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INFORMATION SOURCES AND EQUILIBRIUM WAGE OUTCOMES*

by

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1 Introduction

Workers seek information from a variety of sources when searching for a job and workers of different sex and race use different search method mixes.[1] Although one expects that the method combinations actually used reflects a comparison of relative net returns as Holzer (1988) argues, the gross returns of the different information sources used is endogenous to the market--a fact that has not been fully appreciated in the theoretical literature on equilibrium wage determination in search markets. The principal purpose of this study is to derive the equilibrium effects of different mixes of the two information sources that workers most commonly use, direct application to employers and indirect contact through friends and relatives. As an implication of the analysis, we find that systematic differential in the use of and access to information sources across identifiable worker types does induce predictable differentials in equilibrium wage outcomes.

In the model considered, the law of one wage need not hold as a consequence of friction. All workers are equally productive but the distribution of wages that any particular worker is actually offered is a mixture of the distribution of wages paid across employers and the distribution of wages earned across the worker's personal contacts. The mixture is intended to reflect a combination of the two most common search methods--direct application to an employer, modelled as a random draw of an employer, and indirect referral by an employed friend or relative, modelled as a random draw of an employed contact's employer. Formally, wage information

[1] These are well known facts documented by Rees (1966), Rees and Schultz (1970), Holzer (1988), Campbell and Marsden (1988), and D. Staiger (1989).

from either source arrives at some Poison rate. Differential access to information through contacts is reflected in the fraction of offers that are obtained as a consequence of a referral from an employed friend or relative.

In the model studied, workers act optimally by moving from lower to higher paying employers as the opportunities to do so arise. This behavior together with the supposition that job-worker matches dissolve from time to time for exogenous reasons imply that there is a unique steady state distribution of wages earned associated with every offer distribution. As the numbers of employed workers per employer at each wage can be inferred from these two distributions, steady state profit per employer offering any wage can be computed for any given offer distribution. Using this fact, a one-shot wage posting game played among the employers is formulated. We show that a non-cooperative Nash solution exists and is unique given diminishing marginal productivity of labor in each employing firm. Equilibrium wage offers generally differ across employers when the fraction obtained through contacts is small enough.[2]

For any defined collection of workers, a dispersed equilibrium distribution of wages earned by employed workers always stochastically dominates the equilibrium distribution of wage offers across employers because workers only move from lower to higher paying jobs once employed. In other words, wages earned by personal contacts are more likely to be higher than wage offers obtained through direct application. We show that this fact has three important implication: (1) Given a non-degenerate distribution of

[2]The equilibrium concept is the same as that used in Burdett and Vishwanath (1988) and Burdett and Mortensen (1989). Furthermore, the model is a synthesis of those studied in these two papers.

employer offers, the steady state distribution of wages earned is stochastically increasing in the probability that any offer is from a contact. (2) When the equilibrium distribution of wages earned is non-degenerate, it is stochastically increasing in the probability that an offer is from a contact. (3) Finally, a strictly positive critical fraction exists such that for all values of the probability that an offer is from a contact less than the critical fraction the offer distributions is non-degenerate and for every value greater than or equal to the critical fraction all employers offer the competitive wage.

These three formal results have the following respective interpretations: First, even if two groups, say men and women, face the same wage offer distribution, men earn higher wages than women on average if men have greater access to contacts and if male (female) contacts are predominantly male (female). Second, even if members of the same group are equally productive but individuals do not participate in the same markets, e.g., there are "women's" and "men's" occupations, wages earned in the men's market are higher on average than the wages earned in the women's market if men have greater access to jobs through contacts. Finally, differential access to jobs through employed contact can support overt wage discrimination in a common market, at least in the special case of constant marginal productivity at the firm level.

The paper also contains a contribution to the recent and growing economic literature on the role of social networks in the labor market.[3] Montgomery (1989) develops an equilibrium model in which workers are

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[3]See Granovetter (1974) for early work on this topic.

heterogeneous in productivity and contacts provide information about the productivity of potential workers to the firm. His model implies that workers with more contacts receive higher wages, and firms hiring through contacts make higher profits. Alternatively, Staiger (1990) views jobs as experience goods and models contacts as providing up front information about match quality. The model developed in this paper suggests that personal contacts are important even when workers are identical in productivity and jobs are inspection goods. Furthermore, the implications of the model are consistent with the existing empirical findings in this literature.

2 Contacts, Wage Offers, and Earnings

Initially, we consider the case of one worker type in the sense that all have the same access to wage information, have the same opportunity cost of accepting employment, and are equally productive. Each is an expected wealth maximizing worker who repeatedly engages in search, both when unemployed and employed, and acquires information about job opportunities through both contacts and direct application. Worker mobility attributable to the wage search process and exogenous job-worker separation processes induce a steady state earnings distribution for any given distribution of wage offers. The relationship between the earnings and the offer distribution derived here is both of interest in its own right and will be needed in the equilibrium analysis that follows.

Consider a large fixed number (formally a continuum) of infinitely lived homogeneous workers and identical employing firms. Let $m > 0$ denote the measure of this set of workers and let, without loss of generality, the unit interval represent the set of employers. At any point in time, a worker is either unemployed or employed at a certain wage. Let b represent the income equivalent attributable to unemployment that must be forgone while employed, i.e., the unemployment benefit flow less any search cost. Whether unemployed or employed, information about job opportunities arrives sequentially as a Poisson process. To keep the analysis simple it is assumed that the offer arrival rate, denoted by $\lambda > 0$, is the same in both states.[4] Finally, a fraction α of the offers received are referrals from the worker's contacts. In other words, $\alpha\lambda$ and $(1 - \alpha)\lambda$ are the information arrival rates through contacts and applications respectively where α is the probability that a random arrival is a referral.

The distributions of the wages received depends on the information source in the following way. Let F(w) denote the fraction of firms offering wage rates w or less. An offer generated by direct application is viewed as a random draw from this offer distribution. Let G(w) represent the fraction of employed workers earning wage rate w or less. (This distribution is itself determined by the search process; it is derived later.) An offer received through employed contacts is regarded as a random draw from this distribution. This specification implies that referral offers are more likely to arrive from firms with greater number of employees. Specifically, the probability that a particular firm's offer is generated through contacts is equal to the ratio of number of employees at that firm to the total number employed. In contrast, an offer generated via direct application is equally likely to be from any firm. Although this distinction between the two offer distributions

[4]Different arrival rates in the two labor market states and endogenous search intensities can be considered. Doing so makes the analysis more complicated but has no important consequence for the principal results.

associated with the two search methods may be an exaggeration of reality, the qualitative results derived in the sequel continue to hold provided that indirect referral is more likely to yield higher wage offers than direct application, i.e., that the distribution generated through contacts stochastically dominates that generated through direct application.

Workers respond to each offer sequentially and instantaneously by either accepting or rejecting it and have no memory of past offers. When employed, a worker's income is the wage rate earned on the job. Jobs may differ with respect to wages offered but are otherwise identical. Any worker-job match dissolves at an exogenous rate $\delta > 0$, i.e., job duration is a random exponential variable characterized by parameter δ. A separated worker becomes unemployed. Because search conditions (the offer arrival rate, the access to contacts, and the separation rate) are independent of employment status, income maximizing workers move from unemployment to employment upon receipt of a wage offer higher than the unemployment income (b) forgone. In addition, they move from lower to higher paying jobs whenever opportunities arise.

The mobility attributable to worker search and exogenous separation induce a steady state distribution of wages earned; indeed, it is unique for any employer offer distribution. A characterization of the relationship between the two distributions, as well as of the steady state sizes of employed and unemployed worker stocks is the next task. First note that offer distributions such that $F(w) = 0$, for $w < b$, need only be considered, since employers offering less than the common reservation wage of unemployed (b) hire no workers. But then the rate at which unemployed workers become employed in equilibrium is the offer arrival rate λ because all worker accept any offer at or above the reservation wage. Consequently, the time derivative of the unemployed stock is $du(t)/dt = -\lambda u(t) + \delta(m - u(t))$, where the first term on the right side represents the flow out of unemployment (via both indirect contact and direct application) and the second term is the job separation flow into unemployment. It follows that the unique steady state measures of unemployed and employed are given by:

$$u = m/(1 + k) \quad ; (m - u) = mk/(1 + k) \tag{1}$$

where $k^{-1} = \delta/\lambda$, may be interpreted as a measure of search friction. Indeed, as $k^{-1} \to 0$, the frictional unemployment vanishes in steady state.

Given any offer distribution, the aggregate steady state labor force employed by the set of firms offering wage w or less can be derived by equating the inflow with the outflow of workers to this set. Since offers arrive at frequency λ, the distribution of wage information received is the mixture $(1-\alpha)F + \alpha G$ where α is the probability that a wage offer received is a referral from a contact, and the number of unemployed workers is u, the flow from unemployment to employment at a wage w or less is simply the product of these three factors. The outflow from this same set takes two forms: The first is the separation flow, the product of the separation rate, δ, and the number of workers employed at wage w or less, which is equal to $(m-u)G(w)$. The second is the flow of quits to higher paying jobs, which is the product of the offer arrival rate, the probability that an alternative offer exceed the specified wage w, and the number of workers employed at wage w or less. Since the aggregate inflow and outflow are equal in steady state, G must solve

$$\lambda[(1 - \alpha)F(w) + \alpha G(w)]u = \delta(m - u)G(w) +$$

$$\lambda[(1 - \alpha)(1 - F(w)) + \alpha(1 - G(w))](m - u)G(w) \quad . \tag{2}$$

Finally, (1) and (2) yield:

$$F(w) = \frac{(1 + k)G(w)}{1 + kG(w)} + \frac{\alpha k}{1 - \alpha}\,\frac{[1 - G(w)]G(w)}{1 + kG(w)} \quad . \tag{3}$$

For each F, $0 \le F \le 1$, there are two values of G that satisfy this (quadratic in G) equation. However, only one of these lies in the unit interval with one exception. If $k\alpha/(1 - \alpha) > 1$, both $G = 1$ and $G = (1 - \alpha)/\alpha k$ satisfy (3) at $F = 1$. Define

$$G(w) = h(F(w),\alpha) \tag{4}$$

such that for $0 \le F < 1$, $h(F,\alpha)$ is the unique value of G in the unit interval that satisfies (3), and $h(1,\alpha) = 1$. Equation (4) specifies the unique relationship between the earnings and offers distributions. It is easily seen that $h(0) = 0$,

$$h'(0) = 1/[1 + k + \alpha k/(1 - \alpha)] \quad , \tag{5.1}$$

$$h'(x) > 0, \text{ and } h''(x) > 0 \text{ for } 0 \le x < 1 \quad ,^{5} \tag{5.2}$$

[5]Differentiating (3), we get

$$\frac{\partial F}{\partial G} = \frac{1 + k}{(1 + kG)^2}\left[1 - \frac{\alpha k}{1 - \alpha}\,G^2\right] + \frac{\alpha k}{1 - \alpha}\,\frac{(1 - G)^2}{(1 + kG)^2} > 0$$

for $\alpha k < (1 - \alpha)$. This implies $h'(x) > 0$ for $0 \le x < 1$. Differentiating again, yields the claim about the second derivative.

$$\lim_{x \uparrow 1} h(x) = \min\,[1,\ (1-\alpha)/\alpha k],\ \text{and}\ h(1) = 1. \tag{5.3}$$

Thus, the only possible discontinuity is at $x = 1$, which occurs when $\alpha k/(1-\alpha) > 1$ or equivalently when $\alpha > 1/(1+k)$. This discontinuity is an important feature of the matching process involving worker contacts. It implies that a fraction of the employed no less than $1-(1-\alpha)/\alpha k$ must earn the highest wage in any equilibrium if the odds of receiving an offer through contacts ($\alpha/(1-\alpha)$) exceeds search friction (k^{-1}).

Figure 1 depicts the convex relationship between earnings (G) and offers (F). As the contact probability, α, increases, (3) and (4) imply that the curve of $h(F,\alpha)$ shifts down. Hence, our first result:

<u>Proposition 1</u>: Given F, G is stochastically increasing in α, i.e., $\partial h(F,\alpha)/\partial\alpha \leq 0$.

The steady state density of workers employed at wage w, denoted as $g(F(w))$, can be derived as the slope of the relation between the number of workers earning and the number of firms offering wage w or less. As the notation suggests, this "supply of labor" function depends only on the fraction of other firms offering the

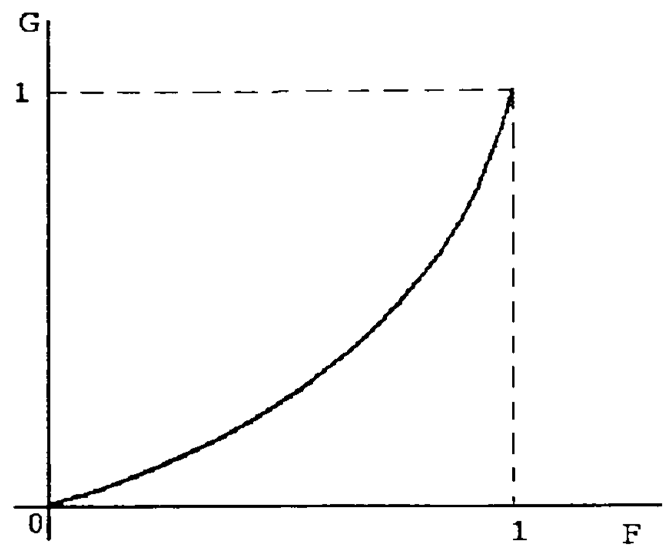

Fig 1: Wages Earned vs. Offered Distributions

same wage or less, provided that the wage offered is acceptable. This construct is needed to define market equilibrium in the next section.

Since the total number of workers employed at wage w or less is

$$(m - u)G(w) = (m - u)h(F(w)) = \int_{\underline{w}}^{w} g(F(x))dF(x)$$

where $\underline{w}$ denotes the lower support of F, equation (1) implies

$$g(F(w)) = \begin{cases} \dfrac{mk}{1+k}h'(F(w)) & \text{, } \textit{if } F(w) \textit{ is continuous at } w \geq \underline{w} \\[2ex] \dfrac{mk}{1+k}\dfrac{h(F(w)) - h(F^{-}(w))}{F(w) - F^{-}(w)} & \textit{if } F(w) \textit{ isn't continuous at } w \geq \underline{w} \\[2ex] 0 \textit{ if } w < \underline{w}. \end{cases} \tag{6.1}$$

where $F(w) - F^{-}(w)$ equals the mass of firms offering wage w. Expression (6.1) is valid whenever $h(\cdot)$ is differentiable. When $\alpha > (1 + k)^{-1}$, to reflect the discontinuity at $x = 1$, i.e., the fact that any single firm, although of zero measure, will attract a mass of workers with an offer larger than any other, the following definition of the (right) derivative is adopted:

$$h'(1) = \begin{cases} \dfrac{(1+k)}{1 - \alpha k/(1-\alpha)} & \textit{if } \alpha/(1-\alpha) < 1/k \\[2ex] \infty & \textit{otherwise.} \end{cases} \tag{6.2}$$

Finally, note that g(F(w)) also specifies the steady state labor force of a firm offering wage w outside as well as inside the support of F, when F describes the wage choices of all other firms.

3 Market Equilibrium

It is assumed that a large number of identical firms, measure normalized to unity, participate in the market. Each firm selects its wage on the basis of its expectation about the behavior of workers (as described in section 2) and the wages chosen by other firms, to maximize its steady state profit flow. Worker productivity is the same at any firm.

Let $\pi(w,n)$ represent the profit flow of a firm expressed as a function of its wage choice w and labor force n. Each firm faces the maximization problem:

$$\max_{w,n} \pi(w,n), \text{ subject to } n \leq g(F(w)).$$

The profit function is assumed to have the form:

$$\pi(w,n) = pf(n) - wn, \tag{7}$$

where $p > 0$ is the output price. By assumption, the production function is strictly concave, i.e., $f'(n) > 0$, $f''(n) < 0$, and $f(0) = 0$.

Each employer selects a wage offer that maximizes steady state profit taking as given the wage offers of the other firms, as characterized by the offer distribution, and the workers' optimal acceptance and mobility behavior. As all firms are identical, all must make the same profit in equilibrium.

<u>Definition 1</u>: A profit level π and an offer distribution F describe an equilibrium if the pair satisfies the following conditions: For all w in the support of F,

$$\pi = \pi(w,g(F(w)) \tag{8.1}$$

and

$$g(F(w)) = d(w) \equiv \underset{n}{argmax}\{\pi(w,n) \mid n \leq g(F(w))\} \text{ if } w > b. \tag{8.2}$$

Otherwise (for w outside of support of F),

$$\pi > \max_{n}\{\pi(w,n) \mid n \leq g(F(w))\}. \tag{8.3}$$

In other words, all firms receive the same profit in equilibrium, the profit maximizing employment level given the wage, the demand d(w), equals the available labor supply at every strictly acceptable wage in the support of F, and there is no incentive for any firm to deviate by offering a wage outside the support.

Given any offer distribution F, the total labor force of a firm paying wage w, g(F(w)), is increasing in the wage offered and jumps up at discontinuity points of F by virtue of (5.2) and (6). Specifically, strict convexity of h(F) together with (6) and (7) imply that if F(•) has a mass point at w, then

$$\lim_{\varepsilon \downarrow 0} g(F(w-\varepsilon)) < g(F(w)) < \lim_{\varepsilon \downarrow 0} g(F(w+\varepsilon)) \quad . \tag{9}$$

This property has the following consequence for the equilibrium offer distribution.

Lemma 1: An equilibrium offer distribution F cannot have a mass point at wage w, if the profit function $\pi(w, n)$ is continuous in both arguments, and strictly increasing in n at n = g(w, F).

Proof: Let $\pi^* = \pi(w,g(F(w)))$ be the profit of a firm offering wage w. From (9) and the hypothesis

$$\pi^* < \pi(w, \lim_{\varepsilon \downarrow 0} \{g(F(w + \varepsilon))\}) = \lim_{\varepsilon \downarrow 0} \{\pi(w + \varepsilon, g(F(w + \varepsilon)))\} \tag{10}$$

if w is a mass point of F. The first step is implied by (9) and the hypothesis that profit is strictly increasing in n at $n = g(F(w))$ and the second step follows from continuity of $\pi(w,n)$. As (10) violates (8.3), F cannot be an equilibrium. Specifically, a deviant firm can earn a larger profit by offering a wage only slightly larger than w. □

As a consequence of Lemma 1, mass points can only occur at the upper support of the distribution and at such a mass point the wage is equal to the value of marginal product, i.e., $\partial\pi/\partial n = 0$. Moreover, if there is dispersion, the lowest wage is the reservation wage because only at the reservation wage is the supply at the lowest wage offer sensitive to its value. To establish this claim some notation is needed

Let

$$\omega(n, \pi) = (pf(n) - \pi)/n \tag{11.1}$$

and

$$\omega^0(\pi) = \max_n \{\omega(n, \pi)\} \tag{11.2}$$

respectively represent the isoprofit curve and the Marshallian wage associated with a given profit π. The latter is the unique wage, associated with π, on the labor demand curve, $w = d^{-1}(n)$.

Furthermore, for any given π, the isoprofit curve defined by $w = \omega(n,\pi)$ is increasing in the size of the labor force from zero up to the number of workers a competitive employer would demand were the given wage $w = \omega^0(\pi)$, denoted as $n^0(\pi)$ in Figure 2. The relationship between the iso-profit curve, labeled II, and the demand curve, labeled DD are illustrated in the figure.

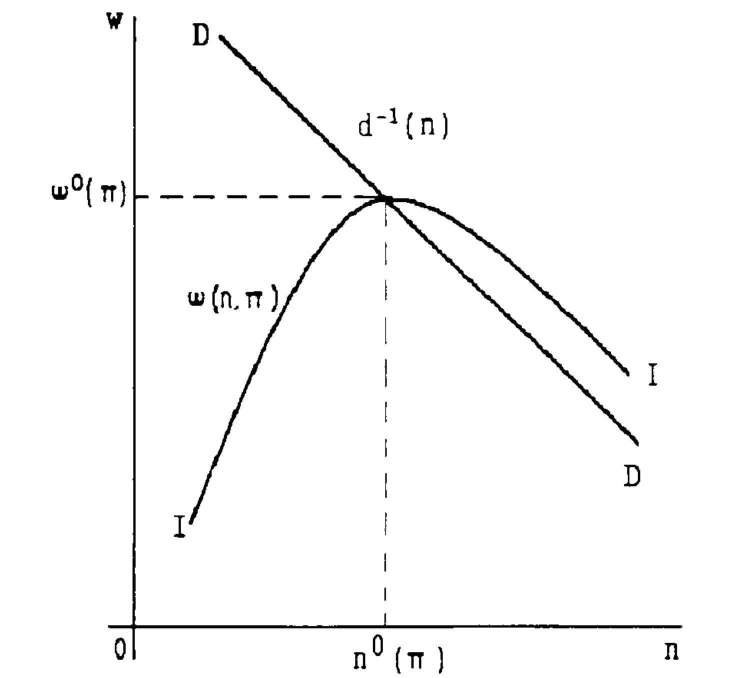

Fig 2 : Iso-profit and Demand Curves

Lemma 2: If (π, F) is an equilibrium pair and F is non-degenerate, then (i) the upper support $\bar{w}$ of F is no greater than $\omega^0(\pi)$; (ii) the lower support $\underline{w}$ of F equals b; and (iii) F is continuous except possibly at the upper support and then only when the upper support $\bar{w}$ is equal to $\omega^0(\pi)$ if f(n) is continuous.

Proof: Claim (i) is trivial as $\omega^0(\pi)$ is the highest wage on the equilibrium iso-profit curve defined by π by definition. To prove (ii), suppose $\underline{w} > b$. Then, the non-degeneracy of F implies $F(\underline{w}) = 0$ due to the following argument. Lemma 1 and (8.2) imply that F can have a mass point at w, only if $d(w) = g(F(w))$ or equivalently if $w = \omega^0(\pi)$. Thus if $F(\underline{w}) > 0$, then $\underline{w} = \omega^0(\pi) \geq \bar{w}$, which cannot occur if F is non-degenerate. Now, from (6.1) it follows that $g(F(w)) = g(0) = mkh'(0)/(1+k)$ for $b \leq w \leq \underline{w}$. Consequently, if $b < \underline{w}$, then a firm by decreasing its wage from $\underline{w}$ can strictly increase its profits as its labor force remains the same which violates the equilibrium condition (8.3). Part (iii) is a direct implication of Lemma 1 and equation (7) under the hypothesis. □

A degenerate equilibrium is competitive in the following sense.

Definition 2: A competitive equilibrium is a profit-wage pair (π^*,w^*) such that

$$\pi^* = \max_n \pi(w^*,n) \quad , \tag{12.1}$$

and $$w^* = \max\{b,\ d^{-1}(mk/(1 + k))\} \quad . \tag{12.2}$$

At a competitive equilibrium, all firms have the same labor force size, wage equals marginal product, and there is no earnings differential across employed workers. If $w^* > b$, then the labor demand of each firm equals the steady state labor supply per firm, equal to $g(1) = mk/(1 + k)$ by virtue of (4) and (6.2). If $mk/(1 + k) \geq d(b)$, then $w^* = b$ since $g(F(w)) = 0$ at any lower wage. Because $d'(w) < 0$, the competitive equilibrium is unique.

Lemma 3: If an equilibrium (π,F) is degenerate at w, then (π,w) is a competitive equilibrium given $f(n)$ concave.

Proof: As $\pi(w,n) = pf(n) - wn$ is concave in n and maximal at $n = d(w)$ given w, $\pi(w,n)$ is increasing (decreasing) in n as $n <$ (>) $d(w)$. Hence, (12.1) holds. Because all firms offer the same wage w, the labor force per firm is $g(F(w)) = g(1) = mk/(1+k)$ if $w \geq b$ and zero otherwise by virtue of (4) and (6.2). Hence, if $w > b$, then (8.2) and Lemma 1 imply that the equilibrium level of employment per firm is equal to demand, i.e., $mk/(1+k) = d(w)$. Analogously, if $w = b$, then $n = d(b) \leq mk/(1+k)$. Hence, (12.2) holds. □

With these properties in mind, we proceed to establish that a unique market equilibrium exists and to specify the conditions that determine its characteristics. To gain an intuitive insight, suppose all firms offer the reservation wage b. If the common steady state labor force, which is $g(1) = mk/(1+k)$ by virtue of (5.3) and (6.1), equals or exceeds the labor demand at $w = b$, $d(b)$, then no firm has an incentive to deviate from this wage--a lower offer draws no workers and supply, $g(1)$, exceeds demand, $d(w)$, at any higher wage. Thus, a market equilibrium is the competitive equilibrium with $w^* = b$ in this case.

On the other hand, if $d(b) > g(1) = mk/(1 + k)$, a firm raising its wage can increase its profits by virtue of Lemma 1. The firm posting a wage b loses some of its labor force to other higher paying firms (note Lemma 1 implies $F(b) = 0$ in this context), but retains a steady state labor force equal to

$$g_0 \equiv g(F(b)) = g(0) = \frac{mk}{1+k} h'(0) > 0 \tag{13}$$

and has an associated profit

$$\pi_b = \pi(b, g^0). \tag{14}$$

These observations suggest the possibility of a non-degenerate equilibrium offer distribution with the properties specified in Lemma 2 and equilibrium profit equal to π_b. In the following, it will be shown that if $\pi_b > \pi^*$ then the market equilibrium is characterized by a unique non-degenerate offer distribution, and if $\pi_b \leq \pi^*$, the competitive equilibrium is the unique solution to this wage posting game.

Let $n^0(\pi) = d(\omega^0(\pi))$ and let $n(w;\pi)$ represent the positively sloped branch of the inverse of the isoprofit curve defined by $w = \omega^0(n; \pi)$.

Theorem 1: If $\pi_b > \pi^*$, then the unique solution to the market equilibrium is characterized by: (i) equilibrium profit equal to π_b; (ii) the smallest offer $\underline{w} = b$ and (iii):

(a) A continuous offer distribution F(w) defined by

$$F(w) = g^{-1}(n(w; \pi_b)), \tag{15}$$

for all $w \in [\underline{w}, \overline{w}]$, with upper support defined by $\pi_b = \pi(\overline{w}, g(1))$ if $n^0(\pi_b) \geq g(1)$.

(b) A continuous offer distribution defined by (15) for all $w \in [\underline{w}, \hat{w}]$, where $\hat{w}$ is the second highest wage defined by

$$\pi_b = \pi(\hat{w}, g(\hat{F})), \tag{16}$$

with a measure $(1 - \hat{F})$ of firms offering the highest wage $\overline{w} = w^0(\pi_b)$ where $\hat{F}$ is the solution to the equation

$$\frac{mk}{1 + k}[1 - h(\hat{F})] = (1 - \hat{F})n^0(\pi_b) \tag{17}$$

if $n^0(\pi_b) < g(1)$.

Proof: First, we rule out degenerate equilibrium offer distributions. If F is degenerate, then it must be the unique competitive equilibrium by Lemma 3. If this is the case, then a firm deviating from w^* and offering a wage b, gets a labor force equal to g_0 and strictly increases its profits as $\pi_b > \pi^*$. Hence equilibrium F must be dispersed and parts (i) and (ii) follow from (8.1) and Lemma 2.

To prove (iii)(a), note that $n^0(\pi_b) \geq g(F)$ for $0 \leq F \leq 1$ follows from (5.2) and (6). Hence, an equilibrium F cannot have a mass point by virtue of

Lemmas 1 and 2. Equation (15) is obtained as the unique solution to the equal profits condition (8.1) (satisfying (8.2))

$$\pi_b = \pi(w, g(F(w)) = pf(g(F(w)) - wg(F(w)) \quad .$$

The equation above also characterizes the upper support $\bar{w} = [pf(g(1)) - \pi_b]/g(1)$. Under the conditions specified in (iii) (a), the support is connected, indeed equal to the interval $[\underline{w}, \bar{w}]$, because any larger offer yields no more workers for an employer of measure zero.

To prove (iii) (b), first note that the solution to $\pi_b = \pi(\bar{w}, g(1))$ for $\bar{w}$ is less than $\omega^0(\pi_b)$ and is located on the downward sloping branch of the iso-profit curve when $g(1) > n^0(\pi_b)$. But, then $g(1) > d(\bar{w})$ which violates equilibrium condition (8.2). Consequently, $\bar{w} = \omega^0(\pi_b)$ and a strictly positive measure of firms must also be offering this highest wage. Equation (17) defines the measure of firms offering the highest wage such that the profit of each is equal π_b. That this equation has a unique solution $0 < \hat{F} < 1$ follows from property (5). The argument to show the uniqueness of distribution (16) in the interval $[\underline{w}, \hat{w}]$ is the same as that used for (15). The equilibrium offer distribution, thus, is continuous in the interval $[\underline{w}, \hat{w}]$, is constant on $[\hat{w}, \bar{w}]$, and has a jump equal to $1 - \hat{F}$ at the highest wage $\bar{w}$. No firm has an incentive to increase its wage to w where $\hat{w} < w < \bar{w}$, as the labor force does not increase due to the fact that $g(F(\hat{w})) = g(F(w))$. Hence, (iii) (b) is proved. □

A graphical sketch of the unique equilibrium for cases (a) and (b) are illustrated in two four-quadrant diagrams, Figure 3a and Figure 3b respectively. Quadrant II contains the steady state relation between the distributions of earned and offered wages, h(F), introduced in Figure 1 earlier. The steady state number of workers available to an employer offering wage w, a function proportional to the slope of the wage earned and offer relation in quadrant II, is drawn in quadrant III. Quadrant IV illustrates the iso-profit curve given the profit level determined by the lowest wage offer, $w = b$, and the available labor supply at the lowest offer, $n = g(0)$, denoted as π_b. Finally, the equilibrium distribution of wages earned determined by this relations is illustrated in quadrant I.

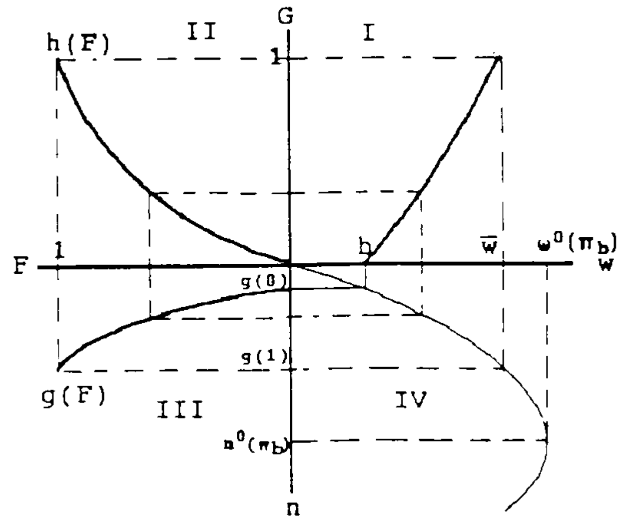

Fig 3a: Equilibrium Distributions

The curve of the equilibrium cumulative distribution of earnings can be derived graphically as follows: In case (a), every wage employment pair (w,n) on the upward sloping portion of this iso-profit curve is an equilibrium wage offer and an associated labor force size. As the labor force size must equal to the available supply at the wage offered and the latter depends on the value of the offer c.d.f., the associated equilibrium value of g(F(w)), F(w), and G(w) are found completing the rectangle determined by any such (w,n), the g(F) curve, and the h(G) curve. The dashed rectangles in Figure 3(a) and 3(b) are illustrative examples. The difference in case (b) is that only $w = w^0(\pi_b)$ and wage offers between b and $\hat{w}$ are in the support of the equilibrium offer distribution. In other words, both F and G are flat between the highest and second highest wage offer and a mass of workers are employed at the highest.

If the production function is assumed to be constant returns to scale (i.e., $f'(n) = 1$), then equilibrium offers are always dispersed. (Note the iso-profit line in Figure 2, and quadrant IV of Figure 3(a), will be strictly increasing in n; and under the condition of theorem 3(b), a single firm will employ a mass of workers. Details of such an equilibrium are omitted here.)

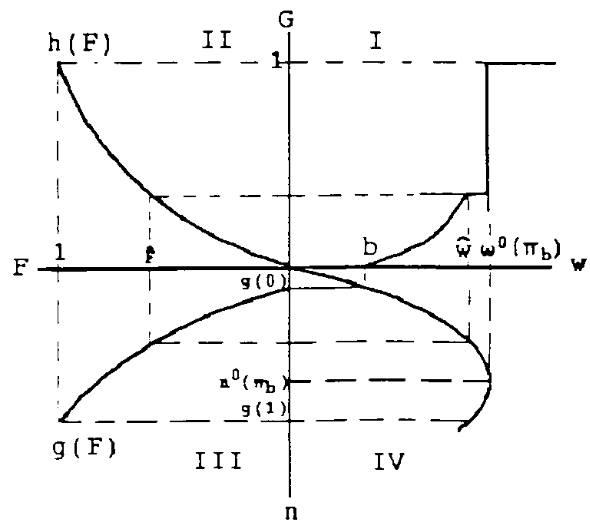

Fig 3b: Equilibrium

<u>Theorem 2 (a)</u>: If $\pi_b \le \pi^*$, then the unique market equilibrium is the competitive equilibrium (π^*, w^*).

2 (b): Furthermore, there exists an α^*, $0 < \alpha^* < 1$, such that for all $\alpha \ge \alpha^*$ competitive equilibrium is the only solution to the wage posting game.

<u>Proof</u>: (a) First, no employer has an incentive to deviate from w^* given all others offer w^* and receive profit π^* when $\pi^* \ge \pi_b$. Thus (π^*, w^*) is an equilibrium. As there is no other degenerate equilibrium by Lemma 3, it is enough to show that a dispersed equilibrium does not exist.

Suppose the equilibrium F is non-degenerate. It follows from Lemma 2 and (8.1) that (π_b, F) is the equilibrium pair. Let $\bar{w}$ denote the upper support of F. If $\bar{w} = \omega^0(\pi_b)$, then because $\pi_b \le \pi^*$ implies

$$d(\omega^0(\pi_b)) \leq mk/(1 + k) < h'(1)mk/(1 + k) = g(F(\bar{w})) \quad ,$$

it follows that for firms offering the highest wage, the labor supply (see (7)) is strictly greater than the demand i.e., equilibrium condition (8.2) is violated. Now suppose $\bar{w} < \omega^0(\pi_b)$ and (8.2) is satisfied with $d(\bar{w}) \geq g(F(\bar{w}))$. Then, as in (18)

$$n(\bar{w};\ \pi_b) < d(\omega^0(\pi_b)) < g(F(\bar{w}) \leq d(\bar{w}) \quad ,$$

and a firm that offers the highest wage has a profit level strictly greater than π_b, violating equal profits condition (8.1). Hence equilibrium offers must be degenerate.

(b) The labor force of a firm paying b, which equals

$$g_0 = \frac{mk}{1 + k} \frac{1}{[1 + k + \alpha k/(1 - \alpha)]} \quad , \tag{19}$$

is continuous and strictly decreasing in α. Then, $\pi_b \equiv \pi(b,\ g_0(b))$ is continuous and strictly decreasing in α. As $\alpha \to 1$, it is evident that $g_0 < (m - u)$, and hence $\pi_b < \pi^*$ in this limit. Thus, given all other parameters, there exists an $\alpha^* < 1$ such that for all $\alpha \geq \alpha^*$, $\pi_b \leq \pi^*$, and hence part (b) of the theorem. Note that $\alpha^* = 0$ only if $w^* = b$.

In Proposition 1, we established that the earnings distribution increases with the probability that an offer is from a contact given the offer distribution, a partial equilibrium result. The following is the comparative equilibrium analogue:

Proposition 2: If (π, F) is a non-degenerate equilibrium associated with a given value of α, the associated equilibrium earnings distribution $G = h(F,\alpha)$

is strictly stochastically increasing in G, i.e., $\partial G(w)/\partial\alpha = \partial h/\partial\alpha + (\partial h/\partial F)(\partial F(w)/\partial\alpha) < 0$ for all w less than the upper support.

<u>Proof</u>: For the purpose, define

$$\ell(G,\alpha) = \frac{\partial h(F)}{\partial F}(m-u) = \frac{[1+kG]^2}{1+k+k[\alpha/(1-\alpha)][(1-G)^2-(1+k)G^2]}\cdot\frac{mk}{1+k} \tag{20}$$

using equations (3) and (4). If follows that $g(F,\alpha) = \ell(h(F,\alpha),\alpha)$ by virtue of (6.1) at any non-mass point of F. Consequently, if w is a non-mass point in the support of F,

$$\pi_b = \pi(b,\ell(0,\alpha)) = \pi(w,\ell(G(w),\alpha)) \quad . \tag{21}$$

As $\pi(w,n) = pf(n) - wn$, differentiation of (21) with respect to α yields

$$\frac{\partial\ell(G,\alpha)}{\partial G}\,\frac{\partial G}{\partial\alpha} = \frac{pf'(\ell(0,\alpha)) - b}{pf'(\ell(G(w),\alpha) - w}\,\frac{\partial\ell(0,\alpha)}{\partial\alpha} - \frac{\partial\ell(G,\alpha)}{\partial\alpha} \quad . \tag{22}$$

The fact that $(b,\ell(0,\alpha)) < (w,\ell(G(w),\alpha))$ and that both are on the same indifference curve implies

$$[pf'(\ell(0,\alpha))-b]\ell(0,\alpha) > [pf'(\ell(G(w),\alpha)-w]\ell(G(w),\alpha) > 0 \tag{23}$$

since $d([pf'(n)-w(n)]n)/dn = npf''(n) < 0$ given that $w(n)$ is defined by $pf(n)-w(n)n = \pi$, a constant and $f'(n)$ is decreasing. Since (20) implies $\ell(G,\alpha)$ increasing in G and decreasing in α, the claim holds when $\ell(G,\alpha)$ is increasing in α. When not, the following implication of (20) and $1 \geq G(w) > 0$ completes the proof:

$$\frac{-\partial \ln(\ell(G,\alpha))}{\partial \alpha} = \frac{k[(1-G)^2 - (1+k)G^2]}{1 + k + k[\alpha/(1-\alpha)][(1-G)^2 - (1+k)G^2]} \frac{\partial(\alpha/(1-\alpha))}{\partial \alpha} \tag{24}$$

$$< \frac{k}{1 + k + k[\alpha/(1-\alpha)]} \frac{\partial(\alpha/(1-\alpha))}{\partial \alpha} = \frac{-\partial \ln(\ell(0,\alpha))}{\partial \alpha} . \qquad \square$$

The intuition behind the above proposition is that if the contact probability, α, increases, lower paying firms lose workers to those employers posting higher wage rates (i.e., $\ell(G,\alpha)$ is decreasing in α for lower values of G and increasing for higher values; see proof of the proposition). Consequently, some lower paying firms will have to increase their wages so as to satisfy the equal profits condition, thus inducing a stochastically higher equilibrium earnings distribution.

With regard to the dependence of the equilibrium outcomes on other parameters of the model considered, if the nonemployment income b falls (all other parameters held fixed), then the equilibrium profit π_b falls (in Figure 3, the iso-profit curve in the fourth quadrant shifts to the right); and this generates stochastically higher earnings and offer distributions. (This is evident from inspection of Figure 3.) The effect on earnings and offers is similar if the labor demand (due to increase in output price p or production function shifts) is higher. An increase in the measure m (workers/firm ratio) increases the equilibrium profit π_b. It also increases the labor force at each firm (percent increase being the same; note the linearity in (20.1) which implies $\partial \ln \ell(G)/\partial m = 1/m$ for all G). Following the arguments used to prove proposition 2, it is straightforward to show that $\partial G/\partial m \geq 0$, i.e., equilibrium earnings distribution is stochastically smaller when the worker/firm ratio increases. Because the earnings-offer relationship is not affected, we can conclude that the equilibrium offers are also stochastically smaller. An increase in the information rate k (or reduction in search friction) produces opposing effects: first, it increases the steady state stock of employed (similar to the effect of increasing m), and second, it increase the labor supply of employers paying higher wage rates relative to those who pay less (similar to the effect of increase of α). The net effect is ambiguous. An increase in the minimum wage (not explicitly considered in the model here) has the same effect as an increase in nonemployment income b.

4 Conclusions: Differential Contacts and Outcomes

Although common access to information for all is assumed above, the results are easily extended to allow for differential access through contacts provided that any individual worker's contacts are of the same type;[6] e.g., sex, race etc. Let m_i and α_i respectively represent the measure of workers and the probability that an offer is from a contact for type $i = 1,2,...$ For comparison, assume that all workers are otherwise identical, i.e., have the same opportunity cost of employment b, offer arrival rate λ, and separation rate δ, and are equally productive.

First, suppose that the firms do not discriminate on the basis of type, i.e. F is the common offer distribution. Let $G_i(w)$ denote the steady state earnings distribution for group $i = 1, 2$. Under the assumptions made, (in particular that a worker's employed contacts are of his or her own type), the

[6]Empirical evidence suggests that contacts are overwhelmingly within identifiable types. See Holzer (1988), Staiger (1990).

relationship between earnings and a common offer distribution, given by (3) and (4), holds for each type. Let

$$h_i(F) \equiv h(F;\ \alpha_i) \tag{25}$$

be as defined by (4) for $\alpha = \alpha_i$. As a corollary of Proposition 1, it follows that a type with greater access earns higher wages on average. Formally, if $\alpha_1 > \alpha_2$, then G_1 stochastically dominates G_2. In particular,

$$G_2(w) = h(F(w),\alpha_1) \leq h(F(w),\alpha_2) = G_2(w) \quad , \tag{26}$$

with strict inequality if $0 < F(w) < 1$.

Generalizing the equilibrium results to many types is a straightforward task. Simply note that theorems 1 and 2 hold given the generalized definition

$$g(F(w) = \sum_i g_i(F(w)) \quad , \tag{27}$$

where the steady state density of workers, g_i, for type $i = 1, 2$ is defined by (6) for all i, with $h_i(F)$ replacing $h(F)$ and m_i and α_i respectively replacing m and α. If the equilibrium wages are nondegenerate, then, given employers to not discriminate between types in the wages they offer, the equilibrium earnings of those with higher access to contacts will be (stochastically) higher (in the sense of (26)). However, in such a market of mixed types, whether an increase in the contacts probability of one type (stochastically) increases the earnings of that type and the cross-effects on other types remains to be solved (analogue of proposition 2 for several types).[7]

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[7]The principal difficulty here is due to the nonlinearity of the labor supply function (see equation (20)). In simpler models, e.g., no offers via contacts during employee search, it can be

Nevertheless, if the contact probability of any one type is sufficiently high, theorem 2 does apply, and competitive equilibrium (in the sense of definition 2) results.

However, if there are two separate non-competing markets with one type in each, then again greater access to contacts in one market than in the another implies greater earnings on average in the former even when workers in the two are equally productive. In this case, $\alpha_1 > \alpha_2$, implies

$$G_1(w) = h(F_1(w),\alpha_1) \leq h(F_2(w),\alpha_2) = G_2(w) \quad , \tag{28}$$

with strict inequality if $0 < F_2(w) < 1$. This case might apply when comparing earnings in "men's" occupations with those in "women's."

Now consider a market with mixed types with marginal productivity constant (i.e., $f'(n) = 1$). Given that employers are permitted to discriminate with respect to wage offers, the equal profit condition implies equal profit by type. Hence, in this case, greater access to contacts generates higher earnings again in the sense of (28). Although it seems likely that employers also offer higher wages to the type with greater access to contacts in both the case of non-competing markets and a common market with different types, at least under constant returns, we have not yet been able to establish that F_1 dominates F_2.[8]

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shown that increasing the contact probability of one type has a positive effect on earnings of all types.

[8]Again, if we consider a simpler model which is similar to that considered here except that all offers in employee search are via direct applications, then one can show that F_1 stochastically dominates F_2.

The model also has several other empirically relevant implications. First, upon transition from unemployment to employment, the wages are higher an average for those who obtained jobs via contacts than via direct application. Second, for those with higher access to contacts post-employment wages, and wage growth during job-job transitions are also higher. Finally, turnover in jobs filled through contacts is less (as they are more likely to be higher wage jobs).

In summary, the paper is a contribution to the growing literature on the role of social networks, personal contacts, and search methods on labor market outcomes. Considering a reasonably simple model, we have analyzed the influence of a number of these aspects on equilibrium wage outcomes. Extending the model to incorporate other heterogeneities (e.g., offer arrival and turnover rates, worker productivity differentials, nonidentical firms, etc.) will perhaps be helpful in generating other useful results.

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