Sequential and Simultaneous Budgeting Under Different Voting Rules - II : Contingent Proposals

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September 2008

Abstract

When agenda setters can make explicit contingent proposals, budgets' dependence on the procedural aspects decreases considerably. Only the strongest voting rule is effective throughout, and the order of decision making is immaterial. When only one member proposes contingently, the simultaneous procedure is equivalent to the sequential setting, with the member making the contingent proposal proposing first. Collective efficiency in a sequential framework is achieved if and only if unanimity is a requirement for either of the issues.

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1 Introduction:

When an agenda setter cannot make a proposal that is explicitly a function of the other decision variable (over which he may or may not have proposal power), we have seen that different procedures result in different budgets. As shown in Boranbay 2008(a), the interaction between the order of decisions, the voting rules employed at each stage or for each issue, and the distribution of proposal power shapes the outcomes in a specific way. We have studied three main budget procedures, each with a different order of decision making: the framework where tax rate is decided first; the framework where allocation is decided first; and finally the framework where both issues are decided simultaneously. Among these three, the one used to decide the US federal and the EU budget, settles the tax rate first, and it is also the framework most affected by the other procedural aspects: for instance, the order and the relative strength of the voting rules influence none of the other procedures, but this framework. Moreover, the equivalence between the framework where allocation is decided first, and the simultaneous framework (established in Boranbay 2008(a)), that holds when there is a fixed agenda setter, fails to extend to the procedure that has tax decision first.

Introducing contingent proposals reduces the degree of budgets' dependence on institutional characteristics. The particular order ceases to matter, but, whether the decisions are made in a sequence or altogether, is important. Similarly, the order of voting rules is irrelevant as well, the only outcome relevant voting rule is the one that imposes the strongest requirement. In sequential procedures, budget sizes and levels of public good production are independent of the second stage agenda setter. A simultaneous procedure, where only one agenda setter is allowed to make a contingent proposal, is identical to the sequential procedure that has that member making the contingent proposal propose first. In general, whoever makes a contingent proposal, essentially proposes infinitely many budgets, which can alter the final outcome, as long as the other member cannot respond with such contingencies. This is because, the member allowed to make a contingent proposal, is able to prevent the other agenda setter from choosing a different proposal than the former would have chosen.

In Section 2, I revisit the sequential procedures. First, I study the framework, where the first agenda setter can base his allocation proposal on the tax rate that is the next decision. In the second part of this section, I study budgets formed when the first agenda setter's proposal is a tax schedule. Section 3 investigates the properties of the simultaneous procedure under each possible assumption on who can make contingent proposals. Finally, I conclude in Section 4. The Propositions and their proofs can be found in the Appendix.

2 Sequential Procedure with Contingent Proposals:

A contingent tax proposal specifies a tax rate as a function of allocation, and similarly an allocation proposal is stated as a function of tax rate. A contingent proposal gives the agenda setter flexibility, by allowing the associated decision variable to be a function of the complementary decision variable, which is the content of the other proposal. When proposals are made sequentially, explicit contingencies are relevant if and only if they are allowed in the first stage; since, given a contingent proposal, the next agenda setter's task is, essentially, to pick one budget offer among the many, the first proposal inherently contains. Given the first stage proposal, in equilibrium the second proposal has to constitute a best response to the first one.

I reverse the order in which I consider the sequential frameworks with respect to Boranbay 2008(a), by starting with the framework that has allocation preceding taxation.

2.1 First Allocation, Then Taxation:

In this new framework, denoted by $(v_A, v_T)^C$, an allocation proposal is a function that specifies how much of the budget is spent on each of the four categories of spending, for *every* feasible tax rate: $\{\alpha(\tau)\}_{\tau\in[0,1]}$. Hence, the second agenda setter's proposal can be interpreted as choosing one allocation among uncountably many ones, by picking the tax rate. A member approves any proposal if and only if that proposal results in a budget that leaves him at least indifferent with respect to status quo.

When contingent proposals are allowed, in equilibrium the voting rule facing each agenda setter is the highest voting requirement.[1] Furthermore, if the effective voting rule is unanimity, the final budget depends only on the member making the allocation proposal. In other words, the first proposer can bring about identical budgets, regardless of the next proposer. The first agenda setter, i_1, is able to do so, because he makes sure that, i_2 is at most indifferent at the tax rates other than i_1 would have proposed. And, since unanimity requirement forces the first agenda setter to leave every other member indifferent (irrespective of i_2), the budgets, which are decided under the same first agenda setter, and which impose unanimity at some stage, are equivalent. This finding and the equilibrium budgets under unanimity, are summarized in the following two results.

Result $\acute{1}$(i): When the effective voting rule is unanimity ($(v_A, v_T) \in \{\left(N,\ \frac{N+1}{2}\right), \left(\frac{N+1}{2},\ N\right), (N,\ N)\}$) and the first agenda setter makes a contingent allocation proposal, then the equilibrium budgets are invariant to the second agenda setter for tax.

Result $\acute{1}$(ii): When the effective voting rule is unanimity ($(v_A, v_T) \in \{\left(N,\ \frac{N+1}{2}\right), \left(\frac{N+1}{2},\ N\right), (N,\ N)\}$) and the first agenda setter makes a contingent allocation proposal, then the equilibrium budgets are efficient and the budget size can lie anywhere between $\underline{b}$ ($\approx 3H(g_3)$) and y. The first agenda setter's utility increases approximately by $3H(g_3) - g_3$, and the other members are left indifferent.

Next, I study the procedure where majority rule is effective. When allocation is decided first, allowing contingent proposals increases the efficiency of the final budgets. Moreover, if contingent proposals are available, level of public spending is the same whether the agenda setters are distinct or not. when majority rule is effective and there are distinct agenda setters, the first proposer, (here $i_1 = i_A$), chooses the second proposer, (here $i_2 = i_T$), to be his coalition partner; whereas, if there is a fixed agenda setter, the first proposer's majority coalition includes the poorest other member. Nevertheless, the same coalition supports both proposals. Moreover, even when there are distinct agenda setters, the first proposer is able to secure transfers, by appropriately calibrating the second proposer's utility at off-the-equilibrium budgets. The next result describes the equilibrium budgets when majority is the effective voting rule.

[1]This holds true for *all* the procedures considered here. However, as seen before, when allocation precedes taxation, contingent proposals are not necessary to produce this effect.

Result 1́(iii): If the effective voting rule is majority and there is a fixed agenda setter, then the procedure where allocation is decided first and contingent proposals are allowed, is identical to the previous frameworks studied in Boranbay 2008(a) (sequential or simultaneous), that do not allow contingencies: The budget is equal to total income: $b = y$; level of public spending is equal to g_2; the poorest other member is left indifferent; the agenda setter receives the residual budget leaving the remaining member worse off. If the agenda setters are distinct, the only change to this outcome is that the second proposer becomes the first agenda setter's coalition partner.

So, overall, fixing the effective voting rule, the first proposer is able to induce the same budget size and, more or less, the same utility (but this is due to the negligibly small income differences), irrespective of the second proposer. The first agenda setter can achieve this by specifying suboptimal (for himself) allocations at off-the-equilibrium tax rates to control the second agenda setter's proposal choice. When explicit contingencies are not permitted, the first proposer cannot prevent the second proposer from choosing the tax rate that maximizes the latter's income and, therefore, enjoying increased utility at the expense of the first agenda setter.

2.2 First Taxation, Second Allocation:

The sequential framework, where the first agenda setter can make a contingent tax proposal, is denoted by $(v_T,\ v_A)^C$. The tax proposal is a function $\tau : [0,1]^4 \mapsto [0,1]$ that specifies a tax rate for each allocation $\alpha = (\alpha_0,\ \alpha_p,\ \alpha_m,\ \alpha_r)$. A member votes for any proposal if and only if that proposal leads to a budget that leaves him at least indifferent.

As mentioned before, in equilibrium, a contingent proposal incorporates an approvable budget, hence, the effective voting requirement any agenda setter faces is the stronger of the two voting rules. It is important to note that, when taxation precedes allocation, contingent proposals are necessary and sufficient to convert the strongest voting requirement into the effective one for the entire budget process.[2]

[2]When allocation is decided first, the timing of decisions automatically integrates the voting rules and therefore contingent proposals are not necessary.

Given an agenda setter pair and an effective voting requirement, if proposals are contingent, then budgets decided in any order are identical. This is because, the first agenda setter can guarantee that the second agenda setter does not propose an allocation that the first one would not pick. To prevent the second agenda setter from picking an allocation other than the first agenda setter would choose, i_1 can set a tax rate low enough (setting the tax rate to zero is always a unanimously acceptable option) at the allocations i_2 can possibly deviate to. So, as long as the effective voting rule and the second agenda setter are the same, then there is no difference from the first agenda setter's perspective (and for that matter from the second agenda setter's perspective, too) between proposing a tax or an allocation schedule. The arguments here and in the previous section, suggest that, if the first agenda setter is constrained only by the effective voting requirement (influencing his coalition choice), the order of decision making is irrelevant. The statement below summarizes this result.

Result 2́: Given an agenda setter pair and any two voting rules, if tax rate and allocation are decided sequentially and contingent proposals are available, then the order of decision making and the voting rules employed at each stage are irrelevant.

The above result also suggests that, introduction of contingent proposals allows efficiency to be attained for a larger set of budgetary arrangements (chiefly including those with two distinct agenda setters and those that have unanimity requirement only over tax).

The following table summarizes budget outcomes under both sequential procedures. The notation is identical to the one used in Boranbay 2008(a).

Table 1:

Equilibrium under	$(v_A, v_T)^C$:				$(v_T, v_A)^C$:			
Agenda setters:	*SAME*		*DISTINCT*		*SAME*		*DISTINCT*	
Effective voting rule:	*U*	*M*	*U*	*M*	*U*	*M*	*U*	*M*
Transfers to i_T :	0	0	0	0;	$\check{e}$	$\hat{e}$	$\check{e}$	$\approx \hat{e}$
Transfers to i_A.	$\check{e}$	$\hat{e}$	$\check{e}$	$\approx \hat{e}$;	0	0	0	0
Level of public good:	g_3	g_2	g_3	g_2;	g_3	g_2	g_3	g_2
Tax rate:	$\check{\tau}$	1	$\check{\tau}$	1;	$\check{\tau}$	1	$\check{\tau}$	1

Note: $\hat{e} > \check{e} > 0$; $g_3 > g_2 > 0$; $1 > \check{\tau} > 0$.

3 Simultaneous Procedure with Contingent Proposals:

Unlike the sequential procedure, the simultaneous framework is characterized by which agenda setter can make such a proposal. Conditional on the member(s) making a contingent proposal, the simultaneous procedure is identical either to one of the frameworks above, $(v_A,\, v_T)^C$ or $(v_T,\, v_A)^C$, or to the original simultaneous procedure without contingencies. As before, the binding voting rule for both proposers is the most stringent one. Note that, as long as proposals are simultaneous, they need not be contingent for this to hold true.

If distinct agenda setters can make contingent proposals, then this is equivalent to letting both proposers to explicitly announce their best response functions. Consequently, the procedure dictates the budget to be characterized by the two agenda setters' mutual best responses. The contingent tax or allocation proposals that are observed in $(v_T,\, v_A)^C$ and $(v_A,\, v_T)^C$, respectively, cannot be an equilibrium once *both* agenda setters employ contingent proposals. To understand this point, suppose the tax proposer specifies a tax rate for an allocation, which he does not want to see implemented, so that the other proposer's utility is equal to, at most, his income. As discussed in the earlier sections, when the allocation agenda setter is unable to respond to each such tax proposal, and has to pick only one allocation, the tax schedule in $(v_T,\, v_A)^C$ can be part of an equilibrium. On the other hand, when the allocation agenda setter is able to respond to each tax proposal, the allocation he picks at such a tax rate, does not coincide with the allocation the tax proposer conditions that tax rate on, and, therefore, this tax schedule can no longer part of an equilibrium. Hence, when both proposers can make contingent proposals, their proposal strategies can only be their best responses.

Result $\acute{3}$(i): Consider two simultaneous procedures both of which have distinct agenda setters. In one, both proposals are contingent, and in the other, neither is. Then the budgets that originate from both procedures are identical and characterized by Result 3(ii) of Boranbay 2008(a): $b = g_1 \frac{y}{y_{i_T}},\ g = g_1$; the agenda setter over allocation enjoys a utility increase by $H(g_1) + g$, the rest of the committee members' utilities increase by $H(g_1) - g_1$.

The model's predictions change considerably, if there are distinct agenda setters and only one of them can make a contingent proposal. In this case the agenda setter making the contingent proposal, preempts the other agenda setter in exactly the same way he would, if he were the first agenda setter in the corresponding sequential framework: the member making the contingent proposal is able to prevent any deviation of the other member through reducing the latter's utility from doing so. In general, whoever makes the contingent proposal, can be thought of as the first proposer in a sequential setup. Therefore, a contingent proposal is valuable to an agenda setter only to the extent that, the other agenda setter cannot make a counteracting contingent proposal. On the other hand, any individual who is not making the contingent proposal, prefers either both or none of the proposals to be contingent. This is because, even though public good level is lower in this symmetric case, the tax rate is sufficiently low to let such a member enjoy an increased utility. The following result documents this argument.

Result $\acute{3}$(ii): Suppose the simultaneous procedure involves distinct agenda setters and permits only one to make a contingent proposal. Then, the ensuing procedure is equivalent to a sequential setup with an order of decisions that dictates the member making the contingent proposal, to propose first

Contingency of proposals is immaterial to the budget process as long as there is a fixed proposer: the budget decision can be viewed as the agenda setter's optimization problem, constrained only by the effective voting rule. Hence, as long as there is a fixed agenda setter, (v_A, v_T), $(v_A, v_T)^C$, $(v_T, v_A)^C$, and the simultaneous framework with and without contingent proposals, generate identical budgets. This leads to the following equivalence result.

Result $\acute{3}$(iii): If contingent proposals are allowed and only one member has proposal power, then the budget procedure is invariant to the timing of decisions. All procedures discussed so far, which enable the fixed agenda setter to consider the strongest voting rule as effective at each stage, are equivalent; and the equilibrium budgets under unanimity and majority as effective voting rules, are depicted in Result $\acute{1}$(ii) and (iii)**.**

The properties of budgets in the simultaneous framework are summarized in Table 2. The notation is identical to the one used in Boranbay 2008(a).

Table 2:

Contingent proposals are made by:	***Equilibrium under simultaneous procedure***											
	i_T				i_A				i_T & i_A			
Agenda setters:	*SAME*		*DISTINCT*		*SAME*		*DISTINCT*		*SAME*		*DISTINCT*	
Effective voting rule:	***U***	***M***	***U***	***M***	***U***	***M***	***U***	***M***	***U***	***M***	***U***	***M***
Transfers to i_T :	$\check{e}$	$\hat{e}$	$\check{e}$	$\approx \hat{e}$	0				$\check{e}$	$\hat{e}$	0	
Transfers to i_A.	0				$\check{e}$	$\hat{e}$	$\check{e}$	$\approx \hat{e}$	$\check{e}$	$\hat{e}$	$\bar{e}$	
Level of public good:	g_3	g_2	g_3	g_2	g_3	g_2	g_3	g_2	g_3	g_2	g_1	
Tax rate:	$\check{\tau}$	1	$\check{\tau}$	1	$\check{\tau}$	1	$\check{\tau}$	1	$\check{\tau}$	1	$\acute{\tau}$	

Note: $\hat{e} > \check{e} > \bar{e}$; $g_3 > g_2 > g_1$ where $H'(g_1)=1$.

4 Conclusion:

So, how does the introduction of contingent proposals to the agenda setting process affect the budgets? The theory supports the use of contingent proposals for certain budget procedures due to the enhanced efficiency that such proposals provide. With contingent proposals, the size of the budget and the level of public good are the same under any agenda setter configuration. This contrasts with the procedures that do not allow explicit contingencies, since, then having distinct proposers generally reduces public good production when compared to the case with a fixed proposer. Therefore, with contingent proposals, efficiency is attained for a larger set of parameters: it is sufficient to allow only one member with the authority to make contingent proposals and unanimity is required at some point.

Despite the efficiency advantages, the member making the only contingent proposal always receives transfers that make him strictly better off, including those procedures, where he would not be able to do so, if he could not propose contingently. Every member other than the agenda setter making the contingent proposal is left at most indifferent. The changes contingent proposals create, are most visible when tax rate is decided first in a sequential setup with distinct agenda setters: in the EU model, the supply of public good is at its lowest level (among those observed in this work), whereas the framework which allows contingent proposals, and that is otherwise identical to the EU model, provides the highest level of public good, i.e. the efficient level.

Finally, requiring a tax or allocation schedule seems too extreme, considering the disproportionately high bargaining power the member, who can make such proposals, has. However, the representation of the several actual budget regimes, namely the US model and the individual EU country model with a single party government, are characterized by the same unbalanced budget authority the agenda setters have.

APPENDIX

A First Allocation, Then Taxation ($(v_A, v_T)^C$)

Let $I = \{1, ..., N\}$ denote the set of committee members. Let i_1 and i_2 be the members making the first and second proposals, respectively. Let $u_i(\alpha^i(\tau), \tau)$ be member i's utility resulting from $(v_A, v_T)^C$. Starting with the second stage, given $\alpha(\cdot)$, i_T picks τ

that

$$\begin{array}{l}\max_{\tau\in[0,1]} u_{i_T}(\alpha^{i_T}(\tau),\tau)\\ \text{subject to}\\ \quad \left|\left\{i\in I : u_i(\alpha^i(\tau))\geq y_i\right\}\right| \geq \max\{v_A, v_T\}.\end{array}$$

Call this problem $P_{i_T}(\alpha(\tau),\tau)$. Note that $u_{i_T}(\alpha^{i_T}(\tau^*),\tau^*)\geq y_{i_T}$ for all $\tau^*\in\arg\max P_{i_T}(\alpha(\tau),\tau)$ since $\tau=0$ guarantees him y_{i_T}. Going back to the first stage, i_A's allocation choice is the solution to the following problem, denoted by $P_{i_A}(\tau,i_T)$:

$$\begin{array}{l}\max_{\alpha(\tau)} u_{i_A}(\alpha^{i_A}(\tau),\tau)\\ \text{subject to}\\ \quad \tau\in\arg\max P_{i_T}(\alpha(\tau),\tau)\\ \quad \sum_i \alpha_i(\tau)\leq 1, \quad \alpha_i(\tau)\geq 0\ \forall i,\tau\\ \quad \left|\left\{i\in I : u_i(\alpha^i(\tau))\geq y_i\ \forall\ \tau\in\arg\max P_{i_T}(\alpha(\tau),\tau)\right\}\right| \geq \max\{v_A, v_T\}.\end{array}$$

Let $P_{i_a}(\alpha(\tau),\tau)$ denote the single agenda setter's decision problem. Let $C_J(S)$ be the set of poorest J members in the set $I\backslash S$. As a reminder, g_k satisfies $H'(g_k)=1/k$.

Proposition 1 *Under* $(v_A, v_T)^C$*:*

If $\max\{v_A,v_T\}=N$*, then* $\tau^*\in\left[\frac{H(g_N)}{y_{i_l}},1\right]$ *and the equilibrium allocation comprises* $\alpha_0^*=\frac{g_N}{\tau^* y},\ \alpha_i^*=\frac{\tau^* y_i - H(g_N)}{\tau^* y}$ *for* $i\neq i_A,\ \alpha_{i_A}^*=1-\sum_{i\notin\{0,i_A\}}\alpha_i^*$*, irrespective of* i_A *and* i_T*.*

If $\max\{v_A,v_T\}<N$*, then*

$i_A\neq i_T$ *implies* $\tau^*=1$ *with* $\alpha_0^*=\frac{g_{\max\{v_A,v_T\}}}{y}$*;* $\alpha_i^*=\frac{y_i-H\left(g_{\max\{v_A,v_T\}}\right)}{y}$ *for* $i\in C_{\max\{v_A,v_T\}-2}(\{i_T,i_A\})\cup\{i_T\}$*;* $\alpha_i^*=0$ *for* $i\notin C_{\max\{v_A,v_T\}-2}(\{i_T,i_A\})\cup\{i_T\}$*; and* $\alpha_{i_A}^*=1-\sum_{i\notin\{0,i_A\}}\alpha_i^*$*.*

$i_A=i_T$ *implies* $\tau^*=1$ *with* $\alpha_0^*=\frac{g_{\max\{v_A,v_T\}}}{y}$*;* $\alpha_i^*=\frac{y_i-H\left(g_{\max\{v_A,v_T\}}\right)}{y}$ *for* $i\in C_{\max\{v_A,v_T\}-1}(\{i_a\})$*; and* $\alpha_i^*=0$ *for* $i\notin C_{\max\{v_A,v_T\}-1}(\{i_a\})$*; and* $\alpha_{i_a}^*=1-\sum_{i\notin\{0,i_a\}}\alpha_i^*$*.*

Proof. First suppose there is a fixed agenda setter.

$\boldsymbol{i_A=i_T}$**:** Initially, we characterize the allocation the agenda setter would pick if he were to propose any tax rate in the next stage, such that both his proposals pass. Then we find the budget that maximizes his utility. Note that once the agenda setter's optimal budget is found, there are infinitely many allocation choices that are available to him (and many tax proposals as well, if $\max\{v_A, v_T\} = N$) that can implement his preferred budget.

The question of which allocation i_a picks if he wants to set any $\tau \in [0,1]$ in the next stage is no different than the question of what allocation i_a picks if the tax rate is already set at τ (as long as the equilibrium voting requirement in each stage is the same). Therefore, the construction of i_a's optimal allocation schedule for each $\tau \in [0,1]$ is described in the proof of Proposition 1 in Boranbay 2008(a), *given that* v_A in (v_T, v_A) equals $\max\{v_A, v_T\}$ in $(v_A, v_T)^C$. Calculating the agenda setter's utility from each such budget shows that, if $\max\{v_A, v_T\} = N$, then i_a's income is maximized at any $\tau \in \left[\frac{H(g_N)}{y_{i_l}},\ 1\right]$ (remember that i_l is the poorest member in the committee other than i_a) and equal to

$$u_{i_a}^{N^*} = y_{i_a} + NH(g_N) - g_N.$$

The allocation schedule that gives him $u_{i_a}^{N^*}$ is characterized by the following: i_a picks any subset T of $\left[\frac{H(g_N)}{y_{i_l}},\ 1\right]$, compensates every one else for $\tau \in T$, and then proposes a $\tau \in T$. He can assign any allocation $\alpha(\tau)$ for $\tau \in [0,1] \setminus T$ under the following off-the-equilibrium restriction that avoids defections by i_a :

$$\text{If } \exists\, i \text{ and } \tau' \in [0,1] \setminus T \text{ such that } u_i\left(\alpha^i(\tau'), \tau'\right) \ < \ y_i, \text{ then } u_{i_a}\left(\alpha^{i_a}(\tau'), \tau'\right) \ < \ u_{i_a}^{N^*}. \tag{1}$$

If $1 < \max\{v_A, v_T\} < N$, similar calculations show that i_a's utility is maximized at $\tau = 1$, and is given by

$$u_{i_a}^{*} = y_{i_a} + \sum_{i \notin C_{\max\{v_A, v_T\}-1}(\{i_a\})} y_i \ + \max\{v_A, v_T\}\ H(g_{\max\{v_A,v_T\}}) - g_{\max\{v_A,v_T\}}.$$

He can propose any $\alpha(\tau)$ at $\tau < 1$ under a similar restriction to (1):

$$\text{If } \exists\, \tau' < 1 \text{ and } i \in C_{\max\{v_A,v_T\}-1}(\{i_a\}) \text{ such that } u_i(\alpha^i(\tau'), \tau') < y_i, \text{ then } u_{i_a}(\alpha^{i_a}(\tau'), \tau') < u_{i_a}^{*}.$$

If $\max\{v_A, v_T\} = 1$, then i_a can choose any $\alpha(\tau)$ for $\tau < 1$.

$\boldsymbol{i_A \neq i_T}$**:** Notice that, i_A's utility when $i_A \neq i_T$, cannot exceed the utility he achieves when $i_A = i_T$. It is because, in the former case his proposal needs to satisfy one more constraint: $\tau \in \arg\max P_{i_T}(\alpha(\tau), \tau)$. Below is the construction of the allocation schedules that give i_A almost the same utility he achieves when he is the sole agenda setter. 'Almost' refers to the fact that income differences are negligible and there may be instances where i_A would not have included i_T as a coalition partner if i_A were the only agenda setter

If $\max\{v_A, v_T\} = N$, then i_A can achieve $u_{i_a}^{N*}$ by proposing an allocation schedule $\alpha(\tau)$ that satisfies the following:

[1] It gets unanimous approval at some $\tau \in T$.

[2] It satisfies (1).

[3] For $\tau \in (0, \frac{H(g_1)}{y_{i_T}}] : \alpha_0(\tau) \leq \frac{H^{-1}(\tau y_{i_T})}{\tau y}$; $0 \leq \alpha_i(\tau) \leq \max\left\{0, \frac{\tau y_i - H(\alpha_0 \tau y)}{\tau y}\right\}$ for $i \neq i_A$; and $\alpha_{i_A}(\tau) = 1 - \sum\limits_{i \notin \{i_A, 0\}} \alpha_i(\tau)$ (to avoid defections by i_T).

If $\max\{v_A, v_T\} < N$, then i_A can achieve approximately $u_{i_a}^*$ by proposing $\alpha(\tau)$ with the following properties:

[1′] It is given by $\alpha_0(1) = \frac{g_{\max\{v_A, v_T\}}}{y}$, $\alpha_i(1) = \frac{y_i - H(g_{\max\{v_A, v_T\}})}{y}$ for $i \in C_{\max\{v_A, v_T\}-2}(\{i_T, i_A\}) \cup \{i_T\}$, $\alpha_i(1) = 0$ for $i \notin C_{\max\{v_A, v_T\}-2}(\{i_T, i_A\}) \cup \{i_T\}$, and $\alpha_{i_A}(1) = 1 - \sum_{i \notin \{0, i_A\}} \alpha_i(1)$.

[2′] If, for some $i \in C_{\max\{v_A, v_T\}-2}(\{i_T, i_A\}) \cup \{i_T\}$ and $\tau' < 1$, $u_i(\alpha^i(\tau'), \tau') < y_i$, then $u_{i_A}(\alpha^{i_A}(\tau'), \tau') < u_{i_a}^*$.

[3′] = [3] For $\tau \in (0, \frac{H(g_1)}{y_{i_T}}] : 0 \leq \alpha_0(\tau) \leq \frac{H^{-1}(\tau y_{i_T})}{\tau y}$; $0 \leq \alpha_i(\tau) \leq \max\left\{0, \frac{\tau y_i - H(\alpha_0 \tau y)}{\tau y}\right\}$ for $i \neq i_A$; and $\alpha_{i_A}(\tau) = 1 - \sum\limits_{i \notin \{0, i_A\}} \alpha_i(\tau)$.

Note that, i_A can always increase $\alpha_{i_T}(\tau)$ at $\tau \in T$ ($\max\{v_A, v_T\} = N$) or $\tau = 1$ ($\max\{v_A, v_T\} < N$) by a very small amount, say $\epsilon > 0$, and induce i_T to pick such τ, hence, in equilibrium, i_T always proposes one such τ. ■

B First Taxation, Second Allocation ($(v_T, v_A)^C$)

Let $u_i(\alpha^i, \tau(\alpha))$ be member i's utility from the budget process.

Corollary 1 *Under any (i_1, i_2) and $\{v_A, v_T\}$, i_1 can implement identical budgets under both $(v_A, v_T)^C$ and $(v_T, v_A)^C$.*

Proof. The following is the construction of a tax schedule that executes i_1's optimal outcome in $(v_T,\, v_A)^C$ which is identical to that under $(v_A,\, v_T)^C$. The order of decision making is not outcome-relevant, precisely, because, i_1 solves for his optimal income, and the order is important only to the extent that, it determines the formulation of his proposal.

If $i_A = i_T$ and $\max\{v_A, v_T\} = N$, i_a sets $\tau(\alpha') \in \left[\frac{H(g_N)}{y_{i_l}},\, 1\right]$ for α' consisting of $\alpha'_0 = \frac{g_N}{\tau y}$, $\alpha'_i = \frac{\tau y_i - H(g_N)}{\tau y}$ for $i \neq i_a$, such that $\sum_i \alpha'_i = 1$. For $\alpha \neq \alpha'$, $\tau(\alpha) \in [0,1]$ with the following restriction:[3]

$$\text{If } \exists\ \bar{\alpha} \text{ such that. } u_i(\bar{\alpha}^i, \tau(\bar{\alpha})) < y_i \text{ for some } i \neq i_a, \text{ then } u_{i_a}(\bar{\alpha}^{i_a}, \tau(\bar{\alpha})) < u_{i_a}^{N^*}.$$

When $\max\{v_A, v_T\} < N$, i_a sets $\tau(\alpha') = 1$ if $\alpha'_0 = \frac{g_{\max\{v_A, v_T\}}}{y}$, $\alpha'_i = \frac{y_i - H(g\max\{v_A, v_T\})}{y}$ for $i \in C_{\max\{v_A,v_T\}-1}(\{i_a\})$, $\alpha'_i = 0$ for $i \notin C_{\max\{v_A,v_T\}-1}(\{i_a\})$, and $\alpha'_{i_a} = 1 - \sum\limits_{i \notin \{0, i_a\}} \alpha_{i_T}$. For $\alpha \neq \alpha'$, $\tau(\alpha) \in [0,1]$ with the following restriction:

$$\text{If } \exists\ i \in C_{\max\{v_A,v_T\}-1}(\{i_a\}) \text{ and } \bar{\alpha} \text{ s.t. } u_i(\bar{\alpha}^i, \tau(\bar{\alpha})) < y_i, \text{ then } u_{i_a}(\bar{\alpha}^{i_a}, \tau(\bar{\alpha})) < u_{i_a}^*.$$

Consider next $i_A \neq i_T$. When $\max\{v_A, v_T\} = N$, the tax schedule that maximizes i_a's utility, also maximizes i_A's utility when agenda setters are distinct. Similarly for $\max\{v_A, v_T\} < N$, i_T sets $\tau(\alpha') = 1$ if $\alpha'_0 = \frac{g_{\max\{v_A, v_T\}}}{y}$, $\alpha'_i = \frac{y_i - H(g\max\{v_A, v_T\})}{y}$ for $i \in C_{\max\{v_A,v_T\}-2}(\{i_T, i_A\}) \cup \{i_T\}$, $\alpha'_i = 0$ for $i \notin C_{\max\{v_A,v_T\}-2}(\{i_T, i_A\}) \cup \{i_T\}$, and $\alpha'_{i_T} = 1 - \sum\limits_{i \notin \{0, i_T\}} \alpha_{i_T}$. except that the following restriction replaces the one above:

$$\text{If } \exists\ i \in C_{\max\{v_A,v_T\}-2}(\{i_T, i_A\}) \cup \{i_T\} \text{ and } \bar{\alpha} \text{ s.t. } u_i(\bar{\alpha}^i, \tau(\bar{\alpha})) < y_i, \text{ then } u_{i_A}(\bar{\alpha}^{i_a}, \tau(\bar{\alpha})) < u_{i_a}^*.$$

To see why i_A proposes no other allocation than α', note that, i_T can alter $\tau(\cdot)$ by setting $\tau(\mathring{\alpha}) = 1$, where $\mathring{\alpha}$ differs from α' with α'_{i_T} reduced only very slightly and either α'_0 or α'_{i_A}, or both, increased only by small amounts, such that $u_{i_A}(\mathring{\alpha}^{i_A}, \tau(\mathring{\alpha}))$ is marginally above y_{i_A}. Hence, as long as same member makes the first proposal, $(v_A,\, v_T)^C$ and $(v_T,\, v_A)^C$ result in identical outcomes for any voting rule. ■

[3] $\tau(\alpha) = 0$ for $\alpha \neq \alpha'$ always works.

C Simultaneous Procedure with Contingent Proposals

If both agenda setters are allowed to make contingent proposals, then, given any $\alpha \in [0,1]^4$, i_T's proposal $\tau^*(\alpha)$ solves

$$\begin{array}{l} \max_{\tau(\alpha)\in[0,1]} u_{i_T}(\alpha^{i_T}, \tau(\alpha)) \\ \text{subject to} \\ \quad \left|\left\{i \in I : u_i((\alpha^*)^i, \tau^*(\alpha^*)) \geq y_i\right\}\right| \geq \max\{v_A, v_T\}, \end{array} \tag{2}$$

and given any $\tau \in [0,1]$, i_A's proposal $\alpha^*(\tau)$ solves

$$\begin{array}{l} \max_{\alpha(\tau)} u_{i_A}(\alpha^{i_A}(\tau), \tau) \\ \text{subject to} \\ \quad \sum_i \alpha_i(\tau) \leq 1, \quad \alpha_i(\tau) \geq 0 \; \forall i \\ \quad |\{i \in I : u_i(\alpha^*(\tau^*), \tau^*)) \geq y_i\}| \geq \max\{v_A, v_T\}. \end{array} \tag{3}$$

If i_T cannot make a contingent proposal, whereas i_A can, then i_T proposes τ^* that

$$\begin{array}{l} \max_{\tau\in[0,1]} u_{i_T}((\alpha^*(\tau))^{i_T}, \tau) \\ \quad |\{i \in I : u_i(\alpha^*(\tau^*), \tau^*)) \geq y_i\}| \geq \max\{v_A, v_T\}. \\ \quad \alpha^*(\tau) \text{ solves (3)} \end{array} \tag{4}$$

Analogously, if i_A cannot make a contingent proposal, whereas i_T can, then i_A proposes α^* that

$$\begin{array}{l} \max_{\alpha} u_{i_A}(\alpha^{i_A}, \tau^*(\alpha)) \\ \text{subject to} \\ \quad \sum_i \alpha_i^* \leq 1, \quad \alpha_i^* \geq 0 \; \forall i \\ \quad |\{i \in I : u_i(\alpha^*(\tau^*), \tau^*)) \geq y_i\}| \geq \max\{v_A, v_T\} \\ \quad \tau^*(\alpha) \text{ solves (2).} \end{array} \tag{5}$$

Proposition 2 $i_T \neq i_A$: *If neither i_T nor i_A makes a contingent proposal, or both make contingent proposals, then $\tau^* = \frac{g_1}{y}$ and $\alpha_0^* = \frac{y_{i_T}}{y}$, $\alpha_i^* = 0$ for $i \neq i_A$ and $\alpha_{i_A}^* = 1 - \frac{y_{i_T}}{y}$, for all v_A and v_T.*

If i_T makes a contingent proposal and i_A does not, then the simultaneous framework yields identical outcomes to $(v_T, v_A)^C$.

If i_A makes a contingent proposal and i_T does not, the simultaneous framework yields identical outcomes to $(v_A, v_T)^C$.

$i_T = i_A$: *The simultaneous framework is outcome equivalent to* $(v_T, v_A)^C$ *and* $(v_A, v_T)^C$.

Proof. Solving (2) and (3) for any $\max\{v_A, v_T\}$, shows that the only equilibrium is given by an allocation and tax pair, α^* and τ^*, that satisfies

$$H'(\alpha_0^* \tau^* y) = 1 = \frac{y_{i_T}}{\alpha_0^* y}.$$

This is the same equilibrium when no member makes a contingent proposal. As noted then, if $\max\{v_A, v_T\} < N$, and $i_T \notin C_{\max\{v_A,v_T\}-1}(\{i_A\})$, then it can never be the case that $\tau^* > \frac{H(g_1)}{y_{i_T}}$ (since i_A never compensates i_T). If $\max\{v_A, v_T\} < N$ and $i_T \in C_{\max\{v_A,v_T\}-1}(\{i_A\})$, or if $\max\{v_A, v_T\} = N$, there exists no allocation and tax pair that satisfies $H'(\alpha_0 \tau y) = \frac{y_{i_T}}{\alpha_0 y} - \frac{\alpha_{i_T}}{\alpha_0} < 1$, where $\alpha_{i_T} = \max\left\{0, \frac{\tau y_{i_T} - H(\alpha_0 \tau y)}{\tau y}\right\}$

The case,where i_A (i_T) can make a contingent proposal and i_T (i_A) cannot, is given by the solution to (4) ((5)). In either case, the agenda setter, who does not make a contingent proposal, has to, essentially, pick one budget among the many inherent in the contingent proposal of the other agenda setter.

Finally, finding equilibrium under $i_T = i_A$ is equivalent to solving for i_a's optimal budget under $(v_T, v_A)^C$ or $(v_A, v_T)^C$. ■