Discussion Paper No. 957

PARALLEL SEARCH: AN APPLICATION TO R & D

by

Tara Viswanath
Department of Economics
Northwestern University
Evanston, IL 60208

Revised August 1991

An earlier version of this paper was presented at the 1991 Econometric Society Summer meetings, Philadelphia, and at the 1991 International Conference on game theory and economics, Stonybrook, New York.

1 Introduction

This paper examines R & D race models viewing the problem of project investments by firms in a search framework. Such an approach is appropriate when the outcome or performance level of each project is uncertain.

The general context of the R & D problem considered may be roughly described as follows. Each firm has a certain number of, possibly different, projects. The uncertainty associated with each can be removed by paying a search or an exploration cost. Competing firms invest in search, simultaneously, selecting one or more projects each time. The outcomes produced from search can be carried over into the future. Reward is collected when all firms terminate search. The reward structure is winner-take-all, i.e., the firm which has produced the highest outcome from the search it undertook, gets all of it; and the reward for other firms (producing a lesser outcome) is zero. The objective for each firm is to maximize the expected discounted reward net of the search cost it pays, by selecting an appropriate search strategy to explore its set of projects.

Several R & D situations fit into (albeit in a highly simplified way) the above scenario. Consider for example, the patent context. If the outcome of each project is a distinct product with a certain quality index, and if the product with the highest quality appropriates all the market surplus, then patenting a product may not be profitable (especially, if there is a cost to get the patent) if it expects a competing firm to generate a higher quality product in the near future. In this situation, the search problem described above is relevant for when to seek a patent.

As an another example, consider several firms that are in a position to deliver a product or service (for example, a communication service). Each project is a substitutable technology, and its search or developmental trial reveals its service value. The firm that generates the highest service value is licensed to deliver the service. (or if more than one is licensed to deliver the best technology, the post-adoption rents are high for the innovator). [1]

The framework and approach are different from the R & D studies that focus on effort levels to finish first in a time race to get a fixed known prize(for example, the studies of Loury (1979), Lee and Wilde (1980), Harris and Vickers (1985), Dixit (1987), and others). It is a race in terms of generating the highest reward from exploring projects with uncertain benefits. The models fall into the class of R & D problems where information production is involved (Allen (1991), Dasgupta (1990), Reinganaum (1983)).[2]

The models analyzed in this paper seek to examine race under uncertainty in a search framework. Investing in R & D projects are viewed as search decisions. Issues addressed include project investments and search decisions in the race, race termination conditions and completion times, conditions for parallel search activity, the influence of distributions characterizing project uncertainty, the effect of continuing R & D opportunities (repeated search), project selection order, and how race outcomes may differ from that of planner's decisions. Models considered include finite and infinite horizon races.

In Section 2.0, a single stage game is analyzed with each firm having access to a single R & D opportunity. The firms enter the game with their own initial technologies. The equilibrium search decisions are characterized and it is shown that, in all cases, there is project overinvestment in the race relative to the planner's problem. It is also shown that there exists a threshold level of initial technology lead beyond which no firm will invest in R & D activity, in this single stage game.

[1](To cite an example of this situation, in the telecommunication industry, at present, there are more than fifty license holders to investigate alternative technologies for delivering personal communication services, via pocketable phones and an appropriate network infrastructure. The Federal Communication Commission is interested in the technology delivering the highest consumer surplus. There is a great deal of uncertainty about which technology will actually provide the best surplus. The regulatory questions are not of interest in this paper. Rather, the interest is in getting some insights into how firms respond in the presence of projects with uncertain outcomes.)

[2]In many instances, one can equate time with reward, but more insights can be gained by keeping them separate. Additionally, in most of the effort-time models, time as a function of efforts is assumed to be an exponential random variable. No such specific assumption is made here.

In Section 2.1, repeated R & D activity is considered wherein each firm can invest in a project per period (each project yielding after one period). Equilibrium strategies, race termination condition, and completion times are analyzed for finite and infinite horizon games (with identical distributions in each period for repeated search). During the course of the race the lead (advantage in terms technology secured thus far) and the leading firm may change from stage to stage. The race termination can be characterized in terms of a threshold lead (which depends on the distributions, search cost, and the discount rate). When the lead is larger than this threshold, the race ends. Additionally, there is project overinvestment in the race, and the race takes longer than the planner's search duration. Conditions under which parallel search and sequential search are undertaken in the race are characterized.

When both firms have continuing prospects of innovation(extending or infinite horizon), and if the search costs(per project) are low, then both search in parallel and stop together(pure strategy Nash equilibrium). In this case of low costs, rents to each firm from the race dissipate. Higher costs result in the lagging firm dropping out of the race (at a certain lead) when it is still better for the leading firm to continue search and gain positive returns. Thus, rival's threat is continued and dissipating when costs are low; whereas if costs are high and R & D prospects are good, a firm has positive returns once it gets past its rival. (This model is different from Harris and Vickers(1985), who consider effort-time race with intermediate stages and fixed termination points for either player. Among other differences, the termination points are endogenous here). It is also shown that better discovery environments lead to longer race times. Additionally, the invariance result of Sah and Stiglitz(1986), race investments coinciding with that of planner, is shown to not hold in repeated R & D environments; if the discount rate is high enough, the planner always prefers sequential search, whereas there can be parallel search in the race.

In Section 3, race with firms having heterogeneous alternatives is considered. Given rival firm's behavior, each firm faces a systematic search problem, i.e., search involving distinct alternatives. For the (partial equilibrium) search problem of a firm, Weitzman's reservation rule (1979) applies, if only one project at a time search is considered. This rule is not optimal, in general, when time is discounted. In general, a parallel search strategy (allowing for each firm to select one or more of its projects at a time) is optimal, as shown in Vishwanath (1991). Although parallel search problem is complex in general, when the distributions associated with projects satisfy certain stochastic ordering conditions, the optimal search order can be characterized in a simple manner. The approach taken here in analyzing the R & D race considered is via the application of results in Vishwanath (1991). The stochastic ordering conditions enable the analysis of risk choices of firms. (Bhattacharya and Mookherjee (1986), consider, in their analysis of risk choices, a one stage game with each firm choosing only one project from its set. Without cost considerations, as is the case in their model, the best strategy for each firm is to do all projects simultaneously regardless of risk considerations. The model addressed here incorporates cost considerations, and is dynamic. The ordering conditions on the distributions given here are also slightly more general.) The (pure strategy) equilibrium search order is shown to be predetermined, and race termination here is shown to have a simple myopic characterization. Concluding remarks are in Section 4.

2 A Single Stage R & D Game

In this section, a single-stage R & D game involving two firms is considered, with each firm having one R & D project. This example seeks to illustrate the nature of search decisions, equilibrium outcomes, and how the research investment in the race may differ from that of a social planner. It also illustrates some distributional effects on the outcomes (i.e., how the distributions associated with the projects influence equilibrium outcomes). In addition, it is useful for the race completion time analysis undertaken later. Some aspects of the relationship of the model considered here to other models in the literature will also be discussed.

Suppose that each firm has an initial technology, the reward (or the performance value) from which is known, and an R & D opportunity, the reward from which is uncertain. Each firm faces the decision of whether or not to invest in (or search) its project. Let $y_i \geq 0$ denote the initial reward for firm $i = 1, 2$. That is, the two firms enter the game with these values. If neither firm undertakes any further R & D effort, the firm with the higher of the two values is the winner. The reward, then, for firm $i = 1, 2$ is y_i if $y_i > y_j$, $j \neq i$; and equals zero if $y_i < y_j$. (If the two are tied, assume both get an equal division: the assumption of division matters little if the probability of ties is zero, as will be the case in the analysis later.) To begin with, suppose the reward distributions associated with the projects are identical and independent. Let $F(x)$ denote this distribution (assumed continuous throughout a bounded support $[0, b]$). To explore each project, a (contractual) cost $c > 0$ must be paid at the beginning. Assume that the results of the exploration will be known after one period, and let β $(0 < \beta \leq 1)$ represent the discount rate for a period. Single-stage means that all R & D activity takes place in one period (and it ceases after the first period).

Consider, first, the planner's problem. From this viewpoint there are two projects, and the initial reward before any search is undertaken is $y = \max(y_1, y_2)$. Then, the value[3] or the expected discounted reward to the planner from searching k projects $(k = 1, 2)$, denoted $V_p(y; k)$ is given by

$$V_p(y; k) = -kc + \beta F^k(y)y + \int_y^b x dF^k(x), \text{ for } k = 1, 2. \tag{1}$$

Note that if k projects are chosen, cost kc is incurred at the beginning. If the outcome from

[3]Both the planner and the firms are assumed risk neutral throughout the paper. Also, in all the models analyzed here yield time of a project is assumed to be one period, for simplicity. More details of project features, including pay-as-you-go or running costs, can be built in.

search (the distribution of the maximum of the rewards from k projects is $F^k(x)$) is no better than the fallback, then the initial technology is implemented though after one period. The third term on the right side of (1) accounts for better outcomes from search.

The value to the planner from optimal search is $V_p(y) = \max_k V_p(y;k)$. Thus search is undertaken if $y < V_p(y)$; and is abandoned if $y \geq V_p(y)$.

To analyze the planner's search investments further, for $k = 1, 2$, let R_k denote the reservation reward, determined (uniquely) by:

$$y \gtrless R_k \quad \text{as} \quad y \gtrless V_p(y;k). \tag{2}$$

Let $R = \max(R_1, R_2)$. Rewriting (1) as

$$V_p(y;k) = -kc + \beta y + \beta \int_y^b [1 - F^k(x)]dx,$$

it is easy to see that the marginal returns from undertaking the second project (over and above undertaking one) is

$$V_p(y;2) - V_p(y;1) = -c + \beta \int_y^b F(x)[1 - F(x)]dx, \tag{3}$$

which is decreasing in y. (It is of some interest to observe that the marginal returns to search need not decline with the number of projects; the returns represented by (3) may or may not be less than $V_p(y;1) - y$, which is the marginal returns to exploring the first project. This is due to the fact that search involves an opportunity cost from delaying the initial reward when $\beta < 1$, whereas once the search decision is undertaken additional projects do not incur such costs). Let $\tilde{y}$ denote the initial reward such that $V_p(\tilde{y};2) - V_p(\tilde{y};1) = 0$.

The planner's optimal search decision may now be characterized as follows:

(a) if $\tilde{y} > R_2$, then (4)

search two projects if $y < R_2$;

and abandon search if $y \geq R_2$.

$$\text{(b) if} \quad 0 \leq \tilde{y} < R_2, \text{ then} \tag{5}$$

search two projects if $y < \tilde{y}$;

search one project if $\tilde{y} \leq y < R_1$;

and abandon search if $y \geq R_1$.

In the first case, $R = R_2$, and in the second, $R = R_1$. For case (b), note that for $y > \tilde{y}$, $V_p(y; 1) > V_p(y; 2)$ by virtue of the property that (3) is decreasing in y, and $V_p(\tilde{y}; 1) > \tilde{y}$ unless $\tilde{y} = R_2$. Hence the reservation price (indifference between searching and not searching) for this case is R_1, which is greater than R_2.

Next, the equilibrium outcomes of the game are analyzed.

Let n (no search) and s (search) denote the actions of a firm. Let $y = \max(y_1, y_2)$ be the initial reward (lead) of the leading firm. If neither firm searches, then the payoff for the leading firm, denoted by $V^w_{nn}(y)$ equals y; and the payoff for the lagging firm denoted $V^l_{nn}(y) = 0$. If only the lagging firm searches, then the expected payoffs are:

$$\begin{aligned} V^w_{ns}(y) &= \beta F(y)y, \\ \text{and } V^l_{ns}(y) &= -c + \beta \int_y^b x dF(x) \equiv \Delta(y) \end{aligned} \tag{6}$$

In this case, the lagging firm has nonzero benefits only if the outcome of its search is greater than the lead y.

If only the leading firm searches, then

$$\begin{aligned} V^w_{sn}(y) &= \beta F(y)y + \Delta(y) \\ &= V_p(y; 1), \\ \text{and} V^l_{sn}(y) &= 0. \end{aligned} \tag{7}$$

Finally, if both search,

$$V^l_{ss} = -c + \beta \int_y^b xF(x)dF(x) \equiv \Delta_1(y), \tag{8}$$

$$\text{and } V^w_{ss}(y) = \beta F^2(y)y + \Delta_1(y). \tag{9}$$

In this case, the lagging firm has positive benefits only if the outcome (x) from its search is greater than the lead, and the leading firm's search outcome is less (probability $F(x)$). (Mixed strategy payoffs are determined in a similar way.) The game matrix is shown below:

		Lag n	Lag s
Lead	n	y, 0	$\beta F(y)y$, $\Delta(y)$
Lead	s	$\beta F(y)y + \Delta(y)$, 0	$\beta F^2(y)y + \Delta_1(y)$, $\Delta_1(y)$

The lagging firm's payoffs (first element in each entry) are decreasing in the lead y; and those of the leading firm are increasing. In addition, search of one firm negatively affects the payoff of the other. The sum of the payoffs in each entry equals the planner's value (given the same number of projects is undertaken as implied in the strategy pair).

To analyze Nash equilibrium outcomes, define $\bar{y}$ as the lead at which the lagging firm has no incentive to search, i.e.,

$$\Delta(\bar{y}) = 0, \tag{10}$$

and let $k_g(y)$ and $k_p(y)$ represent the investment levels (number of projects chosen) in equilibrium and of the planner, respectively.

Theorem 1 *In the single-stage game described above, $k_g(y) \geq k_p(y)$ for all y. Furthermore, $\bar{y} > R$, and neither firm searches or (n, n) is NE iff $y \geq \bar{y}$.* □

Proof: First, consider the case when parallel research is beneficial to the planner, when search is undertaken, i.e., case (a) when $\tilde{y} > R_2$. It is shown that there exists a lead $\hat{y} > R_2$ such that for all $y < \hat{y}$, the unique NE is (s, s); and for $\hat{y} \leq y < \bar{y}$ only the lagging firm searches; and for $y \geq \bar{y}$, (n, n) is NE. Since $y < \tilde{y}$ implies

$$V_p(y; 2) > V_p(y; 1), \tag{11}$$

we have $3F^2(y)y + 2\Delta_1(y) > 3F(y)y + \Delta(y)$, which in turn implies $\Delta_1(y) > \Delta(y) - \Delta_1(y) > 0$ since $\Delta(y) > \Delta_1(y)$. Thus for $y \leq \tilde{y}$, $V^l_{ns}(y) = \Delta(y) > 0$. Furthermore, from (11),

$$V^w_{ss}(y) > V^w_{ns}(y) + \Delta(y) - \Delta_1(y) > V^w_{ns}(y), \tag{12}$$

for all $y \leq \tilde{y}$. Since given the lagging firm searches, the returns to leader search, given by $V^w_{ss}(y) - V^w_{ns}(y)$, is decreasing in y, it follows from (12) that the lead $\hat{y}$ at which the leader is indifferent between search and not search ($V^w_{ss} = V^w_{ns}$ at $\hat{y}$) is such that $\hat{y} > \tilde{y}$. Moreover, $\Delta_1(\hat{y}) = 3F(\hat{y})\hat{y}[1 - F(\hat{y})] > 0$, and hence $V^l_{ns}(\hat{y}) = \Delta(\hat{y}) > 0$. Since $\Delta(y)$ is decreasing in y, it follows that $\hat{y} < \bar{y}$. To sum up, $\hat{y} > R$ (since $\hat{y} > \tilde{y} > R_2 = R$), and for all $y < \hat{y}$, equilibrium involves both firms searching (note at $\hat{y}$, not search weakly dominates search for the leader), and hence $k_g(y) = 2$; for $\hat{y} \leq y < \bar{y}$, only the lagger searches, and $k_g(y) = 1$; and for $y \geq \bar{y}$, $k_g(y) = 0$ as neither search. In contrast, $k_p(y) = 2(0)$ for $y < (\geq) R_2$, thus proving the proposition for this case. See Figure 1.

Arguments are similar for the case when parallel search is not always beneficial to the planner (case of (5)). See Figure 2. There exists $\hat{y}$ such that $\tilde{y} \leq \hat{y} < R_1$ and both search for $y < \hat{y}$; for $\hat{y} \leq y < R_1$ there are two pure strategy equilibria (n, s) and (s, n), and the mixed strategy is not Pareto superior; for $R_1 \leq y < \bar{y}$, only the lagging firm searches; and neither search for $y \geq \bar{y}$. □

Thus competitive R & D in this single stage gave leads to overinvestment. For termination of the R & D activity, threshold lead is higher for the game than that for the planner.

(The implication of this on completion times is analyzed in the next section.) Higher R & D costs reduce both $\bar{y}$ and R (but the behavior of their difference is related to distributional properties). If the distributions of the projects of the two firms are different, say, firm 1 has stochastically better discoveries than firm 2, then the game termination is characterized by two thresholds, $\bar{y}_1$ (which applies when firm 1 is leading) and $\bar{y}_2$ (firm 2 leading), with $\bar{y}_1 < \bar{y}_2$ (both being greater than the planner's threshold).

2.1 R & D Race as Repeated Search

In many instances, R & D activity is a process that evolves over time. In this section, some of the issues addressed above are viewed in the context of a multi-stage game. This also enables the analysis of race completion time, and how it is influenced by the various parameters. First, a finite multi-stage game is considered; an infinitely repeated game is discussed later.

Suppose that each firm can invest in one R & D project in a period. For simplicity, all projects are assumed to be independent having the same distribution $F(x)$, and search cost $c > 0$ (as in previous section). (Heterogeneous projects, and parallel search strategy for each firm wherein it can invest in more than one project in a period, is considered in a later section.) Since search is with full recall of past observations and only the best observed technology is adopted when race terminates, the state (if the horizon is infinite) at the beginning of any period is (y, a), where y is the lead (maximum reward observed thus far) and $a \in \{1, 2\}$ denotes the leading firm. Suppose that search decisions are to be taken at the beginning of the period, the moves are simultaneous (alternative assumptions analyzed later), and that the results of the projects undertaken in a period become known at the end of that period. The state transition function for this stochastic game, if both search in period t, is

$$Pr[(y_{t+1} = y, a)|(y_t = y, a)] \quad = \quad F^2(y), \text{ and}$$

$$Pr[(y_{t+1} \geq x, a_1)|(y_t = y, a_2)] \quad = \quad F(x)[1 - F(x)], \tag{13}$$

for $x \geq y$, and $a_1, a_2 \in \{1, 2\}$. Similarly, the transition function when only the leader or the lagging firm searches can be determined.[4]

A strategy (or policy)[5] for firm $i = 1, 2$, is $\sigma_i : [0, b] \times \{1, 2\} \rightarrow \{n, s\}$, where s (resp. n) denotes the action search (resp. no search). The notation $\sigma^w(\cdot)$ and $\sigma^l(\cdot)$ is used to denote respectively the strategies of leading and lagging firms, whenever it is convenient to do so. (Also whenever it is relevant for discussion, H_t denotes the history up to the beginning of period t, i.e., set of all actions chosen from time 0 up to $t - 1$, and the rewards observed up to t.) Note that the race here is a nonstationary repeated game in the sense that the lead and the leading firm may change from one period to another; each stage is not an exact replica of another. Additionally, the reward structure here is different from the conventional models in repeated game literature (discounted sum of rewards in each period), as a firm can collect the reward only when race terminates, that too only if it is fortunate to be the winner.

Given any strategy combination, the returns to each firm can be determined via dynamic programming. Let $(\sigma_*^w(\cdot), \sigma_*^l(\cdot))$ denote a Nash Equilibrium strategy. A state (y, a) is a race termination state, with firm a winning the race, if $\sigma_*^w(y, a) = \sigma_*^l(y, a) = n$, and at all future times. (In a finite horizon version, the race terminates in state (y, a, t) at time t if no search is the equilibrium choice for states (y, a, τ), $\tau \geq t$.)

Finite-horizon Race: Suppose the maximum duration of the race is T periods, where T is finite and given. To analyze equilibrium, consider a lead $y \geq \bar{y}$, where $\bar{y}$ is as defined in

[4]In models of R & D projects where past stages of development provided information about future stages, (such as in learning models, see Roberts and Weitzman (1981)), then the history of each firm is relevant. Here the stages are taken independent for simplicity.

[5]Attention can be restricted to policies in this model (see, for example, Friedman (1990), Ch.5, pp.179–180).

(10). Clearly, the returns to search in any period is negative for either firm regardless of the strategy of the other. (At $y = \bar{y}$, the returns for the lagging firm equals zero if leader does not search; and the returns to the leader search is strictly negative.) Hence, race terminates in period t, if the state (y, a, t) is such that $y \geq \bar{y}$. (This can be formally shown from backward induction strategy from the last period T.)

Now, consider $y < \bar{y}$. At such a lead y, the race does not terminate, since one of the firms, at least the lagging firm if not both, has strictly positive returns to further search. To see this, consider state (y, a, t) at some period $t \leq T$, with $y < \bar{y}$. Then a one-shot deviation from termination by the lagging firm, i.e., it invests in search at period t and no more, has the expected returns

$$-c + \int_{\bar{y}}^{b} x dF(x) + \int_{y}^{\bar{y}} x F^{T-t}(x) dF(x),$$

since an outcome above $\bar{y}$ results in termination; and any outcome $y \leq x < \bar{y}$ results in a change of leaders, and from period $t+1$ onwards the other firm (which is now lagging) will search. From (18), it follows that the above payoff is strictly positive. Thus, race proceeds if $y < \bar{y}$ (unless, of course, it is the end of the horizon). The properties of the equilibrium in the finite horizon game are similar to the one-stage game analyzed earlier (details omitted here). The above observations are summarized below.

Theorem 2 *In a race with a finite horizon of T periods, the race terminates in any period $t \leq T$, if and only if the lead y in the beginning of period t is such that $y \geq \bar{y}$, where $\bar{y}$ is defined by (18).* □

The probability of race termination in any period $t \leq T$, given race survived up to t, equals $1 - F^k(\bar{y})$ where k is the number of projects chosen in period t.

Infinite-horizon Race: Suppose, now, that at the end of any period both firms always have one more project which they can choose to explore.

Analyzing the equilibrium here, for $y \geq \bar{y}$ neither firm searches for the same reason as in earlier cases. However, for $y < \bar{y}$, the situation is somewhat different. Let $\bar{y}_1$ be such that

$$\Delta_1(\bar{y}_1) = 0,$$

where Δ_1 is as defined in (9). Then $\bar{y}_1 < \bar{y}$. Furthermore, $\bar{y}_1 < \check{y}$ where $\check{y}$ is defined in (4), since the returns to search(in one stage) for the lagging firm is positive when the marginal returns from the second project at $\check{y}$ is zero for the planner. (Note at $\bar{y}_1$, (3) is equal to $\beta F^2(y)y - V_p(y;1) < 0$). Two cases are distinguished in the following. In the first, $\bar{y}_1 \geq R$. This case holds under condition (4), since $R = R_2 < \bar{y} < \bar{y}_1$. The second case is $\bar{y}_1 < R$, which may occur under condition (5), i.e., when parallel search is not always preferred by the planner for the one-stage game.

Suppose $\bar{y}_1 > R$. In the region $\bar{y}_1 \leq y < \bar{y}$, no further search is subgame perfect, since the leading firm has no incentive to search even if the other stops (note $\bar{y}_1 > R$), and if the lagging firm invests in search and becomes the leader the other firm will start search nullifying all returns. To see this, define

$$\mu_0 \equiv (\sigma^w(y,a), \sigma^l(y,a)) = \begin{cases} (s,s) & \text{for all } y < \bar{y}_1 \\ (n,n) & \text{for all } y \geq \bar{y}_1. \end{cases}$$

This may be viewed as a path which prescribes search by both till the lead exceeds $\bar{y}_1$ when both switch to no search. For any $y \in [\bar{y}_1, \bar{y}]$, define the path μ_1 as follows: if the state is (y, a), then the leading firm (a) does not search and the lagging firm ($\neq a$) searches till it becomes the leader. The pair $\sigma_* \equiv (\mu_0, \mu_1)$ represents a simple strategy combination with the interpretation that a deviation from μ_0 when $y \geq \bar{y}_1$ results in μ_1 being applied; and any deviation from current path μ_1 results in its restart[6]. (Note that this strategy combination is defined over histories, H_t, $t \geq 0$.) This construction is intended to show that race terminates

[6]See Friedman (1990, Ch.4) and Abreu (1988). In this problem, there is no need to specify paths for deviations from μ_0 when $y < \bar{y}$, as deviations result in either delayed or zero payoff. It is also not necessary to have two separate punishment paths for the firms.

when $y \geq \bar{y}_1$. To establish this, observe that the leader's payoffs when μ_1 is progress, denoted $\gamma^w(\mu_1)$, equals zero form the definition of this path. Furthermore, any one stage deviation by the leader when μ_1 is in progress results in an expenditure of c, but no benefits accrue since μ_1 is restarted. Hence, leader has no incentive to deviate when μ_1 is in progress. Now, suppose μ_0 is in progress and $y \geq \bar{y}_1$. Clearly the leader has no incentive to search ($\bar{y}_1 > R$). In addition, the returns to one-period deviation for the lagging firm equals (note race terminates if outcome is greater than $\bar{y}$; and path μ_1 is in effect if outcome is between y and $\bar{y}$).

$$-c + \int_{\bar{y}}^{b} x dF(x) + \int_{y}^{\bar{y}} \gamma^w(\mu_1) dF(x) = 0,$$

which implies race terminates for $y \geq \bar{y}_1$. The race does not terminate at any $y < \bar{y}_1$, for the following reason. If $y < R$, leader has incentive to search. If $y \geq R$, consider the returns to search of the lagging firm, say, firm 2. If search produces an outcome greater than $\bar{y}_1$, race terminates; an outcome between y and $\bar{y}_1$ triggers firm 1, which now becomes lagging, to search from the following period. In the latter case, by continuing search, firm 2 can assure itself a positive payoff. Its overall payoff is given by

$$-c + \int_{\bar{y}_1}^{b} x dF(x) + \gamma = \Delta(\bar{y}_1) > 0,$$

since γ, which represents the returns to each firm when both search till race terminates at $\bar{y}_1$, equals zero. Note that sum of the payoffs to both firms from μ_0 equals zero[7]. The above arguments establish that the strategy combination σ_* with initial path μ_0 is a subgame perfect NE, and that the rents dissipate. These are summarized in the theorem below.

Now, considering the case $\bar{y}_1 < R$, which can arise only when $R = R_1$ (condition (5)),

[7] If ν denotes this sum, then

$$\nu = -2c + 3F^2(\bar{y}_1)\nu + 3\int_{\bar{y}_1}^{b} x dF^2(x) = 0,$$

since $\Delta_1(\bar{y}_1) = 0$.

if the lead $y \in [\bar{y}_1, R]$, then the leader by itself would like to invest in further search: and given the leader searches, the lagging firm will not invest in search since for any termination condition (above $\bar{y}_1$) its value from continuing search will be negative(since $\Delta_1(y) < 0$). If $y > R$, then both cease to search for the same reasons given earlier (i.e., any incentive to search by the lagging firm is annulled by the retaliation from the other). Hence, a pure strategy equilibrium for this case is: (s,s) for $y < \bar{y}_1$; (n,s) for $\bar{y}_1 \leq y < R$: and (n,n) for $y \geq R$. The lagging firm drops out of the race when lead is above $\bar{y}_1$, and the leader continues if the lead is below R. In this case rents are positive since the value, v, for either firm when $y < \bar{y}_1$, equals

$$\nu = -c + \beta F^2(\bar{y}_1)\nu + \beta \int_{\bar{y}_1}^{R} RF(x)dF(x) + \beta \int_{R}^{b} xF(x)dF(x)$$

$$= \beta \int_{\bar{y}_1}^{R} \frac{(R-x)F(x)dF(x)}{[1-\beta F^2(\bar{y}_1)]} > 0 \,. \tag{14}$$

In the above expression, note that if a firm becomes the leader with an outcome in the region $[\bar{y}_1, R]$, then its further(sequential) search yields a return of R(from definition). A firm producing an outcome x above R becomes the winner if the rival's outcome is less. The second step is obtained by rearranging, and using $\Delta_1(\bar{y}_1) = 0$.

The equilibrium derived above describes the behavior of a firm in the continued presence of a rival. Although there are other equilibrium in which rents vanish(expected value from this infinite horizon race is zero for both), this is an equilibrium consistent with incentives in each period once entry is assumed (for example, a slightly lower search cost when lagging will keep a firm in the race). Main conclusion from the above analysis is summarized below. Let $k_g(y)$ denote the number of projects undertaken in the race in a period which begins with lead y. At $t = 0$, the initial lead is assumed to be such that both firms search.

Theorem 3 *In an infinite horizon race,* *a) if $\bar{y}_1 \geq R$, then race terminates when the lead $y \geq \bar{y}_1$ and both firms search in each period whenever $y < \bar{y}_1$; in addition expected returns to the race is zero.*

b) if $\bar{y}_1 < R$, both firms search in each period when lead $y < \bar{y}_1$; only the leader searches if $\bar{y}_1 \leq y < R$; and the race terminates for $y \geq R = R_1$. The expected return of a firm at the start of the race is given by (14).

If R & D costs per period is low(high) enough then $\bar{y}_1$ will be greater(less) than R. Rent dissipation in the race considered here is not due to post-race price competition [8] as there is only one winner. Rather, it arises due to continued prospects(existence of R & D opportunities as well as low costs) of better innovation by a competing firm. This part of the result is similar to vanishing of rents in multi-patent bidding models(see Dasgupta (1988)) wherein an incumbent monopolist and a new entrant bid for a process innovation.

That the returns to the race may be positive(part (b) in Theorem 3) is due to the fact that rival's threat disappears at a lead when it is still profitable for the leading firm to continue search on its own.

In all cases there is project overinvestment in race in the sense that the number of projects in any period is no less than what a planner would choose (see Vishwanath (1991) for analysis of single agent parallel search). In the special case when $\beta = 1$, planner would undertake sequential search i.e., at most one project per period, whereas in the race there is parallel research whenever $y < \bar{y}_1$.

As a variation of the above problem, consider a race with many firms. Suppose, in each period, firms enter so long as their single stage returns are positive. Then, the number of firms n in a period beginning with lead y satisfies [9]

$$c - \beta \int_y^b x F^{n-1}(x) dF(x) = 0.$$

[8]That rent may dissipate due to post-innovation competition is one of the reasons for protection mechanisms such as patents.

[9]Strictly speaking n is least integer such that the difference is still positive.

In this scenario, the number of parallel projects would be decreasing over time(n being a decreasing function of y), and the race would terminate when the lead is $\bar{y}$.

The race termination points($\bar{y}, \bar{y}_1$ and R in the various cases) decrease if cost c increases, and increase if β increases. The latter effect is also induced for improvements in the distribution F(first order dominance, increase in upper tail risk - more about this in next section). The influence of these parameters on race completion time is analyzed next.

Race Completion Time: To analyze the distributional effects on the completion time, consider an increase in the mean (denoted m) with the shape remaining the same. If $F(x;m)$ denotes a distribution with mean m, then $F(x;m) = F(x-m;0)$. Race termination at $\bar{y}$ is analyzed below. The effect on the completion time, when race terminates at $\bar{y}_1$ or R (as in theorem 3) is similar. The hazard rate of the race (probability of completion in a period, given race has survived thus far) is given by

$$h_k \equiv 1 - F^k(\bar{y};m), \tag{15}$$

when $k = 1$ or 2 projects are undertaken. An increase in the mean, increases $\bar{y}$ but the distribution also shifts to the right. Hence the overall effect on the hazard rate is not readily obvious. From the definition of $\bar{y}$, and changing variables, we get

$$c = \beta \int_{\bar{y}-m}^{b} (m+z)dF(z;0). \tag{16}$$

Taking derivatives with respect to m yields, (the density is denoted by $f(x;m)$),

$$1 - F(\bar{y}-m;0) = \bar{y}\frac{\partial(\bar{y}-m)}{\partial m} f(\bar{y}-m;0). \tag{17}$$

Furthermore,

$$\frac{\partial h_k}{\partial m} = -kF^{k-1}(\bar{y}-m;0)f(\bar{y}-m;0)\frac{\partial(\bar{y}-m)}{\partial m}. \tag{18}$$

Combining (17) and (18) yields, for $k = 1$,

$$h_1 + \bar{y}\frac{\partial h_1}{\partial m} = 0;$$

and for $k = 2$,

$$h_2[1 + F(\bar{y} - m;0)]^{-1} + kF^{k-1}(\bar{y} - m;0)\bar{y}\frac{\partial h_2}{\partial m} = 0.$$

Hence, $\partial h_k / \partial m < 0$ for $k = 1, 2$. Thus an upward shift in the mean decreases the hazard rate, and consequently the race takes longer. Similar arguments also establish that the race takes longer for termination at $\bar{y}_1$ in the infinite horizon case (note that replacing c by $c/2$, and F by F^2 in (15) and (17), defines $\bar{y}_1$ and the hazard rate for this case), and for termination at R. Increase in cost c, or a decrease in β, has the effect of increasing the hazard rate and reducing the length of the race.

Example: In the following, the case of binary project outcomes are considered and it is shown that invariance of project investment (between race and planner's choice, derived by Sah and Stiglitz(1986)) is a special case. In particular, it holds for a stage game (with additional assumptions) and is generally not the case with repeated search.

Suppose that each project has binary outcomes, 1 (denoting success with unit reward) with probability p, and 0 (failure) with probability $1 - p$. Considering a stage, the marginal benefits from undertaking a second project equals (see (3))

$$-c + \beta p(1 - p),$$

since the second project improves on the first only when the latter is unsuccessful. Hence, in a single stage R & D with no repeated search, the planner prefers to invest in one project if $-c + \beta p > 0$, and two iff $-c + \beta p(1 - p) > 0$. The reward structure for this binary case is winner-take-all, but in case of a tie (both successful) the payoff for each firm is zero due to post-race Bertrand competition. Then, in a single stage game, the payoff (expected reward) for each firm, when both search, equals $-c + \beta p(1 - p) \equiv \Delta_1$. The equilibrium is (s, s) if $\Delta_1 > 0$. There are two equilibrium, (n, s) and (s, n), if $\Delta_1 < 0$ and $\beta p > c$.

Hence in the single stage game, there is invariance – investment in the game coincides with that of the planner.

Now, consider repeated R & D activity (say, infinite horizon) with race and planner's search terminating when a project is successful. In the race, two projects are invested in each period if $\Delta_1 > 0$. However, if R_i denotes the value from planner's search when $i = 1, 2$ projects are undertaken in each period, then the invariance result holds if $R_2 > R_1$ when $\Delta_1 > 0$. It is easily verified that (see (1) and (2))

$$R_1 = [-c + \beta p]/[1 - \beta(1 - p)].$$

and

$$R_2 = [-2c + \beta p(2 - p)]/[1 - \beta(1 - p)^2]$$

and as $\beta \to 1$, $R_2 \to 1 - 2c/[p(2 - p)]$, and $R_1 \to 1 - c/p$. Hence, when β is large enough, planner prefers sequential search (regardless of sign of Δ_1), whereas the race generates parallel search if $\Delta_1 > 0$. The invariance result of Sah and Stiglitz (who only consider a stage game) is not in general applicable for repeated R & D situations.

3 Heterogeneous Alternatives

In this section, we address some aspects of the race when firms have heterogeneous alternatives i.e., the distributions of projects are different. While a complete characterization of the equilibria can become complex in general, insights into the equilibrium project selection order and risk choices are obtained imposing stochastic ordering conditions on project distributions. It is also shown that race termination has a simple myopic characterization.

Suppose each firm has access to a finite set of R & D projects. As before, exploration of each project generates a random social surplus assumed to be revealed after one period, and search cost $c > 0$ for each project is incurred at the beginning of the period in which it is explored. Let $F_{ij}(x)$ denote the probability distribution (assumed independent, atomless with support $[0, b]$) of social benefit associated with project j ($1 \leq j \leq m_i$) of firm $i = 1, 2$.

Each firm explores its projects over time selecting some, possibly more than one in each period, from its unexplored set as long as it is beneficial to do so. Furthermore, each firm can recall its past observations. When firms stop any further research, the firm which has generated the highest benefit is the winner, and as before, the reward structure is winner-take-all. This process stops when firms individually have no incentive to search further.

It is assumed that all projects and their features and distributions are common knowledge. Furthermore, at any stage, each firm knows the results of the other firm's investigation thus far and its remaining projects. That is, if, at a certain time, y_i is the highest benefit secured by firm i, and S_i is its set of unexplored projects at that time, then the common information state is $(y_1, y_2; S_1, S_2)$. Since a lagging firm (lower y_i) must produce from any further search a value higher than the leading firm (higher y_i), for any possibility of positive eventual reward, it is enough to write the state as $(y, m; S_1, S_2)$ where $m \in \{1, 2\}$ denotes the label of leading firm which has collected y.

A search strategy for a firm specifies a set of projects that are explored in any given information state. If σ_i denotes a pure strategy for firm i, then $\sigma_i(y, m; S_1, S_2) \subseteq S_i$. (If σ_i is empty, no search is undertaken in this state.) Given a strategy pair (σ_1, σ_2), let

$$V^w(y, m; S_1, S_2|\sigma_1, \sigma_2) \text{ and } V^l(y, m; S_1, S_2|\sigma_1, \sigma_2) \tag{19}$$

respectively represent the values, expected disconnected reward, from continuing search for the leading and the lagging firms, given information state $(y, m; S_1, S_2)$. Each strategy pair induces a set of stopping states or termination points of the R & D game, defined by the relations:

$$y \geq V^w(y, m; S_1, S_2|\sigma_1, \sigma_2) \tag{20}$$

$$0 \geq V^l(y, m; S_1, S_2|\sigma_1, \sigma_2). \tag{21}$$

Let $I_0 = (y_0, m_0; \bar{S}_1, \bar{S}_2)$ be the initial state, where $\bar{S}_i$ denotes the complete set of projects possessed by firm $i = 1, 2$. A strategy pair (σ_1^*, σ_2^*) is a Nash equilibrium if the returns to continuation of search for firm 1 following σ_1^* is no less than the returns to search for any other strategy σ_1, given firm 2 follows σ_2^*. The same holds for firm 2 as well.

To characterize equilibrium order of project selection, the following stochastic ordering-conditions are imposed.

(A–1) Projects 1 through m_k of firm $k = 1, 2$ are such that for each y $(0 \leq y \leq b)$, and $1 \leq i \leq m_i - 1$

$$\int_y^b [1 - F_{i1}(x)]dx \geq \int_y^b [1 - F_{i+1,1}(x)dx \tag{22}$$

(A–2) For $1 \leq i \leq m_1$, and $1 \leq j \leq m_2$, $yF_{ij}(y)$ is convex in y.

Assumption (A–1) is a form of second order stochastic dominance. (See Meyer (1987).) In particular, for each firm, if project i is better than project $(i+1)$ either in the sense of first order stochastic dominance (i.e., $F_{i1}(x) \leq F_{i+1,1}(x)$) or a mean preserving spread (project i is riskier), then (22) is satisfied [10].

With the above restrictions, the equilibrium search order is unique, although the actual number of projects chosen may differ depending on the state. This is shown via the following lemma (proved in Vishwanath (1991)), related to single agent (partial equilibrium) search.

Lemma 1 *Given independent search opportunity distributions $G_1, \cdots, G_r$, all with same search cost, if G_i is better than G_{i+1} in the sense of (A–1), then the best parallel search (with recall) order is (1,2,$\cdots$,r).* □

The intuition for the above lemma follows from the 'option' effect. If project i or $i+1$ is searched in parallel or simultaneously with other projects, then in one period i improves, or has better marginal returns, on other projects more than $i+1$, as is evident from (A–1). This myopic selection is the best for any parallel search strategy.

[10]If projects are ordered via first order dominance, then A-2 is not required. This assumption enables derivation of stronger results when projects differ in their riskiness(See also Bhattacharya and Mookherjee(1986))

Theorem 4 *Given restrictions (A), the equilibrium project selection order for firm i is $(1, 2, \cdots, m_i)$.* □

Proof : Consider any state $(y, m; S_1, S_2)$. Suppose firm 1 is leading , i.e., $m = 1$. Consider any strategy σ_2 of firm 2, and suppose that σ_2 prescribes selection of projects from S_2 whose composite distribution (i.e., distribution of maximum of their outcomes) is $G(x)$. Consider two projects i and j in S_1, such that i is better than j in the sense of (A–1). Then, in a period beginning with this state, the outcomes of search for firm 1 from selecting project i are: if rival's outcome is better than y and that of firm 1 is less than y, then firm 1 loses leadership; if both draw less than y, then the state is retained; and conditional on outcome x, leadership is retained if the rival's outcome is less. Since, $xG(x)$ is increasing, the value from search of project i is

$$-c + \beta \int_0^b \max[yG(y), xG(x)] dF_{1i}(x)$$

Since the integrand is convex increasing (by virtue of (A–2)), it follows that the above one period returns is higher for project i than for j. The argument is similar when firm 1 picks more than one project. It remains to show this selection order is better even when future evolution is considered, the details of which are similar in nature to proof of lemma 1 (Vishwanath (1991)). The idea is as follows. Fix an order for firm 2. Consider an order for firm 1 which violates the one specified in proposition 1. Then in some period j is selected and in the next period i is picked, for some i and $j > i$. Then a strategy which interchanges the order of selection of i and j holding all other selections the same, yields a better value. This implies, for any strategy of the other firm, the best order for a firm is the one specified in proposition. The case when firm 1 is lagging is also similar. □

Further characterization of equilibrium is in general hard. In the following, attention is focussed on the symmetric case of both firms having the same set of projects $\{F_1, F_2, \cdots, F_n\}$, and race termination is analyzed. It is assumed that F_i is better than F_{i+1} is the sense of (A-1). Since the equilibrium order is as in Theorem 4, the information state can now be written as $(y, m; n_1, n_2)$ where n_i, $i = 1, 2$ denotes the label of the remaining project first in the selection order. That is, in this state the remaining projects for firm i are n_i through m.

Let $\bar{y}(m; n_1, n_2)$ denote the race termination threshold.

Define $\bar{y}_0(m; n_1, n_2)$ as the termination lead for a one stage game starting in state $(\cdot, m; n_1, n_2)$. That is, $\bar{y}_0$ satisfies

$$c = \int_{\bar{y}_0}^{b} x dF_{n_i}(x), \tag{23}$$

if firm i is lagging.

Theorem 5 *If both firms have the same set of projects* $\{F_1, F_2, \cdots, F_n\}$ *with* F_i *better than* F_{i+1} *in the sense of (A-1), then*

$$\bar{y}_0(m; n_1, n_2) = \bar{y}(m; n_1, n_2)$$

for $m \in \{1, 2\}$, *and* $1 \leq n_1, n_2 \leq m$. $\square$

Proof: First, if $y < \bar{y}_0$ in any state, then for at least one of the firms it is better to continue for at least one more stage, which implies no termination and hence $y < \bar{y}$.

To show $y > \bar{y}_0$ implies $y > \bar{y}$, consider a state $(y, m; n_1, n_2)$ at the beginning of some period t. Without loss of generality, let $m = 1$, so that firm 2 is lagging. Two separate cases are identified: i) $n_2 \geq n_1$, leader has more projects remaining, and ii) $n_2 < n_1$, lagging firm has more remaining projects.

Consider the first case. Here, if $y > \bar{y}_0(m; n_1, n_2)$, then the lagging firm drops out of the race. This can be proved via induction as follows: first, it is true for $(n_1, n_2) \equiv (0, 1)$ or (1,0) or (1,1) from the single stage game results (section2); second, assuming it is true from period $\tau + 1$ onwards, it is also true in period τ, since the condition of fewer remaining projects is carried over from τ to $\tau + 1$ with leader not searching in period τ. Thus with $n_2 \geq n_1$ and $y > \bar{y}_0(m; n_1, n_2)$ at the beginning of period t, lagging firm does not search. The leading firm also does not search beginning period t because of the following reasons: first, $\bar{y}_0$ determined from n_2 is the same in period t and $t+1$; second, for this reason search will terminate from $t+1$ onwards; and hence, given leader search does not continue beyond $t+1$, its returns in t to search is less than y.

Now suppose $n_2 < n_1$ at the beginning of period t, and $y \geq \bar{y}_0(m; n_1, n_2)$. Assume theorem holds from period $t+1$ onwards. If search continues in period t, then conditions at the beginning of period $t+1$ are: i) lead is no less than y, and ii) the lagging firm's first project n_l in its order is such that $n_l \geq n_2$. These two hold regardless of outcomes in period t. Since the distributions satisfy (A-1), the one stage termination leads in t and $t+1$ satisfy $\bar{y}_0(t+1) \leq \bar{y}_0(t)$. Hence, race does not continue from $t+1$ onwards (from induction hypothesis that Theorem holds from $t+1$ onwards, or in other words it holds for states with fewer than n_i projects remaining). Thus, the race continuation value equals the value from the one stage game occurring in period t. But neither will search in period t, since $y > \bar{y}_0(m; n_1, n_2)$. Hence, race terminates at the beginning of period t. This completes the proof. □

Let $V^w(\cdot)$ and $V^l(\cdot)$ denote the values to the leading and lagging firms, respectively. Let $\sigma_i^*(y, m; m_1, m_2) = k_i$, for $i = 1, 2$. That is, equilibrium choices of firms in the first period are the first k_1 and k_2 projects. (Note that $k_i = 0$ indicates no search is undertaken in the first period. Thus, V^w and V^l represents the values at a given state, whether or not search is undertaken.) Let $G_{i,k_i} \equiv \prod_{j=1}^{k_i} F_{ij}(x)$ for $i = 1, 2$, which represents the distribution of the maximum of the research outcomes, for each firm, in the first period. The equilibrium values to the firms may now be written as follows using dynamic programming. Given firm

1 leading initially, if both firms draw an outcome from search less than y, firm 1 continues to be the leader retaining y, and in the event one or the other draw greater than y, a new lead is established. In any event, the state of remaining projects moves to $(m_1 - k_1, m_2 - k_2)$. Hence,

$$
\begin{aligned}
&V^w(y, 1; m_1, m_2) = \\
&\quad -k_1 c + \beta V^w(y, 1; m_1 - k_1, m_2 - k_2) G_{1,k_1}(y) G_{2,k_2}(y) + \\
&\quad \beta \int_y^b [V^w(x, 1; m_1 - k_1, m_2 - k_2) G_{2,k_2}(x) + \\
&\quad \int_x^b V^l(z, 2; m_1 - k_1, m_2 - k_2) dG_{2,k_2}(x)] dG_{1,k_1}(x) \qquad (24)
\end{aligned}
$$

For the lagging firm,

$$
\begin{aligned}
V^l(y, 1; m_1, m_2) = -k_2 c + \beta \int_y^b [V^w(x, 2; m_1 - k_1, m_2 - k_2) G_{1,k_1}(x) + \\
\int_x^b V^l(z, 1; m_1 - k_1, m_2 - k_2) dG_{1,k_1}(x)] dG_{2,k_2}(x) \qquad (25)
\end{aligned}
$$

The above equations apply for game continuation, i.e., when k_1 and k_2 are not both zero. If $k_i = 0$, for $i = 1, 2$, then

$$V^w(y, 1; m_1, m_2) = y \text{ and } V^l(y, 1; m_1, m_2) = 0 \qquad (26)$$

It can be shown (via an induction argument) that

$$
\begin{aligned}
&V^w(y, m; n_1, n_2) + V^l(y, m; n_1, n_2) \\
&\leq V_p(y, m; n_1, n_2),
\end{aligned}
$$

that is, in all states, the sum of the values of the two firms is no greater than the planner's value from optimal parallel search. In addition, at the planner's margin (at indifference between search and no search) the lagging firm in the race has incentive to undertake further search (if it has at least one more project, and n is finite), as in Section 2.

To get on insight into race investment and termination when each firm adopts a parallel search strategy, suppose all projects have the same distribution $F_i \equiv F$. Consider first a one-stage game. for this illustration, if the leading firm selects k projects, and the lagger selects k_2 projects, then (conditional on k_1 and k_2) the values are

$$V_0^w(y|k_1, k_2) = -k_1 c + \beta F^{k_1+k_2}(y)y + \beta \int_y^b x F^{k_2}(x) dF^{k_1}(x) \tag{27}$$

$$= \beta F^{k_1+k_2}(y)y + k_1 \Delta(k_1 + k_2 - 1; y) \tag{28}$$

where,

$$\Delta(k; y) \equiv -c + \int_y^b x F^k(x) dF(x). \tag{29}$$

For the lagging firm,

$$V_0^l(y|k_1, k_2) = k_2 \Delta(k_2 + k_1 - 1; y) \tag{30}$$

Let $k_1(y)$ and $k_2(y)$ represent the choices corresponding to equilibrium values V_0^w and V_0^l of the one stage game, beginning with lead y. Taking derivatives in (28)–(30), yields $k_1' \leq 0$ and $k_2' \leq 0$. That is, the number of projects chosen by individual firms decreases as the lead increases. In general, there are many equilibria (even pure strategy) in the one-stage game.

The value for the social planner, conditional on the choice of k projects is

$$V^p(y|k) = \beta F^k(y) + k\Delta(k - 1; y). \tag{31}$$

Let $k(y)$ represent the planner's optimal choice of k. Using (28)–(31), it is easy to check that at the planner's margin, the lagging firm's marginal value is nonnegative. Hence, for all y

$$k_1(y) + k_2(y) \geq k(y), \tag{32}$$

implying project overinvestment in the race.

Considerations similar to those in Section 2.1 apply in extending the above results to repeated situations. For a neck-to-neck race ($k_1(y) = k_2(y)$, at each stage), all the results derived in Section 2.1 hold. In this case of parallel search by each firm, the race will complete sooner (than the race when each firm follows a sequential search strategy). Further examination of equilibrium (issues including identifying other equilibria, how the project investments of leading and lagging firms may differ, possibilities of supporting coordination outcomes etc) are issues for further study.

4 Concluding Remarks

In this paper, R & D race with uncertainty is addressed viewing project investment decisions in a search framework. Equilibrium search strategies, race termination condition, race completion times, project selection order, and other issues are analyzed in single stage and repeated game models using concepts and results from parallel search (Vishwanath (1991)).

In general, there is project overinvestment in the race relative to the planner's level, and the race takes longer. Equilibrium strategies are analyzed to determine the conditions under which there is parallel search activity in the race. Among several other results derived, it is shown that improvement in the prospects of better discoveries lead to longer race times; the invariance (race and planner's investments coinciding) does not hold in general in repeated situations; equilibrium project selection order (with each firm adopting parallel search strategy) is predetermined when project distributions satisfy certain stochastic ordering conditions. A result of interest when both firms have continuing R & D opportunities (infinite horizon race) is that the returns to each firm from the race may vanish if costs per project are low(firms facing identical environments). This is due to the fact that race is prolonged. Both firms search till a draw above termination does them apart (one a winner, the other a loser). However, if costs are not too low and prospects are good, the returns to the race is positive (the rival gets off the contest at a lead when the leader is still running).

In addition to investigating other equilibrium properties, the models can be extended in several useful directions. One is the study of spillover effects. Suppose the firm producing the best technology y at termination gets αy and the loser gets $(1-\alpha)y$. If α is close to unity, the properties of the race will be qualitatively similar to results in previous sections, although the termination thresholds and completion times are reduced. If $\alpha > 1/2$, the race becomes a waiting game. At $\alpha = 1/2$, race and planner's decisions coincide (with condition (4): under condition (5), decisions are the same if firms can also agree to share costs). Underinvestment could result if spillovers are significant. A precise analysis is a topic for further study.

Another direction is to introduce correlation of projects across firms and positive externality. Such an analysis may result in useful insights into the study of incentives for standardization. (Note that, for analytical tractability, retaining the independence of projects within each firm i.e., bandit assumption, simplifies a great deal, and may serve as a good starting point). Supporting cooperative outcomes and coordination is yet another topic for further study.

The implications of the race structure and outcomes on issues related to growth (see Jovanovic and Rob (1990)) is a topic for future research. Search framework has been quite useful in empirical studies in other areas; models like those studied here may prove useful in empirical R & D studies also.

Finally, the R & D models in this paper are studied by applying the results and concepts of parallel search (Vishwanath (1991)). The history of R & D itself is replete with instances of parallel research. This provides added motivation for further study of parallel search.

References

[1] Allen, B., "Choosing R & D Projects: An Informational Approach," *American Economic Review*, May (1991).

[2] Aoki, R., "R & D Completion for Product Innovation: An Endless Race," *American Economic Review*, May (1991).

[3] Bhattacharya S., and D., Mookherjee, "Portfolio Choice in Research and Development," *Rand Journal of Economics*, vol.17, No.4, (1986).

[4] Dasgupta, P., "Patents, Priority and Imitation Or, the Economics of Race and Waiting Games", *Economic Journal*, 98, 66-88, (1988).

[5] Dasgupta, P., "Economics of Parallel Research," in *Economics of Missing Markets, Information, and Games* (ed. F. Hahn), Oxford University Press, (1990).

[6] Freidman, W.J., *Game theory with Applications to Economics*, Oxford University Press, (1990).

[7] Harris, C. and Vickers, J., "Racing with Uncertainty," *Review of Economic Studies*, 54, 1–21, (1987).

[8] Jovanovic, B. and R. Rob, "Long Waves and Short Waves: growth through intensive and extensive search", *Econometrica*, 58, (1990).

[9] Kamien, M.I. and Schwartz, N.L., *Market Structure and Innovation*, Cambridge University Press, (1982).

[10] Lee, T. and Wilde, L., "Market Structure and Innovation: A Reformulation," *Q.J.E.*, 94, 429–436, (1980).

[11] Meyer, J., "Choice among Distributions," *Journal of Economic Theory*, vol.14, No.2, (1977).

[12] Mortensen, D.T., "Job Search and Labor Market Analysis", in *Handbook of Labor Economics*, North Holland, (1984).

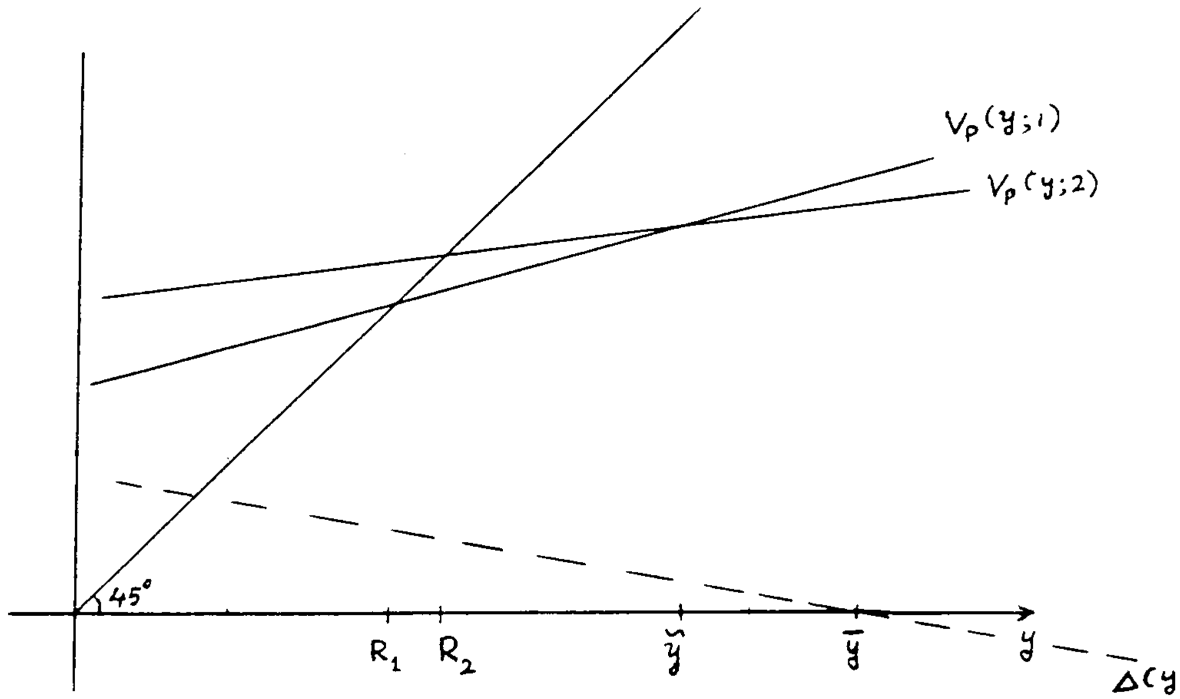
[13] Reinganaum, J.F., "Nash Equilibrium Search for the Best Alternative," *Journal of Economics Theory*, 30, 139–152, (1983).

[14] Roberts, K.W.S. and M.L. Weitzman, "Funding Criteria for Research, Development, and Exploration Projects", *Econometrica*, 49, No. 5, 1261-1288 (1981).

[15] Sah, R.K. and J.E. Stiglitz, "The invariance of market innovation to the number of firms," *Rand Journal of Economics*, vol.18, No.1, (1987).

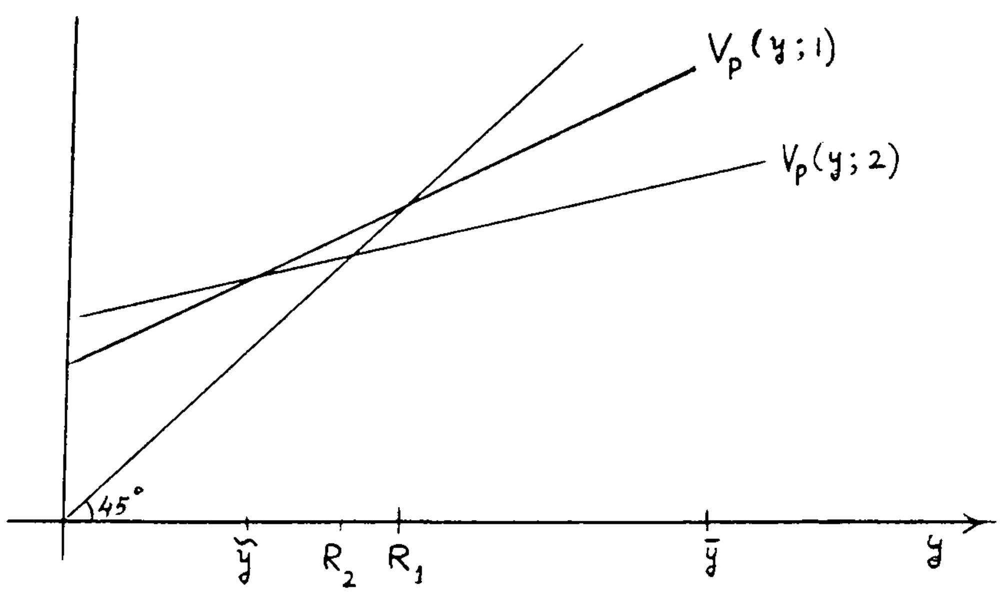
[16] Vishwanath, T., "Parallel Search for the Best Alternative," (1990) forthcoming in *Economic Theory*.

[17] Weitzman, M.L., "Optimal Search for the Best Alternative," *Econometrica*, 47, 641–654, (1979).



$V_p(y;1)$
$V_p(y;2)$
45°
R_1
R_2
$\tilde{y}$
$\bar{y}$
y
$\Delta(y)$

FIG. 1



$V_p(y;1)$
$V_p(y;2)$
45°
$\tilde{y}$
R_2
R_1
$\bar{y}$
y

FIG. 2