Reputation for Quality

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Overview

Investment and Reputation

- "Firm" can invest into future quality
- Moral hazard due to imperfect observability
- Reputation gives firm incentive to invest

Modeling Innovation

- Persistent quality: function of past investments
- Reputation: belief over endogenous state variable

Project Analyzes

- Reputational investment incentives
- Reputational dynamics

Learning Processes

Perfect Good News - Labor markets

- Market discovers high quality via "breakthroughs"
- Work-Shirk Equilibrium & Convergent Dynamics

Perfect Bad News - Computer industry

- Market discovers low quality via "breakdowns"
- Shirk-Work Equilibria & Divergent Dynamics

Imperfect Learning - Automotive

- Gradual market learning through consumer reports
- Work-Shirk Equilibrium & Convergent Dynamics ...

Literature

Theory - Reputation

- Kreps, Wilson (1982)
- Holmstrom (1999)
- Mailath, Samuelson (2001)

Theory - Repeated Games

- Kreps (1990)
- Abreu, Milgrom, Pearce (1991)
- Sannikov, Skrzypacz (2007)

Empirical

- eBay: Cabral, Hortacsu (2008); Resneck et al. (2006)
- Airlines: Bosch et al. (1998); Chalk (1987)
- Restaurant Hygiene: Jin, Leslie (2009)

Outline

1. Introduction
2. **Model**
3. Equilibrium Analysis
4. Perfect Good News
5. Perfect Bad News
6. Imperfect Learning
7. Quality Choice

Bare-Bones Model

Players: One long-lived firm, many short-lived consumers

Timing: Continuous time $t \in [0, \infty)$, discount rate r

- Quality $\theta_t \in \{L = 0, H = 1\}$
- Invest $\eta_t \in [0; 1]$ at marginal cost c
- Utility = Signal $dZ_t(\theta_t, \varepsilon_t)$ with $\mathbb{E}[dZ_t] = \theta_t$
- Reputation $x_t = \mathbb{E}[\theta_t]$

MPE: Beliefs $\tilde{\eta} = \tilde{\eta}(x)$, strategies $\eta = \eta(\theta, x)$ with

(1) $\eta(x_t, \theta_t)$ maximizes value $V_\theta(x) = \int e^{-rt}\mathbb{E}[x_t - c\eta_t]\, dt$

(2) Correct beliefs: $\tilde{\eta}(x) = \mathbb{E}[\eta(\theta, x) | x]$

Fleshing out the Model

Technology: Poisson shocks with intensity λ

- At shock, effort determines quality $\Pr(\theta_t = H) = \eta_t$
- Otherwise, quality is constant $\theta_t = \theta_{t-dt}$

-> $\Pr(\theta_t = H) = \int_0^t e^{\lambda(s-t)} \lambda \eta_s ds + e^{-\lambda t} \Pr(\theta_0 = H)$

Information: Consumers update reputation x_t:

(1) Realized utility dZ_t

(2) Believed effort $\widetilde{\eta}_{t+dt}$

-> $dx_t = x_t(1 - x_t) \frac{\Pr(dZ_t|H) - \Pr(dZ_t|L)}{x_t \Pr(dZ_t|H) + (1-x_t) \Pr(dZ_t|L)} + \lambda(\widetilde{\eta}_{t+dt} - x_t)dt$

Levy Decomposition of Market Learning

Poisson Learning: y arrives with intensity $\mu_{\theta,y}$

$$dx = x(1-x)\sum_y \mu_y \begin{cases} (\cdots) & \text{at arrival } y \\ -dt & \text{otherwise} \end{cases}$$

- Good News: $\mu_{H,y} > \mu_{L,y}$
- Bad News: $\mu_{H,y} < \mu_{L,y}$
- Imperfect Learning: $\mu_{H,y}, \mu_{L,y} > 0$

Brownian Learning: $dZ = \mu_B \theta dt + dW$

$$d_\theta x = x(1-x)\left(\mu_B^2(\theta - x)\,dt + \mu_B dW\right)$$

First-Best Effort

Lemma: First-best effort $\eta \in [0;1]$ satisfies

$$\eta(x) = \begin{cases} 1 & \text{if } c < \frac{\lambda}{\lambda+r} \\ 0 & \text{if } c > \frac{\lambda}{\lambda+r} \end{cases}$$

Proof: Social benefit of effort is:

- ... social benefit of high quality 1, times
- ... probability ot technology shock λdt, annuitized by
- ... effective discount rate $r + \lambda$.

Always assume that effort is socially beneficial, i.e. $c < \frac{\lambda}{\lambda+r}$.

Equilibrium Characterization

Lemma: Optimal effort $\eta(x)$ is:

- Independent of quality θ,
- Bang-bang in reputation:

$$\eta(x) = \begin{cases} 1 & \text{if } c < \lambda\Delta(x), \\ 0 & \text{if } c > \lambda\Delta(x), \end{cases}$$

where $\Delta(x) := V_H(x) - V_L(x)$ is value of quality.

Proof:

- Probability of technology shock: λdt
- Benefit in case of shock: $\Delta(x)$

Asset Value of Quality

$$\Delta(x) = V_H(x) - V_L(x)$$

Theorem: In any MPE, Δ is present value of $D(x_t)$:

$$\Delta(x) = \int_0^\infty e^{-(r+\lambda)t} \mathbb{E}_{x_0=x,\theta_{\leq t}=L}\left[D(x_t)\right] dt.$$

Specifically

$$D(x) = V_H(1) - V_H(x) \qquad \text{(Good)}$$

$$D(x) = V_L(x) - V_L(0) \qquad \text{(Bad)}$$

$$D(x) = x(1-x)\, V_H'(x) \qquad \text{(Brownian)}$$

Asset Value of Quality

$$\Delta(x) = (1-(r+\lambda)dt)\mathbb{E}[V_H(x+d_H x) - V_L(x+d_L x)]$$

Theorem: In any MPE, Δ is present value of $D(x_t)$:

$$\Delta(x) = \int_0^\infty e^{-(r+\lambda)t}\mathbb{E}_{x_0=x,\theta_{\leq t}=L}[D(x_t)]dt.$$

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Asset Value of Quality

$$\Delta(x) = (1-(r+\lambda)dt)\mathbb{E}[V_H(x+d_H x) - V_H(x+d_L x)] + (1-(r+\lambda)dt)\mathbb{E}[V_H(x+d_L x) - V_L(x+d_L x)]$$

Theorem: In any MPE, Δ is present value of $D(x_t)$:

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Asset Value of Quality

$$\begin{aligned}\Delta(x) = &(1-(r+\lambda)dt)\mathbb{E}[V_H(x+d_Hx) - V_H(x+d_Lx)] \\ &+ (1-(r+\lambda)dt)\mathbb{E}[\Delta(x+d_Lx)]\end{aligned}$$

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Asset Value of Quality

$$\begin{aligned}\Delta(x) =& (1-(r+\lambda)dt)\mathbb{E}[V_H(x+d_Hx) - V_H(x+d_Lx)] \\ &+ (1-(r+\lambda)dt)\mathbb{E}[\Delta(x+d_Lx)] \\ =& \textbf{Reputational Dividend} + \textbf{Cont Value}\end{aligned}$$

Theorem: In any MPE, Δ is present value of $D(x_t)$:

$$\Delta(x) = \int_0^\infty e^{-(r+\lambda)t}\mathbb{E}_{x_0=x,\theta_{\leq t}=L}[D(x_t)]dt.$$

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Asset Value of Reputation

Reputation x has asset value:

- Current revenue x
- Future revenue $x_t|_{x_0=x}$

Lemma: In MPE firm value $V_\theta(x)$ is strictly increasing in x.

Proof:

- Firm $x' > x$ can mimick x
- Same effort & quality $\Rightarrow x'_t \geq x_t$ for all t
- In MPE firm x' does at least as good

General Properties of Equilibrium Effort

Corollary (No effort at top):
Absent perfect bad news signals, a firm with perfect reputation shirks in MPE: $\eta(1) < 1$.

Proof: Otherwise $x_t = 1$ and reputational dividend $D(x_t) = 0$.

Corollary (Some effort somewhere):
For low costs c, pure shirking $\eta(x) = 0$ for all x is not a MPE.

Proof: If $\eta \equiv 0$ then $\lambda\Delta(x)$ is bounded away from 0, indep. of c.

Perfect Good News

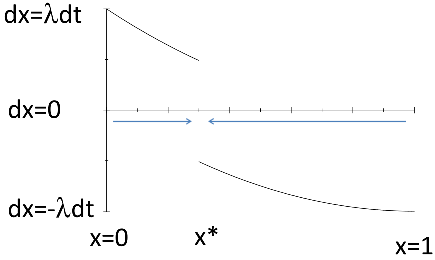
Updating & Dynamics

Reputational Updating: Arrival rate $\mu_{\theta,y} = \theta$ of breakthrough

- Breakthrough: x_t jumps to 1
- Otherwise: $dx = \lambda\left(\tilde{\eta}\left(x\right) - x\right)dt - x\left(1 - x\right)dt$

"Work-Shirk" profile with cut-off x^*:

$$\eta\left(x\right) = \begin{cases} 1 & \text{for } x < x^* \\ 0 & \text{for } x > x^* \end{cases}$$



Work-Shirk

Proposition: Every equilibrium is work-shirk.

Proof:

$$\Delta(x) = \int e^{-(r+\lambda)t} \mathbb{E}_{x_0 = x, \theta_{\leq t} = L} [D(x_t)] dt$$

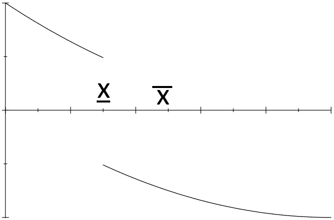
- Dividend $D(x) = V_H(1) - V_H(x)$ decreasing in x
- Future reputation $x_t|_{x_0=x}$ increasing in x (as $\theta_{s\leq t} = L$)
- $\Delta(x)$ decreasing in x

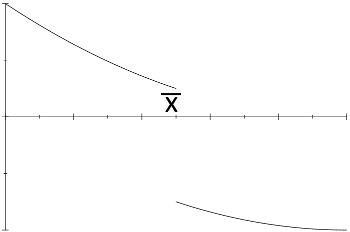
Corollary: Reputational dynamics converge to cycle

Unique Equilibrium

Proposition: Equilibrium is unique, if $\lambda > 1$.

Proof: Consider two cutoffs $\underline{x}$ and $\overline{x}$





- $\Delta_{\underline{x}}(\underline{x}) > \Delta_{\underline{x}}(\overline{x})$: Value of quality increasing in reputation
- $\Delta_{\underline{x}}(\overline{x}) > \Delta_{\overline{x}}(\overline{x})$: $\overline{x}$ has more to gain if he is drifting further

Perfect Bad News

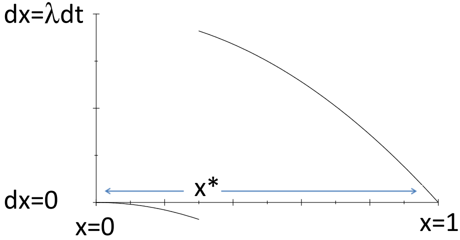
Updating & Dynamics

Reputational Updating: Arrival rate $\mu_{\theta,y} = 1 - \theta$ of breakdown

- Breakdown: x_t jumps to 0
- Otherwise: $dx = \lambda\left(\tilde{\eta}\left(x\right) - x\right)dt + x\left(1 - x\right)dt$

"Shirk-Work" profile with cut-off x^*:

$$\eta\left(x\right) = \begin{cases} 0 & \text{for } x < x^* \\ 1 & \text{for } x > x^* \end{cases}$$



Shirk-Work

Proposition: Every equilibrium is shirk-work.

Proof:

$$\Delta(x) = \int e^{-(r+\lambda)t}\mathbb{E}_{x_0=x,\theta_{\leq t}=H}[D(x_t)]dt$$

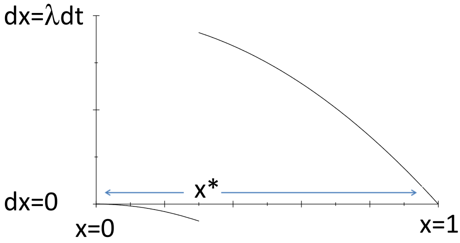
- Dividend $D(x) = V_L(x) - V_L(0)$ increasing in x
- Future reputation $x_t|_{x_0=x}$ increasing in x (as $\theta_{s\leq t} = H$)
- $\Delta(x)$ increasing in x

Corollary: Reputational dynamics diverge

Multiple Equilibria

Proposition: There is $[\underline{x}, \overline{x}]$ s.t. every $x^* \in [\underline{x}, \overline{x}]$ can be equilibrium cutoff, if $\lambda > 1$.

Proof:



x^* is not indifferent:

- $x^* + \varepsilon$ drifts up, has lot to loose
- $x^* - \varepsilon$ drifts down, is lost anyway

$$\lambda \Delta_x^-(x) < c < \lambda \Delta_x^+(x)$$

Work vs. shirk is self-fulfilling prophecy

Imperfect Learning

Fundamental Asymmetry

Reputational Dividend

$$D_\theta(x) = \sum_y \mu_y \left(V_\theta \left(x + \mu_y x (1-x)(\cdots) \right) - V_\theta(x) \right) + \mu_B^2 x (1-x) V_\theta'(x)$$

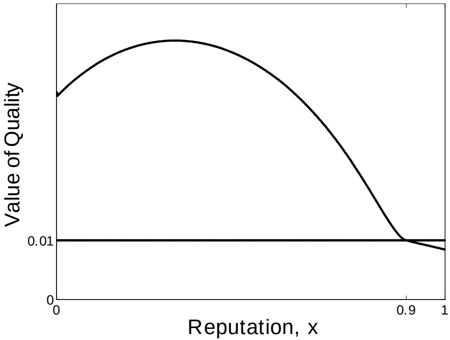
If learning imperfect, $\lim_{x \to 0;1} D_\theta(x) = 0$.

Fundamental Asymmetry

- Work at top $\eta(1) = 1$ not sustainable in MPE: $\longrightarrow$ Reputation stuck at $x = 1$; dividend low
- Work at bottom $\eta(0) = 1$ sustainable in MPE: $\longrightarrow$ Reputation drifts to $x \approx \frac{1}{2}$; dividend high

Work-Shirk Equilibrium

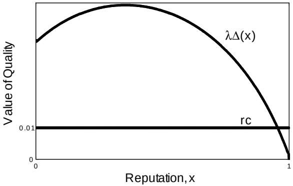
Theorem:
For imperfect learning and low c, a work-shirk equilibrium exists.



Corollary: Dynamics converge to cycle.

Idea of Proof - Layer 1

$\Delta_1(x)$ has correct shape:



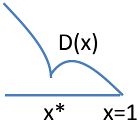
Looks like "by continuity":

$$\lambda\Delta_{x^*}(x) \begin{cases} > c & \text{for } x < x^* \quad \text{(Low types shirk),} \\ = c & \text{for } x = x^* \quad \text{(Cutoff type indifferent),} \\ < c & \text{for } x > x^* \quad \text{(High types work).} \end{cases}$$

Idea of Proof - Layer 2

Focus on $\mu_y = 0$, $\mu_B = 1$. For $x^* < 1$:

- $V'(x) = \int e^{-rt}\mathbb{E}\left[\frac{dx_t}{dx}\right] dt$ can have local minimum at x^*.
- $D(x) = x(1-x)V'(x)$ can have local minimum at x^*.
- $\Delta(x)$ as well?



Lemma: If $x^* \approx 1$ and $x^* < x$, then $\Delta(x^*) > \Delta(x)$.

$$d_L(1-x) \approx \begin{cases} -\lambda(1-x)\,dt - (1-x)\,dW & \text{for } x < x^* \\ \lambda x dt & \text{for } x > x^* \end{cases}$$

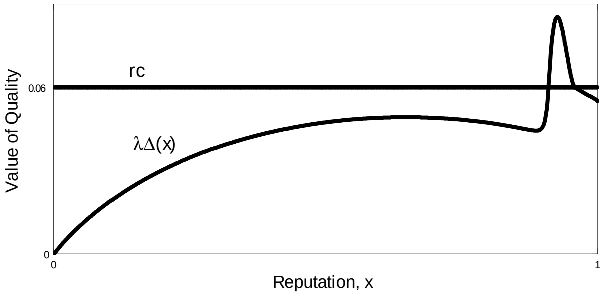
Proof: $\Delta_{x^*}(x)$ for $x > x^*$ convex combination of:

- Small dividends for $x' \in (x, x^*)$,
- $\Delta_{x^*}(x^*)$.

Shirk-Work-Shirk

Simulation Results:

For intermediate c, there exists a shirk-work-shirk equilibrium.



But for low c, there is no shirking in the middle

$$\lambda\Delta\left(\cdot\right) > c \text{ on } \left[\varepsilon; 1-\varepsilon\right]$$

Unique Equilibrium

HOPE: Market Learning dZ satisfies

$$\Pr\left[dx > 0 | \widetilde{\eta} = 0\right] > 0 \text{ for all } x$$

- Non-trivial Brownian or good news signals $\mu_B, \mu_y > 0$
- Bad news with drift $-\sum \mu_y > \lambda$

Theorem: With imperfect learning, HOPE and low c, the work-shirk equilibrium is essentially unique.

Proof:

- $\lambda\Delta\left(\cdot\right) > c$ on $\left[\varepsilon; 1-\varepsilon\right]$
- HOPE: $\lambda\Delta\left(x_*\right) > c$ for shirk-work cutoff x_*

No HOPE: Two Types of Equilibria

Proposition: Assume no HOPE, and c small.
Work-Shirk equilibrium and Shirk-Work-Shirk equilibria co-exist.

Idea:

- Adding shirk-hole at bottom is incentive compatible
- Divergent dynamics make work self-fulfilling

Non-monotonic incentives in SWS equilibrium:

- One breakdown increases incentives: Hot-seat
- Multiple breakdowns destroy incentives: Shirk-hole

Quality Choice

Effects of High Obsolescence Rate

Link model to literature

- As $\lambda \to \infty$, quality is effectively chosen instantaneously
- Limit game is continuous-time repeated game

Countervaling effects on incentives:

$$\lambda \Delta(x) = \lambda \int_0^\infty e^{-(r+\lambda)t} \mathbb{E}[D(x_t)] dt \approx \underbrace{\frac{\lambda}{r+\lambda}}_{\leq 1} \underbrace{D(x_{future})}_{\to 0}.$$

- Returns are front-loaded
- Reputational dividends may disappear

Bad News is Good

Theorem: For λ large:

(1) There is no work-shirk equilibrium.

(2) $\eta(x) = 0$ for all x is an equilibrium.

(But) Perfect bad news: Any $x^* \in (0, 1]$ defines shirk-work eqm.

Mechanisms distinguishing bad news:

- Bounded likelihood ratios of defection (AMP and SS)
- Divergent reputational dynamics (here)

Good News is Bad

Perfect Good & Bad news case

- Bad product has breakdown at rate μ_b
- Good product has breakthrough at rate $\mu_g > \mu_b$

-> Equilibria are work-shirk.

Corollary: For λ large:

(1) Effort sustainable with perfect bad news.

(2) Effort not sustainable with perfect good & bad news.

-> **More information can lead to less effort**

Idea:

- Breakthrough gives firm second chance
- Undermines incentives to avoid breakdowns

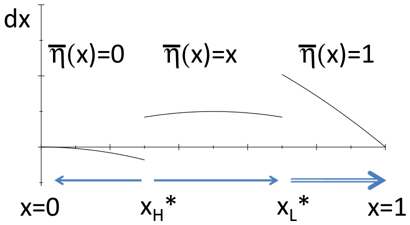
Robustness to Differential Costs - Bad News

Model Variation

- Quality cheaper to maintain than to build: $c_H \leq c_L$
- Bad news learning

Results Robust

- Updating absent shocks: $dx = \lambda\left(\overline{\eta}\left(x\right) - x\right) dt + x\left(1 - x\right) dt$
- Equilibria characterized by two cutoffs $x_H^* \leq x_L^*$



Conclusion

Modeling Innovation:

- Reputation as belief about *endogenous* quality
- Positive effort and dynamics without exogenous type changes
- Reputation spent as well as built up

Role of learning process

- Perfect Good: Work-Shirk
- Perfect Bad: Shirk-Work
- Imperfect: Work-Shirk ...

Extensions

- Competition
- Entry & Exit

Coming Soon: Reputational Theory of Firm Lifecycle

- Market Entry and Exit driven by Reputational Capital
- Repercussions of Exit on Investment

Methodological Innovation

- Exit depends on actual quality $\Rightarrow$ Private Monitoring
- Self-esteem: $z = \Pr(\theta = H)$ as judged by the firm
- Investment incentives: $\partial_z V(x, z)$

Shirk-Work-Shirk