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Imperfect Competition, Foreign Trade, and the Multipliers:
Machlup-Metzler Fifty Years Later

By

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Abstract

This paper constructs a general equilibrium model of imperfect competition and international trade and investigates the international transmissions of country specific aggregate demand shocks, such as changes in taste and fiscal policies. The impacts of coordinated fiscal expansion and of transfer payments are also discussed. The model has some curious resemblance with earlier Keynesian models of open economies, notably of the Machlup-Metzler variety, which emphasize the role of foreign trade multipliers in income determination in interdependent national economies.

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1. **Introduction**

This paper constructs a simple general equilibrium model of foreign trade with imperfect competition. The model is used to examine the international transmissions of country specific shocks, such as changes in taste and fiscal policies. The effects of coordinated fiscal expansion and of transfer payments are also discussed. The model deviates from the standard Walrasian framework only by allowing for the monopoly power in the goods market. The presence of profit margin plays a central role in the international transmissions of country specific shocks in this paper.

To grasp the intuition behind this transmission mechanism, imagine that demand for domestic goods suddenly rises due to, say, a change in the national fiscal policies. In the presence of monopoly power, prices exceed marginal costs; hence such a shift in demand increases the level of monopoly profits in the economy and thus the national income. This increased income will generate additional demand for domestic products, which further raises profits and income, and so on. To the extent that this induced increase in demand falls on foreign goods and raises aggregate profits abroad, it also creates similar chain reactions and lead to an increase in income abroad. Thus, under imperfect competition, there are positive spillover effects of country specific demand shocks and their magnitude depends on the profit margin as well as the marginal propensity to consume foreign goods.

The model therefore suggests equilibrium behavior very much like that predicted by the traditional Keynesian models of open economies, developed by Machlup (1939, 1942) and Metzler (1942a, 1942b|1973|), which emphasize the role of foreign trade multipliers in income determination in interdependent national economies. The purpose of this paper is thus partly pedagogical: to demonstrate how a general equilibrium model of imperfect competition can
generate the results with Keynesian flavor without resorting to the standard assumptions of sticky nominal prices or unemployment. In presenting the model below, particular emphasis will be placed on the analogy to the Robinson (1952) graphic analysis, which appears in many standard textbooks, such as Dornbusch (1980, Ch. 3).

Many recent studies have addressed the role of imperfect competition and aggregate demand spillovers in macroeconomic issues: see, for example, Blanchard and Kiyotaki (1987), Hart (1982), Kiyotaki (1985), Mankiw (1988), Murphy, Shleifer and Vishny (1989), Startz (1989) and Weitzman (1982). Imperfect competition also plays the eminent role in the so-called new trade theory; see, for example, Helpman and Krugman (1985). Rather surprisingly, virtually no attempt has been made in exploring the fruitful synergism between these two strands of literature, from which the present analysis draws its insight significantly. It is hoped that this paper will help to bridge the gap between the two fields.

The rest of the paper is organized in five parts. Section 2 lays out the basic model. Preference shocks are examined in section 3. Section 4 extends the model to incorporate the government sectors and the effects of fiscal policies and of transfer payments are discussed. Section 5 offers a reinterpretation of the model as a two-period model, which makes the analogy to the Keynesian model even closer. Section 6 concludes.

2. The Model

Consider the world economy that consists of two countries: Home and Foreign. Each country is inhabited by the representative consumer. There are three classes of tradeable goods: alpha, beta, and numeraire, all produced by labor. The numeraire good is homogeneous and can be produced by both
economies. The alpha goods consist of a continuum of variety, indexed by \( z \in [0,1] \); they can be produced only in the Home country, and \( p(z) \) denotes the price of variety \( z \). Likewise, the beta goods consist of a continuum of variety, indexed by \( z \in [0,1] \), and can be produced only in the Foreign country: \( p^*(z) \) denotes the price of variety \( z \).

A. Consumption

The Home agents supply labor, \( L \), inelastically and earn wage income, \( wL \). They also own every Home firm and thus receive the aggregate profits in the Home economy, \( \Pi \). The Home national income is thus equal to \( Y = wL + \Pi \). Let \( c(z) \), \( d(z) \), and \( N \) denote their consumption of the alpha good of variety \( z \), the beta good of variety \( z \) and the numeraire good, respectively. Home consumption is determined by the following problem: choose \( c(z) \), \( d(z) \), and \( N \) to maximize

\[
\ln(U) = \alpha \ln(C - C_0) + \beta \ln(D - D_0) + (1 - \alpha - \beta) \ln(N - N_0),
\]

subject to

\[
\int_0^1 p(z)c(z)dz + \int_0^1 p^*(z)d(z)dz + N \leq Y,
\]

where \( C \) and \( D \) are the quantity indices defined by

\[
C = \exp\left[\int_0^1 \ln(c(z))dz\right], \quad D = \exp\left[\int_0^1 \ln(d(z))dz\right].
\]

Thus, \( \alpha, \beta \) and \( 1 - \alpha - \beta \) are the marginal budget shares parameters and are assumed to be between zero and one. The parameters, \( C_0 \), \( D_0 \), and \( N_0 \), could be interpreted as the subsistence requirements, but do not need to be positive; they are introduced to build in preference shocks.

Similarly, the Foreign agent's consumption decision is given by the
following problem: to choose \( c^*(z), d^*(z), \) and \( N^* \) to maximize

\[
\ln(U) = \alpha^\prime \ln(C^*-C_0^*) + \beta^\prime \ln(D^*-D_0^*) + (1-\alpha^\prime-\beta^\prime) \ln(N^*-N_0^*),
\]

subject to

\[
\int_0^1 p(z) c^*(z) dz + \int_0^1 p^*(z) d^*(z) dz + N^* \leq Y^*,
\]

where

\[
C^* = \exp\left[\int_0^1 \ln(c^*(z)) dz\right], \quad D^* = \exp\left[\int_0^1 \ln(d^*(z)) dz\right],
\]

where \( 0 < \alpha^*, \beta^*, \alpha^* + \beta^* < 1; \) \( Y^* = w^*L^* + \Pi^* \) is the Foreign national income, and equal to the sum of the labor income and the aggregate profits earned by the Foreign firms. Note that the Home agents and the Foreign agents face the identical goods prices, which implies there is neither tariffs nor transportation costs.

Solving the consumption problems, one can obtain the demand for each variety of alpha and beta goods,

\[
q(z) = c(z) + c^*(z) = P(C+C^*)/p(z), \quad q^*(z) = d(z) + d^*(z) = P^*(D+D^*)/p^*(z),
\]

where \( P \) and \( P^* \) are the price indices of alpha goods and beta goods, given by

\[
P = \exp\left[\int_0^1 \ln(p(z)) dz\right], \quad P^* = \exp\left[\int_0^1 \ln(p^*(z)) dz\right],
\]

and

\[
C = C_0 + (\alpha/P)(Y-PC_0-P^*D_0-N_0), \quad C^* = C_0^* + (\alpha^*/P^*)(Y^*-PC_0^*-P^*D_0^*-N_0^*),
\]

\[
D = D_0 + (\beta/P^*)(Y-PC_0-P^*D_0-N_0), \quad D^* = D_0^* + (\beta^*/P^*)(Y^*-PC_0^*-P^*D_0^*-N_0^*),
\]

\[
N = N_0 + (1-\alpha-\beta)(Y-PC_0-P^*D_0-N_0), \quad N^* = N_0^* + (1-\alpha^*-\beta^*)(Y^*-PC_0^*-P^*D_0^*-N_0^*),
\]
B. Production

A particular variety of alpha good, z, can be produced by two types of Home firms. First, there is a competitive fringe of firms that convert one unit of labor input into one unit of output with constant returns to scale technology. Second, there is a unique monopolist firm with access to an increasing returns to scale technology. This firm alone can produce q units of output by using \((1-\mu)q + F\) units of labor input, where \(0 < \mu \leq 1\), and \(F\) represents the fixed cost. Taking the demand function (1) given, this firm chooses \(p(z)\) to maximize its profit, \(\pi(z) = p(z)q(z) - w[(1-\mu)q(z)+F]\). In doing so, it treats \(C\), \(C^*\), and \(P\), as fixed parameters; although this firm has some monopoly power over its own variety, it is negligible relative to the aggregate economy. Because of the unit elasticity of demand and the competitive fringe, the monopolist practices the limit pricing, \(p(z) = w\), and thus \(\mu = (p(z)-w(1-\mu))/p(z)\) can be interpreted as the profit margin. Since all monopolists face the same incentive, \(P = p(z) = w\) from (2), and thus, from (1), \(q(z) = C + C^*\) for all \(z \in [0,1]\). The aggregate profits are therefore equal to \(\Pi = \pi(z) = w[C+C^*-F]\). Note that higher aggregate demand increases aggregate profits.

The beta goods sector in the Foreign country is organized in an analogous way. Thus, each variety of beta good is supplied by a monopolist firm, which practices the limit pricing, \(p^*(z) = w^*\), because of the competitive fringe that can convert one unit of labor input to one unit of output. Thus, \(p^*(z) = P^* = w^*\), and \(q^*(z) = D + D^*\) for all \(z \in [0,1]\). The aggregate profits in the Foreign economy is \(\Pi^* = w^*[\mu(D+D^*]-F]\).

In both economies, the numeraire goods sector is competitive and can produce one unit of output by employing one unit of labor. This implies that
w ≥ 1 and w* ≥ 1.

C. Equilibrium

In order to find the equilibrium, let us assume that both countries produce the numeraire good, which is the case if the parameters of the two countries are not too far away from the symmetry case. The assumption of non-specialization ensures that w = w* = 1. As a result, P = P* = 1, and

\[ Y = L + \Pi = (L-F) + \mu(C+C^*) , \quad Y^* = L^* + \Pi^* = (L^*-F) + \mu(D+D^*) , \]

and (3) becomes

\[ C = C_0 + \alpha(Y-C_0-D_0-N_0) , \quad C^* = C_0^* + \alpha^*(Y^*-C_0^*-D_0^*-N_0^*) , \]
\[ D = D_0 + \beta(Y-C_0-D_0-N_0) , \quad D^* = D_0^* + \beta^*(Y^*-C_0^*-D_0^*-N_0^*) , \]
\[ N = N_0 + (1-\alpha-\beta)(Y-C_0-D_0-N_0) , \quad N^* = N_0^* + (1-\alpha^*-\beta^*)(Y^*-C_0^*-D_0^*-N_0^*) . \]

Combining (4) and (5) yields

\[ Y = A_0 + (1-s-m)Y + m^*Y^* , \]

and

\[ Y^* = A_0^* + mY + (1-s^*-m^*)Y^* , \]

where

\[ A_0 = (L-F) + \mu[(1-\alpha)C_0 + (1-\alpha^*)C_0^* - \alpha(D_0 + N_0) - \alpha^*(D_0^* + N_0^*)] , \]
\[ A_0^* = (L^*-F) + \mu[(1-\beta)D_0 + (1-\beta^*)D_0^* - \beta(C_0 + N_0) - \beta^*(C_0^* + N_0^*)] , \]

which summarize the aggregate demand parameters for the alpha and beta goods, respectively, and

\[ s = 1 - \mu(\alpha + \beta) , \quad m = \mu\beta , \quad s^* = 1 - \mu(\alpha^* + \beta^*) , \quad m^* = \mu\alpha^* . \]

Here, 1-s-m = \mu\alpha (m = \mu\beta) is the Home country's marginal propensity to consume
the alpha (beta) goods, multiplied by the profit margin, and hence represents the extent to which marginal increase in the Home national income would contribute to the aggregate profits in the Home (Foreign) country. Note that $0 < \alpha, \beta, \alpha + \beta < 1$, and $0 < \mu \leq 1$ ensure that $0 < s, m, s + m < 1$. Likewise, $1 - s^* - m^* = \mu \beta^*$ and $m^* = \mu \alpha^*$ represent the extent to which marginal increase in the Foreign national income contributes the aggregate profit in the Foreign and Home countries, respectively: they satisfy $0 < s^*, m^*, s^* + m^* < 1$.

Figure 1 illustrates how the Home and Foreign national incomes are jointly determined. Line $HH$ represents equation (6): its slope is equal to $(s + m)/m^*$. An increase in $A^*_H$ shifts this line to the right. Line $FF$ represents (7), whose slope is $m/((s + m)^*)$: an increase in $A^*_F$ shifts it upward. The equilibrium income in each country is given at the intersection. To solve explicitly,

\[
\begin{bmatrix}
Y \\
Y^*
\end{bmatrix} = \frac{1}{(s + m)(s^* + m^*) - mm^*} \begin{bmatrix}
s^* + m^* & m^* \\
m & s + m
\end{bmatrix} \begin{bmatrix}
A^*_H \\
A^*_F
\end{bmatrix}.
\]  

(9)

Once the equilibrium incomes are solved for, it is easy to evaluate the levels of the national welfare. They are equal to $U = k(Y - C_0 - D_0 - N_0)$ and $U^* = k^*(Y^* - C_0^* - D_0^* - N_0^*)$, respectively, where $k = \alpha^* \beta^* (1-\alpha-\beta)(1-\alpha^*-\beta^*)$ and $k^* = (\alpha^*)^* (\beta^*)^* (1-\alpha^*-\beta^*)(1-\alpha^*-\beta^*)$, which can be verified by inserting (5) back into the utility functions.

Because of the static nature of the model, the trade balances of these economies are always in balance: Walras's Law ensures it once the budget constraints of the agents are respected.\(^1\) One may be interested, however, in

\(^1\)However, the model could be reinterpreted as a two-period model, in which one could talk about the trade balance in the current period; see Section 5.
Figure 1:
The Robinson Diagram
predicting the patterns of trade or in calculating the sectoral trade balance. Let $B$ be the Home country's trade balance in the imperfectly competitive sector. From (5), it is equal to

$$B = C' - D = B_0 + \alpha'Y' - \beta Y.$$  (10)

where

$$B_0 = (1-\alpha')C_0' - \alpha'(D_0' + N_0') - (1-\beta)D_0 + \beta(C_0 + N_0).$$  (11)

The locus of the zero sectoral balance, $B = 0$, has a positive slope, which is equal to $\beta/\alpha = m/m^*$: it is steeper than FF, and less steep than HH. If HH and FF intersect on the $B = 0$ line, as depicted in Figure 1, the sectoral trade balance is zero. If they intersect to the right (left) of the $B = 0$ line, the Home country becomes a net importer (exporter) of imperfectly competitive goods; that is, it runs a trade account deficit (surplus) in that sector.

3. **Comparative Statics: Preference Shocks**

   Let us now examine the effects of preferences shocks. Throughout this exercise, it is assumed $\Delta C_0 + \Delta D_0 + \Delta N_0 = 0$, and $\Delta C_0' + \Delta D_0' + \Delta N_0' = 0$, so that the welfare of agents will not be affected for the original levels of the national incomes. From (8) and (11), this restriction implies

$$\Delta A_0 = \mu(\Delta C_0 + \Delta C_0'), \quad \Delta A_0^* = \mu(\Delta D_0 + \Delta D_0'), \quad \Delta B_0 = \Delta C_0' - \Delta D_0.$$  (12)

A. **A Shift in HH Line: An Autonomous Increase in Demand for Home (Alpha) Goods**

   Consider first the impacts of a rise in the demand for alpha goods. In a traditional Keynesian model, such an autonomous increase in demand comes
from a reduction in saving. Here, let us assume that it is accompanied by a reduction in demand for the numeraire good. More specifically, suppose that the tastes of the Home agents shift from the numeraire good to the alpha goods: \( \Delta C_0 = -\Delta N_0 > 0 \), and \( \Delta D_0 = 0 \). (Note that, from (5), this implies the demand for the alpha goods rises by \( \Delta C_0 \) for a given level of the Home income.) Then, \( \Delta A_0 = \mu \Delta C_0 > 0 \) and \( \Delta A_0^* = 0 \); HH shifts to the right, and FF stays the same. As shown in Figure 2, the equilibrium moves from \( e \) to \( e' \); \( Y \) increases by \( e \) and \( Y^* \) by \( ge' \). From (9),

\[
\begin{align*}
\frac{\Delta Y}{\Delta C_0} |_{\Delta C_0 = -\Delta N_0} &= \frac{\mu (s^* + m^*)}{(s + m)(s^* + m^*) - mm'} = \frac{\mu (1 - \mu \beta^*)}{(1 - \mu \alpha)(1 - \mu \beta^*) - \mu^2 \alpha^* \beta} > 0, \\
\frac{\Delta Y^*}{\Delta C_0} |_{\Delta C_0 = -\Delta N_0} &= \frac{\mu m}{(s + m)(s^* + m^*) - mm'} = \frac{\mu^2 \beta}{(1 - \mu \alpha)(1 - \mu \beta^*) - \mu^2 \alpha^* \beta} > 0.
\end{align*}
\] (13)

The multipliers thus depend on the profit margin and the marginal propensities. They are increasing in the profit margin. Note that, given \( s, m, s^*, \) and \( m^* \), they are equal to \( \mu \) times the open economy multipliers of the standard Keynesian models: see, for example, equation (14) in Dornbusch (1980, Ch.3). This is because a one dollar increase in demand leads to an increase in income only by \( \mu \) dollar in this model. Because of this, the own multiplier, \( \Delta Y/\Delta C_0 \), is not necessarily greater than one. A sufficient condition for \( \Delta Y/\Delta C_0 > 1 \) is given by \( \alpha > (1 - \mu)/\mu \).\(^2\) Otherwise, these expressions have the standard properties. The own multiplier is smaller than \( \mu/s - \mu/(1 - \mu(\alpha + \beta)) \), the multiplier of the "closed economy": that is, the

\(^2\)If the Home labor endowment and hence the Home wage income increases by one unit, then \( A_0 \) increases by one, so that the multipliers would be the same with those of the standard textbook. In particular, the own multiplier is necessarily greater than one. Of course, no difference exists if \( \mu = 1 \); that is, if marginal labor requirement of the imperfectly competitive goods is equal to zero. Note that, in a traditional Keynesian model, labor is not fully employed so that marginal social costs of labor is considered to be zero.
Figure 2:
An autonomous increase in demand for Home goods
multiplier in the case where an increase in the Home spending on the imperfectly competitive goods, either autonomous or induced, would fall entirely on the Home goods. This is because the power of the demand increase to "multiply" income is limited by "leakages" into imports. Not all imports, however, will be a loss to the domestic multiplier process, as first pointed out by Machlup (1939), because the induced increase in Home imports raises income abroad and leads to a partial "reinjection" to the demand for the Home goods and thus the Home income stream through a rise in exports. Thus, the own multiplier is larger than the multiplier in the absence of "foreign repercussion effects" ($m^* = 0$, or $\alpha^* = 0$), which is equal to $\mu/(s+m) = \mu/(1-\mu\alpha)$ and represented by ef in Figure 2. The foreign repercussion effect is equal to fg. The spillover effect of the autonomous increase in the demand in the Home country is captured by ge' in Figure 1. This cross multiplier, $\Delta Y^* / \Delta C_0$, can be either greater or smaller than the own multiplier in general, depending on whether the slope of FF is greater or smaller than one. It is smaller if $\mu(\beta+\beta^*) < 1$. Thus, the own effect always dominates the cross effect in the symmetry case ($\alpha = \beta^*$ and/or $\beta = \alpha^*$).

Note that this exercise suggests a fallacy of composition. Although each Home agent increases consumption of the alpha good and reduces consumption of the numeraire good at the rate which, on the margin, keeps the agent indifferent at the original level of equilibrium income. Such a change in behavior, however, raises the aggregate profits, which will be paid out to the owners of the firms, and initiate the multiplier process. As a result, every agent, including those abroad, will be made better off.

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3The closed economy multiplier can be obtained by setting $\beta = 0$ and then replacing $\alpha$ by $\alpha + \beta$ in equation (13).
The locus $B = 0$ does not shift after the change ($\Delta B_0 = 0$). Since the equilibrium moves along FF, the Home trade balance in the imperfectly competitive sector turns to a deficit, $B < 0$. Algebraically,

$$-1 < \frac{\Delta B}{\Delta C_0} |_{C_0^* - N_0} = \frac{-ms^*}{(s+m)(s^*+m^*)-mm^*} = \frac{\mu \beta (\mu (\alpha^*+\beta^*)-1)}{(1-\mu^*) (1-\mu^*+\mu^*)-\mu^*\alpha^*\beta} < 0. \quad (14)$$

The impacts of an autonomous increase in the Home exports (i.e., the Foreign demand for alpha goods) may also be analyzed by letting $\Delta C_0^* = -\Delta N_0^* > 0$. This implies $\Delta A_0 = \mu C_0 > 0$ and $\Delta A_0^* - 0$, and thus the multiplier effects on the national incomes are the same with the increase in the Home demand for alpha goods; The result would not depend on where the shock is originated.

The effect on the sectoral trade balance differs, however. From (12), $\Delta C_0^* = -\Delta N_0^* > 0$ implies that $\Delta B_0 = \Delta C_0^* > 0$; the $B = 0$ locus shifts down. As a result, the Home economy runs a surplus, as shown below:

$$0 < \frac{\Delta B}{\Delta C_0} |_{C_0^* - N_0^*} = \frac{s(s^*+m^*)}{(s+m)(s^*+m^*)-mm^*} = \frac{(1-\mu (\alpha+\beta))(1-\mu^*+\mu^*)}{(1-\mu^*) (1-\mu^*+\mu^*)-\mu^*\alpha^*\beta} < 1.$$

Note that $\Delta B < \Delta B_0$, because the induced change of the sectoral balance is negative for the same reason why equation (14) is negative.

B. A Shift in FF Line: An Autonomous Increase in Demand for (Foreign) Beta Goods:

The impacts of an autonomous increase in the demand for beta goods may be analyzed in a similar way. Whether it is due to a shift in export demand, $\Delta D_0 = -\Delta N_0 > 0$, or a shift in domestic demand, $\Delta D_0^* = -\Delta N_0^* > 0$, such a shock moves FF upward, while HH stays the same. As a result, both Home and Foreign incomes rise. The multiplier can be obtained from (13), by exchanging $s$ for $s^*$ and $m$ for $m^*$ as follows:
\[ \frac{\Delta Y}{\Delta D_0} \bigg|_{\Delta D_0 - \Delta \mu_0} = \frac{\Delta Y}{\Delta D_0} \bigg|_{\Delta D_0^* - \Delta \mu_0^*} = \frac{\mu m}{(s+m)(s^*+m^*) - \mu m^2} = \frac{\mu^2 \alpha}{(1-\mu \alpha)(1-\mu \beta^*) - \mu^2 \alpha^* \beta} > 0, \]
\[ \frac{\Delta Y^*}{\Delta D_0} \bigg|_{\Delta D_0 - \Delta \mu_0} = \frac{\Delta Y^*}{\Delta D_0} \bigg|_{\Delta D_0^* - \Delta \mu_0^*} = \frac{\mu (s+m)}{(s+m)(s^*+m^*) - \mu m^2} = \frac{\mu(1-\mu \alpha)}{(1-\mu \alpha)(1-\mu \beta^*) - \mu^2 \alpha^* \beta} > 0. \]

For the effect on the Home trade balance,

\[ -1 < \frac{\Delta B}{\Delta D_0} \bigg|_{\Delta D_0 - \Delta \mu_0} = \frac{-s+m}{(s+m)(s^*+m^*) - \mu m^2} = \frac{(1-\mu \alpha)}{(1-\mu \alpha)(1-\mu \beta^*) - \mu^2 \alpha^* \beta} \]
\[ 0 < \frac{\Delta B}{\Delta D_0} \bigg|_{\Delta D_0^* - \Delta \mu_0^*} = \frac{m^2 s}{(s+m)(s^*+m^*) - \mu m^2} = \frac{\mu \alpha}{(1-\mu \alpha)(1-\mu \beta^*) - \mu^2 \alpha^* \beta} < 1. \]

Thus, it is negative (positive) if the demand shift originates from the Home (Foreign) country.

C. **Simultaneous Shifts in HH and FF Lines: An Autonomous Demand Switch from Foreign (Beta) to Home (Alpha) Goods:**

Finally, consider the effects of an expenditure "switching," while the level of spending on the imperfectly competitive goods is held constant.

Suppose that the Home agent's tastes shifts from beta goods to alpha: that is, \( \Delta C_0 = -\Delta D_0 > 0 \). Then, \( \Delta A_0 = -\Delta A_0^* = \mu \Delta C_0 > 0 \); HH shifts to the right and FF shifts down: see Figure 3. The Home national income rises and the Foreign income declines. Algebraically,

\[ \frac{\Delta Y}{\Delta C_0} \bigg|_{\Delta C_0^* = \Delta D_0} = \frac{\mu s^*}{(s+m)(s^*+m^*) - \mu m^2} = \frac{\mu (1-\mu (\alpha^*+\beta))}{(1-\mu \alpha)(1-\mu \beta^*) - \mu^2 \alpha^* \beta} > 0, \]  \hspace{1cm} (15)
\[ \frac{\Delta Y^*}{\Delta C_0} \bigg|_{\Delta C_0^* = \Delta D_0} = \frac{-\mu s^*}{(s+m)(s^*+m^*) - \mu m^2} = \frac{\mu (\alpha + \beta - 1)}{(1-\mu \alpha)(1-\mu \beta^*) - \mu^2 \alpha^* \beta} < 0. \]

The own multiplier, eg in Figure 3, is now smaller than \( \mu/(s+m) \), the multiplier in the absence of foreign repercussion effect; the difference is given by \( g/f \). This is because this shock has a negative spillover effect on the Foreign country, equal to \( ge' \). Furthermore, it turns the Home sectoral
Figure 3:
An autonomous demand switch from Foreign goods to Home goods
balance into a surplus:

\[ 0 < \frac{\Delta B}{\Delta C_0} \Delta C_0 = \frac{ss^* (s + m)(s^* + m^*)}{(1 - \mu (s + \beta))(1 - \mu (\alpha^* + \beta^*))} < 1. \]

The effect of this shock on the world income, \( \Delta(Y + Y^*) \), depends on the difference on the marginal propensities. From (15), it is positive if \( s^* > s \) or \( \alpha + \beta > \alpha^* + \beta^* \); it is negative if \( s^* < s \), or \( \alpha + \beta < \alpha^* + \beta^* \). Thus, the demand switch from the foreign goods to the Home goods increases the world income if and only if the Home agent has a higher marginal propensity to consume the imperfectly competitive goods. The effects of the demand switch originated in the Foreign country, \( \Delta C_0^* = -\Delta D_0^* > 0 \), are the same with those originated in the Home country, as is clear from (12).

4. **International Transmissions of Fiscal Policies**

This section considers the effects of fiscal policies. In order to reduce the notational burdens, it is assumed \( C_0 = D_0 = N_0 = C_0^* = D_0^* = N_0^* = 0 \) below. The Home government imposes a lump sum tax, \( T \), and spends \( pG_C/p(z) \) on the alpha good of variety \( z \), \( p^* G_D/p^*(z) \) on the beta good of variety \( z \), \( G_N \) on the numeraire good, and transfers \( R \) to the Foreign government. The Foreign government is modelled in an analogous way. The budget constraints are given by

\[ PG_C + P^* G_D + G_N + R = T, \quad PG_C^* + P^* G_D^* + G_N^* = R = T^*. \]

Given \( G_C, G_D, G_N, G_C^*, G_D^* \), and \( G_N^* \), all monopolists face unit elastic demand curves, and therefore \( p(z) = P = w = p^*(z) = P^* = w^* = 1 \). And the demand functions for alpha goods and beta goods are given by
\[ C + G_c = \alpha(Y - T) + G_c, \quad D + G_D = \beta(Y - T) + G_D. \]
\[ C^* + G_c^* = \alpha^*(Y^* - T^*) + G_c^*, \quad D^* + G_D^* = \beta^*(Y^* - T^*) + G_D^*. \]

Aggregate profits are then
\[ \Pi = \mu(C + G_c + C^* + G_c^*) - F, \quad \Pi^* = \mu(D + G_D + D^* + G_D^*) - F. \]

Thus, the equilibrium incomes are determined by (6) and (7), or (9), where
\[ A_c = (L - F) + \mu(G_c + G_c^* - \alpha T - \alpha^* T^*), \]
\[ A_c^* = (L^* - F) + \mu(G_c^* + G_D^* - \beta T - \beta^* T^*). \quad (16) \]

and the sectoral balance is given by (10), where
\[ B_c = (G_c - \alpha^* T^*) - (G_D - \beta T). \quad (17) \]

A. The Government Purchase Multipliers

By inspecting equations (12), (16), and (17), it should be immediately clear that, as long as the levels of taxes are held constant, any change in government purchases of the imperfectly competitive goods are similar to the corresponding preference shocks discussed in the previous section. For example, an increase in \( G_c \), financed by a reduction in \( G_D \), is identical to the effect of \( \Delta G_c = -\Delta N_c > 0 \). Thus, equation (13) gives the multiplier associated with an increase in government spending on the Home goods. Similarly, if the Home government switches its spending from the Foreign goods to the Home goods, \( Y \) rises, \( Y^* \) declines and the Home sectoral balance turns to a surplus: such a fiscal policy may thus be considered as beggar-thy-neighbor policies. If both Home and Foreign governments pursue such a policy to the same extent, net results would be zero: if \( \Delta G_c = -\Delta G_D = \Delta G_c^* = -\Delta G_D^* > 0 \), \( \Delta A_c = \Delta A_c^* = \Delta B_c = 0 \).
One may be interested in the effects of coordinated fiscal expansions. Suppose, for example, that both governments agree to increase their purchases of foreign goods by the same amount, financed by a reduction in purchases of the numeraire good; that is, \(\Delta G_D = \Delta G_N = -\Delta G_D^* = -\Delta G_N^* = \Delta E > 0\). This implies \(\Delta A_0 - \Delta A_0^* - \Delta E > 0\) and \(\Delta B_0 = 0\). (The effect of coordinated fiscal expansion, targeted to the domestic goods, \(\Delta G_C = \Delta G_D^* = -\Delta G_N = -\Delta G_N^* - \Delta E > 0\), is identical, as can be seen from (16) and (17).) Both HH and FF shift outward and the \(B = 0\) locus stays intact, as in Figure 4. As a result, the national income rises in each economy. More specifically,

\[
\frac{\Delta Y}{\Delta E} = \frac{\mu(1+\alpha^*-\mu\beta^*)}{(s+m)(s^*+m^*) - \mu\alpha^*\beta} > 0,
\]

\[
\frac{\Delta Y^*}{\Delta E} = \frac{\mu(1-\mu\alpha^*+\mu\beta^*)}{(1-\mu\alpha)(1-\mu\beta^*) - \mu^2\alpha^*\beta} > 0.
\]

and

\[
\frac{\Delta B}{\Delta E} = \frac{s\alpha^*-\mu s^*}{(s+m)(s^*+m^*) - \mu\alpha^*\beta} = \frac{\alpha^*-\beta + \mu((\beta^*-\alpha^*)}{(1-\mu\alpha)(1-\mu\beta^*) - \mu^2\alpha^*\beta}.
\]

To see what is involved in these expressions, consider the symmetry case: \(\alpha = \beta^*\) and \(\alpha^* = \beta\). Then, \(\Delta Y/\Delta E = \Delta Y^*/\Delta E = \mu/s - \mu/(1-\mu(\alpha+\beta))\) and \(\Delta B = 0\). The multipliers of the coordinated fiscal policies are thus equal to the closed economy multiplier. Coordinated fiscal policies lead to an expansion without any consequence on the trade balance.

B. The Tax Multipliers

Consider now an increase in taxes, holding government purchases on the imperfectly competitive goods fixed. The budget constraint implies that government purchases of the numeraire good need to be raised. A tax increase in the Home country, \(\Delta T = \Delta G_N > 0\), leads to \(\Delta A_2 = -(1-s-m)\Delta T < 0\), \(\Delta A_2^* = -m\Delta T < 0\) and \(\Delta B_0 = \beta\Delta T > 0\). Both HH and FF shifts inward, and as a result,
Figure 4:
Coordinated Fiscal Expansion
\[
\frac{\Delta Y}{\Delta T} \bigg| \Delta T \Delta G_N = \frac{-1(s^* + m^*)(1-s-m)*mm^*}{(s+m)(s^*+m^*)-mm^*} < 0, \\
\frac{\Delta Y^*}{\Delta T} \bigg| \Delta T \Delta G_N = \frac{-m}{(s+m)(s^*+m^*)-mm^*} < 0.
\]

and

\[
0 < \frac{\Delta B}{\Delta T} \bigg| \Delta T \Delta G_N = \frac{s^*\beta}{(s+m)(s^*+m^*)-mm^*} < 1.
\]

It should be noted that these expressions for the tax multipliers on incomes are exactly the same with those of the standard Keynesian model. (By contrast, the other multipliers derived above differ from those of the standard model in that \( \mu \) appears in the numerators.) This is because tax changes directly affects the disposal income of agents. In absolute terms, the own multiplier is smaller than the closed economy multiplier, \( 1 - 1/s \), while it is larger than the multiplier without the foreign repercussion effect, \( 1 - 1/(s+m) \). The cross multiplier captures the negative spillover of the tax increase. The cross multiplier could be either larger or smaller than the own multiplier, even in the symmetric case. That is, the condition for the own effect to dominate the cross effect is more stringent than what is obtained in section 3.A, because the tax increase directly reduces the demand for the Foreign good. The condition is generally given by \( (1-\mu \alpha^*)\beta < \alpha(1-\mu \beta^*) \), which is equal to \( \beta^* = \alpha > \beta = \alpha^* \) in the symmetry case: the agents need to have preferences for local goods.

C. The Balanced Budget Multiplier

Consider next the effect of an increase in government purchase on the Home goods financed by an increase in taxes: \( \Delta G_C = \Delta T > 0 \). This implies that \( \Delta A_0 = \mu(1-\alpha) \Delta G_C > 0, \Delta A_0^* = -\mu \beta \Delta G_C \), and \( \Delta B_0 + \beta \Delta G_C > 0 \). And therefore,
\[
\frac{\Delta Y}{\Delta G_C} |_{\Delta G_C = \Delta \tau} = 1 - \frac{(1-\mu)(s^*+m^*)}{(s+m)(s^*+m^*)-mm^*},
\]

\[
\frac{\Delta Y^*}{\Delta G_C} |_{\Delta G_C = \Delta \tau} = \frac{(\mu-1)m}{(s+m)(s^*+m^*)-mm^*} \leq 0,
\]

and

\[
0 \leq \frac{\Delta B}{\Delta G_C} |_{\Delta G_C = \Delta \tau} = \frac{(1-\mu)\beta s^*}{(s+m)(s^*+m^*)-mm^*} < 1.
\]

If the marginal cost of output expansion in the Home goods sector is equal to zero \((\mu = 1)\), these expressions are quite similar to the standard textbook ones. That is, the multiplier on Home income is equal to one, and there is no effect on the Foreign income or the trade balance. In general, however, it has a smaller effect on the Home income, (and in fact, the effect could be negative) and would lead to a reduction in income abroad, and a Home surplus in the imperfectly competitive sectors.

D. The Transfer Problem

Finally, the present model can be used to analyze the transfer problem: the question which originally motivated Metzler (1942a[1973]) to develop the foreign trade multipliers. Suppose that the Home country is a donor and the Foreign country a recipient. The effects of a transfer, of course, depend on the budgetary consequences. Metzler discussed three different cases. His Case I assumes that a transfer is "accompanied by increased taxes in the paying country and reduced taxed in the receiving country (1973, pp.52-3)"; that is, \(\Delta T = -\Delta T^* = \Delta R > 0\). This implies \(\Delta A_0 = \mu(\alpha - \alpha)\Delta T\) and \(\Delta A_0^* = \mu(\beta^* - \beta)\Delta T\), and \(\Delta B_0 = (\alpha^* + \beta)\Delta T > 0\). Therefore,

\[
\frac{\Delta (Y-R)}{\Delta R} = \frac{-s^*}{(s+m)(s^*+m^*)-mm^*} < 0, \quad \frac{\Delta (Y^*+R)}{\Delta R} = \frac{s}{(s+m)(s^*+m^*)-mm^*} > 0.
\]
which implies that the national income after the transfer declines in the Home country, and increases in the Foreign country; that is, there is no transfer paradox in this model. Whether the transfer imposes additional burdens on the donor depends on the parameters. The condition for $\Delta Y < 0$ is given by $s^*\alpha > s\alpha^*$. Likewise, the transfer brings additional benefits to the recipient, $\Delta Y^* > 0$, if $s\beta^* > s^*\beta$. In the symmetry case, these conditions are simply $\beta^* - \alpha > \beta - \alpha^*$. Thus, the income adjustment process magnifies the effect of the transfer, if the agents have preferences for local goods. The effect on the world income depends on the relative size of $s$ and $s^*$, or $\alpha + \beta$ and $\alpha^* + \beta^*$. The world income increases after the transfer if $s > s^*$, or $\alpha + \beta < \alpha^* + \beta^*$, or when the recipient has a higher marginal propensity to consume the imperfectly competitive goods than the donor. The effect on the sectoral trade balance in the imperfectly competitive sector is given by

$$0 < \frac{\Delta B}{\Delta R} = \frac{m^*s + m's}{(s+m)(s^*+m^*)-mm'} < 1.$$ 

Note that $\Delta B < \Delta R$ means that the Home sectoral balance in the numeraire goods sector also improves by $\Delta R - \Delta B > 0$. The other two cases in Metzler's analysis can also be examined in this model by either letting $\Delta T = \Delta C_N = \Delta R > 0$ (Case II), whose effects on the national incomes are the same with a tax increase in the Home country, or $-\Delta T^* = -\Delta C_N = \Delta R > 0$ (Case III), whose effects on the national incomes are the same with a tax reduction in the Foreign country. In any case, the donor's trade balance in the imperfectly competitive sectors improves, but not as much as the transfer payment, and therefore its trade balance in the numeraire goods sector also improves.

5. Reinterpreting the model

One may be skeptical about the usefulness of a static model for
understanding some of the macroeconomic issues, such as saving, investment, and the current account, which are inherently dynamic. To those skeptics, it seems worth pointing out that the model presented above can be indeed reinterpreted as a two-period model.

Imagine that all agents live for two periods: 1 and 2. They are endowed with labor only in period 1. Both alpha and beta goods are produced and consumed in period 1 only. On the other hand, the numeraire good is produced with one period gestation lag. Thus, the technology requires use of labor in period 1 and output is consumed in period 2.

With this interpretation, $\alpha + \beta$ can be read as the Home agent's marginal propensity to consume; consumption of the numeraire good is saving in period 1, while its production is investment; $B$, which is also equal to the difference between consumption and production of the numeraire good, can be interpreted as the Home country's trade balance in period 1. If $B$ is negative, the Home country runs a surplus in the numeraire sector. This means that the Home runs a trade deficit or is a debtor country today, and therefore it needs to run a trade surplus in the future in order to service its debt obligation. The exercises in Section 3.A can be considered as looking at the effects of an autonomous increase in demand for the Home goods, financed by a reduction in saving. Those in Section 3.C are the effects of a change in the compositions of expenditure, holding the current expenditure fixed. Similarly, $G_N$, the government consumption of the numeraire good can be now considered as the government saving in period 1. Thus, a rise in government spending on the imperfectly competitive goods financed by a reduction in $G_N$ can be viewed as a debt-financed increase in government spending. The results in Section 4.A therefore suggests that a debt-financed government spending on
the domestic goods, if done unilaterally, increases income, both at home and abroad, and turns the economy's current account into a deficit, while if it is coordinated, a much larger rise in the national incomes will result, without any significant current account imbalances, etc. The exercises in Section 4.C can be interpreted as looking at the effects of a government purchase increase, holding the government saving constant, which justifies the expression, the balanced budget multiplier. For the transfer problem, $\Delta B < \Delta R$ implies that the trade balance of the donor improves less than the transfer payment, so that the donor runs a current account deficit: The positive marginal propensity to save precludes the real transfer of the resources from being completed contemporaneously. As Metzler concluded in his classic article, "real-income movements induced by shifts of purchasing power may be expected to create only a part of the surplus required for capital transfers (1973, p.68)."

6. **Concluding Remarks**

More than a half century ago, Kindleberger wrote at the end of his book, "In conclusion, the opinion may be hazarded that perhaps the time has come to rewrite the theory of international trade in terms of the national money income (1937, p.237)." The theory of multipliers in open economies, developed by Machlup and Metzler, was the initial effort in this direction. Fifty years later, one may argue that perhaps the time has come to rewrite the theory of open economy macroeconomics in terms of general equilibrium models of international trade. It is to be hoped that this work would stimulate further research in the microfoundations of international macroeconomics.
References:


