CREDIT CARDS AND BUYER PRICE PROTECTION

by

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ABSTRACT

In a duopoly model with homogeneous products, I show that allowing credit cards to offer buyer price protection will have both pro- and anti-competitive effects. Because of the buyer price protection, customers will be indifferent between purchasing an inexpensive and an expensive product as long as the difference in price is less than the cap on the refund allowed by the credit card. In order to obtain the entire market, a more efficient firm must charge prices lower than the marginal cost of the more inefficient firm. This is in contrast to the results obtained in pure Bertrand competition or when the firms themselves offer "meet the best price" clauses. The anti-competitive effects are similar to the results obtained when firms themselves offer "meet the best price clauses." The anti-competitive effects strengthen as the cost difference between the firms decreases and as the cap on the refund allowed by the credit card increases.
I. Introduction

It is well known in the industrial organization literature that allowing firms to offer "most-favored customer" or "meet the best price" clauses may be a facilitating practice that allows firms to charge a higher than competitive price.¹ With Bertrand or price competition in the absence of a "meet the best price" clause, even with only two firms in the market, the equilibrium price is the competitive price. If firms charge a higher than competitive price, there is an incentive to undercut each other. Only when profits are driven to zero is there no incentive for undercutting.

If there are symmetric firms offering "meet the best price" clauses, then they may be able to sustain higher prices. Firms have no incentive to reduce their price since lowering their price will not generate any more customers. If firms have asymmetric costs, even if they are allowed to offer "meet the best price" clauses, there will be no incentive to do so if one firm is substantially more inefficient than the other firm. In this case, only the efficient firm will remain in the market, although it will charge its monopoly price. If the inefficient firm is not "too" inefficient, then the clause allows firms to charge higher than Bertrand prices.

In this paper, I explore the effects of credit cards offering similar price guarantees. Recently, Citibank MasterCard has started offering to refund the difference

¹The "most favored customer" clauses facilitate a higher than competitive price by discouraging discounts. If one customer is offered a lower price, then the firm must offer all customers the lower price. By removing the incentive to discount, higher prices are sustained in the market. See Cooper (1986) or Salop (1986) for the results on "most favored customer" clauses.
in price if a customer purchases the good with their credit card and then sees it advertised for less later on. There are equilibria to this game only if there is a ceiling or cap on the amount that credit cards will refund to customers. I find that allowing credit cards to offer this price protection will have both pro- and anti-competitive effects. The equilibrium price offered by the firms will depend not only on the degree of inefficiency of the firms but also on the amount that the credit card company will refund. If the cap is in an “intermediate” range and the marginal cost of the inefficient firm is not too small, then the efficient firm will choose its price so that the inefficient firm will not enter the market. The price that the efficient firm charges in order to discourage entry is less than the inefficient firm’s marginal cost. When the firms offer “meet the best price” clauses, the lowest price seen in the market is the marginal cost of the inefficient firm.² If the cap is sufficiently large, regardless of the value of the marginal cost of the inefficient firm, then both firms will remain in the market. This is in contrast to the result when the firms alone offer a “meet the best price” clause. If the firms themselves are offering a “meet the best price” clause, then an inefficient firm with very high marginal costs will not enter the market.

The paper is divided into the following three sections: Section II specifies the Bertrand equilibria dependent on the marginal cost of the more inefficient firm. In Section III, I consider the equilibria that arise when firms offer “meet the best price” clauses. In Section IV, the effect of a third-party, such as a credit card vendor, offering a

²However, Belton(1987) shows that firms offering heterogeneous products may charge less than Bertrand prices when they offer a “meet the best price” clause.
lowest price guarantee is explored. Finally, the conclusion and extensions are considered in Section V.

II. Bertrand Competition

Consider two firms producing a homogeneous good in the market, with a market demand curve of \( Q = a - bP \). If \( P_i < P_j \) then \( Q_i = a - bP_i \), if \( P_i = P_j \) then \( Q_i = \frac{a - bP_i}{2} \) and if \( P_i > P_j \) then \( Q_i = 0 \) for \( i,j = 1,2 \) and \( i \neq j \). The two firms have asymmetric costs, an efficient one (firm E) with zero costs, and an inefficient firm (firm I) with total cost \( cQ_i \) (there are no fixed costs). The firm with the lower price gets the entire market and if they tie they split the market. I assume that all consumers are perfectly informed about prices.

If firms engage in Bertrand competition, one of two results will be obtained. If the inefficient firm is "very" inefficient, i.e. if the marginal cost of the inefficient firm is greater than the monopoly price of the efficient firm, than the efficient firm will charge its monopoly price, the inefficient firm will charge a price equal to its marginal cost and the efficient firm will capture the entire market. The inefficient firm has no incentive to lower its price since it will make a negative profit at prices lower than its marginal cost. The efficient firm has no incentive to raise or lower its price; it is the only firm in the market so the best it can do is charge the monopoly price.

If the inefficient firm is not very inefficient, i.e. if its marginal cost is less than the monopoly price of the efficient firm, then the efficient firm will charge a price slightly
less than the marginal cost of the inefficient firm, the inefficient firm will again charge a price of \(c\), and again the efficient firm will service the entire market. In the limit the price the efficient firm charges is the marginal cost of the second firm. Note that the monopoly price for the efficient firm is \(\alpha/(2b)\). In summary, with Bertrand competition the results are as follows:

**TABLE 1: EQUILIBRIUM PRICE, QUANTITY AND PROFITS UNDER BERTRAND COMPETITION**

<table>
<thead>
<tr>
<th>(c &gt; \alpha/(2b))</th>
<th>(P_E = \alpha/(2b);)</th>
<th>(Q_E = \alpha/2;)</th>
<th>(\Pi_E = \alpha^2/(4b);)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(P_1 = c)</td>
<td>(Q_1 = 0)</td>
<td>(\Pi_1 = 0)</td>
</tr>
<tr>
<td>(c \leq \alpha/(2b))</td>
<td>(P_E = c; P_1 = c)</td>
<td>(Q_E = a-bc;)</td>
<td>(\Pi_E = ac-bc^2;)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Q_1 = 0)</td>
<td>(\Pi_1 = 0)</td>
</tr>
</tbody>
</table>

III. Bertrand Competition with "Meet the Best Price" Clauses

If firms are allowed to offer a "meet the best price" clause, then again I obtain different results depending on the how large the marginal cost for the inefficient firm is. We can imagine this as a two-stage game, in the first stage, firms choose whether or not they will have the policy to "meet the best price" and in the second stage they choose
prices. Let \( \sigma_i = (x_i, P_i) \) denote the strategies of firm \( i = I, E \) where \( x_i = 0 \) if firm \( i \) offers a "meet the best price" clause and \( x_i = N \) otherwise. For the game tree, see Figure 1.

If both firms offer a "meet the best price" clause, then the demand facing each firm is given by the equation \( Q_i = \frac{a - b\{\min(P_E, P_i)\}}{2} \) for \( i = E, I \). If only one firm offers the clause then for that firm \( Q_i = \frac{a - bP_i}{2} \) if \( P_i \geq P_j \) and \( Q_i = a - bP_i \) if \( P_i < P_j \). The equilibria of this game depend on the degree of inefficiency of the lesser firm. To specify the equilibria of this game, we let \( \sigma = (\sigma_E, \sigma_I) = ((x_E, P_E); (x_I, P_I)) \). The strategies where the semicolon separates the strategies of the two firms. The first strategy in the set refers to the efficient firm. The second refers to the inefficient firm. An equilibrium in this game will be defined as a strategy \( \sigma_i \triangleright v_i, v_0 \).

\[ \Pi_i(\sigma_i^*, \sigma_{-i}^*) \geq \Pi_i(\sigma_i, \sigma_{-i}^*) \] where \( \Pi_E = P_EQ_E \) and \( \Pi_I = P_IQ_I - Cq_I \) where \( Q_i \) is defined as above and \( P_i = P_j \) if \( x_i = N \) or if \( x_i = O \) and \( P_i \leq P_j \); \( P_i = P_j \) if \( x_i = O \) and \( P_i > P_j \).

The following proposition shows the conditions under which allowing firms to offer "meet the best price" clauses yields the same equilibrium as ordinary Bertrand competition.

**Prop.1:** If \( c > \frac{a}{75} \), then there are two sub-game perfect Nash equilibria: \( ((O, \frac{a}{75}); (N, c)) \) and \( ((N, \frac{a}{75}); (N, c)) \).

If the inefficient firm's marginal cost exceeds the monopoly price of the efficient firm, then the inefficient firm will choose not to offer a "meet the best price" clause; the
efficient firm will be indifferent between offering and not offering to meet the best price. \( P_E \) is equal to its monopoly price, \( P_I \) is equal to its marginal cost and the efficient firm grabs the entire market. Neither firm has any incentive to deviate from this strategy. The inefficient firm will not meet the price of the efficient firm since this will generate negative profits. The efficient firm charges its monopoly price and therefore does the best it possibly can.

Proposition 2 shows conditions under which firms are able to charge higher than Bertrand prices.\(^3\)

\[
\text{Prop.2: If } c < \frac{a}{2b} - \frac{\sqrt{2a}}{4b}, \text{ then a sub-game perfect Nash Equilibrium is}
\]
\[
((O, \frac{a}{2b}); (O, \frac{a}{2b})).
\]

If the second firm is not very inefficient, then it will offer a "meet the best price" clause only if the price offered by the efficient firm is so high that the inefficient firm’s profit is non-negative. The efficient firm will offer the clause only if the profit that it gets from splitting the market is greater than the profit it can obtain alone by charging a price slightly below the inefficient firm’s marginal cost. The efficient firm can guarantee itself a profit of \( ac - be^2 \) so it is willing to offer a "meet the best price clause" if \( P[(a-bP)/2] \geq (a-bc)c \). Note that since the right hand side of that inequality does not depend on \( P \) and the lefthand side is equal to 1/2 times the monopoly profit, the

\(^3\) key strategy combination of the for \(((0,P_E);(0,a/(2b)))\) for \( P_E \geq a/(2b) \) or \(((0,a/(2b));(0,P_E))\) for \( P_E \geq a/(2b) \) will be a sub-game perfect Nash equilibrium.
equilibrium price that the efficient firm will charge if it offers a "meet the best price" clause is the monopoly price. The profit that the efficient firm gets when it splits the market with the inefficient firm at the monopoly price is greater than the profit that the efficient can guarantee itself by driving the inefficient firm out of the market if

\[
\frac{a^2}{8b} \geq c(a - bc) \Rightarrow c \leq \left[ 0, \frac{a}{2b} - \frac{a\sqrt{2}}{4b} \right]
\]

Proposition 3 details the values of the marginal cost of the inefficient firm that lead to equilibrium prices equal to Bertrand prices. However, the prices here are below the prices in Proposition 1.

\[
\text{Prop. 3: if } \frac{a}{2b} \geq c \geq \frac{a}{2b} - \frac{a\sqrt{2}}{4b}, \text{ then } P_E = c \text{ and } P_I = c.
\]

If \( a/2b \geq c \geq a/(2b) - a(2)^{5/4}/(4b) \), then the efficient firm will choose a price slightly less than the marginal cost of the inefficient firm (in the limit this price is equal to c), the inefficient firm chooses a price equal to its marginal cost. The efficient firm does better if it retains the entire market if \( a/2b \geq c \geq a/(2b) - a(2)^{5/4}/(4b) \), and therefore will set a price so as to force the inefficient firm to stay out of the market. The inefficient firm does not choose to offer a "meet the best price" clause, and the efficient firm is indifferent between offering and not offering the clause. In summary, the results from Bertrand competition where "meet the best price" clauses are allowed
TABLE 2: BERTRAND COMPETITION WITH "MEET THE BEST PRICE"

**CLauses:**

<table>
<thead>
<tr>
<th>Condition</th>
<th>( P_E = a/(2b); )</th>
<th>( Q_E = a/2; )</th>
<th>( Q_I = 0 )</th>
<th>( \Pi_E = a^2/(4b); )</th>
<th>( \Pi_I = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a/(2b) \geq c \geq \frac{a/(2b) - a(2)^{-5}/(4b)}{a/(2b) - a(2)^{-5}/(4b)} )</td>
<td>( P_E = c; P_I = c )</td>
<td>( Q_E = a - bc; Q_I = 0 )</td>
<td>( \Pi_E = ac - bc^2; )</td>
<td>( \Pi_I = 0 )</td>
<td></td>
</tr>
<tr>
<td>( c &lt; \frac{a/(2b) - a(2)^{-5}/(4b)}{a/(2b) - a(2)^{-5}/(4b)} )</td>
<td>( P_E = a/(2b); )</td>
<td>( Q_E = Q_I = a/4 )</td>
<td>( \Pi_E = a^2/(8b); )</td>
<td>( \Pi_I = a^2/(8b) - ac/4 )</td>
<td></td>
</tr>
</tbody>
</table>
IV. Credit Cards and "Meet the Best Price" Clause

I consider a scenario where the efficient firm picks its price first and then the inefficient firm acts as a Stackelberg follower. In other words, prices are chosen sequentially rather than simultaneously. If prices are chosen simultaneously, there will not be a pure strategy equilibrium in prices. The credit card vendor refunds the difference between the lowest price and the highest price up to an exogenously set cap on the refund amount. Therefore, consumers are indifferent between purchasing from firm E at $P_E$ and purchasing from firm I at a price of $P_E + \text{cap}$. The results can be divided into five cases which depend on the marginal cost of the inefficient firm.

Case 1: $c \geq a/(2b)$

If the inefficient firm has marginal cost greater than the monopoly price of the efficient firm, then there are three results that can be obtained depending on the cap that the credit card vendor places on the refund allowed to consumers. If the cap is very small, i.e. if $\text{cap} < c - a/(2b)$, then the efficient firm will charge its monopoly price. The inefficient firm will charge its marginal cost. All consumers will purchase from the efficient firm since $c - \text{cap} > a/(2b)$, or the actual price if a consumer purchases from the inefficient firm is greater than the price if the consumer purchases from the efficient firm.

If the cap is very large (but not infinite), then the efficient firm will be forced to split the market and will therefore charge its monopoly price while the inefficient firm
charges a price of $a/(2b) + \text{cap}$. There is no incentive for either firm to deviate from these prices. Even at $P_E = 0$, the inefficient firm charges $P_T = \text{cap} > c$ and makes a profit.

The efficient firm will lower its price so as to obtain the entire market only if the profit it gets with the whole market at this lower price is greater than the profit it obtains by splitting the market. If the efficient firm splits the market it will charge its monopoly price of $a/(2b)$ and will therefore generate profits of $a^2/(8b)$. Therefore, the efficient firm will limit the profit of the inefficient firm out of the market only if the profit it gets by supplying the whole market is greater than the profit by splitting the market. This means that the efficient firm is looking for a price, $P_E$, such that $P_E (a - bP_E) \geq a^2/(8b)$. Therefore, the efficient firm will "limit price" the second firm out of the market only if the price that it must charge to get the entire market is above $a/(2b) - a(2)^{-1}/(4b)$.

The inefficient firm will not enter the market if its profits are less than zero. In other words, if $\Pi_I = (P_E + \text{cap})(a - bP_E)/2 - c(a - bP_E)/2 < 0$, then the inefficient firm will not enter the market (or will enter the market but not get any customers). But $\Pi_I < 0$ if $(P_E + \text{cap} - c)(a - bP_E)/2 < 0$ or if $P_E + \text{cap} - c < 0$. Therefore the zero profit condition for the inefficient firm is $\text{cap} = c - P_E$. The inefficient firm will make negative profits if $\text{cap} < c - P_E$. See Figure 2.

This generates three areas of credit card caps. In Area 1 (again see Figure 2), $\text{cap} < c - a/(2b)$. The efficient firm will charge its monopoly price $P_E = a/(2b)$. Since the profits of the inefficient firm are negative at $P_T = a/(2b) + \text{cap}$, the inefficient firm
charges its marginal cost \( c \) but gets no customers. In Area II, \( \text{cap} \geq c - a/(2b) \) but \( \text{cap} < c - [a/(2b) - a(2)^{-5/4}(4b)] \). In this region the efficient firm will limit price the inefficient firm out. This is because the profit that the efficient firm obtains when it splits the market is less than the profit it obtains when it charges the highest effective limit price. This limit price is epsilon less than the price that would give the inefficient firm zero profits. Since the zero profit condition for the inefficient firm is \( \text{cap} = c - P_E \), the efficient firm charges a price of \( P_E = c - \text{cap} - \epsilon \) (as \( \epsilon \) goes to zero, \( P_E = c - \text{cap} \)). The inefficient firm charges \( P_I = c \) and the efficient firm gets the entire market.

Note that the price that customers face in this situation is less than both the Bertrand price and the price that appeared when the firms alone offered "meet the best price" clauses. With homogeneous goods, the efficient firm never charges less than the marginal cost of the inefficient firm to get the entire market, even when one or both of the firms is offering a "meet the best price" clause. Here, the efficient firm needs to charge less than the marginal cost of the inefficient firm in order to guarantee itself the entire market. The credit card vendor is not paying out anything since customers are only purchasing from the efficient firm. The presence of the credit card vendor is enough to generate prices below Bertrand equilibrium prices.

Finally, in Area III of Figure 2, both firms are in the market. The efficient firm would prefer to split the market since the price that it would have to charge to ensure itself the entire market is so low that its profits are greater by charging the monopoly price and splitting the market with the inefficient firm. In this case, \( P_E = a/(2b) \) and \( P_I = a/(2b) + \text{cap} \). Note that if \( \text{cap} \geq bc^{-2/(a-bc)} \), then the efficient firm would prefer to
be a Stackelberg follower and charge the inefficient firm’s monopoly price plus the cap. However, \( P_E = a/(2b) + c/2 + \text{cap} \), \( P_I = a/(2b) + c/2 \) cannot be an equilibrium if the efficient firm must pick its price first. This is because the inefficient firm would prefer to pick a price slightly less than its monopoly price and get the entire market.

The results of a credit card vendor offering a "meet the best price" clause when \( c > a/(2b) \) can be summarized in the following proposition.

**Prop. 4:** If \( c > \frac{a}{2b} \), then

a. If \( \text{cap} \geq c - \left[ \frac{a}{2b} - \frac{a \sqrt{2}}{4b} \right] \), then \( P_E = \frac{a}{2b} \) and \( P_I = \frac{a}{2b} + \text{cap} \).

b. If \( c - \left[ \frac{a}{2b} - \frac{a \sqrt{2}}{4b} \right] \geq \text{cap} > c - \frac{a}{2b} \), then \( P_E = c - \text{cap} \) and \( P_I = c \).

c. If \( \text{cap} < c - \frac{a}{2b} \), then \( P_E = \frac{a}{2b} \) and \( P_I = c \).

Case 2: \( a/(2b) > c \geq \varepsilon \)

If \( c < a/(2b) \), then the line \( \text{cap} = c \cdot P_E \) shifts inward until Area I disappears. So if the cap is sufficiently small (\( \text{cap} \leq c - a/(2b) + a(2)^{-5}/(4b) \)), the efficient firm will limit price the inefficient firm out of the market (this corresponds to Area II in Figure 3). If \( c > a/(2b) + a(2)^{-5}/(4b) \) the efficient firm would prefer to split the market at its monopoly price corresponding to Area III in Figure 3. In Case 2 we must consider whether the inefficient firm will choose a price of \( P_I = a/(2b) + \text{cap} \) or will undercut.
the efficient firm by charging \( P_1 = a/(2b) - \text{cap} - \epsilon \) and get the entire market. We did not have to consider this additional constraint in Case 1 since the marginal cost of the inefficient firm in Case 1 is less than \( a/(2b) \) and the inefficient firm would make negative profits by undercutting the efficient firm. This constraint for the inefficient firm is that its profit from charging \( P_E + \text{cap} \) and splitting the market must be greater than or equal to its profit from charging \( P_E - \text{cap} \) and obtaining the entire market.

\[
\Pi_I (\text{splitting market}) \geq \Pi_I (\text{whole market}) \Rightarrow \\
\left[ \frac{a - bP_E}{2} (P_E + \text{cap} - c) \right] \geq [a - b(P_E - \text{cap})](P_E - \text{cap} - c)
\]

This relationship can be rewritten as follows:

\[
(a - bP_E)(P_E - c) + (a - bP_E)\text{cap} \geq \\
2(a - bP_E)(P_E - c) - 2(a - bP_E)\text{cap} + 2b\text{cap}(P_E - c) - 2b\text{cap}^2 = \\
(a - bP_E)(P_E - c) - 3(a - bP_E)\text{cap} + 2b\text{cap}(P_E - c) - 2b\text{cap}^2 \leq 0
\]

When this constraint is graphed it generates two areas. See Figure 4. We can restrict attention to the area above the upper hyperbola since the lower area corresponds to
negative values of the credit card cap.\textsuperscript{4}

If \( c \) is sufficiently large, then at \( \text{cap} = c - a/(2b) + a(2)^{-5}(4b) \) the constraint given in the equation above is less than or equal to zero at \( P_E = a/(2b) \). This means that the inefficient firm will not choose to undercut the efficient firm when the efficient firm chooses to switch from limit pricing the inefficient firm out of the market to splitting the market with the inefficient firm. In fact we can solve explicitly for a lower bound of \( c \) so that at \( c \) the profit difference given by the above equation is equal to zero. If this profit difference is monotonically decreasing in \( c \) evaluated at \( \text{cap} = c - a/(2b) + a(2)^{-5}(4b) \), then we know that for \( c > c \), the inefficient firm will not undercut the efficient firm when it charges \( P_E = a/(2b) \). The exact functional form of \( c \) is a complicated function of the choke price \( a/b \).\textsuperscript{5} The value of \( c \) is real only if the choke price \( a/b \) is between 0.393 and 302.487. Also, it is an increasing function of \( a/b \). See Table 1 for some calculated values of \( c \). In Figure 6, I have included the constraint

\textsuperscript{4}In Figure 5 I have graphed the constraint that the profit for the inefficient firm is greater when it splits the market than when it undercut the whole market for \( a = 1 \), \( b = 50 \). In this case, the lower hyperbola cuts the price axis. However, this does not mean that for a cap of zero, the efficient firm can charge a price greater than zero. If the efficient firm charges a high price, the profits for the inefficient firm may be the higher if the inefficient firm charges a price of \( P_E + \text{cap} \) and splits the market rather than charging a price of \( P_E - \text{cap} \) and getting the entire market. However, the inefficient firm need not charge a price as high as \( P_E - \text{cap} \) instead it may charge a much lower price. Again, we need only look at the area defined by the upper hyperbola.

\textsuperscript{5}The exact functional form of \( c \) is as follows:

\( c = -F/48 - (-F^2/2304 + G/48b)/((H + I)^{1/3}) \times (1/(H + I))^{1/3} + \sqrt{(H + I)^{1/3}} \)

where \( F = 16 - 16a/b + 8(2)^{-5}a/b \), \( G = 6a(2)^{-5} + 6a/b - 6a(2)^{-5}b \), \( H = -1/27 - 5a^2/(42b^3) + 7(2)^{-5}a^2/(1864b^2) + a^2/(16b^2) - (2)^{-5}a^2/(188b^2) + a/(9b) - a(2)^{-5}b/(772b) \), \( I = (1/6 - 10(2)^{-5})a^2 - (11/6 - 11a(2)^{-5})a^2b^2 + (472 - 168(2)^{-5})a^2b^3 - (144 - 32(2)^{-5})a^2b^5 \), and \( J = 68(6)^{-5}b^2 \).
on the inefficient firm. Figure 6 uses the demand function \( Q = 1 - P \) when \( c = \varepsilon = .25866 \). Notice that at cap = \( c - a/(2b) + a(2)^{-5}/(4b) \) and the profit difference for the inefficient firm is equal to zero. In other words, the inefficient firm is indifferent between splitting the market or undercutting the efficient firm when the efficient firm charges its monopoly price and the cap = \( c - a/(2b) + a(2)^{-5}/(4b) \). In the Appendix I show that this profit difference is monotonically decreasing in \( c \).

Case 3: \( \varepsilon > c \geq a/(2b) - a(2)^{-5}/(4b) \)

If the cap is small then the efficient firm will choose to limit price the inefficient firm out of the market. This corresponds to Area II in Figure 7 for cap = \( c - a/(2b) + a(2)^{-5}/(4b) \). If the cap is sufficiently large, then the two firms will share the market with the efficient firm choosing \( P_E = a/(2b) \) and the inefficient firm choosing \( P_I = a/(2b) + \text{cap} \). This corresponds to Area III in Figure 7. There is also an intermediate range in the cap where the efficient firm will either limit price the inefficient firm out of the market or will choose to split the market with the inefficient firm at a price that makes the inefficient firm indifferent between splitting the market and undercutting the efficient firm. This corresponds to Area IV in Figure 7. For low values of the cap in Area IV, the efficient firm will choose to limit price the inefficient firm out of the market. As the cap increases, the profit from limit pricing decreases and the profit from splitting the market increases. Since the limit pricing profit is monotonically decreasing and the profit from splitting the market is monotonically increasing, we know that there is a unique cap* (dependent on \( a/b \) and \( c \)) at which the
efficient firm will switch from limit pricing to splitting the market. In either case, the inefficient firm is choosing its price $P_I = P_E + \text{cap}$ but is not getting any customers until the cap $> \text{cap}^*$.

Case 4: $a/(2b) - (a(2)^{5/4}b^2) > c > 0$

Case 4 differs from Case 3 in that there is no region where the efficient firm definitely prefers to limit price the inefficient firm out. If the cap is very small (cap $< c$), the efficient firm will compare the profit from limit pricing to the profit from splitting the market. This corresponds to Area IV in Figure 8. For small values of the cap, limit pricing is the preferred strategy for the efficient firm. As in Case 3, there is a unique cap* where the efficient firm will prefer to split the market rather than limit price the inefficient firm out.

For cap $\geq c$, the efficient firm will split the market with the inefficient firm choosing a price to make the inefficient firm indifferent between splitting the market and undercutting the efficient firm. This corresponds to Area V in Figure 8.\textsuperscript{6}

The efficient firm will never choose to price higher than its monopoly price even when it splits the market with the inefficient firm. Therefore for sufficiently high values of the cap, $P_E = a/(2b)$ and $P_I = a/(2b) + \text{cap}$. This corresponds to Area III in Figure 8.

\textsuperscript{6}There may be an area corresponding to Area V in Case 3 also depending on c and a/b.
Case 5: \( c = 6 \)

If the inefficient firm has marginal costs of zero then the efficient firm can never guarantee itself the entire market no matter what price it charges. In this case the efficient firm must charge a price, \( P_E \), so that the inefficient firm's profit from splitting the market is greater than or equal to the profit it gets by undercutting the efficient firm and getting the entire market. If the inefficient firm charges \( P_I = P_E + \text{cap} \), then the two firms will split the market and the inefficient firm's profit will be

\[
\Pi_I = \frac{(a - bP_I)}{2} \times (P_E + \text{cap}).
\]

If it undercuts the efficient firm, it will charge \( P_I = P_E - \text{cap} \) (in the limit, this is equal to \( P_E - \text{cap} \)) and the inefficient firm will serve the entire market. In this case, the inefficient firm's profit will be

\[
\Pi_I = \frac{(a - b(P_E - \text{cap}))}{(P_E - \text{cap})}.
\]

Therefore, the efficient firm cannot always guarantee itself half the market if it charges its monopoly price. The efficient firm must charge a price lower than its monopoly price if the cap is too low. This is because if the cap is too low, the inefficient firm would prefer to undercut the efficient firm and get the entire market rather than charge the higher price and split the market with the efficient firm. The condition that the profit for the inefficient firm is greater when it splits the market than when it undercuts and gets the entire market is given below:

\[
\Pi_I(\text{splits market}) \geq \Pi_I(\text{whole market}) \Rightarrow
\]

\[
\frac{a - b}{2} (P_E + \text{cap}) \geq [c - b(P_E - \text{cap})] (P_E - \text{cap})
\]
This relationship can be rewritten as follows:

\[(a - bP_E)P_E - 3(a - bP_E)\text{cap} + 2b\text{cap}P_E - 2b\text{cap}^2 \leq 0 \Rightarrow\]

\[(P_E - 3\text{cap})(a - bP_E) - 2b\text{cap}(\text{cap} - P_E) \leq 0\]

Notice from the above expression that a sufficient condition for the inefficient firm to prefer to charge \(P_E + \text{cap}\) is that \(\text{cap} > P_E\). In other words, if the \(\text{cap}\) is higher than the efficient firm’s monopoly price, then the efficient firm will charge its monopoly price and the two firms will split the market. However, \(\text{cap} > P_E\) is not a necessary condition for the inefficient firm to prefer to split the market. In Figure 9 I have graphed the conditions given above for the example \(a = 1, b = 1\). This graph tells us the relationship between the \(\text{cap}\) and the price the efficient firm can charge. If the \(\text{cap}\) is low, then the efficient firm will charge a price so that the inefficient firm is indifferent between charging a high price and splitting the market with the efficient firm or charging a lower price and getting the whole market. This corresponds to Area V on Figure 9. If \(\text{cap} \geq a/(4b)\) then \(P_E = a/(2b)\). If \(\text{cap} < a/(4b)\), then \(P_E = a/(2b) - 2.5\text{cap} - (c(a/b - 5\text{cap})^2 - 4\text{cap}(3a/b + 2\text{cap}))^{1/2}/2\). In either case, \(P_I = P_E + \text{cap}\).

Notice that with a credit card vendor, even if \(c > a/(2b)\) both firms can remain in the market if the credit card \(\text{cap}\) is sufficiently high. This is contrast to the result obtained when the firms alone offer a "meet the best price" clause. If only firms can offer a "meet the best price" clause, then both firms remain in the market only if \(c < a/(2b) - ab(2)\).
IV. Conclusion and Extensions

I have derived an equilibrium where when a third-party, such as a credit card vendor, is allowed to offer a buyer price protection by giving refunds, there are both pro- and anti-competitive effects. The pro-competitive results come from the fact that with homogeneous products, the efficient firm will never have to charge a price below the marginal cost of the inefficient firm in order to keep the inefficient firm out of the market. If a credit card vendor is making up the difference in price to consumers, then the efficient firm must charge a price less than the marginal cost of the inefficient firm in order to keep the entire market. The anti-competitive results follow because an inefficient firm can remain in the market even when the inefficient firm has marginal cost greater than the monopoly price of the efficient firm. In this case, even though the number of competitors increased, the amount of competition decreased with the introduction of this clause by the credit card companies. I am currently working on two extensions of the above model. First, I am looking at the effects of the decreased competition on the amount of research and development engaged in this market. The second extension I am working on is to include the role of advertising in the model. In reality, consumers are not completely informed about prices in the market. To gain consumers, firms must advertise their prices. However, by advertising their prices, consumers become indifferent between which firm they purchase from since the difference between the advertised price and the actual price paid by consumers is refunded by the credit card.
It is important to understand why credit cards offer services such as the buyer protection discussed in this paper. Credit card interest rates seem to be sticky downward relative to the cost of funds.\textsuperscript{7} When prices are sticky downward, firms compete on other dimensions (as evidenced by the airline industry before deregulation). Credit card vendors may be unwilling to compete on interest rates because of moral hazard and adverse selection problems. Therefore, non-price competition such as buyer price protection may become more prevalent in the market.

\textsuperscript{7}See Ausubel(1991).
FIGURE 3

\[ \text{cap} = c \cdot P_E \]

\[ c \]

\[ c - P_E \]

\[ P_E \quad c \quad a/(2b) \]

Price of Efficient Firm
FIGURE 4

$Q = 1, b = 1, c = 0.25866$
\( a = 1, \ b = 50, \ c = 0 \)
Example: $Q = 1 - P (a = 1, b = 1)$. Here, $a/(2b) = .5$ and $c = .25866$ and $P_E = .164447$
$c = \xi$

Area II: $0 \leq \text{cap} \leq P_E$
Area III: $c - P_E \geq \text{cap} \geq c$
FIGURE 7

\[ C = 0.17, \quad a = 1, \quad b = 1 \]
FIGURE 8

$c = 0.09, a = 1, b = 1$
FIGURE 9

\[ P_E = \frac{a}{2b}, \quad P_I = \frac{a}{2b} + \text{cap} \]

\[ P_E = \frac{a}{2b} + 2.5 \text{cap} - \frac{\sqrt{\left(-\frac{9a}{b}\right) - 5 \text{cap}}}{4 \text{cap} \left(\frac{3a}{b} + 2 \text{cap}\right)} \]

\[ P_I = P_E + \text{cap} \]
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<th>$a/b$</th>
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<th>$\bar{c}$</th>
<th>$c \cdot a/(2b) + a(2)^{5/4}$</th>
<th>$(4b)$</th>
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Proof of Proposition 1:

Suppose $P_E^* = a/(2b)$. Then it is a dominant strategy for $i$ to choose $x_i = N$ since $0 = \Pi_i((x_E^*, a/(2b)); (N, P_i)) > \Pi_i((x_E, a/(2b)); (O, P_i)) > 0 \forall P_i$. Therefore, $(x_i^*, P_i^*) = (N, c)$. Suppose $(x_E^*, P_E^*) = (N, c)$. For any $P_E \leq c$, $\Pi_E((x_E, P_E); (N, c)) = P_E(a-bP_E)$. $P_E(a-bP_E)$ is maximized at $P_E = a/(2b)$ yielding $P_E = a^2/(4b)$. If $P_E < c$ then $\Pi_E((N, P_E); (N, c)) = 0$ and $\Pi_E((O, P_E); (N, c)) = c(a-bc)/2$. Since $c > a/(2b)$ then $a^2/(4b) > c(a-bc)/2$. Therefore $(x_E^*, P_E^*) = \{(O, a/(2b), (N, a/(2b))\}$.

Proof of Proposition 2:

Suppose $(x_E^*, P_E^*) = (O, a/(2b))$. Claim: $(x_i^*, P_i^*) = (O, a/(2b))$.

\[ \Pi_i((O, a/(2b)); (O, a/(2b))) > \Pi_i((O, a/(2b)); (O, P_i)) \forall P_i \text{ since} \]
\[ \Pi_i((O, a/(2b)); (O, a/(2b))) > \Pi_i((O, a/(2b)); (O, P_i)) \text{ for } P_i \leq a/(2b) \text{ and} \]
\[ \Pi_i((O, a/(2b)); (O, a/(2b))) = \Pi_i((O, a/(2b)); (O, P_i)) \text{ for } P_i \geq a/(2b). \]

Also, $\Pi_i((O, a/(2b)); (N, a/(2b))) \geq \Pi_i((O, a/(2b)); (N, P_i)) \forall P_i$ since $\Pi_i((O, a/(2b)); (N, a/(2b))) \geq \Pi_i((O, a/(2b)); (N, P_i))$ for $P_i \leq a/(2b)$ since $d\Pi_i/dP_i \geq 0$ for $P_i \leq a/(2b) + c/2$ and $P_i((O, a/(2b)); (O, a/(2b))) = \Pi_i((O, a/(2b)); (O, P_i)) = 0$ for $P_i > a/(2b)$.

Finally, $\Pi_i((O, a/(2b)); (O, a/(2b))) \geq \Pi_i((O, a/(2b)); (N, a/(2b)))$. Therefore, $\Pi_i((O, a/(2b)); (O, a/(2b))) \geq \Pi_i((O, a/(2b)); (N, P_i)) \forall P_i$.

Suppose $(x_E^*, P_E^*) = (O, a/(2b))$. Claim: $(x_i^*, P_i^*) = (O, a/(2b))$.

If $P_E > c$ then
\[ \Pi_E((O, P_E); (O, a/(2b))) = P_E(a-bP_E)/2 < a^2/(8b) \text{ if } P_E < a/(2b) \]
\[ \Pi_E((O, P_E); (O, a/(2b))) = P_E(a-bP_E)/2 \text{ which is maximized at } P_E = a/(2b) \text{ yielding} \]
\[ a^2/(8b) \text{ if } P_E \geq a/(2b). \]
\[ \Pi_E((N, P_E); (O, a/(2b))) = 0 \text{ if } P_E > a/(2b) \]
\[ \Pi_E((N, P_E); (O, a/(2b))) = P_E(a-bP_E)/2 \leq a^2/(8b) \text{ if } P_E \leq a/(2b) \]
If \( P_E \leq c \), then

\[
\Pi_c((O,P_E);(O,a/(2b))) = P_E(a-bP_E)/2 < a^2/(8b) \text{ for all } P_E < c
\]

(since \( c < a/(2b) - a(2)^{-5}/(4b) \)).

Since \( a^2/(8b) \geq P_E(a-bP_E)/2 \), \( (x_E^*,P_E^*) = (O,a/(2b)) \).

Show that the profit difference for the inefficient firm between splitting the market and undercutting the efficient firm is monotonically decreasing in \( c \) evaluated at \( \text{cap} = c - a/(2b) + a(2)^{-5}/(4b) \).

\[
\begin{align*}
\text{PD} &= \text{Profit Difference} = (a - bP_E)(P_E - c) - 3(a - bP_E)\text{cap} + 2bcP_E\frac{P_E - c}{2} - 2bcP_E^2 \\
\text{dPD/dc} &= -(a - bP_E) - 3(a - bP_E)\frac{\text{cap}}{dc} + 2bP_E\frac{\text{dcap}}{dc} - 2bc\frac{\text{dcap}}{dc} \\
&\quad - 2b(c - a/(2b) + a(2)^{-5}/(4b)) - 2bc \\
&\quad - 2b(c - a/(2b) + a(2)^{-5}/(4b))^2 - 4bc(c - a/(2b) + a(2)^{-5}/(4b)) \\
\text{Evaluating } \text{dPD/dc at } P_E = a/(2b) \text{ yields:} \\
\text{dPD/dc} &= -a/2 - 3a/2 + a - 2bc - a/(2b) + a(2)^{-5}/(4b)(1 + c - a/(2b) + a(2)^{-5}/(4b)) \\
&\quad - 2bc < 0
\end{align*}
\]
References:


