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EFFICIENCY IN BARGAINING WITH INFORMATION EXTERNALITIES

by

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Abstract

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Efficiency in Bargaining

Bargaining is examined for the situation in which each party has private information regarding their valuation of the good as well as the value of the good to the other party. The k-double auction and the first-and-final offer bargaining game are shown not to be ex ante incentive efficient. This result contrasts with the independent private values case. A trading process based on priority pricing is shown to implement the ex ante incentive efficient mechanism.

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1. Introduction

Bargaining is generally studied using the assumption of independent private values.[1] However, in many trading situations, both parties may possess private information regarding both their own value of the good and the value of the good to the other party.[2] This paper shows that information externalities then result that can cause standard bargaining procedures to be inefficient. In particular, it is shown that the k-double auction, and the first-and-final offer bargaining game have inefficient outcomes. A trading process based on priority pricing is presented and shown to be efficient.

It is easy to imagine situations where both the buyer and the seller have private information regarding the value of a good to be traded--that is, each party's information is required to correctly value the good. A manufacturer may have information about the characteristic of a good, while a retailer seeking to purchase the good may have information regarding the market or resale value of the good. Consider a partner is a business seeking to buy out another partner's share. Both partners may possess private information regarding the value of the business. In forming a principal-agent relationship, both the principal and the agent may have

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[1]See Myerson and Satterthwaite (1983), Chatterjee and Samuelson (1983), and Linhart, Radner and Satterthwaite (1989).

[2]See Myerson (1985b) for a general presentation of Bayesian games that allows for such general information structures. This is referred to by Johnson, Pratt and Zeckhauser (1990) as "Mutually Payoff Relevant Private Information." Bilateral asymmetric information about a common value is not generally examined. If only one of the parties has information about the common value, the "lemons" problem is obtained. This would correspond in the present model to a commonly observed value of either x or y, with the other parameter privately observable. General valuation markets are studied by Cremer and McLean (1985, 1988), and Gresik (1991).

information regarding the potential benefits for the principal and the potential costs for the agent. In out-of-court settlement of a tort case, which represents the sale of the potential plaintiff's damage claim to the defendant, the plaintiff may have information on the size of the damages while the defendant may have information on the likelihood of an award.[3]

The k-double auction is examined in the independent values case by Chatterjee and Samuelson (1983) and is shown to be ex ante incentive efficient by Satterthwaite and Williams (1989) (see also Myerson (1985a) and Williams (1987)). In the presence of information externalities, it is shown here that the k-double auction does not make sufficient use of available information to achieve an efficient outcome, and thus fails to achieve gains from trade.

The first-and-final offer bargaining game is shown by Myerson (1985a) to be interim incentive efficient in the independent values case where it is a special case of the k-double auction.[4] With information externalities, the first-and-final offer bargaining game involves signaling by the party making the offer, and a random outcome since the party receiving the offer also has private information. The outcome of the game is shown not to achieve ex ante incentive efficiency. The first-and-final offer bargaining game is extended by allowing the buyer or seller to offer a nonlinear priority price schedule such that the party receiving the offer chooses the

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[3]See, for example, P'ng (1983) and Bebchuk (1984) for models with one-sided asymmetric information.

[4]Samuelson (1984) shows that the ex ante efficient outcome cannot be achieved by any simple mechanism, such as a first and final offer, due to the presence of a common value in a one-sided asymmetric information problem. This differs from the present analysis which involves bilateral asymmetric information.

likelihood of trade. At the sequential equilibrium of this game, both parties reveal their private information and the outcome is ex ante incentive efficient.

The paper is organized as follows. Section 2 gives the basic framework. Section 3 characterizes efficient mechanisms. Section 4 examines the efficiency of the k-double auction, and Section 5 examines the first-and-final offer bargaining game. Section 6 presents the trading process based on priority pricing. Section 7 concludes.

2. The Basic Framework

Consider a bargaining problem in which a seller and buyer possess mutually payoff-relevant information. Bilateral asymmetric information exists. The seller's beliefs about the buyer's private information y are represented by the cumulative distribution function $G(y)$ on $[0,1]$ with density $g(y)$ positive on $(0,1)$. The buyer's beliefs about the seller's private information x are represented by $F(x)$ defined on $[0,1]$ with density $f(x)$ positive on $(0,1)$. Let F and G be common knowledge.

The seller's value of the good is xy and the buyer's value of the good is $w + xy$, where $w > 0$ represents potential gains from trade. The interaction term xy allows x and y to represent, for example, the probability x of obtaining a payoff of y, so that xy can be an expected value. The multiplicative form can be used to represent a wide variety of situations and plays a role in the analysis of the information externality.

By the revelation principle,[5] without loss of generality, any Bayesian

[5]See Myerson (1979), Dasgupta, Hammond and Maskin (1979), Gibbard (1973), and Harris and Townsend (1981).

equilibrium of the settlement bargaining game may be represented as a direct revelation bargaining game. A direct mechanism for the bargaining game consists of an expected payment $S(x,y)$ and a probability of trade $\Pi(x,y)$. For convenience, define the (interim) functions $S_x(x) \equiv \int_0^1 S(x,y)dG(y)$, and $S_y(y) \equiv \int_0^1 S(x,y)dF(x)$. The expected returns to the seller and buyer from reporting $\hat{x}$, respectively $\hat{y}$, are:

$$(1) \quad U(x,\hat{x}) = S_x(\hat{x}) - \int_0^1 \Pi(\hat{x},y)xydG(y),$$

$$(2) \quad V(y,\hat{y}) = \int_0^1 \Pi(x,\hat{y})(w + xy)dF(x) - S_y(\hat{y}).$$

Let $U(x) \equiv U(x,x)$ and $V(y) \equiv V(y,y)$. Incentive compatibility of the direct mechanism (S,Π) requires $U(x) \geq U(x,\hat{x})$ for all $x,\hat{x}$ in $[0,1]$ and $V(y) \geq V(y,\hat{y})$ for all $y,\hat{y}$ in $[0,1]$. Individual rationality requires $U(x) \geq 0$ and $V(y) \geq 0$. A mechanism (S,Π) is feasible if it is both incentive compatible and individually rational.

Attention is restricted to mechanisms (S,Π) that are integrable functions. This is without loss of generality since sufficient conditions are stated such that the efficient mechanism is a step function. By standard arguments, incentive compatibility implies that[6]

$$(3) \quad \int_0^1 \Pi(x,y)(x' - x)ydG(y) \geq U(x) - U(x') \geq \int_0^1 \Pi(x',y)(x' - x)ydG(y).$$

Taking limits on both sides of (3) it follows that for almost all x,

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[6]By equation (3), $\int_0^1 \Pi(x,y)ydG(y)$ is nonincreasing in x and, analogously, $\int_0^1 \Pi(x,y)xdF(x)$ is nondecreasing in y. Further, $U(x)$ is monotone decreasing in x and $V(y)$ is monotone increasing in y on $[0,1]$. Thus, by Royden (1968, Thm. 2, p. 96), $U(\cdot)$ and $V(\cdot)$ are differentiable almost everywhere and the derivatives dU/dx and dV/dy are measurable.

$dU(x)/dx = -\int_0^1 \Pi(x,y)y\,dG(y)$ and $dV(y)/dy = \int_0^1 \Pi(x,y)x\,dF(x)$. Integrating over x and y implies that

$$(4)\qquad U(x) = U(1) + \int_x^1 \int_0^1 \Pi(\tilde{x},y)y\,dG(y)\,d\tilde{x},$$
$$V(y) = V(0) + \int_0^y \int_0^1 \Pi(x,\tilde{y})x\,dF(x)\,d\tilde{y}.$$

<u>Expected gains from trade</u>, given the mechanism (S,Π), are

$$(5)\quad B(\Pi) = \int_0^1 \int_0^1 [U(x) + V(y)]dF(x)dG(y) = w \int_0^1 \int_0^1 \Pi(x,y)dF(x)dG(y).$$

Clearly, expected gains from trade are lowered by asymmetric information, $B(\Pi) \leq w$.

The expected returns from trade for each of the parties include <u>information rents</u>, which are the payments required to induce truth-telling in the direct revelation game and are defined by $R^U(\Pi) = \int_0^1 U(x)dF(x) - U(1)$ and $R^V(\Pi) = \int_0^1 V(y)dG(y) - V(0)$. Taking expectations in equation (4) and applying integration by parts yields

$$(6)\qquad R^U(\Pi) = \int_0^1 \int_0^1 y\Pi(x,y)(F(x)/f(x))dF(x)dG(y),$$
$$R^V(\Pi) = \int_0^1 \int_0^1 x\Pi(x,y)((1 - G(y))/g(y))dF(x)dG(y).$$

Let the inverse hazard rate for the seller, $F(x)/f(x)$, be continuously differentiable and increasing in x. Let the inverse hazard rate for the buyer, $(1 - G(y))/g(y)$, be continuously differentiable and decreasing in y. These assumptions are satisfied by the uniform distribution and the exponential distribution $F(x) = (1 - e^{-\lambda x})/(1 - e^{-\lambda})$ for $\lambda > 0$.

The following characterization result extends Myerson and Satterthwaite (1983) to allow for external effects of information. The proof is given in the Appendix.

Proposition 1: The mechanism (S,Π) is feasible if and only if $\int_0^1 \Pi(x,y)ydG(y)$ is nonincreasing in x, $\int_0^1 \Pi(x,y)xdF(x)$ is nondecreasing in y and

$$(7) \qquad B(\Pi) \geq R^U(\Pi) + R^V(\Pi).$$

Given full information, it is always optimal to trade since $w > 0$. To attain the full-information optimum by a feasible mechanism under asymmetric information would require $\Pi(x,y) = 1$ for all x and y. Proposition 1 then implies the following.

Corollary 1: The full information outcome is feasible if and only if $w \geq \int_0^1 ydG(y)$.

For example, if y is uniformly distributed on [0,1], the full information outcome is feasible for all $w \geq 1/2$. Alternatively, trade always occurs with a subsidy equal to $\max\{0, w - \int_0^1 ydG(y)\}$.

3. Efficient Mechanisms

The ex ante expected utility of the buyer and seller are

$$(8) \qquad \tilde{U}(S,\Pi) = \int_0^1 U(x;S,\Pi)dF(x)$$

(9) $$V(S,\Pi) = \int_0^1 V(y;S,\Pi)dG(y).$$

A feasible mechanism (S^*,Π^*) is <u>ex ante incentive efficient</u>[7] if no feasible allocation mechanism (S,Π) exists such that $\hat{U}(S,\Pi) \geq \hat{U}(S^*,\Pi^*)$ and $\hat{V}(S,\Pi) \geq \hat{V}(S^*,\Pi^*)$ with a strict inequality for the buyer or the seller. The following characterizes all ex ante incentive efficient allocation mechanisms, hereafter referred to as efficient mechanisms.

<u>Proposition 2</u>: A feasible allocation mechanism (S^*,Π^*) is efficient if and only if

(10) $$B(\Pi^*) - R^U(\Pi^*) - R^V(\Pi^*) = 0,$$

<u>and scalars</u> $\alpha,\beta \in [0,1]$ <u>exist such that</u>

(11) $$\Pi^*(x,y) = \begin{cases} 1 & w \geq \alpha yF(x)/f(x) + \beta x(1 - G(y))/g(y), \\ 0 & \text{otherwise.} \end{cases}$$

Define the trading boundary as follows:

$$l(x,y,\alpha,\beta) = w - \alpha yF(x)/f(x) - \beta x(1 - G(y))/g(y).$$

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[7] Ex ante incentive efficiency is defined by Holmstrom and Myerson (1983).

Then, note that I is monotonic in x.

$$\partial I(x,y,\alpha,\beta)/\partial x = -\alpha y\ (\partial(F(x)/f(x))/\partial x) - \beta(1 - G(y))/g(y) < 0.$$

Since $F(x)/f(x)$ is increasing in x by assumption.[8]

<u>Corollary 1</u>: The efficiency criterion for any weights α and β corresponds to a trading boundary $x = x^*(y,\alpha,\beta)$ that solves $I(x^*,y,\alpha,\beta) = 0$. Then, the efficient mechanism is given by

$$(12) \qquad \Pi^*(x,y) = \begin{cases} 1 & x \le x^*(y,\alpha,\beta) \\ 0 & \text{otherwise} \end{cases}$$

4. <u>The k-double Auction is Not Efficient</u>

The <u>k-double auction</u> is a Bayes-Nash game in offers. The seller and buyer submit sealed bids to an arbitrator. Trade occurs if and only if the seller's request $r = r(x)$ is less than or equal to the buyer's offer $c = c(y)$. If $r(x) \le c(y)$, trade occurs at the price given by the weighted average $s = kc + (1 - k)r$, where k takes values between zero and one and the value of k is common knowledge. The k-double auction thus corresponds to a direct mechanism for any value of k: $S(x,y) = kc(y) + (1 - k)r(x)$ if $c(y) \ge r(x)$, and $S(x,y) = 0$ otherwise; and $\Pi(x,y) = 1$ if $c(y) \ge r(x)$, and $\Pi(x,y) = 0$ otherwise. The seller and buyer net benefits are defined as

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[8] $I(x,y,\alpha,\beta)$ is differentiable in x and y. Given that $\Pi^*(x,y)$ is a step-function as defined in equation (12), $\Pi^*(x,y)$ is piecewise continuously differentiable and is therefore integrable.

follows.

$$(13)\qquad U(x;r,c) = \int_{\{y:c(y)\geq r\}} [kc(y) + (1-k)r - xy]dG(y).$$

$$(14)\qquad V(y;r,c) = -\int_{\{x:c\geq r(x)\}} [(w + xy) - (kc + (1-k)r(x))]dF(x).$$

The strategies $r(x)$ and $c(y)$ are equilibrium best responses, given the objective functions U and V. Incentive compatibility of the strategies implies

$$\int_{\{y:c(y)\geq r(x)\}} (x' - x)ydG(y) \geq \int_{\{y:c(y)\geq r(x')\}} (x' - x)ydG(y),$$

$$\int_{\{x:c(y)\geq r(x)\}} (y' - y)xdF(x) \geq \int_{\{x:c(y')\geq r(x)\}} (y' - y)ydF(x).$$

Given $x' > x$, $r(x') \geq r(x)$ so that $r(x)$ is nondecreasing in x. Similarly, $c(y)$ is nondecreasing in y. Individual rationality holds for all k if $r \geq xY(r(x))$ and $c \leq w + X(c(y))y$, where

$$Y(r) \equiv \int_{\{y:c(y)\geq r\}} ydG(y)/ \int_{\{y:c(y)\geq r\}} dG(y),$$

$$X(c) \equiv \int_{\{x:c\geq r(x)\}} xdF(x)/ \int_{\{x:c\geq r(x)\}} dF(x).$$

Note that $\{y: c(y) \geq r\} = \emptyset$ if $r > c(1)$ and $\{x: c \geq r(x)\} = \emptyset$ if $c < r(0)$.

Attention is restricted to regular strategies. The following restrictions extend the analysis of equilibrium strategies in Chatterjee and

Samuelson (1983) and Satterthwaite and Williams (1989).[9] The strategies $r(x), c(y)$ are said to be regular if they are continuous and strictly increasing, r is C^1 on $[0, c(1)]$, c is C^1 on $[r(0), T(a + 1) + 0]$, and the following hold:

$$(15) \qquad xY(r(x))) \leq r(x) \leq 1 \text{ for all } x \in [0,1].$$

$$(16) \qquad 0 \leq c(y) \leq w + X(c(y))y \text{ for all } y \in [0,1].$$

Also, the first inequality in (15) is binding if $xY(r(x))) \geq c(1)$ and the second inequality in (16) is binding if $w + X(c(y))y \leq r(0)$. The maximization problems of the seller and buyer, given their objective functions (13) and (14), yield the following:

Proposition 3: If $\hat{r}(x), \hat{c}(y)$ are regular equilibrium strategies in the k-double auction with $k \in [0,1]$, then in the range defined by (15)-(16), the strategies satisfy the differential equations

$$(17) \qquad \hat{r}(x) = x\hat{y} - (1 - k)\hat{c}'(\hat{y})[(1 - G(\hat{y}))/g(\hat{y})],$$

$$(18) \qquad \hat{c}(y) = w + \hat{x}y - k\hat{r}'(\hat{x})F(\hat{x})/f(\hat{x})),$$

where $\hat{y} = \hat{y}(x)$ and $\hat{x} = \hat{x}(y)$ are on the trading boundary, $\hat{r}(x) = \hat{c}(y)$.

The efficiency of the equilibrium of the k-double auction is now

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[9] The k-double auction may have multiple equilibria. See Satterthwaite and Williams (1989) and Leininger, Linhart and Radner (1989).

examined.

<u>Proposition 4</u>: The Bayes-Nash equilibrium of the κ-double auction for all $\kappa \in [0,1]$ is not interim incentive efficient.

<u>Proof</u>: From Proposition 3, the trading boundary for the κ-double auction $y = \hat{v}(x)$ is

$$(19) \qquad w - \hat{\kappa} r'(x)F(x)/f(x) + (1 - \hat{\kappa})c'(y)[(1 - G(y))/g(y)] = 0.$$

Compare equation (19) with the interim incentive efficient trading boundary $y = y^*(x)$ or $w - \alpha y^* F(x)/f(x) + \beta x(1 - G(y^*))/g(y^*) = 0$. Suppose that the trading boundaries are equal. Then, combining (19) and (20) implies that

$$(20) \qquad \frac{\alpha y - \hat{\kappa} r'(x)}{\alpha y(1 - \hat{\kappa})c'(y) - \beta x \hat{\kappa} r'(x)} = \frac{1 - G(y)}{g(y)}\frac{1}{w}.$$

Note that $r'(x) > 0$, $c'(y) > 0$. The equation implies that the left side does not depend on x, which is a contradiction. Q.E.D.

The efficient trading boundary involves multiplicative interaction terms involving x and y while in the Bayes-Nash equilibrium of the κ-double auction, the trading boundary is additively separable in x and y. This suggests that the κ-double auction does not make efficient use of the information possessed by the seller and buyer.

5. The First-and-Final Offer Bargaining Game is Not Efficient

This section examines the sequential equilibrium of a first-and-final offer bargaining game. The game is studied for the case in which the buyer makes an offer, and the seller then decides whether to accept or reject the offer. All of the conclusions apply equally to the case in which the seller makes an offer and the buyer makes a decision whether or not to meet the offer. This symmetry occurs since both parties have private information about the common value. This differs from the standard approach in many bargaining games in which only one party has private information about the common value. There, the party making the first-and-final offer is either the "informed" or "uninformed" party.[10]

For ease of presentation, let the distribution of the seller's information, $F(x)$, be uniform on $[0,1]$. The buyer makes a settlement offer $c(y)$. The seller observes $c(y)$ and forms expectations about the buyer's type y. Since the seller is risk-neutral, it is sufficient to represent the seller's beliefs by the estimate of the expected value of y, $\hat{y} = \gamma(c)$. The seller accepts the settlement offer, $c = c(y)$, if and only if it exceeds the value xy, that is, $c \geq x\gamma(c)$. Thus, the seller accepts $c(y)$ if and only if the seller's information x is below a critical value: $x \leq x(c) \equiv c/\gamma(c)$. The expected net benefit to a buyer of type y obtained by offering c is then

$$V(c) = \int_0^{x(c)} (w - xy - c)dx.$$

The sequential equilibrium for the settlement bargaining game consists

[10] This is the case, for example, in the models examined by Bebchuk (1984), P'ng (1983), Reinganum and Wilde (1985), and Nalebuff (1987). See Spulber (1985) for further discussion.

of strategies $c^*(y)$ and $x^*(c)$, and beliefs $\gamma^*(c)$ such that:

(A) The buyer chooses $c^*(y)$ to maximize expected net benefits, $V(c,x^*,\gamma^*;y) = \int_0^{x(c)} (w + xy - c)dx$ for all y in $[0,1]$;

(B) The seller chooses the critical acceptance value $x^*(c) = c/\gamma^*(c)$ that maximizes expected net benefits; and

(C) The seller's beliefs, $\gamma^*(c)$, on the equilibrium path, $c^{*^{-1}}(c) \neq \emptyset$, are consistent with Bayes' rule and the buyer's equilibrium strategy $c^*(y)$.

Attention is restricted to equilibria that are separating across seller types. Characterization of the set of separating equilibria does not require application of refinements to determine whether there is a unique equilibria since the result holds for all equilibria of the sequential equilibria game that are separating across seller types.

It can be shown by incentive compatibility that $c^*(y)$ is nondecreasing in y. The buyer's strategy satisfies the first order condition

$$(21) \qquad -x^*(c) + (w + x^*(c)y - c)x^{*\prime}(c) = 0.$$

At a fully separating equilibrium, the seller is able to invert the buyer's equilibrium strategy $c^*(y)$ and correctly infer y, $y = \gamma^*(c^*(y))$. Thus, for values of c on the equilibrium path, the critical value $x^*(c)$ is given by $x^*(c) = c/y$. Solve for y and substitute into (21) to obtain

$$(22) \qquad x^{*\prime}(c)/x^*(c) = 1/w.$$

This differential equation yields the <u>seller's equilibrium strategy</u>, $x^*(c) = Ke^{c/w}$, where K is a constant of integration. Note that $x^{*\prime}(c) = (K/w)e^{c/w}$. Substitute for $x^*(c)$ and $x^{*\prime}(c)$ in the buyer's first order condition to obtain an implicit solution of the <u>buyer's equilibrium strategy</u> $c^*(y)$,

$$(23) \qquad c^*(y) - yKe^{c^*(y)/w} = 0.$$

The trading boundary is simply $x = x^*(c^*(y))$ or $y = y^*(x)$, which solves

$$(24) \qquad c^*(y^*(x)) - y^*(x)x = 0.$$

Assume that the buyer's strategy satisfies the second order condition for an interior solution, $V''(c) < 0$, where

$$V''(c) = (w + x^*(c)y - c)x^{*\prime\prime}(y) + (x^{*\prime}(c))^2 y - 2x^{*\prime}(c).$$

Note that $x^{*\prime\prime}(c) = x^{*\prime}(c)/w$, $x^{*\prime}(c) = x^*(c)/w$, and $c = x^*(c)y$ so that the second order sufficient condition is $V''(c) = (x^*(c)/w^2)(x^*(c)y - w) > 0$.

<u>Proposition 5</u>: The sequential equilibrium of the first-and-final offer bargaining game is not ex ante incentive efficient.

<u>Proof</u>: Since the settlement boundary in the sequential equilibrium game is invariant with respect to the distribution, G, we need only consider the efficient solutions that do not depend on G. In particular, let $\beta = 0$.

Since $F(x)$ is uniform, the efficient trading boundary expressed as a function of y is simply $w - \alpha x(y)y = 0$. Compare with the trading boundary in the sequential equilibrium, $c^*(y) - x^*(y)y = 0$. These are not equivalent since $c^*(y)$ is decreasing in y. So, the sequential equilibrium (c^*, x^*, γ^*) is not ex ante efficient. Q.E.D.

6. An Efficient Trading Mechanism

Ex ante efficient mechanisms can be implemented by a trading process based on priority pricing. The buyer and seller face payment schedules based on their announcements of the likelihood of trade. The announcements are then used to determine whether or not trade takes place. The announcements of the likelihood of trade are consistent with the likelihood of trade in equilibrium.

We restrict attention to regular mechanisms. This is a restriction on the ratio of the scalar weights α and β.

Definition: An efficient mechanism (Π^*, S^*) is said to be regular if and only if

$$\frac{\partial}{\partial y}\left[\frac{1 - G(y)}{g(y)}\right] > \frac{\alpha}{\beta}\,\frac{F(x)}{xf(x)}$$

for all x and y.

Suppose, for example, that the distributions F and G are uniform. Then, the regularity condition requires $\beta > \alpha$.

For regular mechanisms, the interim expected probability of trade can be written as follows.

(25) $$\Pi_x^*(x) = \int_0^1 \Pi^*(x,y)dG(y) = 1 - G(y^*(x))$$

(26) $$\Pi_y^*(y) = \int_0^1 \Pi^*(x,y)dF(x) = F(x^*(y))$$

where $x = x^*(y)$ is the trading boundary and $y = y^*(x)$ is the inverse function.

Proposition 6: For regular mechanisms, the probability of trade, $\Pi^*(x,y)$, is such that $\Pi_x^*(x)$ is decreasing in x and $\Pi_y^*(y)$ is increasing in y.

Proof: By regularity,

$$\frac{\partial x^*(y)}{\partial y} = \frac{-I_y(x^*,y,\alpha,\beta)}{I_x(x^*,y,\alpha,\beta)} > 0,$$

since $I_y > 0$ by regularity and $I_x < 0$. So, $\partial\Pi_x^*(x)/\partial x = -g(y^*(x))\partial y^*(x)/\partial x < 0$ and $\partial\Pi_y^*(y)/\partial y = f(x^*(y))\partial x^*(y)/\partial y > 0$. Q.E.D.

The trading boundary $y = y^*(x)$ for a regular mechanism is increasing in x by Proposition 6. This implies that the unit square can be partitioned into regions where the probability of trade equals one or zero for a particular mechanism. The probability of trade equals one above the line $y^*(x)$ and zero below it. Two cases are illustrated in Figures 1a and 1b. In Figure 1a, for example, for $y > y''$, bargaining will always succeed. Figure 1b, in contrast, shows that with $x > x'$, bargaining will not result in trade for any y. In both cases, trade requires $y > y'$ for the buyer.

Additional cases are possible if $y^*(x)$ intersects the lower boundary.

A trading process can be designed based on priority pricing. Priority pricing is a pricing system for allocating randomly available capacity (see Harris and Raviv (1981)). Under priority pricing, consumers choose a rank order of service, paying more for a higher rank. After the available capacity has been observed, the capacity is allocated to consumers in rank order. This can be shown to be equivalent to a system in which consumers choose a probability of obtaining a unit of the good, referred to as reliability, and pay more for greater reliability.

A trading process based on priority pricing can be constructed as follows. Let π_x and π_y represent the likelihood of trade from the point of view of the seller and buyer, respectively. The seller and buyer face (nonlinear) priority price schedules $P_x(\pi)$ and $P_y(\pi)$. A trading rule determines whether or not trade occurs, $H(\pi_x, \pi_y) \in \{0,1\}$. Given the price schedules and the trading rule, the seller and buyer announce their estimates of the likelihood of trade, π_x^*, π_y^*. The following is proved in the Appendix.

<u>Proposition 7</u>: A regular efficient mechanism (Π^*, S^*) is implemented by a trading process consisting of priority price schedules

$$(27)\qquad P_x(\pi_x) = \int_{y^*(1)}^{1} y\,dG(y) + U(1) - \int_{X(\pi_x)}^{1} x(\partial\Pi_x^*(x)/\partial x)y^*(x)dx$$

$$(28)\qquad P_y(\pi_y) = wF(x^*(Y(\pi_y))) - V(0) + \int_{0}^{Y(\pi_y)} y(\partial\Pi_y^*(y)/\partial y)x^*(y)dy$$

and the trading rule H.

$$(29) \qquad H(\pi_x, \pi_y) = \begin{cases} 1 & X(\pi_x) < x^*(Y(\pi_y)), \\ 0 & \text{otherwise}, \end{cases}$$

where $X(\pi_x)$ is the inverse of $\pi_x = \Pi_x^*(x)$ and $Y(\pi_y)$ is the inverse of $\pi_y = \Pi_y^*(y)$.

It can be demonstrated that, given the trading rule H, a higher π_x or π_y raises the equilibrium likelihood of trade. It also can be verified that both priority price schedules are increasing in the announcements of the likelihood of trade. Therefore, the buyer pays more to obtain a higher likelihood of trade in equilibrium. The seller receives a higher revenue by increasing the equilibrium likelihood of trade. Thus, the priority pricing trading mechanism provides incentives for ex ante efficient trade.

7. Conclusion

The bargaining problem has been examined in the presence of information externalities. Efficiency results obtained with independent private values were shown not to carry over to more general settings. The k-double auction does not make full use of available information. Sequential signaling in the first-and-final offer bargaining game allows the party making the offer to communicate information. However, the decision to accept or reject the offer by the party receiving the offer does not adjust the likelihood of trade in a manner that reflects the information about the good's value.

The equilibrium of the priority pricing game addresses these problems. The priority price schedules induce the seller and buyer to reveal their types through their choices of the expected likelihood of trade. The transfer payment and trading rule then depend on the private information of the seller and buyer in an efficient way. This suggests that more general information structures, such as the one considered here, may require more complex bargaining procedures.

Appendix

Proof of Proposition 1: Let (S,Π) be a feasible mechanism. Then, incentive compatibility implies that $\int_0^1 \Pi(x,y)ydG(y)$ is nonincreasing in x and $\int_0^1 \Pi(x,y)xdF(x)$ is nondecreasing in y by eq. (3). Individual rationality requires $U(1) \geq 0$ and $V(0) \geq 0$. From eq. (4),

$$\text{(A.1)} \qquad U(1) + V(0) = \int_0^1 \int_0^1 [U(x) + V(y)]dF(x)dG(y)$$

$$- \int_0^1 \int_0^1 [(\int_x^1 \Pi(\tilde{x},y)yd\tilde{x}) + (\int_0^y \Pi(x,\tilde{y})xd\tilde{y})]dF(x)dG(y).$$

Integration by parts and the definition of U and V yields

$$\text{(A.2)} \qquad U(1) + V(0) = B(\Pi) - R^U(\Pi) - R^V(\Pi)$$

so that eq. (7) holds.

Conversely, suppose that $\int_0^1 \Pi(x,y)ydG(y)$ is nonincreasing in x and $\int_0^1 \Pi(x,y)xdF(x)$ is nondecreasing in y, and that eq. (7) holds. Let the expected payment $S(x,y)$ have the following form:

$$\text{(A.3)} \qquad S(\hat{x},\hat{y}) = \int_0^1 \Pi(\hat{x},y)dG(y)$$

$$+ \int_0^1 [\int_{\hat{x}}^1 y\Pi(x,y)dx]dG(y)$$

$$+ \int_0^1 \Pi(x,\hat{y})(w + x\hat{y})dF(x)$$

$$+ \int_0^1 \int_0^1 \Pi(x,y)[(w + xy) - x((1 - G(y))/g(y))]dF(x)dG(y)$$

$$- \int_0^{\hat{y}} [\int_0^1 x\Pi(x,y)dy]dF(x).$$

Taking the expectation of $S(x,y)$ over y and over x yields

$$\text{(A.4)} \qquad S_x(\hat{x}) = \int_0^1 \Pi(\hat{x},y)\hat{x}y\,dG(y)$$

$$+ \int_0^1 [\int_{\hat{x}}^1 y\Pi(x,y)dx]dG(y),$$

$$\text{(A.5)} \qquad S_y(\hat{y}) = -B(\Pi) - R^P(\Pi) + R^D(\Pi)$$

$$+ \int_0^1 \Pi(x,\hat{y})(w + x\hat{y})dF(x)$$

$$- \int_0^1 [\int_0^{\hat{y}} x\Pi(x,y)dy]dF(x).$$

Therefore,

$$\text{(A.6)} \qquad U(x,x) - U(x,\hat{x}) = S_x(x) - S_x(\hat{x})$$

$$- \int_0^1 [\Pi(x,y) - \Pi(\hat{x},y)]xy\,dG(y)$$

$$= \int_0^1 \Pi(x,y)xy\,dG(y)$$

$$- \int_0^1 \Pi(\hat{x},y)\hat{x}y\,dG(y)$$

$$+ \int_0^1 [\int_x^1 y\Pi(\tilde{x},y)d\tilde{x} - \int_{\hat{x}}^1 y\Pi(\tilde{x},y)d\tilde{x}]dG(y)$$

$$+ \int_0^1 [\Pi(x,y) - \Pi(\hat{x},y)]xydG(y).$$

Rearranging terms and applying the definition of integration gives

$$U(x,x) - U(x,\hat{x}) = \int_0^1 [\int_x^{\hat{x}} [\Pi(\tilde{x},y) - \Pi(\hat{x},y)]d\tilde{x}]ydG(y). \tag{A.7}$$

Since $\int_0^1 y\Pi(x,y)dG(y)$ is nonincreasing in x, reversing the order of integration in equation (A.7) implies $U(x,x) - U(x,\hat{x}) \geq 0$ for all $x,\hat{x} \in [0,1]$, which establishes incentive compatibility of Π in x. Similiar arguments hold for y.

It remains to be shown that (S,Π) is individually rational for both parties since (S,Π) is incentive compatible, U(x) is nonincreasing in x and V(y) is nondecreasing in y. So, it is sufficient to show that $U(1) \geq 0$ and $V(0) \geq 0$. From equations (A.4) and (4) and the definition of U,

$$U(1) = S_x(x) - \int_0^1 \Pi(x,y)xydG(y)$$

$$- \int_x^1 [\int_0^1 \Pi(\tilde{x},y)ydG(y)]d\tilde{x} = 0.$$

From equations (A.5) and (4), and the definition of V,

$$V(0) = -S_y(y) + \int_0^1 \Pi(x,y)(w + xy)dF(x)$$

$$- \int_0^y [\int_0^1 \Pi(x,\tilde{y})xdF(x)]d\tilde{y}$$

So, $V(0) = B(\overline{\Pi}) - R^{U}(\overline{\Pi}) - R^{V}(\overline{\Pi})$. Thus, by eq. (7), $V(0) \geq 0$. Therefore, the mechanism $(S, \overline{\Pi})$ is individually rational and incentive compatible.

Q.E.D.

<u>Proof of Proposition 7</u>: Given the trading process (P_x, P_y, H), let $\pi_x^*(x), \pi_y^*(y)$ represent a Bayes-Nash equilibrium. The equilibrium strategies for the seller and buyer maximize the following objective functions.

$$U(\pi_x, \pi_y^*, x) = P_x(\pi_x) - \int_0^1 H(\pi_x, \pi_y^*(y))dG(y)$$
$$V(\pi_x^*, \pi_y, y) = P_y(\pi_y) - \int_0^1 H(\pi_x^*(x), \pi_y)dF(x).$$

Using integration by parts, it can be shown that

$$P_x(\pi_x) = U(1) + \int_{X(\pi_x)}^1 \int_{y^*(\tilde{x})}^1 y dG(y) d\tilde{x} + \int_{y^*(X(\pi_x))}^1 X(\pi_x) y dG(y)$$

$$P_y(\pi_y) = \int_0^{x^*(Y(\pi_y))} (w + xY(\pi_y))dF(x) - \int_0^{Y(\pi_y)} \int_0^{x^*(\tilde{y})} x dF(x) d\tilde{y} - V(0).$$

It can then be shown that the expected efficient probabilities of trade, $(\overline{\Pi}_x^*(x), \overline{\Pi}_y^*(y))$ are Nash equilibrium strategies. Furthermore, $H(\overline{\Pi}_x^*(x), \overline{\Pi}_y^*(y)) = \overline{\Pi}^*(x,y)$, $P_x(\overline{\Pi}_x^*(x)) = S_x^*(x)$ and $P_y(\overline{\Pi}_y^*(y)) = S_y^*(y)$. By the revelation principle, the trading process implements the efficient mechanism $(\overline{\Pi}^*, S^*)$.

Q.E.D.

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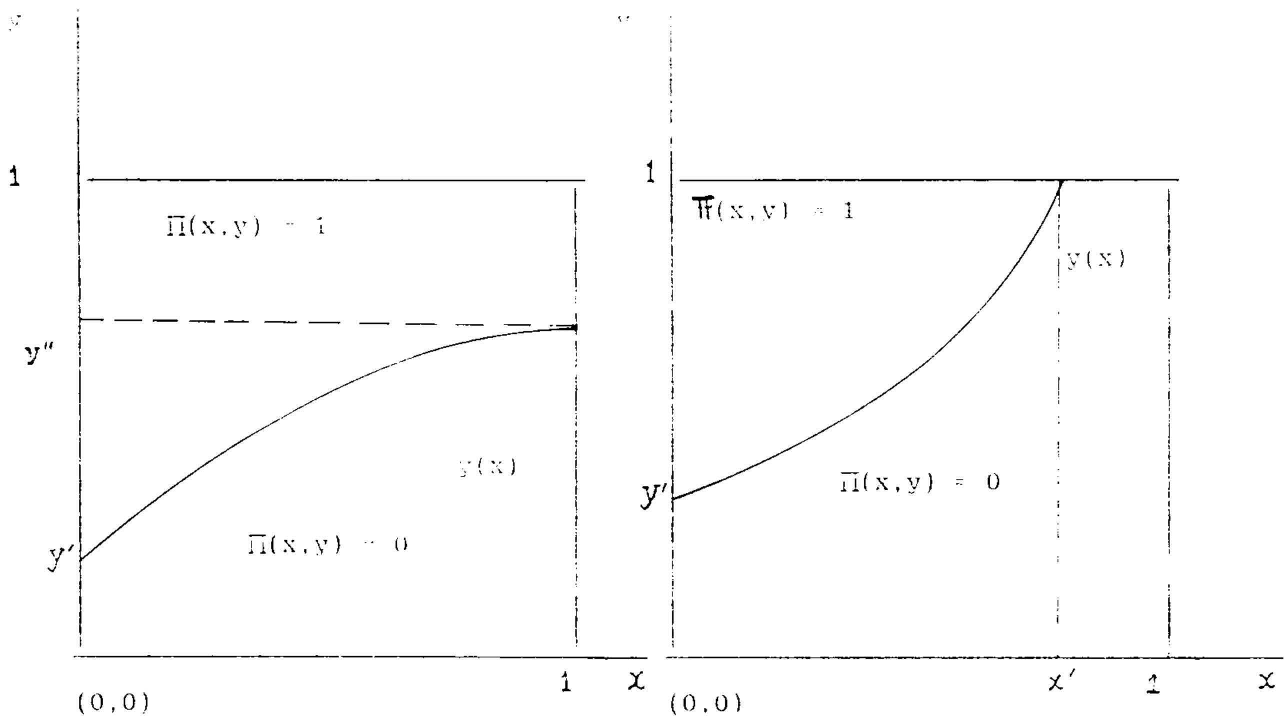

Figure 1a   Figure 1b

<u>The Trading Boundary</u>