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THE CYCLICAL BEHAVIOR OF JOB CREATION AND JOB DESTRUCTION[1]

by

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ABSTRACT

Panel studies show that job creation and job destruction coexist at all phases of the business cycle. In this paper, we develop a model of endogenous job destruction in response to persistent idiosyncratic shocks and incorporate the model into the transactions cost (matching) approach to equilibrium job creation and wage determination. Second, we examine the dynamic stochastic implications of the model for co-movement between job creation, job destruction, and employment growth induced by a common aggregate shock to productivity. Finally, a simulation of the model for a reasonable parameterization demonstrates that it can explain cyclical properties of US Manufacturing data.

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1. INTRODUCTION

Recent microeconomic evidence from the United States and other countries has shown that job creation and job destruction coexist at all phases of the business cycle.[2] Even within narrowly defined sectors, individual establishments have diverse employment experiences regardless of the state of aggregate conditions. Still, both series when aggregated have distinct cyclical behavior. In their study of US manufacturing establishments, Davis and Haltiwanger (1990, 1991) find job destruction is more variable than job creation over the cycle, the two are negatively correlated, and job reallocation, defined as their sum, is counter cyclic.

In a companion paper, Mortensen and Pissarides (1991), we develop a model of endogenous job creation and job destruction and incorporate it into the transactions cost (matching) approach to equilibrium unemployment and wage determination. In the extended model, establishments have diverse experiences because of persistent idiosyncratic shocks. In this paper, we examine the dynamic stochastic implications of the model for co-movement between job creation, job destruction, and employment induced by a common aggregate shock to productivity. A demonstration that the cyclical properties of the Davis and Haltiwanger data are implied by our model is the principal contribution of this paper.

The economy we examine has a continuum of jobs that differ with respect to values of labor product. For concreteness, suppose that each job is located on a separate island and that the technology on the island is subject to idiosyncratic risk associated say with the weather. New jobs can be freely created on any of the islands but the investment is irreversible, so an existing job cannot move among the islands once created. We model the idiosyncratic risk for existing jobs as a jump process characterized by a Poisson arrival frequency and a drawing from a common distribution of relative prices. Large negative shocks induce job destruction but the choice of when to destroy the job is the firm's.

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[2] For the United States see Leonard (1987), Davis and Haltiwanger (1991) and Blanchard and Diamond (1990), for Germany Boeri and Cramer (1991) and for Italy Contini and Revelli (1988).

New jobs are created with value of labor product at the upper support of the idiosyncratic distribution. In other words, they locate on the island which is expected to have the "best weather" in the foreseeable future. As the matching process is viewed as taking place between individual jobs and workers, there is an exante zero-profit condition for a new job vacancy. Given our assumption of constant returns in the matching technology, zero expected profit on a new vacancy is equivalent to a marginal productivity condition for the most productivity jobs.[3]

In this paper, only the version of the model in which workers are paid the opportunity cost of time is considered.[4] Equilibrium is a time path for the number of job/worker matches (employment) implied by the matching law and optimal non-cooperative behavior by individual workers and employers based on rational forecasts of relevant future events. The principal purpose of the analysis that follows is to illustrate that the dynamic equilibrium behavior implied by the model is broadly consistent with the aggregate data on job creation and job destruction, particularly that constructed by Davis and Haltiwanger (1990, 1991) from US manufacturing establishment data.

2. CONCEPTS AND NOTATION

A job is in one of two states, filled and producing or vacant and searching. Each job is characterized by a fixed irreversible technology which together with a single worker produces a value of product flow $p + \sigma\varepsilon$. p is an aggregate productivity parameter common to all jobs and ε is the idiosyncratic component of productivity. The parameter σ reflects dispersion in the idiosyncratic component, an increase in σ represents a symmetric mean preserving spread in its distribution of the idiosyncratic "shock" or

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[3] For more discussion of the unemployment model see Pissarides (1990) and Mortensen (1990).

[4] In other words, the employer captures all of the match surplus. In Mortensen and Pissarides (1991) the more general case of any bilateral bargaining outcome at fixed shares is considered. Job to job movement induced by employed worker search is the phenomena added in this more realistic case.

equivalently an increase in variance. The process that changes the idiosyncratic component of price is Poisson with arrival rate λ. When there is change, the new value of ε is a drawing from the fixed distribution F(x), which has finite upper support ε_u and no mass points. Without further loss of generality, F(x) can be endowed with zero mean and a unit variance so that σ is the standard deviation of the job-specific component $\sigma\varepsilon$.[5]

In general, the aggregate shock, p, is modelled as an arbitrary Markov process with persistence. However, for expositional purposes, the theoretical discussion considers only the deterministic case and the case of a two point Markov chain defined on a high value p' and a low value with a common transition rate from one to the other denoted as μ. In the subsequent simulations, a three point process is considered. Although overly simple, these processes captures the most important feature of cyclical shocks, a positive probability less than one that boom or recession will end within any finite period of time.

Job creation and job destruction are costless, but the search process that matches workers to vacant jobs is costly. Firms create jobs that have value of product equal to the upper support of the value of product distribution, denoted as $p+\varepsilon_u$, until the marginal vacancy yields zero expected profit. When a vacant job experiences an idiosyncratic shock, it exits so that all active vacancies are characterized by this same value. Filled jobs do not always exit when hit by a shock because the cost of recruiting, modeled here as a fixed vacancy cost of c. Existing filled jobs are destroyed only if the idiosyncratic component of its value is less than some critical value $\varepsilon_d < \varepsilon_u$. In sum, job births take place at ε_u and job deaths at ε_d.

The rate at which vacant jobs and unemployed workers meet is determined by the homogeneous of degree one matching function m(v,u), where v and u represent the numbers of

[5]Note, except for the lack of mass points and a finite upper support restriction, there are no other shape requirements.

vacancies and unemployed job seeking workers respectively, normalized by the fixed labor force size. Vacancies are filled at the rate $q = m(v,u)/v = m(1,u/v)$ and job seekers find employment at the rate $vq/u = m(v/u,1)$.

3. STEADY STATE EQUILIBRIUM

In this section of the paper we consider the case in which the common productivity component p is fixed and unchanging. As a consequence of this assumption, unemployment converges to and remains forever at a fixed steady state or "natural" rate that depends on the value of p. The review of the comparative static effects of p on vacancies and unemployment, derived and discussed in more detail in our companion paper, is the principal purpose of the section. These results serve as a benchmark for comparison with the comparative dynamic effects of changes in p that occur when the aggregate shock is regarded as a stochastic process rather than a fixed parameter.

We assume that employers capture all the rents associated with a job-worker match by paying workers the common alternative value of their time, b. As Diamond (1971) has shown, this outcome is an equilibrium in a wage setting game played among employer's when workers have only the power to accept or reject offers and workers search sequentially at some positive cost. Given the outcome, workers have no incentive to search on the job and their parameters, other than b, do not affect the equilibrium.

The assumptions that vacancies cost c per unit time to hold and that they are the most productive jobs imply

$$rV(\varepsilon_u) = -c + q[J(\varepsilon_u) - V(\varepsilon_u)]. \tag{1}$$

where $V(\varepsilon)$ and $J(\varepsilon)$ are respectively the asset values of a vacancy and of a filled job with idiosyncratic component ε. Job creation until the exhaustion of all rents yields

$$rV(\varepsilon_u) = 0. \tag{2}$$

Since firms have the option of closing jobs at no cost, a filled job continues in operation for as long as its value is above zero. Hence, filled jobs are destroyed when a productivity shock ε arrives for which the value of a job $J(\varepsilon)$ is negative. For any realization ε, $J(\varepsilon)$ solves,

$$rJ(\varepsilon) = p + \sigma\varepsilon - b + \lambda \int \{\max[J(x), 0] - J(\varepsilon)\} dF(x). \tag{3}$$

As he solution to (3) is unique and strictly increasing, jobs are destroyed whenever the idiosyncratic shock falls below the unique reservation value ε_d, defined by

$$J(\varepsilon_d) = 0. \tag{4}$$

A more explicit expression for the reservation value of the idiosyncratic shock can be derived by integrating (3) by parts. Differentiation of (3) gives $J'(\varepsilon) = \sigma/(r+\lambda)$, so (3) and (4) and integration by parts yield

$$\begin{aligned}(r+\lambda)\ J\ (\varepsilon) &= p + \sigma\varepsilon - b + \lambda \int_{\varepsilon_d}^{\varepsilon_u} J(x)\, dF(x) \\ &= p + \sigma\varepsilon - b + \lambda \int_{\varepsilon_d}^{\varepsilon_u} J'(x)\,[1 - F(x)]\, dx \\ &= p + \sigma\varepsilon - b + \frac{\lambda\sigma}{r+\lambda} \int_{\varepsilon_d}^{\varepsilon_u} [1-F(x)]\, dx.\end{aligned} \tag{5}$$

Therefore, by (4) and (5) the reservation shock is the unique solution to

$$p + \sigma\varepsilon_d = b - \frac{\sigma\lambda}{r+\lambda}\int_{\varepsilon_d}^{\varepsilon_u}[1-F(x)]\,dx. \tag{6}$$

The left side in this expression is the job productivity acceptable to firms with a filled job. Note that it is less than the opportunity cost of employment because of the existence of a hiring cost. The second term on the right side of (6) is a measure of the extent to which the employer is willing to incur an operational loss now in anticipation of a future improvement in the value of the match's product, i.e., it is the option value of retaining an existing match. That this is positive is indicative of the existence of "labor hoarding" at low price realizations.[6]

It is easily established by differentiation that the reservation shock ε_d decreases with the difference between the aggregate productivity parameter, p, and the common opportunity cost of employment, b. Because the decrease in ε_d increases the option value of a job, equation (6) also implies that an increase in p reduces the reservation productivity p + $\sigma\varepsilon_d$: the range of productivity observed expands with the common component of productivity.

An increase in λ increases the option value of a job because job-specific product values are now less persistent. So, an employer experiencing a bad shock is more willing to maintain the match. In contrast, a higher discount rate, r, reduces future profitability in the event of any change in value, reducing the option value of waiting for an improvement. Thus, ε_d decreases with λ and increases with r.

The effect of an increase in the variance of the idiosyncratic shock, σ, on ε_d is ambiguous in general. An increase in σ increases the profitability of good shocks but decreases the profitability of bad. Because the employer has the option of ending the match when the shock is sufficiently bad, his response to an increase in σ depends on the opportunity cost of

[6]It can easily be checked that if c = 0, $J(\varepsilon_u) = 0$ from (1) and (2) and so $\varepsilon_d = \varepsilon_u$: given the exogeneity of p, b and σ, jobs are either created at the upper support of the productivity distribution or destroyed without limit.

employment. Differentiating (6) with respect to σ gives,

$$\sigma\frac{\partial\varepsilon_d}{\partial\sigma} = \frac{r + \lambda}{r+\lambda F(\varepsilon_d)} \times \frac{p-b}{\sigma}. \tag{7}$$

So ε_d increases in σ if p > b and decreases if p < b.

The solution for the other two unknowns of the model, vacancies and unemployment, is obtained from (1) and (2) and the steady-state condition for unemployment. To write (2) in a more convenient form, substitute from (6) into (5) to obtain,

$$J(\varepsilon) = \frac{\sigma(\varepsilon - \varepsilon_d)}{r + \lambda}. \tag{8}$$

Therefore, (1), (2), and (6) imply

$$m(1, u/v)\frac{\sigma(\varepsilon_u - \varepsilon_d)}{r + \lambda} = c. \tag{9}$$

Equation (9), the job creation condition, uniquely determines v/u given the solution for ε_d obtained from (6).

The final equation of the model, the Beveridge curve, is the steady-state condition for unemployment. The flow out of unemployment equals the flow into unemployment at points on the curve. The endogenous job separation rate is $\lambda F(\varepsilon_d)$ and the job matching rate per unemployed worker is m(v/u,1), so the equation for the Beveridge curve is

$$u = \frac{\lambda F(\varepsilon_d)}{\lambda F(\varepsilon_d) + m(v/u, 1)}. \tag{10}$$

Since from (6) the reservation shock is independent of v/u, in vacancy-unemployment space the Beveridge curve has the usual convex-to-the-origin shape as illustrated in Figure 1 by the curve labeled BB. The job creation condition (9) is a ray through the origin labeled 0F in the figure. Given the reservation shock, the steady state equilibrium vacancy and unemployment combination, (v^*, u^*) in Figure 1, lies at the intersection of the job creation condition and the Beveridge curve.

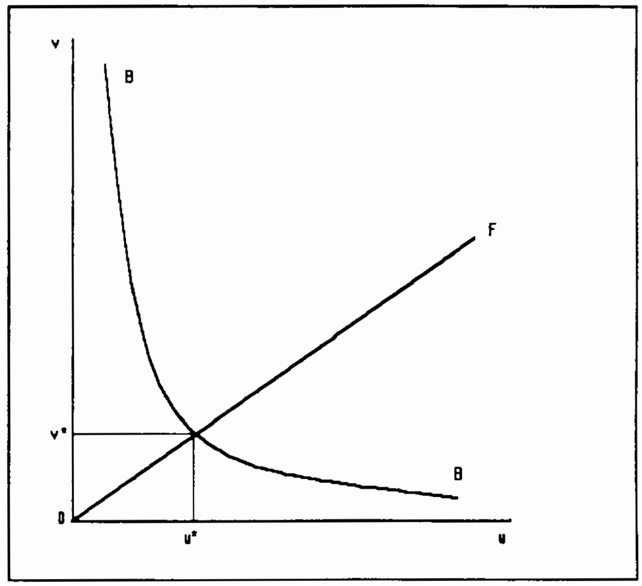

Figure 1: Steady State Vacancy and Unemployment Rates

The job creation flow is $m(v/u,1)u$ and the job destruction flow is $\lambda F(\varepsilon_d)(1-u)$. The analysis that follows derives the initial impact of an increase in the aggregate shock parameter conditional on current unemployment, u. Obviously, unemployment eventually adjusts to equate the two in steady state. Note that an increase in job creation shifts the job creation condition (9) up in v-u space while an increase in job destruction shifts the Beveridge curve out.

A positive net aggregate productivity shock, represented by either an increase in p or a fall in b, rotates the job creation condition (9) up and shifts the Beveridge curve (10) in, both in response to the induced fall in ε_d implied by equation (6). In other words, job creation increases while job destruction decreases in response to a positive macro shock. Eventually, unemployment falls to its new steady state value but the effect on steady state vacancies is ambiguous. The difference between these implications and those of the pure matching model is the shift in the Beveridge curve and the ambiguity of the vacancy effect implied by the sensitivity of the job destruction decision to the value of the aggregate productivity component.

4. STOCHASTIC EQUILIBRIUM

The empirical facts that motivate our analysis concern the cyclical behavior of job creation and job destruction. In this section we extend the static model of section 3 to the case in which the aggregate component of productivity changes stochastically. For simplicity of exposition, the aggregate productivity parameter p takes two values, a high value p' and a low value p, and the transition rate from one to the other is μ.

The steady-state equilibrium solutions for ε_d and v/u (and hence job destruction and job creation) for a given p are determined by equations (6) and (9). As neither of the expressions contain sticky variables, the solutions for the two variables jump between the pair ε_d and v/u on the one hand and ε'_d and v'/u' on the other, as the common productivity component jumps between p and p'. In contrast, unemployment is a sticky variable, since it changes according to the laws governing the matching technology. The differential equation describing the unemployment path for any given p is

$$\dot{u} = (1-u)\lambda F(\varepsilon_d) - um(v/u) . \tag{11}$$

The steady-state analysis leads us to expect that since $p' > p$, $\varepsilon'_d \le \varepsilon_d$ and $v'/u' \ge v/u$. Therefore, when price changes from p' to p, some marginal jobs are immediately destroyed and some vacancies close down. In contrast, when price increases from p to p', new vacancies are opened up but nothing happens to employment on impact. This asymmetry has important cyclical implication for the behavior of the job creation and job destruction rates.

As before, jobs are destroyed whenever their value falls below zero. Equations (1), (2) and (4) of the steady state model still hold for each p. The expression for the returns from a filled job, equation (5), needs to be modified to reflect the fact that aggregate productivity component may now change. Denoting by $J'(\varepsilon)$ the expected return from a filled job when aggregate shock is p', and noting that if $J(\varepsilon) < 0$ the job is destroyed, (5) becomes,

$$(r+\lambda+\mu)J(\varepsilon) = p + \sigma\varepsilon - b + \lambda\int_{\varepsilon_d}^{\varepsilon_u} J(x)\,dF(x) + \mu J'(\varepsilon), \qquad \varepsilon \geq \varepsilon_d, \tag{12}$$

$$(r+\lambda+\mu)J'(\varepsilon) = p' + \sigma\varepsilon - b + \lambda\int_{\varepsilon'_d}^{\varepsilon_u} J'(x)\,dF(x) + \mu J(\varepsilon), \qquad \varepsilon \geq \varepsilon_d, \tag{13}$$

$$(r+\lambda+\mu)J'(\varepsilon) = p' + \sigma\varepsilon - b + \lambda\int_{\varepsilon'_d}^{\varepsilon_u} J'(x)\,dF(x), \qquad \varepsilon_d > \varepsilon \geq \varepsilon'_d, \tag{14}$$

The difference between (13) and (14) is due to the fact that all jobs whose idiosyncratic component of productivity in the interval $\varepsilon_d > \varepsilon \geq \varepsilon'_d$ are destroyed when common price falls from p' to p.

The reservation idiosyncratic shock ε'_d solves $J'(\varepsilon'_d) = 0$. The relevant expression for $J'(\varepsilon'_d)$ is (14), where by differentiation, $\partial J'(\varepsilon)/\partial\varepsilon = \sigma/(r+\lambda+\mu)$. Hence, integration of (14) by parts gives,

$$(r+\lambda+\mu)J'(\varepsilon) = p' + \sigma\varepsilon - b + \frac{\lambda\sigma}{r+\lambda+\mu}\int_{\varepsilon'_d}^{\varepsilon_u}[1-F(x)]\,dx, \qquad \varepsilon_d > \varepsilon \geq \varepsilon'_d. \tag{15}$$

The reservation shock in the "boom" (when price is p' > p) therefore solves,

$$p' + \sigma\varepsilon'_d = b - \frac{\lambda\sigma}{r+\lambda+\mu}\int_{\varepsilon'_d}^{\varepsilon_u}[1-F(x)]\,dx. \tag{16}$$

A comparison of (16) with the equivalent expression in the steady state, (6), shows that the only change is a reduction in the option value of the job. The fact that in the boom there is a positive probability that price will fall reduces the option value of continuing a match that is less productive than the opportunity cost of employment. But obviously the reduction in the option value of the job does not affect any of the qualitative comparative static properties of the reservation shock previously derived.

In "recession" the expressions determining the value of a job are (12) and (13). Differentiation with respect to ε gives,

$$\frac{\partial J(\varepsilon)}{\partial \varepsilon} = \frac{\partial J'(\varepsilon)}{\partial \varepsilon} = \frac{\sigma}{r+\lambda}, \qquad \varepsilon \geq \varepsilon_d, \tag{17}$$

and so integration of (12) by parts yields,

$$(r+\lambda+\mu)J(\varepsilon) = p + \sigma\varepsilon - b + \frac{\lambda\sigma}{r+\lambda}\int_{\varepsilon_d}^{\varepsilon_u}[1-F(x)]\,dx + \mu J'(\varepsilon). \tag{18}$$

From (13) and (14) it follows that for $\varepsilon = \varepsilon_d$,

$$J'(\varepsilon_d) = \frac{\sigma(\varepsilon_d - \varepsilon_d')}{r+\lambda+\mu}. \tag{19}$$

Substitution of (19) into (18) produces the expression for the reservation shock at low common p,

$$p + \sigma\varepsilon_d = b - \frac{\lambda\sigma}{r+\lambda}\int_{\varepsilon_d}^{\varepsilon_u}[1-F(x)]\,dx - \frac{\mu\sigma}{r+\lambda+\mu}(\varepsilon_d - \varepsilon_d'). \tag{20}$$

In contrast to the reservation shock during boom, the probability that price will increase increases the option value of the marginal job in recession. Thus, firms destroy fewer jobs in recession the higher the transition rate to the boom.

Conditions (16) and (20) show that when cyclical shocks are anticipated, the gap between the reservation prices at high and low common price is less than implied by the steady-state analysis. The gap grows as the probability of changing state (measured by the Poisson rate μ) falls. Apart from that change, however, the previous analysis still holds, with more jobs destroyed in recession than in the boom.

Job creation is found by computing the value of jobs at the upper support of the value of product distribution. From (12) we can write

$$(r+\lambda+\mu)J(\varepsilon) = \sigma(\varepsilon - \varepsilon_d) + \mu[J'(\varepsilon) - J'(\varepsilon_d)], \tag{21}$$

and from (13),

$$(r+\lambda+\mu)[J'(\varepsilon) - J'(\varepsilon_d)] = \sigma(\varepsilon - \varepsilon_d) + \mu J(\varepsilon). \tag{22}$$

Solving (21) and (22) gives,

$$J(\varepsilon) = \frac{\sigma(\varepsilon - \varepsilon_d)}{r + \lambda}. \tag{23}$$

Given the reservation shock, this is the same expression as the one holding in the steady state, (8). Therefore vacancy creation in recession solves an expression similar to (9), rewritten here as,

$$m(1, u/v) = \frac{c(r + \lambda)}{\sigma(\varepsilon_u - \varepsilon_d)}. \tag{24}$$

Equations (13) and (14) imply that the value of jobs in the boom, when $\varepsilon > \varepsilon_d$, is,

$$J'(\varepsilon) = \frac{\sigma(\varepsilon - \varepsilon_d')}{r+\lambda+\mu} + \frac{\mu}{r+\lambda+\mu}J(\varepsilon). \tag{25}$$

Making use of (23) we get, for vacancy creation at high p,

$$m(1, u'/v') = \frac{c(r+\lambda)}{\sigma(\varepsilon_u - \varepsilon_d) + (r+\lambda)\sigma(\varepsilon_d - \varepsilon_d')/(r+\lambda+\mu)} . \tag{26}$$

Therefore, comparing with (9), even for given reservation prices job creation in the boom is less when there is the expectation of cyclical change.

Now a comparison of (26) with (24) shows that $u'/v' < u/v$, i.e. that there is more job creation in the boom than in recession. The fact that $\varepsilon_d > \varepsilon'_d$, however, implies that when the cyclical shocks are anticipated (i.e. when $\mu > 0$), the job creation rate is likely to exhibit less cyclicality than when $\mu = 0$. For given values of the reservation shock, a larger μ leaves job creation at low p unaffected, as in (24), but reduces the higher job creation rate at high p, as in (26). Of course, μ also influences the reservation values but an examination of the job creation conditions shows that influence is not likely to influence job creation in one state differently from job creation in the other state.

The anticipation of cyclical shocks narrows the gap between the reservation shocks at the two values of the aggregate productivity component, p and p', so job destruction fluctuates less for a given change than in the steady state. But the dynamics of job destruction changes are likely to increase the cyclicality of the job destruction rate, at least for a short period after a the arrival of a new value. Consider first what happens when aggregate component increases from p to p'. Firms open up more job vacancies and hold on to more jobs after unfavorable job-specific shocks. Thus the job creation rate, $um(v/u,1)$, increases and the job destruction rate, $(1-u)\lambda F(\varepsilon_d)$, decreases, inducing a fall in unemployment through (11). Since neither v/u nor ε_d has dynamics of its own at given p, the decrease in unemployment induces a fall in job creation and an increase in job destruction until there is convergence to a new steady state, or until there is a new cyclical shock.

When aggregate component falls from p' to p the dynamics of job creation follow a pattern similar to that after an increase in p: v/u falls once for all, job creation falls on impact but again increases as unemployment begins to rise. The dynamics of job destruction, however, are different, because the rise in the reservation shock from ε_d' to ε_d leads to an immediate destruction of all jobs with idiosyncratic component between the two reservation shock values. Job destruction also rises for reasons similar to the ones that led to its decrease when productivity increased, since with higher reservation shock firms are more likely to destroy jobs as they are hit by job- specific shocks. But the increase in job destruction immediately after the cyclical downturn has no counterpart in the behavior of the job destruction rate when p increases, or in the behavior of the job creation rate. This imparts a cyclical asymmetry in the job destruction rate and in the dynamic behavior of unemployment. The short-run cyclicality of the job destruction rate increases, the job destruction rate leads the job creation rate as a cause of the rise in unemployment and the speed of change of unemployment at the start of recession is faster than its speed of change at the start of the boom.

In this section we demonstrate that the model is consistent with at least some of the findings of Davis and Haltiwanger and others. Job creation and job destruction move in opposite directions as the economy cycles and job creation fluctuates less than job destruction in our model. We have also shown that if we take the steady-state analysis as a yardstick, the anticipation of cyclical shocks causes an asymmetry in job creation, reducing it in the boom but not in recession. Since in the boom job creation is already higher, this result is consistent with the finding that it does not fluctuate as much. Finally, we find an implied asymmetry in job destruction which is consistent with the finding that job destruction is more cyclical. Job destruction increases more rapidly and by a larger extent at the start of recession than it decreases at the start of the boom. The latter claim is also consistent with observations on the behavior of unemployment, that entry into unemployment leads exit as the cause of the rise in unemployment.

5. IMPLEMENTING THE MODEL

The quantitative implication of the model is the focus of this section. In particular, we ask whether an operational version might replicate at least the magnitudes of movements and correlations observed in the quarterly US manufacturing data constructed by Davis and Haltiwanger for reasonable parameter values. These data are illustrate in Figure 2..[7] The fact

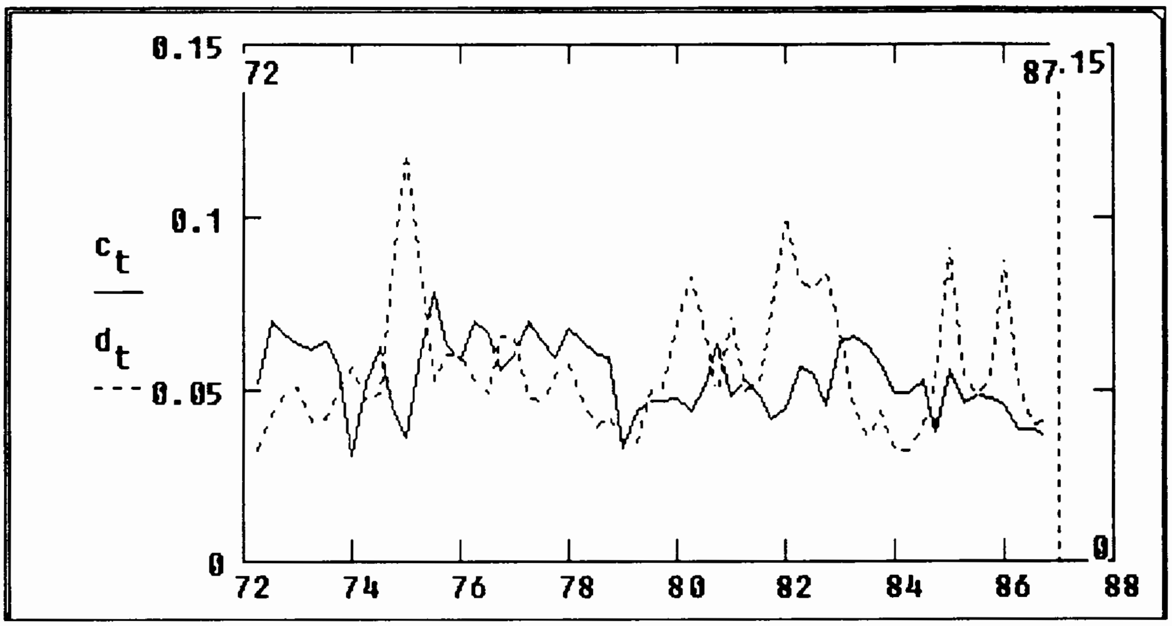

Figure 2: US Manufacturing Job Creation (c) and Job Destruction (d) Rates, 1972, II, to 1986, IV. Source: Davis and Haltiwanger (1990)

that cyclic deviations in job destruction are generally of opposite direction and larger than those of job creation is an obvious property of these data. Furthermore, the large spikes in the job destruction series that occur at the beginning of an employment downturns are not matched by correspondingly high rates of job creation in upturns. Finally, job reallocation, the sum of job creation and destruction, is counter cyclic, negatively associated with employment growth,

[7] The data (See Davis and Haltiwanger (1990).) are quarterly job creation and job destruction rates expressed as a percentage of the arithmetic average of US manufacturing employment as of the beginning and end of each quarter. The simulation results reported in the sequel use the same normalization rule.

because cyclic movement in job destruction dominate those in job creation.

The model used in the simulation that follows is a generalization of that considered in the previous section, one that allows for three aggregate productivity states. Specifically, the aggregate shock is a three-state Markov chain defined by p-b = x where x can take on the values in the set {-z, 0, z} and where the state to state transition probability matrix is

$$\Pi = \begin{bmatrix} \phi & \gamma & 1-\phi-\gamma \\ \varphi & 1-2\varphi & \varphi \\ 1-\phi-\gamma & \gamma & \phi \end{bmatrix} . \tag{27}$$

As Christiano (1990) notes, this specification has a Wold representation for $\{x_t\}$ of the form

$$x_t = \rho x_{t-1} + v_t, \text{ where } Ev_t x_{t-1} = Ev_t = 0 , \tag{28}$$

the variance of the innovation is

$$\sigma_v^2 \equiv Ev_t^2 = z^2(1 - \rho^2)/\kappa , \tag{29}$$

the auto correlation coefficient is

$$\rho = 2\phi + \gamma - 1 , \tag{30}$$

and the kurtosis of the process is

$$\kappa = 1 + .5\gamma/\varphi . \tag{31}$$

For each value of the aggregate shock, there are two equilibrium parameters that determine the associated steady state level of unemployment: The probability per quarter that an unemployed worker becomes employed and the probability per quarter than an employed worker transits to unemployment, denoted as α and δ respectively. By virtue of equations (1) and (2) and a log linear specification of the matching function m(u,v)

$$\alpha = m(u,v)/u = a[J(\varepsilon_u)]^{(1-\theta)/\theta} \tag{32}$$

where a is a scale parameter and θ is the elasticity of the matching function with respect to unemployment. After the initial effect of a change in the aggregate shock, the job destruction rate under the assumption that the idiosyncratic shock is uniformly distributed about zero on the interval [-1,1] is

$$\delta = d + \lambda F(\varepsilon_d) = d + \lambda \frac{1+\varepsilon_d}{2} . \tag{33}$$

where λ is the arrival frequency and d represents the exogenous rate of job turnover attributable to worker quits.

To compute the values of α and δ for each aggregate state, one needs the aggregate state contingent value of a job function. The three functions, denoted as $J_i(\varepsilon),\ i = 1,2,3$, solve the following system of continuous time Bellman equations:

$$(r+q+\lambda+\mu_i)J_i(\varepsilon) = x_i+\sigma\varepsilon + \lambda\int \max[J_i(y),0]\,dF(y) + \sum_{j\neq i}\pi_{ij}\max[J_j(\varepsilon),0] \tag{34}$$

$$\text{where } \mu_i = \sum_{j\neq i}\pi_{ij},\ dF(y) = .5dy\ \forall\ y \in [-1,1]$$

π_{ij} is the transition probability from aggregate state i to j, the typical element of the matrix given in equation (27), x_i is the aggregate productivity component in state i, the typical element of the vector (-z, 0, z), and r+d is the discount rate, the sum of the interest rate and the exogenous turnover rate.

After dividing both sizes of (34) by $r+d+\lambda+\mu_i$, one can easily see that the right side of the result is a contraction map defined on the set of continuous and piece wise linear functions of ε on the support of F with "kinks" at the aggregate state-contingent reservation values of the idiosyncratic shock. Hence, a unique solution exists in this set by virtue of the contraction map theorem. For the purpose of the simulations that follows, we solved (34) for each of the three aggregate shock contingent value functions associated with the assigned parameter values using the method outlined in the Appendix. Given computed value functions, one solves for the value of a vacancy in each state, $J_i(\varepsilon_u)$, and the reservation idiosyncratic shock, ε_d^i . Substituting these numbers into equations (33) and (34) for each aggregate shock, one obtains the aggregate state contingent transition rate between employment and unemployment.

Let t = 0, 1,... represent a quarterly sequence of time periods and let N_t, C_t, and D_t respectively denote the associated employment, job creation, and job destruction series. Because all jobs below the higher reservation value of the idiosyncratic productivity component are destroyed in response to a negative aggregate shock, one needs to keep track of the distribution of filled jobs by productivity in order to simulate these sequences. For this purpose, we let $N_t(\varepsilon)$ be the fraction of the unit labor force employed in jobs with idiosyncratic component ε or less at the beginning of period t.

As we have specified the aggregate shock process, changes in the common productivity component occur only at the beginning of each time period, each quarter in our particular case. This is an assumption of analytic convenience, meant only as an approximation. Because in fact new information may be recognized throughout the period and because reaction lags are common, we assume that the jump response in job destruction to new information about the aggregate state takes place at the end of each quarter. Hence, the within period employment dynamic is determined by the linear differential equation

$$\frac{dn}{ds} = \alpha_t(1 - n) - \delta_t n \;\forall\; s \in [t, t+1),\; n_t = N_t \tag{35}$$

where of course the parameters α_t and δ_t are the values associated with the aggregate shock prevailing during period t. The particular within period solution of interest can be written as

$$n_{t+s} = \frac{\alpha_t}{\delta_t+\alpha_t} + [n_t - \frac{\alpha_t}{\delta_t+\alpha_t}]e^{-(\delta_t+\alpha_t)s},\; s\in[0,1]\;. \tag{36}$$

Similarly, the within period fraction of the labor force employed at any given value of the idiosyncratic component ε or less solves the following differential equation when ε is strictly less than the upper support:

$$\begin{aligned}\frac{dn(\varepsilon)}{ds} &= \lambda\max\left[F(\varepsilon)-F(\varepsilon_d(x_t)),0\right][n - n(\varepsilon)]\\ &-\lambda[1-F(\varepsilon)]n(\varepsilon) - \delta(x_t)n(\varepsilon)\;\forall\; s\in[t,t+1),\; n_t(\varepsilon) = N_t(\varepsilon)\end{aligned} \tag{37}$$

As unemployed workers are only employed in vacancies, the only inflow into the set of less productive job, the first term on the right side of (37), are jobs whose idiosyncratic component fall from some higher value. The outflow, the last two terms, includes those jobs that experience an increase in their idiosyncratic component and those that are destroyed. As the destruction rate is d + $\lambda F(\varepsilon_d)$ the solution to (37) can be written

$$\begin{aligned}n_{t+s}(\varepsilon) &= n_t(\varepsilon)e^{-(d+\lambda)s}\\ &+\lambda\max\left[F(\varepsilon)-F(\varepsilon_d(x_t)),0\right]\int_0^s n_{t+\tau}e^{-(d+\lambda)(s-\tau)}d\tau,\; s\in[0,1]\end{aligned} \tag{38}$$

Finally, accounting for the response of job destruction to the change in the aggregate state at the end of the period implies

$$N_{t+1} = n_{t+1} - n_{t+1}(\varepsilon_d(x_t)) \tag{39}$$

and

$$N_{t+1}(\varepsilon) = \max[n_{t+1}(\varepsilon) - n_{t+1}(\varepsilon_d), 0] . \tag{40}$$

Since created jobs are those filled by the matching process during any quarter

$$C_t = \alpha_t \int_0^1 (1 - n_{t+s}) ds . \tag{41}$$

Job destruction includes the flow that occurs during the quarter plus any jump induced by a negative aggregate shock.

$$D_t = \delta_t \int_0^1 n_{t+s} ds + n_{t+1}(\varepsilon_d(x_t)) . \tag{42}$$

Of course,

$$N_{t+1} \equiv N_t + C_t - D_t . \tag{43}$$

6. AN ILLUSTRATIVE SIMULATION

For the purposes of the simulation reported in this section, the parameters of the aggregate shock process, ρ, σ_v, γ, and κ were set equal to 0.9 per quarter, 0.007 per quarter, 0 . 1 per quarter, and 3 respectively. As kurtosis of the normal distribution is 3, this specification is similar to that for total factor productivity, the Solow residual, typically assumed in the real business cycle literature (See Prescott (1986) and Hansen (1985).) The only difference is that the process assumed here is somewhat less persistent.

The elasticity of the matching function with respect to unemployment, θ, was set equal to 1/3, a quantity close to the point estimate obtained by Blanchard and Diamond (1989) using US data. The idiosyncratic shock was assumed to be uniformly distribution about zero with upper support $\sigma = 0.3$. A comparison of relative shock magnitudes can be made by using the fact that the parameters assumed for the aggregate productivity process specified above imply that the deviation in the aggregate shock about zero, z, is 0.121. The interest rate, r, and exogenous separation rate, d, were set at 0.01 per quarter and 0.02 per quarter per quarter.

Finally, the remaining two parameters, the frequency of the idiosyncratic shock process λ and the scale parameter of the transition to employment probability a were "calibrated" to the data. Specifically, parameters values were found that approximately matched the means of the standard deviations of the job destruction and job creation rates over 100 sample paths of 60 quarters each to the values found in the US Manufacturing data. The values obtained were λ = 0.065 and a = 0.02. As the autocorrelation coefficient for the idiosyncratic process is $1-\lambda$ = 0.935, the model requires considerable persistence in the idiosyncratic process to explain the data as expected. Furthermore, this value for the arrival frequency together with an assumed exogenous separation rate of 0.02 match the mean value of the job destruction rate, 0.056 per quarter, almost exactly. Finally, given these parameter values and the value assumed for the quarter, almost exactly. Finally, given these parameter values and the value assumed for the elasticity of the matching function, 1/3, the calibrated value a = 0.02 implies that the steady state employment ratio, $\alpha/(\alpha+\delta)$, fluctuates between 0.8 and 0.95 over the range of the aggregate shock (See the Appendix, Table II.). Since non-participation (as discouraged workers) and employment in non-manufacturing are included in the non-employment (unemployment) in manufacturing state, these values seem reasonable.

Comparisons of the magnitudes of standard deviations and correlations found in the job creation and job destruction data with their theoretical counterparts (and standard errors) obtained by simulating the parameterized model are reported in Table I. As listed in the first row of the table, variation in US manufacturing destruction rates over the sample period is slightly less than twice that of creation as measured by standard deviations expressed in % per quarter. The two series are negatively correlated with each other and their sum, job reallocation, is negatively correlated with their difference, employment growth. The second row of the table are means of the statistics (with standard errors in parentheses) computed from 100 simulated samples of 60 quarters each.

Table I: Comparison of Data and Simulation Results

Statistics	StDev(c) % per qrt	StDev(d) % per qrt	Corr c,d	Corr c+d,c-d
US Manufacturing 1972,II to 1986,IV	1.045	1.816	-0.221	-0.512
Model Results (Std Errors)	1.057 (.469)	1.737 (.716)	0.129 (.222)	-0.440 (.206)

As noted in the theoretical section on stochastic equilibrium, the difference in the magnitudes of variation in the two series and the asymmetry in their cyclical behavior are explained in our model by the fact that destruction is a jump process that takes place immediately in response to new information while job creation requires matching time. Both of these features of our model are clearly illustrated by the computed impulse response diagrams illustrated in Figures 3 and 4. The positive shock is a movement in x from zero to z = 0.121 while the negative shock considered is a change of equal magnitude from zero to -0.121 given that the system is in the steady state associated with x = 0 in both cases.

The initial response of job destruction to a negative aggregate shock is much larger than the increase in job creation to a positive shock of equal absolute magnitude because the entire set of jobs less than the new lower value of the reservation shock are immediately eliminated. Furthermore, because the large jump in job destruction increases unemployment, it is subsequently followed by a positive response in job creation which actually "overshoots" its new steady state value in the process of adjusting to it. This kind of reaction appears quite consistent with the patterns observed in the data illustrated in Figure 2.

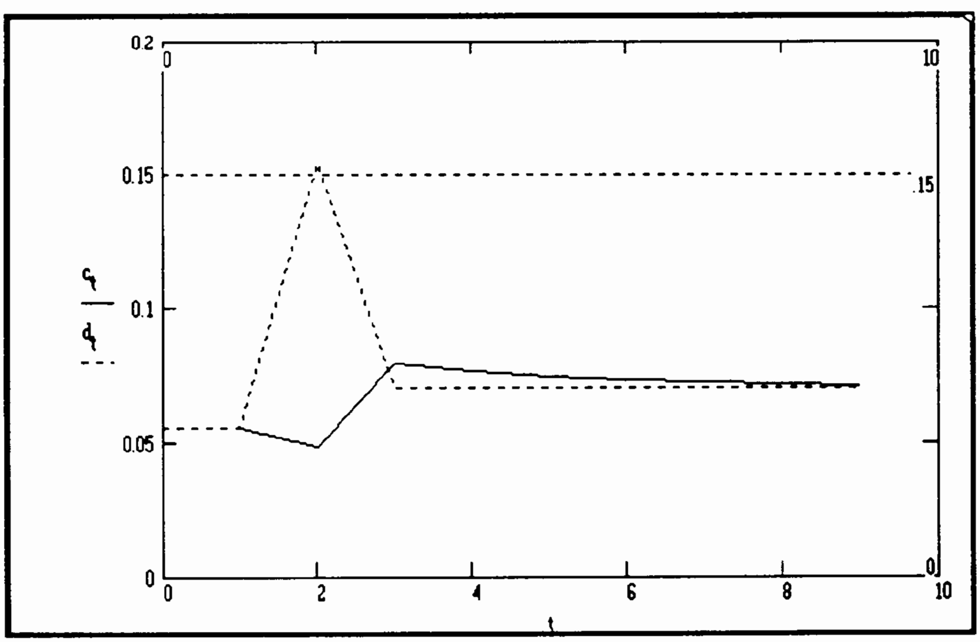

Figure 3: Impulse Responses to a Negative Aggregate Shock

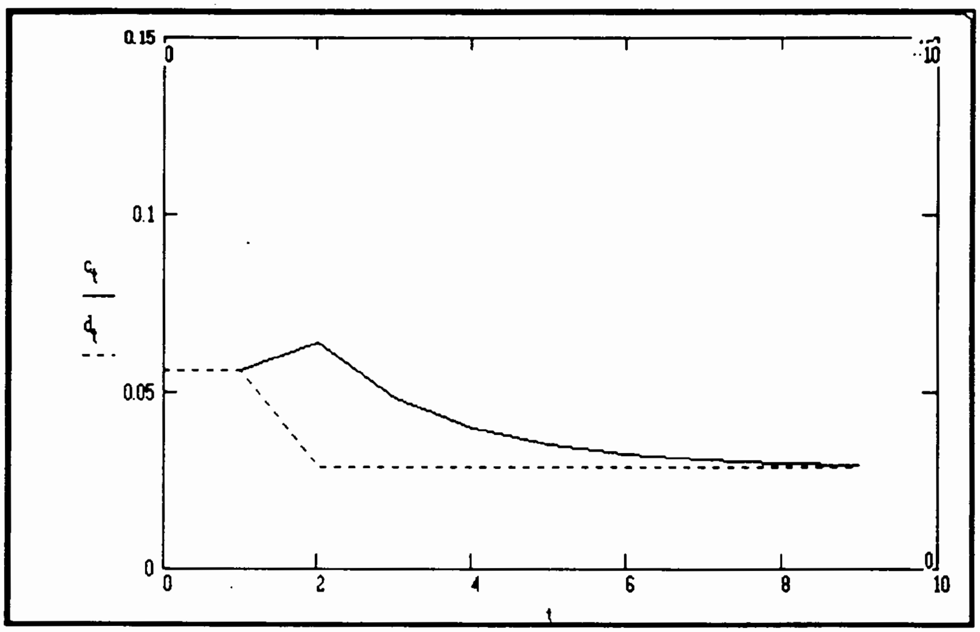

Figure 4: Impulse Responses to a Positive Aggregate Shock

In contrast, a positive shock of equal magnitude induces an initial increase in job creation and an initial reduction in job destruction of approximately equal absolute magnitude, but which is only a fraction of the initial impact of a negative shock on the job destruction rate. In this case adjustment to the new steady state is smoother and slower. Finally, the counter cyclic nature of job reallocation, the negative correlation between the sum of and difference the two series, is also explained by these features of the model.

Although the results reported in Table I establish that the extent of the correlation between job creation and job destruction found in the data is within two standard errors of the mean of the simulated samples, the typical value of the correlation coefficient implied by the model is positive. This result would seem to be inconsistent with the impulse response diagrams in Figures 3 and 4 which suggest that creation and destruction initially respond in opposite directions to any aggregate shock. To resolve the paradox, one must realize that the aggregate shock contingent steady state values of the two series are equal by definition and that the two value, equal to $\alpha\delta/(\alpha+\delta)$, generally varies with x, the "long run" positive association between the creation and destruction rates can offset the negative "short run" relationship. It is more likely to do so the more responsive is the common steady state creation-destruction rate to the aggregate shock and the more rapid is the process of adjustment to it. In our particular case, the common steady state value falls with the aggregate shock as indicated by Figures 3 and 4. (Also see the actual values reported in the Appendix, Table II.)

The importance of the dependence of the steady state value on the aggregate shock for the model's prediction regarding the correlation between creation and destruction is indicated by Figure 5, which is a typical scatter diagram generated by a 120 quarter simulation of the model. Although the scatter illustrates the opposite initial responses to changes in the aggregate state, the positive "long run" relationship between two series is clearly indicated by the many observations that are near the three steady state values along the diagonal.

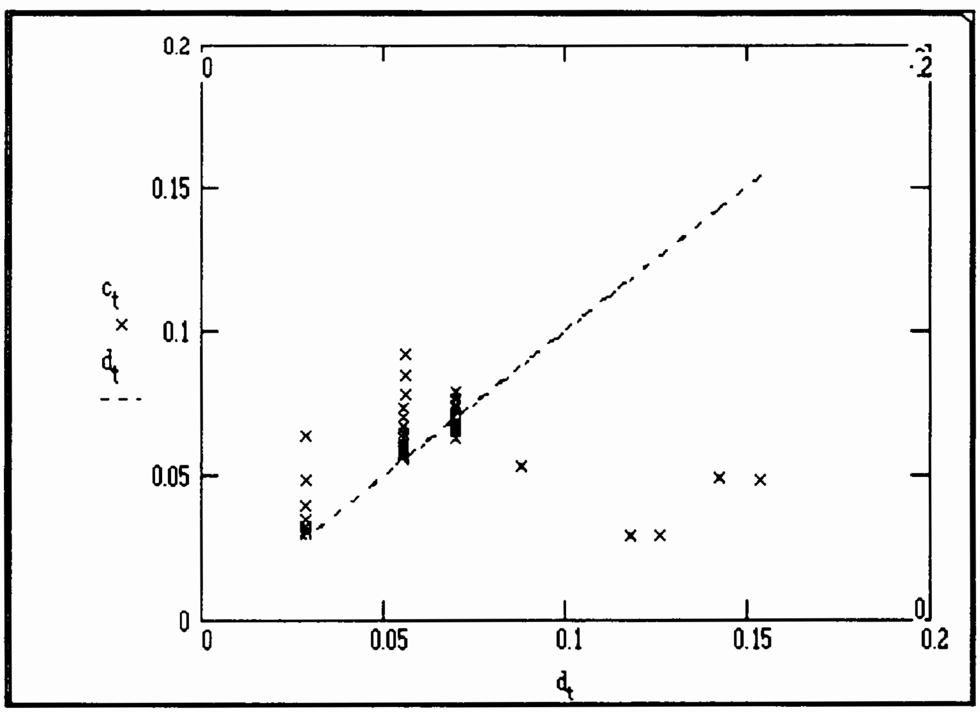

Figure 5: Simulated Creation and Destruction Rate Scatter

7. SUMMARY

There are two lessons to draw from the simulation exercise. A reasonable parameterization of the model is capable of explaining the observed cylical variation in job creation and job destruction rates in US manufacturing. However, the particular parameters that allow the model to match these features of the data also imply that variation in aggregate productivity induces a positive steady state relationship between the job creation and job destruction rates which dominates the contemporaneous theoretical relationship, contrary to the evidence. Whether this apparent contradiction can be explained away is an important question for future research.

APPENDIX: VALUE FUNCTION SOLUTION ALGORITHM

Given any finite number of aggregate states, equal to n, the state contingent value functions, $J_i(\varepsilon)$, solve the system of n functional equations (34). As noted in the text, the solution is unique and each function is piece wise linear in ε with "kinks" occurring at the n reservations values, the zeroes of the n value functions. Let the n vector **a** represent the reservation values of the idiosyncratic shock for the n aggregate states and let the ith row of the n by n matrix **b** represent the slope coefficients associated with the n linear segments of the function $J_i(\varepsilon)$. Formally, the typical element of **a** solves

$$J_i(a_i) = 0, \; i = 1, 2, \ldots, n \tag{44}$$

and the typical element of **b** is defined as

$$b_{ij} = J_i'(\varepsilon) \forall \varepsilon \in (a_j, a_{j-1}) \text{ where } a_0 = \varepsilon_u, \; . \tag{45}$$

As (34) implies $a_i < a_{i-1}$ where $x_i > x_{i-1}$ without loss of generality, (45) has meaning.

Indeed, a differentiation of (34) evaluated at values of ε on each of the intervals specified in (45) implies that column $j \in \{1,\ldots,n\}$ of the matrix **b** is the solution to the following system of n linear equations:

$$(r + d + \lambda + \sum_{k \neq 1} \pi_{ik}) b_{ij} = \sigma + \sum_{k \geq j,\, k \neq 1} \pi_{ik} b_{kj}, \; i = 1, \ldots, n \; . \tag{46}$$

Having solved (46) for **b**, one can define the n piece wise linear functions

$$f_i(\varepsilon, a) = \max[J_i(\varepsilon), 0] = \sum_{j=k+1}^{i} b_{ij}(a_{j-1} - a_j) + b_{ik}(\varepsilon - a_k)$$

$$\forall\ \varepsilon \in [a_k, a_{k-1}),\ k = 1, \ldots, i \tag{47}$$

for i = 1,...,n. In the second step in the solution algorithm, the vector **a** is obtained as the unique solution to the following system on n non-linear equations implied by (34), (44), (46) and (47):

$$0 = x_i + \sigma a_i + \lambda \int_{a_i}^{\varepsilon_u} f_i(\varepsilon, a)\, dF(\varepsilon) + \sum_{j=1}^{i-1} \pi_{ij} f_j(a_i, a) \tag{48}$$

In the case at hand, n = 3. Given r+d = 0.01+0.02, λ = 0.065, σ = 0.3, and the transition matrix implied by ρ = 0.9, σ_v = 0.007, γ = 0.1, κ = 3 and the equations of (27)-(30), the solution to (46) is

$$b = \begin{bmatrix} 3.158 & 1.538 & 1.538 \\ 3.158 & 2.561 & 2.069 \\ 3.158 & 2.852 & 1.538 \end{bmatrix}. \tag{49}$$

Given x = (-.121,0,.121) and dF(ε) = .5 on [-1,1], and as assumed in our example and (49), one obtains the following solution to (48):

$$a = \begin{bmatrix} -.228 \\ -.448 \\ -.870 \end{bmatrix} \tag{50}$$

The contents of Table II, the aggregate state contingent transition rates to and from employment, the associated steady state employment ratioes, and the associated steady state job creation-destruction rates, are obtained by substituting $\varepsilon_d = \mathbf{a}$ and $J(\varepsilon_u) = f(1,\mathbf{a})$ into equations (32) and (33).

Table II: Aggregate State Contingent Equilibrium Parameters

x	α	δ	$\alpha/(\alpha+\delta)$	$\alpha\delta/(\alpha+\delta)$
-0.121	0.301	0.070	0.811	0.057
0	0.394	0.056	0.876	0.049
0.121	0.531	0.029	0.949	0.027

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