Discussion Paper No. 996

An Analysis of Post-Product Development Market Research and its Effects on Firms and Consumers

by

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August 1992

Acknowledgements: The author greatly benefitted from numerous discussions with Ron Braeutigam, Bill Rogerson, and especially Kyle Bagwell. Comments and suggestions from the members of the Graduate Student Seminar were also extremely helpful. Any errors that remain are, of course, solely the author's responsibility.

Abstract

This paper examines a model of post-product development market research that allows firms to improve their information about demand curves. The process does not allow perfect information at finite expenditure levels. The model examines oligopolistic behavior and social welfare in the heterogeneous product world.

The basic results derived are that investment of this type bears no externality to other firms in the market and a negative externality to consumers in the market from the improved information of the firm. These results are robust across several extensions which are presented in the paper.

Conclusions for policy toward firms engaging in post-product market research are drawn for several different international market scenarios.

I. INTRODUCTION

It is common practice in some markets for firms to invest in market research to improve their expected profitability. This market research occurs at several stages of the product development process. At the most basic level of differentiation, a firm can invest in market research before or after their product has been developed. Firms are observed carrying out both types of market research. Market research, performed after a product is developed, is evidence that firms are uncertain about demand curves. This paper proposes a model of post-product development market research that allows firms to discover market demand through research. The effects of market research expenditures will be examined. The analysis will include both an oligopolistic examination of market behavior and an examination of social welfare.

This model will allow a determination of social welfare benefits, or losses, from this investment in the discovery of market parameters. Through such an analysis, it will be possible to determine whether free market levels of investment of this type are optimal, sub-optimal, or supra-optimal. In order to resolve this issue, two externalities must be examined as the free market level of investment[1], though optimal for the firm, may affect both consumers and the other firms.

It is shown that firms do not care about the investment levels of other firms. Thus, there is no externality to other firms from this investment. The reason for this is that a firm's investment cannot actually affect demand. So, ex ante, the expected result is unaffected by the level of investment. It is also shown that there is a negative externality to the consumer. The negative externality is due to the ability of the firm to better match demand with price after investment. Without market research there will be "bargains" and "rip-offs" and after investment the probability of facing these is reduced. The reduction of this probability reduces the expected surplus of the neoclassical consumer.

The early literature dealing with information gathering concerned itself mainly with the concept of information pooling. These papers are divided into two groups: cost information sharing and demand information sharing.

Li (1985), Shapiro (1986), Gal-or (1986), and Li, McKelvey, and Page (1987) are concerned with

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[1]From here onward "investment" will refer to post-product development market research expenditures.

the issue of cost information sharing. Vives (1984), Gal-or (1985), and Novshek and Sonnenschein (1982) focus on the issue of demand information sharing. Novshek and Sonnenschein look at the welfare effects of increasing the number of signals about demand that a firm receives. The welfare conclusions of their paper are contrasted with the results of this model.

This paper does not concern itself with the issue of pooling information. The goal is to examine a single firm's actions and determine whether they are socially optimal. All of the papers mentioned above have firms receiving signals with a fixed level of noise.[2] The model presented here allows the firm to affect the noise level of their signal by investing more money in the process (to be defined later). This allows for a continuous decision space rather than the (0,1) decision to pool or not to pool.[3] Also, this model allows the firms to have separate demand intercepts rather than the common demand intercept assumption of the papers previously mentioned. The model will also lead to policy conclusions regarding international industries. This will allow for an analysis of information gathering subsidies or taxes under various scenarios.

The paper is organized into six sections. Section II introduces a model of post-product development market research that allows for improvement in the information that a firm possesses about the market, without granting the firm full knowledge of the environment. This is contrasted with the perfect information equivalent. Section III solves the game for a two firm equilibrium. The behavioral analysis leads to the result that firms do not care about the investment levels of other firms, either ex ante or ex post. Section IV deals with the issue of social welfare. A welfare analysis is carried out utilizing a general form for the welfare function. The effect of this type of investment on consumers is examined. It is found that consumers are hurt by investment in market research. This leads directly to the conclusion that the equilibrium levels of investment are supra-optimal.

Section V extends the model in three ways. First, the firm is allowed to invest in research about

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[2]Gal-or allows firms to disguise their signal with a variable level of noise, but the original signal received contains a fixed level of noise. Novshek and Sonnenschein (N&S) allow the firm to purchase a number of signals of the same noise level.

[3]N&S allow a discrete decision set in the number of signals as well as the (0,1) decision to pool.

model in which firms invest sequentially is examined. It is shown that none of these modifications affect the earlier results.

Section VI discusses the findings in the context of international industries. It is shown that the findings transfer to these circumstances as well. The conclusions of this paper are also presented in this section. An appendix contains several long equations and some proofs that are not given within the text of the paper.

II. THE MODEL

The model developed here will involve a duopoly competing in a differentiated product market. Extensions to more general environments are discussed later. Each firm is assumed to have previously developed a product and incurred any fixed cost associated with development or production. There is no uncertainty concerning firms' cost functions or the number of competitors. The firms also know that they face a common representative consumer in a single time period game.[4]

The game has two stages:

Stage I : Each firm privately chooses an investment level and receives a signal, s_i, about its demand.

Stage II: Firms observe competitors' investment levels, and then chose prices simultaneously.

In order to solve for equilibrium behavior, both the consumer's preferences and the market research process must be described. I begin with the introduction of preferences and proceed to describe the process by which firms receive information about the unknown parameters.

[4]There exist forms of multiple period games that reduce to a repetition of the game developed here. In such games the results in each period are identical to those presented in the single period context.

Preferences

The preferences of the representative consumer are represented by a quadratic utility model similar to that of Shubik and Levitan (1980) and Bagwell and Staiger (1990). The consumer gets utility from a numeraire good and the good sold in the market being examined. The utility function has the following form:

$$U = M + \frac{\alpha_i q_i}{\beta} + \frac{\alpha_j q_j}{\beta} - \frac{(q_i + q_j)^2}{2\beta} - \frac{(q_i - q_j)^2}{2\beta(1+\gamma)}$$

where:

M is quantity of numeraire good consumed.

α_i is a "perceived quality" variable that affects the demand intercept of firm i.

q_i is the amount of good i purchased.

β is the negative of the slope of the firm's demand curve.[5]

γ is the degree of product differentiation inherent in the market.[6]

This utility function is maximized subject to the budget constraint:

$$M + p_i q_i + p_j q_j = Y$$

After substituting for M, the indirect utility function can be defined as :

$$U^* = \max_{q_i, q_j} Y + \frac{\alpha_i q_i}{\beta} + \frac{\alpha_j q_j}{\beta} - \frac{(q_i + q_j)^2}{2\beta} - \frac{(q_i - q_j)^2}{2\beta(1+\gamma)} - p_i q_i - p_j q_j$$

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[5]Obviously a generalization of this would be to allow each firm to have a different β. The results herein are not sensitive to changes of β.

[6]When $\gamma = 0$ the products are unrelated and each firm is a monopolist. When $\gamma = \infty$, the products are perfectly homogeneous and the Bertrand result is obtained.

It is this representation of the consumer's indirect utility function that is used. In the context of this model it will be α_i and α_j that are unknown to the firm ex ante. The consumer, of course, knows the value of the parameters in its utility function.

Perfect Information Case

In the case of perfect information, where the firms have a priori knowledge of α_i and α_j, the firms know the consumer's First Order Condition and Nash prices can be found directly. The demand for good i is:

$$q_i = \frac{2+\gamma}{4}\alpha_i - \frac{\beta(2+\gamma)}{4}p_i - \frac{\gamma}{4}\alpha_j + \frac{\beta\gamma}{4}p_j$$

In order to solve for Nash prices the profit function must be defined. In this case, since the fixed costs have already been incurred, a variable profit function is appropriate.[7] An assumption maintained throughout this paper is that there are constant returns to scale in the variable production process. Thus the profit function can be represented as: $\Pi_i = (p_i - c) * q_i$. Allowing the firm to maximize profits, the Nash price is defined by:

$$p_i = \frac{2(2+\gamma)^2 - \gamma^2}{\beta(4+\gamma)(4+3\gamma)}\alpha_i + \frac{2(2+\gamma)^2 + \gamma(2+\gamma)}{(4+\gamma)(4+3\gamma)}c - \frac{\gamma(2+\gamma)}{\beta(4+\gamma)(4+3\gamma)}\alpha_j$$

Imperfect Information in Market Parameters

The assumption that firms can ex ante observe α_i and α_j will now be relaxed. The firm has inherited α_i from its product development process. The firm knows which market the good is sold in but not the exact value of α_i. Since both the first derivatives of U^* and q_i with respect to α_i are positive, α_i can be interpreted as a product quality parameter. It is important to note that the parameter registers quality

[7] Later the cost of investing in market research will enter the profit function as a fixed cost.

components that are not strictly predictable. For example, should a manufacturer of basketball shoes expect David Robinson or Clyde Drexler to be the more popular signee? There will be a difference in demand for these shoes, and this is represented by the value of α_i.

α_i is drawn from a distribution of possible product parameters, G. G is defined by its mean, $\bar{\alpha}$, nd its variance, V. Since firm i and firm j are selling closely related products, α_i and α_j are both drawn from G. Thus, ex ante, $E_i[\alpha_i] = E_i[\alpha_j] = E_j[\alpha_j] = E_j[\alpha_i] = \bar{\alpha}$.

Investment in Market Parameters

Without a market research technology, the firms would have to be content with their existing beliefs about relative product quality. Firm i, however, would like to know the actual value of α_i. While the market research technology proposed can provide information about the market parameters, it is limited in that it cannot provide the actual parameters themselves at a finite cost to the firm. The type of information the market research technology makes available to the firms is a signal, s_i, about the true parameter α_i. In order to describe the process that this market research technology determines, four distributional assumptions are made.

First, it is assumed that s_i is drawn from a distribution F, with mean α_i. Thus s_i is an unbiased estimator of α_i. This will be true at <u>any</u> level of investment. In order for the signal to be meaningful to the firm, all that is needed is that the bias of the signal be known. It is most convenient analytically for this known bias to be zero. In order to keep the firm from receiving perfect information, the variance of the signal must, in general, be greater than zero. It is also desirable for the firm to be able to choose its level of investment and for this to affect the noise of the signal.

The signal is always unbiased so the level of investment in market research, denoted I_i, does not affect the mean of F. The variance of F, denoted $V(I_i)$, will vary with I_i. The greater the level of investment; the lower the noise level in s_i. For one of the proofs, a slightly stronger condition is needed and thus, the second assumption is that an increase in I_i induces a mean-preserving reduction in F. A weaker assumption, $V'(I_i) < 0$, would suffice for all but the proof of Proposition 4.

If it is also desired that the technology yield smooth returns to investment, so that s_i is not always "bad" until I_i reaches a certain level, and is then always "good", the next two assumptions are sufficient to guarantee a degree of "smoothness" in the choice of I_i.

Without any investment at all, there is no signal. To approximate this at very low levels of investment, the third assumption is $\lim_{I_i \to 0} V(I_i) = \infty$. If a firm could spend an infinite amount on market research, then they should receive α_i as their signal. Thus, the fourth assumption is that $\lim_{I_i \to \infty} V(I_i) = 0$.

Our assumptions about the market research process can be summarized by the following: s_i is drawn from a distribution, F, with mean α_i and variance $V(I_i)$; $V(I_i)$ is differentiable and strictly decreasing in I_i. So, s_i improves smoothly with the level of investment and allows the firm to form a more precise expectation of α_i.

The firm is concerned with the term $E_i[\alpha_i \mid s_i, I_i]$. As long as F and G are members of a certain set of conjugate families,[8] the expectation is linear in s_i. The results of this paper are derived, partially, based upon this distributional property. Staying within this set of conjugate families, the expectation of α_i, after investing in the signal, is formed as follows:

$$E_i[\alpha_i \mid s_i, I_i] = \left(\frac{K}{K+K(I_i)}\right)\bar{\alpha} + \left(\frac{K(I_i)}{K+K(I_i)}\right)s_i$$

where $K = 1/V$ and $K(I_i) = 1/V(I_i)$, so K and $K(I_i)$ are the precisions of G and F respectively. Note that despite coming from the same distribution, s_i and s_j are independent signals. They are signals about an uncommon intercept so this implies that $(s_i - \alpha_i)$ is uncorrelated with $(s_j - \alpha_j)$.

III. EQUILIBRIUM BEHAVIOR

This new problem facing the firms is significantly different from the perfect information case. Since the problem facing firm i is identical to that facing firm j we will focus on a symmetric equilibrium.

[8]This is satisfied if both are Normal; or if one is Beta and the other Binomial; or if one is Gamma and the other Poisson.

however, observed prices and quantities will generally differ due to the realizations of α_i, α_j, s_i, and s_j. The firms have already inherited their products, and correspondingly α_i, from some product development process.[9] Both firms know $\bar{\alpha}$, V, and the functions $U^{*}(\bullet)$ and $V(\bullet)$.

Beginning with Stage II the firm's problem can be solved. Taking first order conditions from $E_i[U^{*}]$ and solving yields firm i's expectation of equilibrium demand:

$$E_i[q_i]=(\frac{2+\gamma}{4})[(\frac{K}{K+K(I_i)})\bar{\alpha} + (\frac{K(I_i)}{K+K(I_i)})s_i] - \frac{\beta(2+\gamma)}{4}p_i - \frac{\gamma}{4}\bar{\alpha} + \frac{\beta\gamma}{4}E_i[p_j]$$

Note that $E_i[\alpha_j] = \bar{\alpha}$. This is because there is no information available to firm i about firm j's product. This assumption will be relaxed later, in an extension. Substituting this demand into the profit function from above and setting $\frac{dE_i[\pi_i]}{dp_i} = 0$, the price reaction curve of firm i is found to be:

$$p_i = \frac{2K - \gamma K(I_i)}{2\beta(2+\gamma)(K+K(I_i))}\bar{\alpha} + \frac{K(I_i)}{2\beta(K+K(I_i))}s_i + \frac{c}{2} + \frac{\gamma}{2(2+\gamma)}E_i[p_j]$$

where:

$$E_i[p_j] = \frac{1}{\beta(2+\gamma)}\bar{\alpha} + \frac{c}{2} + \frac{\gamma}{2(2+\gamma)}E_i[E_j[p_i]].$$

This equation for $E_i[p_j]$ can be solved by recognizing that in a symmetric equilibrium $E_i[p_j] = E_i[E_j[p_i]]$. Solving the equation above yields: $E_i[p_j] = \dfrac{2(\frac{\bar{\alpha}}{\beta}) + c(2+\gamma)}{4+\gamma}$. Substituting this into firm i's price reaction function, the Nash price is easily found to be:

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[9]The product development process was originally the focus of this model. It is taken as exogenous for this phase of market research, but there is clearly a need to model pre-product development market research. Such a model incorporates many of the characteristics present in this paper along with the ability to change product "location". Such a model would also contain this post-product development market research technology as a possible strategy for the firm.

$$p_i = \frac{2K - \gamma K(I_i)}{2\beta(2+\gamma)(K+K(I_i))}\bar{\alpha} + \frac{K(I_i)}{2\beta(K+K(I_i))}s_i + \frac{c}{2} + \frac{2(\frac{\bar{\alpha}}{\beta}) + c(2+\gamma)}{4+\gamma}$$

Proposition 1: $\frac{dp_i}{dI_j} = \frac{dI_i}{dI_j} = 0$. Both the price and investment reaction curves of firm i are independent of the investment level of firm j.

Proof: This follows directly from the fact that I_j does not enter the equations for either q_i or p_i. The investment by firm j, as shown above does not affect firm i's expectations. Because s_j is unbiased at any level of investment the actual level of I_j is not important to firm i.

Proposition 2: $E_i[p_i - c]$ and $E_i[q_i]$, the expected price markup and the expected quantity are independent of I_i.

Proof: The only way that investment enters the expected price markup or the expected demand is through the expectation of α_i. Examining this term

$$E_i[(\frac{K}{K+K(I_i)})\bar{\alpha} + (\frac{K(I_i)}{K+K(I_i)})s_i] = (\frac{K}{K+K(I_i)})\bar{\alpha} + (\frac{K(I_i)}{K+K(I_i)})E_i[s_i] = (\frac{K+K(I_i)}{K+K(I_i)})\bar{\alpha} = \bar{\alpha}$$

So, I_i does not enter either of these terms since it falls out of the expectation of α_i. Since investment is not expected to change a firm's expectation of α_i the ex ante expected price markup and demand are unaffected by the level of I_i. Note that profits contain the second moment of F and thus are a function of I_i.

IV. THE SOCIAL PLANNER'S PROBLEM

Now that the firm has optimally chosen I_i, we can look to the consumer. It is convenient to change the definition of the profit function to include the cost of investment.[10] The representation of the profit function will now be as follows:

$\pi_i = (p_i - c)q_i - I_i$. Proposition 1 leads directly to a corollary that is important in the social welfare analysis.

Corollary 1: Assuming that the two firms do not share information about demand, $\hat{I}_i$, and $\hat{I}_j$ (where $\hat{I}_i$ is firm i's equilibrium level of investment) are the joint profit maximizing levels of investment.

Proof: This is a direct result of Proposition 1 and profit maximization by firm i. Formally,

$$\frac{d(\pi_i + \pi_j)}{dI_i'} |_{\hat{I}_i} = \frac{d\pi_i}{dI_i} + \frac{d\pi_j}{dI_i} |_{\hat{I}_i} = 0 + 0 = 0.$$

As long as joint profit maximization does not imply the sharing of signals, there is no externality to firm j from firm i's investment.

Welfare Function

Even in the absence of firm level externalities there can be externalities to consumers from the investment. To evaluate the expected welfare effects of the investment, an extremely general form of a welfare function is utilized,

$$W = \omega(E[\pi_i], E[\pi_j], E[U^*])$$

The assumptions about ω are kept to an absolute minimum. It is assumed only that ω is increasing in all of its arguments. With this assumption, the following result is obtained:

Proposition 3: At the equilibrium levels of investment, the sign of dW/dI_i is the same as the sign of $dE[U^*]/dI_i$.

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[10] The assumption that market research does not affect demand allows us to discard $C(I_i)$ for the previous sections and then bring it back for this section. The consumer's decision is not swayed by I_i.

Proof: This is easily shown using the chain rule:

$$\frac{dW}{dI_i} \equiv \frac{\partial\omega}{\partial E[\pi_i]}\frac{dE[\pi_i]}{dI_i} + \frac{\partial\omega}{\partial E[\pi_j]}\frac{dE[\pi_j]}{dI_i} + \frac{\partial\omega}{\partial E[U^*]}\frac{dE[U^*]}{dI_i}$$

As shown in the proof to Corollary 1, the first two terms reduce to 0 at $\hat{I}_i$. So the original identity reduces to $\frac{dW}{dI_i} = \frac{\partial\omega}{\partial E[U^*]}\frac{dE[U^*]}{dI_i}$. Since ω is increasing in all of its arguments, the sign of dW/dI_i is the same as the sign of $dE[U^*]/dI_i$.

Denoting the social welfare maximizing level of market research I^*, it can be determined whether $\hat{I}_i$ is >, =, < I^* by finding the value of $dE[U^*]/dI_i$.[11]

Investment and Expected Utility

In order to examine the term $dE[U^*]/dI_i$, a formal description of $E[U^*]$ is required.

$$E[U^*] = \int_{\alpha_j}\int_{s_j}\int_{\alpha_i}\int_{s_i} U^* f(s_i \mid I_i, \alpha_i)\, g(\alpha_i)\, f(s_j \mid I_j, \alpha_j)\, g(\alpha_j) ds_i\, d\alpha_i\, ds_j\, d\alpha_j$$

Taking the derivative with respect to I_i, the following expression is obtained:

$$\frac{dE[U^*]}{dI_i} = \int_{\alpha_j} g(\alpha_j) \int_{s_j} f(s_j \mid I_j, \alpha_j) \int_{\alpha_i} g(\alpha_i) \int_{s_i} [U^* \frac{df(s_i \mid I_i, \alpha_i)}{dI_i} + f(s_i \mid I_i, \alpha_i)\frac{dU^*}{dI_i}] ds_i\, d\alpha_i\, ds_j\, d\alpha_j$$

It is convenient to look at the two terms of this expression separately. The first term of the expression

$$\int_{\alpha_j} g(\alpha_j) \int_{s_j} f(s_j \mid I_j, \alpha_j) \int_{\alpha_i} g(\alpha_i) \int_{s_i} U^* \frac{df(s_i \mid I_i, \alpha_i)}{dI_i} ds_i\, d\alpha_i\, ds_j\, d\alpha_j$$

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[11] $C(I_i)$ can be made as convex as is necessary to guarantee that profits are concave in I_i. Also, though closed-form solutions to the problem in this paper are difficult to obtain due to the number of parameters, a grid search can produce reasonable levels of parameters that lead to positive investment levels. Therefore, the existence of an interior is assumed for the analysis in this section.

is termed the risk effect, the effect of I_i on utility strictly through the change exacted on the distribution F. The changing of the distribution represents the lessening of the risk, through investment, on the part of the firm.

The second term of the expression

$$\int_{\alpha_j} g(\alpha_j) \int_{s_j} f(s_j \mid I_j, \alpha_j) \int_{\alpha_i} g(\alpha_i) \int_{s_i} f(s_i \mid I_i, \alpha_i) \frac{dU^*}{dI_i} ds_i \, d\alpha_i \, ds_j \, d\alpha_j$$

is termed the matching effect. This term picks up the effect investment has on utility through price changes alone. The firm is better able to anticipate the actual demand and thus prices higher in high demand markets and lower in low demand markets.

Proposition 4: The risk effect is negative.

Proof: Assuming that F is differentiable with respect to I_i, $V'(I_i) < 0$, and that I_i induces a mean preserving reduction in F this proof is straightforward. Integrating by parts:

$$\int_{\alpha_j} g(\alpha_j) \int_{\alpha_j} f(s_j \mid I_j, \alpha_j) \int_{\alpha_i} g(\alpha_i) \int_{s_i} U^* \frac{df(s_i \mid I_i, \alpha_i)}{dI_i} ds_i \, d\alpha_i \, ds_j \, d\alpha_j$$

$$=$$

$$\int_{\alpha_j} g(\alpha_j) \int_{s_j} f(s_j \mid I_j, \alpha_j) \int_{\alpha_i} g(\alpha_i) \int_{s_i} [U^* \frac{dF(s_i \mid I_i, \alpha_i)}{dI_i} \Big|_{\underline{s_i}}^{\overline{s_i}} - \frac{dF(s_i \mid I_i, \alpha_i)}{dI_i} \frac{\partial U^*}{\partial s_i}] \, ds_i \, d\alpha_i \, ds_j \, d\alpha_j$$

$$=$$

$$- \int_{\alpha_j} g(\alpha_j) \int_{s_j} f(s_j \mid I_j, \alpha_j) \int_{\alpha_i} g(\alpha_i) \int_{s_i} \frac{dF(s_i \mid I_i, \alpha_i)}{dI_i} ds_i \, d\alpha_i \, ds_j \, d\alpha_j$$

Note that $\int_{\underline{s_i}}^{\overline{s_i}} \frac{dF(z \mid I_i, \alpha_i)}{dI_i} dz = 0$. A proof of this is given in the appendix. Intuitively, at the upper and lower bounds of integration the cumulative equals 0 and 1, respectively, no matter what the level of investment. Integrating by parts once more:

$$- \int_{\alpha_j} g(\alpha_j) \int_{s_j} f(s_j \mid I_j,\alpha_j) \int_{\alpha_i} g(\alpha_i) \int_{s_i} \frac{dF(s_i \mid I_i,\alpha_i)}{dI_i} \frac{dU^*}{ds_i} ds_i \, d\alpha_i \, ds_j \, d\alpha_j$$

$$=$$

$$\int_{\alpha_j} g(\alpha_j) \int_{s_j} f(s_j \mid I_j,\alpha_j) \int_{\alpha_i} g(\alpha_i) \int_{s_i}$$

$$=$$

$$\int_{\alpha_j} g(\alpha_j) \int_{s_j} f(s_j \mid I_j,\alpha_j) \int_{\alpha_i} g(\alpha_i) \int_{s_i} \frac{\partial^2 U^*}{\partial s_i \partial s_i} \frac{dF(s_i \mid I_i,\alpha_i)}{dI_i} ds_i \, d\alpha_i \, ds_j \, d\alpha_j$$

Examining this expression we find that two terms must be signed: $\int_{\underline{s_i}}^{s_i} \frac{dF(z \mid I_i,\alpha_i)}{dI_i} dz$ and $\frac{\partial^2 U^*}{\partial s_i \partial s_i}$. We can

see that $\int_{\underline{s_i}}^{s_i} \frac{dF(z \mid I_i,\alpha_i)}{dI_i} dz$ is negative from the properties of a mean preserving reduction. A diagram

depicting this is shown in the appendix. Now, s_i enters utility only through the price so we can use the convexity of U^* in p_i and the linearity of p_i in s_i to show the fo..owing:

$\frac{\partial^2 U^*}{\partial s_i \partial s_i} = \frac{\partial(\frac{\partial U^*}{\partial p_i}\frac{dp_i}{ds_i})}{\partial p_i} \frac{dp_i}{ds_i} = \frac{\partial(k\frac{\partial U^*}{\partial p_i})}{\partial p_i} k = k^2 \frac{\partial^2 U^*}{\partial p_i \partial p_i}$ where k is the value of $\frac{dp_i}{ds_i}$, so the entire risk effect is negative.

Now attention can be turned to the matching effect. The following proposition allows us to unambiguously determine the welfare effects of post-product development market research.

Proposition 5: The matching effect is negative.[12]

Proof: Evaluating the matching effect expression, it is found that:

[12]The analysis of the matching effect here, and later in the paper, assumes the absence of income effects.

$$\int_{\alpha_j} g(\alpha_j)\int_{s_j} f(s_j \mid I_j,\alpha_j)\int_{\alpha_i} g(\alpha_i)\int_{s_j} f(s_i \mid I_i,\alpha_i)\frac{dU^*}{dI_i}\, ds_i\, d\alpha_i\, ds_j\, d\alpha_j$$
$$=$$
$$-\left(\frac{2+\gamma}{16\beta}\right)\left(\frac{K\,K(I_i)K'(I_i)}{(K+K(I_i))^3}\right)(V+V(I_i))$$

γ, β, K, V, $K(I_i)$, $V(I_i)$, and $K(I_i)$ are all positive. $K'(I_i)$ is positive as investment induces a mean preserving reduction. Equivalently, $K'(I_i) = \dfrac{-V'(I_i)}{V(I_i)^2}$ and $V'(I_i) < 0$ by the definition of market research. So, the matching effect is negative.

Barring income effects, the neoclassical consumer is risk loving in prices and thus this last result seems a bit counter-intuitive. However, a price equal to $p(\bar{\alpha})$ that does not change when demand shifts, is actually a "changing" price. When demand (α_i) is high, the consumer gets a bargain and when demand is low, the consumer gets "ripped-off". The consumer wishes to gamble over these prices rather than get a "right" price every time. The consumer is aided more by the bargain than hurt by the "rip-off". Investment in market research leads to the price changing for every different demand and reduces the number of bargains and "rip-offs" to be found.

Proposition 6: Evaluated at the equilibrium investment levels, $\frac{dW}{dI_i} < 0$, and thus $\hat{I}_i > I^*$.

Proof: This is a direct result of Corollary 1 and Propositions 5 and 6. The sign of $\frac{dW}{dI_i}$ is the same as the sign of $\frac{dE[U^*]}{dI_i}$ which is negative because both terms of this expression are negative.

This result is not the same as that obtained by Novshek and Sonnenschein. They find that the total welfare is increasing in the number of signals so long as the cost of a signal is zero. The model that Novshek and Sonnenschein find this result has two Cournot competitors in a homogeneous product market, whereas this model concerns itself with heterogeneous products. It should also be pointed out that the welfare function of Novshek and Sonnenschein is the sum of consumer surplus and profits. Their result may not generalize to other welfare functions even in the single demand intercept world.

V. EXTENSIONS

To examine the robustness of these results, several extensions are considered. The extensions examined are investment in the other firm's parameters, expansion to a general n-firm scenario, and a note about sequential games. Each of the extensions provides insight into a slightly different structural scenario, and the model and its results prove robust.

Investment in other firms' parameters

It is possible that the firm will choose to invest in determining the other firm's demand as well as its own, since this also affects profits. In other words it may be beneficial for firm i to invest in discovering s_j. This will have different effects on the consumer than did investment in s_i. It would be expected, however, that there would still be a negative externality to the consumer. The problem changes only slightly as s_i and s_j enter profits and utility similarly.

Propositions 1 and 2 still hold because I, which denotes investment in both parameters, still only enters profits through the second moment of the distribution. Propositions 3 and 4 are not changed. The derivative of the social welfare function becomes:

$$\frac{dW}{dI} = \frac{\partial\omega}{\partial E[\pi_i]}\frac{dE[\pi_i]}{dI} + \frac{\partial\omega}{\partial dE[\pi_j]}\frac{dE[\pi_j]}{dI} + \frac{\partial\omega}{\partial E[U^*]}\frac{dE[U^*]}{dI}$$

Proposition 7: Investment in other firms' market parameters is supra-optimal.

Proof: Proposition 4 still holds so the risk effect is negative. After some algebra it is seen that the matching effect can be written as:

$$\int_{\alpha_j} g(\alpha_j)\int_{s_j} f(s_j \mid I,\alpha_j)\int_{\alpha_i} g(\alpha_i)\int_{s_j} f(s_i \mid I,\alpha_i)\frac{dU^*}{dI}\, ds_i\, d\alpha_i\, ds_j\, d\alpha_j$$
$$=$$
$$-\left(\frac{\gamma}{16\beta}\right)\left(\frac{K\,K(I)K'(I)}{((K+K(I))^3}\right)(V+V(I))$$

By the same analysis used in the proof of Proposition 5, this term is negative and thus Proposition 6 holds as well. Investment in market research is still supra-optimal. Investment in just s_j is another possible market research strategy for the firm. Once again the risk effect is negative and the matching effect is:

$$\int_{\alpha_i} g(\alpha_i) \int_{s_i} f(s_i \mid I_i, \alpha_i) \int_{\alpha_j} g(\alpha_j) \int_{s_j} f(s_j \mid I_j \alpha_j) \frac{dU^*}{dI_j} ds_j \, d\alpha_j \, ds_i \, d\alpha_i$$

$$=$$

$$-\left(\frac{\gamma}{16\beta}\right)\left(\frac{K K(I_j) K'(I_j)}{((K+K(I_j))^3}\right)(V+V(I_j))$$

and thus negative, Thus Proposition 6 still holds and investment levels are supra-optimal.

Generalized n-firm Results

The results of this paper are not sensitive to the number of firms in the market. The results are not altered when the framework is expanded to include oligopoly. Though actual levels of investment will change, the propositions will still hold.

The representation of expected demand becomes:

$$E_i[q_i] = \frac{N+\gamma(N-1)}{N^2} E_i[\alpha_i] - \frac{\beta(N+\gamma(N-1))}{N^2} p_i - \frac{\gamma}{N^2} \sum_{j \neq i} E_i[\alpha_j] + \frac{\gamma\beta}{N^2} \sum_{j \neq i} E_i[p_j]$$

Similarly, the Nash price becomes:

$$p_i = 2\beta \frac{E_i[\alpha_i]}{2\beta} + \frac{c}{2} - \frac{\gamma}{2\beta(N+(N-1))} \sum_{j \neq i} E_i[\alpha_j] + \frac{\gamma}{2(N+\gamma(N-1))} \sum_{j \neq i} E_i[p_j]$$

where

$$E_i[p_j] = p(\bar{\alpha}) = \frac{c(N+\gamma(N-1)) + N(\frac{\bar{\alpha}}{\beta})}{2N+\gamma(N-1)}$$

It can be seen that only trivial changes have been made in the expressions contained in Propositions 1-4. The proofs of these propositions are still valid. The matching effect for own-parameter market research becomes:

$$\int_{\alpha_1} g(\alpha_1)\int_{s_1} f(s_1 \mid I_i,\alpha_1) \cdots \int_{\alpha_n} g(\alpha_n)\int_{s_n} f(s_n \mid I_n,\alpha_n)\frac{dU^*}{dI_i} ds_n\, d\alpha_n \cdots ds_1\, d\alpha_1 =$$
$$-(V+V(I_i))\left(\frac{N+\gamma(N-1)}{4\beta N^2}\right)\left(\frac{KK(I_i)(K'(I_i)}{(K+K(I_i))^3}\right)$$

which, for the reasons given in the proof of Proposition 5, is negative for all N >= 1. Proposition 6 holds and investment is still supra-optimal.[13]

Sequential Games

From Proposition 1 it is easily shown that the sequential structure of the game, in investment levels, is of no importance for the results of this paper. The sequential structure of the price game is , however, important to the results. If a Stackelberg price game is played, the leader will convey all of its information through the price choice. The only exception to this is if there are multiple solutions to Stage II.

In investment levels, however, the model survives any ordering of firm action. For example, if firm i were to invest such that firm j observed I_i, prior to choosing I_j, there would be no change in behavior. Not only would the basic results still hold, the equilibrium investment levels and prices would not change either. Thus a Stackelberg dominant firm, or a tight oligopoly, can be examined in the same manner as a

[13] Investment in just one other firm's parameters is straightforward and the result is that the matching effect becomes:

$$\int_{\alpha_1} g(\alpha_1)\int_{s_1} f(s_1 \mid I_{i,1},\alpha_1) \cdots \int_{\alpha_n} g(\alpha_n)\int_{s_n} f(s_n \mid I_{i,1},\alpha_n)\frac{dU^*}{dI_{i,1}} ds_n\, d\alpha_n \cdots ds_1\, d\alpha_1 =$$
$$-(V+V(I_{i,1}))\left(\frac{\gamma^2}{4\beta N^2(N+\gamma(N-1))}\right)\left(\frac{KK(I_i)(K'(I_{i,1})}{(K+K(I_{i,1}))^3}\right)$$

Thus market research to discover a single competitor's parameters is supra-optimal. Investment in every firm's parameter is an unwieldy problem. Carrying out the above suggested that the expressions would retain the same sign.

large number of small firms. The number of firms, N, obviously matters, but their configuration and the method of choosing the investment levels does not.

VI. CONCLUSION

The results in this paper are derived from fairly weak assumptions and are able to suggest policy for many situations. In a simple two country model of world trade, the following can be said about policy.

Suppose the firms are both located in country A and consumers are also located in country A. Then, we have the model described in the paper and market research levels are supra-optimal. Thus if any policy is to be enacted, and that is not clear from the analysis above, it should be a tax and not a subsidy. Now suppose that the firms are both located in A but all of the consumers are in B. What should the government policies be in this case?

The negative externality associated with market research is felt in B, so if possible B's government may wish to tax the market research of the firms in A. This may not be feasible. How about the government of A? Should the government in A now subsidize the research since it does not have any consumers? No. Since there is no externality to the firms, the market research levels are already optimal for A. Any subsidy, or tax, would only lead to a deviation from the welfare maximizing levels of investment.

This implies that if any post-product development market research that is subsidized in any way by a government, even in a purely export country, the government is actually moving the economy away from the welfare maximizing point. This policy prescription does not apply to pre-product development market research as that is an entirely different process with different effects on consumers and other firms.

A model of pre-product development market research would allow for the possibility that improved information would lead to higher quality goods and perhaps even to a positive externality to the consumers. This issue, the subject of another paper, is possibly of greater interest than the one posed here, depending on the actual proportion of market research expenditures that is post-product development. However, even if pre-product development market research is allowed, this post-product development process is still available to the firms and thus the results will hold even in the presence of pre-product development market research.

VII. APPENDIX

In the proof of Proposition 4, it is claimed that $\int_{s_i} \frac{dF(s_i \mid I_i,\alpha_i)}{dI_i} ds_i = 0$. A formal proof of this is presented here. Since I_i induces a mean preserving reduction, we know the following about the mean:

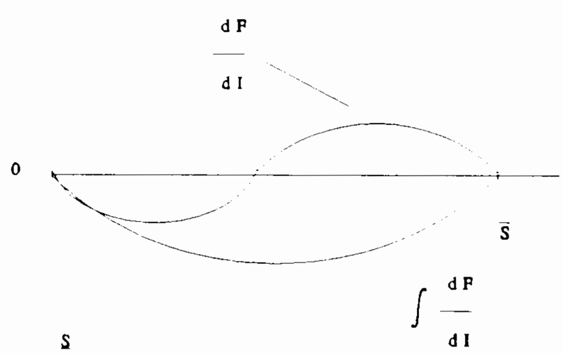
$$E[s_i] = \int_{s_i} s_i f(s_i \mid I_i',\alpha_i) ds_i = s_i F(s_i \mid I_i,\alpha_i) \mid_{s_i} - \int_{s_i} F(s_i \mid I_i,\alpha_i) ds_i =$$
$$\bar{s}_i - \int_{s_i} F(s_i \mid I_i,\alpha_i) ds_i$$

And the derivative of the mean with respect to I_i:

$$\frac{dE[\alpha_i]}{dI_i} = \frac{d\bar{s}_i}{dI_i} - \frac{d}{dI_i}\int_{s_i} F(s_i \mid I_i,\alpha_i) ds_i = 0$$

But since the upper bound of the distribution of the signals is not changed, the second term is zero as well.

A diagram depicting $\int_{s_i} \frac{dF(s_i \mid I_i,\alpha_i)}{dI_i} ds_i$ is shown to provide visual evidence that when evaluated over its entire domain it equals zero, yet when evaluated up to any point below the upper end of its range it is less than zero.



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