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SIMULATIONS AND SPATIAL VOTING MODELS*

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Simulations and Spatial Voting Models

We develop a model of spatial elections that departs from the standard model in three important respects. Our parties' information of voters' preferences is limited to polls; our parties can be either office-seeking or ideological; and our parties are not perfect optimizers, i.e. they are modelled as boundedly rational actors. Since our imperfect parties do not necessarily find optimal positions, rather than concern ourselves with existence and location of equilibria, we trace the trajectory of winning party positions. The outcomes are subsequently evaluated with respect to a measure of social welfare, centrality. Our results suggest that in fair voting systems, two party elections lead to normatively appealing outcomes.

We are seeking to introduce the role of computers generally and adaptive artificial agents (AAA) specifically, to the study of parties, voters, and elections in spatial models. We argue for using adaptive parties to add behavioral complexity to standard formal models of politics without sacrificing a logical foundation. Doing so may not only revise our judgments about the relevance of spatial voting models to real elections, but it may lead us to important insights about what occurs in those elections.

Introduction

Since Anthony Downs' Economic Theory of Democracy (1957), a spatial theory of elections has occupied a prominent theoretical status within political science. Practitioners use the intuitive notion of ideological distance to develop plausible explanations for observable electoral trends. The most famous of these trends is the idea that in a two party system, given certain strict assumptions, the parties will converge towards a median position on the continuum of possible voter positions. Yet further research in spatial modeling has led to a very different conclusion. Following the voting paradox and the results of Plott (1967) and McKelvey (1976), some scholars have speculated that chaotic results are possible, and in some cases, likely. In three or more dimensions, given strict assumptions, parties can move all over the space to defeat their opponent(s). Bates (1990, p. 45) summarizes the chaotic result: "The principle lesson is that, in general, one cannot expect an equilibrium to exist; and, because any outcome can be defeated, political decisions represent arbitrary outcomes." Whereas some scholars lament the predicted instability in multidimensional voting models (Riker, 1982), others see the Downsian convergence of parties on issues as a more accurate description of reality, not to mention more stable and (perhaps) more normatively desirable. There is a search, therefore, for reasonable modifications of multidimensional models that produce more stable results.

Theorists who are generally inclined to believe that electoral chaos is extremely unlikely have incorporated various complexities to explain stable, often centrist, outcomes. Coughlin (1990a) divides these models into four general categories: a) models that allow for mixed strategies by parties, b) models that track dynamic trajectories of party locations,

c) models that search for uncovered or undominated sets, and d) models that include candidate uncertainty over voters' behavior (probabilistic voting models). Coughlin writes: "(I)t is hard to resist the alternative inference that the primary contribution of recent work on the majority rule relation is as a grand 'reductio ad absurdum' that tells us to go back to the basic model that has been used to see how it should be modified in order for theory and empirical observations to match up" (p. 164).

Electoral outcomes in fact are more stable empirically than the chaotic results predict, so these revisionist scholars are on the right track in seeking alternative assumptions to get more realistic outcomes. Nevertheless, both the original spatial models and contemporary revisions rely on unrealistic assumptions to produce equilibria. They require parties to have complete knowledge and to perform superhuman calculations in locating good regions of the space. Even probabilistic voting models require parties to know the probabilities of voters' actions. Is it possible to generate realistic results without relying on unrealistic assumptions about the information and computational abilities of parties?

We advocate a new approach to study the dynamics of spatial elections, the use of adaptive artificial agents. Underlying our methodology is the notion that there exist important classes of generic behavior that can be captured in models too complex for traditional mathematical analysis. Absence of equilibria in a model (or equilibria that require hyperrational agents to locate) does not necessarily imply a lack of predictability. As we will argue in the next section, using adaptive artificial agents allows us to search previously inaccessible models for patterns of generic behavior. The model that we put forth does not stretch the boundaries of our technique and may even be amenable to

equilibrium analysis. We envision future research which presents more complicated and descriptively accurate models. Our present purposes, however, showing the strength of our approach and relaxing informational assumptions, are best served by a simple model.

Our model incorporates most of the assumptions of spatial voting models, with some important exceptions. First, modelling parties as strictly office-seeking has troubled many political scientists, particularly party theorists. It was popular in the 1950s to urge the United States towards more responsible parties, with strong ideological positions (APSA, 1950), rather than office-seeking parties who follow public opinion. Some, most notably Barry Goldwater, have formulated the debate as being between a "choice and an echo" (see Page, 1978, p. 21). We consider both kinds of parties, ideological and purely office seeking.

Second, we relax the assumption of identical voter preference intensities. Early spatial models assumed all voters have circular indifference curves, or that any deviations from symmetry will average out (Davis, et al., p. 434). Attempts to allow different intensities of preferences, (i.e. noncircular indifference curves), have led to two general results: a) equilibrium sets, such as minmax, top cycle, or uncovered, that can be disjoint or even encompass the whole space of positions (see Kramer, 1977, and Cohen, 1979, for discussion); and b) the requirement of strict assumptions, such as lexicographic preferences (Taylor, 1970) or nonmean strategies that are not necessarily democratic (Jackson, 1973), to generate stable outcomes. Those who have tried to measure the importance of intensity of preferences on vote choice in the real world have had mixed success as well (Rabinowitz, et al., 1982; Niemi and Bartels, 1985; and Rivers, 1988), but empirical evidence does indicate that preferences often correlate with how much voters care about issues (Jackson,

1973, p. 234; Dalton, 1988, pp. 193-200). So we include what we call issue "strengths" in our model below. Strength is just the degree that voters care about a particular issue in relation to other issues.

Third, most important results rely on perfectly informed and optimizing voters and candidates (Davis, Hinich, and Ordeshook, 1970). Even probabilistic models assume that the candidates know perfectly the expected outcome and that they can position themselves optimally. These models require that the candidates know voters' probabilistic decision rules, and therefore they do not reduce the informational assumptions (Coughlin 1990b). Our parties are not perfectly informed. They do not explicitly know the individual voter utility functions. Rather, they obtain vote totals in an election against the incumbent for a finite number of tested positions. It follows from the fact that the parties have a finite amount of time to move that they will not be able to find an optimal location.

The unique feature of our approach is the use of boundedly rational agents. Spatial models assume that parties act as if they perfectly make all relevant calculations. Because each election can be viewed as a unique representation of a standard problem, this assumption may be incorrect. The idiosyncracies of individual elections may complicate the problem to such an extent that the positioning strategies of parties cannot possibly be optimal. We will consider three procedures for candidate positioning and show that our results are invariant to the procedure chosen. Our results tend to agree with the small body of literature on experimental research involving spatial voting models (McKelvey and Ordeshook, 1990). For the most part, experiments confirm the analytical result that parties move toward the center of voters' preferences. Yet the external validity of the experiments is suspect.

Subjects are constrained in the ways they can interact, their choices are limited, and the space of options is typically one dimensional. While our complex systems approach will allow for many dimensions, by far a more important extension will be the inclusion of internal and external forces on the behavior of voters and candidates.

The remainder of this paper is organized into six sections. Section 2 discusses the use of artificially intelligent agents in the social sciences. Section 3 presents the basic model, and Section 4 describes in detail the two types of parties we consider. Section 5 presents the three ways our parties learn to move towards strategic positions in the issue space. Section 6 presents our results, and the conclusion discusses the possibility of future research.

Artificially Adaptive Agents in the Social Sciences

Why use computers? Why create boundedly rational, adaptive artificial agents (AAA)? Holland and Miller (1991) provide the best answer. They concur with Coleman (1989) that simulation games can be useful to the construction of social theory. They argue that AAA models offer us tools to link mathematical rigor with the contingencies of the real world, while at the same time allowing us to observe emergent phenomena not predicted a priori.

Models based on pure linguistic descriptions, while infinitely flexible, often fail to be logically consistent. Mathematical models lose flexibility, but gain a consistent structure and general solution techniques. AAA models, specified in a computer language, retain much of the flexibility of pure linguistic models, while having precision and consistency enforced by the language. The resulting models are dynamic and they are "executable" in the sense that the

unfolding behavior of the models can be observed step-by-step. This makes it possible to check the plausibility of the behavior implied by the assumptions of the model...The ability to explore a wide range of phenomena involving learning and adaptation...provides a powerful modeling technique (p. 366).

There are precedents for the use of AAA in the social and behavioral sciences. Axelrod has simulated punishment strategies in his paper on the evolution of norms (Axelrod 1986). He has also used genetic algorithms to develop strategies against a representative field from his famous repeated prisoner's dilemma tournament (Axelrod 1984, 1987). The genetic algorithm developed a strategy roughly equivalent to Tit for Tat, which, incidentally, was the winning strategy in his tournament. Miller (1987) has explored the coevolution of strategies using genetic algorithms as learning rules in the repeated prisoner's dilemma game with both perfect and imperfect information. Genetic algorithms will be one of the three search procedures we employ in this paper.

The use of AAA has important consequences for selection from among equilibria and from among equilibrium concepts. Economic learning literature began with least squares learning (Bray, 1982), a technique that allows mathematical analysis but often demands unrealistic prior knowledge and computational capabilities on the part of the agents. Adaptive search techniques such as simulated annealing, hill climbing, and genetic algorithms offer a more behavioral and evolutionary approach to modelling the learning process (see Holland et al. 1986). Economists have begun to apply AAA to traditional economic problems. Marimon, McGrattan, and Sargent (1989) use classifier systems with genetic algorithms for rule selection in a special class of exchange economies. Arifovic (1989) models learning by economic agents with genetic algorithms in four types of

economic environments. One goal of this line of research is to determine whether AAA will find the equilibria predicted by rational expectations.

Our approach differs from previous AAA models in one important respect: rather than of rely on one search technique, we compare *three* techniques with different known strengths and weaknesses and show that the normative conclusions are strikingly similar for all three. We proceed in this fashion because our dynamic interaction is not a game in the formal sense. Instead, our parties face a sequence of decisions against fixed environments, and our concern is with trajectory of those decisions. Were we to cast the interaction as a game where both parties move simultaneously, we would generically have no equilibria. Since our parties suffer from informational and computational shortcomings, this chaotic possibility may not be manifested in actual election trajectories. Of great interest will be whether we find a pattern of convergence to good regions of the issue space or a trajectory of random positions. In other words, do our parties converge, or do they move aimlessly about the space constantly in search of the "best" position?

Our method for measuring the "goodness" of the democratic outcome is the centrality measure described below. Even though all positions are beatable, the restrictions we place on parties may make some of them more difficult to beat than others; therefore, we record probabilities of victory for different types of adaptive artificial parties (AAP) and different party incentives. In many ways our model is in the spirit of Kramer (1977), who, in the absence of an available model of pure strategy equilibria, sought to explain trajectories of party positions in a sequence of elections, and to define the set to which these trajectories converge (the minmax set).

The Basic Model

For the reasons noted above, we begin with a skeletal model for ease of interpretation. The basic model below does not escape the criticism of unrealistic assumptions, nor does it deal with the many contingencies we hope eventually to incorporate. The voters and parties in this paper have simple preferences and uncomplicated actions. Of course, we anticipate more descriptively accurate AAP models in the future.

There are two parties competing for V votes in an n -dimensional issue space. Each voter is represented by a vector of $2n$ integers, which are the voter's ideal points and strengths on the n issues. We assume that there are k possible positions on each issue $\{0,1,\dots,k-1\}$ and s possible strengths $\{0,1,\dots,s-1\}$. The inclusion of strengths means that the indifference curves will be ellipses, a departure from the standard spatial model. A strength is the amount of importance that a voter attaches to each issue. The utility to a voter from a party's position vector, $y \in \mathbb{R}^n$, is given by the negative of the squared weighted Euclidean distance, where the weight on the i th issue is the strength associated with that issue by the voter. If s_{ji} denotes the j th voter's strength on the i th issue, and x_{ji} is her ideal point, then her utility is given by:

$$u_j(y) = -\sum_i s_{ji} \cdot (x_{ji} - y_i)^2 \quad (1)$$

In this model we assume that both strengths and ideal points are independently and uniformly distributed. Our election results, which will be explained in Section 6, consider 251 voter types, 15 issues, 7 positions per issue, and 3 possible strengths. So on average each voter will have five issues of major importance ($s_{ji}=2$), five of minor importance ($s_{ji}=1$), and five of no importance ($s_{ji}=0$). The assumption that voter ideal points are uniformly distributed does not necessarily imply regularity. A relatively small number of voters are generated in a large space. A spray of points is a more appropriate way to think of the distribution than a continuous uniform distribution. Central limit theorems and the like are not appropriate given the relatively small number of voters and the size of the space. With 15 issues, 7 positions and 3 strengths, there are 21^{15} ($\approx 10^{20}$) possible voter types, so limit theorems do not apply when taking a sample of 251.

Each voter casts a ballot for the party giving her the higher utility. To evaluate the trajectory of democratic outcomes, we need to develop a measure of the goodness of each outcome. Without such a measure we cannot compare our model analytically to any other model, nor can we compare outcomes across elections. We introduce the notion of centrality as follows: first, we compute the sum of the utilities of the individual voters if the winning party were located at the median on all issues; second, we compute the sum of utilities resulting from the winning party in the election; third, we divide the first number by the second.

$$c(y) = [\sum_j u_j(\text{median})]/[\sum_j u_j(y)] \quad (2)$$

It follows that $c(\text{median}) = 1$. This normalization has the following interpretation: the higher the centrality, the closer the winning candidate is to the weighted center of voter preferences, and therefore the better the democratic outcome. The median need not give the highest utility, a crucial caveat which implies that there may exist points with centralities greater than one. Moreover, we attach no normative significance to the median as an outcome. We merely exploit the fact that it will generally be of very low average distance. Ideally we would find the point of minimal average distance and use its utility as the numerator, but the costs in computer time outweigh any advantages. Regardless of the numerator, we have a measure of aggregate utility, or the average weighted distance to a voter. See Appendix 1 for more about centrality.

Our model is a dynamic model of elections in which the incumbent party is fixed and the challenger party attempts to find a position in the issue space which defeats the incumbent. In the first election the incumbent party (arbitrarily chosen) sits at its ideal point. Thereafter, the incumbent sits at the point where it won its last election. The challenger party attempts to defeat the incumbent by choosing a candidate to represent it. How a party chooses a candidate is a crucial component of our model, for unlike most spatial models, we assume that candidates do not have any information about voters' preferences other than vote totals. This implies that our parties will not have explicit knowledge of the mean or median position of voters on an issue. Our parties, though, will have some information. The challenger party, during a finite campaign, tests position vectors on the voters and receives feedback in the form of vote totals. Voters will be assumed to have perfect information about candidate positions, so these tests are like

opinion polls about candidate popularity. While standard spatial models attempt to show where and why parties locate where they do, our model answers the following questions: How quickly (if at all) do boundedly rational parties converge towards a central location? and how do different motivations and search techniques on the part of parties alter this convergence?

Choices and Echoes

Parties will be represented by randomly generated ideal points from the same distribution as the voters. Party issue strengths will equal one on all issues. We consider two types of parties: *ambitious parties* and *ideological parties*. Ambitious parties care only about winning elections, and their party ideal points serve only as starting points for the initial campaign. Ideological parties also want to win the elections, but they want to win with a position that is close to their party ideal point. Formally, if $v(y:x)$ is the number of votes the candidate receives if it takes position y and the incumbent is at x , the objective functions for the two types of parties can be written as:

$$\begin{aligned} \text{Obj}_{\text{am}}(y) &= v(y:x) \\ \text{Obj}_{\text{id}}(y) &= \begin{cases} v(y:x) & v(y,x) \leq V/2 \\ V/2 + n \cdot k^2 - \text{dist}(y, x_{\text{ideal}}) & v(y:x) > V/2 \end{cases} \end{aligned}$$

Recall that V is the total number of voters, n the number of issues, and k the number of positions per issue. This implies that $n \cdot k^2$ exceeds $\text{dist}(y,x)$ for any x and y . Ambitious parties attempt to maximize their vote total in the hopes that a larger margin of victory

makes them more difficult to defeat in subsequent elections. Ideological parties have lexicographic preferences. Their primary goal is to win the election. Once this is accomplished, $v(y:x) > V/2$, they attempt to get as close to their ideal point as possible. This construction allows an ideological party to choose a candidate on a platform that is less preferred by the party than the incumbent's platform. However, this will only happen when the distance between party ideal points is small, an unlikely event.

Both ambitious and ideological AAP will be constrained in how they search the issue space for good positions. There is no such constraint in standard spatial models where ambitious parties, assumed to know the decision functions of each voter, move in a positive direction for a maximal number of gradient vectors of voter utility functions. Since standard spatial models tend to ignore ideological parties, no equivalent "standard" rule exists for them.

The three types of AAP we consider (described below) are restricted in the following three ways: first, the campaigns are of finite length, so parties are limited in the number of polls that they can take. For example, a party may only be able to take forty polls before the election. Second, our parties are limited by the number of issues they can change during any position adaptation; and third, our parties are constrained in the degree of position change on an issue during each position adaptation. The latter two constraints will become clearer after the AAP have been formally defined (See Appendix 2). An example will clarify their effects. If in a five issue, seven position space, the current party position vector is $(6,2,5,4,1)$, then the adapting party will be able to test the vector $(5,2,5,4,2)$ but not the vector $(1,6,1,3,6)$, given their relative distances to the current position.

The positioning constraints imply that our parties will fail to fulfill their goals optimally. On the one hand, ideological challengers will rarely lose, but will be forced to accept positions further from their party ideal points than necessary to win. On the other hand, ambitious challengers' limitations will be manifested in losses. One reason for ambitious party losses (besides well positioned opponents) is the lack of information about the median, a position that, if known, would easily defeat incumbents in the first few elections. Despite this ignorance, ambitious parties who do win move quickly to positions near the median anyway. As the number of elections increases, the median has less informational value because it is less likely to defeat the incumbent (See Section 6.b). The next section defines the three types of AAP and details how each type locates positions.

How Parties Find Platforms

Once we relax the assumptions of perfect information and perfect rationality on the part of parties, we can model our parties in a myriad of ways. There are many ways to be imperfectly rational. We choose parties with three policy location procedures: random adaptive parties (RAP), climbing adaptive parties (CAP), and genetic adaptive parties (GAP). For the most part, we consider these to be crude approximations of actual procedures. But more important, they provide reasonable upper and lower bounds on the ability of parties to locate positions. The procedures themselves are mechanisms for the party to choose the candidate (or candidate positions) it will present to the voters against the incumbent. As noted in the previous section, the procedures contain restrictions on

movement from the previous party position, and therefore will not correspond to perfect candidate mobility. All procedures will be discussed below within the context of ambitious parties. The extension to ideological parties is easy. The formal presentation of the procedures for both types of parties is in Appendix 2.

RAP are the least adaptive of our AAP. RAP randomly generate forty position vectors in a neighborhood of their previous position vector and choose the point (candidate) that receives the most votes against the incumbent. The analogy would be a party who chooses a candidate from among volunteers. The member who fares best against the incumbent carries the mantle of the party into the election. Neither the party nor the candidate herself alters positions on issues to improve the vote total. We do not dispute the contention that this underestimates the ability of parties. RAP are intended as lower bounds on the ability of parties to position themselves.

In contrast to RAP, CAP and GAP refine their positions to improve vote totals. Challenger CAP begin with their current party position and experiment, slightly changing positions on a few issues. If the new position vector fares better against the incumbent than did the previous one, the candidate switches to the new position. We assume that the candidate gets perfect information about position performance against the incumbent. These position tests will be called hill climbing iterations. The number of hill climbing iterations that a party performs before the election will be called the campaign length. We will present data on campaigns of length forty, and summarize results of increasing the length to sixty. CAP enter the election with their final, and therefore, best-to-date position. CAP represent parties who select a candidate and then adapt the candidate's positions to the electorate's

views by testing alterations in positions with focus groups and speeches. After finitely many refinements, the improved challenger faces the incumbent.

GAP, the third type of parties we consider, employ a genetic algorithm to guide their search. A genetic algorithm begins with a population of m position vectors, in our case twelve. A new population of m vectors is generated using three genetic operators: reproduction, crossover, and mutation. The reproduction operator creates a new population based upon the relative fitness of the vectors in the previous population. We use a tournament selection process in our reproduction operator. Twelve pairs of candidates are randomly selected from the population. Whichever member of each pair has the higher fitness (obtains the most votes against the incumbent) is represented in the next population. For example, suppose that candidate 1 obtains the most votes against the incumbent, candidate 2 the second most, candidate 3 the third most, and so on with candidate 12 faring worst. If the first three pairs generated are: $\{1,5\}$, $\{6,3\}$ and $\{8,3\}$, then the next population will have two copies of candidate 3 and one copy of candidate 1. After reproduction, the crossover operator is applied. Crossover begins by randomly pairing each candidate with another. Exactly half of the time these pairs will be allowed to trade positions on a few issues. For example, suppose candidate 1 and candidate 2 are paired and are allowed to trade. For each issue, with probability 0.2, they will trade positions, i.e. candidate 1 will assume candidate 2's position and candidate 2 will assume candidate 1's position. The final genetic operator, mutation, introduces a small probability (0.07) of a random change on each issue for a candidate who modifies her position using crossover.

Each application of the reproduction, crossover, and mutation operators is known as

a generation. Both crossover and mutation involve candidate position alterations, therefore each generation of the genetic algorithm will be considered as two units of campaign length. After twenty generations of the genetic algorithm, a campaign of length forty, the best-to-date position vector will be chosen by the party. Note that this position may be different than the best in the last population. Refer to Appendix 2 and Goldberg (1989) for a complete description of a genetic algorithm.

GAP represent parties whose members learn from each other and teach one another which issue positions help garner votes against the incumbent. A successful party member is one whose position is well received with respect to the incumbent's position. This process corresponds to a party evolving a candidate during the interim period; survival of the fittest as it were. The notion of evolving is used very loosely here.

RAP, CAP, and GAP should be viewed as types of boundedly rational parties. The similarities and differences in outcomes resulting from their use will be covered in the next section. In the most general of terms, our findings suggest that, despite the absence of equilibria, and the lack of perfect information and computational abilities necessary to find optimal positions, two party democratic elections lead to normatively appealing outcomes. Moreover, after restricting their information and abilities, parties converge as spatial models predicted all along.

Results

Our research supports three primary conclusions. First, democratic elections lead to good outcomes, where goodness is measured by centrality. Second, ambitious parties reach

higher centrality than ideological parties; echoes may be better than choices given the assumptions of our model. And third, the type of AAP does not effect the broad characteristics of outcomes. This section proceeds in three parts. We begin with a caveat on our parameter choices and a recapitulation of the model and conclude with analyses of ambitious and ideological party results.

Parameters and a Summary

The robustness of computer simulation results often hinges upon sensitivity to parameter values. A result unique to a small subset of parameter values may be more anomaly than result. The spatial election results we present below appear invariant to reasonable parameter alterations. An increase in the number of voter types to 1000 or positions to 25 does not fundamentally shift the results. In particular, the number of issue strengths appears to have little effect, provided the number of strengths is small relative to the number of voter types. Large increases in the number of positions or issues requires altering the search algorithms proportionately. For example, an increase in the number of positions per issue from 7 to 56 would necessitate increasing the size of each hill climbing iteration from only one position to up to 8 positions. The parameter values that we use:

Voter types (V)	251
Number of issues (n)	15
Positions per issue (k)	7
Strengths (s_{ji})	3

fall safely within the ranges for which we observed no significant changes in the conclusions.

The party ideal points are randomly generated from this voter landscape with the lone exception that parties do not have different strengths on issues. We arbitrarily designate one of the parties as the fixed incumbent. The challenger party then adapts its position in an attempt to maximize its objectives, which may be ambitious or ideological. The winner in each election then becomes the incumbent, who is fixed at the winning point. We measure the goodness of election outcomes by evaluating the centrality of the winning party's position. Centrality measures have greater significance when viewed with respect to the distribution of all centralities. Figure 0 in Appendix 3 shows approximate distribution and density functions for centrality of position vectors. We compare elections outcomes to the cumulative distribution function (cdf). For example, a winning party with a position having centrality of .55 lies in the upper 17% of the distribution. The numerical results are summarized in Appendix 3. We note the ambitious party elections first.

(place Figure 0 about here)

Ambitious Parties

Tables 1 through 8 can be interpreted as follows: we ran 200 trials of a twelve election sequence for each party and AAP type. The columns denote average centrality of the winning party, probability that the incumbent is defeated, and in the ideological case, the distance to the party ideal point. Three features deserve attention. First, for all types of AAP, centrality increases over time (see Figure 1). By the sixth election, CAP (Table 1) and

GAP (Table 2) have expected centralities above .9 and RAP (Table 3) above .8, which places all three types of parties in the top 0.01% of all positions! Convergence to high centrality and the increase in centrality over time are invariant to the particular type of AAP. This result contrasts with Kramer (1977), whose minmax set is not a dynamic attractor; i.e. for Kramer the trajectory of winning party positions may jump in and out of the minmax set. While we have no way of knowing whether our parties leave the minmax set, on average our parties do not move to positions of lower centrality; the effects on centrality of (possible) movements out of the minmax set are negligible.

(place Figure 1 about here)

Second, CAP and GAP have higher centrality than RAP. Yet when these centralities are viewed with respect to the distribution, the differences are not very great (Figure 0). CAP and GAP, in the first few periods, have a higher probability of winning than RAP, but thereafter these probabilities decrease more quickly. For all three types of parties the probability of winning decreases from almost 1 to below .4 by the twelfth election. Our results suggest that incumbents' advantages may be partly attributable to challengers' lack of information. Typically, spatial models resort to an exogenous incumbency advantage to explain winning incumbents. In our elections, a strong incumbent wins because the opposing party cannot locate a winning position, as shown in Tables 1 through 3.

Third, increasing the length of the campaign from forty to sixty, and hence allowing more time for adaptation, leads to higher centrality for each type of AAP, as might be expected. We add, though, that this increase is very small. Likewise, a longer campaign only marginally effects the other results. The effect on probability of winning is unclear. Here our

results contradict intuition. We would expect the probability of wins to increase with campaign length. But this does not necessarily occur. We offer two explanations. First, margin of victory increases with campaign length; therefore, unseated incumbents, who begin their search from the previous party position, face a tougher opponent. Second, the aforementioned centrality increase resulting from longer campaigns implies candidates are more difficult to beat regardless of their starting point.

In Tables 7 and 8, we briefly consider a second scenario in which both parties' ideal points are the median. We find that after the first election the median would not be a good position against the incumbent, i.e. on average the incumbent defeats the median. Also, the probability of winning decreases for the first few elections, and the centrality of winning positions hovers around 1 in this scenario, and the probability of winning decreases to around .1 for CAP and .05 for GAP. In short, these results suggest that parties who use issue polls to find the median would be defeated by the AAP and be unable to defeat the AAP in subsequent elections.

Ideological Parties

Ideological parties concern themselves with both winning and holding true to their ideals. We expect the centrality of outcomes in elections between ideological parties to be lower than if the parties were solely ambitious. As a consequence, the probability of winning should be much higher in ideological contests. Our results confirm these expectations (see Tables 4, 5, and 6). The probability of winning is higher in the ideological case, while centrality is lower. As was the case for ambitious parties, changing the campaign length from

forty to sixty does not have a significant effect on centrality for the ideological CAP.

Note that centrality increases with time for all three AAP (see Figure 2). There is surprisingly little difference between RAP and GAP. CAP perform marginally better than the others. Because they are less able to fine tune positions towards their ideal points, ideological RAP and GAP obtain higher centrality than ideological CAP. The magnitude of this centrality difference is small; at the end of twelve elections, all three are in the top 3% of all positions. The probability of winning is approximately 1 for both GAP and CAP in the first ten elections. The probability of winning for RAP drops off more quickly because the centrality is higher and RAP cannot fine tune as well.

(place Figure 2 about here)

For ambitious parties the satisfaction of party objectives can be measured by winning percentage. For ideological parties, the distance of winning positions from the party ideal can also be taken into account. For all AAP we observe over time that the distance to party ideal increases by small amounts, while the distance to the median decreases (see Figure 3 for GAP example). We refer to this positioning behavior as the dumbbell waltz. The challenging party dances in the neighborhood of its ideal point until it finds a winning position. A chart of the winning positions would consist of two disjoint neighborhoods, one near each of the party ideal points, and resemble a dumbbell. The ends of the dumbbell slowly converge as the number of elections increases.

(place figure 3 about here)

In conclusion, the main result that democratic elections lead to normatively appealing outcomes appears invariant to both party motivation and method of position search. All six

types of parties yielded similar results. While these results do not prove that democratic elections lead to good outcomes, they do substantiate the intuition that democracy ensures leaders who reflect voter preferences.

Discussion

This paper addresses several important research topics in political science. By using computer simulations to explore the dynamic behavior of boundedly rational political parties, we offer important findings on subjects ranging from spatial voting models, to genetic algorithms, to computers in social science. We see the future of political science reaching out in all three areas.

We began this paper with a discussion of two classes of results in standard spatial models. Convergence results offer an intuitively appealing explanation for the stability of two party systems. Chaotic results raise doubts about the legitimacy of democratic outcomes. Our results support the former; two party democratic elections yield good outcomes with respect to our centrality measure. Moreover, our results suggest that even ideological parties will move towards the center in efforts to win. These conclusions are even more convincing given that they appear invariant to parameter choice within reasonable bounds and type of boundedly rational party.

Our results mimic the results of previous spatial models, but our methods do not. Relaxing assumptions to make spatial models more realistic can create difficult, often impossible, mathematics. Complex mathematical models, of course, are not useless. Instead, we must construct new theoretical tools with which to analyze them. On the one hand, AAP

models allow for more descriptively accurate and robust characterizations of parties and incentives than the traditional spatial models. On the other hand, they retain a logical foundation through the formal computer language; computer simulations place constraints on our model that do not exist for purely verbal treatments of elections.

One motivation of this paper moves considerably beyond the goal of presenting a model of elections. We seek to introduce the role of computers generally, and AAA models, specifically, to the study of parties and voters in electoral models. Our technique is designed to analyze the behavior of a complex adaptive system, a system which exhibits a dynamic aggregate behavior that emerges from the knowledge based behavior of individual agents. We claim that our model can be extended in many directions, incorporating a variety of components known to exist in the real world. We hope to include interest groups, issue polling, term limitation for incumbents, boundedly rational voters, voters who are influenced by party positions, and three or more parties in future AAP models. Further, we can consider the case where voters are lumped into groups, rather than scattered throughout a large space. Finally, we can model elections with coadaptive parties, although doing so often requires problematic assumptions about levels of strategic complexity. These and other possibilities may not only let us revise our judgement about the relevance of spatial voting models to real elections, they may ultimately lead us to important insights about what occurs in those elections.

Appendix 1: The Centrality Measure

The centrality measure defined in the paper is mathematically equivalent to a utilitarian social choice function. We normalize our measure so that it lies in the interval $[0, 1 + \delta]$ where δ is small. The centrality of an outcome is the number by which the average voter utility (squared weighted distance) must be multiplied to get the average voter utility of the median, in other words:

$$\text{centrality}(x) \cdot (\text{ave utility}(x)) = \text{ave utility}(\text{median})$$

Example: Let there be 3 voters, 4 issues, 7 positions, and 3 strengths. Ideal points and strengths are given below:

	ideal points	strengths	utility from median
voter 1	{4,3,5,1}	{2,1,0,1}	$-(2+0+0+4) = -6$
voter 2	{1,5,3,3}	{1,2,2,0}	$-(4+8+0+0) = -12$
voter 3	{3,1,2,5}	{2,0,1,2}	$-(0+0+1+8) = -9$

The median is {3,3,3,3} and total utility from the median to the voters is -27. Similarly the utilities from the position vector {2,4,5,2} to the three voters are -10, -12 and -29 respectively which sums to -51. The centrality measure is then $27/51 = .53$.

Appendix 2: The Policy Location Procedures

The discussion will include both ambitious and ideological parties. A position will be more preferred by an ambitious party if it obtains a higher vote total. For an ideological party, a position vector x will be preferred to x' if either of the following two conditions hold:

- (1) x and x' both win the election and x is closer to the party ideal point
- (2) x' loses the election and x obtains a higher vote total.

Note that for each type of AAP the ideological parties will search closer to the status quo point. This is intended to reflect their incentive not to stray too far from the party ideal point. For ambitious parties the only constraint on positioning should be credibility, which explains both why they search a wider area and why that area is not the entire space.

Random Adaptive Parties (RAP):

Forty position vectors $^0\mathbf{x}, ^1\mathbf{x}, \dots, ^{39}\mathbf{x}$ are created using the rules given below. Whichever position vector gives the highest vote total against the incumbent becomes the party position. If $\mathbf{x}$ is the current party position, then $^j\mathbf{x}$ is created by applying the probabilistic operator T to the vector $\mathbf{x}$ a total of $\lfloor j/3 \rfloor$ times, where $\lfloor a \rfloor$ = largest integer less than or equal to a . For example, $^7\mathbf{x} = T^3(\mathbf{x})$.

T is defined as follows:

- a. Randomly select an issue i .
- b. T then maps x_i into $\{x_i-1, x_i+1\}$ with the following probabilities:

$$T(0) \rightarrow \{-1, 1\} \text{ with prob. } (0, 1)$$

$$T(j) \rightarrow \{j-1, j+1\} \text{ with prob. } (1/2, 1/2) \quad j \in \{2, \dots, k-1\}$$

$$T(k) \rightarrow \{k-1, k+1\} \text{ with prob. } (1, 0)$$

- c. $T(\mathbf{x}) = (x_{-i}, T(x_i))$.

Climbing Adaptive Parties (CAP):

In the first period of the campaign, the current position vector x is compared to $T(x)$, whichever gives the higher vote total becomes the position vector. In the second period of the campaign, the position vector, x , is compared to $T^2(x)$ with the higher vote getter becoming the position vector. In the third period, the comparison is between x and $T^3(x)$. In the fourth period, the comparison is again between x and $T(x)$. More precisely, in period k of the campaign the comparison is between x and $T^{l(k)}(x)$ where $l(k) = (k-1) \bmod(3) + 1^1$, with the vector obtaining the most votes against the incumbent becoming the party position. This climbing algorithm is carried out for m iterations where m is the length of the campaign.

Genetic Adaptive Parties:

An initial population of 12 position vectors, $^0x, ^1x, \dots, ^{11}x$ is randomly created, where jx is generated by applying the operator T j times. The fitness, $f(x_i)$, (the number of votes) is assigned to each position vector. The fitness functions for ambitious and ideological parties are given in the text of the paper. Each generation consists of one application of the three genetic operators: reproduction, crossover, and mutation.

¹ $a \bmod(b)$ or "a modulo b" is simply the remainder that results from dividing a by b, i.e. $7 \bmod(3) = 1$.

Reproduction: Twelve random pairs of positions are chosen from the population of positions. From each pair the position with higher fitness is selected for inclusion in the next generation.

Crossover: The twelve selected position vectors are randomly placed in six sets of pairs for possible crossing (mating). With probability .5 the strings are crossed. If the strings are crossed then with probability .13 they begin trading positions on each issue, and with probability .13 they cease trading.

Mutation: With probability .07 each position may go up or down one position on each issue. On average, each position vector will mutate on less than one issue.

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Tables

Note: These data are included for reviewers. They can easily be summarized and put into better tabular form.

Table 0
Distribution of Centrality

C	F(C)	C	F(C)
33	0.0000	57	0.8919
34	0.0002	58	0.9152
35	0.0007	59	0.9339
36	0.0017	60	0.9489
37	0.0044	61	0.9611
38	0.0097	62	0.9705
39	0.0191	63	0.9778
40	0.0342	64	0.9838
41	0.0564	65	0.9880
42	0.0875	66	0.9912
43	0.1277	67	0.9935
44	0.1771	68	0.9954
45	0.2341	69	0.9967
46	0.2972	70	0.9977
47	0.3655	71	0.9984
48	0.4351	72	0.9989
49	0.5049	73	0.9993
50	0.5719	74	0.9996
51	0.6345	75	0.9998
52	0.6922	76	0.9999
53	0.7499	77	0.9999
54	0.7904	78	1.0000
55	0.8302	79	1.0000
56	0.8640	80	1.0000

Table 1
CAP: Ambitious

40 iterations of hill in search

12 elections 251 voters 15 issues 7 positions 3 strengths 200 trials

(numbers in parentheses are standard deviations).

Elec	P(win)	centrality	vs loser	vs median	winner		dist(ideal)	dist(median)
1	0.99 (0.07)	0.74 (0.05)	149.72 (42.36)	-148.32 (21.95)	25.37 (8.03)	21.11 (5.66)		
2	0.58 (0.49)	0.78 (0.05)	43.66 (31.39)	-134.07 (21.09)	26.25 (8.13)	17.86 (5.05)		
3	0.95 (0.21)	0.87 (0.04)	66.54 (36.83)	-100.35 (22.59)	44.87 (12.57)	9.77 (3.60)		
4	0.76 (0.43)	0.91 (0.04)	42.72 (29.98)	-78.47 (23.02)	46.85 (12.47)	6.83 (2.82)		
5	0.77 (0.42)	0.94 (0.03)	38.81 (25.63)	-57.83 (23.68)	55.43 (13.85)	4.45 (2.41)		
6	0.69 (0.46)	0.96 (0.03)	32.09 (22.12)	-40.02 (24.40)	55.88 (14.38)	2.94 (1.93)		
7	0.65 (0.48)	0.97 (0.02)	25.11 (18.51)	-26.49 (22.16)	58.80 (14.16)	2.13 (1.53)		
8	0.60 (0.49)	0.98 (0.02)	18.67 (15.29)	-18.88 (20.02)	59.65 (13.78)	1.71 (1.34)		
9	0.54 (0.50)	0.99 (0.01)	14.30 (13.93)	-13.79 (16.65)	59.53 (14.26)	1.39 (1.09)		
10	0.47 (0.50)	0.99 (0.01)	11.04 (10.65)	-9.53 (16.19)	59.44 (14.15)	1.21 (1.04)		
11	0.42 (0.50)	0.99 (0.01)	8.90 (9.93)	-6.69 (14.20)	60.48 (14.25)	1.08 (0.99)		
12	0.40 (0.49)	0.99 (0.01)	7.87 (9.53)	-5.63 (12.84)	60.76 (13.43)	1.05 (0.96)		

Table 2
GAP: Ambitious

10 iterations 20 ga pop size 0.07 ga mut rate 0.50 ga modification rate 0.13 ga cross rate
12 elections 251 voters 15 issues 7 position 3 strengths 200 trials

Elec	P(win)	centrality		vs loser	vs median	winner		dist(ideal)	dist(median)
						dist(ideal)	dist(median)		
1	1.00 (0.00)	0.72 (0.05)		148.54 (44.78)	-155.75 (20.05)	41.75 (19.29)	24.21 (5.97)		
2	0.61 (0.49)	0.75 (0.05)		40.97 (30.38)	-143.79 (20.21)	40.10 (16.55)	20.52 (5.06)		
3	0.87 (0.34)	0.82 (0.04)		47.72 (29.57)	-121.44 (19.46)	57.15 (20.95)	14.21 (3.94)		
4	0.74 (0.44)	0.85 (0.04)		34.27 (26.09)	-105.19 (20.94)	56.19 (18.39)	11.23 (3.49)		
5	0.77 (0.43)	0.88 (0.03)		27.87 (20.74)	-93.78 (19.34)	61.67 (18.88)	9.01 (2.68)		
6	0.67 (0.47)	0.90 (0.03)		25.00 (18.04)	-81.98 (18.89)	60.69 (17.77)	7.51 (2.26)		
7	0.60 (0.49)	0.92 (0.03)		19.54 (15.51)	-74.97 (18.56)	61.68 (17.01)	6.51 (2.14)		
8	0.59 (0.49)	0.93 (0.02)		18.93 (14.25)	-67.29 (16.89)	61.60 (16.83)	5.63 (1.77)		
9	0.56 (0.50)	0.93 (0.02)		16.29 (12.98)	-62.24 (18.50)	62.92 (15.98)	5.14 (1.72)		
10	0.53 (0.50)	0.94 (0.02)		13.68 (11.95)	-58.51 (18.26)	62.02 (16.06)	4.76 (1.63)		
11	0.48 (0.50)	0.95 (0.02)		13.09 (11.69)	-55.62 (18.50)	62.73 (18.36)	4.45 (1.58)		
12	0.40 (0.49)	0.95 (0.02)		13.11 (12.26)	-51.13 (19.12)	62.69 (17.58)	4.08 (1.51)		

Table 3
RAP: Ambitious

40 iterations of random in search

12 elections 251 voters 15 issues 7 positions 3 strengths 200 trials

Elec	P(win)	winner						
		centrality	vs loser	vs median	dist(ideal)	dist(median)		
1	0.94 (0.25)	0.64 (0.06)	95.58 (51.05)	-182.05 (17.78)	12.93 (5.26)	34.75 (7.81)		
2	0.50 (0.50)	0.66 (0.05)	40.48 (29.90)	-173.35 (15.30)	14.71 (6.33)	31.08 (6.72)		
3	0.81 (0.40)	0.71 (0.04)	37.91 (24.20)	-159.62 (16.03)	26.34 (10.03)	24.64 (4.97)		
4	0.77 (0.42)	0.75 (0.04)	29.04 (18.15)	-147.52 (16.46)	32.56 (11.98)	21.00 (4.57)		
5	0.77 (0.43)	0.78 (0.04)	26.07 (19.22)	-136.22 (16.95)	39.04 (13.07)	17.96 (4.15)		
6	0.71 (0.45)	0.81 (0.04)	26.14 (21.10)	-125.41 (18.92)	45.33 (14.00)	14.98 (3.83)		
7	0.58 (0.49)	0.83 (0.04)	21.61 (19.45)	-116.52 (17.78)	48.87 (15.01)	13.37 (3.62)		
8	0.59 (0.49)	0.84 (0.03)	20.30 (16.26)	-110.50 (18.08)	53.46 (15.77)	11.97 (3.07)		
9	0.46 (0.50)	0.85 (0.03)	16.82 (13.43)	-107.28 (17.22)	54.45 (16.45)	10.95 (2.64)		
10	0.45 (0.50)	0.86 (0.03)	15.07 (13.30)	-102.26 (16.96)	56.27 (15.86)	10.40 (2.74)		
11	0.44 (0.50)	0.87 (0.03)	15.88 (14.50)	-98.83 (18.19)	60.33 (16.83)	9.69 (2.39)		
12	0.40 (0.49)	0.88 (0.03)	13.71 (12.96)	-96.14 (17.01)	62.15 (17.95)	9.30 (2.34)		

Table 4
CAP: Ideological

40 iterations of hill in search

12 elections 251 voters 15 issues 7 positions 3 strengths 200 trials

Elec	P(win)		centrality		vs loser		vs median		winner	
									dist(ideal)	dist(median)
1	0.98	(0.12)	0.54	(0.06)	28.59	(34.25)	-204.21	(15.56)	2.53	(4.08) 52.42 (11.35)
2	0.98	(0.14)	0.55	(0.06)	8.05	(6.19)	-202.51	(14.85)	3.59	(4.82) 49.94 (11.21)
3	0.99	(0.07)	0.56	(0.06)	8.03	(6.27)	-199.86	(16.37)	3.86	(4.23) 48.62 (10.95)
4	0.99	(0.07)	0.57	(0.06)	7.88	(6.06)	-198.09	(16.05)	4.68	(4.84) 46.54 (10.85)
5	0.99	(0.10)	0.58	(0.06)	6.60	(5.68)	-196.06	(17.51)	5.30	(4.73) 45.68 (10.70)
6	0.98	(0.12)	0.58	(0.06)	6.49	(5.20)	-194.54	(17.54)	5.68	(4.98) 44.06 (10.60)
7	0.99	(0.10)	0.59	(0.06)	5.55	(4.50)	-192.81	(17.32)	6.26	(5.04) 43.30 (10.42)
8	1.00	(0.00)	0.60	(0.06)	6.67	(5.08)	-191.68	(17.11)	6.59	(5.09) 41.84 (10.26)
9	0.99	(0.07)	0.60	(0.06)	5.97	(4.62)	-189.62	(18.22)	6.81	(4.76) 41.26 (10.15)
10	0.99	(0.10)	0.61	(0.06)	6.11	(5.06)	-188.07	(17.71)	7.72	(5.40) 39.61 (10.06)
11	0.99	(0.10)	0.61	(0.06)	5.39	(4.48)	-186.32	(18.61)	7.78	(4.73) 39.12 (10.01)
12	0.99	(0.10)	0.62	(0.06)	5.93	(4.52)	-185.38	(18.13)	8.25	(5.16) 37.93 (9.71)

Table 5
GAP: Ideological

10 iterations 20 ga pop size 0.07 ga mut rate 0.50 ga modification rate 0.13 ga cross rate
12 elections 251 voters 15 issues 7 positions 3 strengths 200 trials

Elec	P(win)	centrality	vs loser	vs median	winner	
					dist(ideal)	dist(median)
1	0.99 (0.07)	0.55 (0.06)	34.77 (35.10)	-203.53 (15.67)	4.32 (7.50)	51.17 (11.32)
2	0.99 (0.07)	0.57 (0.06)	17.02 (13.70)	-198.12 (16.54)	7.23 (8.15)	46.95 (11.07)
3	1.00 (0.00)	0.59 (0.07)	19.16 (16.25)	-191.49 (19.85)	8.38 (7.44)	42.55 (11.48)
4	0.99 (0.07)	0.62 (0.07)	18.43 (14.16)	-186.75 (21.30)	11.35 (7.81)	38.81 (11.06)
5	0.99 (0.10)	0.64 (0.07)	16.66 (13.93)	-179.88 (22.74)	13.29 (9.15)	35.27 (10.83)
6	0.98 (0.14)	0.66 (0.07)	15.85 (14.57)	-172.66 (23.91)	15.27 (8.68)	32.30 (10.27)
7	0.99 (0.07)	0.68 (0.08)	15.61 (13.38)	-165.92 (25.35)	17.66 (10.31)	29.41 (10.01)
8	0.98 (0.14)	0.70 (0.08)	15.04 (12.94)	-159.96 (25.37)	19.95 (9.57)	27.11 (9.54)
9	1.00 (0.00)	0.72 (0.08)	14.31 (13.74)	-153.55 (26.55)	22.66 (11.89)	24.66 (9.09)
10	0.99 (0.07)	0.74 (0.08)	14.45 (11.60)	-147.47 (27.26)	25.26 (12.34)	22.43 (8.67)
11	0.97 (0.16)	0.76 (0.08)	14.21 (13.09)	-141.71 (29.50)	26.72 (12.72)	20.53 (8.38)
12	0.92 (0.27)	0.77 (0.08)	12.31 (11.48)	-135.19 (29.60)	29.84 (12.28)	18.96 (7.78)

Table 6
RAP: Ideological

40 iterations of random in search

12 elections 251 voters 15 issues 7 positions 3 strengths 200 trials

Elec	P(win)	winner						
		centrality	vs loser	vs median	dist(ideal)	dist(median)		
1	0.95 (0.21)	0.55 (0.06)	37.69 (32.64)	-202.81 (16.17)	3.50 (4.81)	51.43 (10.96)		
2	0.96 (0.20)	0.57 (0.06)	20.50 (16.54)	-197.68 (18.79)	7.69 (5.27)	46.73 (10.38)		
3	0.96 (0.18)	0.59 (0.06)	20.52 (17.83)	-191.16 (18.48)	9.60 (6.36)	42.39 (10.45)		
4	0.97 (0.17)	0.62 (0.06)	18.47 (15.38)	-185.66 (20.88)	12.46 (5.70)	38.18 (9.60)		
5	0.97 (0.17)	0.64 (0.06)	17.32 (14.47)	-178.85 (21.45)	14.49 (7.82)	34.96 (9.28)		
6	0.97 (0.16)	0.67 (0.07)	18.98 (15.88)	-172.28 (23.75)	18.45 (8.30)	31.23 (8.96)		
7	0.91 (0.29)	0.69 (0.07)	16.80 (13.80)	-164.84 (24.39)	19.97 (8.34)	28.79 (8.93)		
8	0.92 (0.28)	0.71 (0.07)	15.71 (13.84)	-159.85 (25.61)	24.48 (10.62)	26.05 (8.33)		
9	0.89 (0.31)	0.73 (0.07)	17.12 (13.06)	-151.54 (26.61)	27.98 (12.52)	23.82 (8.40)		
10	0.84 (0.36)	0.74 (0.07)	14.61 (14.50)	-146.51 (27.05)	31.45 (12.05)	21.95 (7.73)		
11	0.83 (0.38)	0.76 (0.07)	15.66 (15.35)	-139.57 (26.32)	33.79 (12.94)	20.00 (7.33)		
12	0.77 (0.42)	0.78 (0.07)	14.23 (12.77)	-134.56 (25.81)	38.31 (14.35)	18.25 (6.77)		

Table 7
CAP: Middle

40 iterations of hill in search

12 elections 251 voters 15 issues 7 positions 3 strengths 200 trials

Elec	P(win)		centrality		vs loser		vs median		winner	
									dist(ideal)	dist(median)
1	0.41	(0.49)	1.00	(0.00)	2.57	(4.33)	2.57	(4.33)	0.47 (0.61)	0.47 (0.61)
2	0.30	(0.46)	1.00	(0.01)	2.53	(4.50)	1.38	(7.13)	0.71 (0.78)	0.71 (0.78)
3	0.26	(0.44)	0.99	(0.01)	2.85	(5.13)	0.22	(7.78)	0.79 (0.81)	0.79 (0.81)
4	0.20	(0.40)	0.99	(0.01)	2.73	(6.08)	0.02	(8.12)	0.81 (0.85)	0.81 (0.85)
5	0.17	(0.38)	1.00	(0.01)	2.72	(5.17)	0.14	(8.08)	0.74 (0.80)	0.74 (0.80)
6	0.15	(0.36)	1.00	(0.01)	2.56	(4.73)	0.39	(8.14)	0.76 (0.89)	0.76 (0.89)
7	0.14	(0.35)	1.00	(0.01)	2.35	(5.25)	0.40	(7.77)	0.74 (0.84)	0.74 (0.84)
8	0.12	(0.32)	0.99	(0.01)	2.06	(4.95)	0.30	(8.37)	0.78 (0.86)	0.78 (0.86)
9	0.15	(0.36)	0.99	(0.01)	2.52	(5.04)	0.44	(8.34)	0.79 (0.86)	0.79 (0.86)
10	0.11	(0.31)	1.00	(0.01)	2.29	(5.12)	0.53	(7.61)	0.76 (0.81)	0.76 (0.81)
11	0.11	(0.31)	1.00	(0.01)	2.12	(4.45)	0.46	(7.59)	0.76 (0.82)	0.76 (0.82)
12	0.12	(0.33)	1.00	(0.01)	2.20	(4.56)	0.28	(8.43)	0.78 (0.86)	0.78 (0.86)

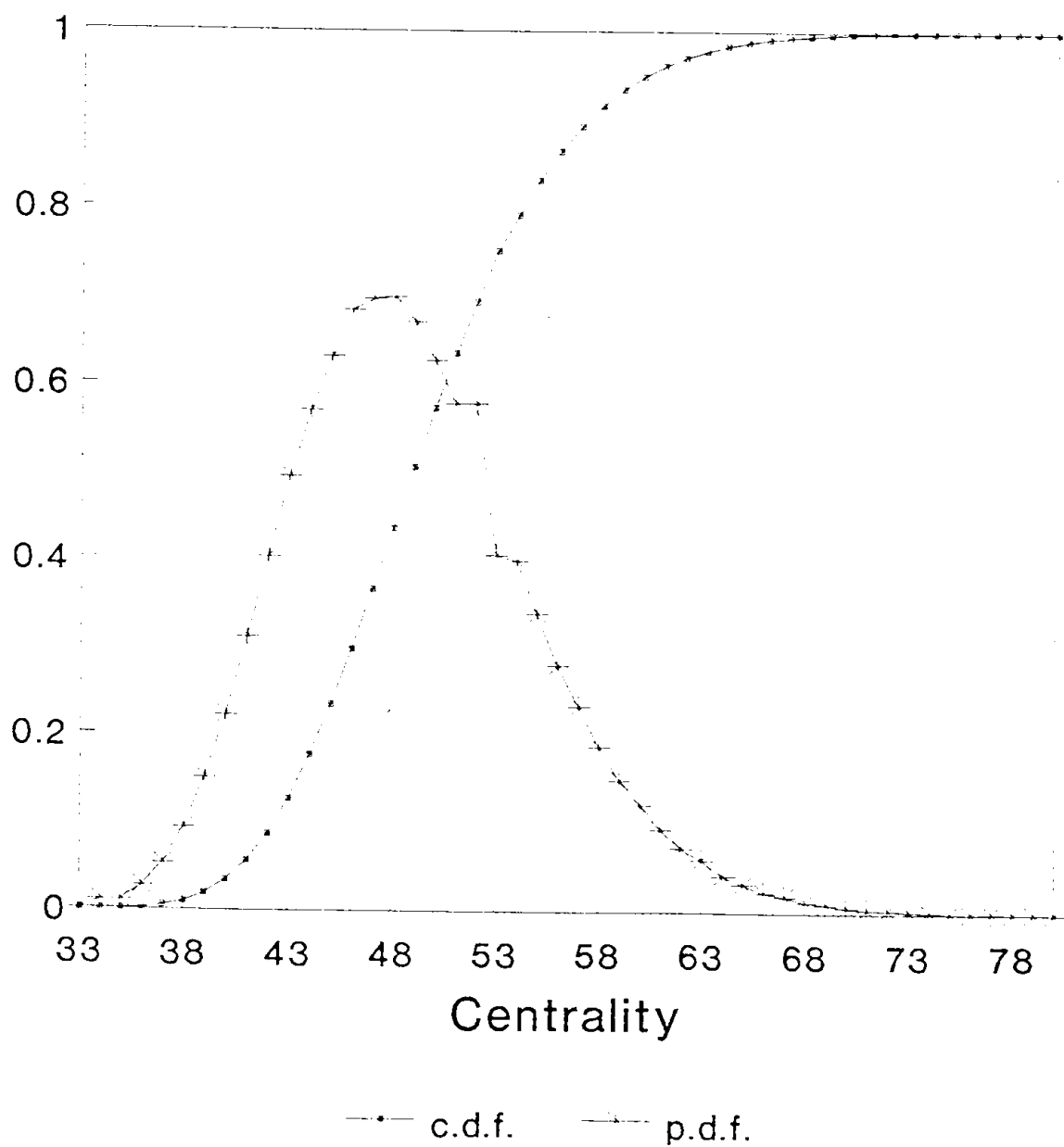
Table 8
GAP: Middle

10 iterations 20 ga pop size 0.07 ga mut rate 0.50 ga modification rate 0.13 ga cross rate
12 elections 251 voters 15 issues 7 positions 3 strengths 200 trials

Elec	P(win)	centrality		vs loser		vs median		winner	
								dist(ideal)	dist(median)
1	0.04 (0.20)	1.00 (0.00)		0.14 (0.89)		0.14 (0.89)		0.04 (0.23)	0.04 (0.23)
2	0.06 (0.24)	1.00 (0.00)		0.61 (2.69)		0.57 (2.68)		0.10 (0.32)	0.10 (0.32)
3	0.05 (0.22)	1.00 (0.00)		0.79 (2.89)		0.70 (2.94)		0.14 (0.41)	0.14 (0.41)
4	0.06 (0.23)	1.00 (0.00)		0.96 (3.07)		0.83 (3.06)		0.20 (0.49)	0.20 (0.49)
5	0.04 (0.21)	1.00 (0.00)		1.10 (2.91)		1.06 (3.32)		0.23 (0.51)	0.23 (0.51)
6	0.03 (0.17)	1.00 (0.00)		1.08 (2.87)		1.03 (3.47)		0.26 (0.53)	0.26 (0.53)
7	0.03 (0.17)	1.00 (0.00)		1.22 (2.98)		1.11 (3.77)		0.28 (0.56)	0.28 (0.56)
8	0.04 (0.20)	1.00 (0.00)		1.40 (3.27)		1.00 (4.14)		0.30 (0.56)	0.30 (0.56)
9	0.03 (0.17)	1.00 (0.00)		1.53 (3.36)		1.06 (4.30)		0.32 (0.57)	0.32 (0.57)
10	0.07 (0.25)	1.00 (0.00)		1.85 (3.85)		1.18 (5.11)		0.36 (0.59)	0.36 (0.59)
11	0.03 (0.17)	1.00 (0.00)		1.79 (3.80)		1.11 (5.20)		0.38 (0.59)	0.38 (0.59)
12	0.05 (0.22)	1.00 (0.00)		1.74 (3.42)		1.20 (4.99)		0.40 (0.62)	0.40 (0.62)

Figure 0

Centrality Distribution/Density



10,000 trials

Figure 1

Centrality of Ambitious Parties

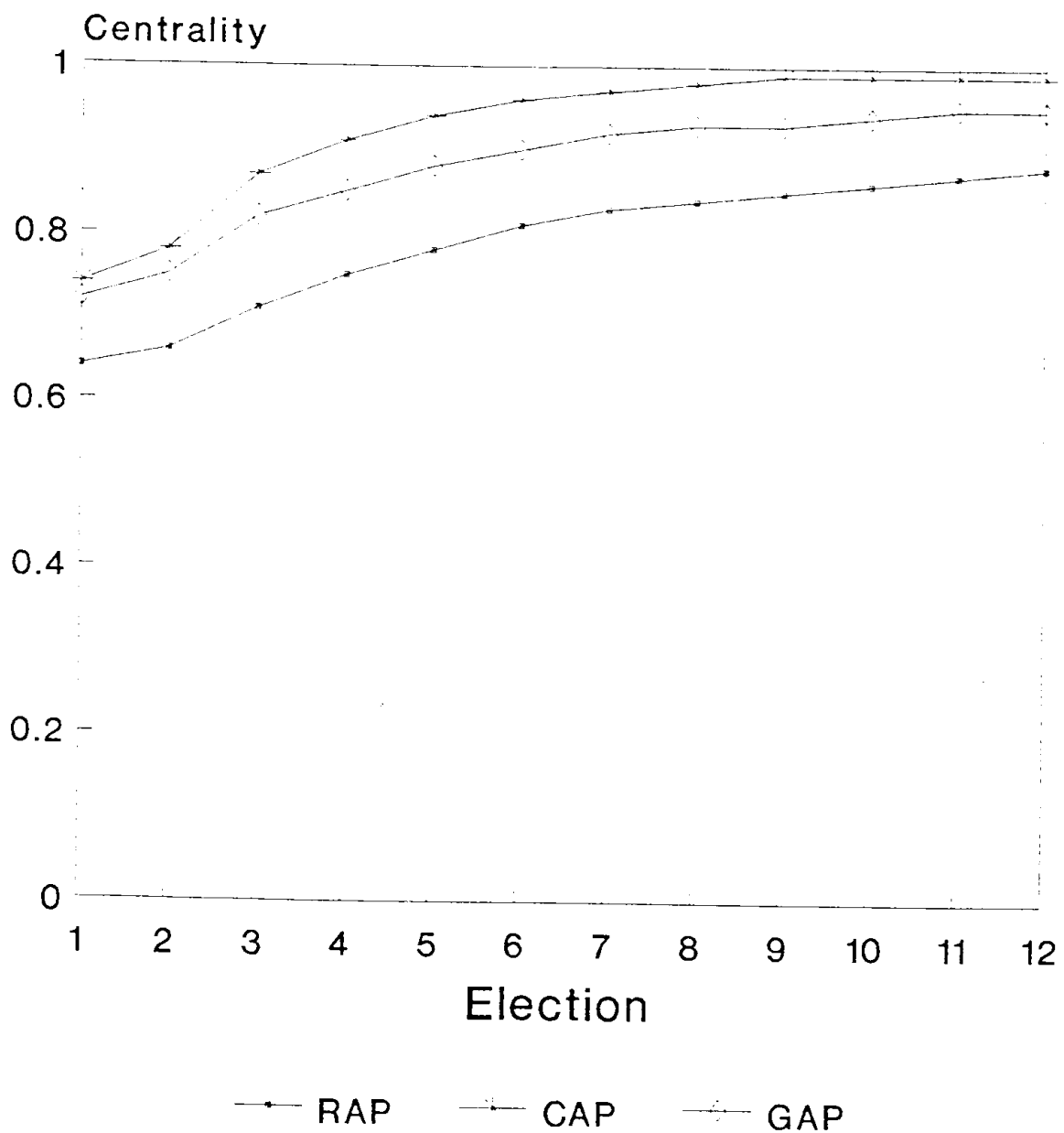


Figure 2

Centrality of Ideological Parties

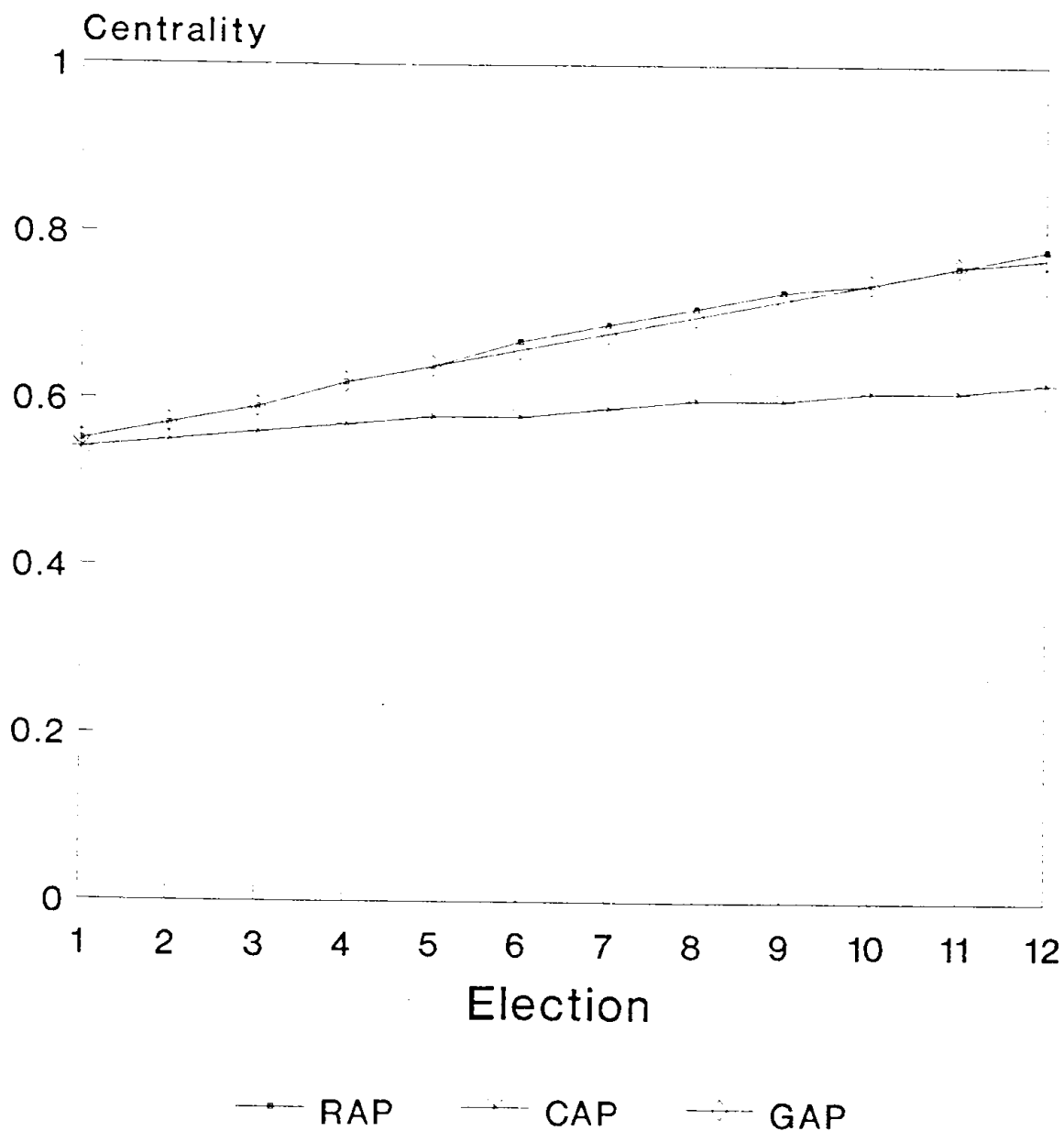


Figure 3

Distance from Ideal and Median for Ideological GAP

