics implied by our model are qualitatively similar to those implied by models with convex adjustment costs, a standard feature in workhorse macroeconomic models. We consider several commonly used convex adjustment cost formulations and identify the conditions under which our model delivers dynamics consistent each formulation. For example, adjustment cost specifications that penalize change in the fraction of capital stock – implying a declining rate of reallocation – are consistent with the model with adverse selection in which the economic gain from reallocation is decreasing in capital quality. Specifications that penalize the change in the rate of reallocation – implying an increasing rate of reallocation – are consistent with a model in which the economic gain from reallocation is increasing in capital quality. The capital reallocation dynamics are further affected by the distribution of capital quality and inherit its shape. In summary, our model is flexible enough to generate dynamics that are consistent with multiple different adjustment cost models.

Recent papers argue that time-variation in these adjustment costs may be important in explaining features of the data. For instance, Eisfeldt and Rampini (2006) argue that counter-cyclical reallocation costs are needed to reconcile the fact that reallocation activity is pro-cyclical, while the gains from reallocation – measured as the dispersion in productivity or Tobin's Q is counter-cyclical. Further, Justiniano, Primiceri, and Tambalotti (2011) estimate a medium-scale DSGE model and find that shocks to marginal adjustment costs account for a substantial fraction of business cycle fluctuations. By endogenizing the costs of reallocating capital to the economic environment, our paper allows us to interpret these shocks in the context of parameter shifts in our model. To illustrate this connection, we conduct impulse responses to shifts in three key parameters in our model: the dispersion in capital quality, the frequency of sectoral shocks and the level of the interest rate. When the dispersion of capital quality increases, the degree of adverse selection increases which reduces the allocative efficiency and therefore aggregate productivity and output. Perhaps surprisingly, a similar result obtains in response to reduction in the interest rate. Increasing the frequency of sectoral shocks can increase or decrease the rate of reallocation but leads to an unambiguous drop in aggregate output.

An alternative mechanism that delivers persistent misallocation is financial constraints. Financial constraints – typically based on moral hazard – prevent poor but potentially highly productive entrepreneurs from efficiently deploying capital (for a workhorse model, see Banerjee and Newman, 1993). However, several pieces of evidence suggest it is unlikely that financial constraints are the only reason for persistent misallocation. First, Gilchrist, Sim, and Zakrajek (2013) study the empirical role of moral-hazard-based models of financial constraints in generating realistic levels of misallocation using data on borrowing costs of a sample of US manufacturing firms. They find that, for misallocation due to financial constraints to account for a significant fraction of measured TFP differentials across countries, the dispersion in borrowing costs has to be an order of magnitude higher than that observed in the data. Second, the quantitative performance of these models in generating sizable losses from reallocation is mixed (Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2010). The reason is that if the sole force preventing reallocation is a moral-hazard style borrowing constraint, high-type entrepreneurs quickly save out of their borrowing constraint. Along the same lines, Banerjee and Moll (2010) argue that the persistence of misallocation – especially on the intensive margin – is puzzling. Our theory provides an explanation for the persistence in misallocation based on adverse selection rather than financial constraints. Adverse selection may play a key role in preserving the large differences in productivity that have been documented even in countries where financial constraints may be less important, such as the US.[3]

In financial economics, models with adverse selection are the dominant framework used to study the sale of claims on firms' capital (see, for instance Leland and Pyle, 1977; Myers and Majluf, 1984; Brennan and Kraus, 1987; Lucas and McDonald, 1990; Korajczyk, Lucas, and McDonald, 1991). Our paper contributes to a small but growing literature that introduces adverse selection into dynamic macro-finance models. The most closely related papers are Eisfeldt (2004), House and Leahy (2004) and Kurlat (2013). Eisfeldt (2004) and House and Leahy (2004) study the problem of equity issuance and consumer's choice of a durable good in an environment with adverse selection. Both papers find that increasing the variance of the underlying shock increases non-informational motives for trade and thus ameliorates the adverse selection problem. Kurlat (2013) studies a setting in which entrepreneurs have private information that lasts for one period. He shows that this is mathematically equivalent to a tax on capital, which leads to an amplification mechanism in response to aggregate shocks. By contrast, the duration of the information asymmetry is endogenously determined in our model and our focus is on how the information friction affects both the duration and the magnitude of the effect on aggregate dynamics in response to a shock.

That adverse selection can generate a delays in trade between buyers and sellers is, by now, well understood within the dynamic adverse selection literature. Janssen and Roy (2002) derive a competitive equilibrium in which the price mechanism sorts sellers of different qualities into different (discrete) time periods. Hörner and Vieille (2009) use a game-theoretic approach to investigate the implications of public versus private offers in a discrete-time model. Daley and Green (2012) study trade dynamics in a continuous-time model in which information about the seller's quality is revealed gradually. Formally, the approach used here is most similar to Fuchs and Skrzypacz (2013), who study the costs and benefits of temporarily closing the market. The contribution of this paper is to embed this mechanism into a macroeconomic model and study the implications for aggregate dynamics.

The remainder of the paper is organized as follows. In the next section, we provide evidence and motivation for studying reallocation. In Section 2, we illustrate how adverse

[3]For instance, Syverson (2004) reports that, within narrowly defined industries in the U.S., the ratio between the 90-th and the 10-th percentiles of the firm-level productivity distributions is approximately equal to two.

selection generates slow movements in capital across sectors and describe the relation to various convex adjustment cost models. In Section 3, we embed the mechanism into a model which incorporates general equilibrium effects allowing us to derive capital values endogenously. In Section 4, we explore the impulse response of output and productivity to a variety of shocks. Section 5 extends the model to study new investment. Section 6 relates the prediction of our theory to the empirical literature. Section 7 concludes. Proofs are located in the Appendix.

2 A Motivating Example

To illustrate the main ideas in the paper, we start with a motivating example. Consider an economy with two productive sectors, $i \in \{A, B\}$. This is a small open economy, so the interest rate is constant and equal to r. There is a fixed mass $M > 1$ firms in each sector. Firm cannot migrate across sectors. They are risk neutral, have an infinite horizon, and maximize total discounted profits, which includes the purchase or sale of any capital.

There is a unit mass of capital. Capital is heterogenous in its *quality*, also referred to as *type* and denoted by θ, which is distributed according to F on the support $\Theta = [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}_{++}$. Output of the capital stock depends on sector productivity x_i and capital quality. Quality is observable only to the firm who owns and operates the capital. If the firm does not have any capital, it remains idle and generates a constant output normalized to zero. For simplicity, we assume here that capital does not depreciate and there is no inflow of investment (the model in Section 3 incorporates such features).

A unit of capital of quality θ, henceforth a "θ-unit," operated by a firm in sector i, generates a flow of output per unit time of

$$\pi_i(\theta) = (\beta\theta^{\alpha} + (1-\beta)x_i^{\alpha})^{\frac{1}{\alpha}}. \tag{1}$$

Here, β captures the importance of capital quality in production, and $(1-\alpha)^{-1}$ represents the elasticity of substitution between capital quality and productivity.

We focus here on the process by which capital is reallocated from sector A to sector B. To do so, assume that at $t = 0$, all capital is allocated to firms in sector A and that productivity is higher in sector B, $x_B > x_A$.[4] Prior to analyzing the role of adverse selection, it is useful to establish two benchmarks: (1) the frictionless benchmark, and (2) a model with exogenously specified adjustment costs.

[4] Perhaps, due to a demand shock or recent technological innovation in sector B.

2.1 Benchmark 1: Frictionless Economy

In the absence of adjustment costs, a social planner would immediately reallocate all capital from sector A to sector B. In the decentralized economy, the same outcome obtains without the information friction. To see this, suppose that θ is perfectly observable and therefore prices can be conditioned on capital quality θ.

At any point in time, a sector B firm is willing to pay up to $\pi_B(\theta)/r$ to buy a θ-unit of capital. Since capital is scarce and sector B firms are identical and competitive, the price for a θ-unit will get bid up to exactly this amount. Each sector A firm will sell at $t = 0$ at a price equal to the present value of the output the capital generates in sector B. Since there is no informational friction and there are gains from reallocation, all capital is immediately and efficiently reallocated.

2.2 Benchmark 2: Exogenous Adjustment costs

A second useful benchmark is the case in which capital is homogeneous (i.e., $\underline{\theta} = \overline{\theta}$), but there are exogenous costs to reallocating capital. We consider three formulations for these adjustment costs that correspond to the cases most commonly used in the literature. In line with the literature on convex adjustment costs, we specify these costs as a function of the aggregate mass of capital being reallocated at a point in time and focus on the central planner's problem. We denote by k the capital stock in sector B.

The first formulation corresponds to the case where adjustment costs are convex in the rate of reallocation $\dot{k}$.

$$c(\dot{k}) = \frac{1}{2} c \left(\dot{k}\right)^2. \tag{2}$$

These costs are in line with the formulation in Abel (1983). We refer to this as the 'kdot' model. The second formulation is closely related to (2), except that it specifies the adjustment cost in terms of the growth rate of capital being reallocated

$$c(k, \dot{k}) = \frac{1}{2} c \left(\frac{\dot{k}}{1-k}\right)^2 (1-k). \tag{3}$$

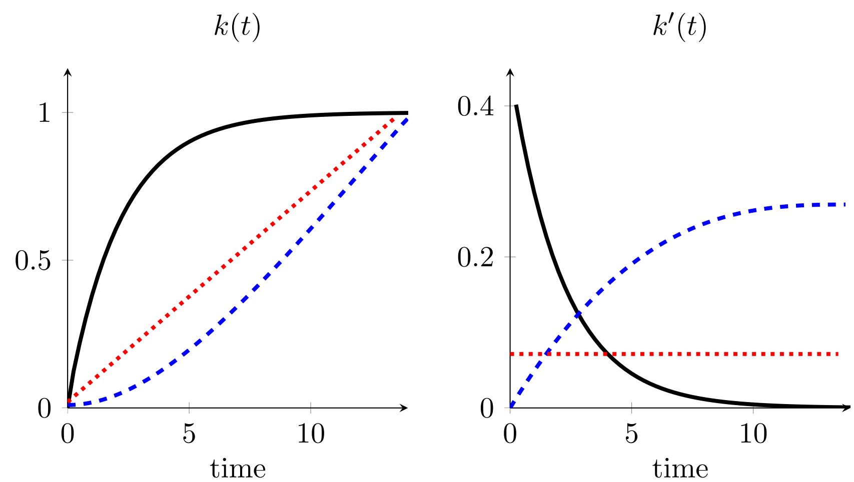
This type of adjustment costs is commonly used in the literature studying investment and reallocation dynamics (Abel and Eberly, 1994; Eisfeldt and Rampini, 2006; Eberly and Wang, 2009). We refer to these costs as the 'ik' model.

The last adjustment cost formulation penalizes *changes* in the flow rate of reallocation $\dot{k}$

$$c(\ddot{k}) = \frac{1}{2} c \left(\ddot{k}\right)^2, \tag{4}$$

and is based on the adjustment costs proposed by Christiano, Eichenbaum, and Evans (2005). We refer to these costs as the 'idot' model.

We compare the reallocation dynamics across the three adjustment cost models in Figure 1. In terms of capital stock, the 'kdot' model implies a linear response of capital. By contrast, the 'ik' model generates strictly concave dynamics for the capital stock, whereas the 'idot' model generates S-shaped path for the capital stock. Relative to the first two formulations, the 'idot' model generates more delayed responses of capital flow to a sectoral productivity shift. The rate of capital reallocation in the 'ik' model spikes on impact and decays smoothly over time. By contrast, in the 'idot' model, the rate of capital reallocation increases slowly over time. This slow increase occurs because the formulation in (4) severely penalizes large adjustments to the rate. Christiano, Eichenbaum, and Evans (2005) argue that this feature is crucial in explaining the response of aggregate investment to shocks. In what follows, we show that adverse selection can generate dynamics similar to all three of the convex adjustment costs models and derive the economic conditions under which each one of them obtains.



(a) Fraction of capital reallocated (b) Rate of reallocation

Figure 1: Comparison across the 'kdot' (red dotted), 'ik' (black solid), and 'idot' (blue dashed) adjustment cost models. The left panel illustrates the capital quality that switches at time t, the right panel illustrates the rate at which capital is reallocated. See Appendix for details.

2.3 Heterogeneous Capital and Adverse Selection

Having established two important benchmarks, here and from now on, capital is heterogenous in quality ($\underline{\theta} < \overline{\theta}$) and is privately observed by the firm who owns it. Further, assume that

$\pi_A(\overline{\theta}) > \int \pi_B(\theta)dF(\theta)$, so that a firm with the highest quality capital in sector A would prefer to retain her capital rather than trade at the average value to firms in sector B. We focus on the competitive equilibrium of the decentralized economy in which reallocation decisions are made by firms.

In order for a unit of capital to be reallocated, a transaction must take place: a firm in sector B must purchase the capital from a firm in sector A. This occurs in a dynamic market; at every $t \geq 0$ a firm in sector A who wishes to sell its unit of capital can trade with firms in sector B who wish to purchase capital. There are no institutional frictions in the market (e.g., transactions costs or search). The only friction is an informational one. That is, buyers cannot observe the quality of capital in the market prior to purchasing it (or, alternatively, that it is too costly to do so). Therefore, sector B firms face a potential adverse selection problem in the market for capital.

A competitive equilibrium of this environment can be characterized by (1) a path of prices P_t, and (2) a time for each θ, $\tau(\theta)$, at which a θ-unit of capital is reallocated. We allow for the possibility that certain types of capital are never reallocated, in which case $\tau(\theta) = \infty$. We formalize our notion of equilibrium in Section 3 (see Definition 1). Roughly, it requires that (i) given the path of prices, sector A firms with capital choose the optimal time to trade, (ii) firms in sector B make zero expected profits and (iii) that the market for capital clears.

Remark 2.1 (Human Capital)**.** *The model has an alternative interpretation related to the allocation of human capital. Relabel 'capital' as 'workers', 'quality' as 'ability' and 'prices' as 'wages'. Workers are privately informed of their ability θ. Rather than the firms decision of when to sell its capital, it becomes the worker's decision of when to migrate to sector B. Firms from sector B do not observe workers ability, but compete for workers from sector A through the timing and the wage they offer. In the context of technological progress, the elasticity of substitution between worker quality and productivity $(1-\alpha)^{-1}$ can be interpreted as the technology being skill biased $(\alpha < 1)$ or 'unskill'-biased $(\alpha > 1)$.*

Since quality is unobservable, prices cannot be conditioned on θ and the first-best reallocation cannot be part of an equilibrium. To see why, suppose that all sector A firms sell their capital at $t = 0$. For sector B firms to break even requires that $P_0 = \frac{1}{r}\int \pi_B(\theta)dF(\theta)$. But given this price, a firm with capital of quality $\overline{\theta}$ in sector A would prefer to retain her capital than trade it away to sector B at the current market price. An alternative conjecture is that all firms with capital quality below some threshold $\theta^c \in (\underline{\theta}, \overline{\theta})$ trade at $t = 0$. In this case, the remaining capital in sector A is of discretely higher quality and the equilibrium price would jump upward. Clearly then firms that sold capital at $t = 0$ would prefer to wait.

Having ruled out a mass of reallocation at date zero, we will show that the unique separating equilibrium involves smooth and gradual reallocation of capital; firms trade off the immediate gains from reallocation versus preserving the option to sell it in the future. Firms with lower quality capital are effectively more anxious to sell – since their capital is less productive – and do so sooner than firms with high quality capital. Since higher quality capital gets reallocated later, the market price of capital will gradually increase over time.

To construct this equilibrium, let $\chi(t)$ denote the quality of capital that is reallocated at date t. In order for sector B firms to break even, it must be that

$$P_t = \frac{\pi_B(\chi(t))}{r}. \tag{5}$$

For this to be an optimal strategy, the firm who owns a $\chi(t)$-unit of capital must be locally indifferent between trading immediately or waiting an instant for a higher price:

$$\underbrace{rP_t - \pi_A(\chi(t))}_{\text{Cost of Delay}} = \overbrace{\frac{d}{dt}P(t)}^{\text{Benefit}}. \tag{6}$$

The left hand side of (6) corresponds to the cost that a firm with a $\chi(t)$-unit in sector A gives up by delaying trade. Using (5), the right hand side can be rewritten as:

$$\frac{d}{dt}P_t = \frac{\pi_B'(\chi(t))}{r}\,\chi'(t), \tag{7}$$

where $\chi'(t) = \frac{d\chi(t)}{dt}$ represents the *rate* at which capital is reallocated to the more productive sector. Combining (6) and (7), we have that

$$\chi'(t) = \frac{r\,\left[\pi_B(\chi(t)) - \pi_A(\chi(t))\right]}{\pi_B'(\chi(t))} \tag{8}$$

This simple differential equation characterizes the equilibrium rate at which capital transitions to sector B. It is based on the first two equilibrium requirements, (i) that sector A firms optimize their selling decisions and (ii) that sector B firms break even. One immediate observation from (8) is that the rate of reallocation is proportional to the gains from doing so (i.e., $\pi_B - \pi_A$).

The boundary condition is pinned down by the market clearing condition, which requires the price at time zero to be at least $\pi_B(\underline{\theta})/r$. This implies that the lowest quality capital trades immediately

$$\chi(0) = \underline{\theta}. \tag{9}$$

For any set of production technologies, (8) and (9) pin down the equilibrium reallocation dynamics. The main takeaway is that adverse selection inhibits the reallocation of capital, resulting in a slow transition of resources to the more productive sector.

The exact shape of the equilibrium reallocation dynamics depends, in part, on the production technology and specifically, on the elasticity of substitution between capital quality and productivity. Using our CES formulation, (1), we focus on three values for the elasticity, $\alpha \in \{0, 1, 2\}$, the first two of which permit analytic solutions.

First, in the case $\alpha = 1$, the production technology is linear, and as a result, there are constant gains from reallocation. Equation (8) becomes

$$\chi'(t) = \left(\frac{1-\beta}{\beta}\right)(x_B - x_A)r.$$

Since the right hand side is a constant, the equilibrium reallocation rate is constant over time. Combining with (9), the solution is given by

$$\chi(t) = \underline{\theta} + \left(\frac{1-\beta}{\beta}\right)(x_B - x_A)rt.$$

where the above holds for $t \leq \tau(\overline{\theta})$, where $\tau = \chi^{-1}$. At this point all capital has been reallocated to sector B and the transition dynamics terminate.

Second, as $\alpha \to 0$, the production technology tends to a Cobb-Douglas. In this case, the gains from reallocation are increasing with quality. Equation (8) becomes

$$\chi'(t) = \kappa\chi(t),$$

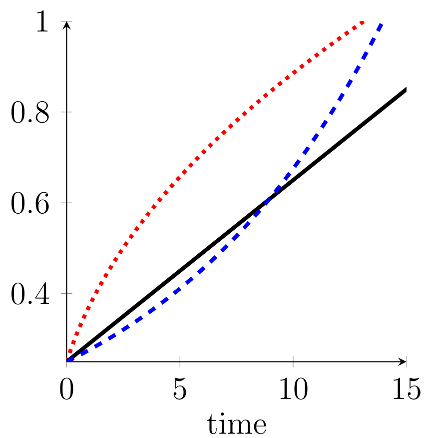
where $\kappa = \left(1 - \left(\frac{x_A}{x_B}\right)^{\beta}\right)(1-\beta)^{-1}r$. Combining with (9), for $t \leq \tau(\overline{\theta})$, the solution is given by

$$\chi(t) = \underline{\theta}e^{\kappa t},$$

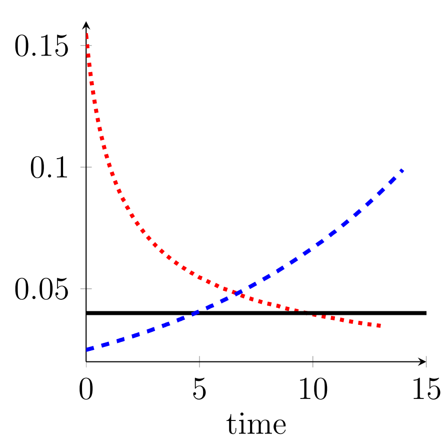
In this case, the equilibrium reallocation rate is increasing exponentially over time.[5]

It is worth noting that the distribution of capital quality affects the equilibrium rate only through the support, Θ. That is, $\chi(t)$ is independent of the exact shape of $F(\theta)$. However, to study the implications for capital reallocation dynamics, we need to specify a distribution over capital quality. For now, we assume a uniform distribution over quality, in which case the rate of capital reallocation is proportional to the rate at which the lowest quality types transition out of the inefficient sector, $k'(t) \propto \chi'(t)$. More generally, the rate at which capital flows from A to B also depends on the distribution over capital quality (i.e., it is given by

[5] Figure 2 also illustrates the case in which $\alpha = 2$. In this case, the differential equation does not admit an analytic solution, however, it is straightforward to compute it numerically.



(a) Fraction of capital reallocated



(b) Rate of reallocation

Figure 2: Equilibrium reallocation with CES production technology for $\alpha = 0$ (blue dashed line), $\alpha = 1$ (black solid line) and $\alpha = 2$ (red dotted line). The left panel illustrates the capital quality that switches at time t, the right panel illustrates the rate at which capital is reallocated. We assume a uniform distribution of quality.

$\chi'(t)dF(\chi(t)))$. We maintain the uniform specification here to highlight the intuition. We consider alternative distributions in Section 3.3 and when conducting impulse responses in Section 4.

We plot the implied reallocation dynamics for the three cases in Figure 2. As we see in panel (a), the quality of capital that is reallocated increases over time in all three cases. This property is true regardless of the production technology; lower quality capital will reallocate sooner than higher quality capital for all specifications of the model. More importantly, panel (b) shows that the qualitative features of the equilibrium reallocation rate depend on the elasticity of substitution between factors. Comparing Figure 2 to Figure 1, we see that increasing gains from trade ($\alpha = 0$) generate an increasing rate of reallocation in line with the 'idot' models of adjustment costs, while decreasing gains from trade ($\alpha = 2$) generate a decreasing rate of reallocation in line with the 'ik' models of adjustment costs. When the gains from reallocation are constant ($\alpha = 0$), the dynamics match those of the 'kdot' model. In the next section, we study the behavior of χ' in more detail, as well as the implications for capital prices in an environment with stochastically recurring productivity shifts.

3 The Model

Our motivating example in the previous section considers a single transitionary period since reallocation occurs only once. Here, we allow productivity to vary across sectors and over time in a stochastic manner. In this case, firms internalize the possibility of costly future reallocation in their decisions. Further, the frequency of these shocks affect the equilibrium price of capital.

We retain most of the key elements from the example: this is a small open economy with two sectors (A and B), a mass $M > 1$ of firms located in each sector; capital is heterogenous in its quality, which is privately observed by the firm that owns and operates it; capital quality is distributed according to $F(\theta)$, which is continuous with strictly positive density over the support $\Theta = [\underline{\theta}, \overline{\theta}]$; reallocation of capital occurs in a competitive market; this market is open continuously at all $t \geq 0$ and firms have an infinite horizon.

Unlike in the previous section, we let capital depreciate at rate δ and allow new capital to flow into the most productive sector of the economy at rate $\gamma \geq \delta$. The quality of capital inflows are also distributed according to F. Except when otherwise noted, we assume there is no growth ($\gamma = \delta$) in the economy.

Sectoral Productivity Shifts. The key additional feature here is that we incorporate shocks to the model by allowing the production technology to vary (stochastically) over time. To do so, we introduce a Markov switching process $\Phi(\omega) = \{\Phi_t(\omega), 0 \leq t \leq \infty\}$ defined on the underlying probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\Phi_t(\omega) \in \{\phi_A, \phi_B\}$ represents the state of the economy at date t.[6] The (flow) output of a firm in sector i, operating a θ-unit of capital in the state ϕ is given by

$$dy_t^i(\theta) = \pi_i(\theta, \phi)dt,$$

where π_i is strictly positive, increasing and twice differentiable in θ, with uniformly bounded first and second derivatives.[7] The state determines the sector to which capital is most efficiently allocated. That is, capital is most productive in sector i, when the aggregate state is ϕ_i. Correspondingly, $\pi_i(\theta, \phi_i) > \pi_i(\theta, \phi_j)$ for $i \neq j$. The transition matrix is denoted by Λ with arbitrary element, λ_{ij}, denoting the rate at which the state of the economy switches from state ϕ_i to state ϕ_j.

Information and Prices. All firms observe the path of the exogenous state variable $\Phi = \{\Phi_s, 0 \leq s \leq \infty\}$. We let $\{\mathcal{F}_t\}_{t \geq 0}$ denote the filtration encoding the information observed by all firms prior to date t. In addition, a firm who currently owns a unit of capital privately observes its quality. The quality of each unit of capital is unobservable to all other firms. However, firms can observe to which sector the capital is currently allocated. For this reason, at each point in time t, there will be two prices in the market; one for capital currently located in sector A, denoted by P_t^A, and one for capital currently located in sector B, denoted by P_t^B.

[6] Henceforth, we omit the argument ω, and use a t subscript as a place holder for the argument (t, ω).

[7] That is, there exists a, A such that $0 < a < A < \infty$ and $\frac{\partial}{\partial q}\pi_i$, $\frac{\partial^2}{\partial q^2}\pi_i \in (a, A)$ for all (i, q, x).

The Firms Problem. Consider a sector i firm who purchases a unit of capital at date t. Upon doing so, the firm will observe the capital quality, θ, and operate the capital until it is no longer optimal to do so. The decision facing the firm is when to reallocate (i.e., sell) their existing capital. Let $V_t^i(\theta)$ denote the firm's value for the unit of capital. Given an ($\mathcal{F}_t$-adapted) price process, P_t^i, the firm's problem can be written as

$$V_t^i(\theta) = \sup_{\tau \geq t} E_t \left[\int_t^\tau e^{-(r+\delta)(s-t)} \pi_i(\theta, \Phi_s) ds + e^{-(r+\delta)(\tau - t)} P_\tau^i \right]. \tag{10}$$

Equilibrium Concept. To formally define an equilibrium of the economy, we will need the following notation and definitions. Denote the *policy* of the firm as a stopping time $T_t^i(\theta)$. The policy is *admissible* if it is both adapted to the filtration $\{\mathcal{F}_s\}_{s\geq 0}$ and weakly larger than t. The policy is *optimal* if it solves (10). Let $\Theta_t^i \equiv \{\theta : T_s^i(\theta) = t, s \leq t\}$ denote the set of capital qualities sold at date t from sector i. Finally, let $\theta_t^i \equiv \inf\{\theta : T_s^i(\theta) \geq t, s \leq t\}$ denote the lowest quality of capital currently allocated to sector i.

Definition 1. *A* ***competitive equilibrium*** *of the decentralized economy consists of admissible stopping times, $T_t^i(\theta) : \Omega \to \mathbb{R}_+$ and $\mathcal{F}_t$-adapted price processes $P_t^i : \Omega \to \mathbb{R}$ such that for each $i \in \{A, B\}$, $t \geq 0$, $\theta \in \Theta$ and $j \neq i$:*

1. *Firm Optimality. $T_t^i(\theta)$ is optimal,*
2. *Zero Profit. If $\Theta_t^i \neq \emptyset$ then $P_t^i = \mathbb{E}\left[V_t^j(\theta) | \theta \in \Theta_t^i, \mathcal{F}_t\right]$,*
3. *Market Clearing. If $\Theta_t^i = \emptyset$, $P_t^i \geq \inf\{V_t^j(\theta) : \theta \geq \theta_t^i\}$.*

Condition 1 is straightforward. Condition 2 says that the price at time t must be equal to the expected value of the reallocated capital at time t, which implies a firm who purchases a unit of capital cannot make positive (or negative) expected profits. Condition 3 is justified by a market clearing reasoning. Specifically, the price for a unit of capital in sector i cannot be less than the lowest possible value for that unit of capital in sector j. If the price was strictly less, than all firms in sector j would demand capital at that price and demand would exceed supply. Besides being intuitive, this condition rules out trivial candidate equilibria, such as one in which prices are always very low and trade never takes place.[8]

One property that is typical of environments like this is the so called *skimming* property, which says that lower quality capital gets reallocated sooner than higher quality capital.

Lemma 3.1 (Skimming)**.** *In any equilibrium, $T_t^i(\theta)$ is weakly increasing in θ for all (i, t).*

[8]Condition 3 has the same implications as the market clearing condition used in Fuchs and Skrzypacz (2013).

The intuition is the same as in the motivating example; firms with lower quality capital are more anxious to sell their capital, because their outside option to wait is less valuable due to lower output in the interim.

Symmetric Economies

For both tractability and ease of exposition, we illustrate the main ideas of this section within the class of *symmetric economies*. In a symmetric economy, the output of a firm depends only on the quality of its capital and whether that capital is allocated efficiently (i.e., to the more productive sector given the current state).

Definition 2. *The economy is* ***symmetric*** *if there exists a pair of functions* $\{\pi_0, \pi_1\}$ *and scalar* λ *such that* $\pi_i(\theta, \phi_i) = \pi_1(\theta)$ *for* $i \in \{A, B\}$*,* $\pi_i(\theta, \phi_j) = \pi_0(\theta)$*, and* $\lambda_{ij} = \lambda$ *for* $i \neq j$*.*

For the remainder of the this section, we will restrict attention to symmetric economies. It is straightforward, though more notationally cumbersome, to extend results to a setting in which the economy is not symmetric. A symmetric economy is fully described by $\Gamma \equiv \{\pi_0, \pi_1, r, \delta, \gamma, \lambda, F\}$. We refer to the *production technology* as a pair of functions $\{\pi_0, \pi_1\} : \Theta \to \mathbb{R}$. We refer to the *efficient sector* as the sector in which capital is currently most productive.

Within the class of symmetric economies, the natural extension of the equilibrium from Section 2 can be characterized by two functions. The first is $\tau(\theta)$, which represents how long a θ-unit of capital takes to be reallocate following a productivity shock provided no other shocks arrive in the interim. The second is $V(\theta)$, which is the (endogenous) value of efficiently allocated capital as it depends on θ. As in Section 2, we look for *fully separating*, which requires that τ is strictly increasing in θ. Here again, it will sometimes be easier to use the inverse of τ, which we denote by $\chi(t) \equiv \tau^{-1}(t)$.

To formalize the connection to the equilibrium objects in Definition 1, let $m_t \equiv t - \sup\{s \leq t : \phi_{s^+} \neq \phi_{s^-}\}$ denote the amount of time that has elapsed since the last shock arrived.

Definition 3. *The strategies and prices that are* ***consistent*** *with* (τ, V) *are given by:*

$$T_t^i(\theta) = \inf\{s \geq t : m_s = \tau(\theta), \phi_s \neq \phi_i\} \tag{11}$$

$$P_t^i = \begin{cases} V\left(\chi(m_t)\right) & \text{if } \phi_t \neq \phi_i \text{ and } m_t < \tau(\overline{\theta}) \\ V(\underline{\theta}) & \text{otherwise} \end{cases} \tag{12}$$

The main result of this section is the following.

Theorem 3.2. *In a symmetric economy, there exists a unique* (τ^*, V^*) *such that the strategies consistent with* (τ^*, V^*) *constitute a fully separating equilibrium.*

To sketch the argument, we proceed with a heuristic construction of the equilibrium based on necessary conditions, which can be reduced to a single initial value problem. This initial value problem can be shown to have a unique solution (under regularity conditions), which proves that a unique candidate exists. We then verify that these necessary conditions are also sufficient.

According to the candidate equilibrium, the value a firm derives from capital depends only on its quality if it is efficiently allocated. If it is inefficiently allocated, the value derived also depends the lowest quality of capital remaining in the inefficient sector (or equivalently, m_t). Let $V_0(\theta, \chi)$ denote the value of an inefficiently allocated θ-unit when the lowest remaining quality of capital in the inefficient sector is $\chi \leq \theta$. According to (τ, V), the firm waits until $\chi = \theta$ to trade. Therefore, the evolution of V_0 for $\chi < \theta$ is given by

$$rV_0(\theta, \chi) = \pi_0(\theta) - \delta V_0(\theta, \chi) + \lambda(V(\theta) - V_0(\theta, \chi)) + \frac{\partial}{\partial \chi} V_0(\theta, \chi)\chi'(t) \tag{13}$$

When $\theta = \chi$, a firm with a misallocated θ-unit sells at a price equal to $V(\chi)$. We abuse notation by letting $P(\theta)$ denote the price at which a firm in the inefficient sector sells a θ-unit to a firm in the efficient sector. Hence, a necessary boundary condition for V_0 is given by

$$V_0(\theta, \theta) = P(\theta). \tag{14}$$

The (local) optimality condition (required to ensure that *Firm Optimality* holds) is that when $\theta = \chi$, the firm with a θ-unit is just indifferent between selling immediately and waiting an "instant". In other words, the firm's value function must smoothly paste to the path of prices.

$$P'(\chi) = \frac{\partial}{\partial \chi} V(\theta, \chi)\bigg|_{\theta = \chi} \tag{15}$$

In order for the *Zero Profit* condition to hold, the price at which capital transacts must be equal to its value in the efficient sector. This requires that

$$P(\theta) = V(\theta). \tag{16}$$

Evaluating (13) using (14)-(16), we arrive at

$$\chi'(t) = \frac{\rho V(\chi(t)) - \pi_0(\chi(t))}{V'(\chi(t))}, \tag{17}$$

where $\rho = r + \delta$, represents the effective discount rate. Note that (17) is analogous to (8), where π_B/r is replaced with V. It is also worth noting that the rate at which productivity shocks arrive, λ, does not enter directly into (17). This is because the price the firm gets upon selling capital is equal to the value of that capital if another shock were to arrive (in which case the firm would retain possession). Nevertheless, λ does play an important role in determining the equilibrium capital values and prices.

Equilibrium Value of Capital

Consider an arbitrary (τ, V) and note that the value of a unit of inefficiently allocated capital when $\chi = \underline{\theta}$ can be written as

$$V_0(\theta, \underline{\theta}) = f(\tau(\theta))\frac{\pi_0(\theta)}{\rho} + (1 - f(\tau(\theta))V(\theta), \tag{18}$$

where

$$f(\tau) \equiv \int_0^\tau (1 - e^{-\rho t})\lambda e^{-\lambda t} dt + e^{-\lambda\tau}(1 - e^{-\rho\tau}), \tag{19}$$

denotes the expected discount factor until either (i) the state switches back, or (ii) the capital gets reallocated to the other sector. Similarly, for an arbitrary $V_0(\theta, \underline{\theta})$, the value of an efficiently allocated θ-unit is given by

$$V(\theta) = \frac{\rho}{\rho + \lambda}\frac{\pi_1(\theta)}{\rho} + \frac{\lambda}{\rho + \lambda}V_0(\theta, \underline{\theta}). \tag{20}$$

Solving (18) and (20) jointly, we arrive at

$$V(\theta) = g(\tau(\theta))\frac{\pi_0(\theta)}{\rho} + (1 - g(\tau(\theta)))\frac{\pi_1(\theta)}{\rho}, \tag{21}$$

where $g(\tau) \equiv \frac{\lambda}{\rho + \lambda f(\tau)} f(\tau)$. The expression in (21) has an intuitive form. Capital spends some fraction of the time allocated efficiently and some fraction of the time misallocated. Therefore, its value is simply a weighted average of the value were it to be permanently efficiently allocated (i.e., $\frac{\pi_1}{r}$) and permanently misallocated (i.e., $\frac{\pi_0}{r}$). The amount the time it takes to get reallocated is determined by (17), which in turn depends on V; this illuminates the nature of the fixed point. The solution turns out to be quite tractable. By substituting $\chi(t)$ for θ into (21) and substituting back into (17), we arrive at

$$\chi'(t) = \frac{r\left(1 - g(t) + \frac{g'(t)}{r}\right)(\pi_1(\chi) - \pi_0(\chi))}{g(t)\pi_0'(\chi) + (1 - g(t))\pi_1'(\chi)}. \tag{22}$$

Note that both the numerator and denominator are strictly positive ensuring that χ (and therefore τ) are strictly increasing. As before, the boundary condition is pinned down by the fact that in any separating equilibrium, the lowest type must reallocate immediately after the productivity shock and therefore

$$\chi(0) = \underline{\theta}. \tag{23}$$

The regularity conditions imposed on π_1 and π_0 ensure a unique solution exists as demonstrated by the following lemma.

Lemma 3.3. *There exists a unique χ^* that satisfies (22) and (23). Furthermore, χ^* is strictly increasing.*

The last step in the proof of Theorem 3.2 is to verify that the candidate satisfies the three equilibrium conditions. That *Zero Profit* holds follows from the fact that the equilibrium is fully separating and capital of quality θ trades at a price of $V(\theta)$. That *Market Clearing* holds follows immediately from (12) and that $V(\theta)$ is equal to the value derived from a θ-unit. Finally, we demonstrate that a firm who owns capital does not have a profitable deviation by showing that the Spence-Mirlees condition holds for firms' objective function, which verifies *Firm Optimality*.

3.1 Reallocation with Permanent Shifts

A special case of the model is when the productivity shift is *permanent*. To study the transition dynamics for this case, let $\lambda = 0$, assume that all capital is originally allocated to sector A, and the productivity shift occurs at $t = 0$ so that B is the more productive sector for all $t \geq 0$.

This situation is effectively the same as that in Section 2: because sector B is more productive, capital will transition from A to B; due to adverse selection, the reallocation process occurs slowly over time. Since there is no further technological progress, firms in sector B retain the capital until it fully depreciates. Hence, firms have a value $\pi_1(\theta)/\rho$ for a θ-unit of capital. Thus, in any separating equilibrium, the rate at which at θ-unit of capital is reallocated does not impact the price at which it trades.

Proposition 3.4. *Suppose that the productivity shift is permanent. Then, $g(t) = 0$ for all t and (22) reduces to*

$$\chi'(t) = \rho \frac{\pi_1(\chi(t)) - \pi_0(\chi(t))}{\pi_1'(\chi(t))}. \tag{24}$$

Notice that the expression for χ' in (24) is effectively the same as (8) in the example. Therefore, the unique separating equilibrium in the case of permanent productivity shifts is precisely the one characterized in Section 2. We revisit it here because it is particularly useful for highlighting the economic environments under which various patterns in the rate of reallocation obtain. The numerator in (24) measures the magnitude of the productivity gains from reallocation as they depend on the quality of the capital; the larger the benefit of reallocation, the faster it takes place. The denominator measures the marginal productivity of capital quality in the efficient sector. Perhaps surprisingly, the higher the marginal productivity of quality, the *slower* the reallocation rate and the longer the reallocation process takes. The intuition for this comes from the indifference condition of the cutoff type. Recall that the total change in prices with respect to time is given by

$$dP_t = \frac{\pi_1'(\chi)}{\rho} \cdot \chi'(t)dt$$

Fixing $\chi'(t)$, increasing the marginal productivity of capital quality increases the rate at which prices increase over time. In order for the cutoff type to remain indifferent, the reallocation rate must decrease. Using (24) and noting that $\chi'(t)$ is strictly positive, we have the following result.

Proposition 3.5. *Suppose that the productivity shift is permanent. Then, the equilibrium rate of reallocation will increase (decrease) over time until all capital has been reallocated (i.e., $t \in (0, \tau(\overline{\theta}))$) if and only if $(\pi_1 - \pi_0)/\pi_1'$ is increasing (decreasing) over $\theta \in \Theta$.*

Returning to the example with a CES production technology, Proposition 3.5 implies the following corollary.

Corollary 3.6. *Suppose that the economy is symmetric, the productivity shift is permanent, and the production technology is CES. Then, until all capital has been reallocated to the efficient sector:*

- *If $\alpha < 1$, the equilibrium rate of reallocation is strictly increasing over time.*
- *If $\alpha > 1$, the equilibrium rate of reallocation is strictly decreasing over time.*
- *If $\alpha = 1$, the equilibrium rate of reallocation is constant over time.*

This corollary formalizes the result illustrated in Figure 2. When $\alpha < 1$, reallocation and capital quality are complementary; reallocation is more beneficial for higher types. In this case, like the 'idot' model, the model with adverse selection predicts an increasing path of reallocation. In contrast, when reallocation and capital quality are substitutes, equilibrium reallocation is decreasing over time as in the 'ik' model.

3.2 Reallocation with Transitory Shifts

When the sectoral productivity shift is transitory, firms investing in capital today will become sellers of capital at some point in the future. Therefore, in considering their willingness to pay for a θ-unit of capital, firms must account for the potential costs associated with reallocation, which are then incorporated into equilibrium prices. Recall that when the shift is permanent, the price at which a θ-unit trades is equal $\pi_1(\theta)/\rho$, which is the present value of the future output that it generates for a firm in the efficient sector. With transitory shifts, this is no longer the case.

Proposition 3.7. *If the productivity shift is transitory ($\lambda > 0$), the price at which a θ-unit of capital trades is strictly less than $\pi_1(\theta)/\rho$ for all $\theta > \underline{\theta}$.*

Capital trades at an (information-driven) illiquidity discount relative to the value that it could generate were it always efficiently allocated. Further, the discount is endogenously determined by the degree of adverse selection. One way to see this is through a comparative static on the dispersion of capital quality, as measured by an increase in the support of capital quality. Increasing the support of capital quality increases buyer's uncertainty about capital quality and hence the severity of the adverse selection problem. Reallocation will then be slower and more costly leading to a larger discount. To measure the discount, we compute the difference between the full information price and the price at which capital sells when firms are privately informed,

$$\text{Discount}(\theta) = \frac{\pi_1(\theta)}{\rho} - V_1(\theta).$$

Figure 3 illustrates the illiquidity discount as it depends on θ and the dispersion of capital quality.

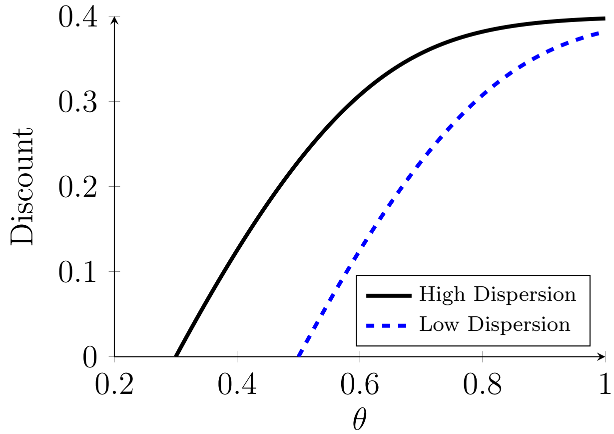


Figure 3: The effect of capital quality dispersion on the illiquidity discount. The solid line represents an economy with higher dispersion ($\Theta = [0.3, 1]$) whereas the dashed line involves lower dispersion ($\Theta = [0.5, 1]$). The figure uses CES production technology with $\alpha = 1$.

Naturally, the illiquidity discount affects the price at which a firm can sell its capital and affects its eagerness to do so. Therefore, we now explore how changes in the frequency of sectoral shifts, λ, impacts the rate of reallocation. It may be intuitive to think that firms will have less incentive to reallocate because the shock is only temporary and hence the rate of reallocation should decrease with λ. As the next result demonstrates, this intuition is not entirely correct.

Proposition 3.8. *Consider any two symmetric economies Γ_x and Γ_y, which are identical except that $\lambda_x < \lambda_y$. There exists a $\bar{t} > 0$ such that the rate of reallocation is strictly higher in Γ_y than in Γ_x prior to $\bar{t}$, i.e., $\chi'_y(t) > \chi'_x(t)$ for all $t \in [0, \bar{t}]$.*

We should note that this behavior is in stark contrast to what one would expect in a standard model with exogenous reallocation costs, in which increasing the volatility of sectorial shocks typically leads to more delay as the option value of waiting increases. In contrast, in our model, increasing volatility leads to an increase in the rate of reallocation for at least some types and can do so for all types.

To see the intuition behind this result, recall that the lowest-quality capital is always efficiently allocated and therefore $\rho V(\underline{\theta}) = \pi_1(\underline{\theta})$ in both Γ_x and Γ_y. Fixing the equilibrium strategies from Γ_x, consider the effect of an increase from λ_x to λ_y. Since the state is now switching more frequently and the rate of reallocation remains unchanged, firms with capital of quality $\theta > \underline{\theta}$ endure more misallocation, which gives them more incentive to imitate the lowest type. Now recall that by construction, types arbitrarily close to $\underline{\theta}$ were indifferent in Γ_x between accepting $P(\underline{\theta})$ or waiting an instant. Hence, the increase in λ will cause these types to strictly prefer to imitate $\underline{\theta}$. To restore the equilibrium in Γ_y, types near $\underline{\theta}$ must trade faster and the reallocation of capital increases, as we see in Figure 5.

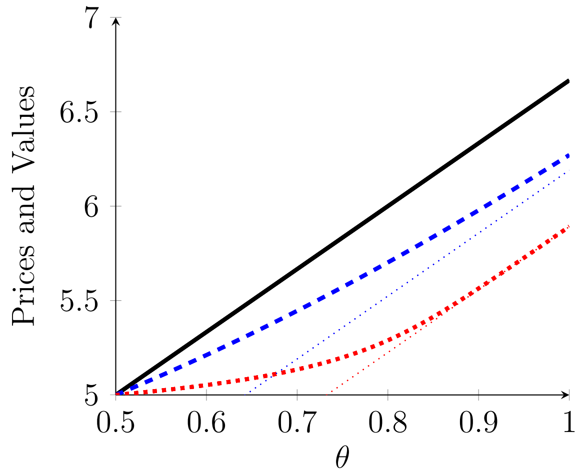


Figure 4: The effect of transitory shocks on the price of capital. The dashed blue corresponds to $\lambda = 0.1$ and the dotted red lines corresponds $\lambda = 1$. The black line represents the case when the shock is permanent ($\lambda = 0$), which also corresponds to the fully efficient value of capital. The fainter blue (red) dotted lines represent the hypothetical value of a unit of capital if it is never reallocated for $\lambda = 0.1$ ($\lambda = 1$), which approaches for $\lambda = 1$ and θ large. The figure uses CES production technology with $\alpha = 1$.

This behavior implies that equilibrium prices are becoming less sensitive to capital quality for θ near $\underline{\theta}$. As a result, there is endogenously *less* adverse selection at the bottom of the market. Despite this decrease in the level of adverse selection, since the frequency with which costly reallocation occurs increases, the overall efficiency decreases leading to lower equilibrium capital prices as illustrated in Figure 4.

Finally, using the CES technology, we study the interaction of the complementary of quality and productivity of the sector with the persistence of the shocks. Figure 5 illustrates how the transitory nature of shocks affects the reallocation dynamics. In particular, the transitory nature of shocks tends to make $\chi'(t)$, decreasing offsetting the effects of complementarity between quality and productivity ($\alpha = 0$) which generates an increasing rate of reallocation when $\lambda = 0$. This effect is new to this model and highlights the importance of endogenously deriving the costs of adjustment as a function of the economic environment. Another important consideration which will shape the aggregate dynamics is the distribution of capital quality, which we explore next.

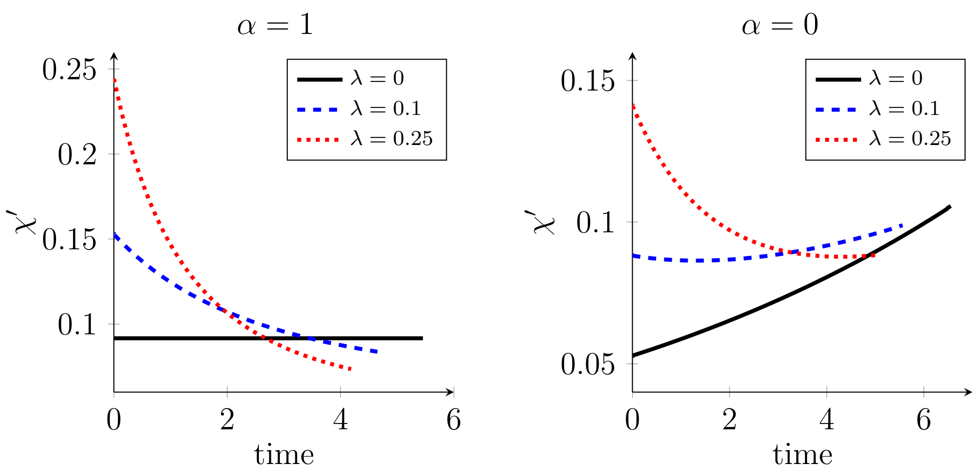


Figure 5: Equilibrium reallocation with transitory shocks and CES production technology for $\alpha = 1$ (left) and $\alpha = 0$ (right). The other parameters used are $\beta = 0.45$, $r = 0.15$, $x_A = \frac{1}{2}$, $x_B = 1$, $\Theta = [0.5, 1]$.

3.3 Reallocation and the distribution of capital quality

Our previous discussion implicitly assumed that the distribution of capital quality was uniform. Here, we relax this assumption and consider more general distributions of quality. In general, the total measure of capital allocated in sector i equals

$$k_t^i = \int dF_t^i(\theta), \tag{25}$$

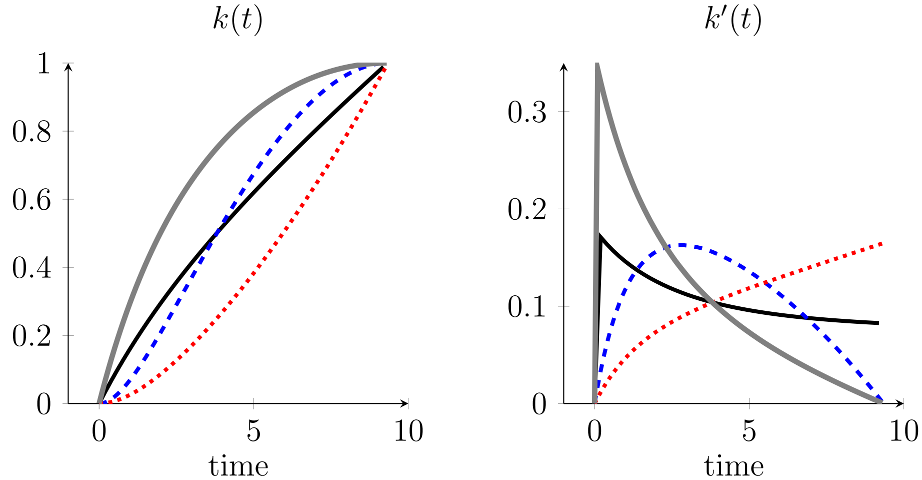
where $F_t^i(\theta)$ is the cumulative distribution of capital quality in sector i at time t. Hence, given the equilibrium rate of skimming $\chi(t)$, the rate of capital reallocation from sector i to sector j equals

$$k^{i\prime}(t) = \chi'(t)\, dF^i(\chi(t)). \tag{26}$$

Holding constant the rate at which types transition across sectors, varying the distribution of capital quality leads to different reallocation dynamics. For instance, in the case where the rate of type transition $\chi'(t)$ is a constant, the dynamics of capital reallocation are driven entirely by the shape of F. In the special case of uniform quality, $dF^i(\theta) = (\overline{\theta} - \underline{\theta})^{-1}$, the capital reallocated to sector B is proportional to $\chi(t)$, and the rate of capital reallocation is equal to $\chi'(t)$.

We compare the dynamics implied by the uniform distribution for $F(\theta)$ in $[\underline{\theta}, \overline{\theta}]$ to those of a beta distribution with the same support. We consider three parameterizations of the beta distribution: a right-skewed $(2, 1)$, a symmetric $(2, 2)$, and a left-skewed $(1, 2)$ version.

To illustrate the effect of the distribution of quality on the dynamics of capital reallocation, we focus on the case of constant gains from trade, $\alpha = 1$.



(a) Fraction of capital reallocated (b) Rate of capital reallocation

Figure 6: Comparison of capital reallocation dynamics across different distributions of capital quality, F. The solid black line refers to the case of uniform distribution of quality. The dashed blue line corresponds to a beta (2,2); the dotted red line refers to a beta (2,1); the thick gray line corresponds to a beta (1,2). The figure uses constant gains from trade $\alpha = 1$ and transitory shocks $\lambda = 1/10$.

The dynamics of capital reallocation inherit the dynamics of $\chi(t)$ along with the shape of the distribution as shown in Figure 6. Comparing Figure 6 to Figure 1, we see a striking qualitative similarity between the 'ik' and 'idot' adjustment cost models and the beta (1,2) and beta (2,1) cases respectively. When the distribution of quality is hump shaped (as is the case quality follows a beta (2,2), the rate of reallocation is hump-shaped and the dynamics of capital have an S-shape.

4 Response of output and productivity to shocks

So far, we have been focusing on the dynamics of capital reallocation. Here, we examine the behavior of aggregate and sectoral output and productivity, in response to several types of shocks. The output of sector i at time t depends on the current distribution of project quality in that sector

$$Y_t^i = \int y_t^i(\theta)\, dF_t^i(\theta), \tag{27}$$

where $y_t^i(\theta)$ denotes the output of a unit of capital of quality q in sector i at time t. Aggregate output is then equal to $Y_t = \sum_i Y_t^i$.

We also compute the average productivity of capital in each sector as

$$X_t^i = \frac{Y_t^i}{k_t^i}. \tag{28}$$

Since aggregate capital is constant, aggregate productivity is equal to total output, $X_t = Y_t$.

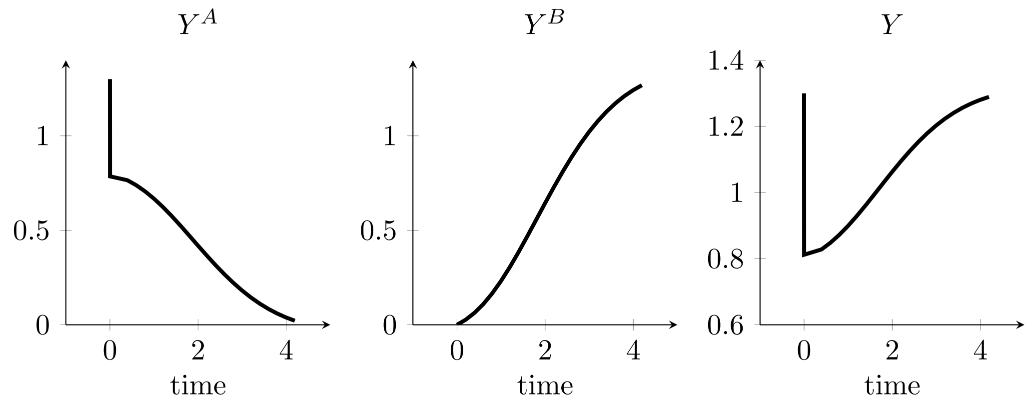
4.1 Response to a sectoral productivity shift

First, we examine the response of the economy to a sectoral productivity shift. We focus on the case where the gains from trade are constant, $a = 1$, and the overall distribution of quality is distributed as a truncated normal on Θ. We show the results in Figure 7.

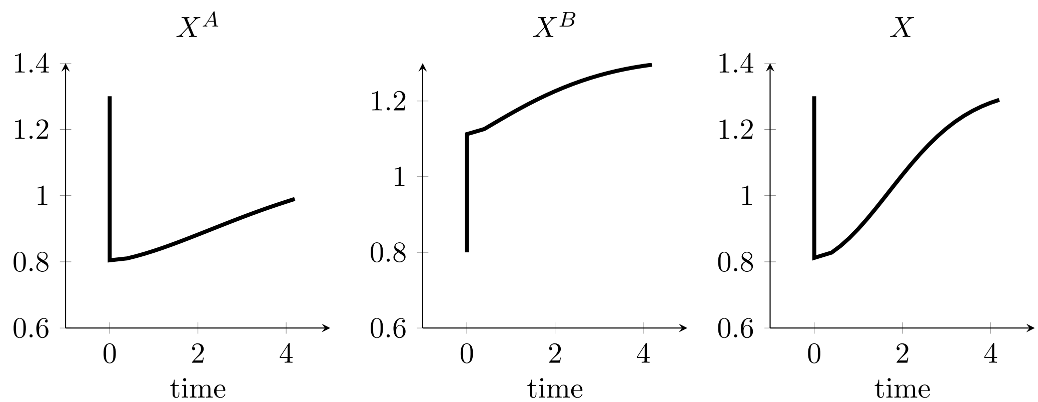
Recall that a productivity shift causes the sectoral productivity of A to fall and of B to rise. Since all capital is initially allocated in sector A, aggregate output falls on impact, as we see in Panel (c). As the economy reallocates capital, output in sector A continues to fall while output in sector B rises. Once all capital is reallocated from sector A to sector B, total output is restored to the pre-shock level. In this model, the response of output to a sectoral shift is qualitatively similar to that of a model with adjustment costs. However, the behavior of total factor productivity exhibits dynamics that are markedly different to a model with adjustment costs. In particular, Panels (d) and (e) show that productivity rises over time in *both* the sector from which capital exits (A) and in the sector to which it is being reallocated (B). In contrast, in the standard adjustment cost models, productivity would either be flat or display opposite patterns in each sector.[9]

The behavior of average productivity in response to a shock is a distinguishing feature of our model. Productivity improves in both sectors because the distribution of capital quality θ across sectors varies over time. As time passes, the quality of the marginal unit being transferred from sector A to B is higher, implying the average quality for both sectors is increasing. This prediction is in line with the data. Collard-Wexler and Loecker (2013) study the reallocation of resources in the steel industry following the introduction of the minimill technology. They find that productivity increased over time among plants that used minimills, but also among vertically integrated plants (the old technology). The improvement in productivity among vertically integrated plants took place primarily through the exit of low productivity plants. Further, this mechanism may help to explain a puzzling fact from the international literature, which is that developing countries experience measured increases in productivity along side capital outflows (Gourinchas and Jeanne, 2006; Prasad, Rajan,

[9] Specifically, with constant returns average productivity of capital would be flat. With decreasing returns, productivity in sector A would increase while average productivity in sector B would decrease. Increasing returns to scale would generate the opposite pattern.



(a) output in sector A (b) output in sector B (c) total output



(d) productivity in sector A (e) productivity in sector B (f) total productivity

Figure 7: Response to a sectoral productivity shift, where at $t = 0$, sector B becomes the more productive sector. The distribution of quality $F(q)$ is beta in $\underline{\theta}$ and $\overline{\theta}$ with shape parameters $a = b = 2$. The figure uses constant gains from trade $\alpha = 1$ and transitory shocks $\lambda = 1/10$.

and Subramanian, 2007).

4.2 Response of the economy to structural changes

Here, we consider the effect of an unanticipated change in the model's structural parameters on aggregate output, the level of misallocation, and the rate of capital reallocation. Several papers have recently argued that shocks to adjustment costs can be useful for explaining features of the data (Eisfeldt and Rampini, 2006; Justiniano et al., 2011). Yet, one may find it difficult to interpret the underlying economic environment in which adjustment costs are exposed to such shocks. One benefit of conducting this exercise is to link a change in adjustment costs to a shift in the parameters of our model.

We consider three types of shocks. First, we examine an increase in the dispersion of capital quality. Second, we consider unanticipated changes to the frequency of sectoral productivity shifts, modeled as an increase in λ. We can interpret this change as an increase in the volatility of sectoral shocks. Finally, we consider the effect of a change to the interest rate.

To understand the dynamic response of these shocks, we compute the level of misallocation at time t as the percent of total potential output lost due to misallocation of capital:

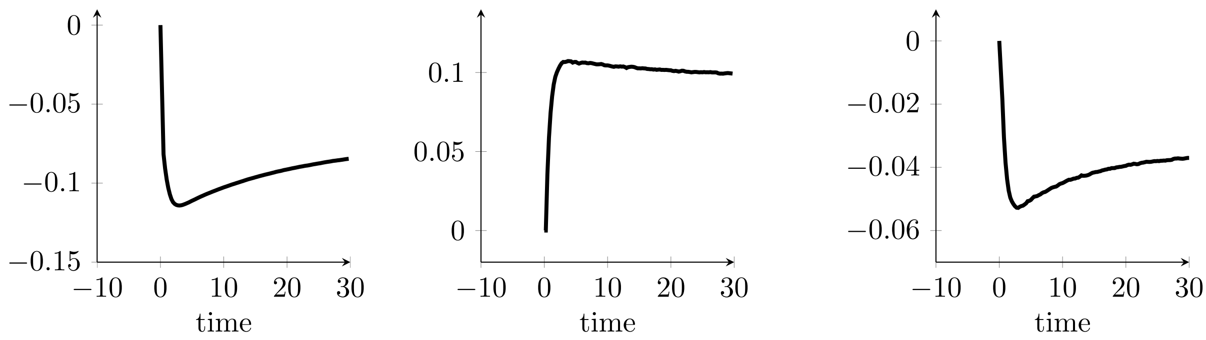
$$M_t = \frac{\int \pi_1(\theta) dF(\theta) - Y_t}{\int \pi_1(\theta) dF(\theta)}. \tag{29}$$

We construct impulse responses with respect to these structural changes as follows. We first simulate a sequence of shocks assuming no structural shifts. Holding the sequence of shocks fixed, we then permute the model by introducing an unanticipated parameter change at time 0 and compute the deviation across the two paths. We repeat this procedure 1,000,000 times and report mean deviations over all simulations in the rate of reallocation ($\Delta R_t = R_t - R_t^{SS}$), the percentage of misallocation (ΔM_t), and total output ($\Delta \log(Y_t)$).

4.2.1 Increase in quality dispersion

First, we consider an unanticipated increase in the dispersion of capital quality. As before, we model this as an expansion in the support of the quality distribution of new capital inflows holding the mean quality constant. The results are plotted in Figure 8.

Increasing the dispersion of quality for new capital has no effect on impact, since new capital flows in slowly and is initially efficiently allocated. However, upon the arrival of the next productivity shift, the distribution of quality in the divesting sector is now greater. This increase in the degree of adverse selection implies that the rate of reallocation is slower. As buyers become more uncertain about capital quality, sellers need to wait longer to sell in order to signal their type. This decrease in the speed of transaction leads to a higher likelihood of capital misallocation, and therefore to lower aggregate output and productivity. Consistent with Eisfeldt and Rampini (2006), the model predicts that the dispersion in capital productivity is counter-cyclical



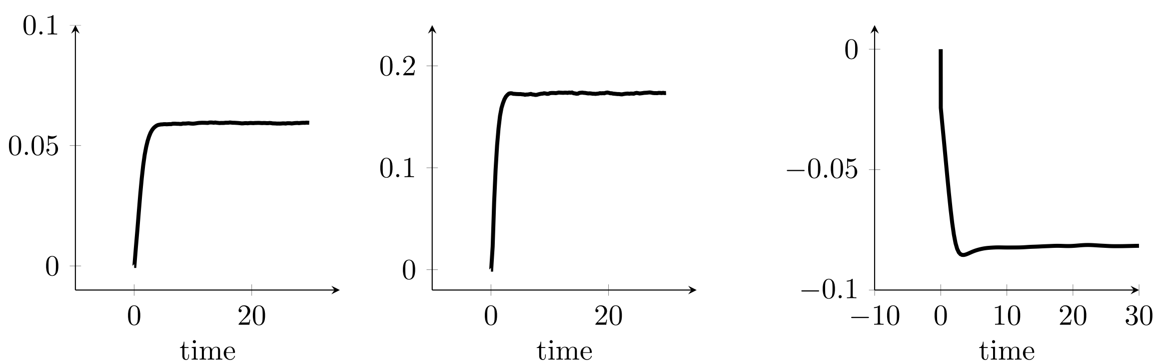
(a) ΔR_t (b) ΔM_t (c) $\Delta \log(Y_t)$

Figure 8: Response to an increase in the dispersion of capital quality. Figures plot mean difference from steady state across simulations.

4.2.2 Increase in volatility

Next, we consider the effect of an unanticipated increase in the frequency of sectoral shifts λ on equilibrium outcomes. We show the results in Figure 9. An increase in the volatility of sectoral shifts λ increases the rate of reallocation.

This increase in the rate of reallocation is not purely the result of the increased frequency of shocks. It also results from the fact that lower quality types reallocate *faster* as λ increases, as we saw in Proposition 3.8. However, despite the increase in the rate that capital is reallocated, the likelihood that it is misallocated is still increasing with λ. As a result, average output is lower following an increase in λ.



(a) ΔR_t (b) ΔM_t (c) $\Delta \log(Y_t)$

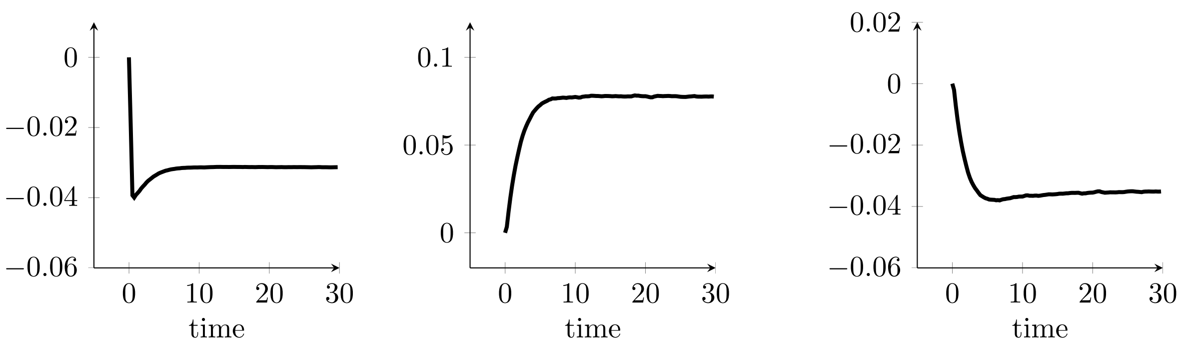
Figure 9: Response to an increase in volatility of sectoral shocks.

In summary, our model generates a negative causal link from volatility of sectoral productivity to the degree of misallocation, and consequently to the level of aggregate productivity.

Consistent with this prediction, Collard-Wexler, Asker, and Loecker (2013) document a negative relation between the time-series volatility of sectoral TFP shocks and the degree of resource misallocation – measured by the dispersion in TFP. Further, the negative relation between volatility – or adverse selection – and output is an alternative mechanism that is consistent with the findings of Bloom (2009), who documents a negative relation between measures of uncertainty and aggregate productivity.

4.2.3 Expansionary monetary policy

Last, we analyze the impact of an unanticipated expansion in monetary policy, modelled as a reduction in the interest rate r. In standard models, lowering the rate at which agents discount the future increases the present value of the benefits from reallocating capital. This increase in valuations leads to faster reallocation of capital and an increase in efficiency. By contrast, in our setting, lowering the discount rate lowers the opportunity cost of delay for firms in the less productive sector (i.e., the left-hand side of (6)). To separate themselves, firms with higher quality capital must wait even longer. As we see in Figure 10, lowering the discount rate leads to a lower rate of capital reallocation, more misallocation and thus lower productivity. This comparative static not only illustrates the benefit of endogenizing the costs of reallocation, but also has implications on the impact of monetary policy on the efficient allocation of capital. Indeed, despite the fact that the Federal Reserve dropped the interest rate to zero following the recent housing and financial crisis, turnover in many asset markets remained extremely low.



(a) ΔR_t (b) ΔM_t (c) $\Delta \log(Y_t)$

Figure 10: Response to a decrease in the interest rate.

5 New Investment and technology adoption

So far, we have focused on the dynamics of capital reallocation. Here, we incorporate the same mechanism into a model of new investment. What changes is the type of capital that gets reallocated. Specifically, rather than physical capital or labor, the factor in limited supply is *entrepreneurial talent*, namely the ability to take advantage of investment opportunities to create productive projects. Below, we flesh this out in a simple model and discuss the several novel findings.

The economy has a mass of entrepreneurs and investors. When an *investment opportunity* or *innovation* arrives, entrepreneurs can take advantage of it to create projects. Entrepreneurs can also manage the projects they create. Investors can manage projects, but cannot create them. Projects are heterogeneous in their profitability, they have a quality θ distributed according to $F(\theta)$ with a continuous and strictly positive density on the support $\Theta = \left[\underline{\theta}, \overline{\theta}\right]$. A project of quality θ using innovation i produces a flow of cash flows $\pi_i(\theta)$, where $\pi_{i+1}(\theta) \geq \pi_i(\theta)\ \forall i, \theta$. Entrepreneurs exhibit persistence in their ability to create projects. Specifically, with probability $\kappa \in [0,1]$ the next project the entrepreneur creates is of the same quality, θ, as her current project and with probability $(1-\kappa)$ the quality of the new project is drawn from $F(\theta)$.

For simplicity, assume all entrepreneurs are initially managing projects from innovation 0 and an investment opportunity (using innovation 1) arrives at $t = 0$. Since entrepreneurs and investors generate the same cash flows from managing projects, there are no gains from trade prior to $t = 0$. Entrepreneurs have limited capacity; they cannot create a new project while they are currently managing an older one. Hence, in order to be able to take advantage of the new investment opportunity, the must sell their current project to an investor. However, as before, entrepreneurs are privately informed about the quality of their current project.

As long as an entrepreneur with the highest quality project is not willing to trade at the price for the average quality firm, not all entrepreneurs are willing to sell their firms immediately. As before, the equilibrium will have a gradual sale of gradual sale of firms in sector 0 to investors, and consequently, gradual investment in the new technology.

5.1 Equilibrium

The equilibrium construction follows similar formal arguments to those in the previous sections so we will avoid some of these formal steps. The equilibrium satisfies the gradual skimming property, hence there is only one type trading each instant. Investors are competitive, hence their break-even condition implies that, the price of a project of quality θ in sector $i = 0$ is $P(\theta) = \frac{\pi_0(\theta)}{r}$.

The next step involves determining the time $\tau(\theta)$ that an entrepreneur owning a project of type θ will sell to investors. Once the entrepreneur has started his new firm of quality $\tilde{\theta}$, the firm will generate a profit flow of $\pi_1(\tilde{\theta})$ forever. Hence, his valuation of the new firm once it is created equals

$$V(\tilde{\theta}) = \frac{\pi_1(\tilde{\theta})}{r}. \tag{30}$$

After the arrival of the innovation, but prior to the creation of a new project, the entrepreneur's expected payoff is equal to his conditional expectation of (30),

$$E_{\tilde{\theta}}\left[V(\tilde{\theta})\right] = \kappa\, V\left(\theta\right) + \left(1-\kappa\right)\int V(\theta)\, dF(\theta), \tag{31}$$

discounted for the fact that the entrepreneur needs to wait until $\tau(\theta)$ to sell. Consequently, an entrepreneur of type θ has a continuation value that is a function of the lowest remaining entrepreneur in sector $i=0$, denoted by χ

$$V_0\left(\theta,\chi\right) = \frac{\pi_0\left(\theta\right)}{r} + e^{-r(\tau(\theta)-\tau(\chi))}\left(\kappa\, V\left(\theta\right) + \left(1-\kappa\right)\int V(\theta)\, dF(\theta) - I\right). \tag{32}$$

In equilibrium, the entrepreneur of type χ must be indifferent between locally speeding up or delaying the sale of his project. This indifference condition can be written as

$$P'\left(\chi\right)\chi'(t) = r\left(\kappa V_1(\theta) + \left(1-\kappa\right)\int V(\theta) dF(\theta) - I\right). \tag{33}$$

Combining the two equations above yields a differential equation in χ

$$\chi'(t) = r\,\frac{\kappa\,\pi_1(\chi(t)) + (1-\kappa)\int \pi_1(\theta) dF(\theta) - I}{\pi_0'(\chi(t))} \tag{34}$$

Equation (34), along with the boundary condition that $\chi(0) = \underline{\theta}$, pins down the unique equilibrium. From this, one can begin to see the role played by the persistence parameter, κ. If there is no persistence, the entrepreneurs' expected return from investing in the new technology (i.e., the numerator in (34)) is constant in θ. For $\kappa > 0$ the numerator is increasing in θ and increasingly so as κ becomes higher, or equivalently, as entrepreneurial talent and the new technology become more complementary. Recalling our discussion of the case where $\alpha < 1$ in the CES example in Section 3.1, when gains from trade are increasing in quality, trade is slower with the low types and then speeds up with higher types, or equivalently the rate of investment in the new technology $\chi'(t)$ is increasing over time. Comparing (34) to (8) illustrates the fact that the investment rate is increasing with time even when the gains from moving to the new technology, $\pi_1(\theta) - \pi_0(\theta)$, are independent of quality θ, a feature which is in stark contrast with the motivating example.

5.2 Output and productivity

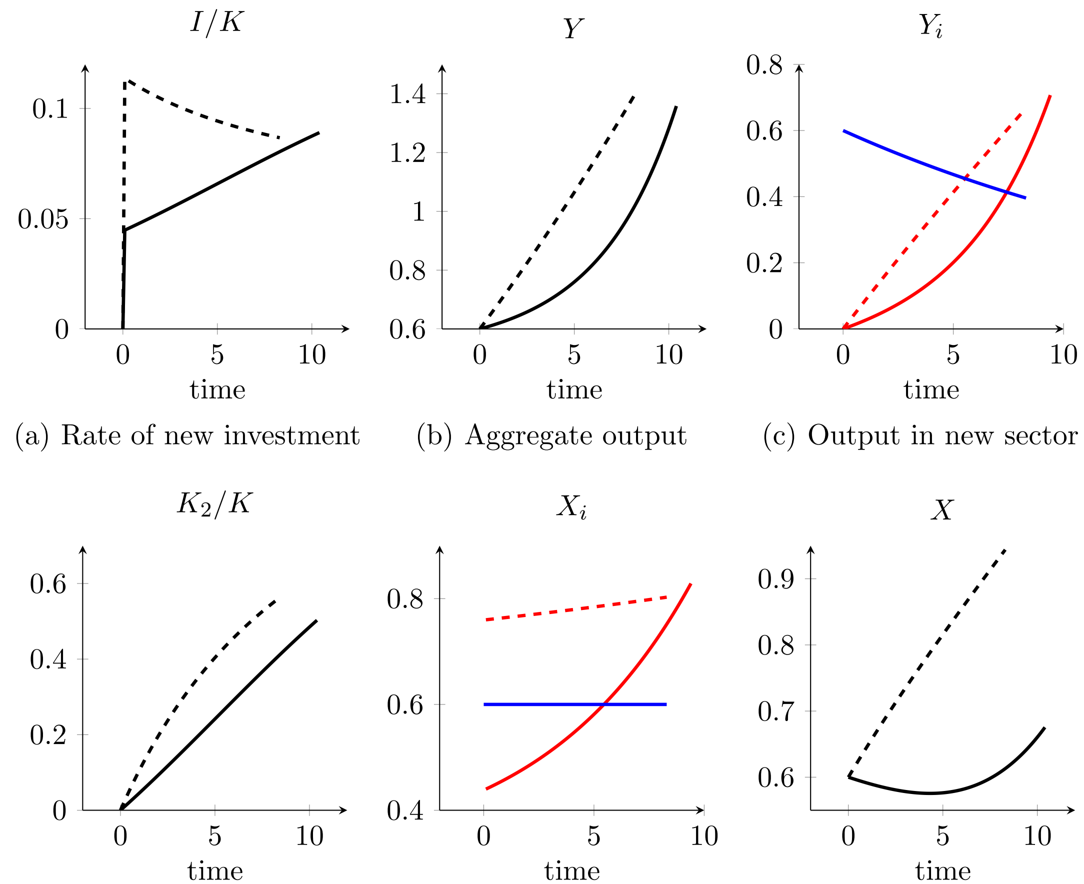
Next, we analyze the model's implications for the dynamics of output and TFP of each technology and in the economy as a whole, defined as in (27) and (28) respectively.

Once the innovation becomes available and entrepreneurs start investing in the new technology, total factor productivity in the new sector is slowly increasing over time. This gradual increase in productivity occurs as progressively more talented entrepreneurs sell their old firms and create projects using the new technology. However, once the new technology becomes available, aggregate TFP can actually *decrease*. This productivity drop can occur because, even though the new sector is on *average* more productive, the first projects created using the new technology sector are of below average quality – since they are created by below-average entrepreneurs. The higher the persistence, the greater the drop in measured TFP. The following proposition states the conditions under which this drop in productivity occurs.

Proposition 5.1. *Upon the arrival of an innovation, economy wide TFP is initially decreasing (and eventually increasing) over time if and only if* $\kappa\pi_1(\underline{\theta}) + (1-\kappa) E_{\tilde{\theta}}[\pi_1(\tilde{\theta})] < E[\pi_0(\theta)]$.

Furthermore, the total magnitude of the TFP drop will be higher the greater the persistence in quality κ. As we see in Figure 11, when entrepreneurial talent is more transferable to the new technology (high κ), the process of technology adoption is further delayed; investment responds with a lag, and aggregate productivity dips on impact.

The possibility that measured total factor productivity might drop at the onset of the arrival of a new technology is consistent with several empirical studies (David, 1990; Jovanovic and Rousseau, 2005). In their investigation of two major technological innovations, electrification and information technology (IT), Jovanovic and Rousseau (2005) point out that both technologies were accompanied by continued use of the old technology, slow adoption of the new technology, continued productivity improvements, and an initial decline in aggregate productivity. These patterns are consistent with panels (c) through (f) of Figure 11, respectively. Further evidence is provided by Collard-Wexler and Loecker (2013) who study the effect of the introduction of the minimill, a superior technology, to the US steel industry. They find a significant, but slow, increase in the productivity of the industry due to, first, reallocation of resources from the old to the new technology, and second, by improvement in productivity in plants employing the old technology due to the exit of inefficient producers. Further, despite the widespread belief in the superiority of the new



(a) Rate of new investment (b) Aggregate output (c) Output in new sector

(d) Technology diffusion (e) Productivity in new sector (f) Aggregate TFP

Figure 11: Output, productivity and rate of technology adoption. Black is aggregate, red is new sector, blue is old sector. A solid line represents the high persistence case ($\kappa = 0.75$) and the dotted line represents lower persistence ($\kappa = 0.25$)

technology, plants employing the old technology were 20% more productive in terms of output per worker compared to plants that adopted the new technology, but this difference evaporated over time.

6 Discussion and connection to the literature

Our model delivers predictions that are consistent with empirical evidence. For instance, Ramey and Shapiro (2001) argue that the process of capital reallocation is fraught with significant frictions In their study of capital reallocation in the aerospace industry, they document the following stylized facts that are consistent with our model: i) capital sells at a substantial discount relative to its replacement cost; ii) this discount is smaller if capital sells to other aerospace firms (which presumably have better ability to evaluate its quality); iii) the process of selling used equipment is lengthy.

Most estimates of resource misallocation – based on the dispersion in revenue-based productivity – suggest that financial market development is a key factor. For instance, Hsieh and Klenow (2009) compare the dispersion in productivity between the US, India and China and find substantially lower dispersion in the US. Using a fairly general model, they argue that if the dispersion in TFP in India and China were equal to US levels, TFP would be 30-60% higher. Of course, these countries differ in many other respects – for instance, institutions – in addition to financial market development. However, Ziebarth (2013) finds similar levels of TFP dispersion in the 19th century US as China and India today. The fact that the US had similar institutions in the 19th century as today suggests that the level of financial development is likely important. Further, informational asymmetries may play an important role, as evidenced by certain similarities in the forms of financing between India and 19th century US – such as, the prevalence of family financing or the fact that banks lending to persons they had personal relationships with (Lamoreaux, 1986).

Point estimates of the magnitude of adjustment costs vary substantially. For example, Collard-Wexler, Asker, and Loecker (2013) estimate an investment model with adjustment costs using Census data; their point estimates of the convex component implies that a plant that wishes to double its scale in a month would incur costs equal to 8.8 times its cost of investment. For comparison, the estimates in Bloom (2009) based on a cross-section of publicly traded firms imply an additional cost of one times the cost of investment. By providing a micro-foundation for adjustment costs our model provides an alternative interpretation of existing estimates of the magnitude of adjustment costs. This difference in point estimates is consistent with our theory to the extent that estimates of convex adjustment costs include the effects of adverse selection in addition to physical adjustment costs, we would expect to see lower estimates for firms with lower information asymmetry.

In the context of human capital, one of the biggest challenges in the literature studying the mobility of human capital is to explain why emigration rates were not always highest for countries with the largest wage differential – that is, countries whose populations have the most to gain from the move – and why emigration rates often rose from low levels as successful development (e.g.,rising wages) took place at home (Hatton and Williamson, 1994). Further, the immigration literature has documented that immigration rates are highly persistent over time. The consensus explanation for this persistence is the 'friends and family' effect, where the presence of friends and family that have already immigrated lowers the transition cost for newly-arrived immigrants. However, the presence of an existing group of immigrants also likely reduces information asymmetries, either due to learning or referrals.

Focusing on technology diffusion, major technological breakthroughs are typically accompanied by an initial decline in productivity, a slow adoption of new technologies, and continued use of the old technology (Griliches, 1957; David, 1990; Jovanovic and Rousseau, 2005; Atkeson and Kehoe, 2007). Collard-Wexler and Loecker (2013) study the effect of the introduction of the minimill, a superior technology, to the US steel industry. They find a significant, but slow, increase in the productivity of the industry due to, first, reallocation of resources from the old to the new technology, and second, by improvement in productivity in plants employing the old technology due to the exit of inefficient producers. Further, despite the widespread belief in the superiority of the new technology, plants employing the old technology were 20% more productive in terms of output per worker compared to plants that adopted the new technology, but this difference evaporated over time.

Another stylized feature of the process of technology adoption is that diffusion curves – the fraction of users that adopt the new technology – is S-shaped (Griliches, 1957; Jovanovic and Rousseau, 2005). If the distribution of capital quality is hump-shaped, then our model can generate S-shaped diffusion curves similar to those in the left panel of Figure 6.

7 Conclusion

In this paper, we have incorporated adverse selection into a competitive decentralized economy to study the dynamics of capital allocation and new investment. The information friction leads to slow movements in capital reallocation, lagged investment following technological innovations, and provides a micro-foundation for convex adjustment cost models. The model generates a rich set of dynamics for reallocation, investment, output and productivity.

Clearly, our mechanism is not the only one that can generate some of these patterns. Physical (convex) costs, search, financial frictions, learning, time-to-build and other factors are surely important components in the allocation of new and existing capital. We have abstracted away from these considerations in order to highlight the key ideas of the paper. Further developing this framework into models that are suitable for calibration seems a promising direction for future work.

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Appendix

Proof of Lemma 3.1. If the skimming property did not hold, then there exists (i, t) such that $t_1 \equiv T^i_t(\theta) > t_0 \equiv T^i_t(\theta')$ for some $\theta' > \theta$. Since q prefers to wait until $T^i_t(\theta)$ then $V^i_{t_0}(\theta) \geq P^i_{t_0}$. Since θ' accepts at t_0, $V^i_{t_0}(\theta) = P^i_{t_0}$. But since π_i is increasing, θ' could do strictly better by mimicking the type θ, which violates (10). □

The proof of Theorem 3.2, relies on Lemma 3.3, which we prove below.

Proof of Lemma 3.3. Note first that (22)-(23) is an initial value problem of the form

$$\chi'(t) = f(t, \chi(t)), \quad \chi(0) = \underline{q} \tag{35}$$

To verify existence and uniqueness of a solution, we will apply the Picard-Lindelof Theorem (see Zeidler (1998), Theorem 3.A.) To do so, it is sufficient to verify several properties of f : (i) $f\,(t, x)$ is continuous on $[0, T] \times [\underline{q}, \bar{q}]$; (ii) f is bounded (iii) that $f(t, x)$ is Lipschitz. Property (i) is by inspection (since both g and π_i are continuously differentiable). Property (ii) follows immediately from the expression for g and the conditions placed on π_i. To demonstrate (iii), it suffices to show that $\frac{d}{dx} f(t, x)$ is bounded, which follows from the restriction that π_i have bounded first and second derivatives. □

Proof of Theorem 3.2. From Lemma 3.3, there is a unique candidate separating equilibrium. Thus, in order to prove the theorem, it suffices to check that the candidate satisfies the three equilibrium conditions. The *Zero Profit* and *Market Clearing* condition are satisfied by construction. To verify that firms optimize, note that no firm in the efficient sector strictly prefers to sell their capital since the price is $V(\underline{\theta})$, which is the least a firm can expect to earn by continuing to operate their capital. It remains to verify that there are no profitable deviations for firms in the inefficient sector. Since $\tau\,(\chi)$ and $P\,(\chi)$ are monotonic functions and the sellers' payoff satisfies the single crossing property. To see this, note that the sellers objective can be written as

$$u_\theta(t, P) = (1 - f(t))\frac{\pi_0(\theta)}{\rho} + f(t)P$$

and therefore

$$\frac{\partial}{\partial\theta}\left(\frac{\partial u_\theta/\partial P}{\partial u_\theta/\partial t}\right) = \frac{f(\tau)}{f'(t)(P - \pi_0(\theta)/\rho)} > 0$$

Since this condition holds, the local IC constraint is sufficient to guarantee that there are no profitable global deviations. Note that $\chi'(t)$ was constructed as to ensure that the local IC constraint is satisfied. Therefore, there are no profitable deviations for firms in the inefficient sector. □

Proof of Proposition 3.4. That $g(t) = 0$ for $\lambda = 0$ is by inspection. That (22) then reduces to (24) follows immediately. □

Proof of Proposition 3.5. Taking the total derivative of the RHS of (24) with respect to time we get that

$$\chi''(t) = \rho \cdot \frac{d}{d\chi}\left(\frac{\pi_1(\chi) - \pi_0(\chi)}{\pi_1'(\chi)}\right) \cdot \chi'(t)$$

Since $\chi'(t)(t) > 0$ for all $t \in [0, \tau(\overline{\theta})]$. The derivative of $\chi'(t)$ with respect to time has the same sign as the derivative of $\frac{\pi_1(\theta)-\pi_0(\theta)}{\pi_1'(\theta)}$ with respect to θ. □

Proof of Corollary 3.6. Follows immediately from Proposition 3.5 and the fact that for CES production technology, $\frac{d}{d\theta}(\frac{\pi_1 - \pi_0}{\pi_1'})$ is strictly positive for $\alpha < 1$, strictly negative for $\alpha > 1$, and equal to zero for $\alpha = 1$. □

Proof of Proposition 3.7. This follows from (21) and the fact that (i) $\tau(\theta) > 0$ for all $\theta > \underline{\theta}$, and (ii) $g(t) > 0$ for all $t > 0$. □

Proof of Proposition 3.8. Using a subscript to represent elements of the relevant economy, we have that

$$\begin{aligned}
\chi_2'(0) - \chi_1'(0) &= (1 - g(0;\lambda_2) + g(0;\lambda_2)) - (1 - g(0;\lambda_1) + g'(0;\lambda_1)) \\
&= g'(0;\lambda_2) - g_1'(0;\lambda_1) > 0
\end{aligned}$$

Where the inequality follows from the fact that $\frac{d}{d\lambda}g'(0;\lambda) > 0$. Therefore, $\chi_2'(0) - \chi_1'(0) > 0$. By the continuity and boundedness of χ_1' and χ_2', there must exist $\bar{t} > 0$ such that the inequality holds for $t \in [0, \bar{t}]$. □

Proof of Proposition 5.1. The expected productivity of a new firm is $\rho\pi_1\left(\underline{\theta}\right) + (1-\rho)\, E_{\tilde{\theta}}\left[\pi_1(\tilde{\theta})\right]$ and the average productivity when all firms are in the original sector is just the average productivity of the sector, $E\left[\pi_0\left(\theta\right)\right]$. Hence, when the former is smaller than the latter the firms created upon arrival of new vintage are of below average TFP and thus lower the average measured TFP of the economy. □