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Competitive Limit Pricing Under Imperfect Information\*

by

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**ABSTRACT:** This paper offers a new theory of limit pricing. Incumbents from different markets or regions "compete" against one another, with each attempting to price in a manner that deflects entry into the others' markets. An entrant is imperfectly informed as to the incumbents' respective investments in cost reduction and seeks to enter markets in which incumbents have high costs. In the focal equilibrium, the entrant uses a simple "comparison strategy," in which it enters only the highest-priced markets, and incumbents engage in limit-pricing behavior. The influence on pricing of the number of markets and the scope of entry is also reported. Finally, the theory indicates that limit pricing may in fact deter entry, with the entrant choosing to enter no market whatsoever. Throughout, the central feature of the analysis is that an incumbent's price affects its investment incentives, with lower prices being complementary to greater investment.

**Keywords:** Limit Pricing, Entry Deterrence, Mixed Strategies, Endogenous Costs

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## I. Introduction

The basic notion of limit pricing seems simple enough. If an incumbent firm is threatened with the prospect of entry, then it might price low in order to suggest that entry is unprofitable. This loose logic, however, does not expose the specific linkage through which a low pre-entry price affects the entrant's expected post-entry profit.

Two linkages have been proposed. According to Bain (1956), Modigliani (1958), and Sylos-Labini (1962), an incumbent may commit to sustain its pre-entry price (or output) if entry occurs. Here, a commitment linkage is hypothesized, whereby a low pre-entry price deters entry, because the incumbent is committed to the same low price in future periods. More recently, Milgrom and Roberts (1982) have introduced the idea of an informational linkage. This theory derives from a game of incomplete information, in which a lone incumbent is privately informed of its exogenous production costs. By selecting a low pre-entry price, the incumbent signals that its costs are low and thus that entry prospects are unfavorable.<sup>1</sup>

This paper is intended as a further contribution to the theory of informational limit pricing. It offers two key departures from the Milgrom-Roberts framework. First, the model developed here allows incumbents to privately choose their respective cost technologies. As cost technologies are fully endogenous, the limit pricing theory given below relies only on imperfect information. Second, it seems natural to suppose that an entrant might be simultaneously evaluating several markets, hoping to determine the markets in which entry would be most profitable. This introduces some

competition between incumbents of different markets, as each may attempt to price in a manner that deflects entry into the others' markets.

The model takes a simple, two-period form. There are  $N$  identical markets, each inhabited by a single incumbent. The incumbents are aware that a sole (large-scale) entrant is comparing the markets. Initially, it is assumed that entry into each market is profitable, but that the entrant's financial resources are sufficient for entry into only one market. Unable to deter entry in aggregate, therefore, incumbents each attempt to redirect entry away from their respective markets. An intuitive interpretation is that the different incumbents service separate regions of a national or global market. Each incumbent then fears that the entrant may elect to attack its region.

In the pre-entry period, the incumbents simultaneously choose pre-entry prices as well as investments in cost reduction. The entrant sees all  $N$  pre-entry prices but is unable to observe the (endogenous) cost technologies of the incumbents. Having studied the pre-entry prices, the entrant next selects a market to enter, marking the beginning of the post-entry period. This period is characterized by  $N-1$  monopolies and one duopoly, and the resulting payoffs are represented in a general way.

An important conclusion of this paper is that the presence of limit pricing need not imply a sophisticated entrant.<sup>2</sup> To emphasize this point, it is supposed that the entrant employs a simple and natural comparison strategy, whereby the entrant selects for entry the market whose pre-entry price is highest. Not surprisingly, this rule engenders limit-pricing behavior in the pre-entry period. As always, however, the question arises as to why the entrant links low pre-entry prices with poor post-entry profit. The

fundamental finding here is that the comparison strategy is fully rational; that is, a focal Nash equilibrium exists in which markets with lower pre-entry prices also offer the entrant smaller post-entry profit.

This finding derives from two complementary effects. First, when the entrant adopts the comparison strategy, an incumbent that prices low is less likely to face entry.<sup>3</sup> Assuming only that an incumbent's post-entry output is larger under monopoly than duopoly, it then must be that a low pre-entry price raises an incumbent's expected post-entry sales. Consequently, the incumbent's benefit from cost reduction is enhanced by a low pre-entry price. The second effect is in fact independent of the entrant's strategy. The simple idea here is that an incumbent with a low pre-entry price stimulates sales in the pre-entry period. This, too, sparks the incumbent's incentive to reduce costs with investment. Thus, for both of these reasons, lower pre-entry prices will be associated with greater investment. As an entrant fares better against less-efficient (higher-cost) incumbents, it follows at once that the simple comparison strategy is fully optimal.

The basic framework is quite rich, being amenable to comparative statics analysis. The results are counter-intuitive. As the number of markets increases, representing an ostensible lessening in competitiveness across markets, incumbents select lower pre-entry prices and higher investment levels more often. Similarly, if the entrant's financial resources were to expand, enabling it to concurrently enter several markets, competition among incumbents would be expected to intensify. In fact, however, an expansion in the scope of entry causes higher pre-entry prices and lower investment levels to be chosen with greater frequency.

A further modification of the model allows that entry may be unprofitable. Specifically, it is assumed that a critical investment level exists, where entry into a market is unprofitable if the corresponding incumbent invests beyond this level. In this setting, incumbents may strategically deter aggregate entry with limit pricing.<sup>4</sup> The key point is that a low pre-entry price "commits" an incumbent to a high investment level, since the associated growth in pre-entry sales enhances investment benefits. If all incumbents adopt limit prices of this nature, the entrant's best option is to enter no market whatsoever. This finding indicates an essential difference between incomplete- and imperfect-information limit-pricing theories. In the separating equilibria emphasized by Milgrom and Roberts, limit pricing does not lead to entry deterrence. By contrast, when costs are endogenous, limit pricing serves as an influence on investment incentives that makes possible actual entry deterrence. It is important to stress that this limit-pricing/entry-deterrence equilibrium exists even if entry into only one particular market is contemplated by the entrant.

The general orientation of this paper has been influenced by the previous research of Rogerson (1981). He extends Milgrom-Roberts' incomplete-information framework into a multi-market context and proposes that the entrant adopt the comparison strategy.<sup>5</sup> In focusing exclusively on pure-strategy equilibria, however, Rogerson is unable to find limit-pricing behavior in a separating equilibrium.<sup>6</sup> This paper builds on Rogerson's basic view, although in a symmetric, endogenous-cost formulation. A richer framework is thus provided, since the theory is capable of explaining pricing and technology choices. Furthermore, the comparison rule has a "loser lose

all" flavor and thus naturally nominates mixed-strategy equilibria. Unlike the incomplete-information literature, the current model endows incumbents with the same initial technology. This in turn makes possible symmetric mixed-strategy equilibria, which are of course much easier to characterize.

From a methodological standpoint, this paper draws heavily from the price-dispersion literature. In particular, the mixed-strategy analysis presented below is closely related to that developed previously by Varian (1981), Rosenthal (1980), Stahl (1989), and Bagwell and Ramey (1991b). It should be noted that counter-intuitive comparative statics findings have also been reported in this literature.

The paper has five sections. The basic model is presented in Section 2. Following the provision of comparative statics findings in Section 3, the model is modified in Section 4 to generate strategic aggregate entry deterrence. Section 5 concludes.

## II. A Model of Competitive Limit Pricing

### A. Framework

A single entrant desires to enter one market, where there are  $N \geq 2$  markets from which to choose. To simplify, it is assumed that a separate incumbent monopolist inhabits each market and that the markets are initially identical. One interpretation is that the different incumbents service separate regions of a large market. The incumbents know of a watchful entrant and seek to redirect the entrant away from their respective regions.

The game entails two periods. In the pre-entry period, the  $N$  incumbents simultaneously choose pre-entry prices and levels of cost-reducing investments. The entrant observes the  $N$  prices but does not observe the

investment choices. The post-entry period commences when the entrant selects a market to enter, after which the entrant and corresponding incumbent earn some general duopoly profits whereas incumbents in other markets receive monopoly profits. The key point here is that the entrant's duopoly profit is positively related to the incumbent's cost of production. Thus, the entrant monitors the incumbents' pre-entry prices, hoping thereby to infer the identity of the incumbent with the least investment in cost reduction.

Formally, a pure strategy for an incumbent is a price-investment pair,  $(P, I) \in [0, P_u] \times [0, I_u]$ , where  $P_u$  and  $I_u$  are some maximal price and investment levels, respectively. An incumbent may also choose to mix over price and/or investment selections. Let  $c(I) > 0$  denote an incumbent's constant unit cost of production, where  $c'(I) < 0$ . The incumbent's (discounted) post-entry profit when entry occurs is denoted  $\Pi_e(c(I))$ , while  $\Pi_N(c(I))$  gives the corresponding monopoly profit when there is no entry. For a given probability of entry  $\rho \in [0, 1]$ , an incumbent's total expected profit function then may be written

$$(1) \quad \Pi(P, I, \rho) \equiv (P - c(I))D(P) - rI + \rho\Pi_e(c(I)) + (1 - \rho)\Pi_N(c(I))$$

where  $r > 0$  is the constant unit cost of investment and  $D(P)$  is a pre-entry period demand function. Assume that  $P_u > c(0)$ , where  $D(P) = 0$  for  $P \geq P_u$  while  $D'(P) < 0$  for  $P \in [0, P_u)$ .

A pure strategy for the entrant is a function mapping from the set of observed prices,  $[0, P_u]^N$ , into the set of markets or regions,  $\{1, 2, \dots, N\}$ . In other words, the entrant observes all prices and picks a single market to enter.<sup>7</sup> The entrant may also choose to mix over markets for a given

observation of prices. It is convenient to assume that the entrant observes the corresponding incumbent's costs upon entry. The entrant's profit from entry is then denoted  $V(c)$ , where  $c = c(I)$  is the incumbent's unit cost of production. To capture the notion that higher-cost industries are more inviting for entrants, it is assumed that  $V'(c) > 0$ . Assume also that  $V(c) > 0$  so that aggregate entry cannot be deterred.

In solving this game, it is sufficient to look for Nash equilibria. Thus, taking as given the price and investment strategies of rival incumbents and the entrant's strategy, an incumbent may determine the probability of entry associated with any price and thereby choose an expected-profit-maximizing price-investment pair. Similarly, the entrant enters the market whose expected entry profit is highest, conditional on the observed pre-entry prices, when the observed prices lie in the support of the firms' equilibrium strategies. When prices are "off the equilibrium path," the entrant's strategy may be specified arbitrarily.<sup>8</sup>

As a general matter, a large set of Nash equilibrium outcomes may be constructed for this game, since the entrant is free to draw rather severe inferences following a deviation from the equilibrium strategies. To refine the set of equilibria, attention is restricted to equilibria in which the entrant optimizes (over the full set of entry strategies) in using a "comparison strategy," whereby the entrant compares incumbents' prices and enters the market whose price is highest. (If more than one incumbent charges the highest price, the entrant randomizes uniformly over the set of highest-priced incumbents.) This rule-of-thumb is attractive as it represents a



simple and natural decision procedure. Henceforth, an equilibrium refers to a Nash equilibrium in which the entrant adopts the comparison strategy.

Before proceeding to an analysis of equilibrium behavior, some additional structure is required. An important assumption is

$$(2) \quad \Pi'_N(c) < \Pi'_e(c) < 0 < \Pi_e(c) < \Pi_N(c) \quad .$$

This is easily motivated. An incumbent certainly prefers no entry to entry. In addition, if entry occurs, the incumbent's (post-entry) output should contract. It follows that a unit cost reduction is most valuable when entry does not occur, since the reduction then applies to a larger output.<sup>9</sup>

Next, it is useful to define monopoly price and investment levels. A key observation is that no single monopoly price exists: the monopoly price depends on the level of investment, which is in turn sensitive to the probability of entry. Assume, then, that  $\Pi(P, I, \rho)$  is uniquely maximized over  $P \in [0, P_u]$  and  $I \in [0, I_u]$  by  $(P_M(\rho), I_M(\rho))$ , with  $P_M(\rho) \in (c(I_M(\rho)), P_u)$  and  $I_M(\rho) \in (0, I_u)$ . These values jointly satisfy

$$(3) \quad \Pi_P(P, I, \rho) = (P - c(I))D'(P) + D(P) = 0$$

$$(4) \quad \Pi_I(P, I, \rho) = -r + [\rho \Pi'_e(c(I)) + (1 - \rho) \Pi'_N(c(I)) - D(P)]c'(I) = 0$$

Assume further that the expected profit function is jointly and globally concave in  $P$  and  $I$ . Using (3) and (4), this means that, for all  $P$  and  $I$ ,

$$(5) \quad \Pi_{PP}(P, I, \rho) = (P - c(I))D''(P) + 2D'(P) < 0$$

$$(6) \quad \begin{aligned} \Pi_{II}(P, I, \rho) &= [\rho \Pi_e'(c(I)) + (1 - \rho) \Pi_N'(c(I))] (c'(I))^2 \\ &+ [\rho \Pi_e(c(I)) + (1 - \rho) \Pi_N(c(I)) - D(P)] c''(I) < 0 \end{aligned}$$

$$(7) \quad J(P, I, \rho) \equiv \Pi_{PP}(P, I, \rho) \Pi_{II}(P, I, \rho) - (\Pi_{PI}(P, I, \rho))^2 > 0$$

where  $J(P, I, \rho)$  is the Jacobian determinant associated with (3) and (4).

It is now convenient to define  $\tilde{I}(P, \rho)$  as the optimal investment level for a given  $P$  and  $\rho$ ; that is,  $\tilde{I}(P, \rho)$  is the unique solution to (4). Of course, the optimal investment level when a monopoly price is selected is the corresponding monopoly investment level, and so  $\tilde{I}(P_M(\rho), \rho) = I_M(\rho)$ . The final assumptions are that  $\tilde{I}(P, \rho) \in (0, I_u)$  and that profit is always negative at  $P = 0$ :

$$(8) \quad \Pi(0, \tilde{I}(0, \rho), \rho) < 0$$

It is direct to argue that these assumptions are consistent.<sup>10</sup>

With these suppositions in place, further properties of the expected profit function are easily uncovered. In particular, for all  $P$  and  $I$ , (1)-(4) give

$$(9) \quad \Pi_\rho(P, I, \rho) = \Pi_e(c(I)) - \Pi_N(c(I)) < 0$$

$$(10) \quad \Pi_{PI}(P, I, \rho) = -c'(I) D'(P) < 0$$

$$(11) \quad \Pi_{I\rho}(P, I, \rho) = [\Pi_e'(c(I)) - \Pi_N'(c(I))] c'(I) < 0$$

$$(12) \quad \Pi_{P\rho}(P, I, \rho) = 0$$

Thus, a higher probability of entry is detrimental to expected profit, as an incumbent prefers that entry not occur; a lower pre-entry price raises current sales and thus the incentive to invest in cost reduction; a lower probability of entry increases expected future sales and hence the benefits from cost reduction; and finally there is no direct interaction between the pre-entry price and the probability of entry.

It is now straightforward to compute comparative statics derivatives. Calculations reveal that (5)-(7) and (9)-(12) yield

$$(13) \quad P'_M(\rho) = \frac{\Pi_{PI}(P_M(\rho), I_M(\rho), \rho) \Pi_{I\rho}(P_M(\rho), I_M(\rho), \rho)}{J(P_M(\rho), I_M(\rho), \rho)} > 0$$

$$(14) \quad I'_M(\rho) = \frac{-\Pi_{PP}(P_M(\rho), I_M(\rho), \rho) \Pi_{I\rho}(P_M(\rho), I_M(\rho), \rho)}{J(P_M(\rho), I_M(\rho), \rho)} < 0$$

Intuitively, as the probability of entry rises, expected future sales drop, leading to a reduction in investment. This in turn prompts an increase in the unit cost of production, causing the pre-entry monopoly price to climb.

Finally, using (5)-(7) and (9)-(12) once more, it is important to record that

$$(15) \quad \tilde{I}_\rho(P, \rho) = \frac{-\Pi_{I\rho}(P, \tilde{I}(P, \rho), \rho)}{\Pi_{II}(P, \tilde{I}(P, \rho), \rho)} < 0$$

$$(16) \quad \tilde{I}_P(P, \rho) = \frac{-\Pi_{PI}(P, \tilde{I}(P, \rho), \rho)}{\Pi_{II}(P, \tilde{I}(P, \rho), \rho)} < 0$$

Thus, a higher probability of entry diminishes expected future sales and hence investment incentives, for any fixed price. Similarly, when the pre-entry price rises, pre-entry sales fall and investment incentives decline.

## B. Equilibrium

Consider first the possibility of a pure-strategy equilibrium. Certainly, it cannot be the case that  $k \geq 2$  firms tie for the highest price,  $P_{\max}$ . One of the highest-priced firms could then deviate from  $(P_{\max}, I)$  to  $(P_{\max} - \epsilon, I)$  for  $\epsilon > 0$  and small and increase expected profit, since the probability of entry would decline from  $1/k$  to 0. Suppose then that exactly one firm selects  $P_{\max}$ . This firm faces entry with probability one and so cannot be deterred from selecting the monopoly price  $P_M(1)$ ; thus,  $P_{\max} = P_M(1)$  in this case. The other  $N - 1$  firms are sure to avoid entry so long as prices lie below  $P_M(1)$ ; therefore, since  $P_M(0) < P_M(1)$  follows from (13), these firms select  $P_M(0)$  and  $I_M(0)$ . But this is inconsistent with equilibrium behavior: by choosing  $(P_M(0) - \epsilon, I_M(0))$  with  $\epsilon > 0$  and small, the high-priced firm gains approximately

$$\Pi(P_M(0), I_M(0), 0) - \Pi(P_M(1), I_M(1), 1) > 0$$

where the inequality follows from (9) and the envelope theorem. Thus, the "loser lose all" feature of the entrant's comparison strategy precludes a pure-strategy equilibrium.

Proposition 1: There is no pure-strategy equilibrium.

The discussion above indicates that some randomization in the price choice is necessary for equilibrium. The characterization of mixed-strategy equilibria is simplified in a rather natural way if attention is restricted to symmetric equilibria. Observe first that such equilibria cannot entail point masses, where a particular price is played with positive probability.

Proposition 2: There is no symmetric equilibrium with point masses in the pricing strategy.

The idea is simple. If in a symmetric equilibrium a price were played with positive probability, then a positive chance of a tie at this price would exist. By shading its price downward slightly, a firm could avoid entry in such events.<sup>11</sup>

Letting  $\hat{F}(P)$  be the distribution function for a symmetric equilibrium pricing strategy, the absence of point masses guarantees that  $\hat{F}(P)$  is continuous over its domain. The investment action is also well-behaved. For a given  $P$ , an equilibrium probability of entry,  $\hat{\rho}(P)$ , is determined. Since (6) ensures that  $\Pi(P, I, \rho)$  is concave in  $I$ , a unique  $I$  value maximizes  $\Pi(P, I, \rho)$  when  $P$  and  $\hat{\rho}(P)$  are given; this value is, of course,  $\tilde{I}(P, \hat{\rho}(P))$ , where  $\hat{\rho}(P) = (\hat{F}(P))^{N-1}$  in a symmetric equilibrium.

Observe, however, that the lack of point masses implies that each  $P$ , with its associated  $I$ , is chosen with probability zero. This means that a given  $P - I$  combination has no actual effect on expected profit, from which it follows that the density function for  $P$  and  $I$  may exhibit arbitrary behavior over a set of measure zero. To direct emphasis away from such anomalies, the results below are expressed in terms of the distribution function  $\hat{F}(P)$ , and it is assumed that  $I$  is selected to maximize expected profit, conditional on a given  $P$  having been selected. This ensures that equilibrium price-investment pairs take the form  $(P, \tilde{I}(P, \hat{\rho}(P)))$ .

As  $\hat{F}(P)$  is continuous over the compact set  $[0, P_u]$ , there must exist numbers  $\underline{P}$  and  $\bar{P}$  such that  $\underline{P}$  is the largest price for which  $\hat{F}(\underline{P}) = 0$  while  $\bar{P}$  is the smallest price satisfying  $\hat{F}(\bar{P}) = 1$ . In fact,  $\bar{P}$  is easily found.

Proposition 3: In any symmetric equilibrium,  $\bar{P} = P_M(1)$  .

The essential argument is direct. Since there are no point masses, a firm selecting  $\bar{P}$  is sure to face entry. There is then no obstacle to a deviation to the price  $P_M(1)$ , if  $P_M(1) \neq \bar{P}$  .<sup>12</sup>

In characterizing the remaining features of the distribution  $\hat{F}(P)$ , it is useful to define

$$(17) \quad \tilde{\Pi}(P, \rho) = \Pi(P, \tilde{I}(P, \rho), \rho) \quad .$$

which is the expected profit value when I is set at its optimal level for a given P and  $\rho$ . Some properties of this maximized function are readily determined:

$$(18) \quad \tilde{\Pi}_P(P, \rho) = \Pi_P(P, \tilde{I}(P, \rho), \rho)$$

$$(19) \quad \tilde{\Pi}_{PP}(P, \rho) = \Pi_{PP}(P, \tilde{I}(P, \rho), \rho) + \Pi_{PI}(P, \tilde{I}(P, \rho), \rho) \tilde{I}_P(P, \rho) < 0$$

$$(20) \quad \tilde{\Pi}(0, \rho) = \Pi(0, \tilde{I}(0, \rho), 0) < 0$$

$$(21) \quad \tilde{\Pi}_\rho(P, \rho) = \Pi_\rho(P, \tilde{I}(P, \rho), \rho) < 0 \quad .$$

where these relationships employ (17), (5), (7), (8), and (9). Recalling that  $\tilde{I}(P_M(\rho), \rho) = I_M(\rho)$  and using (18), it is clear that  $\tilde{\Pi}(P, \rho)$  is maximized at the price  $P_M(\rho)$ .

With these properties in place, the following lemma is direct to establish.

Lemma: There exists a unique price  $P_L$  for which  $P_L < P_M(0)$  and

$$\tilde{\Pi}(P_L, 0) = \tilde{\Pi}(P_M(1), 1) \quad .$$

As shown in Figure 1,  $P_L$  is sure to exist, since (20) and (21) guarantee that  $\tilde{\Pi}(P, 0)$  lies above  $\tilde{\Pi}(P, 1)$  and that  $\tilde{\Pi}(0, 0) < 0$  . Finally, the concavity of  $\tilde{\Pi}(P, 0)$  in  $P$  as given in (19) ensures that  $P_L$  is unique.

Consider now the value  $\underline{P}$  which defines the lower support bound for the symmetric pricing strategy. When a firm selects this price, it deters entry with probability one. Since a firm which employs a mixed strategy must be indifferent over the support of the strategy, this implies:<sup>13</sup>

Proposition 4: In any symmetric equilibrium,  $\underline{P} = P_L$ .

The upper and lower bounds for the mixed pricing strategy are thus  $\bar{P} = P_M(1)$  and  $\underline{P} = P_L$ , respectively.

It remains to determine the properties of  $\hat{F}(P)$  over the interval,  $[P_L, P_M(1)]$ . A first point is that gaps in the pricing strategy are not possible in a symmetric equilibrium. To see this, suppose a gap  $(P_1, P_2)$  exists such that  $\hat{F}(P_1) = \hat{F}(P_2)$  and  $\underline{P} < P_1 < P_2 < \bar{P}$ . Suppose further that  $(P_1, P_2)$  is the largest such gap, in that  $\hat{F}(P_1 - \epsilon) < \hat{F}(P_1)$  and  $\hat{F}(P_2 + \epsilon) > \hat{F}(P_2)$  for all

$\epsilon > 0$ . Since prices just below  $P_1$  and just above  $P_2$  are then selected with positive probability, a mixed-strategy equilibrium is possible only if  $\tilde{\Pi}(P_1, \hat{\rho}(P_1)) = \tilde{\Pi}(P_2, \hat{\rho}(P_1))$ , where  $\hat{\rho}(P_1) = (\hat{F}(P_1))^{N-1} = (\hat{F}(P_2))^{N-1}$  is the equilibrium probability of entry under  $P_1$  and  $P_2$ . But, using (19), this is possible only if  $P_1 < P_M(\hat{\rho}(P_1)) < P_2$ . The contradiction is now apparent, as a deviation to  $(P_M(\hat{\rho}(P_1)), I_M(\hat{\rho}(P_1)))$  does not change the probability of entry (as compared to  $P_1$  or  $P_2$ ) and increases expected profit.

With gaps ruled out, it must be that all prices in the interval  $[P_L, P_M(1)]$  yield the same expected profit. Letting  $\hat{I}(P)$  denote the equilibrium investment choice paired with a particular price selection, it follows that:

Proposition 5: In any symmetric equilibrium, for any  $P \in [P_L, P_M(1)]$ ,  $\hat{F}(P)$  and  $\hat{I}(P)$  must satisfy

$$(22) \quad \tilde{\Pi}(P, (\hat{F}(P))^{N-1}) = \tilde{\Pi}(P_M(1), 1)$$

$$(23) \quad \hat{I}(P) = \tilde{I}(P, (\hat{F}(P))^{N-1}) \quad .$$

Thus, the loser-lose-all nature of the entrant's comparison strategy forces a symmetric equilibrium in which firms mix over prices in a manner that keeps expected profit constant for all prices between  $P_L$  and  $P_M(1)$ . For any particular price, a firm's investment level is then set optimally, given the implied probability of entry.

The necessary features of symmetric equilibria are further highlighted by the following corollary:



Corollary: In any symmetric equilibrium, for any  $P \in [P_L, P_M(1))$ ,  $P < P_M(\hat{\rho}(P))$  and  $\hat{I}(P) > I_M(\hat{\rho}(P))$ , where  $\hat{\rho}(P) = (\hat{F}(P))^{N-1}$ .

This corollary indicates a sense in which the competition between incumbents induces a downward distortion in pre-entry pricing and an upward distortion in investment. In particular, when an incumbent selects a price  $P$ , an equilibrium probability of entry,  $\hat{\rho}(P)$ , is determined. The corollary states that the incumbent is pricing lower and investing higher than would be desired, given the probability that entry will actually occur.

The corollary is easily proved with reference to Figure 1. Fix  $P_1 \in [P_L, P_M(1))$  and thus  $\hat{\rho}(P_1) = (\hat{F}(P_1))^{N-1} \in [0, 1)$ . As the figure illustrates, satisfaction of (22) requires  $P_1 < P_M(\hat{\rho}(P_1))$ . Using (16) and (23) then gives

$$\hat{I}(P_1) = \tilde{I}(P_1, \hat{\rho}(P_1)) > \tilde{I}(P_M(\hat{\rho}(P_1)), \hat{\rho}(P_1)) = I_M(\hat{\rho}(P_1)) \quad .$$

Intuitively, as competition between incumbents forces downward-pricing distortions, pre-entry sales rise above the corresponding monopoly level, creating additional benefits from cost reduction. In this way, downward distortions in price "spill over" into upward distortions in investment.

The necessary features of a symmetric equilibrium are now characterized. When the entrant uses the comparison strategy, the resulting competition between incumbents necessitates a mixed-strategy equilibrium in which firms distort prices downward with probability one. Of course, it is not completely surprising that the entrant's comparison strategy exerts downward pressure on prices. What is perhaps more subtle is that the comparison rule is fully rational.

Proposition 6: There exists a symmetric equilibrium in which  $\hat{F}(P)$  is defined by (22) and  $\hat{I}(P)$  is defined by (23) for all  $P \in [P_L, P_M(1)]$ .

The proof proceeds by construction with four simple steps. The first step is to show that a probability-of-entry function exists which induces indifference for incumbents over all  $P \in [P_L, P_M(1)]$ . That is, it is first demonstrated that a function  $\hat{\rho}(P)$  exists, satisfying

$$(24) \quad \tilde{\Pi}(P, \hat{\rho}(P)) \equiv \tilde{\Pi}(P_M(1), 1)$$

for  $P \in [P_L, P_M(1)]$ . The construction of this function is easily understood with reference to Figure 1. For a pre-entry price  $P_1 \in [P_L, P_M(1)]$ , a unique  $\rho$  value exists which satisfies (24). Clearly, if  $P_1$  were to increase in the direction of more profitable prices, a larger entry probability would be required to restore indifference. More formally, using (21), the implicit function theorem ensures the existence of a differentiable function  $\hat{\rho}(P)$  satisfying (24) and

$$(25) \quad \hat{\rho}'(P) = \frac{-\tilde{\Pi}_P(P, \hat{\rho}(P))}{\tilde{\Pi}_\rho(P, \hat{\rho}(P))} > 0$$

for  $P \in [P_L, P_M(1))$ . Observe further that  $\hat{\rho}(P_L) = 0 < 1 = \hat{\rho}(P_M(1))$ .

The second step is to prove that a legitimate distribution function  $\hat{F}(P)$  exists which satisfies (22). This is a simple matter, as  $\hat{F}(P)$  may be defined by  $\hat{F}(P) = (\hat{\rho}(P))^{\frac{1}{N-1}}$ . Note that  $\hat{F}(P_L) = 0 < 1 = \hat{F}(P_M(1))$  and  $\hat{F}'(P) > 0$

for  $P \in [P_L, P_M(1)]$ . Thus, a distribution function exists which holds expected profit constant along the price strategy support.

The incumbents' strategy is verified to be optimal in the third step. Certainly, no incumbent can gain by altering the probability with which it plays some  $P \in [P_L, P_M(1)]$ . Further,  $P > P_M(1)$  yields lower expected profit than does  $P_M(1)$  and  $P < P_L$  is similarly inferior to  $P_L$ , taking as given other players' strategies.

The final and most interesting step is to show that the comparison strategy is rational. Observe that

$$(26) \quad \hat{I}'(P) = \tilde{I}_p(P, (\hat{F}(P))^{N-1}) + \tilde{I}_\rho(P, (\hat{F}(P))^{N-1}) \frac{\partial (\hat{F}(P))^{N-1}}{\partial P} < 0$$

for  $P \in [P_L, P_M(1)]$  by (15), (16), (23) and (25). Thus, higher prices are paired with lower investment levels and greater incumbent costs: the comparison strategy is indeed a best response to the incumbent's strategy.

A careful inspection of (26) reveals two complementary effects. First, when the entrant uses the comparison strategy, an incumbent that prices high faces a significant chance of entry. Thus, the incumbent's expected post-entry period output level is lower than if it had priced low and reduced the odds of entry. It follows that high-priced incumbents have an incentive to invest less in cost reduction, precisely because the entrant uses the comparison strategy. The second effect is less subtle, as it is independent of the entrant's strategy. The simple point here is that a high pre-entry price curtails sales in the pre-entry period and thereby reduces the incumbent's incentive to invest in cost reduction.

### C. Entry, Investment, and Price

Consider now the effect of entry on equilibrium pricing and investment. There are three benchmarks that might be considered.

#### Benchmark 1: No entrant.

Absent an entrant, each incumbent would set  $P = P_M(0)$  and  $I = I_M(0)$ . As illustrated in Figures 2A and 2B, the threat of entry may lead to lower or higher pricing and lower or higher investment. The possibility of entry acts to lower investment, as less future sales are anticipated. This in turn raises the pre-entry production cost and price. Thus, an incumbent may price higher and invest less than when no entrant exists. On the other hand, the incentive to redirect entry forces pre-entry prices downward, which stimulates sales and induces greater investment. In short, when costs are endogenous, the threat of entry has ambiguous consequences for price and investment levels.

#### Benchmark 2: Certain Entry.

In this case, each incumbent selects  $P_M(1)$  and  $I_M(1)$ . Here, the various influences are reinforcing. When entry is no longer certain, greater future sales are expected, leading to higher investment, lower production costs, and thus lower prices. Similarly, competition among incumbents lowers prices, which increases pre-entry sales and the benefits from investment. Thus, the potential to redirect entry results in lower pre-entry prices and greater investment.

Benchmark 3: Ex Post Incentives.

This benchmark addresses the incentive that incumbents have to adjust behavior ex post, that is, after the probability of entry is determined. As captured in the Corollary and illustrated in Figures 2A and 2B, each incumbent has an incentive to raise its price, if it thought it could do so without affecting the probability of entry. Furthermore, incumbents also have an ex post incentive to lower investment. Thus, incumbents price lower and invest higher than they would prefer, given the probability that entry will actually occur. This seems a reasonable standard with which to conclude that limit-pricing behavior occurs.

#### D. Competitive Limit Pricing with Delayed Cost Reduction

An alternative hypothesis is that investment has only a delayed effect on production costs. In particular, one might imagine that investment reduces costs only in the post-entry period. Under this hypothesis, an incumbent's expected profit function is

$$\Pi(P, I, \rho) \equiv (P - c(0))D(P) - rI + \rho\Pi_e(c(I)) + (1 - \rho)\Pi_N(c(I))$$

As a formal analysis of this model offers little new insight, only the key findings are reported here.

The novel feature of the delayed-cost-reduction model is the absence of any direct interaction between the pre-entry price and the investment level (i.e.,  $\Pi_{PI}(P, I, \rho) = 0$ ). This has two implications. First, a single monopoly price,  $P_M$ , maximizes  $\Pi(P, I, \rho)$  for all  $I$  and  $\rho$ . Second, pricing distortions no longer "spill over" into investment distortions. Rather, the optimal

investment selection is only a function of the probability of entry, so that  $\hat{I}(P) = I_M(\hat{\rho}(P))$  for any  $P$  that might be chosen in a symmetric equilibrium.

Under assumptions exactly similar to those above, a unique symmetric equilibrium outcome arises. In this outcome, a price  $P_L$  exists with  $P_L < P_M$  and incumbents mix over all prices between  $P_L$  and  $P_M$ , with the exact distribution function being selected to generate the necessary indifference. As illustrated in Figures 3A and 3B, prices are distorted downward but investment is not distorted. Clearly, limit pricing occurs in this model, since the pre-entry price lies below the unique monopoly price with probability one.

This model also serves to isolate the important role played by the entrant's comparison strategy in competitive-limit-pricing equilibria. As an incumbent's pre-entry production costs are fixed and independent of investment, an incumbent will pair a low price and a high investment level only if it anticipates that the associated entry probability is low. But this expectation is in fact correct when the entrant employs the comparison strategy. In this way, the incumbent's optimal response to the comparison strategy in turn rationalizes the rule. Thus, the comparison strategy induces limit pricing and constitutes rational behavior, even when pre-entry production costs are unresponsive to investment.<sup>14</sup>

### III. Comparative Statics

#### A. The Number of Markets

Consider now the possibility that the number of markets rises from  $N$  to  $N+1$ . What effect does this have on the symmetric pricing and investment incentives of incumbents? One might expect incumbents to "ease up" and compete less vigorously, since, all else equal, each incumbent faces a lower probability of entry as the number of markets expands. In fact, the opposite occurs: as an additional market for entry is allowed, pre-entry prices tend to drop and investment tends to rise.

To prove this, observe that  $\underline{P}$  and  $\bar{P}$  are independent of  $N$ . Further, as shown in the first step of the proof of Propositions 6, for any  $P \in [\underline{P}, \bar{P}]$ , a unique value  $\hat{\rho}(P)$  is defined to satisfy  $\tilde{\Pi}(P, \hat{\rho}(P)) = \tilde{\Pi}(\bar{P}, 1)$ , and this value is also independent of  $N$ . Letting  $\hat{F}(P, N)$  be the symmetric equilibrium pricing distribution function when  $N$  markets exist, it follows that

$$(\hat{F}(P, N))^{N-1} = \hat{\rho}(P) = (\hat{F}(P, N+1))^N$$

which in turn implies  $\hat{F}(P, N) < \hat{F}(P, N+1)$  for  $P \in (\underline{P}, \bar{P})$ . As the number of markets grows, more weight is put on lower prices; i.e., a first order stochastic shift toward lower prices occurs. Since the probability of entry at any given price is unchanged and lower prices are selected more often, (23) implies that higher investment levels are chosen with greater frequency as the number of markets expands.

The simple idea here is that a mixed-strategy equilibrium exists only if each price is balanced against a specific entry probability. When the number

of incumbents rises, the probability of entry associated with a given price declines. To restore the entry probability to the necessary level, other incumbents must select lower pre-entry prices more often.

### B. The Scope of Entry

Suppose next that the entrant seeks to enter  $z > 1$  markets. This would appear to make the competition between incumbents more intense, suggesting lower pre-entry prices and greater investment. Careful analysis reveals, however, that prices tend to rise and investment tends to fall as the scope of entry increases.

Assuming that  $N-1 \geq z$ , so that a winner and a loser always exists, Propositions 1-4 may be verified to apply. Thus,  $\underline{P}$  and  $\bar{P}$  are not altered by the expansion of entry scope. Again, it must be true that  $\tilde{\Pi}(P, \hat{\rho}(P)) \equiv \tilde{\Pi}(\bar{P}, 1)$  for all  $P \in [\underline{P}, \bar{P}]$ . Letting  $\hat{F}(P, z)$  denote the symmetric pricing distribution function when the entrant seeks to enter  $z$  markets, it follows that, for any  $P \in [\underline{P}, \bar{P}]$ ,

$$(27) \quad \hat{\rho}(P) = \sum_{k=0}^{z-1} \binom{N-1}{k} (\hat{F}(P, z))^{N-1-k} (1 - \hat{F}(P, z))^k$$

for any integer  $z$  such that  $1 \leq z \leq N-1$ .<sup>15</sup> Here,  $k$  may be understood as the number of incumbents with higher pre-entry prices.

To examine a tractable case, consider the transition from  $z = 1$  to  $z = 2$ ; that is, consider the change in the pricing strategy that occurs when an entrant seeks to enter two markets as opposed to one market. Using (27), a little algebra gives



$$(28) \quad (\hat{F}(P,1))^{N-1} = \hat{\rho}(P) = (\hat{F}(P,2))^{N-2} [(\hat{F}(P,2))(2 - N) + N - 1] \quad .$$

As the right hand side of (28) increases in the variable  $\hat{F}(P,2)$  for all  $\hat{F}(P,2) \in (0,1)$ , the implicit function theorem guarantees the existence of a continuously increasing function,  $\hat{F}(P,2)$ , satisfying (28),  $\hat{F}(\underline{P},2) = 0$ , and  $\hat{F}(\bar{P},2) = 1$ . Thus,  $\hat{F}(P,2)$  is also a well-defined distribution function. It is now straightforward to use (28) to conclude that  $\hat{F}(P,2) < \hat{F}(P,1)$  for  $P \in (\underline{P}, \bar{P})$ . As the scope of entry rises, the pre-entry price distribution experiences a first order shift toward higher prices; hence, higher pre-entry prices and lower investment are associated with a greater entry scope.

Once again, the result is counter-intuitive and yet easy to explain. When the entrant expands its scope, the probability of entry at a given price is higher, all else equal. To restore the critical entry probability at the given price, other incumbents must select higher pre-entry prices more often.

#### IV. Competitive Limit Pricing and Aggregate Entry Deterrence

##### A. Framework

Up to this point, the level of aggregate entry has been certain. The motivating idea has been that the entrant finds all markets profitable but, due to financial constraints or other reasons, is able to select only (say) one market for entry. This section modifies the framework developed above, in order to determine whether limit pricing and aggregate entry deterrence may be associated, if markets are potentially unprofitable for entry.

The model is identical to that developed in Section 2, with an entrant seeking to enter one market and investment affecting pre-entry and post-entry costs, except that it is now assumed that a critical limit cost level,  $c_t \in (c(I_u), c(0))$ , exists at which  $V(c_t) = 0$ . In the modified model, entry is profitable if and only if the corresponding incumbent invests sufficiently little that its unit cost exceeds  $c_t$ . Letting  $I_t \in (0, I_u)$  satisfy  $c(I_t) = c_t$ , the entrant profits from entry only if the relevant incumbent's investment is less than the limit investment level,  $I_t$ . Next, since investment is unobservable and therefore selected to maximize expected profit, if a particular pre-entry price  $P$  is expected to deter entry, then  $\tilde{I}(P, 0)$  will be the matching investment choice. It is convenient therefore to define  $P_t$  by  $\tilde{I}(P_t, 0) = I_t$ ; thus,  $P_t$  is a limit price in that, if no entry is expected and the pre-entry price  $P_t$  is picked, then the incumbent has the incentive to select the entry-detering investment level,  $I_t$ . The key points below are most easily expressed if  $P_t$  is always well-defined; this is guaranteed under the further assumption that  $\tilde{I}(0, 0) \geq I_t \geq \tilde{I}(P_u, 0)$ .<sup>16</sup>

The comparison strategy may be modified for this environment. A natural definition is: for markets with pre-entry prices in excess of  $P_t$ , simply enter the highest-priced market; however, never enter a market in which the pre-entry price is  $P_t$  or less.<sup>17</sup> In other words, the entrant uses the standard comparison rule, unless all pre-entry prices are no greater than  $P_t$ . In this latter case, no entry occurs.

### B. Entry Redirection

To begin, suppose  $c_t < c(\tilde{I}(P_L, 0))$  or, equivalently,  $I_t > \tilde{I}(P_L, 0)$ , where  $P_L$  is defined by the lemma. These relationships are captured in Figures 4A and 4B. Notice that the aggregate entry deterrence outcome is feasible here: if each incumbent sets the pre-entry price  $P_t$  and expects to deter entry, then the optimal investment level is  $I_t$ , which is indeed sufficiently large to make entry unprofitable. The basic idea is that a low pre-entry price stimulates sales and raises the incentive for cost reduction, given that investment has an immediate effect on costs.

While aggregate entry deterrence is feasible, however, it does not represent equilibrium behavior. It is simply too expensive for incumbents to reduce their costs to the low value of  $c_t$ . Formally, recalling the definition of  $P_L$  in the lemma and using  $P_t < P_L$  (as captured in Figure 4B), it follows at once that

$$\tilde{\Pi}(P_t, 0) < \tilde{\Pi}(P_L, 0) = \tilde{\Pi}(P_M(1), 1)$$

and so an incumbent would deviate from  $(P_t, \tilde{I}(P_t, 0))$  to  $(P_M(1), I_M(1))$ .

In fact, continuing to use the term "equilibrium" to refer to a Nash equilibrium in which the entrant uses the (modified) comparison strategy, it is direct to show that all of the conclusions in Section 2 remain valid for the  $c_t < c(\tilde{I}(P_L, 0))$  case. Thus, even if aggregate entry deterrence is feasible, if it requires too great of a reduction in production costs, then incumbents will choose not to deter aggregate entry. Rather, the unique

equilibrium will entail the redirection of entry, as characterized in the mixed-strategy equilibrium of Proposition 6.

### C. Blockaded Entry

Suppose now that  $c_t \geq c(I_M(0))$  or, equivalently,  $I_t \leq I_M(0)$ . This case is illustrated in Figures 5A and 5B. Here aggregate entry deterrence is quite easy, as entry is completely deterred if all incumbents' costs are not terribly high (i.e., at or below  $c_t$ ).

Observe that the mixed-strategy equilibrium of Proposition 6 now fails to exist. This follows since  $P_t > P_L$ : the probability of entry is reduced to zero at prices well above  $P_L$ . In fact, by choosing  $P_M(0)$ , an incumbent is sure to deter entry and monopolize its market. As an incumbent can never do better than this, the unique equilibrium is a blockaded equilibrium, where incumbents act as if there were no entrant, choosing  $(P, I) = (P_M(0), I_M(0))$ , and this behavior in turn deters aggregate entry.

### D. Strategic Aggregate Entry Deterrence

Suppose finally that  $c(I_M(0)) > c_t > c(\tilde{I}(P_L, 0))$  or, equivalently,  $I_M(0) < I_t < \tilde{I}(P_L, 0)$ . This set of relationships is given in Figures 6A and 6B; observe that  $P_t$  satisfies  $P_L < P_t < P_M(0)$  in this case.

An attractive pure-strategy equilibrium exists for this game. In it, each incumbent sets its pre-entry price at  $P_t$ , selects investment level  $I_t$ , and deters entry. The entrant's comparison strategy is a best response, since  $c_t$  is the cost of each incumbent. Further, no incumbent wishes to deviate: pre-entry prices below  $P_t$  are further away from the relevant monopoly price,

$P_M(0)$ , and a pre-entry price above  $P_t$  is sure to attract entry, leading to a decrease in profit since  $P_M(0) > P_t > P_L$  implies

$$(29) \quad \tilde{\Pi}(P_t, 0) > \tilde{\Pi}(P_L, 0) = \tilde{\Pi}(P_M(1), 1)$$

where  $\tilde{\Pi}(P_M(1), 1)$  gives the maximum profit available from certain entry.

In fact, this pure-strategy equilibrium is the unique pure-strategy equilibrium and the unique symmetric equilibrium. It is straightforward to see that the "loser lose all" nature of the comparison strategy precludes the existence of any other pure-strategy equilibrium. Further, symmetric equilibria with point masses above or below  $P_t$  fail to exist, as expected profit rises with slight downward or upward price deviations, respectively. Finally, if a mixed-strategy equilibrium were to exist without point masses (except perhaps at  $P_t$ ), then familiar arguments imply that  $P_M(1)$  is the upper support of the strategy. But this is impossible under (29), which indicates that all probability weight should be given to  $P_t$ . Thus, the described pure-strategy equilibrium is extremely focal for this case.

This equilibrium has several novel features. First, limit pricing occurs, as  $P_t$  is less than all monopoly prices, and aggregate entry is deterred. Intuitively, by setting a pre-entry price that is sufficiently low, an incumbent heightens sales in the pre-entry period to such an extent that the associated optimal investment level makes entry unprofitable. This result, of course, requires an immediate cost reduction from investment, but it does not require that investment be observed by the entrant.

Thus, when costs are endogenous, a linkage arises between limit pricing and aggregate entry deterrence. This linkage underscores important

differences between endogenous- and exogenous-cost models, especially as regards the consequences of public policy against limit-pricing activity. To illustrate, imagine the existence of a fully-informed regulator whose objective is to prohibit all limit pricing. In the exogenous-cost model studied by Milgrom and Roberts, this restriction causes an incumbent to always select its monopoly price, thus revealing its costs. For any given cost type, the probability of entry facing an incumbent is then the same as in the separating, limit-pricing equilibria that Milgrom and Roberts emphasize. In the endogenous-cost model, however, a ban on limit-pricing can induce entry when it would be otherwise deterred. This is because the restriction blocks the investment stimulus that low pre-entry prices provide.<sup>18</sup>

A second comment is that the limit-pricing, entry-deterrence equilibrium also exists when only a single market is available for entry. The entrant's strategy is then interpreted as a "trigger strategy" that specifies entry for any pre-entry price above  $P_t$ .

A third and related remark is that the featured equilibrium continues to exist and remains unique over the class of symmetric equilibria, even if the entrant considers entry into  $z \leq N - 1$  markets (with the obvious modifications in the comparison strategy). Thus, the comparative statics findings reported in Section 3 do not apply when  $c(I_M(0)) > c_t > c(\tilde{I}(P_t, 0))$ .

A convenient summary of the results in this section is given in Figure 7. The entry redirection findings of the previous sections are valid even when aggregate entry deterrence is feasible, provided that such deterrence requires large investment. Strategic limit pricing which deters aggregate

entry occurs when the entrant is more easily deterred.<sup>19</sup> Finally, if very little investment is needed for deterrence, entry is blockaded.

## VI. Conclusion

From the start, limit pricing models have been haunted by a dilemma: why should an entrant respond to limit pricing, if it knows the incumbent has the opportunity to select a new price should entry occur? This paper reconciles this dilemma with a model of endogenous cost technologies, in which lower pre-entry prices are complementary to greater investment in cost reduction. This reconciliation does not require an incomplete-information, Bayesian framework or even sophisticated entrant inferences; portions of the analysis do, however, require mixed strategies.

There are at least two important extensions that remain. First, the possibility of multiple incumbents within a single market is intriguing. Depending upon how the comparison strategy is formulated, some free-riding might develop among incumbents in a given market in the provision of low prices and high output.<sup>20</sup> Second, for manufacturing industries, pre-entry prices may be observable only with noise, since the good's final price may be influenced by the retailer. Pure-strategy equilibria are likely to emerge in this setting; however, if entry is always profitable, then the entrant's comparison strategy is (strictly) optimal only in an asymmetric pure-strategy equilibrium.

## Notes

1. This informational theory has been extended in several directions. See, e.g., Bagwell and Ramey (1988, 1990, 1991a), Harrington (1986, 1987), Matthews and Mirman (1983), Ramey (1987), and Srinivasan (forthcoming).
2. This may be contrasted with the Milgrom-Roberts theory and most of its extensions (as given in Note 1). There, an entrant may require rather specialized information in order to determine the critical price below which a high-cost incumbent would never venture. The implied sophistication level is even larger when standard refinements are used to eliminate equilibria.
3. Thus, this theory rationalizes the exogenous and stochastic entry function specified by Kamien and Schwartz (1971).
4. The comparison strategy must be modified to be consistent with the possibility of unprofitable entry. The modified strategy entails entering the highest-priced market, provided that price exceeds some critical, entry-detering limit price.
5. This extension was originally suggested by Milgrom and Roberts (pp. 449-50). Gal-Or's (1980) dissertation also should be mentioned. She assumes, however, that a single entrant commits to an entry-probability function before incumbents select prices. In a different model, where a fringe of entrants exists, incumbent prices rise as the number of markets expands. This contrasts with the comparative statics results found in this paper.
6. Rogerson does find a pure-strategy equilibrium in which each incumbent sets its monopoly price; this equilibrium exists only if the cost difference is sufficiently severe. He also finds a separating, limit-pricing equilibrium in pure strategies, under the assumptions that the low-cost incumbent picks its price before the high-cost incumbent and that the entrant does not observe the pricing sequence.
7. It is worth noting that the analysis in Sections 2 and 3 requires only that the entrant observe the ranking of prices; i.e., the entrant's strategy depends only on ordinal price information.
8. The imposition of sequential rationality has no additional bite in this game, since a deviant price may be associated with low investment, making it sequentially rational to enter the deviator's market.
9. This discussion ignores the strategic effect of cost reduction on entrant behavior. In particular, a lower incumbent cost level induces a lower entrant price (output) when a price (quantity) game follows entry. Thus, the



strategic effect is supportive of (2) when a price game is played, but this effect tends to work against (2) when quantity competition occurs. Even if a quantity game follows entry, however, (2) holds for linear demand provided only that  $c(I_u)$  is not too far below the entrant's cost level.

10. For example, all of the assumptions hold if

$c'(0) < r/\Pi_e'(c(0))$ ,  $c'(I_u) > r/[\Pi_N'(c(I_u)) - D(0)]$ ,  $c''(I) > 0$  is large,  $D(0)$  is large, and  $|D''|$ ,  $|\Pi_e''(c)|$ , and  $|\Pi_N''(c)|$  are small (e.g., zero). Note that  $D(0)$  must be large enough to satisfy (8) and yet not so large that the second inequality listed above is violated. There need be no conflict here, however, if  $I_u$  is taken sufficiently large that  $|c'(I_u)|$  is small.

11. A more rigorous argument parallels those developed previously by Varian (1981) and Bagwell and Ramey (1991b). If a price  $\bar{P}$  were selected with positive probability, then, since the number of point masses is countable,  $\epsilon > 0$  and arbitrarily small may be found such that  $\bar{P} - \epsilon$  is played with probability zero. Further, (6) guarantees that a unique  $I$  value, denoted  $\bar{I}$ , must accompany  $\bar{P}$  in the hypothesized equilibrium. For  $\epsilon$  sufficiently small, a deviation to  $(\bar{P} - \epsilon, \bar{I})$  is sure to increase expected profit given (9), as events in which a tie occurs are replaced with events in which entry occurs with zero probability.

12. More rigorously, by the definition of  $\bar{P}$ , prices between  $\bar{P} - \epsilon$  and  $\bar{P}$  are played with positive probability; thus, for  $\epsilon$  small, the expected profit earned over this interval is approximately  $\Pi(\bar{P}, \bar{I}(\bar{P}, 1), 1)$ , which is the profit when  $\bar{P}$  is selected. If  $\bar{P} \neq P_M(1)$ , higher expected profit can be achieved if  $(P_M(1), I_M(1))$  is played with probability one, since

$$\Pi(P_M(1), I_M(1), \hat{\rho}(P_M(1))) \geq \Pi(P_M(1), I_M(1), 1) > \Pi(\bar{P}, \bar{I}(\bar{P}, 1), 1)$$

where  $\hat{\rho}(P_M(1))$  is the probability of entry when  $P_M(1)$  is selected.

13. More rigorously, the definitions of  $\underline{P}$  and  $\bar{P}$  ensure that prices between  $\underline{P}$  and  $\underline{P} + \epsilon$  as well as between  $\bar{P}$  and  $\bar{P} - \epsilon$  are played with positive probability. For  $\epsilon$  small, prices in the former set yield approximately  $\bar{\Pi}(\underline{P}, 0)$  and prices in the latter set earn approximately  $\bar{\Pi}(\bar{P}, 1)$ . Thus, if  $\bar{\Pi}(\underline{P}, 0) \neq \bar{\Pi}(\bar{P}, 1)$ , then either  $\underline{P}$  or  $\bar{P}$  would be selected with probability one in an improving deviation.

14. The comparative statics findings in Section 3 also apply for this alternative model; however, the ability to affect costs immediately with investment is important for the entry deterrence results in Section 4. An

earlier draft with all details regarding the alternative model is available from the author.

15. The convention  $0^0 = 1$  is imposed for  $P = \bar{P}$  and  $k = 0$ .

16. Equivalently,  $c_t \in [c(\tilde{I}(0,0)), c(\tilde{I}(P_u,0))] \subset [c(I_u), c(0)]$ ; thus, this assumption further restricts the range of  $c_t$ . This restriction does not eliminate any interesting cases: if  $c_t < c(\tilde{I}(0,0))$ , then aggregate entry deterrence is infeasible and so the entry redirection results below apply; while if  $c_t > c(\tilde{I}(P_u,0))$ , then aggregate entry may be blockaded with the monopoly price  $P_M(0)$ , as discussed more generally below.

17. This modified strategy corresponds to a very safe rule: regardless of the incumbent's perceived entry probability, its optimal investment is always below  $I_t$  if the pre-entry price exceeds  $P_t$ . In other words, thinking of the pre-entry price as moving from high to low levels,  $P_t$  is the first price at which the optimal investment level is  $I_t$  or higher for some perceived entry probability (viz,  $\rho = 0$ ). While the modified strategy has this motivation, it should be noted that the comparison strategy is now more sophisticated, as it is no longer completely based on ordinal information.

18. For example, in Figure 6B,  $P_t$  would be illegal, since  $P_t < P_M(0) \leq P_M(\rho)$  for all  $\rho \in [0,1]$ .

19. When  $c_t = c(\tilde{I}(P_L,0))$ , both entry-redirection and strategic-aggregate-entry-deterrence equilibria exist.

20. For other models of free-riding in entry deterrence, see, e.g., Bagwell and Ramey (1991a), Gilbert and Vives (1986), and Harrington (1987). As Bagwell and Ramey (1991a) discuss, equilibrium behavior may be very sensitive to whether common-market incumbents observe one another's cost levels.

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