Equilibrium Programming and Leader-Follower Games.

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This talk pertains to my research on equilibrium programming motivated by leader-follower games. In the first part of the talk, I use the static moral hazard problem as a motivating example to introduce the single-leader follower(s) game. To compute a solution to the moral hazard problem, economists use a lottery approach that leads to solving a sequence of linear programs. The advantage is that both the principal and the agent are making globally optimal choices. The disadvantages are: (1) one must discretize the economic choice variables that are more naturally assumed to be continuous; (2) the size of the resulting linear programs often grows enormously and quickly when an accurate solution is needed; (3) the curse of dimensionality makes this approach infeasible in many multidimensional problems.

I show that the static deterministic contract can be formulated and solved as a mathematical equilibrium with equilibrium constraints (MPEC). The advantages of using an MPEC formulation are: (1) the size of resulting mathematical program is often orders of magnitude smaller than linear programs derived from the LP lottery approach; (2) one does not need to discretize the choice variables pertaining to compensations. To avoid local solutions for the MPEC formulation, I propose a hybrid procedure that combines both the LP lottery and the MPEC approaches. This procedure obtains a solution that is, if not global, at least as good as an LP lottery solution. The numerical efficiency of the hybrid approach in both computational speed and solution quality (in term of the optimal objective value) is illustrated through an numerical example.

The first part is a joint work with Kenneth L. Judd (Hoover Institution and NBER).

In the second part of the talk, I discuss my work on equilibrium problems with equilibrium constraints (EPECs), which are modeling tools for multi-leader follower(s) games. One can view an EPEC as a mathematical program for finding a pure-strategy Nash equilibrium of a multi-leader follower(s) game, in which each leader is solving a nonconvex MPEC. Two immediate research questions are: (1) the existence of an EPEC solution; (2) developing robust algorithms for solving an EPEC. Numerical approaches used by researchers in engineering fields to solve EPEC models fall into the category of the best reply iteration, a cyclic procedure that solves the MPECs one at a time. One main drawback with this procedure is that it may fail to find an EPEC solution even if one exists. I propose a sequential nonlinear complementarity (SNCP) algorithm for solving EPECs and establish the convergence of this algorithm. While the existence of a solution for a general EPEC is still an open question, I show that the existence result can be established for the Allaz-Vila spot-forward market model and Saloner's two-stage Cournot production game with asymmetric producers, given linear demand and linear cost functions. I demonstrate the use of the SNCP method to Saloner's model with three producers and discuss the numerical results.