Social Choice - Information, Power, Indeterminacy, and Chaos

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Abstract

We will consider (generalized) social welfare functions (SWFs) for N individuals (voters) and M alternatives. Those are functions which associate to every profiles of individual order preference-relations on the alternatives, a social preference relation. We do not assume that the social preferences are transitive; and we try to understand what they are.

We will discuss, power, aggregation of information, collective rationality, indeterminacy and chaos in the context of social choice. We will consider several properties of social welfare functions and the connections between them.

The first property is well known (Condorcet and Arrow).

1) Irrationality of the social preference: "Undesirable outcomes can happen": When there are more than 3 alternatives and the SWF is given by the pairwise majority rule the social outcome can be cyclic: This goes back to Condorcet and a famous result by Arrow asserts that the conclusion cannot be avoided unless one voter has all the power (dictatorship).

The second property also goes back to Condorcet.

2) Information aggregation: the voting rule enables the society to make the right choice with high probability when the voters have weak independent signals on the desirable outcome. Another famous theorem of Condorcet's asserts that this is the case for the majority rule.

One of our main results assrtes that aggregation of information is equivalent to diminishing individual Shapley-Shubik power.

The third property was first considered by McGarvey.

3) Indeterminacy: "Everything can happened". Every preference relation can occur as the social preferences, when there are sufficiently many voters. It turns out that this property follows from aggregation of information.

We also consider,

4) Stochastic indeterminacy: "Everything will happen." If the voters profile is random (chosen uniformly) then every social preference has a probability to occur which is bounded away from zero. It turns out that this property holds if the maximum individual Banzhaf power tends to zero.

The next step is to describe a dichotomy between noise stable and noise sensitive SWFs. Majority is an example of a noise stable SWF. Noise sensitive SWFs can be described in several equivalent ways one of which is an extreme form of indeterminacy:

5) Social chaos : "Everything is equally likely to happen." For random voter profiles when the number of voters tends to infinity all preference relations occur with approximately the same probability.