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**Renegotiation of Sales Contracts under Moral Hazard\***

by

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*Abstract:*

Sales contracts emerge when a principal and an agent in a moral hazard environment cannot prevent themselves from renegotiating their contract. The renegotiation occurs after the agent chooses his unobservable effort, but before its consequences are realized. Unlike previous analyses, a contract is a single sharing rule of the classical variety, and the agent leads the renegotiation. A *sales contract* transfers the random return wholly to the agent, thereby relieving the principal of concern about his effort. Equilibria exist in which an initial sales contract is agreed upon, but subsequently renegotiated to the (second-best) efficient contract. All equilibria satisfying a relatively weak refinement criterion are efficient in this sense; renegotiation does not reduce welfare. When the agent can finely control the probabilities of observable signals, the initial contract in every equilibrium satisfying the criterion must be a sales contract. Two applications are briefly considered, managerial compensation and the timing of new firms' security issues.

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## 1. Introduction

In a principal-agent model of moral hazard, a sales contract is what the principal uses to “sell the firm” to the agent. It gives the principal a certain profit, the sales price, regardless of the agent’s unobservable action. A sales contract generally gives the agent too much risk to be second-best efficient. Yet, the thesis of this paper is that sales contracts still arise naturally as initial contracts to be renegotiated. The precondition is that renegotiation cannot be prevented from occurring between a date at which the agent’s effort is known to be fixed, and a later date at which its consequence is observed.

Sales contracts arise because they place the principal in a strong bargaining position in the renegotiation stage. Regardless of her beliefs about the agent’s effort, the principal knows that her profit will equal the sale price if she refuses to renegotiate. This limits the principal’s vulnerability to the agent’s “opportunism.” In the model of this paper, (second-best) efficient outcomes result from the renegotiation of sales contracts.<sup>1</sup>

Other conclusions are reached elsewhere. In particular, renegotiation decreases efficiency in Fudenberg and Tirole (1990). In that paper’s game, the principal makes a take-it-or-leave-it renegotiation offer. The agent’s equilibrium effort strategy is mixed, and efficiency is less than its second-best level.<sup>2</sup> The reason the agent cannot use a pure strategy and so choose, for example, the efficient effort is the following. If he were to choose his effort deterministically, then in equilibrium his effort would be known to the principal in the renegotiation stage, and she would offer the agent an optimal risk-sharing (first-best) contract for that level of effort. Realizing at the start that the final contract will have such low-powered incentives, the agent would shirk.

However, because information is asymmetric at the renegotiation stage (the principal does not see the agent’s effort), bargaining then does not have to result in

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<sup>1</sup> Henceforth, the adjective *efficient*, when used without a modifier, is meant in the second-best sense of, say, Holmström (1979) or Grossman and Hart (1983).

<sup>2</sup> Ma (1991) obtains similar results in a multiperiod model.

first-best outcomes when the agent's effort is chosen deterministically. It does in Fudenberg and Tirole (1990) only because their agent can make only an accept/reject decision, and so cannot influence the principal's beliefs.

On the other hand, if the agent leads the renegotiation, his offer can reveal information about his effort. He might not offer a first-best contract because doing so can signal that his effort is so low that the offer is rejected. This intuition is verified by Ma (1990), who modifies the game of Fudenberg and Tirole (1990) by having the agent instead of the principal make the renegotiation offer. Ma shows that then efficient equilibria exist, and that only they satisfy a weak refinement criterion.

In both Ma (1990, 1991) and Fudenberg and Tirole (1990), a contract is a menu of rules for sharing the return, one sharing rule for each possible effort. After a contract is fixed, but before the return is realized, the agent's last act is to select a sharing rule from the menu. Appealing to a general revelation principle,<sup>3</sup> attention is restricted to menu contracts which are incentive compatible and renegotiation-proof.

The motivating observation for this paper is that many real-world contracts do not look like menus of sharing rules. They tend instead to resemble simpler, single sharing rules. A sales contract, or a debt contract, or an equity contract, or an insurance contract each specify one sharing rule. Furthermore, theoretical contracts so complicated that they can and, in equilibrium, will be renegotiation-proof are of limited value in explaining observed renegotiations, such as when an executive renegotiates a compensation plan, or a firm changes its financial structure (more on these applications in Section 8). To explain differences between initial and final contracts, a model in which equilibrium renegotiation occurs is essential.

Contracts in this paper are thus single sharing rules. Otherwise, the game is as in Ma (1990). The principal offers the initial contract. If the agent accepts it, he chooses his unobservable effort, and a renegotiation subgame is played in which he makes a new offer for the principal to accept or reject.<sup>4</sup> The first result is that if an

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<sup>3</sup> See Myerson (1982, 1983) and Maskin and Tirole (1990).

<sup>4</sup> The agent's effort is (essentially) fixed and known only to him when he makes his offer in the renegotiation subgame. The subgame is thus an "informed principal"

equilibrium exists and sales contracts are feasible, then an equilibrium in which a sales contract is initially accepted, but later renegotiated, exists and is efficient.

A result similar in spirit is obtained by Hermalin and Katz (1991), who also study the renegotiation of simple contracts in a moral hazard environment. They focus on the case in which the principal observes the agent's effort (but cannot verify it to a third party). Information is symmetric, and renegotiation results in a contract which is first-best given the agent's effort. Because this contract varies with the agent's actual effort, renegotiation does not destroy incentives; an equilibrium exists when the agent leads the renegotiation that is first-best efficient and entails the renegotiation of a sales contract.<sup>5</sup> The first result of this paper is the same, except that the agent's effort is not observable and only second-best efficiency is obtained.

Because the agent's offer signals his effort in this paper, a multitude of equilibria exist. The plausibility of the efficient equilibria, and of the equilibria in which sales contracts are renegotiated, must be considered. Attention is restricted to perfect Bayesian equilibria satisfying a weak refinement criterion, one which requires the principal to believe that the agent has not played a certain kind of dominated strategy.

Three sets of assumptions, or environments, are considered. The role of sales contracts in efficient equilibria is shown most clearly in a General Environment in which contracts are abstract entities. They could, for example, be either single or menus of sharing rules.

A Specific Environment is also defined, employing familiar assumptions about uncertainty, risk aversion, and separable utility. Contracts are single sharing rules.

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game, with the "principal" and "agent" labels reversed, similar to that of Maskin and Tirole (1990). Their results about the subgame cannot be used here, however, most obviously because their contracts are menus ("direct revelation mechanisms"). More subtly, this paper's refinement criterion has no analog in the informed principal game because of the latter's exogenous initial ("status quo") contract.

<sup>5</sup> Demski and Sappington (1991) also show how a sales-like contract can be used to obtain first-best outcomes in a (double) moral hazard model with observable but unverifiable effort. In their model the sales contract is a "buy-out option" the principal can exercise to force the agent to buy the firm if he does not choose the right effort.

The Specific Environment is general in that the revenue probability distribution need not stochastically increase with effort, and the set of possible efforts can range from a finite set to a multidimensional continuum. In this environment, all equilibria satisfying the criterion are efficient. This result depends on sales contracts; the criterion implies that the agent accepts any sales contract with a price which gives the principal less than her optimal expected profit.

A special case of the Specific Environment is the Talented Agent Environment, the key characteristic of which is that the agent can shift the probability distribution of contractible variables small amounts in any direction. This ability, together with the influence he has on the principal's beliefs in equilibria satisfying the criterion, allow the agent to take maximal advantage of the principal. Only if the initial contract is a sales contract, with its guaranteed profit level, is the principal well protected in the renegotiation subgame. In the Talented Agent Environment, the initial agreement in all equilibria satisfying the criterion is shown to be a sales contract.

The remainder of the paper begins with an example in Section 2. The General Environment is considered in Sections 3 and 4, the Specific Environment in Sections 5 and 6, and the Talented Agent Environment in Section 7. Section 8 concludes with a discussions of the game's structure and two possible applications, managerial compensation and the timing of new firms' security issues. Appendix A contains missing proofs of lemmas; Appendix B contains existence results.

## 2. An Example

Many results can be illustrated in a two-by-two example in which the agent's effort determines the probability distribution of an asset's contractible return. The return is either good,  $\pi_g$ , or bad,  $\pi_b < \pi_g$ . The agent's unobservable effort, denoted as  $e$ , directly determines the probability of the good return:

$$\text{Prob}(\pi_g | e) = e.$$

The agent's effort can be low,  $e_\ell$ , or high,  $e_h$ , where  $0 < e_\ell < e_h < 1$ . A *contract* is a sharing rule,  $s = (s_g, s_b)$ , which determines the principal's payment to the agent as a

function of the return. Given an *allocation*  $(s,e)$ , the principal's expected profit is,

$$P(s,e) = e(\pi_g - s_g) + (1-e)(\pi_b - s_b).$$

The principal is risk neutral, and so she acts to maximize  $P(s,e)$ .<sup>6</sup>

The agent has a separable utility function,  $u(y) - c(e)$ , for money and effort, with  $u$  strictly concave and increasing. The agent's disutility of effort is increasing:  $c(e_l) < c(e_h)$ . Given an allocation  $(s,e)$ , the agent's expected utility is,

$$A(s,e) = eu(s_g) + (1-e)u(s_b) - c(e).$$

The agent receives his reservation utility,  $\bar{A}$ , if a contract is not agreed.

The classic principal-agent problem, relevant when the principal can make a non-renegotiable contract offer to the agent, is to maximize the principal's expected profit subject to the agent's individual rationality (participation) and incentive constraints:

maximize  $P(s,e)$  such that  
 $s, e$

$$(IR) \ A(s,e) \geq \bar{A} \text{ and}$$

$$(IC) \ A(s,e) \geq A(s,d) \text{ for } d \neq e.$$

The solution to this problem,  $(s^*, e^*)$  is assumed unique, with  $e^* = e_h$ , and referred to as the (*second-best*) *efficient allocation*. The principal's expected profit from it is  $P^* \equiv P(s^*, e_h)$ .

The principal-agent problem is illustrated in the Edgeworth box of Figure 1. Points in the box are contracts  $(s_g, s_b)$ . The agent's expected utility increases to the northeast, and the principal's expected profit increases to the southwest.

(Figure 1 About Here)

Contracts along the agent's 45° line, the *wage contracts*, perfectly insure the agent. The increasing IC curve indicates the contracts at which the incentive constraint

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<sup>6</sup> Throughout, the principal is referred to as "she", and the agent as "he".

binds.<sup>7</sup> Contracts below the IC curve induce the agent to work hard by rewarding him with a sufficient bonus if the observed return is  $\pi_g$ . The steeper indifference curve gives the agent his reservation utility when he chooses high effort; it is the locus of contracts satisfying,

$$e_h u(s_g) + (1-e_h)u(s_b) = \bar{A} + c(e_h).$$

In the “first-best world,”  $e$  is contractible. In this case the principal would require the agent to choose high effort  $e_h$ . The agent would receive the wage contract labelled  $s^{**}$  and defined by  $s_g^{**} = s_b^{**} = u^{-1}(\bar{A} + c(e_h))$ .

In the “second-best world,” only the return is contractible, but the parties can commit to a contract. In this case the principal must offer the agent a contract below the IC curve if she wants him to choose high effort. In order for the agent to accept it, the contract must be above his high effort reservation indifference curve. The shaded area is thus the set of feasible contracts which induce high effort.

The principal’s isoprofit lines in the shaded area are steep, with slope  $-e_h/(1-e_h)$ , because the agent chooses high effort at these contracts. Contract  $s^*$  maximizes the principal’s expected profit on the shaded set. Her expected profit at  $(s^*, e_h)$  is  $P^*$ , which can be read to the right of the point  $r^*$  at which her isoprofit line through  $s^*$  crosses her 45° line. Her best profit from a contract which induces the agent to choose low effort is  $\hat{P}$ , and it is obtained by offering the wage contract  $\hat{s}$ . Because  $\hat{P} < P^*$ , inducing high effort by offering  $s^*$  is optimal. Thus,  $(s^*, e_h)$ , solves the principal-agent problem. (Figure 1 shows that IR and IC bind, as is well-known.)

In the “third-best world,” the parties cannot commit to the initial contract. Our focus is on the game in which the principal offers the initial contract on a take-it-or-leave-it basis, but then later, after he has chosen his effort, the agent offers to renegotiate to another contract. The principal then decides whether to accept or to reject the agent’s offer in lieu of the initial contract.

The principal now cannot achieve an efficient allocation by offering  $s^*$ . In any

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<sup>7</sup> E.g., if  $u(y) = \ln(y)$ , the IC curve is a line,  $\ln(s_g/s_b) = (c(e_h) - c(e_\ell))/(e_h - e_\ell)$ .



perfect Bayesian equilibrium, the agent accepts  $s^*$ , chooses low effort, and proposes a wage contract which the principal accepts. Figure 2 demonstrates.

(Figure 2 About Here)

Note first that if  $s^*$  is not renegotiated, then the agent's expected utility is  $\bar{A}$ , regardless of whether he chooses high or low effort. But if he chooses  $e_\ell$  and they renegotiate to a contract above his low-effort indifference curve through  $s^*$ , such as any contract in the shaded region, his expected utility will be greater than  $\bar{A}$ .

Renegotiating to a contract in the shaded region also benefits the principal, *regardless of her beliefs about the agent's effort*.<sup>8</sup> If she believes the agent has chosen high effort, she is better off in the shaded region because it is below the isoprofit line through  $s^*$  and  $r^*$ . On the other hand, if she believes the agent has chosen low effort, she is better off in the shaded region because it is below the isoprofit line through  $s^*$  and  $r$ .

**REMARK 1:** It seems paradoxical that both parties can be made better off by switching from the supposedly efficient contract  $s^*$  to, say, some  $s$  in the shaded region of Figure 2. However, it is not  $s^*$ , but rather the entire allocation  $(s^*, e_h)$  which is efficient. Renegotiating  $s^*$  to  $s$  entails switching from  $(s^*, e_h)$  to  $(s, e_h)$ , or from  $(s^*, e_\ell)$  to  $(s, e_\ell)$ , depending on the agent's effort. The efficiency of  $(s^*, e_h)$  implies nothing about the switch from  $(s^*, e_\ell)$  to  $(s, e_\ell)$ , and indeed, this switch is Pareto-improving. The fact that switching from  $(s^*, e_h)$  to  $(s, e_h)$  benefits the principal does not violate the fact that  $(s^*, e_h)$  solves the principal-agent problem, since  $(s, e_h)$  is not feasible for it:  $s$  is above the IC curve, and so  $(s, e_h)$  violates IC.

We have shown that if  $s^*$  is the initial contract, the principal accepts any renegotiation offer in the shaded region of Figure 2. The agent obtains utility greater than  $\bar{A}$  by offering such a contract and choosing low effort. This proves that his equilibrium utility exceeds  $\bar{A}$ , and so  $s^*$  is renegotiated.

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<sup>8</sup> I thank Albert Ma for bringing this property of the two-effort model to my attention.

Renegotiation of  $s^*$  cannot result in a contract below the IC curve, because then either he or the principal would be worse off than at  $s^*$ . Thus, the renegotiated contract must be above the IC curve, and the agent's effort must be low. Because the principal agrees to the renegotiation, the renegotiated contract must lie in the shaded region. It actually must be the agent's best contract in that region, the wage contract  $t$ , since he has the bargaining power. The principal's resulting expected profit is  $P$ , which is lower than the  $\hat{P}$  she would have obtained by offering the wage contract  $\hat{s}$  in the first place.

We conclude that in this example, if the principal offers  $s^*$ , it will be renegotiated to a wage contract and the agent will shirk.

REMARK 2: What if the principal makes the renegotiation offer? Then the game is as in Fudenberg and Tirole (1990), except that contracts are single sharing rules, rather than menus of sharing rules. This game has an equilibrium which starts with the principal offering and the agent accepting  $s^*$ . Then the agent mixes between  $e_\ell$  and  $e_h$ . The principal's renegotiation offer is the wage contract  $\hat{s}$ . The agent accepts (rejects)  $\hat{s}$  if he chose  $e_\ell$  ( $e_h$ ). The principal's expected profit is a convex combination of  $\hat{P}$  and  $P^*$ , with weights equal to the probabilities the agent puts on  $e_\ell$  and  $e_h$ , respectively. Many such equilibria exist, varying only in these probabilities. None of them are efficient because the probability put on  $e_h$  is bounded below one. For, if  $e_h$  is too likely, the principal would offer the first-best wage contract ( $s^{**}$  in Figure 1) instead of  $\hat{s}$ , in which case the agent would never choose  $e_h$ . The principal's payoff in the equilibrium with the highest possible probability put on  $e_h$  is equal to her equilibrium payoff in the Fudenberg and Tirole menu-contract game. It is the maximum payoff she can obtain, given that she leads the renegotiation.<sup>9</sup>

Returning to the game in which the agent leads the renegotiation, in this example

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<sup>9</sup> The principal's equilibrium payoff in the game of Fudenberg and Tirole (1990) is greater than in the game here if the number of possible efforts is greater than two. Then, the agent's choice between two sharing rules cannot substitute for the choice he has, in their game, from a menu of sharing rules indexed by the set of all efforts.

it does have efficient equilibria. In particular, efficient equilibria exist in which the principal “sells the firm” to the agent with the initial contract. This sales contract is then renegotiated to one in which the agent is better insured.

In this example, a *sales contract with price  $p$* , denoted  $r(p)$ , is a contract which transfers the return to the agent for price  $p$ :

$$r_i(p) = \pi_i - p \quad \text{for } i = g, b.$$

The contracts on the principal’s 45° line are the sales contracts. The key feature of a sales contract is that it leaves the principal indifferent as to the agent’s effort. Her profit from contract  $r(p)$  is price  $p$  for any  $e$ :

$$\begin{aligned} P(r(p), e) &= e(\pi_g - (\pi_g - p)) + (1-e)(\pi_b - (\pi_b - p)) \\ &= p. \end{aligned}$$

Of most interest is the *optimal sales contract*,  $r^* \equiv r(P^*)$ , in which the price is equal to the principal’s expected profit in the efficient allocation.

Suppose the optimal sales contract  $r^*$  has been agreed. The contracts to which it can, in equilibrium, possibly be renegotiated are depicted in Figure 3.

(Figure 3 About Here)

Each contract in the shaded region is preferred by the principal to  $r^*$ , given that the effort is the agent’s optimal choice for that contract. Contracts above the IC curve induce low effort, and thus must be below the flatter isoprofit line through  $r^*$  for the principal to prefer them to  $r^*$ . Contracts below the IC curve induce high effort and therefore need be below only the steeper isoprofit line through  $r^*$ . If the contract is renegotiated to  $s$ , and the agent takes low effort, his expected utility is less than  $\bar{A}$ .<sup>10</sup> But if the contract is renegotiated to  $s^*$ , the agent’s expected utility is  $\bar{A}$ . Contract  $s^*$  is thus the agent’s best contract among those which the principal could be persuaded to accept over  $r^*$ .

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<sup>10</sup> To see this, note that the agent would receive  $\bar{A}$  from  $(\hat{s}, e_l)$ . Since  $(s^*, e_h)$  solves the principal-agent problem,  $\hat{s}$  is above  $r^*$ , and thus  $\hat{s}$  is above  $s$ .

This shows that any equilibrium in which the agent accepts  $r^*$  is efficient. For, since the agent has the option of signing no contract, the agent's equilibrium utility can be no less than his reservation utility  $\bar{A}$ . If  $r^*$  is the initial contract, it can only be renegotiated to contracts in the shaded region of Figure 3. The only such contract which gives the agent no less than  $\bar{A}$  is  $s^*$ . For the principal to agree, in equilibrium, to renegotiate  $r^*$  to  $s^*$ , the agent must surely choose  $e_h$ .

It should now be clear, although it has not been rigorously proved, that the following is an equilibrium outcome. First, the principal offers the optimal sales contract  $r^*$ , which the agent accepts. Then the agent chooses effort  $e_h$  and offers  $s^*$ , which the principal accepts.

### 3. The General Environment and Game

Described in this section is the General Environment, as well as the precise game and equilibrium concept to be employed.

#### *The General Environment*

The General Environment is defined in terms of a set  $S$  of feasible contracts, and a set  $E$  of feasible efforts. The exact nature of a contract and an effort is not specified. An *allocation* is a pair  $(s, e)$ , and it gives the principal and agent utilities  $P(s, e)$  and  $A(s, e)$ , respectively. If agreement is not reached, they receive their reservation utilities,  $\bar{P}$  and  $\bar{A}$ .

The agent cannot be prevented from choosing his optimal effort given the contract in force. Hence,  $(s, e)$  is feasible only if  $e \in E(s)$ , where  $E(\cdot)$  is the agent's optimal effort correspondence:

$$E(s) \equiv \operatorname{argmax}_{e \in E} A(s, e). \quad (3.1)$$

Each  $E(s)$  is assumed nonempty. The maximum utility the agent can obtain from contract  $s$  is denoted  $A^*(s)$ , so that  $A^*(s) = A(s, e)$  for  $e \in E(s)$ .

The *principal-agent problem* consists of maximizing the principal's utility subject to the agent's *individual rationality (participation)* and *incentive constraints*:

(P) maximize  $P(s,e)$  such that  
 $s \in S, e \in E$

(IR)  $A(s,e) \geq \bar{A}$  and

(IC)  $e \in E(s)$ .

A solution of this program may not be Pareto optimal if constraint IR is slack. The Pareto optimal solutions are those which are most preferred by the agent, the *(second-best) efficient allocations*. Efficient allocations are denoted with asterisks,  $(s^*, e^*)$ , and their existence is merely assumed in this environment. The utilities of the principal and agent at efficient allocations are denoted  $P^*$  and  $A^*$ , respectively. Assume  $P^* > \bar{P}$ , so that the principal has an incentive to bargain.

Actually, at this level of abstraction, (P) may not be the Pareto problem it appears. It does not allow randomizations between contracts, which may give the principal greater utility than  $P^*$ . This is impossible under a variety of assumptions. For now, randomized rules are simply assumed to have no value.

ASSUMPTION A0: Suppose effort  $e \in E$  is inducible by a randomization between contracts  $s, r \in S$ , which means that for some  $a \in (0,1)$ ,

$$e \in \operatorname{argmax}_{e' \in E} aA(s,e') + (1-a)A(r,e').$$

Then  $t \in S$  and  $d \in E(t)$  exist such that,

$$A(t,d) \geq aA(s,e) + (1-a)A(r,e), \text{ and}$$

$$P(t,d) \geq aP(s,e) + (1-a)P(r,e).$$

### *The Game*

The renegotiation issue arises when the parties cannot prevent themselves from renegotiating after the agent chooses his effort. So, suppose the principal knows the last date at which the agent can choose his effort, and that there is time for renegotiation to occur after that date and before the realization of the effort's consequence. In this paper the agent makes the renegotiation offer.

The game starts with the principal offering a contract  $r$  for the agent to accept or

reject. If it is accepted, a *renegotiation subgame*,  $\Gamma(r)$ , is played. It starts with the agent choosing an effort  $e$  and proposing a contract  $s$ . Then the principal, unaware of the agent's effort, decides between  $s$  and  $r$ . The extensive form is shown in Figure 4.

(Figure 4 About Here)

Two features of the extensive form are relatively unimportant. The first is who makes the initial offer; the principal is assumed to make it primarily so that results will be comparable to the literature. (But some changes are necessary if the agent makes the initial offer, as is discussed in Section 8.) Second, the ordering of the agent's moves is unimportant. All results hold if the agent makes his decisions (whether to accept  $r$ , and his  $s$  and  $e$  choices) in another order or simultaneously. The normal forms of games which vary only in this way are the same; their perfect Bayesian equilibria can differ, but the results in this paper apply to them all.<sup>11</sup>

A strategy for the principal is an offer  $r$ , and an acceptance rule  $a_P(r, s)$  giving the probability with which she accepts the agent's offer  $s$  when the initial contract is  $r$ . The agent's strategy is an acceptance rule  $a_A(r)$  giving the probability with which he accepts the principal's offer  $r$ , and a contract and effort pair  $(s(r), e(r))$  for him to choose in subgame  $\Gamma(r)$ .

The principal chooses between  $r$  and  $s$  according to her beliefs about the agent's effort. Her belief function,  $\beta(e | r, s)$ , is a probability measure on  $E$  conditional on the agent having accepted  $r$  and offered  $s$ .

A *perfect Bayesian equilibrium* is a profile of mixed strategies and a belief function satisfying three conditions. (i) The strategies induce a Nash equilibrium on each subgame. (ii) The belief function is consistent with Bayes' rule and the agent's strategy, whenever possible. (iii) The principal's acceptance rule maximizes her expected utility according to her belief function.

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<sup>11</sup> Perfect Bayesian equilibria are the same as subgame perfect equilibria in the game with  $s$  decided before  $e$ , and that game's set of equilibrium outcomes is a subset of those of the other games. But the game with  $e$  decided before  $s$  is the more natural one for the renegotiation issue.

Lemma 1 shows that the perfect Bayesian equilibrium payoffs are bounded in a natural way by  $A^*$  and  $P^*$ . Let  $A^{eq}$  and  $P^{eq}$  be equilibrium payoffs of the game as a whole, and  $A^{eq}(r)$  and  $P^{eq}(r)$  be equilibrium payoffs of subgame  $\Gamma(r)$ .

**LEMMA 1:** *Given a perfect Bayesian equilibrium, the principal's payoff satisfies  $P^{eq} \leq P^*$ . If the agent accepts  $r$  with positive probability, then  $P^{eq}(r) \leq P^*$ . For any  $r$ , if  $P^{eq}(r) = P^*$ , then  $A^{eq}(r) \leq A^*$ .*

A perfect Bayesian equilibrium is referred to as *efficient* if the equilibrium payoffs are  $A^*$  and  $P^*$ .

#### *A Refinement Criterion*

As will be shown later, some perfect Bayesian equilibria require the principal to have implausible beliefs off the equilibrium path, beliefs which hold that the agent has chosen a dominated strategy. Attention is restricted in this paper to perfect Bayesian equilibria in which the principal never believes the agent has chosen a certain kind of dominated strategy. It is a relatively weak “forward induction” criterion satisfied, for example, by equilibrium outcomes which survive the common procedure of twice removing dominated strategies, first for the agent and then for the principal.<sup>12</sup>

At issue is a strategy of the agent requiring him to choose an effort and propose a contract which, if it is accepted, gives him less than his reservation utility. That is, the strategy is to accept some  $r$  with positive probability, and in subgame  $\Gamma(r)$  to choose  $e$  and propose  $s$  such that  $A(s, e) < \bar{A}$ . This strategy is dominated. The dominating strategy depends on whether  $r$  can give the agent his reservation utility. If  $A^*(r) \leq \bar{A}$ , the dominating strategy rejects  $r$ . If  $A^*(r) \geq \bar{A}$ , the dominating strategy accepts  $r$ , but proposes  $r$  instead of  $s$  in  $\Gamma(r)$ . In each case these strategies give the agent a payoff no less than does the dominated strategy, and they give him strictly more if the principal's strategy is to propose  $r$  and then accept  $s$ .

If the principal believes, at the information set indexed by  $(r, s)$ , that the agent

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<sup>12</sup> See, e.g., Cho and Kreps (1987), Kohlberg and Mertens (1986), or the recent survey by Van Damme (1990).

has not played such a dominated strategy, she assigns probability one to the following set of efforts:

$$\bar{E}(s) \equiv \{e \in E \mid A(s,e) \geq \bar{A}\}.$$

This motivates Criterion C (for want of a better name).<sup>13</sup>

CRITERION C: For all contracts  $r$  and  $s$ ,  $\beta(\bar{E}(s) \mid r, s) = 1$  if  $\bar{E}(s) \neq \emptyset$ .

Henceforth, the unmodified term *equilibrium* refers only to perfect Bayesian equilibria satisfying this criterion.

#### 4. Renegotiating Sales Contracts Is Efficient

In this section it is proved that if any equilibrium exists in the General Environment, and if sales contracts are feasible, then an efficient equilibrium exists in which the initial contract is a sales contract.

Sales contracts first must be defined in the General Environment. The following definition defines them precisely in terms of the required characteristic: a sales contract should leave the principal uncaring about the agent's effort.

DEFINITION: Contract  $r$  is a *sales contract* if  $P(r,e) = P(r,d)$  for all  $e, d \in E$ .

Contract  $r^*$  is an *optimal sales contract* if  $P(r^*,e) = P^*$  for all  $e \in E$ .

PROPOSITION 1: *If equilibria exist and an optimal sales contract is feasible, then any efficient allocation is the outcome of an equilibrium in which an optimal sales contract is initially agreed, but then renegotiated.*

PROOF: Let  $(s^*, e^*)$  be efficient, and let  $r^*$  be an optimal sales contract. For each  $s \in S$ , let  $e(s)$  be an element of  $E(s)$ , with  $e(s^*) = e^*$ . The following strategy-belief profile, which gives rise to  $(s^*, e^*)$ , will be shown to be an equilibrium.

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<sup>13</sup> The criterion is most sensible at the  $(r,s)$  pairs where it will be used. At these pairs,  $P(s,e) > P(r,e)$  for all  $e \in \bar{E}(s)$ , so that it is consistent for the agent to believe the principal will accept  $s$ .



- (a) The principal offers  $r^*$ , and the agent accepts it.
- (b) In subgame  $\Gamma(r^*)$ , the agent chooses  $(s^*, e^*)$ .
- (c) In subgame  $\Gamma(r^*)$ , if the agent offers  $s$  the principal believes his effort is  $e(s)$ , and she accepts  $s$  if and only if  $P(s, e(s)) \geq P^*$ .
- (d) The strategies and beliefs at all information sets following the initial offer of any  $r \neq r^*$  are as in some arbitrarily chosen equilibrium.

Consider the principal in  $\Gamma(r^*)$ . She knows she will receive  $P(r^*, \cdot) = P^*$  if she refuses the agent's offer. Thus, given her belief that the agent has taken effort  $e(s)$  when he offers  $s$ , it is optimal for her to accept  $s$  if and only if  $P(s, e(s)) \geq P^*$ . Her beliefs are correct on the path of play, as the agent chooses  $(s^*, e^*) = (s^*, e(s^*))$ . Her beliefs satisfy Criterion C, since  $e(s) \in E(s) \subseteq \bar{E}(s)$  when  $\bar{E}(s) \neq \emptyset$ .

Consider the agent in  $\Gamma(r^*)$ . He can get  $A^*$  if he offers  $s^*$ . Let  $s$  be either  $r^*$  or a contract which the principal accepts. Then  $P(s, e(s)) \geq P^*$ . Note that  $(s, e(s))$  is feasible for (P) if  $A(s, e(s)) \geq \bar{A}$ . This would imply  $A(s, e(s)) \leq A^*$ , as would  $A(s, e(s)) < \bar{A}$ . Thus, in either case, offering  $s$  gives the agent no more than  $A^*$ . If the agent makes an offer which the principal rejects, he receives at most  $A(r^*, e(r^*))$ , which has just been shown to be no more than  $A^*$ . So no offer gives the agent more than  $A^*$ , proving that  $(s^*, e^*)$  is a best reply for him in  $\Gamma(r^*)$ .

This shows that (b) and (c) define an equilibrium for  $\Gamma(r^*)$ . It gives the principal  $P^*$  and the agent  $A^*$ . Since  $A^* \geq \bar{A}$ , accepting  $r^*$  is optimal for the agent.

Part (d) defines equilibria for all proper subgames in which the principal offers  $r \neq r^*$ . The most she can get by offering  $r \neq r^*$  is the payoff she receives from the equilibrium used in (d), which is no more than  $P^*$  by Lemma 1. She obtains  $P^*$  if she offers  $r^*$ , so this is her best offer. Thus, (a)-(d) define an equilibrium. ■

## 5. The Specific Environment

The remainder of the paper concerns the Specific Environment introduced in this section. It is similar to moral hazard environments often studied. It is more general in that no monotonicity assumption (such as the MLRP) relates the effort variable to the return distribution, and the set of efforts can be multidimensional.

### Assumptions

In the Specific Environment, the set  $E$  of feasible efforts is a nonempty compact subset of  $\Re^m$ , either finite or infinite. All functions on  $E$  are continuous.

The agent has a continuous, separable utility function for money and effort,

$$U(y, e) = u(y) - c(e).$$

The function  $u$  is strictly increasing, strictly concave, and defined on the real line. The agent's reservation utility,  $\bar{A}$ , is small enough that he can be adequately compensated for any effort:  $\bar{y}$  exists such that for all  $y \geq \bar{y}$  and  $e \in E$ ,  $u(y) - c(e) > \bar{A}$ .

The agent's effort determines the probability distribution of a contractible signal. The possible realizations of the signal is a finite set,  $X = \{x_1, \dots, x_n\}$ . Effort  $e$  results in a probability vector,  $f(e) = (f_1(e), \dots, f_n(e))$ , on  $X$ . The usual full support assumption is made:  $f_i(e) > 0$  for each  $e$  and  $i$ .

A contract is a vector  $s = (s_1, \dots, s_n)$ , where  $s_i$  is the payment to the agent if  $x_i$  is realized. The set of feasible contracts is  $S = \Re^n$ , which implies that neither party has limited liability. Contract  $s$  generates utility vector  $u(s) = (u(s_1), \dots, u(s_n))$ . An allocation  $(s, e)$  gives the agent the expected utility,

$$A(s, e) = u(s) \cdot f(e) - c(e).$$

Because this function is continuous in  $e$  on the compact set  $E$ , the set  $E(s)$  of efforts which maximize  $A(s, e)$  is nonempty.

The expected return when the agent takes effort  $e$  is  $R(e)$ . The principal is risk neutral. Her expected profit given an allocation  $(s, e)$  is,

$$P(s, e) = R(e) - s \cdot f(e).$$

Now, a sales contract  $r$  is a contract such that  $P(r, e)$  is independent of  $e$ . A contract with this property generally exists only if the random return can be wholly transferred to the agent, regardless of his effort. To this end, the expected return conditional on signal  $x_i$  and effort  $e$  is assumed to be a constant  $\pi_i$ , independent of  $e$ .<sup>14</sup>

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<sup>14</sup> Let  $\tilde{R}$  and  $\tilde{x}$  be the random return and signal variables. Then

Each  $\pi_i$  is a *realized return*, and  $R(e)$  is their expected value: letting  $\pi = (\pi_1, \dots, \pi_n)$ ,

$$R(e) = \pi \cdot f(e).$$

This formulation essentially requires the return to be part of the verifiable signal, although  $x_i$  may reveal other useful information too, as Holmström (1979) shows.

A *sales contract with price  $p$*  is now a contract  $r(p) = (r_1(p), \dots, r_n(p))$  which transfers the return to the agent for price  $p$ :

$$r_i(p) = \pi_i - p \text{ for each } i = 1, \dots, n.$$

Given a sales contract  $r(p)$ , the principal's profit is  $p$  regardless of the agent's effort:

$$P(r(p), e) = \pi \cdot f(e) - (\pi \cdot f(e) - p) = p.$$

The *optimal sales contract*,  $r^*$ , gives the principal her best expected profit:  $r^* = r(P^*)$ .

In contrast to a sales contract, a *wage contract* perfectly insures the agent by paying him the same amount for each realized signal. As the principal is risk neutral, wage contracts provide first-best risk sharing. To make the moral hazard problem significant, wage contracts, and hence efforts which minimize the agent's cost,  $c(e)$ , are assumed to not be part of any efficient allocation.<sup>15</sup>

The final assumption requires that each inducible effort be strictly inducible.

ASSUMPTION A1: For all efforts  $e$ , if  $e \in E(s)$  for some  $s$ , then  $E(t) = \{e\}$  for some  $t$ .

This assumption is not overly restrictive. Being weakly but not strictly inducible is a knife-edge property for an effort  $e$ , since perturbing the cost  $c(e)$  downwards would make  $e$  strictly inducible, and perturbing  $c(e)$  upwards would make  $e$  not inducible.

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$$P(r, e) = \sum_{i=1}^n \left[ \mathcal{E}(\tilde{R} | \tilde{x} = x_i, e) - r_i \right] f_i(e).$$

Without unduly restricting the behavior of  $f(e)$ ,  $P(r, e)$  does not vary with  $e$  only if each  $\mathcal{E}(\tilde{R} | \tilde{x} = x_i, e)$  does not vary with  $e$ .

<sup>15</sup> The results of this paper generalize to the case of a risk averse principal by replacing the term “wage contract” by “first-best risk-sharing contract,” and assuming that such a contract is not part of an efficient allocation.

*Preliminaries*

(a) As is well known, the IR constraint binds in the principal-agent problem in the Specific Environment. Thus,  $A^* = \bar{A}$ , and every solution of (P) is efficient. Furthermore, solutions exist (see Appendix B).

(b) Because the principal's payoff is now separable in revenue and cost,

$$P(s,e) > P(r,e) \Leftrightarrow s \cdot f(e) < r \cdot f(e).$$

In a renegotiation subgame  $\Gamma(r)$ , the principal's preferred contract is the one she believes has the lower expected cost — the expected revenue  $R(e)$  is irrelevant.

(c) A stronger version of Assumption A0, made earlier to rule out beneficial randomizations between contracts, is provable in the Specific Environment. The following lemma is a consequence of the agent's strict risk aversion.

**LEMMA 2:** *In the Specific Environment, suppose  $r \neq s$  are contracts,  $a \in (0,1)$ , and*

$$e \in \operatorname{argmax}_{e' \in E} aA(s,e') + (1-a)A(r,e').$$

*Then contract  $t$  exists such that  $e \in E(t)$ ,*

$$A(t,e) > aA(s,e) + (1-a)A(r,e), \text{ and} \\ P(t,e) > aP(s,e) + (1-a)P(r,e).$$

(d) Wage contracts are not renegotiable. Any contract the agent might offer which he prefers to a wage contract must give him an expected payment greater than the wage. The risk neutral principal therefore prefers paying the wage to accepting the offer if she has 'rational expectations' about the agent's effort, which she does in equilibrium.<sup>16</sup>

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<sup>16</sup> This is the no-trade theorem of Milgrom and Stokey (1982). Since a wage contract efficiently shares risk, it is common knowledge that one party must lose by renegotiating a wage contract (even though  $e$  is not common knowledge).

**LEMMA 3:** *In the Specific Environment, a wage contract is not renegotiated in any perfect Bayesian equilibrium: if  $r_i = w$  for all  $i$ , then  $a_P(r,s) = 0$  for every  $s \neq r$ .*

(e) *Incentive contracts* (non-wage contracts) are renegotiable. This is not to say (yet) that they are renegotiated, as that depends on the principal's beliefs. However, given any incentive compatible allocation in which the contract is an incentive contract, another incentive compatible allocation exists which both parties prefer. The proof entails perturbing the given incentive contract in a way which better insures the agent and lowers the principal's expected payment. (Recall that  $A^*(s)$  is the maximal utility the agent can obtain from contract  $s$ .)

**LEMMA 4:** *In the Specific Environment, each neighborhood of an incentive contract  $r$  contains a contract  $s$  such that*

- (i)  $A^*(s) > A^*(r)$ , and
- (ii) *if effort  $e$  satisfies  $A(s,e) \geq A^*(r)$ , then  $P(s,e) > P(r,e)$ .*

(f) Because of Assumption A1, every contract is the limit of contracts which induce unique optimal efforts. This will be very useful.

**LEMMA 5:** *In the Specific Environment, given contract  $s$  and effort  $e \in E(s)$ , every neighborhood of  $s$  contains a contract  $\hat{s}$  such that  $E(\hat{s}) = \{e\}$  and  $A(\hat{s},e) = A^*(s)$ .*

(g) Equilibria exist in the Specific Environment, as is shown in Appendix B.

## 6. All Equilibria Are Efficient

In this section, all equilibria in the Specific Environment are shown to be efficient. Sales contracts are central to the proof.

The reason all equilibria are efficient is because Criterion C gives the agent some control over the principal's beliefs. His choice of a contract  $s$  signals that his effort is in the set  $\bar{E}(s)$ . This allows him to make offers the principal cannot refuse, namely, offers  $s$  such that  $P(s,e) > P(r,e)$  for all  $e \in \bar{E}(s)$ . This, in turn, implies that the

principal can formulate initial offers which the agent cannot refuse. In particular, the agent must accept sales contracts with prices less than  $P^*$ .

To see how this works, suppose that in the two-by-two example, the principal offers the sales contract  $r(p)$  shown in Figure 5.

(Figure 5 About Here)

Notice that contract  $s$  satisfies  $A(s, e_h) > \bar{A} > A(s, e_l)$ . Hence, a strategy which accepts  $r(p)$  and plays  $(s, e_l)$  is dominated by, e.g., a strategy which rejects  $r(p)$  (but see Remark 3 below). Thus, Criterion C requires the principal to believe the agent's effort is  $e_h$  if he proposes  $s$  in subgame  $\Gamma(r(p))$ . She then believes her isoprofit line is the indicated steep one through  $r(p)$ ; she accepts  $s$  because it is below this line. The agent's resulting utility exceeds  $\bar{A}$ , and so he accepts  $r(p)$ . This shows that the principal obtains a profit of at least  $p$  if she offers  $r(p)$ . As  $p$  can be chosen arbitrarily close to  $P^*$ , the principal's equilibrium profit must be  $P^*$ .

**REMARK 3:** In this example, efficiency is also implied by the criterion of Ma (1990). Notice in Figure 5 that given either  $s$  or  $r(p)$ , the agent's best effort is  $e_h$ . Thus, in subgame  $\Gamma(r(p))$ , playing  $(s, e_l)$  is dominated by playing  $(s, e_h)$ . More generally, if  $\{e\} = E(r) = E(s)$ , choosing any effort except  $e$  is dominated, and Ma's criterion requires the principal to believe that  $e$  is the agent's effort. So it too requires the principal to believe the agent's effort is  $e_h$  if he proposes the  $s$  shown in Figure 5, and the same argument shows that equilibria satisfying Ma's criterion are efficient. Ma (1990) shows that his criterion implies efficiency if the number of efforts is finite, and a contract is a menu of sharing rules. Here, as contracts are single sharing rules, his criterion does not imply efficiency if the number of efforts exceeds two.<sup>17</sup>

**PROPOSITION 2:** *Every equilibrium in the Specific Environment is efficient.*

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<sup>17</sup> Suppose  $E$  is an interval in this example, and  $c(e)$  is strictly convex. Then,  $\{e\} = E(r) = E(s)$  only if  $r_g - s_g$  and  $r_b - s_b$  have the same sign, which is an irrelevant case for proving efficiency.

PROOF: Let  $\varepsilon > 0$  and  $p = P^* - 2\varepsilon$ . Let  $(s^*, e^*)$  be efficient. Then  $s^*$  is an incentive contract. By Lemma 5, an incentive contract  $\hat{s}$  close to  $s^*$  exists so that

$$P(\hat{s}, e^*) > P(s^*, e^*) - \varepsilon = P^* - \varepsilon, \quad (6.1)$$

$A(\hat{s}, e^*) = A(s^*, e^*) = \bar{A}$ , and  $E(\hat{s}) = \{e^*\}$ . Note that  $\bar{E}(\cdot)$  is upper hemicontinuous, and  $\bar{E}(\hat{s}) = \{e^*\}$ . Hence, a neighborhood  $N$  of  $\hat{s}$  exists such that if  $s \in N$ , then

$$P(\hat{s}, e) > P(\hat{s}, e^*) - \varepsilon \quad \text{for all } e \in \bar{E}(s). \quad (6.2)$$

From Lemma 4 (setting  $r = \hat{s}$ ),  $s \in N$  exists such that  $A^*(s) > A^*(\hat{s})$ , and  $P(s, e) > P(\hat{s}, e)$  if  $A(s, e) \geq A^*(\hat{s})$ . This shows that  $\bar{E}(s) \neq \emptyset$  (since  $A^*(\hat{s}) = \bar{A}$ ), and

$$P(s, e) > P(\hat{s}, e) \quad \text{for all } e \in \bar{E}(s). \quad (6.3)$$

From (6.1) - (6.3),

$$P(s, e) > P^* - 2\varepsilon = P(r(p), e) \quad \text{for all } e \in \bar{E}(s). \quad (6.4)$$

Criterion C requires the principal to believe the agent's effort is in  $\bar{E}(s)$ . Hence, by (6.4) she accepts  $s$  in  $\Gamma(r(p))$ . This implies  $A^{eq}(r(p)) > \bar{A}$  (since  $A^*(s) > \bar{A}$ ). So the agent accepts  $r(p)$ . Hence,  $P^{eq} \geq P(r(p), \cdot) = P^* - 2\varepsilon$ . Letting  $\varepsilon \rightarrow 0$  proves  $P^{eq} = P^*$ . Lemma 1 now implies  $A^{eq} \leq A^*$ , which, as  $A^* = \bar{A}$ , proves  $A^{eq} = A^*$ . ■

REMARK 4: Not all perfect Bayesian equilibria are efficient; the refinement criterion is needed. In some perfect Bayesian equilibria, the agent totally shirks because that is what the principal expects him to do. Consider the following profile in the example of Section 2: The principal believes the effort is  $e_\ell$  after any  $(r, s)$ , and chooses accordingly; in each  $\Gamma(r)$ , the agent chooses  $e_\ell$  and proposes the wage contract  $s_r$  that pays him  $w(r) = e_\ell r_g + (1 - e_\ell)r_b$ ; the agent accepts  $r$  if and only if  $u(w(r)) + c(e_\ell) \geq \bar{A}$ ; and the principal proposes the wage contract  $\hat{s}$  that gives the agent  $\bar{A}$  if he chooses  $e_\ell$ . This describes a perfect Bayesian equilibrium if for each  $r$ , the agent prefers  $(w(r), e_\ell)$  to  $(s, e_h)$ , where  $s$  is any contract the principal will accept in  $\Gamma(r)$ , i.e., any  $s$  satisfying  $e_\ell s_g + (1 - e_\ell)s_b \leq e_\ell r_g + (1 - e_\ell)r_b$ . This is the case if, for example,  $u(y) = \ln(y)$  and high

effort is relatively costly in that,<sup>18</sup>

$$\frac{c(e_h) - c(e_\ell)}{e_h - e_\ell} > \ln \left( \frac{e_h(1-e_\ell)}{e_\ell(1-e_h)} \right).$$

This section closes by characterizing equilibrium outcomes. Shown first is that if they were not renegotiated, equilibrium initial offers would give the agent less than his reservation utility. The proof is easy:  $A^*(r) \leq \bar{A}$  follows from  $A^* = \bar{A}$  and the equilibrium's efficiency, and  $A^*(r) \neq \bar{A}$  follows from the ability that Criterion C gives the agent to make unrefusable offers which he strictly prefers to  $r$ .

**LEMMA 6:** *Given an equilibrium in the Specific Environment, let  $r$  be an equilibrium offer of the principal. Then  $a_A(r) = 1$  and  $A^*(r) < \bar{A}$ . The equilibrium payoffs in subgame  $\Gamma(r)$  are  $P^{eq}(r) = P^*$  and  $A^{eq}(r) = \bar{A}$ .*

The next lemma concerns the following program, which is parameterized by  $r$  and shown in Appendix B to have a solution:

$$(A-r) \quad A^r \equiv \max_{s \in S, e \in E} A(s, e) \text{ such that}$$

$$(IC) \quad e \in E(s) \text{ and}$$

$$(AC) \quad P(s, e) \geq P(r, e).$$

This program maximizes the agent's expected utility subject to the incentive constraint IC, and to an individual rationality constraint for the principal, her *acceptance constraint* AC. Its value,  $A^r$ , can be shown to be an upper bound on the agent's equilibrium utility in subgame  $\Gamma(r)$ .<sup>19</sup> Lemma 7 shows that if  $r$  is an equilibrium initial contract, then the agent achieves  $A^r$  in  $\Gamma(r)$ . This is again because Criterion C allows him to make unrefusable renegotiation offers.

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<sup>18</sup> Even with this condition satisfied,  $e_h$  is efficient if  $\pi_g$  is sufficiently large.

<sup>19</sup> The proof of this is easy for a pure strategy subgame equilibrium, since its allocation satisfies IC and AC. The general proof is involved and not needed here.



**LEMMA 7:** *Given an equilibrium in the Specific Environment, let  $r$  be an equilibrium offer of the principal, and let  $s$  be an equilibrium offer of the agent in subgame  $\Gamma(r)$ . Then  $a_P(r, s) = 1$ , and  $A^{eq}(r) = A^r = \bar{A}$ .*

Proposition 3 completes the characterization.

**PROPOSITION 3:** *Given an equilibrium in the Specific Environment, let  $r$  be an equilibrium offer of the principal. Then the equilibrium allocations of  $\Gamma(r)$  are efficient and solve  $(A - r)$ .*

**PROOF:** Let  $\mu$  be the agent's mixed strategy in  $\Gamma(r)$ , a probability distribution with support  $\text{Supp}(\mu)$ , a subset of  $S \times E \subset \mathfrak{R}^n \times \mathfrak{R}^m$ . The principal accepts the agent's offers in  $\Gamma(r)$ , by Lemma 7. Thus,  $\mu$  is the equilibrium distribution of allocations in  $\Gamma(r)$ .

Define  $g(s, e) \equiv P(s, e) - P(r, e)$ , and then fix  $(s, e) \in \text{Supp}(\mu)$ .

By Lemma 7,  $A(s, e) = A^r$ . As the principal accepts the agent's offers,  $(s, e)$  satisfies IC. Thus,  $(s, e)$  solves  $(A - r)$  if and only if it satisfies AC. It cannot satisfy AC with slack, as AC binds in  $(A - r)$ .<sup>20</sup> Thus,  $g \leq 0$  on  $\text{Supp}(\mu)$ . But the expectation of  $g$  according to  $\mu$  is nonnegative, since the principal accepts the agent's offers. Thus, as  $g$  is continuous,  $g = 0$  on  $\text{Supp}(\mu)$ . So  $(s, e)$  satisfies AC, and hence solves  $(A - r)$ .

By Lemma 7,  $(s, e)$  satisfies IR as well as IC, and so is feasible for (P). Thus,  $P \leq P^*$  on  $\text{Supp}(\mu)$ . As  $P^{eq}(r) = P^*$ , the expectation of  $P$  according to  $\mu$  is  $P^*$ . Since  $P$  is continuous, this shows that  $P = P^*$  on  $\text{Supp}(\mu)$ . Therefore  $(s, e)$  is efficient. ■

## 7. Talented Agents Receive Sales Contracts

The agent in this section is able to shift the signal probabilities in all directions, and the payoffs change smoothly when he does so. The main result is that then, each equilibrium initial contract is the optimal sales contract. Only it sufficiently protects the principal from the opportunism which talent, and Criterion C, allow the agent.

<sup>20</sup> Proof: Let  $(s, e)$  be an IC allocation satisfying AC with slack. Define  $t$  by  $u(t_i) = u(s_i) + \varepsilon$ . For  $\varepsilon > 0$  small enough,  $(t, e)$  satisfies both IC and AC, and  $A(t, e) > A(s, e)$ . Thus,  $(s, e)$  cannot solve  $(A - r)$ .

### *The Talented Agent Environment*

To define this environment, the principal's cost function for inducing effort in the Specific Environment is first defined:

$$C(e) \equiv \min_{s \in S} s \cdot f(e) \text{ subject to IC and IR.}^{21}$$

Then, the *Talented Agent Environment* is a Specific Environment satisfying the following three-part assumption: for any efficient  $(s^*, e^*)$ ,

- (i)  $e^*$  is in the interior of  $E$ ;
- (ii) both  $f$  and  $C$  are continuously differentiable in a neighborhood of  $e^*$ ; and
- (iii) the  $m \times n$  Jacobian matrix  $J(e^*) \equiv [\partial f_j(e^*) / \partial e_i]$  has rank  $n-1$ .

Part (i) implies that a continuum of efforts are possible, and that each efficient  $e^*$  can be shifted in any direction in  $\mathfrak{R}^m$ , the space containing  $E$ . Part (ii) requires the cost function  $C(e)$  to be differentiable, as it generally is if the “first order approach” to the principal-agent problem is valid.<sup>22</sup> Part (iii), in conjunction with (i), implies that the probability vector  $f(e^*)$  can be shifted in any direction in the simplex; the agent is able to finely tune the likelihood of each contractible signal. Notice that  $n-1$  is the maximum possible rank of the Jacobian, since its row sums are zero. It has this rank only if  $m$ , the dimension of an effort vector, is not less than  $n-1$ , where  $n$  is the dimension of the return vector  $\pi$ .

A canonical example of the Talented Agent Environment is obtained when the agent directly controls each signal's probability. Suppose  $e = (e_1, \dots, e_n)$ , and  $f(e) = e$ ; the set  $E$  is a compact and convex subset of the simplex; the agent's cost function  $c$  and utility function  $u$  are twice continuously differentiable;  $c$  is convex and  $u$  is concave; and efficient efforts are in the relative interior of  $E$ . Then, an efficient  $e^*$  is induced by contract  $s$  if and only if a multiplier  $\psi$  exists such that,

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<sup>21</sup> As Grossman and Hart (1983) show, this program has a solution if the constraint set is nonempty. Let  $C(e) = \infty$  if the constraint set is empty.

<sup>22</sup> Rogerson (1985), Jewitt (1988).

$$u(s_i) = \psi + c_i(e^*) \text{ for each } i,$$

where  $c_i(e^*)$  is the partial derivative. The agent's resulting expected utility is

$$\bar{A} = u(s) \cdot e^* - c(e^*) = \psi + \nabla c(e^*) \cdot e^* - c(e^*).$$

Hence,  $s_i = u^{-1}[\bar{A} + c(e^*) - \nabla c(e^*) \cdot e^* + c_i(e^*)]$ , and so

$$C(e^*) = \sum_{i=1}^n e_i^* u^{-1}[\bar{A} + c(e^*) - \nabla c(e^*) \cdot e^* + c_i(e^*)].$$

This proves that  $C$  is continuously differentiable near  $e^*$ . Changing variables to  $d = (e_1, \dots, e_{n-1})$ , so that  $f_n(d) = 1 - \sum_{i=1}^{n-1} d_i$ , results in an effort set  $D \subset \mathfrak{R}^{n-1}$  with an interior which corresponds to the relative interior of  $E$ , and so contains the efficient efforts. The Jacobian  $[\partial f_j(e^*)/\partial d_i]$  has rank  $n-1$ , since it is the  $n-1$  dimensional identity matrix with a column of -1's on its right. This is hence an example of the Talented Agent Environment.

### *The Result*

The result is now proved, after which it is illustrated in the example of Section 2. To start, notice that because  $P(s, e) = \pi \cdot f(e) - s \cdot f(e)$ , the term  $\pi \cdot f(e)$  subtracts from both sides of the AC constraint in problem (A -  $r$ ). It can therefore be written,

$$(A - r) \quad \text{maximize } A(s, e) \text{ subject to IC and } s \in S, e \in E$$

$$(AC) \quad s \cdot f(e) \leq r \cdot f(e).$$

In view of Proposition 3, the relevant case is when the value of (A -  $r$ ) is  $\bar{A}$ . Assuming this, the dual of (A -  $r$ ) is,

$$(DA - r) \quad \text{maximize } r \cdot f(e) - s \cdot f(e) \text{ subject to IC and } s \in S, e \in E$$

$$(IR) \quad A(s, e) \geq \bar{A}.$$

Since (A -  $r$ ) and (DA -  $r$ ) have solutions (Appendix B), and AC and IR bind, a standard exercise shows that their solutions are the same.

Problem (DA -  $r$ ) is a principal-agent problem, the same as (P) except that  $r$

replaces  $\pi$ . Consequently, if  $(s^*, e^*)$  solves both (P) and (DA -  $r$ ), then  $e^*$  solves both of the following problems:

$$(P - \pi) \quad \max_{e \in E} \pi \cdot f(e) - C(e),$$

$$(P - r) \quad \max_{e \in E} r \cdot f(e) - C(e),$$

where  $C$  is the principal's cost function for inducing effort, as defined above. An efficient  $(s^*, e^*)$  therefore solves (A -  $r$ ) only if  $e^*$  solves (P -  $\pi$ ) and (P -  $r$ ).

Now, Proposition 3 showed that an equilibrium allocation is efficient and solves (A -  $r$ ). An equilibrium effort must therefore solve both (P -  $\pi$ ) and (P -  $r$ ), where  $r$  is the equilibrium initial contract. In the Talented Agent Environment, problems (P -  $\pi$ ) and (P -  $r$ ) each have a necessary first order matrix equation. Because of the rank condition, these equations have the same solution if and only if  $r$  is a sales contract. This is the essence of the result's proof.

**PROPOSITION 4:** *In the Talented Agent Environment, every equilibrium initial contract is the optimal sales contract.*

**PROOF:** Let  $r$  be an equilibrium initial contract, and  $(s, e)$  an equilibrium allocation of  $\Gamma(r)$ . By Proposition 3,  $(s, e)$  is efficient and solves (A -  $r$ ). Therefore  $e$  solves (P -  $\pi$ ) and (P -  $r$ ). The necessary matrix equation for  $e$  to solve (P -  $r$ ) is,

$$J(e)r = \nabla C(e).$$

Because  $e$  also solves (P -  $\pi$ ), the same equation holds with  $\pi$  replacing  $r$ . Hence, the vector  $\pi - r$  belongs to the kernel (null space) of  $J(e)$ . The kernel of  $J(e)$  is unidimensional, since  $J(e)$  has rank  $n-1$  and  $n$  columns, and it contains the unit vector, since the row sums of  $J(e)$  are zero. So a scalar  $p$  exists such that  $\pi - r$  is a multiple  $p$  of the unit vector. This proves that  $r$  is the sales contract with price  $p$ . Because AC binds in (A -  $r$ ),  $P(s, e) = P(r, e) = p$ . Since  $(s, e)$  is efficient,  $P(s, e) = P^*$ . Therefore  $p = P^*$ , and  $r$  is the optimal sales contract. ■

### *The Example Revised*

Some intuition for the result can be obtained by considering the example of Section 2. First, notice that it has equilibria in which the initial contract is not a sales contract. In Figure 3, choose an  $r$  on the steep isoprofit line through  $r^*$ , and below the isoprofit line connecting  $\hat{r}$  and  $\hat{s}$ . Then an efficient equilibrium exists in which  $r$  is offered and renegotiated to  $s^*$ , and the agent receives  $\bar{A}$  by choosing  $e_h$ . No acceptable offer the agent might make will give him more than  $\bar{A}$ . In particular, since  $r$  is below the isoprofit line connecting  $\hat{r}$  and  $\hat{s}$ , the principal will not accept any contract on the agent's 45° line slightly above  $\hat{s}$ , because Criterion C requires her to believe that the agent's effort is  $e_\ell$  if he makes such an offer.

Notice that the line connecting  $\hat{r}$  and  $\hat{s}$  cuts the principal's 45° line close to  $r^*$  if  $e_\ell$  and  $c(e_\ell)$  are close to  $e_h$  and  $c(e_h)$ , respectively. In this case the  $r$  of the previous paragraph cannot be too far above  $r^*$ . Similarly, it cannot be too far below  $r^*$  if efforts higher but close to  $e_h$  exist. To see this more clearly, consider the limiting case in which the set of efforts is the unit interval, the efficient  $e^*$  is interior, and the agent's cost function  $c(e)$  is strictly convex and twice differentiable. This is the canonical Talented Agent Environment of one dimension.

The agent's first order condition for choosing effort is now necessary and sufficient. That is, given an interior effort  $e$ , he chooses  $e$  when the contract is  $s = (s_h, s_g)$  if and only if,

$$u(s_g) - u(s_h) = c'(e).$$

The contracts solving this equation form the curve labelled IC( $e$ ) in Figure 6.

(Figure 6 About Here)

Figure 6 indicates why the equilibrium initial contract now must be the optimal sales contract. The efficient allocation is  $(s^*, e^*)$ , and shown in the figure is the principal's isoprofit line and the agent's  $\bar{A}$ -indifference curve through  $s^*$  when his effort is  $e^*$ . Suppose, by way of contradiction, that the indicated  $r$  is an equilibrium initial

contract.<sup>23</sup> Since the equilibrium must be efficient,  $r$  must be renegotiated to  $s^*$  and the agent's effort must be  $e^*$ .

Consider an effort  $e$  slightly less than  $e^*$ . The contract with the lowest expected cost which induces  $e$  is the indicated  $\hat{s}$  at the intersection of the  $IC(e)$  curve and the indifference curve given by  $A(\cdot, e) = \bar{A}$ . The principal's expected profit from  $(\hat{s}, e)$  is  $p$ , as shown. This  $p$  exceeds the profit the principal would obtain from  $(r, e)$ , since  $r$  is above the indicated isoprofit line through  $\hat{s}$  and  $r(p)$ . Because  $r$  is above the principal's 45° line, an effort  $e$  having this property does exist.<sup>24</sup>

Now, suppose the agent offers the indicated  $s$  instead of  $s^*$ . Effort  $e$  is optimal for him at  $s$ , since  $s$  on the  $IC(e)$  curve. His utility at  $(s, e)$  exceeds  $\bar{A}$ , and so the principal must reject  $s$ . But she prefers  $(s, e)$  to  $(r, e)$ . Criterion C requires her to believe the agent's effort is such that he obtains at least  $\bar{A}$  from  $s$ . Because  $s$  is close to  $\hat{s}$ , all efforts for which this is true are close to  $e$ , and the principal prefers  $s$  to  $r$  given any of them. She should therefore accept  $s$ , rather than reject it as the alleged equilibrium requires. Thus,  $r$  cannot be an equilibrium contract.<sup>25</sup>

## 8. Discussion

This paper has shown that renegotiation does not necessarily reduce efficiency in moral hazard environments; that this is so even if only the usual single sharing rule contracts are used; and that sales contracts play a compelling role as initial contracts from which efficient renegotiation can occur.

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<sup>23</sup> Contract  $r$  is drawn far from  $r^*$  only for the sake of clarity. Also, it is put on the isoprofit line through  $r^*$  corresponding to the efficient  $e^*$  only because this is the more difficult case; if  $r$  were above this line the argument would be easier. A symmetric argument holds if  $r$  is below the principal's 45° line, as indicated below.

<sup>24</sup> A first-order lowering of  $e^*$  results in a first-order rotation of the isoprofit line through  $r$ , and hence a first-order shift upward of  $t$ , its endpoint on the principal's 45° line. But profit  $p$  is only a second-order decrease from the optimal profit,  $P^*$ , and so  $r(p)$  is only a second-order shift upwards from  $r^*$ . Thus, if  $e$  is sufficiently close to  $e^*$ ,  $t$  will be above  $r(p)$ .

<sup>25</sup> A similar argument, but with  $e \geq e^*$ , shows that Criterion C is violated if  $r$  is below the principal's 45° line.

The first topic of this section is the game in which the agent makes the initial as well as the renegotiation offer. Two applications then are considered briefly, managerial compensation and the timing of a new firm's security issues. The final topic is the sensitivity of results to the structure of the game.

### *Initial Offer by the Agent*

It makes sense in some applications, such as the new firm application discussed below, for the agent to make the initial contract offer. The relevant principal-agent problem is then to maximize the agent's expected utility subject to the incentive constraint IC, and to a lower bound on the principal's expected profit. An efficient contract is now a solution to this problem, and generally gives the agent an expected utility  $A^*$  which exceeds his non-participation utility  $\bar{A}$ .

Most results extend to the game in which the agent makes the initial offer. Efficient perfect Bayesian equilibria still exist in which the agent offers a sales contract and later renegotiates it.

However, Criterion C no longer implies efficiency; its focus on contracts which give the agent  $\bar{A}$  is not useful if  $A^* > \bar{A}$ . But efficiency is still implied by stronger criteria in the same vein. One such criterion requires the principal to sometimes believe that the agent chose his effort under the assumption that his renegotiation offer would be accepted. The principal should have such beliefs if the offer is credible. The offer is arguably credible if, assuming the agent's effort is optimal for it, both parties prefer it to the initial contract. In this case it is consistent for the agent to expect the principal to accept the offer.<sup>26</sup> The formal statement of this criterion, which implies Criterion C, is as follows.

CRITERION CC: For all contracts  $r$  and  $s$ ,  $\beta(E(s) | r, s) = 1$  if  $A^*(s) > A(r)$  and  $P(s, e) > P(r, e)$  for all  $e \in E(s)$ .

When initial offers are made by the agent, equilibria satisfying this criterion in the

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<sup>26</sup> This criterion is thus similar to that of Farrell (1990) and Grossman and Perry (1986).

Specific Environment are efficient, and in the Talented Agent Environment entail the renegotiation of a sales contract. The proofs are nearly the same, but slightly simpler.

A similar analysis applies when an initial contract is not negotiated, but the agent initially owns the return stream. For example, to foreshadow an upcoming discussion, consider an entrepreneur who uses his own funds and (unobservable, costly) actions to set up a new firm. He then owns the rights to the firm's return without having explicitly signed a sales contract. Suppose he later wants to share risk by making an equity offering, i.e. a "renegotiation" offer, to a potential stockholder (principal). If the potential stockholder declines the offering, she is uncaring about the firm's performance, just as though she had initially sold the firm. This game corresponds to one subgame  $\Gamma(r)$  in this paper, where  $r$  is the sales contract with a zero price. Criterion CC will imply that the equilibrium equity offering and effort chosen by the entrepreneur is efficient, even though the entrepreneur chooses his effort before the equity offering is actually accepted.

### *Managerial Compensation*

Another, prominent application of any incentive model is to managerial compensation. Corporate executives, in negotiation with compensation boards, not infrequently renegotiate the form of their compensation plans. The model suggests that contracts with high powered incentives (sales contracts) will be renegotiated to contracts with lower powered incentives (efficient contracts). Thus, stock option plans should tend to be renegotiated to bonus schemes, or bonus schemes should tend to be renegotiated to increased retirement benefits. To what extent this is true has yet to be definitively addressed. Some anecdotal evidence is affirmative, as Fudenberg and Tirole (1990) indicate.<sup>27</sup>

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<sup>27</sup> Perhaps a first issue to resolve is why observed compensation schemes provide too few incentives, as, e.g., Jensen and Murphy (1990) claim. The renegotiated schemes predicted here provide the correct amount of incentives, not too few. In theory, only if the actions of executives are observable to directors will equilibrium final contracts provide no incentives. (This is true even if their actions are unverifiable, if renegotiation is possible, as Hermalin and Katz (1989, 1991) show.)



### *Financial Structure of New Firms*

Consider again an entrepreneur who wishes to set up a firm and share its risk, but who now needs to borrow start-up funds. Moral hazard is a problem because his unobservable effort is essential. A literature, starting with Jensen and Meckling (1976), has studied the financial structure of such firms. In particular, in a static model in which renegotiation is not possible, and in which securities consist of riskless debt and risky equity, Kihlstrom and Matthews (1990) show that the (second-best) efficient contract consists of a nondegenerate mix of debt and equity which the market can provide in a rational expectations equilibrium. Casual observation, however, indicates that new firms obtain start-up funds almost entirely from the entrepreneur's own savings and by borrowing from banks, i.e., by issuing debt. Only later in the firm's lifecycle does the entrepreneur shed risk by issuing equity.

The model, or an extension of it, may help to explain this timing of security offerings. Debt contracts are the same as sales contracts if the entrepreneur is not subject to limited liability. Even if he is, debt contracts are much like the model's sales contracts in some circumstances, such as when default risk is small, or the entrepreneur's actions only affect the upper tail of the profit distribution. Under these conditions the payback to a debt holder, and hence her expected profit, does not depend on the entrepreneur's actions. To the extent that debt contracts are like sales contracts, the model predicts an initial financial structure consisting wholly of debt, but which later, after the entrepreneur is known to have exerted himself to start the enterprise, is changed to a mixture of equity and debt.

In general, however, debt contracts are not like sales contracts if the entrepreneur has limited liability. Thus, future work should extend the model to include wealth and limited liability constraints, which can make the optimal sales contract infeasible (as  $r^*$  can lie beneath the Edgeworth box in, say, Figure 3). A central question then is whether debt contracts substitute for sales contracts, i.e., whether renegotiated debt contracts arise in equilibrium.

### *The Structure of the Game*

The results of this paper, and of Ma (1990), in comparison with those of Fudenberg and Tirole (1990), illustrate nicely the well-known sensitivity of bargaining results to the structure of the game. Efficiency is obtained if the agent, but not the principal, leads the renegotiation. Renegotiated sales contracts arise naturally from the analysis if contracts are single sharing rules, but not if they are menus of sharing rules. Consequently, in any application the structure of the game must be modeled as accurately as possible.

In the managerial compensation application, the extent to which executives rather than compensation committees lead the renegotiation, and the extent to which compensation schemes resemble single sharing rules rather than menus, are difficult questions to answer. In the new firm application, entrepreneur-owners do seem to lead the renegotiation when they, rather than the future stockholders, instigate an equity offering.

A related issue concerns the implicit commitment assumptions embodied in these games. In Fudenberg and Tirole (1990) and Ma (1990, 1991), even though the agent may use a mixed effort strategy, in equilibrium the principal learns the agent's effort at the point in time when the agent selects his preferred sharing rule from the final contract's menu. His choice reveals his effort because the menu is a separating, incentive compatible mechanism. The principal, at that point, would like to offer the agent an optimal risk-sharing contract. Thus, embodied in the menu approach is an assumption that the principal can commit to not making this incentive-destroying offer, or that the agent can commit to refusing it.

A similar commitment assumption is embodied in the game of this paper, as well as that of Ma (1990). In their equilibria, the agent uses a pure strategy; in equilibrium the principal knows the agent's effort in the renegotiation period. But the game rules do not allow the principal to offer the first-best contract she would like to then offer — all she can do is accept or reject the agent's offer. The implicit commitment assumption is that the principal cannot make offers in the renegotiation period.

It appears that some sort of commitment assumption is required to obtain outcomes in which the agent does not totally shirk. Fortunately, rarely are all forms of commitment impossible. Good arguments for a particular game form which embodies some sort of commitment can often be made. For example, as mentioned above, this paper's assumption that only the agent makes renegotiation offers seems appropriate for the new firm application. Reality departs from the model in this application in two obvious ways: first, the principal in the renegotiation period actually consists of a multitude of individuals (future stockholders), and, second, none of them is likely to have been the initial period principal (an investment banker). (The model is easily extended, with no substantial changes in results, to allow many identical principals in the renegotiation period, all of whom differ from the principal in the initial period.<sup>28</sup>) It is only reasonable then that the entrepreneur-owner make the equity offering, since the future stockholders may not know of his existence until he does so. Furthermore, the multiplicity and individual smallness of the future stockholders make it implausible that they can make serious counteroffers — it seems most plausible that they would be price-takers, passively accepting or rejecting the terms of the offering. Exploring these speculations is left for future work.

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<sup>28</sup> This extension does require the renegotiation period principals to observe the initial period principal's contract with the agent. In the focal equilibrium of this paper, the first period principal is even happy to leave the scene with her sales contract because it renders her uncaring about the firm's later performance (given, of course, that the agent can surely pay the sales price (loan) back.)

## Appendix A

PROOF OF LEMMA 1: Fix an equilibrium, and a contract  $r$ . If the agent's strategy in subgame  $\Gamma(r)$  is mixed, some but not all of his choices may give the principal less than her equilibrium payoff  $P^{eq}(r)$ . Hence, he has an equilibrium choice  $(s, e)$  such that,

$$\hat{P} \equiv aP(s, e) + (1-a)P(r, e) \geq P^{eq}(r),$$

where  $a \equiv a_P(r, s)$  (the principal accepts  $s$  with probability  $a$ ). Because  $(s, e)$  is an equilibrium choice of the agent,

$$\begin{aligned} A^{eq}(r) &= aA(s, e) + (1-a)A(r, e) \\ &= \max_{e' \in E} \{aA(s, e') + (1-a)A(r, e')\}. \end{aligned}$$

Assumption A0 now implies the existence of  $t \in S$  and  $d \in E(t)$  such that  $A(t, d) \geq A^{eq}(r)$  and  $P(t, d) \geq \hat{P} \geq P^{eq}(r)$ .

If  $r$  is acceptable to the agent, then  $A^{eq}(r) \geq \bar{A}$ . Hence,  $A(t, d) \geq \bar{A}$ . This implies that  $(t, d)$  satisfies the constraints of (P), and so  $P^* \geq P(t, d)$ . Thus,  $P^* \geq P^{eq}(r)$ , and this implies  $P^* \geq P^{eq}$ .

Now let  $r$  be any contract, and suppose  $P^* = P^{eq}(r)$ . Then  $P(t, d) \geq P^*$ . If  $A(t, d) > A^*$ , then  $(t, d)$  satisfies the constraints of (P), and hence solves it. This is impossible, since  $A^*$  is the maximal value of  $A(\cdot, \cdot)$  on the solution set of (P). Thus  $A(t, d) \leq A^*$ , and so  $A^{eq}(r) \leq A^*$ . ■

PROOF OF LEMMA 2: Define  $\hat{t}$  by  $u(\hat{t}_i) = au(s_i) + (1-a)u(r_i)$ . Then,

$$A(\hat{t}, d) = aA(s, d) + (1-a)A(r, d) \text{ for all } d \in E.$$

Hence  $e \in E(\hat{t})$ , and  $A(\hat{t}, e) = aA(s, e) + (1-a)A(r, e)$ . As  $u$  is strictly concave,  $\hat{t}_i \leq as_i + (1-a)r_i$ , with the inequality strict for those  $i$ 's satisfying  $s_i \neq r_i$ . Hence,  $P(\hat{t}, e) > aP(s, e) + (1-a)P(r, e)$ . Now a small  $\varepsilon > 0$  exists such that the desired  $t$  is defined by  $u(t_i) = u(\hat{t}_i) + \varepsilon$ . ■

PROOF OF LEMMA 3: Let  $r_i = w$  for all  $i$ . Consider a perfect Bayesian equilibrium of

subgame  $\Gamma(r)$ . Let  $s$  be an equilibrium offer of the agent in this subgame, and let  $a \equiv a_P(r, s)$ . Suppose  $a > 0$ . Because the principal finds  $s$  acceptable, effort  $e$  exists such that  $s \cdot f(e) \leq w$ , and  $(s, e)$  is an equilibrium choice of the agent. The agent could have offered  $r$  instead of  $s$ , and so,

$$a(u(s) \cdot f(e)) + (1-a)u(w) - c(e) \geq u(w) - c(e).$$

Since  $u$  is strictly concave and  $a$  is positive, this implies that  $s \cdot f(e) \geq w$ , with the inequality strict unless  $s = r$ . Hence,  $s = r$ . ■

PROOF OF LEMMA 4: Let  $\hat{e} \in E(r)$  be an effort which minimizes  $r \cdot f(e)$  on  $E(r)$ . For small  $\varepsilon, \delta > 0$ , define a contract  $s^{\varepsilon, \delta}$  by,

$$\begin{aligned} u(s_i^{\varepsilon, \delta}) &= \delta(\varepsilon + u(r) \cdot f(\hat{e})) + (1-\delta)u(r_i) \\ &= \delta(\varepsilon + A^*(r) + c(\hat{e})) + (1-\delta)u(r_i). \end{aligned}$$

The  $s$  we seek is  $s^{\varepsilon, \delta}$  for sufficiently small  $\varepsilon$  and  $\delta$ . Observe that

$$\begin{aligned} A(s^{\varepsilon, \delta}, e) &= u(s^{\varepsilon, \delta}) \cdot f(e) - c(e) \\ &= \delta\varepsilon + A^*(r) - \delta\{c(e) - c(\hat{e})\} - (1-\delta)\{A^*(r) - A(r, e)\}. \end{aligned}$$

Thus,  $A^*(s^{\varepsilon, \delta}) \geq A(s^{\varepsilon, \delta}, \hat{e}) = \delta\varepsilon + A^*(r)$ . This proves (i).

Let  $E(s, r) \equiv \{e \in E \mid A(s, e) \geq A^*(r)\}$ . To prove (ii), we must show

$$(s^{\varepsilon, \delta} - r) \cdot f(e) < 0 \quad \text{for } e \in E(s^{\varepsilon, \delta}, r). \quad (\text{a1})$$

The  $r_i$ 's vary, the distribution  $f(\hat{e})$  has full support, and  $u$  is strictly concave. Thus,  $k > 0$  exists such that, for sufficiently small  $\varepsilon$ ,

$$\varepsilon + u(r) \cdot f(\hat{e}) \leq u(r \cdot f(\hat{e}) - k). \quad (\text{a2})$$

Note that  $s^{\varepsilon, \delta} \rightarrow r$ , and  $E(r, r) = E(r)$ . Also,  $E(\cdot, r)$  is upper hemicontinuous, and the set  $E$  of possible efforts is compact. Thus, as  $\hat{e}$  minimizes  $r \cdot f(e)$  on  $E(r)$ , we can choose  $(\varepsilon, \delta)$  sufficiently small and positive so that both (a2) and the following hold:

$$r \cdot f(\hat{e}) < r \cdot f(e) + k \quad \text{for } e \in E(s^{\varepsilon, \delta}, r). \quad (\text{a3})$$

Now, by the convexity of  $u^{-1}$ , the fact that  $A^*(r) + c(\hat{e}) = u(r) \cdot f(\hat{e})$ , and (a2),

$$\begin{aligned}
s_i^{\varepsilon, \delta} &= u^{-1} \{ \delta(\varepsilon + A^*(r) + c(\hat{e})) + (1-\delta)u(r_i) \} \\
&\leq \delta u^{-1}(\varepsilon + u(r) \cdot f(\hat{e})) + (1-\delta)u^{-1}(u(r_i)) \\
&\leq \delta(r \cdot f(\hat{e}) - k) + (1-\delta)r_i.
\end{aligned}$$

Consequently,  $(s^{\varepsilon, \delta} - r) \cdot f(e) \leq \delta \{ r \cdot f(\hat{e}) - k - r \cdot f(e) \}$ , which by (a3) is negative for  $e \in E(s^{\varepsilon, \delta}, r)$ . This proves (a1) and, hence, (ii). ■

PROOF OF LEMMA 5: Contract  $s$  and effort  $e$  satisfy,

$$A^*(s) = A(s, e) \geq A(s, e') \text{ for all } e' \neq e. \quad (\text{a4})$$

From A1, contract  $t$  exists such that,

$$A^*(t) = A(t, e) > A(t, e') \text{ for all } e' \neq e. \quad (\text{a5})$$

For any  $k$  and sufficiently small  $\delta > 0$ , a contract  $\hat{s}$  is defined by,

$$u(\hat{s}_i) = \delta(k + u(t_i)) + (1-\delta)u(s_i).$$

Since  $\hat{s} \rightarrow s$  as  $\delta \rightarrow 0$ , this  $\delta$  can be chosen small enough to fit  $\hat{s}$  into any given neighborhood of  $s$ . For  $e' \neq e$ , (a4) and (a5) imply,

$$A(\hat{s}, e) - A(\hat{s}, e') = \delta[A^*(t) - A(t, e')] + (1-\delta)[A^*(s) - A(s, e')] > 0.$$

Thus,  $E(\hat{s}) = \{e\}$ . Setting  $k = A^*(s) - A^*(t)$  proves that  $A(\hat{s}, e) = A^*(s)$ . ■

PROOF OF LEMMA 6: Because the equilibrium is efficient,  $P^* = P^{eq}$ . Therefore,

$$P^* = a_A(r)P^{eq}(r) + (1-a_A(r))\bar{P}. \quad (\text{a6})$$

Thus  $a_A(r) = 1$ , since  $\bar{P} < P^*$  and, by Lemma 1,  $P^{eq}(r) \leq P^*$ . So (a6) implies  $P^{eq}(r) = P^*$ . This in turn implies  $A^{eq}(r) = \bar{A}$ , by Lemma 1 and  $A^* = \bar{A}$ . Because the agent has the option of proposing  $r$  in  $\Gamma(r)$ ,  $A^{eq}(r) \geq A^*(r)$ . Hence,  $A^*(r) \leq \bar{A}$ .

Suppose  $A^*(r) = \bar{A}$ . Now,  $r$  is not a wage contract; if it were, it could not be renegotiated (Lemma 3), and so  $r$  would be the equilibrium contract in  $\Gamma(r)$ , contrary to the presumed inefficiency of wage contracts. So Lemma 4 can be applied to  $r$  to show that, since  $A^*(r) = \bar{A}$ , contract  $s$  exists such that  $A^*(s) > \bar{A}$ , and  $P(s, e) > P(r, e)$  for all  $e \in \bar{E}(s)$ . Criterion C requires the principal to accept  $s$  in  $\Gamma(r)$ . This implies

$A^{eq}(r) \geq A^*(s) > \bar{A}$ , contrary to  $A^{eq}(r) = \bar{A}$ . Therefore,  $A^*(r) < \bar{A}$ . ■

PROOF OF LEMMA 7: We first show that  $A^r \leq \bar{A}$ . Assume otherwise. Then  $(\hat{t}, e)$  exists such that  $A(\hat{t}, e) > \bar{A}$ ,  $P(\hat{t}, e) \geq P(r, e)$ , and  $e \in E(\hat{t})$ . Define contract  $t$  by  $u(t_i) = u(\hat{t}_i) - \varepsilon$ , where  $0 < \varepsilon < A(\hat{t}, e) - \bar{A}$ . Then  $A(t, e) > \bar{A}$ ,  $P(t, e) > P(r, e)$ , and  $e \in E(t)$ . Now  $\{e\} = E(t)$  can be assumed, by Lemma 5. Further,  $t$  can be assumed to be an incentive contract.<sup>29</sup> Define  $\hat{s}$  by

$$u(\hat{s}_i) = u(t_i) + \bar{A} - A(t, e).$$

Then  $E(\hat{s}) = E(t) = \{e\}$  and  $A^*(\hat{s}) = \bar{A}$ . Hence,  $\bar{E}(\hat{s}) = \{e\}$ . Also, as each  $\hat{s}_i < t_i$ ,  $P(\hat{s}, e) > P(r, e)$ . Since  $\bar{E}(\cdot)$  is upper hemicontinuous, a neighborhood  $N$  of  $\hat{s}$  exists such that if  $s \in N$ ,

$$P(\hat{s}, \hat{e}) > P(r, \hat{e}) \text{ for all } \hat{e} \in \bar{E}(s). \quad (\text{a7})$$

Notice that  $\hat{s}$  is an incentive contract because  $t$  is an incentive contract. So Lemma 4 implies, given that  $A^*(\hat{s}) = \bar{A}$ , the existence of  $s \in N$  such that  $A^*(s) > \bar{A}$ , and  $P(s, \hat{e}) > P(\hat{s}, \hat{e})$  for all  $\hat{e} \in \bar{E}(s)$ . Because of the latter, in view of (a7),

$$P(s, \hat{e}) > P(r, \hat{e}) \text{ for all } \hat{e} \in \bar{E}(s).$$

Criterion C thus requires the principal to accept  $s$ . This yields a contradiction,  $A^{eq}(r) \geq A^*(s) > \bar{A}$ . Therefore,  $A^r \leq \bar{A}$ .

Let  $a = a_P(r, s)$ . As  $A^{eq}(r) > A^*(r)$ ,  $s \neq r$  and  $a > 0$ . The principal's beliefs about the agent's effort conditional on  $(r, s)$  are correct, since  $(r, s)$  is on the equilibrium path. Thus, since the principal finds  $s$  acceptable,  $e$  exists such that  $(s, e)$  is an equilibrium choice of the agent, and  $P(s, e) \geq P(r, e)$ . Henceforth,  $e$  refers to this effort.

Assume  $a < 1$ . Then, because

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<sup>29</sup> Since  $E(\cdot)$  is upper hemicontinuous and  $\{e\} = E(t)$ , a neighborhood  $N$  of  $t$  exists such that for all  $s \in N$  and  $d \in E(s)$ ,  $A(s, d) > \bar{A}$  and  $P(s, d) > P(r, d)$ . Let  $s$  be an incentive contract in  $N$ , and let  $d \in E(s)$ , and replace  $(t, e)$  by  $(s, d)$ . Then, again by Lemma 5,  $\{d\} = E(s)$  can be assumed.

$$e \in \operatorname{argmax}_{d \in E} aA(s,d) + (1-a)A(r,d),$$

Lemma 2 implies the existence of  $t$  such that  $e \in E(t)$ ,

$$A(t,e) > aA(s,e) + (1-a)A(r,e), \text{ and}$$

$$P(t,e) > aP(s,e) + (1-a)P(r,e).$$

The right side of the first inequality is  $A^{eq}(r)$ , and so  $A(t,e) > A^{eq}(r) = \bar{A}$ . Since  $P(s,e) \geq P(r,e)$ , the second inequality implies  $P(t,e) > P(r,e)$ . Thus,  $(t,e)$  is feasible for  $(A - r)$ . This is contrary to  $A(t,e) > \bar{A}$  and the previous paragraph's conclusion that  $\bar{A} \geq A^r$ . Therefore,  $a = 1$ .

It remains to show  $A^r \geq \bar{A}$ . Since  $P(s,e) \geq P(r,e)$ ,  $(s,e)$  satisfies AC. Since the agent knows (in equilibrium) that  $s$  will be the final contract,  $(s,e)$  satisfies IC. So  $(s,e)$  is feasible for  $(A - r)$ , which implies  $A^r \geq A(s,e) = A^{eq}(r) = \bar{A}$ . ■



## Appendix B

Shown here is the existence of solutions to programs (P) and (A- $r$ ), and then of an equilibrium. The Specific Environment assumptions are made, except for A1, and the assumption that  $R(e)$  takes the form  $\pi \cdot f(e)$  (but  $R$  is assumed to be continuous). (Grossman and Hart (1983) also show that (P) has a solution; the proof here is more elementary, not relying on Bertsekas (1974).)

The first program is:

$$\begin{aligned} \text{(P)} \quad & \text{maximize } P(s,e) \text{ such that} \\ & s \in S, e \in E \\ \text{(IC)} \quad & e \in E(s) \text{ and } \text{(IR)} \quad A(s,e) \geq \bar{A}. \end{aligned}$$

Since each  $E(s)$  is nonempty, and the wage contract paying  $\bar{y}$  necessarily gives the agent a payoff greater than  $\bar{A}$ , the constraint set is nonempty. Thus,  $\gamma' < \infty$  exists such that the constraint  $P(s,e) \geq \gamma'$  can be added to define a new program that has the same solution set. Call the new program (PP).

The second program is, for a given  $r \in S$ :

$$\begin{aligned} \text{(A-} r \text{)} \quad & \text{maximize } A(s,e) \text{ such that} \\ & s \in S, e \in E \\ \text{(IC)} \quad & e \in E(s) \text{ and } \text{(AC)} \quad P(s,e) \geq P(r,e), \end{aligned}$$

The constraint set is nonempty: for any  $e \in E(r)$ , it contains  $(r,e)$ . So  $\alpha' < \infty$  exists such that the constraint  $A(s,e) \geq \alpha'$  can be added to define a new program that has the same solution set. Call the new program (AA- $r$ ). On its constraint set,  $P(s,e)$  is bounded below by the finite minimum of  $P(r, \cdot)$  on the compact  $E$ .

To show that (PP) and (AA- $r$ ) have solutions, we show that their constraint sets are bounded. If  $(s,e)$  is in either set, then  $A(s,e) \geq \alpha'$  and  $P(s,e) \geq \gamma'$ . Let  $\alpha = \alpha' + \min\{c(e) \mid e \in E\}$ , and let  $\gamma = \max\{R(e) \mid e \in E\} - \gamma'$ . Then, a point in either constraint set satisfies,

$$\alpha \leq u(s) \cdot f(e), \tag{b1}$$

and

$$s \cdot f(e) \leq \gamma. \tag{b2}$$

The following lemma implies that (PP) and (AA- $r$ ) have solutions.

**LEMMA B:** *Any sequence  $\{(s^k, e^k)\}$  in  $S \times E$  satisfying (b1) and (b2) is bounded.*

**PROOF:** As  $u$  is concave, nonlinear and increasing, there exists  $x < 0$ ,  $b = u'(x)$ ,  $z > 0$ , and  $c = u'(z)$  such that  $0 < c < b$ . Let  $a$  be a number greater than both  $u(x) - bx$  and  $u(z) - cz$ . Then the concavity of  $u$  implies it is bounded above by a function defined by  $v(y) = a + by$  if  $y \leq 0$ , and  $v(y) = a + cy$  if  $y \geq 0$ . From (b1),

$$\alpha \leq v(s^k) \cdot f(e^k). \quad (\text{b3})$$

For each  $k$ , let  $I(k) = \{i \mid s_i^k > 0\}$ . Then, using (b2),

$$\begin{aligned} v(s^k) \cdot f(e^k) &= a + b \sum_{i \notin I(k)} f_i(e^k) s_i^k + c \sum_{i \in I(k)} f_i(e^k) s_i^k \\ &\leq a + b\gamma + (c - b) \sum_{i \in I(k)} f_i(e^k) s_i^k. \end{aligned} \quad (\text{b4})$$

Let  $M^k = \max_i s_i^k$ . As  $E$  is compact, we may assume  $\{e^k\}$  converges to some  $e \in E$ .

Since  $f$  is continuous and  $f(e)$  strictly positive, a number  $\delta > 0$  exists such that  $f_i(e^k) > \delta$  for all  $i$  and large  $k$ . Then, since  $s_i^k > 0$  for  $i \in I(k)$ :

$$\sum_{i \in I(k)} f_i(e^k) s_i^k \geq \delta \sum_{i \in I(k)} s_i^k \geq \delta M^k. \quad (\text{b5})$$

Combine (b3)-(b5), recalling that  $c < b$ , to obtain

$$\alpha \leq a + b\gamma + (c - b)\delta M^k.$$

This proves  $\{M^k\}$  is bounded above, say by  $M$ . Let  $m^k = \min_i s_i^k$ . Then,

$$\alpha \leq v(s^k) \cdot f(e^k) \leq \delta v(m^k) + (1 - \delta)v(M).$$

Since  $v(y) \rightarrow -\infty$  as  $y \rightarrow -\infty$ ,  $\{m^k\}$  is bounded below. ■

**PROPOSITION B:** *An equilibrium exists in the Specific Environment.*

**PROOF:** For each  $r$ , let  $(s(r), e(r))$  solve (A- $r$ ). The strategies and beliefs defined in (a) - (d) are an equilibrium, i.e., a perfect Bayesian equilibrium satisfying Criterion C.

- (a) At the information set  $(r,s)$ , the beliefs and acceptance rule of the principal are the following. If  $s = s(r)$ , then  $\beta(e(r)|r,s) = a_P(r,s) = 1$ . If  $s \neq s(r)$  and  $e \in E(s)$  exists such that  $P(s,e) \geq P(r,e)$ , then  $\beta(e|r,s) = a_P(r,s) = 1$ . And if  $s \neq s(r)$  and no such  $e$  exists, then  $a_P(r,s) = 0$  and  $\beta(e(r)|r,s) = 1$ .

It is immediate that (a) defines a best-reply for the principal. The only point worth observing is that because  $(s(r),e(r))$  satisfies AC, accepting  $s(r)$  is optimal for the principal when she believes the effort is  $e(r)$ . Criterion C is satisfied because for each  $s$ , the support of the beliefs is in  $E(s)$ , which in turn is in  $\bar{E}(s)$  if the latter is nonempty. The beliefs are correct on the equilibrium path because:

- (b) In subgame  $\Gamma(r)$ , the agent plays  $(s(r),e(r))$ .

If the agent chooses  $(s(r),e(r))$ , he gets  $A'$ , the value of  $(A-r)$ . If he chooses another  $(s,e)$  which is accepted, then from (a) we see that  $e' \in E(s)$  exists such that  $P(s,e') \geq P(r,e')$ . So  $(s,e')$  is feasible for  $(A-r)$ , implying that  $A(s,e) \leq A(s,e') \leq A'$ . If the agent offers an  $s$  which is rejected, or if he offers  $r$ , the most he can get is  $A^*(r) \leq A'$ . So (b) defines a best reply for the agent.

- (c) The agent accepts any  $r$  for which  $A' \geq \bar{A}$ .

This is a best-reply acceptance rule for the agent, since by (a) and (b), the agent's continuation payoff after accepting  $r$  is  $A'$ .

- (d) The principal's initial offer is the optimal sales contract  $r^*$ .

The solution  $(s(r^*),e(r^*))$  of  $(A-r^*)$  gives the agent a payoff of  $A'^* = \bar{A}$ . So the agent accepts  $r^*$  by (c), and the principal gets  $P^*$ . She cannot get more than  $P^*$  by offering another  $r$  accepted by the agent, since the  $(s(r),e(r))$  then chosen by the agent is feasible for (P). Offering an  $r$  which is not accepted by the agent gives the principal  $\bar{P} < P^*$ . Thus, (d) defines a best reply for the principal. ■

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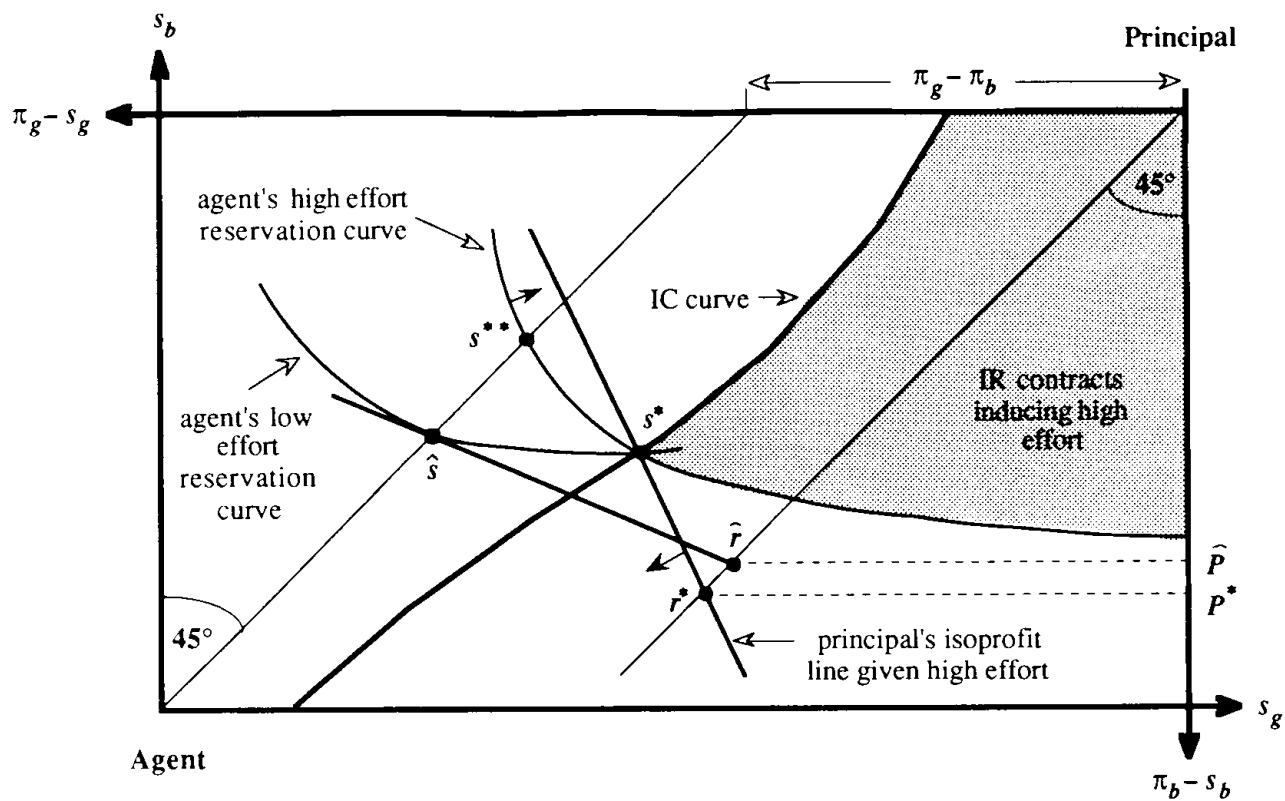


Figure 1

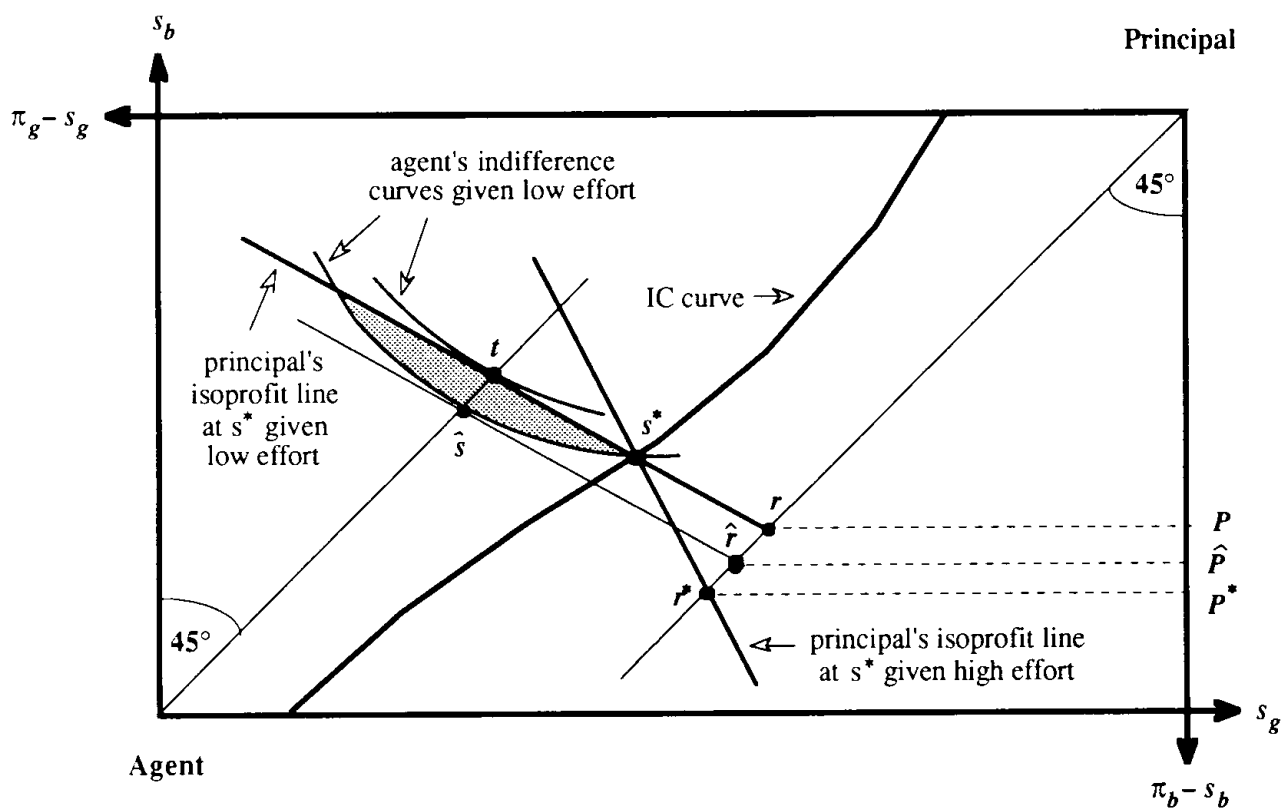


Figure 2

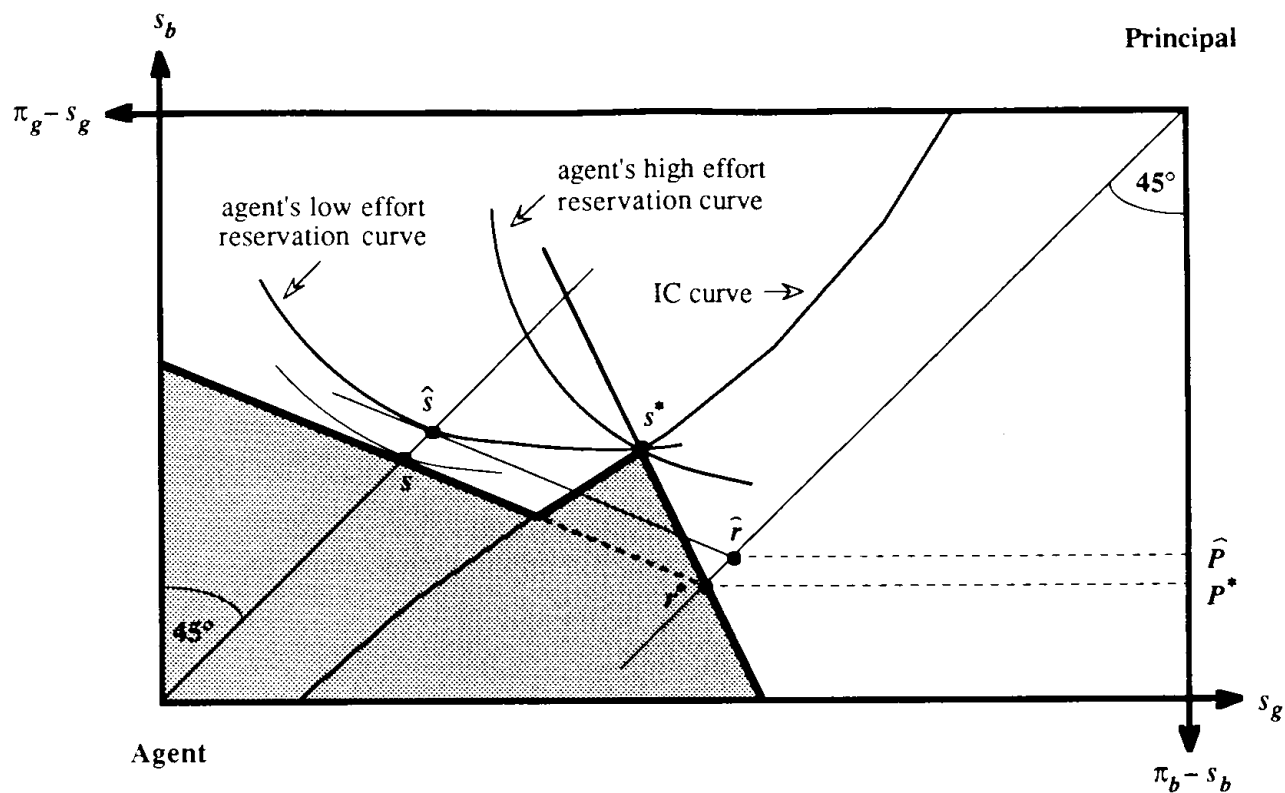
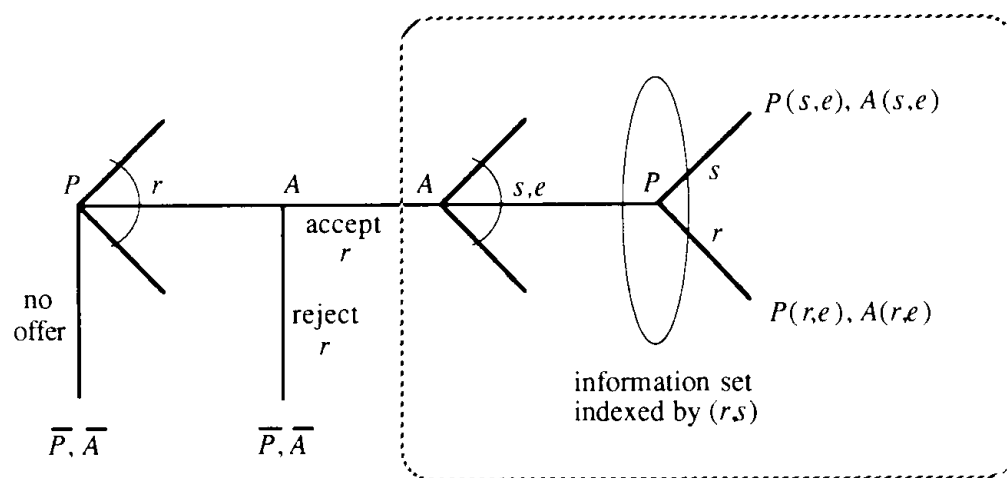


Figure 3



Renegotiation Subgame  $\Gamma(r)$

Figure 4

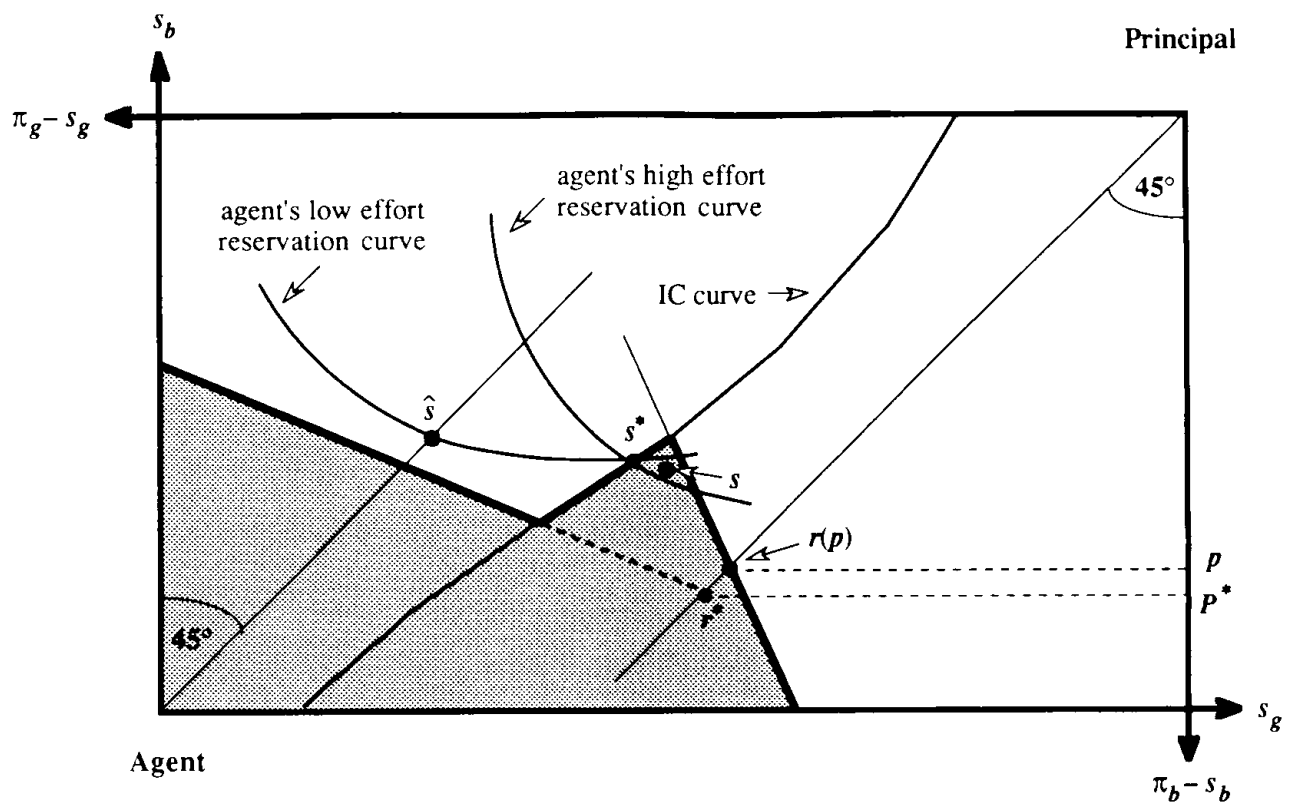


Figure 5

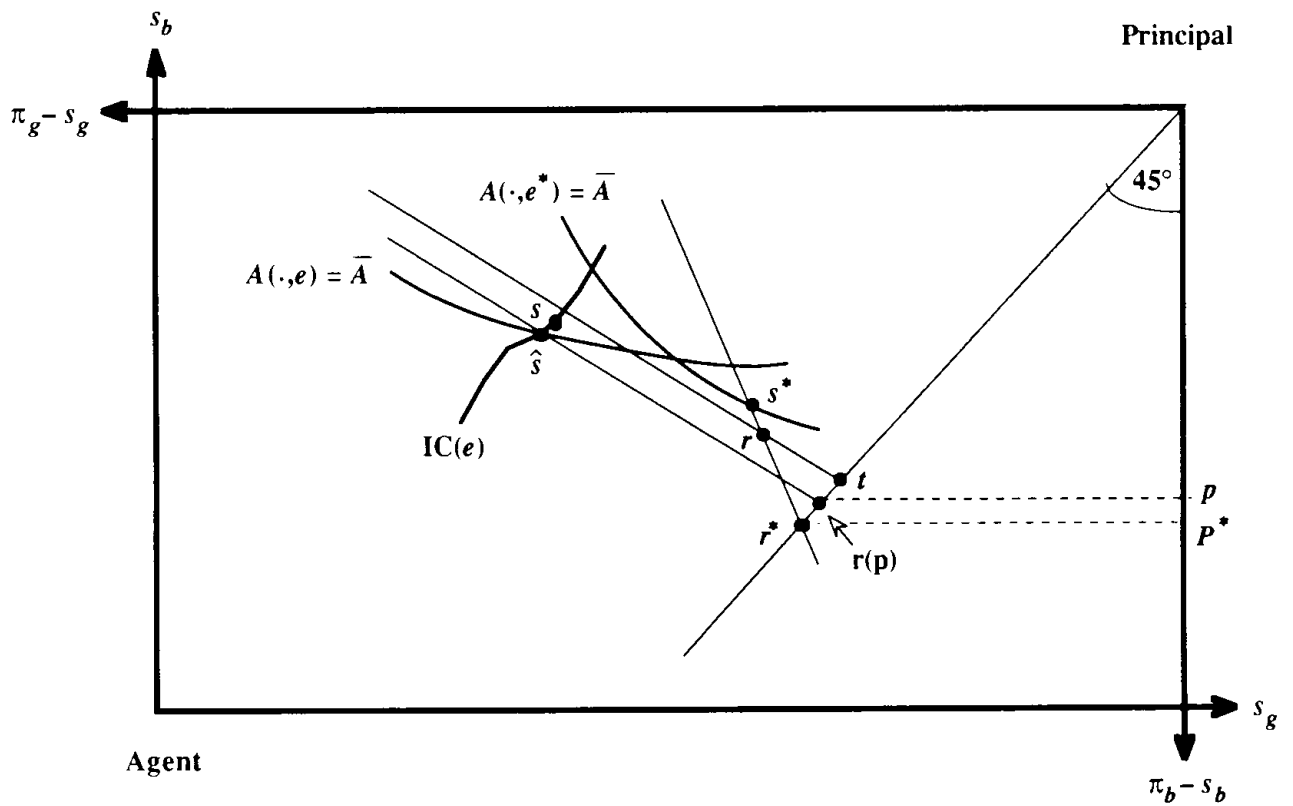


Figure 6