Sequential and Simultaneous Budgeting Under Different Voting Rules - I: Without Contingent Proposals*

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Abstract

A public budget involves decisions on its size and distribution. These aspects are decided by a legislature in any order, using any voting rule pair and under any distribution of agenda setting power. Assuming that proposals cannot be explicitly contingent on each other, this paper shows that budgets depend heavily on the agenda rules, and certain dimensions are pivotal in budget design. The model's predictions are consistent with the relatively larger US federal budgets characterized by extensive redistribution when compared with the EU budgets. This observation suggests that budget procedures are endogenously chosen by political agents. The outcome of the US model does not change if the order of decisions is reversed; the EU model, and any procedure that has the same order and the distribution of agenda setting power as the EU model, are outcome-equivalent. When allocation is decided first under two distinct agenda setters there is tacit collusion between them, making both residual claimants on the budget not devoted to public good production. Collective efficiency is attained only if the effective voting rule over allocation is unanimity and there is a fixed agenda setter.

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1 Introduction:

The determination of a public budget typically involves decisions on both its size and allocation. These two dimensions may be determined simultaneously or sequentially; furthermore, there is no reason in principle why the rules governing collective decisions on each issue need to be the same. Indeed, while both the US and EU now use a sequential protocol under which the budget size is fixed prior to any decision on its allocation, the US uses simple majority rule at both stages with the agenda setter for each step drawn from the majority party, whereas the EU uses majority rule over the distribution of the budget, but unanimity rule on its size with agenda setters representing distinct countries.

Although it has long been understood that collective choices are rarely, if ever, invariant to the institutional rules under which those choices are made (Kramer (1972); Ferejohn, Fiorina and McKelvey (1987)), relatively little is understood about exactly how variations in the budgeting process influence final outcomes. The focus of this paper, then, is to provide some insight regarding the various positive and welfare implications of the different agenda rules for budget determination. Building upon the basic legislative bargaining model with take-it-or-leave it proposals, I compare the implications of the possible sequences for determining a budget, including the degenerate 'sequence' in which both dimensions (size and allocation) are chosen simultaneously. In each case, I consider variations in the choice rules (majority or unanimity) and in the distribution of proposal power under the assumption that budgets are used either to produce a homogenous public good or to redistribute income across legislators or both.

Relative to the EU scheme, in which budget size is fixed under unanimity rule prior to allocation, determined by majority rule, it seems that the US system, which uses the same sequence but majority rule at both stages, is broadly characterized by systematically larger (per capita) budget size and a more extensive pattern of targeted expenditures to legislators' districts (see, for example, Hix (1999) and Schick (2000)).[1] Among other things, the model below yields this stylized empirical comparison. Under

[1] In 1974, the US changed the order in which Congress decided the size and distribution of the federal budget: broadly speaking, prior to 1974 the allocation was determined before budget size, and the order

the (plausible) assumption that agenda setters from the same (national) political party are essentially equivalent, but those drawn from different countries are distinct, there are two institutional distinctions between the US and the EU systems as captured within my setting: the use of unanimity rule at the first stage and distinct agenda setters across stages in the EU, but majority rule and a fixed agenda setter in the US. It turns out that it is the difference in the distribution of proposal power that supports the difference in budget sizes and allocations. In particular, if the US assigned proposal power to different parties at each stage, as in the EU, then the model predicts that the difference in voting rule for the budget size and allocation becomes outcome irrelevant. Indeed, assuming all committee members value using at least some of the budget for a public good sufficiently highly, insisting on distinct agenda setters at each stage implies that the voting rules become immaterial. In effect, the assumption guarantees there is a maximal tax rate at which there is unanimity approval for using the entire budget for the public good; the legislator proposing the budget size recognizes that the person with proposal power over the allocation chooses to redistribute tax revenues beyond this rate rather than supply more public good. Since, in equilibrium, every member of the allocation agenda setter's coalition can at most be left indifferent between being included and excluded, there is nothing for the budget size proposer to gain from proposing a budget (equivalently, tax rate in the model) in excess of the maximal unanimously approvable rate.

If, instead of the US using distinct agenda setters as in the EU, the EU adopted the use of the same agenda setter convention as in the US, the model predicts that the voting rule becomes the key institutional difference: not only whether or not unanimity rule, for instance, is used rather than majority rule, but at which stage the more demanding rule is in effect. Moreover, although there does not exist (to my knowledge) any definitive evidence that either the US or the EU budgets are in fact economically inefficient (although such efficiency seems unlikely), the model predicts that neither the US nor the EU protocols are capable of assuring efficient budgets. Within the framework here, where budget size is determined prior to budget allocation, efficiency can be assured with a protocol that uses the identical agenda setter for both stages but

reversed thereafter. With majority rule used at both stages and a fixed agenda setter, as in the US setting, my model predicts that this change has no impact on the eventual outcomes. This is in line with results of Ferejohn and Krehbiel (1987).

requires the second (i.e. allocation) stage to use unanimity rule. On the other hand, when allocation is decided prior to taxation, even though a fixed agenda setter is still necessary to implement efficient budgets, unanimity requirement can be on either of the issues.

The predictions regarding the US and the EU budget procedures underline the political economy of budget making. The US budget system provides the salient decision makers, who hold the majority of the seats in the Congress, with substantial political gains by allowing them to extract funds from the budget. The legislators, then, can target these transfers at their constituencies, hence increasing their chances of reelection. The political benefits of agenda setting power suggest that the US federal budget procedure is likely to persist, but they also raise the question of why the US budget system is not adopted by the EU. Why the EU espouses a different regime, can be explained by the different political system in which the EU members operate. The budget authority essentially belongs to the Council of Ministers (that control mandatory spending), whose presidency rotates every six months to allow a rich-and-old member country, and a poor-and-new member country hold the presidency consecutively. The revenue legislation is part of a multi annual package (known as 'financial perspectives'), and is made under a different presidency than the allocation decision that succeeds it. As the model suggests, in this way, the richer members of the Union are able to avoid paying large amounts of tax money into a budget that can be appropriated by the poorer members. The Treaty establishing a constitution for Europe, which could not be implemented, involved extending the term of presidency to two and a half years. The model here predicts that, coupled with the proposed transition from unanimity to (qualified) majority rule, this longer term of presidency would have allowed poorer members to earmark transfers from the budget, just as the case in the US system. This is one of the main reasons why the strongest opposition to the Treaty came from the richer members (France and the Netherlands rejected it).

When budget size and allocation issues are considered simultaneously, equilibrium budget size and the share devoted to public good production fall between those that emerge from the sequential procedures. Finally, it is worth observing that the discussion above presumes that no agenda setter can make contingent proposals; for example, the proposer responsible for the allocation is prohibited from offering a menu of allocations dependent on the eventual budget size. Although this presumption reflects empirical reality, it is clearly a theoretical restriction. In a short companion paper, I prove that permitting such contingent proposals renders almost all of the institutional variations discussed herein outcome equivalent: in particular, details of the sequence or voting rules become immaterial (Boranbay 2008(b)).

Among existing contributions to the theory of public budget determination, only Ferejohn and Krehbiel (1987), to my knowledge, explicitly studies a sequential model of collective choice over budget size and allocation. However, unlike the bargaining approach adopted here, Ferejohn and Krehbiel consider a spatial model with issue-by-issue majority preference among legislators with separable preferences over the two issues. Questions of efficiency do not arise in their because, under their assumptions, the outcome is well-defined by the induced median legislator on each issue considered independently. Persson, Roland, and Tabellini (1997) show that voters are better off in a two-stage budgeting, where the Executive makes a size proposal to the Legislature, which then makes an allocation proposal. Their model is more related to the trade off between reelection concerns and rent extraction opportunities of the politicians, mine is a model of bargaining between agents, who are perfect representatives of their districts, under all possible orderings and distribution of agenda setting power. von Hagen and Harden (1995) use the Nash bargaining solution to compare final budget sizes of the EU member states under alternative budget procedures, focusing on the impact of delegating budget preparation to a minister who cares about aggregate social welfare. Among those contributions that build upon the noncooperative legislative bargaining model of Baron and Ferejohn (1989), the most closely related to this paper are Baron (1991); Leblanc, Snyder and Tripathi (2000); Battaglini and Coate (2007); and Volden and Wisemen (2007). While each of these contributions considers some aspect of either budget size or budget allocation or both, in various settings, none have explicitly considered the joint implications of varying institutional rules governing the decision sequence, agenda control and stage-specific voting rules.

The rest of the paper is structured as follows. I start by describing the model in Section 2. In Section 3, I derive the equilibria in of the sequential budgetary framework where tax rate is decided prior to allocation. Section 4 goes on to study the implications of reversing the sequence and Section 4 considers the properties of budgets formed under a simultaneous procedure. Section 6 concludes. An Appendix contains formal statements of, and proofs for, the results upon which the discussions in the text are predicated.

2 Model:

There is a committee $\mathcal{I}$ formed by 3 members who represent different districts.[2] The committee decides on a uniform tax rate, $\tau \in [0,1]$, that applies to all members and a reallocation of the budget across projects and districts. The income of region i, denoted by y_i is common knowledge among the members of the committee. Let $y \equiv \sum_{i \in \mathcal{I}} y_i$ be the total income of the districts. The budget size, denoted by b, cannot exceed the tax proceeds coming from all regions, that is, borrowing from outside sources is not feasible, neither is lending: $0 \leq b \leq y$. The budget is financed by the proportional income taxes collected from each region: $b = \tau y$. The allocation involves dividing the budget across a public good that benefits all regions and district-specific redistributions, or transfers. For example, these transfers may be subsidies for local projects or regional grants. Let g denote the supply of a pure public good with benefits given by an increasing, strictly concave and twice differentiable function, $H(g)$, such that $H(0) = 0$. Let α_0 and α_i denote the proportions of the budget spent on the public good and received by district i, respectively. The utility of district i is given by

$$u_i = (1 - \tau)y_i + H(\alpha_0 \tau y) + \alpha_i \tau y.$$

To avoid having to analyze a variety of substantively immaterial additional cases, I assume throughout that no two members have the same income, so districts can be labeled r, m, p such that $y_r > y_m > y_p$. Moreover, members' income levels are arbitrarily close, that is, there exists $\epsilon > 0$ very small such that $y_r < \epsilon + y_p$.[3] Hence $y_i \approx y/3 \; \forall i$.

[2]The restriction to three committe members is for expository reasons only. As is clear from the formal analysis, all of the results (mutatis mutandis) go through for odd-size committees.

[3]These assumptions can be relaxed with no substantive change in the main results. The reason for adopting them here is to rule out indifferences during coalition formation and, thereby, the need to consider various additional cases.

For future reference, it is useful to define several critical values of public good expenditure. Specifically, for each $k = 1, 2, 3$, let g_k solve $H'(g_k) = 1/k$. Substantively, because of the assumption that individual payoffs are separable, g_3 defines the efficient level of provision for the committee as a whole; g_2 defines the preferred level of provision for any minimal winning coalition; and g_1 defines the utility maximizing level of public good supply for each individual separately. Under the assumptions on H, $g_1 < g_2 < g_3$. Finally, to make the problem non-trivial, assume that the benefits from the (collectively) efficient level of provision, g_3, satisfies

$$H(g_3) < y_1. \tag{1}$$

2.1 Voting Rules:

A tax proposal requires the approval of at least v_T committee members to pass; similarly, an allocation proposal needs the support of at least v_A members. Failure to reach the required number of votes on either issue leads to zero taxation and no redistribution: $\tau = 0$ and $\alpha_i = 0$ for all i. To simplify the analysis without significant loss of insight, assume this outcome defines the *status quo*. [4,5]

2.2 Institutional Frameworks:

The two main budgetary frameworks I consider are sequential and simultaneous budgetary frameworks. In the sequential framework a complete budget decision is made after two rounds of voting. The general sequential setup is a simple two-stage legislative bargaining model, each of which is initiated by the relevant agenda setter making a 'take it or leave it' proposal (Romer and Rosenthal 1978). Assume no discounting and perfect information. For the sequential cases each representative i has probability $\mathbb{P}_i$

[4]Depending on the member(s) status quo favors, a non-zero status quo introduces a lot of subcases, not adding too much further insight. Nonetheless, when the status quo favors some member(s) i disproportionately, i.e. α_i and τ are both sufficiently high, if (1) there is unanimity requirement or any other voting requirement that makes i's approval indispensable; (2) i is an agenda setter at some stage, then there is going to be status quo bias. With a status quo that treats every individual the same, it is easier to see the impact of procedural rules.

[5]For a discussion on status quo points, Romer and Rosenthal (1978).

of being recognized as the proposer in the second stage, where $\mathbb{P}_i \geq 0$ with $\sum_{i \in \mathcal{I}} \mathbb{P}_i = 1$. $(\mathbb{P}_i)_{i \in \mathcal{I}}$ is common knowledge.

In the first nondegenerate sequential framework, denoted (v_T, v_A), the first stage agenda setter, i_T, makes a tax proposal (that is, budget size). If it is not accepted by at least v_T members, the process terminates with the status quo, $\tau = 0$, $\alpha_i = 0$ for all i. If a non-zero tax rate is accepted, the process moves to the second stage. The second stage agenda setter, i_A, proposes a feasible allocation. If the proposed allocation is accepted, the budget and its allocation as chosen are implemented.

In the alternative sequential framework, $(v_A,\, v_T)$, the first stage agenda setter, i_A, makes an allocation proposal. If i_A's proposal is approved by at least v_A members, then the second stage starts with i_T proposing a tax rate, which requires the support of v_T members. Similar to the previous framework where taxation precedes allocation, if either stage proposal is rejected, the status quo remains in place.

Under the simultaneous choice of the two issues, one agenda setter makes a tax proposal, while a possibly distinct agenda setter proposes an allocation. Observing both proposals, the committee votes on the budget, v_A and v_T being the minimum number of required votes for the relevant offers. At this point it does not matter whether the committee votes on these proposals sequentially or simultaneously because, once both proposals are observed, there is no strategic voting that can change the outcome. This point will be clarified as each institution is analyzed in detail.

3 Sequential Budget Procedure:

3.1 First Taxation, Second Allocation:

The equilibrium concept used for sequential budget procedures is subgame perfection. I further assume that both agenda setters are known at the beginning of a budgetary process, that is $\mathbb{P}_i = 1$ for some i. When tax rate precedes allocation. the first agenda setter proposes a tax rate τ and if the proposal is accepted, the second agenda setter proposes a budget allocation. The main result of this paper is stated and proved formally as Proposition 1 in the Appendix. It shows that the critical institutional feature when taxation precedes allocation, is whether or not the same committee member has agenda setting power over both stages. I discuss Proposition 1 through a series of informal results stated below. Although, all the Propositions in the Appendix consider any odd-size committee and study all possible voting rules, the results stated henceforth look at majority and unanimity rules.

Since they are our motivating examples, let us focus first on the model's implications for the US federal and EU budgets. The EU budget procedure imposes $(v_T, v_A) = \left(N, \frac{N+1}{2}\right)$ as the voting requirement and the agenda setting power of tax and allocation belongs to distinct members. The US procedure has $(v_T, v_A) = \left(\frac{N+1}{2}, \frac{N+1}{2}\right)$ and agenda setting power resides in a single member. The following result is consistent with the stylized facts associated with the US and EU budgets: relatively large US federal budgets with substantial transfers, and small EU budgets without transfers.

Result 1(i): If taxation, decided with unanimity, precedes allocation, decided with majority, and distinct members propose at two stages, then the budget is small and spent entirely on public good; specifically, $b = g_1$. Each member enjoys almost the same increase in his utility ($H(g_1) - \frac{g_1}{y} y_i$) relative to the status quo.
If taxation precedes allocation, both decided under majority rule, and the same member proposes at both stages, then the budget is equal to total income, $b = y$, and only an amount $g_2 < b$ is spent on public good supply with the residual devoted to earmarks: The single agenda setter's utility increases approximately to $y/3 + 2H(g_2) - g_2$ relative to the status quo. The utility of poorest individual other than the agenda-setter equals his endowment income. The remaining member suffers a utility loss of $y_i - H(g_2)$ relative to the status quo.

Result 1(ii): When taxation precedes allocation and the proposals are made by distinct agenda setters, the voting rules are irrelevant. The budget $b = g_1$ is spent entirely on public good supply with each member enjoying a utility increase solely due to public good production.

(The word 'approximately', as used in Result 1(i) and below to describe the changes in the utilities, refers to the fact that the each member's income is almost equal and can be approximated by $y/3$. Similarly, all statements regarding changes in final utility payoffs are to be taken as relative to the status quo.)

Together, parts (i) and (ii) of Result 1 demonstrate that the relatively small budgets of the EU model, consisting only of public good spending, are due largely to differences in the distribution of agenda setting power rather than to any differences in the voting rules. The irrelevance of voting rules under distinct agenda setters holds because there exists unanimous agreement among the committee members that all tax revenues should be devoted to public production until a level g_1 of the good is supplied. Once the tax rate is high enough that budgets exceed this level, then the agenda setter for allocation starts redirecting some or all (depending on the voting requirement for allocation) of the residual funds to himself. Note that it is only the second agenda setter making the allocation proposal who, in equilibrium, can appropriate more funds to himself than his tax contribution. Consequently, with distinct agenda setters across stages, the agenda setter over tax proposes the highest tax rate that leads to public good production only.

Result 1(ii) also shows that, even when there is no change in the voting rule pair used in the US federal budget procedure, making different members responsible for tax and allocation is sufficient to produce a budget identical to that of the EU. The natural question, then, to ask is how the predictions of the EU budgetary framework would change if the voting rule pair is kept as it is, but one member has all the agenda setting power. Unlike the previous result that shows the irrelevance of the voting requirements under distinct agenda setters, the analysis under the same agenda setter relies on both the order of voting rules and their relative strength.

Result 1 (i) is consistent with the findings of Persson, Roland and Tabellini 1997, in the sense that separation of agenda setting powers over budget size and allocation between the Executive and the Congress (but requiring both organs to approve the final budget) and having the Executive the first proposer in a two-stage model leads to smaller budgets.

Any individual, say j, is aware that the member making both proposals, say i, would set a budget that maximizes i's own utility, specifically by raising the tax rate sufficiently high to guarantee i earmarked benefits. At such tax rates the most j can expect is not to suffer an income loss. The best response for such a committee member is to oppose any tax rate beyond the level that makes him indifferent. The next result formalizes this intuition.

Result 1(iii): Suppose there is a single agenda setter and tax rate is decided under a strictly stronger voting requirement than allocation, that is $(v_T, v_A) = \left(N, \frac{N+1}{2}\right)$. In this case the budget size is approximately equal to $3H(g_1)$, the public good supply is g_1, and the agenda setter's utility increases by about $2H(g_1) - g_1$.

Under a fixed agenda setter, if there is a committee member who is strictly worse off in equilibrium relative to the status quo (as occurs, for example, in the US model where both decisions need majority approval), or when no member has an equilibrium incentive to veto any tax rate proposal (as, for instance, when the allocation proposal requires unanimous support for passage), the agenda setter can raise the tax rate as high as he wants. However, as expected and proved formally, his utility is a decreasing function of the voting requirement over allocation. This is because the required coalition for approval and, therefore, the required proportion of the budget devoted to public good supply, become larger, leaving less to be appropriated through redistribution.

Result 1(iv): Suppose there is a single agenda setter and allocation is decided under a stronger voting requirement than tax rate, i.e. $(v_T, v_A) \in \left\{\left(\frac{N+1}{2},\ N\right), (N,\ N)\right\}$[6]. Then public good supply is efficient at g_3; the budget size can take any value in (approximately) the interval, $b \in [3H(g_3), y]$; and the agenda setter's utility increases by approximately $3H(g_3) - g_3$.

Table 1 summarizes the results of this section.

[6] (v_T, v_A), where $i_A = i_T$ and $v_A < N$ is the only procedure that would produce a different budget if the agenda setters have identical incomes.($y_i = y'$) Since the agenda setter chooses his coalition partners randomly, no other member can be insured against his tax contribution, and therefore the agenda setter cannot propose a tax rate that makes any other member worse off: in equilibrium he proposes $\frac{H(g_{v_A})}{y'}$, where $H'(g_{v_A}) = \frac{1}{v_A}$.

Table 1.

Equilibrium of (v_T, v_A) **:**						
Agenda setters:	*SAME*				*DISTINCT*	
Voting rule pairs:	*MM*	*MU*	*UU*	*UM*	*MM;MU;UU;UM*	$(\check{e} > \hat{e} > \tilde{e} > 0)$
Transfers to i_A :	$\check{e}$	$\hat{e}$	$\hat{e}$	$\tilde{e}$	0	
Transfers to i_T :	$\check{e}$	$\hat{e}$	$\hat{e}$	$\tilde{e}$	0	
Level of public good:	g_2	g_3	g_3	g_1	g_1	$(g_3 > g_2 > g_1)$
Tax rate:	1	$\check{\tau}$	$\check{\tau}$	$\tilde{\tau}$	$\breve{\tau}$	$(1 > \check{\tau} > \tilde{\tau} > \breve{\tau} > 0)$

Note: U: unanimity, M: majority; e, g, τ: equilibrium levels.

When the tax rate is decided first, a fixed agenda setter and a majority requirement throughout leads to the largest budget, giving the agenda setter the highest utility available given the order of decision making. Efficient budgets are observed if and only if there is a fixed agenda setter and the voting constraint over allocation is unanimity. The fixed agenda setter always manages to receive transfers. Under distinct agenda setters, the outcome is the same regardless of the voting rule and there are no transfers. The two empirically observed versions of this sequential framework are those that generate the smallest budgets with no transfers (EU) and the largest budgets with the highest transfers (US). In both cases, positive public good provision is necessary to create surplus for some redistribution and it is common among all the procedures studied here.

Uncertainty over the agenda setter for allocation can lead to different outcomes than those observed when agenda setters are distinct, only if the first agenda setter and the member, who is never going to be chosen as a coalition partner unless chosen as an agenda setter, ($v_A < N$) both have relatively high chances of becoming the next agenda setter. In this case, given voting rule requirements, the agenda setter for tax proposes one of the corresponding tax rates he would have proposed if he were the only agenda setter.

3.2 First Allocation, Second Taxation:

When allocation precedes taxation, the first stage agenda setter proposes an allocation that specifies the fraction of the budget that goes to each expenditure item:

$\alpha = (\alpha_0, \alpha_p, \alpha_m, \alpha_r)$. If the proposal is accepted, the second agenda setter proposes a tax rate. In the first stage a member votes for an allocation proposal if and only if the current proposal leads to a tax rate that leaves him indifferent.[7] Similarly in the second stage a member votes for the tax proposal if he is weakly better off with the given budget. I explore the main result of this section, Proposition 2 which is stated and proved in the Appendix, through a series of informal statements expressed below.

Before stating the results, let us start by emphasizing two features of the equilibria in this model. First, the individual voting rules governing each stage are irrelevant because the effective voting rule is the strongest voting requirement, $\max\{v_A, v_T\}$. To see this, suppose that the allocation and taxation decisions are governed by majority and unanimity rules, respectively. If the proposed allocation makes a member worse off at a tax rate τ, then he can block the allocation by vetoing τ if it is proposed. In contrast, the agenda setter for allocation has to ensure the tax proposal induced by his allocation proposal receives unanimous approval. Suppose next that the allocation and tax decisions are decided under unanimity and majority, respectively. Since the tax-decision stage is only reached conditional on all committee members approving the allocation proposal, then in equilibrium it must be the case that all acceptable tax proposals leave every individual at least indifferent relative to the status quo. Indeed given an agenda setter pair, any two frameworks where allocation precedes taxation are equivalent as long as the strongest voting requirement in each one is the same. Therefore, instead of studying the implications of four different sets of voting rule pairs, it is sufficient to study budgets under two sets, one that has majority as the highest requirement ($(v_A, v_T) = \left(\frac{N+1}{2}, \frac{N+1}{2}\right)$) and one with unanimity as the highest requirement ($(v_A, v_T) \in \left\{\left(\frac{N+1}{2}, N\right), \left(N, \frac{N+1}{2}\right), (N, N)\right\}$). Having a single effective voting rule throughout the budget procedure implies that the same coalition approves both proposals.

The second feature relates to when a member receives transfers that increase his utility above his income. As discussed earlier, when taxation is decided prior to allocation, only a member who proposes at both stages can extract such transfers Therefore, it is possible to avoid transfers by simply authorizing distinct members to make each

[7]Any allocation proposal that specifies levels of spending for three goods and leaves the level of the fourth good to be determined residually, would lead to equivalent budget outcomes.

proposal. On the other hand, when the allocation of any budget is decided first, the first proposer is able to extract transfers from the budget regardless of the second agenda setter. Moreover if the agenda setters are distinct, then the second agenda setter also enjoys a utility higher than his income. This is because, given that he is one of two agenda-setters, the first agenda-setter designates a higher share to the second than would otherwise be necessary to leave him indifferent; as a result, the second agenda setter picks the tax rate that maximizes both agenda setters' utilities.

I now consider results for the sequence (v_A, v_T).[8]

Result 2(i): If allocation is decided prior to the tax rate under unanimity, which is therefore the effective voting requirement, and there are distinct agenda setters, the budget size b lies in the interval $(3H(g), y]$ where $g \in (g_1, g_3)$. Public good supply is therefore socially inefficient and both agenda setters receive transfers that more than compensate for their tax payments net of public good benefits.

Result 2(ii): If allocation is decided prior to the tax rate under majority as the effective voting requirement and there is only one agenda setter, then the ensuing budget is identical to the one observed when tax rate is decided prior to allocation. That is: $b = y$, $g = g_2$ and i_A's utility increases approximately by $y/3 + 2H(g_2) - g_2$.

Efficiency can be achieved for a broader range of institutional parameters if allocation rather than taxation is resolved first. To insure efficiency with allocation decided first, select a single agenda setter for both stages and use unanimity rule for at least one of the two decisions. In this case we have the following result.

Result 2(iii): If allocation is decided prior to tax rate with unanimous support required for at least one decision and a fixed agenda setter, then the final budget coincides with that of Result 1(iv): $b \in [3H(g_3), y]$, $g = g_3$ and i_A's utility increases by approximately $3H(g_3) - g_3$.

[8]Among other things, the results below support the conclusion of Ferejohn and Krehbiel (1987), that switching the order of decision making cannot unilaterally decrease the budget sizes, as 1974 Budget and Impoundment Control Act was expected to do, and as we see here as long as the preferences of the agenda setters remain the same, the outcome does not change.

A common theme that emerges from the study of sequential frameworks is that, under any effective voting rule and any ordering of decisions, budgets prepared by two agenda setters involve lower public good supply than budgets prepared by a single member. The reason underlying the lower supply of public good when allocation is decided first, however, is quite different than the previous arguments when taxation precedes allocation. When allocation is resolved first, the first agenda setter allocates a share to the second, which exceeds the latter's tax contribution (net of the benefit from public good production). Hence, the second agenda setter proposes a tax rate that secures transfers for both. Specifically, in equilibrium the second agenda setter picks the tax rate that maximizes both agenda setters' utilities. The second agenda setter's transfer is less dependent on the share of public good than that required to induce the support of a coalition partner without proposal power.[9] This creates large budgets with lower levels of public good relative to the single agenda setter case. Among the sequential procedures which have allocation as the first decision and involve distinct agenda setters, the budgets and total benefits derived by the agenda setters are higher, and the supply of public good is lower, if majority is the effective voting rule instead of unanimity. This is because, with a majority requirement being effective, the coalition consists only of agenda setters. The following result summarizes the case with distinct agenda setters where majority is effective.

Result 2(iv): Suppose allocation is decided prior to tax rate under majority as the effective voting rule and the agenda setters are distinct. Then the budget equals total income, $b = y$, and public good supply g lies strictly between g_1 and g_2. Moreover, the level of transfers is the highest among all possible decision procedures, sequential or simultaneous.

Table 2 summarizes the results of this section.

[9] The change in i_T's extra share for a small increase in public good is given by $-\left[\frac{\partial H(\alpha_0 \tau y)}{\partial \alpha_0} + \alpha_0 \frac{\partial^2 H(\alpha_0 \tau y)}{\partial \alpha_0^2}\right] \partial \alpha_0$ versus $-\partial H(\alpha_0 \tau y)$ for a compensating share.

Table 2.

Equilibrium of (v_A, v_T) :					
Agenda setters:	*SAME*		*DISTINCT*		
Effective voting rule:	***U***	***M***	***U***	***M***	
Transfers to i_T :	$\check{e}$	$\hat{e}$	e''	e'	$(e'' > e' > 0)$
Transfers to i_A.	$\check{e}$	$\hat{e}$	$\breve{e}$	$\tilde{e}$	$(\hat{e} > \tilde{e} > 0;\ \check{e} > \breve{e} > 0)$
Level of public good:	g_3	g_2	g'	g''	$(g_3 > g_2 > g' > g'' > g_1)$
Tax rate:	$\check{\tau}$	$\hat{\tau}$	τ'	τ''	$(\check{\tau} < \tau' < \hat{\tau} = \tau'' = 1)$

Note: The ranking of e'' and $\breve{e}$ requires further assumptions on the second order effects of increasing g $(H''(\cdot))$.

In the setting being considered here, (v_A, v_T), the first agenda-setter can commit to include the second agenda setter in his coalition by offering a suitably asymmetric allocation. As remarked earlier, this permits tacit collusion between the two committee members resulting in larger transfers to both. In contrast, small variations in the tax rate affect every committee member symmetrically. Consequently, when proposed first, the tax rate (unlike allocation) cannot be fine-tuned to provide particular incentives for the next agenda setter to collude to their mutual advantage.

When the agenda setter for tax rate is uncertain and there is a subset of committee members, each with a sufficiently high chance of being appointed as the next agenda setter, then the agenda setter for allocation may have to include these members in his coalition by allocating each a higher share than that leaves him indifferent.[10] This can even imply that the first agenda setter's coalition is oversized, that is, it includes more members than that dictated by the strongest voting requirement. Due to the higher number of residual claimants on the budget funds which are not spent on public good, the transfer that the first proposer for allocation can extract from the budget is lower.

[10]The fraction of the budget that any member needs to receive to choose τ is given by $\frac{y_i}{y} - \alpha_0 H'(\alpha_0 \tau y)$.

4 Simultaneous Budget Procedure:

In the simultaneous procedure, tax and allocation proposals are made at the same time.[11] This framework portrays characteristics of budget making in European countries; in these parliamentary regimes the government prepares a budget draft before submitting it to the parliament for approval.[12] If both proposals are submitted at the same time it does not matter whether both bills are voted simultaneously or sequentially, since any member votes for both proposals if and only if he is weakly better off with the proposed budget. Since voting is over a budget, the weaker voting rule is not effective.

The main determinant of simultaneous budgeting is the distribution of agenda setting power. If there is a fixed proposer, then this member proposes an approvable budget that maximizes his utility. Furthermore, fixing an effective voting rule, there is no substantive difference between one member making both proposals at once or one after the other, leading to identical budgets: these observations are stated below.

Result 3(i): Budgets that are generated by a procedure where one committee member proposes tax rate and allocation simultaneously, or a procedure where he proposes allocation first, are identical. Therefore, the equilibrium budget of the simultaneous procedure under a fixed agenda setter is given by Result 2(ii) when the effective rule is majority, or by Result 2(iii) when the effective rule is unanimity.

If distinct members propose a tax rate and an allocation, the simultaneous framework does not resemble any of the sequential procedures studied above. When decisions are made sequentially, there is a first-mover advantage. With no first-mover there can be no such advantage. On the other hand, having agenda control over the allocation does endow an individual with some asymmetric advantage.

[11] Having taxation and allocation in a single proposal is the approach generally taken in the literature (for instance Battaglini and Coate (2007)).

[12] The distribution of agenda setting powers between the executive and the legislature depends on the scope of amendments that can be made by the parliament. In France and the U.K. making amendments to the budget proposal is considerably harder, leading to budgets that are only slightly more different than the original proposal. As in the previous frameworks, I adopt a closed rule hence giving the entire agenda setter power over the budget to the government.

Result 3(ii): When distinct members propose tax rate and allocation in the simultaneous procedure, then, regardless of the voting requirements, the budget is approximately equal to $3g_1$ (specifically, $b = g_1\frac{y}{y_{i_T}}$); an amount g_1 is devoted to public good supply; and the agenda setter for the allocation receives the residual, $b - g_1$, enjoying a utility increase of $H(g_1) + g_1$. Every other member enjoys a utility increase of $H(g_1) - g_1$.

The intuition behind the allocation proposer's advantage here is straightforward. First, the allocation proposer strictly prefers to induce the individually optimal level of public supply, g_1; given any allocation α, the tax proposer best responds by proposing a tax rate such that his marginal utility from the public good supply equals his tax payment; and, finally, these two motivations result in a unique pair of mutual best responses that directly implies Result 3(ii).

Table 3 summarizes the findings for the simultaneous procedure.

Table 3.

Equilibrium under simultaneous procedure					
Agenda setters:	*SAME*		*DISTINCT*		
Effective voting rule:	*U*	*M*	*U*	*M*	
Level of public good:	g_3	g_2	g_1		
Transfers to i_A :	$\check{e}$	$\hat{e}$	$\bar{e}$		$(\hat{e} > \check{e} > \bar{e})$
Transfers to i_T :	$\check{e}$	$\hat{e}$	0		
Tax rate:	$\check{\tau}$	$\hat{\tau}$	$\acute{\tau}$		$(1 = \hat{\tau} > \check{\tau} > \acute{\tau})$

At least for parliamentary regimes, the results summarized in Table 3 suggest that budgets prepared by single-party governments are larger, and involve a higher supply of public good, than those prepared by coalition governments. This observation suggests that the inability to make commitments in coalition governments leads to smaller budgets.

5 Conclusion:

This paper explores the influence of institutions on budgets. Its principal contribution is to show the ways in which agenda rules affect budgets by molding the incentives of those in charge of size and distribution decisions. Differentiation among budgetary regimes is achieved by (i) decomposing the consideration of size and allocation, (ii) letting them be considered separately and sequentially, (iii) employing different voting requirements at each stage and, finally, (iv) varying agenda setter profiles. These lines of distinction are inspired by the budgetary procedures in the US, in the European Union and individual European countries. The model where tax is decided first under non-contingent proposals suggests that, when there are distinct proposers, the US Federal and the EU budget procedures lead to different outcomes, consistent with the most well-known stylized facts: relative to the EU, the US exhibits larger budgets and more extensive redistributive transfers. Furthermore, the model also yields the observed inefficiently low levels of public good supply in the EU.

The analysis reveals that budgets decided under a fixed agenda setter are more efficient than those decided under distinct agenda setters. Indeed, full efficiency is only attained under a fixed agenda setter given that unanimity is effective for deciding on budget allocation. Transfers are generally higher when allocation is decided first; moreover, in this case, the level of transfers is highest if majority is the effective rule. All frameworks generate identical outcomes when there is a single agenda setter and the allocation decision requires at least as many votes as the tax rate for approval. The first mover advantage associated with the sequential procedure ceases to exist if the budget is decided by a simultaneous framework.

In sum, this paper offers an array of possibilities for institutional design. The institutions that lead to efficient budgets are not observed (to the best of my knowledge). However, two of the currently used procedures give rise to two extremes of the budget spectrum: the EU model with the smallest budgets (allowing each member to enjoy a strict increase in welfare) and the US model with the largest budgets (the single agenda setter enjoying the highest possible utility). The study in this paper provides motivation for further empirical analysis of the relationship between political institutions and budget determination. Although I have focused on institutions commonly associated, respectively, with the US and the EU, similar incentives as those identified above seem likely to be more widely relevant (for instance, to intergovernmental organizations such as WTO, NATO,...etc.). Exploring this intuition is left for subsequent research.

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APPENDIX

A First Taxation, Second Allocation

Denote $I = \{p, m, r\}$. Let $\alpha^i = (\alpha_0, \alpha_i)$ be member -i relevant segment of the allocation. Let $u_i(\alpha^i, \tau)$ stand for i's utility from the proposed budget. Using backward induction let us first characterize i_A's behavior: i_A's allocation strategy given any $\tau \in [0,1]$ is a solution to the following problem, denoted by $P_{i_A}(\tau)$,

$$\begin{aligned}
&\max_\alpha H(\alpha_0 \tau y) - \sum_{i \neq i_A} \alpha_i \tau y \\
&\text{subject to} \\
&\quad \alpha_i \geq 0 \ \forall i, \quad \sum_{i \in \{0\} \cup I} \alpha_i \leq 1 \\
&\quad \left| \{ i \in I : u_i(\alpha^i, \tau) \geq y_i \} \right| \geq v_A.
\end{aligned}$$

Given i_A's allocation strategy $\alpha(\cdot)$, i_T's tax proposal solves the problem stated below, denoted by $P_{i_T}(\alpha, i_A)$:

$$\begin{aligned}
&\max_{\tau \in [0,1]} u_{i_T}(\alpha^{i_T}(\tau), \tau) \\
&\text{subject to} \\
&\quad \alpha(\tau) \in \arg\max P_{i_A}(\tau) \\
&\quad \left| \{ i \in I : u_i(\alpha^i(\tau), \tau) \geq y_i \} \right| \geq v_T.
\end{aligned}$$

Let τ^* and α^* denote any equilibrium tax rate and allocation proposal that solve $P_{i_A}(\tau)$ and $P_{i_T}(\alpha(\tau), i_A)$, respectively.

Let $g(\tau)$ be i_A's proposal for public good expenditure at τ, which becomes the actual level of spending if i_T proposes τ and it is accepted. Moreover, let $f_i = \max\{0, \tau y_i - H(g(\tau)\}$ be the compensation received by member i. When it is the same member making both proposals, that member is denoted by i_a. From now on we refer to i_h and i_l as the richest and the poorest other members in the committee than i_A.

First, two lemmas are presented that are used to prove the main result. These lemmas basically state that, for a certain range of tax rates, it is feasible to increase public good level so that no member requires compensating earmarked transfers. (The bounds on tax rates depend on income differences and the larger they are, the tighter these ranges become.) The following lemma shows that, if the allocation proposal involves a level of public good between g_1 and g_2 ($H'(g_1) = 1$ and $H'(g_2) = \frac{1}{2}$), then the middle income member needs no compensation.

Lemma 1 *If* $g(\tau) \in [g_1, g_2]$, *then* $\tau \leq \frac{H(g(\tau))}{y_m}$.

Proof. Suppose $g(\tau) \in [g_1, g_2]$. In addition, consider $i_A = p$ and $v_A = N$ (in other words, p needs every committee member's approval, so cannot leave them worse off). The result follows trivially if $\tau \leq \frac{H(g(\tau))}{y_r}$ ($f_r(\tau) = 0$), since when r, a larger contributor to the budget than m is, does not need transfers, neither does m. So, concentrating on $\tau > \frac{H(g(\tau))}{y_r}$ ($f_r(\tau) > 0$), suppose for a contradiction that $\tau > \frac{H(g(\tau))}{y_m}$ ($f_m(\tau) > 0$) as well. That is, in equilibrium $u_i(\alpha^i(\tau), \tau) = y_i$ for $i \in \{m, r\}$, with p receiving the residual budget: $\acute{b}(\tau) = \tau y - g(\tau) - f_r(\tau) - f_m(\tau)$ ($= \tau y_p - g(\tau) + 2H(g(\tau))$). If p proposes $g(\tau)$ that necessitates positive compensation payments to both m and r, then $g(\tau)$ should solve (assuming $f_i(\tau) > 0$ for $i \in \{m, r\}$): $\max_{g \geq 0} 3H(g) - g$ ($u_p = (1-\tau)y_p + \acute{b} = y_p + 3H(g) - g$). Note that $\acute{b} > 0$ for $g(\tau) \in [g_1, g_2]$, so budget constraint is already satisfied. Hence $H'(g(\tau)) = \frac{1}{3}$, implying $g(\tau) = g_3$. ■

The next lemma concerns the type of allocations budgets of size τy can accommodate. The result shows that, as τ increases beyond $\frac{H(g_1)}{y_{i_h}}$, it is feasible for the agenda setter to increase public good level until g_3 to make the richer member i_h indifferent without having to compensate him directly. Similarly he can also raise public good for tax rates above $\frac{H(g_1)}{y_{i_l}}$ so that he makes the poorer member i_l indifferent while compensating the richer member(s) (including himself, if necessary).

Lemma 2 *Given the assumptions on income levels, the following holds:*

$$H^{-1}(\tau y_i) + \textstyle\sum_{j:y_j > y_i} \tau(y_j - y_i) \leq \tau y \; \forall i \textit{ for } \tau \in (\frac{H(g_1)}{y_i}, \frac{H(g_3)}{y_i}]. \tag{2}$$

Proof. Suppose $i = r$. Let $d_r(\tau) = \tau y - H^{-1}(\tau y_r)$. Observe that $d_r\left(\frac{H(g_1)}{y_r}\right) > 0$ and d_r is a strictly concave function. We have $d_r'(\tau) = y - \frac{y_r}{H'(H^{-1}(\tau y_r))}$, which equals 0 at a tax rate τ' which satisfies $H'(H^{-1}(\tau' y_r)) = \frac{y_r}{y} = \frac{1}{3} + \delta,\ \ \delta > 0$ and arbitrarily small, hence τ can be increased to $\frac{H(g_3)}{y_r}$ such that $d_r\left(\frac{H(g_3)}{y_r}\right) > 0$.

Suppose next that $i = p$. Given $g_1 \leq H^{-1}(\tau y_p) \leq g_3$ we have $\frac{y_p}{y - y_r - y_m + 2y_p} = \left(\frac{y_p}{3y_p} = \frac{1}{3}\right) < \frac{\tau y_p}{H^{-1}(\tau y_p)}$, rewriting this inequality gives (2). The case with $i = m$ follows from $\frac{y_p}{y_r + 2y_p} < \frac{1}{3} < \frac{\tau y_m}{H^{-1}(\tau y_m)}$, where $g_1 \leq H^{-1}(\tau y_m) \leq g_3$. ■

Lemma 1 and 2 can easily be generalized to hold for any committee of size N, as long as the richest and the poorest members' incomes are arbitrarily close, and no two members have the same income. Labeling the committee members so that $y_1 < ... < y_N$ with the assumption that $\exists\ \epsilon > 0$ such that $y_N < \epsilon + y_1$, Assumption (1), then, can be rephrased as

$$H(g_N) < y_1, \tag{3}$$

where g_N is the collectively efficient public good supply for an N - size committee. Subsequently, Lemma 1 reads as

$$\text{if } g(\tau) \in [g_{k-1}, g_k]\,,\ \text{then } \tau \leq \frac{H(g(\tau))}{y_k} \text{ for } N > k \geq 2.$$

Analogously, Lemma 2's conclusion can be restated as

$$H^{-1}(\tau y_i) + \textstyle\sum_{j:y_j > y_i} \tau(y_j - y_i) \leq \tau y \ \forall i \in \{1, ..., N\}\,, \forall \tau \in (\frac{H(g_1)}{y_i}, \frac{H(g_N)}{y_i}].$$

Proposition 1 *Suppose $v_T, v_A \leq N$ along with the assumptions on income levels stated above. Under (v_T, v_A):*

If $i_T = i_A$, then

(A) *$v_T > v_A$ means $\tau^* = \frac{H(g_1)}{y_{i_{v_T - 1}}}$, where i_{v_T-1} is the $(v_T - 1)^{th}$ poorest other member; $\alpha_0^* = \frac{g_1}{\tau^* y}$and $\alpha_{i_a}^* = 1 - \alpha_0^*$.*

(B) *$v_T \leq v_A$ means $\tau^* = 1$ if $v_A < N$, or $\tau^* \in (\frac{H(g_N)}{y_{i_l}}, 1]$ if $v_A = N$; α_0^* satisfies $H'(\alpha_0^* \tau^* y) = \frac{1}{v_A}$ and $\alpha_i^* = \frac{\tau^* y_i - H(\alpha_0^* \tau^* y)}{\tau^* y}$ for any member i among the $v_A - 1$ poorest other members than i_a, and $\alpha_i^* = 0$ otherwise; $\alpha_{i_a}^* = 1 - \sum_{i \neq i_a} \alpha_i^*$.*

If $i_T \neq i_A$, then $\tau^* = \frac{g_1}{y}$ and $\alpha_0^* = 1$.

Proof. I prove the result for $v_A \in \{\frac{N+1}{2}, N\}$ due to the empirical relevance of majority and unanimity rules. I also work with $N = 3$. Nonetheless, as the proof below suggests, the results can be generalized to all possible voting rules and all finite committee sizes.

Suppose $v_A = 1$ (i_A faces no voting constraints). Given τ, i_A's problem is to

$$\max_{\alpha_0 \geq 0, \alpha_i \geq 0 \ \forall i} (1 - \alpha_0 - \sum_{i \neq i_A} \alpha_i)\tau y + (1-\tau) y_{i_A} + H(\alpha_0 \tau y),$$

which implies that if $\tau \leq \frac{g_1}{y}$, then $\alpha_i = 0 \ \forall \ i \neq i_A$ and $\alpha_0(\tau) = 1$ at all τ; for all $\tau > \frac{g_1}{y}$, $\alpha_0(\tau) = \frac{g_1}{\tau y}$ and $\alpha_{i_A}(\tau) = 1 - \alpha_0(\tau)$. Observe that, there is unanimous agreement that $g = \tau y$ should hold at all $\tau \leq \frac{g_1}{y}$ (put differently each member sets α_0 so that $H'(\alpha_0 \tau y) = 1$).

Next, suppose $v_A = \frac{N+1}{2}$. Then i_A sets $\alpha_{i_h}(\tau) = 0$ for all τ. Solving $P_{i_A}(\tau)$ is tidier by using actual outlays rather than fractions of the budget, therefore let $g = \alpha_0 \tau y$ and $t_{i_l} = \alpha_{i_l} \tau y$.[13] Hence given τ, $P_{i_A}(\tau)$ can be restated as (along with the associated nonnegative multipliers):

$$\begin{aligned}
&\max_{g, t_{i_l}} H(g) - g - t_{i_l} \\
&\text{subject to} \\
&\quad g \geq 0 \ (\beta_g), \ t_{i_l} \geq 0 \ (\beta_{i_l}), \\
&\quad g + t_{i_l} \leq \tau y \ (\theta), \\
&\quad H(g) + t_{i_l} \geq \tau y_{i_l} \ (\lambda).
\end{aligned}$$

The first order conditions to $P_{i_A}(\tau)$ are given by ■

$$\begin{aligned}
H'(g(\tau)) &= \frac{1 + \theta - \beta_g}{1 + \lambda} \\
\lambda &= 1 + \theta - \beta_{i_l}.
\end{aligned} \tag{4}$$

Proof. Since $\tau = 0$ is trivial, let us focus on $\tau > 0$. Then $\beta_g = 0$ because $g(\tau) > 0$ for all $\tau > 0$. Since individually optimal level of public good is given by g_1, for $\tau \in (0, \frac{g_1}{y}]$, any member i chooses to spend the tax revenue on public good and does not have to compensate i_l since $H(\tau y) > \tau y_{i_l}$. Then $t_{i_l}(\tau) = 0$ and i_l's individual rationality constraint (IRC) does not bind. Hence, for $\tau \leq \frac{g_1}{y}$, $g(\tau) = \tau y$ and we have (**I**)(**a**). Note that (4) implies $H'(g(\tau)) \geq \frac{1}{2}$, meaning any allocation which solves p's optimization problem, involves public good provision no higher than g_2. Let us divide the rest of

[13] Although working with f_i (compensatory payments) is shorter, for expository purposes I use transfers. In equilibrium any positive transfer to any one other than i_A is equal to the compensatory payment.

the proof in two parts: (1) $t_{i_l} = 0$ and (2) $t_{i_l} > 0$. To begin with, it cannot be the case that $t_{i_l} = 0$ and both budget constraint (BC) and i_l's IRC bind. Because then BC and IRC would suggest $\frac{H(g(\tau))}{g(\tau)} = \frac{y_{i_l}}{y}$. However this is impossible since $\frac{H(g)}{g} > \frac{1}{2} > \frac{y_{i_l}}{y}$ for all $g \leq g_2$ due to strict concavity. Because each i's choice of $g(\tau)$ is identical *once* the tax rate is settled, if there is any incentive to raise g above g_1, it is to keep i_l indifferent in the least costly way. With $t_{i_l} = 0$, binding individual rationality and slack budget constraints we have $\frac{1}{2} \leq H'(g) \leq 1$, where the lower bound is due to the equality $1 = \beta_{i_l} + \lambda_{i_l}$.

Now consider $t_{i_l} > 0$. $\beta_{i_l} = 0$ implies $\lambda = 1 + \theta$, hence i_l's IRC binds. This also means that $\tau > \frac{H(g_1)}{y_{i_l}}$ (that is $t_{i_l} = f_{i_l}$). Consider initially that BC binds as well, that is $g + t_{i_l} = \tau y$ suggesting $t_{i_A} = 0$. If $i_l = p$, then g cannot be part of the equilibrium, otherwise i_A (in that case $i_A = p$) can do better than $(1 - \tau)\, y_{i_A} + H(g)$: for $\tau \leq \frac{H(g_2)}{y_{i_l}}$, i_A can increase his utility by $\tau y - H^{-1}(\tau y_{i_l}) - (\tau y_{i_l} - H(g))$ through raising g to $H^{-1}(\tau y_{i_l})$ and, therefore, setting $t_{i_l} = 0$. For $\tau \leq \frac{H(g_2)}{y_{i_l}}$ provision of $H^{-1}(\tau y_{i_l})$ is feasible by Lemma 1. Moreover, when BC does not bind, that is, $t_{i_A} > 0$, and $t_{i_l} > 0$ and $g < g_2$ cannot all be true, because the first two inequalities imply $g = g_2$. Indeed, for $\tau \in (\frac{H(g_1)}{y_{i_l}}, \frac{H(g_2)}{y_{i_l}}]$, the ratio of public good investment given in (**I**)(**c**) is optimal. So if $\tau \in (\frac{H(g_2)}{y_{i_l}}, 1]$, then $g(\tau) = g_2$ and $t_{i_l}(\tau) > 0$. To summarize i_A's allocation proposal strategy:

- If $\tau \in \left[0, \frac{g_1}{y}\right]$, then $\alpha_0(\tau) = 1$.
- If $\tau \in (\frac{g_1}{y}, \frac{H(g_1)}{y_{i_l}}]$, then $\alpha_0(\tau) = \frac{g_1}{\tau y}$ and $\alpha_{i_A}(\tau) = 1 - \frac{g_1}{\tau y}$.
- If $\tau \in (\frac{H(g_1)}{y_{i_l}}, \frac{H(g_2)}{y_{i_l}}]$, then $\alpha_0(\tau) = \frac{g(\tau)}{\tau y}$ and $\alpha_{i_A}(\tau) = 1 - \frac{g(\tau)}{\tau y}$, where $g(\tau) = H^{-1}(\tau y_{i_l})$.
- If $\tau \in (\frac{H(g_2)}{y_{i_l}}, 1]$, then $\alpha_0(\tau) = \frac{g_2}{\tau y}$, $\alpha_{i_l}(\tau) = \frac{\tau y_{i_l} - H(g_2)}{\tau y}$, $\alpha_{i_A}(\tau) = 1 - \frac{g_2 + \tau y_{i_l} - H(g_2)}{\tau y}$.

Finally suppose $v_A = N$:

- If $\tau \in \left[0, \frac{g_1}{y}\right]$, then $\alpha_0(\tau) = 1$: i_A maximizes not only his, but also every one' utility, where

$$u_i = (1 - \tau) y_i + H(\tau y)\ \forall i. \tag{5}$$

- If $\tau \in (\frac{g_1}{y}, \frac{H(g_1)}{y_{i_h}}]$, then $\alpha_0(\tau) = \frac{g_1}{\tau y}$ and $\alpha_{i_A}(\tau) = 1 - \frac{g_1}{\tau y}$: Having $g(\tau) = g_1$ allows i_A to expropriate the residual budget. For future reference his utility within this range is maximized at $\tau = \frac{H(g_1)}{y_{i_h}}$ and is equal to

$$u_{i_A} = y_{i_A} + H(g_1)(2 + \frac{y_{i_l}}{y_{i_h}}) - g_1. \tag{6}$$

- If $\tau \in (\frac{H(g_1)}{y_{i_h}}, \frac{H(g_2)}{y_{i_h}}]$, then $\alpha_0(\tau) = \frac{g(\tau)}{\tau y}$ and $\alpha_{i_A}(\tau) = 1 - \frac{g(\tau)}{\tau y}$, where $g(\tau)$ is specified in the following. First, any increase in τ above $\frac{H(g_1)}{y_{i_h}}$ makes i_h strictly worse off if the tax increase is not matched by a sufficient increase in g above g_1, or by allotting f_{i_h}, or by doing both. By Lemma 1 we can restrict attention on allocations with $\alpha_{i_l}(\tau) = 0$. Then, for $\tau \in (\frac{H(g_1)}{y_{i_h}}, \frac{H(g_2)}{y_{i_h}}]$, i_A's problem is to

$$\begin{aligned}
&\max_g (1-\tau)y_{i_A} + H(g) + \tau y - g - (\tau y_{i_h} - H(g)) \\
&\text{subject to} \\
&\quad \tau y_{i_h} - H(g) \geq 0 \ (\lambda'_h) \\
&\quad \tau y \geq g + \tau y_{i_h} - H(g) \ (\theta')
\end{aligned}$$

Since the first order condition to the above problem is

$$H'(g) = \frac{1 + \theta' + \lambda'_h}{2 + \theta'},$$

Hence $g(\tau) \leq g_2$. To start with, suppose no constraint binds, that is, i_h receives compensation and i_A receives a positive residual budget ($\lambda'_h = \theta' = 0$). This would imply $g(\tau) = g_2$ and then τ has to be such that $\tau > \frac{H(g_2)}{y_{i_h}}$. So for $\tau \leq \frac{H(g_2)}{y_{i_h}}$, either one of the constraints or both of them bind, and, since i_A wants to increase his utility, it should be i_h's IRC that binds: $\tau y_{i_h} = H(g)$. I i_A sets $g = H^{-1}(\tau y_{i_h})$ (BC is not an issue by Lemma 2). To summarize, $g(\tau) = H^{-1}(\tau y_{i_h})$ for all $\tau \in (\frac{H(g_1)}{y_{i_h}}, \frac{H(g_2)}{y_{i_h}}]$. Hence for $\tau \in (\frac{H(g_1)}{y_{i_h}}, \frac{H(g_2)}{y_{i_h}}]$, i_A's utility is maximized at $\tau = \frac{H(g_2)}{y_{i_h}}$ and equal to

$$u_{i_A} = y_{i_A} + H(g_2)(2 + \frac{y_{i_l}}{y_{i_h}}) - g_2. \tag{7}$$

- If $\tau \in (\frac{H(g_2)}{y_{i_h}}, \frac{H(g_2)}{y_{i_l}}]$, then, as the maximization problem above shows, $\alpha_0(\tau) = \frac{g_2}{\tau y}$, $\alpha_{i_h}(\tau) = \frac{\tau y_{i_h} - H(g_2)}{\tau y}$and $\alpha_{i_A}(\tau) = 1 - \frac{g_2}{\tau y} - \frac{\tau y_{i_h} - H(g_2)}{\tau y}$.

- If $\tau \in (\frac{H(g_2)}{y_{i_l}}, \frac{H(g_3)}{y_{i_l}}]$, then $\alpha_0(\tau) = \frac{g(\tau)}{\tau y}$, $\alpha_{i_h}(\tau) = \frac{\tau(y_{i_h} - y_{i_l})}{\tau y}$ and $\alpha_{i_A}(\tau) = 1 - \frac{g(\tau)}{\tau y} - \frac{\tau(y_{i_h} - y_{i_l})}{\tau y}$, where $g(\tau) = H^{-1}(\tau y_{i_l})$. Once τ exceeds $\frac{H(g_2)}{y_{i_l}}$, i_A has to either increase g above g_2 or compensate i_l and i_h directly, or do both. So the two constraints that have to hold are given by $f_{i_h} \geq 0$ and $f_{i_l} \geq 0$. The corresponding decision problem is identical to the one in Lemma 1, in other words i_A is willing to increase g till g_3 before making direct transfers to i_l and as Lemma 2 shows the budget is large enough to accommodate this allocation. Figure 1 illustrates i_A's choice of public good spending.

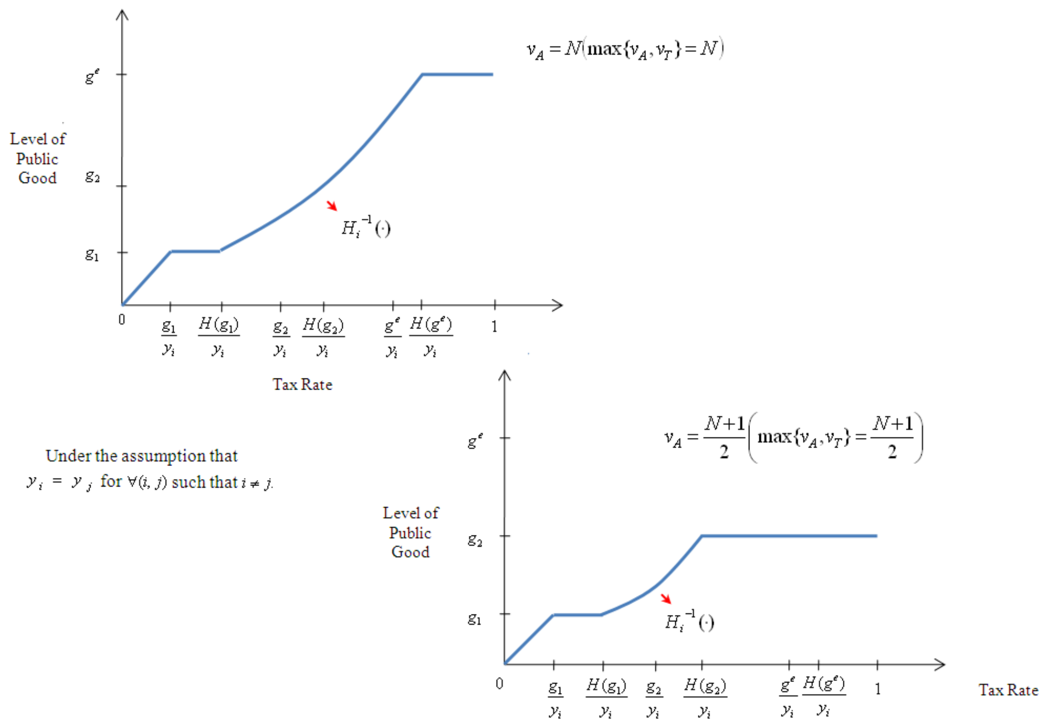

Figure 1: The level of public spending chosen by an agenda setter, given a tax rate τ.

Having characterized i_A's strategy, let us consider $i_T = i_A$ and $i_T \neq i_A$ separately.

(1) $i_T = i_A$:

$(1i)$ $v_T > v_A$:[14] Since i_a needs $v_T - v_A$ more votes to pass his tax proposal, the highest tax rate he can set is $\frac{H(g_1)}{y_{i_{v_T-1}}}$, at which i_{v_T-1}, the $(v_T-1)^{th}$ poorest other member, is indifferent. This is because i_a keeps $g(\tau)$ constant at g_1 until τ reaches $\frac{H(g_1)}{y_{i_{v_A-1}}}$, where i_{v_A-1} is the $(v_A-1)^{th}$ poorest other member in the committee and raises it above g_1 just enough to make i_{v_A-1} indifferent, leaving the members outside the coalition strictly worse off. Hence we have $(\mathbf{A})$.

$(1ii)$ $v_T \leq v_A$: i_a can set as high a tax rate as he wants since his second stage coalition is larger than his first stage coalition and there is no incentive compatibility issues. Therefore $g^* = g_{v_A}$, where $H'(g_{v_A}) = \frac{1}{v_A}$. As long as $v_A < N$, i_a sets $\tau^* = 1$. When $v_A = N$, i_a is indifferent between proposing any τ in $(\frac{H(g_N)}{y_{i_l}}, 1]$. The equilibrium tax rates are as stated because the value function to the second stage optimization problem is given by

$$u_{i_a}(\alpha^{*i_a}(\tau), \tau) \approx y_{i_a} + v_A H(g_{v_A}) - g_{v_A} + \frac{(N - v_A)}{N}\tau y \ \text{ for } \ \tau \geq \frac{H(g_{v_A})}{y_{i_l}}.$$

and (5), (6), and (7) show that when $v_A = N$ ($< N$) the value function is strictly increasing in τ (only for $\tau < \frac{H(g_{v_A})}{y_{i_l}}$). Hence $(\mathbf{B})$.[15]

(2) $i_T \neq i_A$: Given i_A's allocation strategy, each member's (except i_A's) utility is maximized at $\frac{g_1}{y}$. As mentioned in $(1i)$, for $\tau > \frac{g_1}{y}$, i_A appropriates funds at the expense of the member(s) who is left out of the coalition, compensating his coalition partner j (one of the $(v_A - 1)^{th}$ poorest other members) only for those tax rates in $(\frac{H(g_2)}{y_j}, 1]$ at which j incurs a strict income loss in the absence of direct transfers or an increase in public good production. Accordingly, any other member than i_A proposes τ_{g_1} when given the opportunity. Hence the proof. ■

[14]So far the proof has been for $v_A \in \{1, \frac{N+1}{2}, N\}$, $N = 3$. However the next stage in backward induction depends only on the relative strength of voting requirements, and so can be stated generally regardless of the size of the committee and absolute voting requirements.

[15]'$\approx$' is due to the approximation of each member's income by $\frac{y}{N}$.

B First Allocation, Second Taxation

Proposition 2 *Under* (v_A, v_T) :[16]

If $i_A \neq i_T$*, then* α_0^* *satisfies*

$$\max\{v_A, v_T\}\ H'(\alpha_0^* \tau^* y)\ +\ \alpha_0^* \tau^* y\ H''(\alpha_0^* \tau^* y) = 1,$$

and the rest of α^* *is given by:*

$$\alpha_{i_T}^* = \frac{y_{i_T}}{y} - \alpha_0^* H'(\alpha_0^* \tau^* y) \tag{8}$$

$$\alpha_i^* = \begin{cases} \frac{y_i}{y} - \frac{H(\alpha_0^* \tau^* y)}{y}, & \text{if } i \text{ is one of the } \max\{v_A, v_T\} - 1 \text{ poorest other members;} \\ 0, & \text{otherwise.} \end{cases}$$

$$\alpha_{i_A}^* = 1 - \alpha_0^* - \sum_{i \neq \{0, i_T\}} \alpha_i^*.$$

If $\max\{v_A, v_T\} = N$, τ^* *is any tax rate greater than* τ_L *that is defined through* $\alpha_{i_l}^* = 0$:

$$H(\alpha_0^* \tau_L y) = \tau_L y_{i_l}, \tag{9}$$

$\tau^* = 1$ *if* $\max\{v_A, v_T\} < N$.

If $i_A = i_T$ *and* $\max\{v_A, v_T\} = N$*, then* $\alpha_0^* = \frac{g_N}{\tau^* y}$*,* $\alpha_i^* = \frac{\tau^* y_i - H(g_N)}{\tau^* y}$ *for all* $i \neq i_a$ *and* $\alpha_{i_a}^* = 1 - \sum_{i \notin \{0, i_a\}} \alpha_i^*$ *where* $\tau^* \in \left[\frac{H(g_N)}{y_{i_l}},\ 1\right]$*;*

$\max\{v_A, v_T\} < N$*, then* $\alpha_0^* = \frac{g_{\max\{v_A, v_T\}}}{y}$*,* $\alpha_i^* = \frac{y_{i_l} - H(g_{\max\{v_A, v_T\}})}{y}$ *if* i *is one of the* $\max\{v_A, v_T\} - 1$ *poorest other members and* $\alpha_i^* = 0$ *otherwise, and* $\alpha_{i_a}^* = 1 - \sum_{i \notin \{0, i_a\}} \alpha_i^*$*, where* $\tau^* = 1$.

Proof. Assume $N = 3$, $N > 3$ follows from similar arguments. When $i_A \neq i_T$, suppose $\max\{v_A, v_T\} = N$. If, given an allocation α, i_A wants τ' to be implemented, he needs to make sure that i_T proposes it and the remaining member, say i, does not object to it:

[16] As stated before, the intuition behind this result applies to any finite size committee and any voting rule pair.

$\max_{\alpha_0,\alpha_{i_T},\alpha_i}\ (1-\alpha_0-\alpha_{i_T}-\alpha_i)\tau' y+(1-\tau')y_{i_A}+H(\alpha_0\tau' y)$
subject to

$\alpha_0 \geq 0$

$\alpha_{i_T} = \max\left\{0,\ \frac{y_{i_T}}{y} - \alpha_0 H'(\alpha_0\tau' y)\right\}$

$\alpha_i = \max\left\{0,\ \frac{y_i}{y} - \frac{H(\alpha_0\tau' y)}{\tau' y}\right\}$

The last constraint is IRC of the member, who is not a coalition partner. The second constraint is attributable to i_T's optimization problem: Given α_{i_T} and α_0, i_T picks a tax rate that maximizes his utility (since the voting requirement is already taken care of in the first stage, i_T does not have to factor it in his decision):

$$\max_{\tau\in[0,1]} \alpha_{i_T}\tau y+(1-\tau)y_{i_A}+H(\alpha_0\tau y)$$

Consequently, i_T picks τ that satisfies $\alpha_{i_T} = \frac{y_{i_T}}{y} - \alpha_0 H'(\alpha_0\tau y)$ if $\tau \in [0,1]$ or picks $\tau = 1$ if $\alpha_{i_T} > \frac{y_{i_T}}{y} - \alpha_0 H'(\alpha_0 y)$. Therefore i_A chooses an allocation such that his utility is maximized.

Solving i_A's problem under the assumption that its solution satisfies one of the following (and hence exhausting all the possibilities) $(a)\ \alpha_{i_T} = \alpha_i = 0,\ (b)\ \alpha_{i_T} > 0 = \alpha_i,\ (c)\ \alpha_{i_T} > \alpha_i > 0$[17], and then comparing the corresponding value functions reveal that i_A's and i_T's proposals fall in category (b), and are characterized by (8) and (9), respectively.[18]

Suppose next $\max\{v_A,\ v_T\} = \frac{N+1}{2}$. Going through a similar analysis by dropping the third constraint proves that the above allocations evaluated at $\tau = 1$ maximize i_A's utility and induce i_T to pick $\tau^* = 1$.

When $i_A = i_T$, i_a's optimization problem is equivalent to choosing an optimal budget given $\max\{v_A, v_T\}$ as the voting constraint. In this case, the order in which he proposes the components of the budget is irrelevant to the budgetary outcome, hence his allocation and tax proposals are identical to those under $(v_T,\ v_A)$ when $v_A \geq v_T$. ■

[17]Note that $\alpha_{i_0} \leq \alpha_{i_T}$ since income differences are marginally small.

[18]Notice that g^* is at least as large as g_1 since the latter is what i_A would have chosen if he faced no voting constraints. As mentioned several times before, higher public spending than $g_1$is to build the least expensive coalition. Therefore his utility increases beyond y_{i_A} by more than $\frac{H(g_1)}{y_{i_0}}y - g_1$.

C Simultaneous Budget Procedure

If $i_A \neq i_T$, then the equilibrium is characterized by $\tau^* \in \arg\max P_{i_T}(\alpha^*)$ and $\alpha^* \in \arg\max P_{i_A}(\tau^*)$, where $P_{i_T}(\alpha^*)$ and $P_{i_A}(\tau^*)$ are given below in the same order:

$$\begin{aligned} & \max\nolimits_{\tau\in[0,1]} u_{i_T}((\alpha^*)^{i_T}, \tau) \\ & \text{subject to} \\ & \quad \left|\left\{i \in I : u_i((\alpha^*)^{i_T}, \tau) \geq y\right\}\right| \geq \max\{v_A, v_T\} \\ & \quad \alpha^* \in \arg\max P_{i_A}(\tau^*), \end{aligned} \tag{10}$$

and

$$\begin{aligned} & \max\nolimits_{\alpha} u_{i_A}(\alpha^{i_A}, \tau^*) \\ & \text{subject to} \\ & \quad \textstyle\sum_i \alpha_i \leq 1, \quad \alpha_i \geq 0 \; \forall i \\ & \quad \left|\left\{i \in I : u_i(\alpha^{i_T}, \tau^*)) \geq y_i\right\}\right| \geq \max\{v_A, v_T\} \\ & \quad \tau^* \in \arg\max P_{i_T}(\alpha^*). \end{aligned} \tag{11}$$

Proposition 3 $i_T \neq i_A$: *The equilibrium tax rate and allocations are* $\tau^* = \frac{g_1}{y_{i_T}}$ *and* $\alpha_0^* = \frac{y_{i_T}}{y}$, $\alpha_{i_A}^* = \frac{y-y_{i_T}}{y}$ *and* $\alpha_i^* = 0$ *for* $i \neq i_A$ *,and all* v_A *and* v_T *and for all* N.

$i_T = i_A$: *The simultaneous procedure is outcome equivalent to* (v_A, v_T) *and to* (v_T, v_A) *when* $v_T \leq v_A$.

Proof. $i_T \neq i_A$: Solving (10) and (11) shows that the unique equilibrium consists of an allocation and tax pair, α^* and τ^*, that satisfies:

$$H'(\alpha_0^*\tau^*y) = 1 = \frac{y_{i_T}}{\alpha_0^* y},$$

regardless of max$\{v_A, v_T\}$.

A point of clarification is that when max$\{v_A, v_T\} < N$ and i_T is not one of the poorest max$\{v_A, v_T\} - 1$ other members, then it can never be the case that $\tau^* > \frac{H(g_1)}{y_{i_T}}$ (since i_A never compensates i_T). When max$\{v_A, v_T\} < N$ and i_T is one of the poorest

$\max\{v_A, v_T\}-1$ other members, or when $\max\{v_A, v_T\} = N$, there exists no other allocation and tax pair that satisfies $H'(\alpha_0 \tau y) = \frac{y_{i_T}}{\alpha_0 y} - \frac{\alpha_{i_T}}{\alpha_0} < 1$ (hence public good production cannot exceed g_1), where α_{i_T} =max$\left\{0,\ \frac{\tau y_{i_T} - H(\alpha_0 \tau y)}{\tau y}\right\}$ (implying that i_T and i_A's best response functions intersect only once).

$i_T = i_A$: As explained in Proposition 2, equilibrium budget under a single agenda setter is identical to those of $(v_A,\ v_T)$ and $(v_T,\ v_A)$ when $v_T \leq v_A$, since the order of allocation and tax proposals is immaterial once there is a fixed proposer. ■