Trading and Liquidity with Limited Cognition

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"The perception of the intellect extends only to the few things that are accessible to it and is always very limited."

René Descartes

Méditations Métaphysiques

Liquidity shock

Reduces investors' ability to hold assets:

Banks, hedge funds or private equity funds: losses (Khandani & Lo, 08: hedge funds with losses in real estate had to sell in stock market)

Mutual funds: outflows (Coval & Stafford, 07)

Insurance companies: downgrades, delistings (Greenwood, 05, Da & Gao, 05)

Institutions unwind positions, raise new capital, find counterparties => after some time recover

Limited Cognition

Traders must process lots of information (especially around liquidity shocks):

Overall risk position

(what has been sold, hedged, netted,…)

Counterparties

Compliance

Takes time & hard thinking before information collected & processed & decisions can be reached

Issues

How do traders & prices cope with liquidity shocks?

What is the equilibrium process after such shocks?

How are trading and prices affected by cognition limits?

Do the consequences of cognition limits vary with market mechanisms and technologies?

Preview of results

Price drops at time of liquidity shock, then recovers

Limited cognition lengthens recovery, but does not necessarily amplify initial price drop

Traders sell at time of shock, then buy as their expected valuation recovers. Simultaneously place limit orders to sell later at higher price

// market making => round trips
=> raise trading volume

Related literature

Limited cognition // inattention in macro & finance: Mankiw & Reiss (02), Duffie & Sun (90), Gabaix & Laibson (02)

Dynamics of order book // Foucault (99), Parlour (98), Rosu (08), Goettler, Parlour & Rajan (09)

Infrequent changes in trading plans // infrequent contact with financial markets: Duffie, Gârleanu, Pedersen (05, 07), Weill (07), Lagos Rocheteau (09)

1) Model

Mass 1 of risk-neutral competitive institutions

Supply $s < 1$

Discount rate r

Continuous time

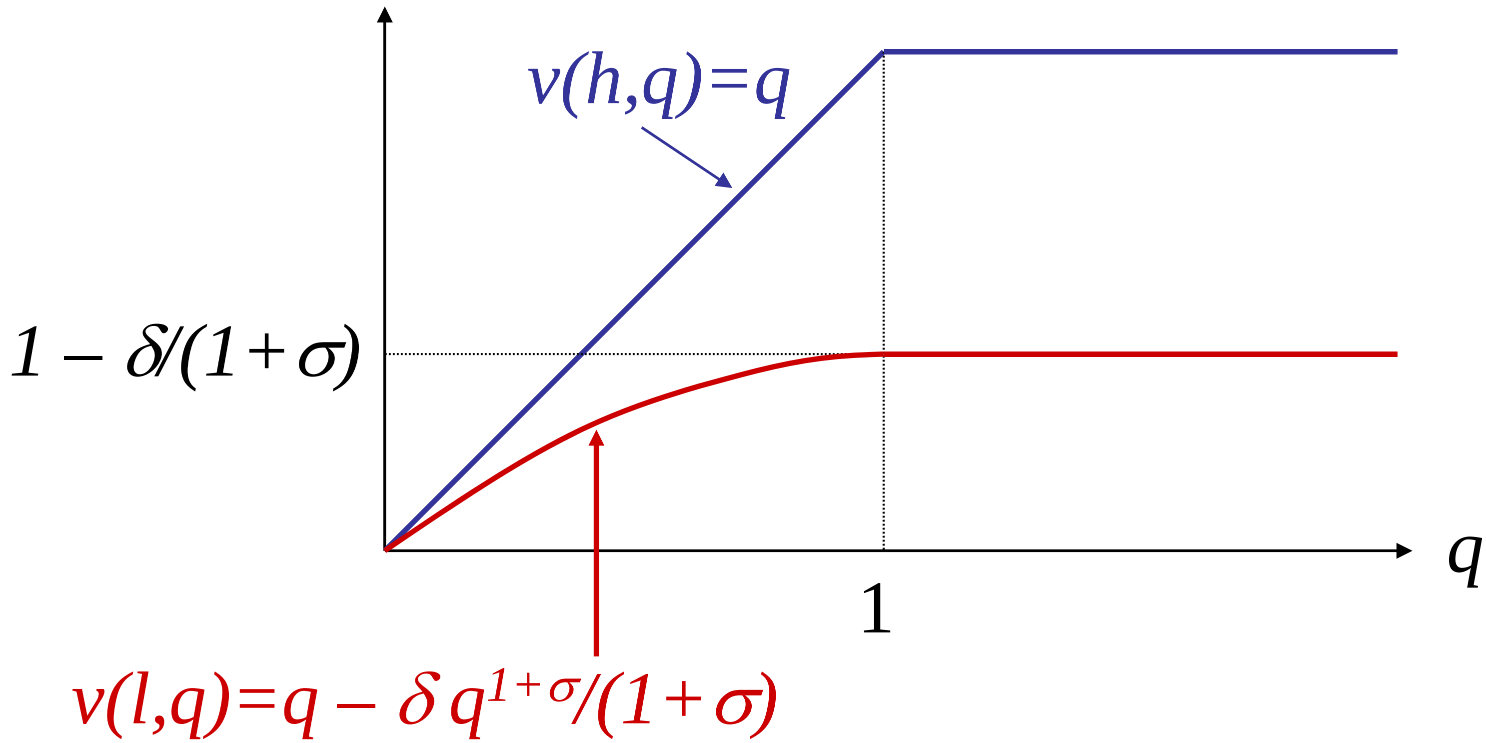
Probability space (Ω, F, P)

Utility flow from holding q units of asset at time t

Before liquidity shock: $\theta = h \Rightarrow v(h,q)$

When hit by liquidity shock $\theta = l \Rightarrow v(l,q)$

Utility flow



Liquidity shock reduces asset holding ability (δ)

Marginal valuation decreases with quantity held (σ)

=> efficient to spread holdings across agents

Liquidity shock

At $t = 0$ all institutions hit by shock: low utility flow

Each institution recovers at first jump of its Poisson process (intensity γ): high utility forever

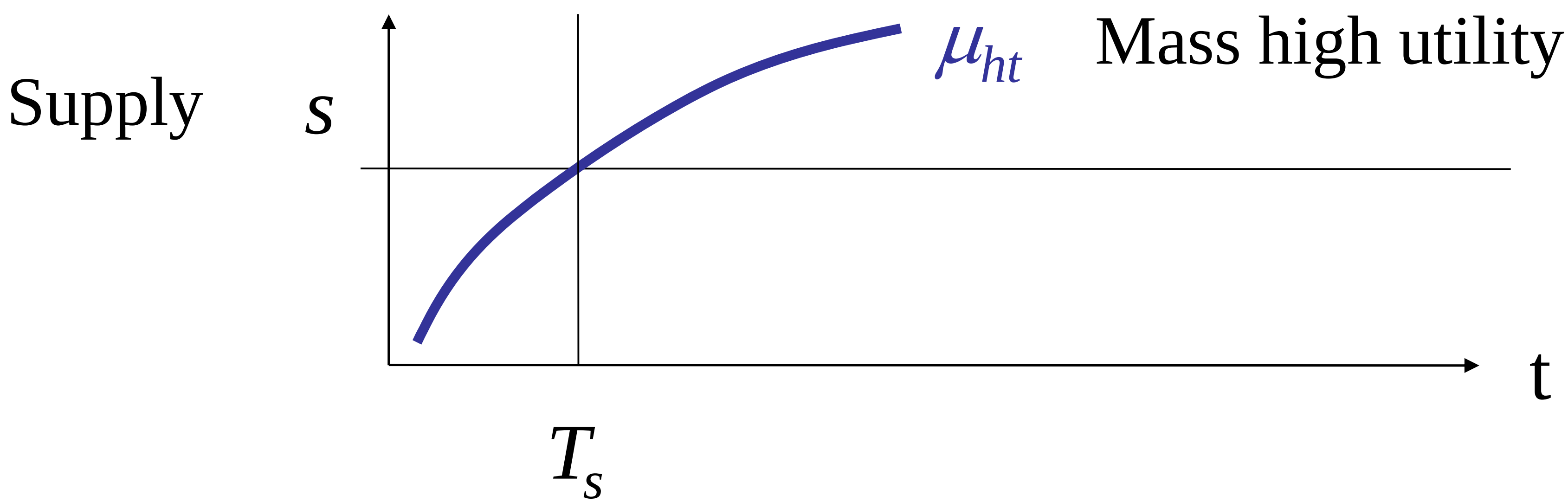
Mass of high utility agents at time $t = \mu_{ht}$ $(\mu_{h0}=0)$

Recovery processes *i.i.d* across institutions
=> by *LLN* aggregate market deterministic:

$$\mu_{ht} = 1 - \exp(-\gamma t)$$

2) Equilibrium with unbounded cognition

Traders continuously observe θ



$t > T_s$: asset held only by high utility

$t < T_s$: also held by low utility: $q_t = (s - \mu_{ht})/(1 - \mu_{ht})$

Opportunity cost of holding asset: ξ_t

At t, borrow p_t to buy asset.
At $t+dt$, resell $p_t + p_t' dt$, reimburse $p_t(1+rdt)$.

$$\xi_t \overset{d}{=} r\,p_t - p_t'$$

Time value of money
(interest paid)

Capital gain

Pricing with perfect cognition

After T_s asset held only by high utility: $p = 1/r$

Before T_s: marginal agent has low utility: $p < 1/r$

Equilibrium: $v_q(l,q_t) = \xi_t$

LLN: aggregate market deterministic => price also

Rise in price = progressive recovery from shock

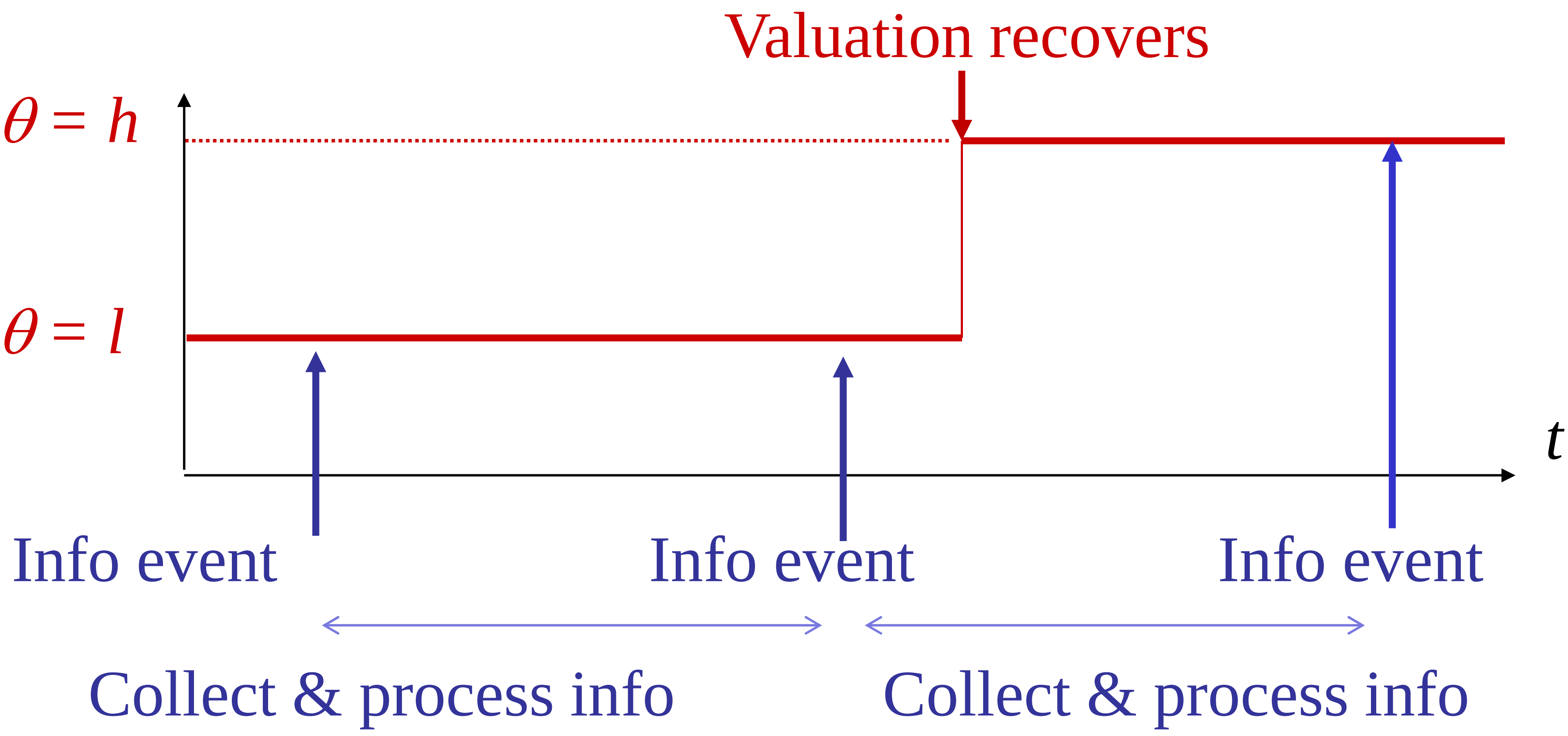
Can't be arbitraged, due to concavity

3) Equilibrium with imperfect cognition

2 Poisson processes per institution (*iid*)

i) Valuation θ for asset: recovers to h with intensity γ

ii) Information process N (observe θ) with intensity ρ



Holding plans

$q_{t,u}$ holdings at time u decided at $t < u$

Bounded variations (technical) & adapted to filtration F_t (info on θ up to time t)

When info process N jumps at time t

reveals refreshed information about θ_t

update plan $q_{t,u}$

Intertemporal value *V(q)* of holding plan $q_{t,u}$

Integrate across *t* & histories

Prob(info event at *t*)

$$E_0\left[\int_{t=0}^{\infty} e^{-rt}\int_{u=t}^{\infty} e^{-(r+\rho)(u-t)}\{E_t[v(\theta_u, q_{t,u})]-\xi_u q_{t,u}\}du\,\rho dt\right]$$

Discounted sum of payoff after info event at *t*

Expected valuation

Opportunity cost

Optimal holding plans & equilibrium

Traders choose q (adapted to F_t & with bounded variations) to maximize $V(q)$

=> pointwise maximization

$$\max_{q_{t,u}} E_t[v(\theta_u, q_{t,u})] - \xi_u q_{t,u}$$

=> $q_{t,u}$ as a function of ξ_u

=> substitute in market clearing condition

=> solve for ξ_u

Market clearing

Cross-sectional average holding = per capita supply

By LLN:

$$E_0\left[q_{\tau_u,u}\right]=s$$

Time of last info event before *u*

Density of traders whose last info event before *u* was at *t*

$$\int_0^u \rho e^{-\rho(u-t)}\left[(1-\mu_{ht})E(q_{t,u}\mid\theta_t=l)+\mu_{ht}E(q_{t,u}\mid\theta_t=h)\right]dt=s$$

Low valuation holdings

High valuation holdings

Supply

Basic properties of holdings

$v_q(\theta, q) = 0$ if $q>1$ => $q_{t,u} > 1$ for some traders only if $\xi_u < 0$

But if $\xi_u < 0$ all want $q_{tu} > 1$: contradicts market clearing

=> $q_{t,u} \leq 1$

$v_q(h,x) > v_q(l,y)$, for all (x,y) in $(0,1)^2$ => 2 regimes:

If some low valuation holding plan > 0
then all high valuations holding plans = 1

If some high valuation holding < 1
then all low valuations holding plans = 0

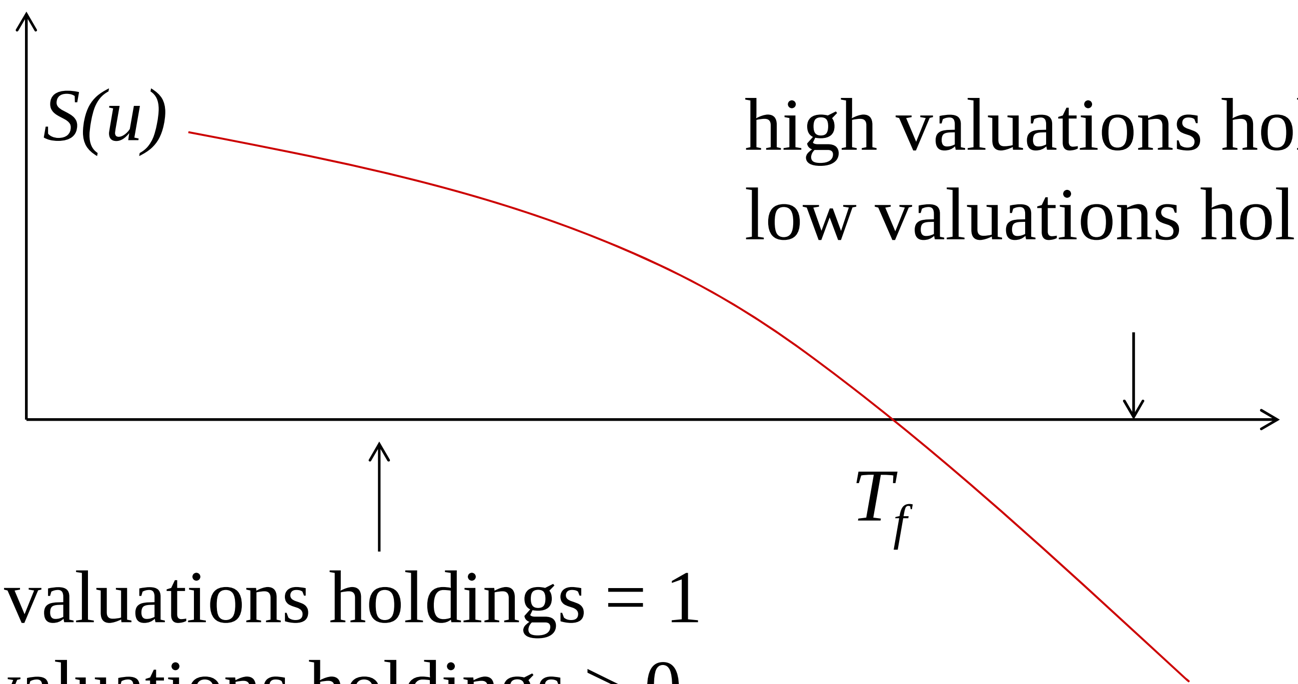
When do we switch from one regime to the other?

Residual supply at time *u*

Gross supply of agents with at least 1 info event
- Maximum possible demand from high valuation

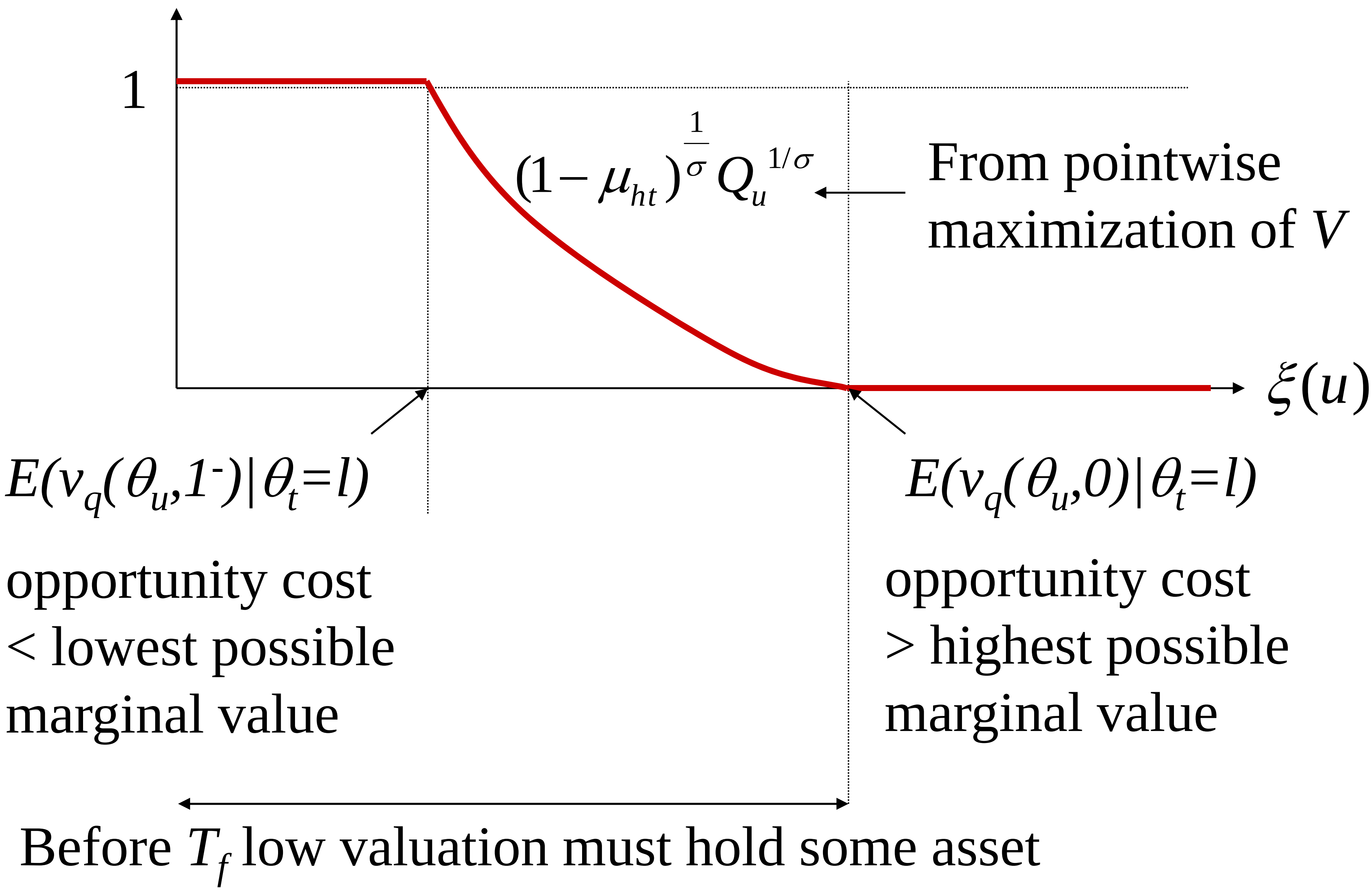
$$\int_0^u \rho e^{-\rho(u-t)}(s-\mu_{ht})dt = S(u)$$

high valuations holdings $\leq$ 1
low valuations holdings = 0

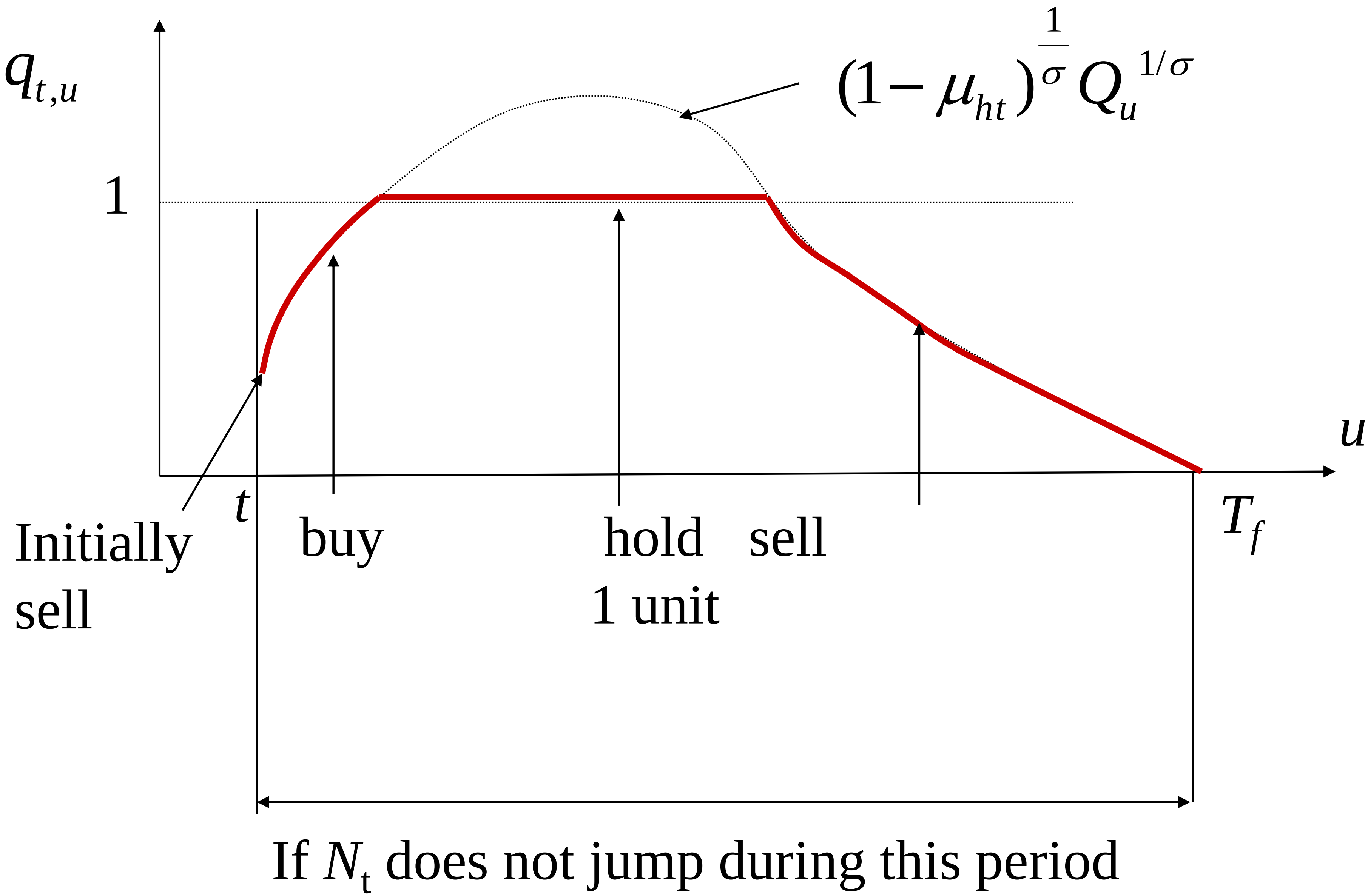


high valuations holdings = 1
low valuations holdings > 0

Low valuation's holding plan



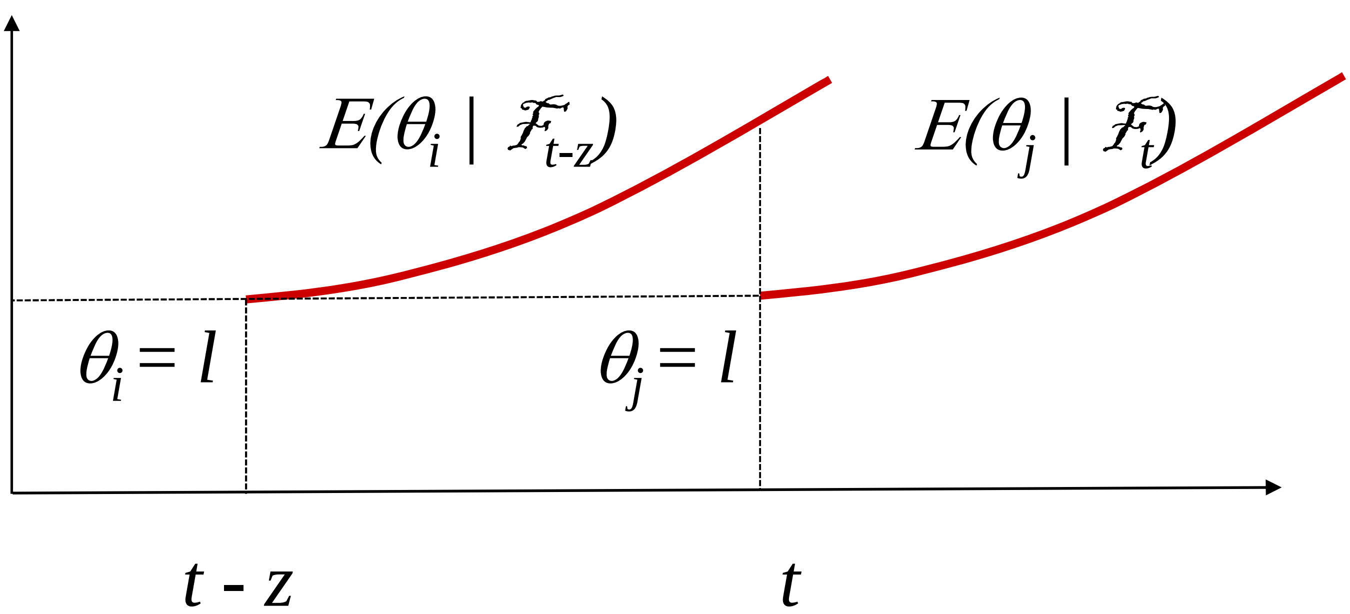
Optimal low valuation's holdings



Why buy back after selling?

To reap gains from trade !

i has higher E(valuation) than *j* at *t* => buys from *j*



Why eventually sell back?

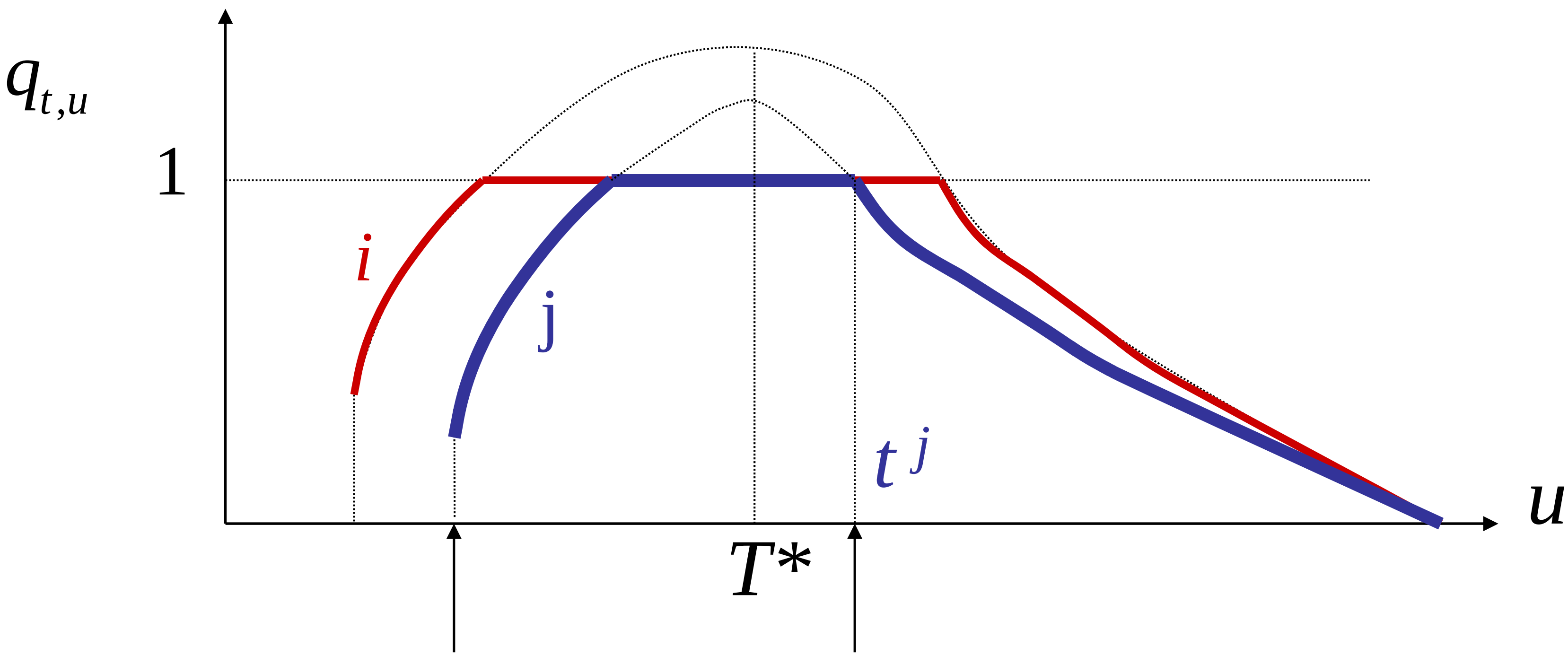
To reap gains from trade !

As time goes by, more and more traders have recovered from shock

Mass of high valuation traders increases: demand increases

When demand is high enough: sell

Holding plans at different points in time



Trader coming later … starts selling earlier

j's orders hit earlier than *i*: at lower prices: *j* undercut

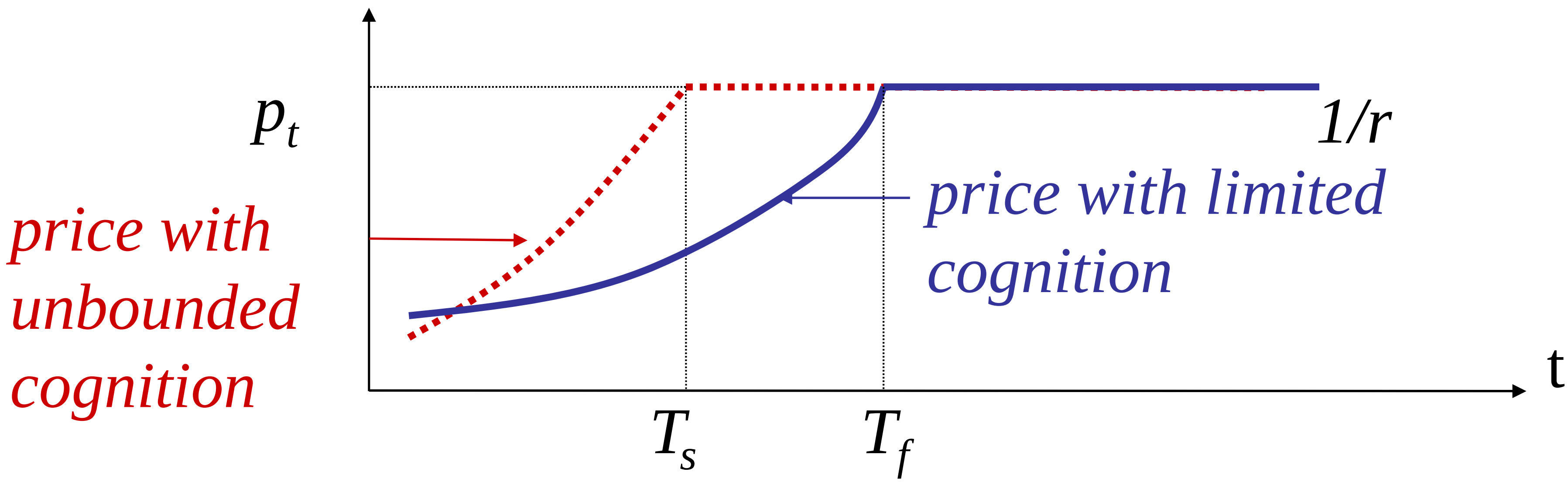
because his expected valuation was lower than *i*'s

Equilibrium price

Substitute $q_{t,u}$ in market clearing:

$$\xi_u \quad = \quad 1 - \delta(1 - \mu_{hu})\, Q_u^{\sigma}$$

Opportunity cost E(marginal value of marginal agent)



Limited cognition => market takes longer to recover

But initial price impact of shock may not be greater

Welfare theorem

Social planner maximizes utilitarian welfare subject to same informational constraints as agents

Competitive equilibrium = Information constrained socially optimal allocation

Equilibrium allocate assets at t to agents with highest expected valuation given F_t

Limited cognition constraint of trader i independent from what others do => no externality

Trading volume

Unbounded cognition: low valuation sells until θ switches back to h

Limited cognition:

on observing $\theta = l$: sell

then buy back as E(valuation) increases

then flat & eventually sell back

at next jump of information process if $\theta = l$ sell

=> round trips

=> limited cognition generates extra trading volume

4) Implementation

Electronic order driven market: Nasdaq, NYSE Euronext …

Limit sell orders request execution at price as large as limit (symmetric for buy orders): stored in order book

Market orders request immediate execution

Limit sell order executed when hit by market (or marketable limit) buy order - price & time priority

Trading algorithms: preprogrammed instructions to place orders in response to market movements

Implementing high valuation's trading plan

When trader observes $\theta = h$ => market buy order

(Also cancel any previously placed limit order)

Implementing low valuation's trading plan

When trader observes $\theta = l$ at time t => market sell order

Simultaneously place limit orders to sell later when price will have recovered (if he already placed limit orders to sell, cancel some & place new orders at lower prices)

Since price rises, increasing part of holding plan cannot be implemented with limit buy placed at time t (would be executed immediately or never): trader programs algo to progressively submit market buy as price moves up

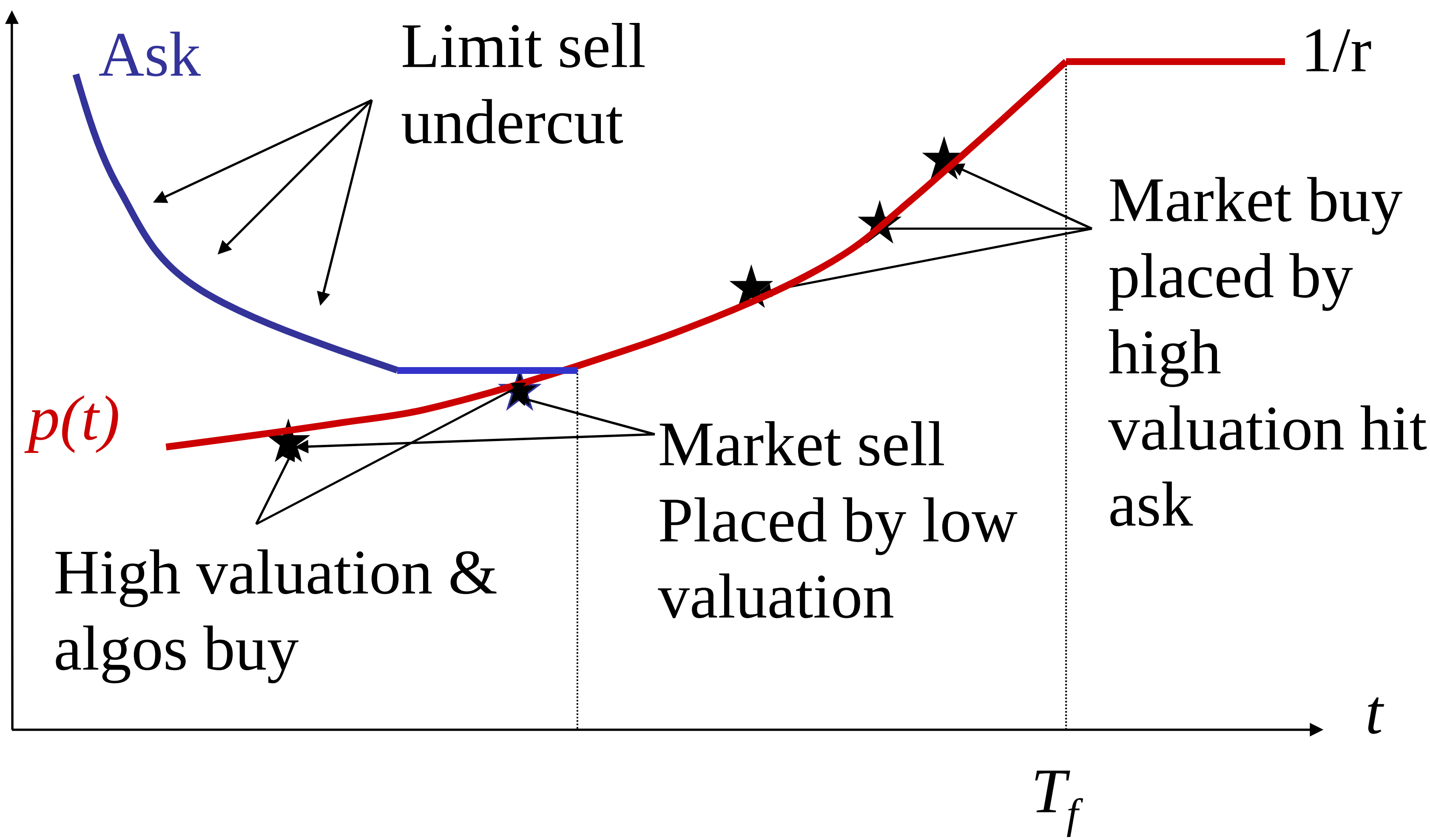
Market making

Initially buy (with algo), simultaneously place limit sell orders to be executed when price has recovered

Similar to market making in Grossman and Miller (1988)

But here traders optimally choose whether to supply or demand liquidity

Equilibrium market dynamics



In line with stylized facts & evidence

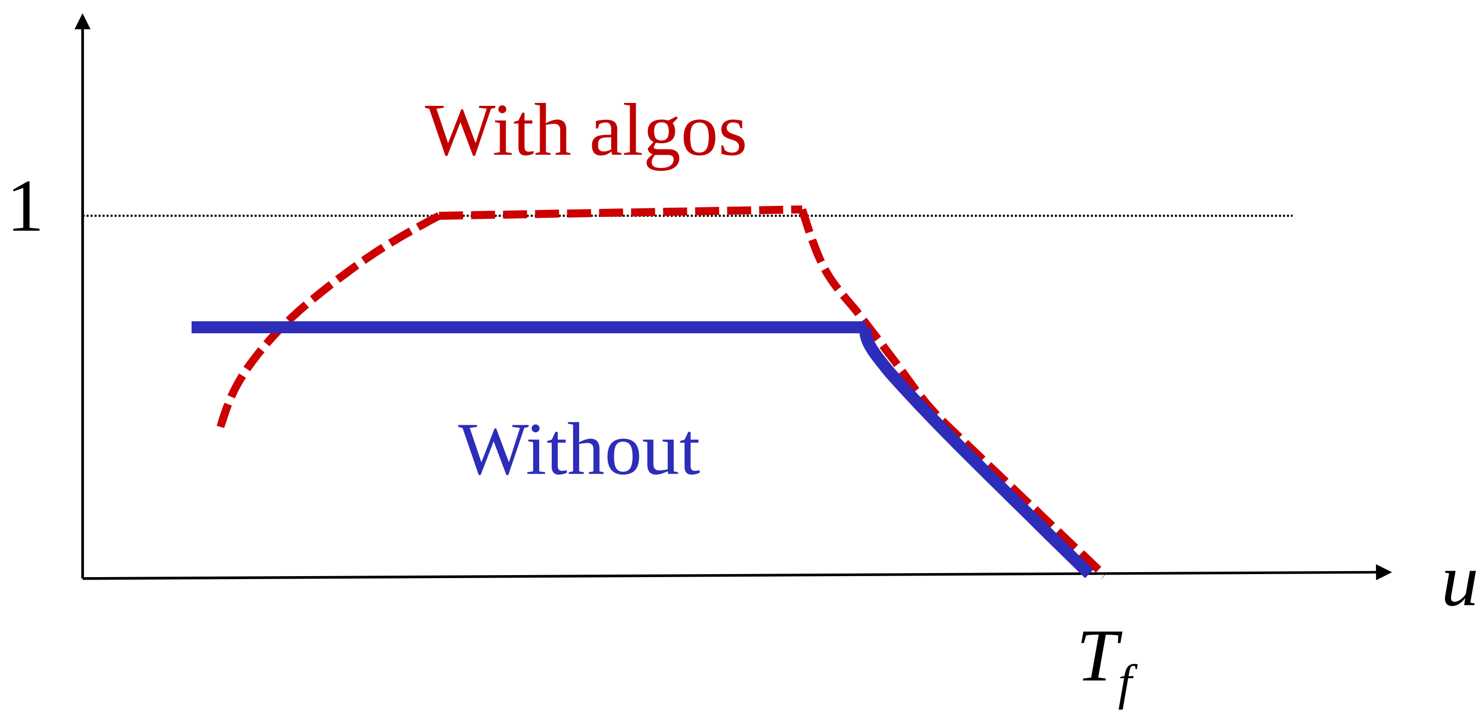
Brogaard (10): algos don’t withdraw after large price drops and take advantage of price reversals

Hendershott Riordan (10): algos provide liquidity when scarce and rewarded

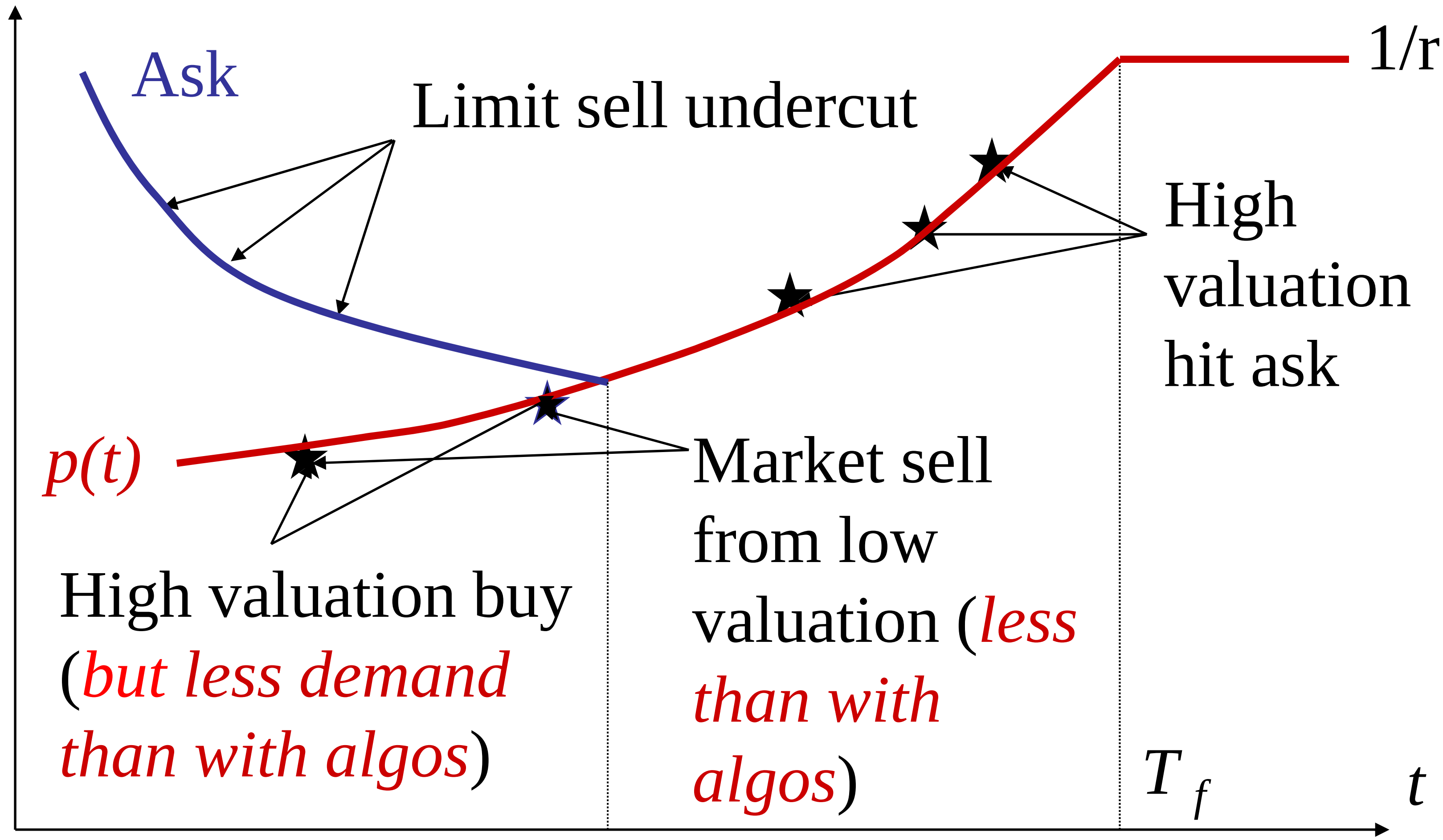
Biais, Hillion & Spatt (95), Griffiths et al (00), Ellul et al (07): limit order undercutting

If traders can only place limit & market orders when information process jumps

Can't implement increasing part of holding plan: iron it out: demand more at beginning, less at the end



Equilibrium dynamics without algos



Market fully recovers at same time as with algos

Comparing price dynamics with and without algos

Market price recovers at time T_f in both cases:

when demand from traders who have observed $\theta = h$
absorbs all supply brought to market
(both independent of whether traders can use algos)

Shortly after shock price can be lower with algos:

traders antcipate they will buy back
hence sell more initially
greater selling presseure on price

Conclusion

Here algos useful: facilitate market making

- Can seem to destabilize market (amplify price drop)
- But equilibrium = Pareto optimum (> equ without algos)
- No externality

No adverse selection in our model

- In practice algos likely to have superior info
  - (Hendershott Riordan (10) & Brogaard (10))
- => negative externality

Extend our dynamic model to information asymmetry ?