Northwestern University
2001 Sheridan Road 580 Leverone Hall Evanston, IL 60208-2014 USA

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"Custom Versus Fashion: Hysteresis and Limit Cycles in a Random Matching Game"

Kiminori Matsuyama
Northwestern University

www.kellogg.nwu.edu/research/math

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CUSTOM VERSUS FASHION: HYSTERESIS AND LIMIT CYCLES IN A RANDOM MATCHING GAME*

by

Kiminori Matsuyama

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Abstract

This paper considers a simple pairwise random matching game in the society populated by two groups of agents: Conformists and Nonconformists. Depending on the relative frequencies of intergroup and intragroup matchings, the best response dynamics show three types of asymptotic behaviors: global convergence, hysteresis and limit cycles. In the hysteresis case, Conformists set the social custom, and Nonconformists revolt against it; what action becomes the custom is determined by "history." In the limit cycle case, Nonconformists become fashion leaders and switch their actions periodically, while Conformists follow with delay.

Keywords: Best response dynamics, Bifurcation, Equilibrium refinement, Evolutionary process, Hysteresis, Limit cycles, Perfect foresight dynamics, Strategic complements and substitutes.

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*Department of Economics, Northwestern University, 2003 Sheridan Road, Evanston, IL 60208, U.S.A. I would like to thank participants of the Math Center Bag Lunch seminar at Northwestern University for their helpful comments.

"Fashion is custom in the guise of departure from custom."

Edward Sapir

"Fashion is evolution without destination."

Agnes Brooks Young

1. Introduction

Fashion is the process of continuous change in which certain forms of social behavior enjoy temporary popularity only to be replaced by others. This pattern of change sets fashion apart from social custom, which is time-honored, legitimated by tradition, and passed down from generation to generation. Fashion is also a recurring process, in which many "new" styles are not so much born as rediscovered: see, for example, Young (1937). This cyclical nature, or its regularity, sets fashion apart from fads, which are generally considered as rather bizarre one-time aberrations.[1] Although most conspicuous in the area of dress, many other areas of human activity are also under the sway of fashion. Among them are architecture, music, painting, literature, business practice, political doctrines, as well as scientific ideas (not least in economic theory). Despite its pervasiveness and its apparent significance as a determinant of variations in demand, very few attempts have been made by economists, supposedly experts of cyclical behavior, to identify mechanisms generating fashion cycles.

On the other hand, there is no shortage of theories in the fields of psychology and sociology; see, for example, Sapir (1930) and Blumer (1968). Two psychological tendencies are often put forward as fundamental forces behind continuous changes and diffusions of fashion. Many observers point out the importance of conformity in the establishment of fashion; that is, the

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[1]Blumer (1968, p.344) wrote: "The most noticeable difference (from fashion) is that fads have no line of historical continuity; each springs up independent of a predecessor and gives rise to no successor."

desire of people to adopt and imitate the behavior of others, or to join the crowd. Such conformist attitudes may result from widely divergent motives. People may imitate out of admiration for one imitated or by the desire to assert equality with her. Those who follow fashions may do so with enthusiasm or may simply be coerced by public opinion exercised through ridicule and social ostracism. Other observers of fashion also emphasize the importance of nonconformity. That is, they find the essence of fashion in the search for exclusiveness or the efforts of people to acquire individuality and personal distinction; they treat fashion as an expression of the desire to escape from the tyranny of the prevailing social custom and to disassociate one's self from the common masses.

It should be immediately clear that, for the recurring process of fashion to emerge and persist, these two fundamentally irreconcilable desires of human beings -- the desire to act or look the same, and the desire to act or look different-- both must operate. We cannot explain continuous changes in the process of fashion by merely pointing out that it is the product of conformity and imitation, because, if everybody conforms, the process would eventually cease, and certain forms of behavior would emerge as the social custom, or convention. The desire for personal distinction or exclusiveness must work against universal adaptation of a fashion. Nor can we adequately explain the regularity of fashion cycles by saying that it is the product of nonconformity, because, if everybody seeks individuality, the result would be disorderly, and utterly unpredictable. The forces of imitation and uniformity need to be strong enough for any discernible patterns to emerge. As Simmel ([1904]1957, p.546) wrote, "two social tendencies are essential to the establishment of fashion, namely, the need of union on the one hand and the need of isolation on the other. Should one of these be absent, fashion will not be formed--its sway will abruptly end."

This paper attempts to demonstrate in a formal model that such a delicate balance between conformity and nonconformity is not only necessary but also sufficient for the emergence of fashion cycles, while too strong conformity would lead to the emergence of the social custom, and too strong nonconformity lead to disorder. To be more specific, I will consider a simple pairwise random matching game, played by two types of agents: Conformists and Nonconformists. All agents are matched with both types of agents with positive probability: there are both intergroup and intragroup matchings. Each agent must take one of two actions, Blue and Red, at any point in time, before knowing the type of the agent s/he will meet. When matched, agents observe the choice made by their partners. One type, the Conformist, gains a higher payoff if he and his partner have made the same choice. The other type, the Nonconformist, gains a higher payoff if she has made the choice different from her partner's.[2]

In the absence of any inertia in changing actions, this game can be analyzed as a static game. The Nash equilibria of this static game depend on the two ratios of intergroup and intragroup matchings, one for a Conformist and the other for a Nonconformist. They are in turn determined by the two parameters of the model: the ratio of Conformists to Nonconformists in the society, and the relative frequency in which a given pair of agents from the

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[2]This game between Conformists and Nonconformists is inspired by Gaertner, Pattanaik, and Suzumura (1990). It should be noted that this game is anonymous in that the type of one's partner in matching does not affect the payoffs. Therefore, the class affiliation of an agent does not play any role in this model. See, however, Karni and Schmeidler (1990) and Matsuyama (in process).

same group is matched, compared to a given pair of agents from the different groups is matched.

In order to investigate the dynamic path of behavior patterns in the presence of inertia, as well as to test the dynamic stability of the Nash equilibria of the static game, the best response dynamics due to Gilboa and Matsui (1991) is considered. This dynamics, which can be considered as the limit case of the perfect foresight dynamics due to Matsuyama (1991a,b), leads to a variety of asymptotic properties, such as global convergence, hysteresis, and a limit cycle, depending on the parameters. These results are used to identify socially stable behavior patterns. In the global convergence case, one half of both Conformists and Nonconformists choose Blue and the other half chooses Red, so that no patterns emerge. In the hysteresis case, Conformists set the social custom, and Nonconformists revolt against it; what action becomes the custom is determined by the initial condition, or by "history." In the limit cycle case, Nonconformists become fashion leaders and switch their actions periodically, while Conformists follow with delay.

The richness of the asymptotic properties may be understood in terms of strategic complements and substitutes: see Bulow, Geanakoplos, and Klemperer (1985) for the definitions. The two actions are strategic complements for a Conformist, and strategic substitutes for a Nonconformist. Therefore, the game, if played by a Conformist and a Nonconformist, or by a pair of Nonconformists, would have a unique Nash equilibrium, in which one half of the population chooses Blue and the other half chooses Red. This equilibrium is globally stable according to the best response dynamics. On the other hand, if played by a pair of Conformists, there would be two additional Nash equilibria, in which every agent chooses the same action. The best response dynamics show that these two equilibria are locally stable, while the equilibrium with mixed strategies is unstable. Allowing for both intergroup and intragroup matchings blends two games with different properties, thereby generating much richer dynamic paths of behavior patterns. In fact, the limit cycle can be generated by two kinds of bifurcation in this model. The first case is when an increase in the ratio of Conformists leads to a loss of stability in the unique Nash equilibrium in the game with strategic substitutes; the regular patterns of fashion cycles emerge from the disorder. The second case is when a decrease in the ratio of Conformists eliminates the two locally stable Nash equilibria in the game with strategic complements; fashion cycles emerge as departure from custom.

The present paper is partly related to the growing literature on evolutionary dynamics and equilibrium refinement in normal form games, such as Friedman (1991), Gilboa and Matsui (1991), Nachbar (1990), Samuelson and Zhang (1990), and van Damme (1987, Ch. 9.4). In this literature, it is typically assumed either that the game is played by the homogeneous population or that, when the population is heterogeneous, consisting of, say, males and females, all matchings are between a male and a female. It should be noted that Gilboa and Matsui have demonstrated the possibility of limit cycles in the best response dynamics, but their example is a two person 3x3 game with the homogeneous population. It is fairly straightforward to show that the best response dynamics do not produce any cycle in a two person 2x2 game if all matchings are either intergroup or intragroup.[3] The reason why a limit cycle occurs in the two person 2x2 game discussed below is that the population is

[3] In this sense, the best response dynamics are similar to the fictitious play; see Miyasawa (1961) and Shapley (1964).

heterogeneous and both inter- and intragroup matchings are possible.

The rest of the paper is in four parts. Section 2 describes the matching game and finds all Nash equilibria (Proposition 1). Section 3 introduces the best response dynamics and characterizes its asymptotic properties (Proposition 2). Section 4 interprets the results. Section 5 discusses related work in the economic literature.

2. The Matching Game

Time is continuous and extends from zero to infinity. There is a continuum of anonymous agents in this society. They are divided into two groups: Conformists, whose measure is equal to $\theta \in (0,1)$, and Nonconformists, whose measure is $1-\theta$. Agents randomly meet each other in pairs. The pairwise matchings occur according to the following Poisson process. At any small time interval, dt, a Conformist runs into another Conformist with probability $(1-\beta)\theta dt$; he runs into a Nonconformist with probability $\beta(1-\theta)dt$, where $\beta \in (0,1)$. On the other hand, a Nonconformist runs into a Conformist with probability $\beta\theta dt$; she runs into another Nonconformist with probability $(1-\beta)(1-\theta)dt$. This matching process assumes that the probability with which one is matched with an agent from a given group is proportional to the size of that group. The matching distribution also depends $\beta/(1-\beta)$, which represents the relative frequency with which a given Conformist-Nonconformist pair is matched compared to a given Conformist-Conformist pair or a given Nonconformist-Nonconformist pair. Thus, if $\beta = 1/2$, the matching process is uniform; if $\beta > 1/2$, it is biased toward intergroup matchings; if $\beta < 1/2$, it is biased toward intragroup matchings.[4]

[4]We have used similar nonuniform matching technologies in Matsui and Matsuyama (in process) and Matsuyama, Kiyotaki and Matsui (1991).

It should be noted that, from each agent's point of view, the relative frequency of intergroup matchings to intragroup matchings depends not only on β but also on $\theta/(1-\theta)$, the relative size of the two groups. For a Conformist, the ratio of inter- versus intragroup meetings is equal to

(1a) $m = \beta(1-\theta)/(1-\beta)\theta$,

while, for a Nonconformist, it is equal to

(1b) $m^* = \beta\theta/(1-\beta)(1-\theta)$.

Note that Equations (1a) and (1b) define a one-to-one mapping between $(m,m^*) \in R_+^2$ and $(\theta,\beta) \in (0,1)^2$. There are six generic cases to be distinguished:

Case 1:	$m < 1 < mm^*$,	or	$1/2 < \beta < \theta$,
Case 2:	$m < mm^* < 1$,	or	$1-\theta < \beta < 1/2$,
Case 3:	$mm^* < m < 1$,	or	$\beta < \theta < 1 - \beta$,
Case 4:	$m > 1 > mm^*$,	or	$1/2 > \beta > \theta$,
Case 5:	$m > mm^* > 1$,	or	$1-\theta > \beta > 1/2$,
Case 6:	$mm^* > m > 1$,	or	$\beta > \theta > 1 - \beta$.

Case 6 is further divided into the two cases:

Case 6a:	$m \geq m^* > 1$,	or	$1/2 \geq \theta > 1 - \beta$,
Case 6b:	$m^* > m > 1$,	or	$\beta > \theta > 1/2$.

For the sake of brevity, I will not discuss nongeneric cases, $\beta = 1/2$ ($mm^* = 1$), $\theta = \beta$ ($m = 1$), or $\theta + \beta = 1$ ($m^* = 1$).

At any moment, every agent has to choose between Blue (B) or Red (R). When matched an agent observes the choice of his/her partner. A Conformist gains the payoff, S, if his partner has made the same choice with his, and otherwise gains D, where S > D. A Nonconformist gains S^* if her partner has made the same choice with hers, and otherwise gains D^*, where $S^* < D^*$. Let (λ_t, λ_t^*) be the <u>behavior patterns</u> in the society as of time t, where λ_t (λ_t^*) denotes the fraction of Conformists (Nonconformists) that chooses Strategy B. Then, the (instantaneous) probability with which a Conformist runs into an agent who chose B and R, are equal to

$$p_{Bt} = \lambda_t(1-\beta)\theta + \lambda_t^*\beta(1-\theta) \ , \qquad p_{Rt} = (1-\lambda_t)(1-\beta)\theta + (1-\lambda_t^*)\beta(1-\theta) \ ,$$

respectively. Likewise, the probability with which a Nonconformist runs into an agent who chose B and R, are

$$p_{Bt}^* = \lambda_t\beta\theta + \lambda_t^*(1-\beta)(1-\theta) \ , \qquad p_{Rt}^* = (1-\lambda_t)\beta\theta + (1-\lambda_t^*)(1-\beta)(1-\theta) \ .$$

Then, a Conformist's expected payoffs per unit of time if he chooses B and R, are given by

$$\Pi_{Bt} = p_{Bt}S + p_{Rt}D \ , \qquad \Pi_{Rt} = p_{Bt}D + p_{Rt}S \ ,$$

and, for a Nonconformist,

$$\Pi_{Bt}^* = p_{Bt}^*S^* + p_{Rt}^*D^* \ , \qquad \Pi_{Rt}^* = p_{Bt}^*D^* + p_{Rt}^*S^* \ .$$

Before proceeding to describe the evolution of the behavior patterns, it is useful to look briefly at the nature of social interactions in this society if there were no inertia in changing the behavior patterns. Agents, being atomistic and anonymous, could play this game at any point in time as a static, one-shot game, in the absence of any inertia. Dropping the time subscript, note that S > D implies $\Pi_B > \Pi_R$ if and only if $p_B > p_R$ and $S^* < D^*$

implies $p_B^* < p_R^*$ if and only if $p_B^* > p_R^*$. The best responses are thus given by $\lambda = 1$ if $p_B > p_R$; $= [0,1]$ if $p_B = p_R$; $= 0$ if $p_B < p_R$ and $\lambda^* = 1$ if $p_B^* < p_R^*$; $= [0,1]$ if $p_B^* = p_R^*$; $= 0$ if $p_B^* > p_R^*$. After some algebra, these conditions can be written as

$$\text{(2a)} \qquad \lambda \in \begin{cases} \{1\} & \text{if} \quad (\lambda-1/2) + m(\lambda^*-1/2) > 0 \ , \\ [0,1] & \text{if} \quad (\lambda-1/2) + m(\lambda^*-1/2) = 0 \ , \\ \{0\} & \text{if} \quad (\lambda-1/2) + m(\lambda^*-1/2) < 0 \ , \end{cases}$$

$$\text{(2b)} \qquad \lambda^* \in \begin{cases} \{0\} & \text{if} \quad m^*(\lambda-1/2) + (\lambda^*-1/2) > 0 \ , \\ [0,1] & \text{if} \quad m^*(\lambda-1/2) + (\lambda^*-1/2) = 0 \ , \\ \{1\} & \text{if} \quad m^*(\lambda-1/2) + (\lambda^*-1/2) < 0 \ . \end{cases}$$

The Nash equilibria of the static game are defined by the fixed points of Equations (2a) and (2b).

Proposition 1. The Nash equilibria of the static game are

Case 1: $(\lambda,\lambda^*) = (1/2,1/2)$, $(1,0)$, $(0,1)$, $((1-m)/2,1)$ and $((1+m)/2,0)$.

Case 2: $(\lambda,\lambda^*) = (1/2,1/2)$, $(1,0)$ and $(0,1)$.

Cases 3 and 4: $(\lambda,\lambda^*) = (1/2,1/2)$, $(1,(1-m^*)/2)$ and $(0,(1+m^*)/2)$.

Cases 5 and 6: $(\lambda,\lambda^*) = (1/2,1/2)$.

Figure 1 shows the Nash equilibria on the (λ,λ^*) space for each of the six generic cases. The two loci, $p_B = p_R$ and $p_B^* = p_R^*$, are both negatively sloped and pass through (1/2, 1/2). The slope of locus of $p_B = p_R$ is $1/m$; a Conformist's best response is B above the locus and R below it. The slope of locus of $p_B^* = p_R^*$ is m^*; a Nonconformist's best response is R above the locus and B below it. The Nash equilibria are depicted by dots.

Multiplicity of Nash equilibria of the static game poses some conceptual problems. To justify studying a Nash equilibrium, it is commonly assumed that all agents know the entire structure of the game and also agree on which equilibrium is being played. In other words, the strategy profile is assumed to be common knowledge among the agents, so that they know how to coordinate or to focus on a specific equilibrium. This assumption, which is often justified by pre-play negotiation, seems too heroic in a game with a continuum of agents, such as ours. In order for agents to hold confident conjectures about the actions of others, some sorts of inertia must be introduced in the behavior patterns of the society. Evolutionary processes are considered appealing as a way of explaining how a particular equilibrium is chosen and emerge in a dynamic context. Although the literature is mainly interested in the power of evolutionary dynamics in equilibrium selection, I will be also concerned with the dynamic path of behavior patterns itself.

3. Best Response Dynamics

In an attempt to examine the dynamic stability of Nash equilibria, many evolutionary dynamics have been proposed, all of which share the following three properties. First, as in a perfectly competitive market, each agent is atomistic and anonymous and thus maximizes his/her expected payoffs without getting involved in complicated strategic interactions such as retaliation or reputation. Second, it is assumed that some sorts of inertia are present in changing one's behaviors, and that only a tiny fraction of agents are changing their actions at each moment. Thus, a change in behavior patterns is made in a continuous way. Third, actions that are more successful, given the current behavior patterns, replace less successful actions. These properties as well as some regularity conditions are often sufficient to test asymptotic stability of Nash equilibria.

In order to investigate the limit properties of evolution of behavior patterns, however, one needs to specify the dynamic process in detail. Here, I will consider the best response dynamics proposed by Gilboa and Matsui (1991). According to this dynamics, the behavior patterns change only in the direction of the best response to the current behavior patterns. For example, if (1,0) is the best response profile (i.e., B is the Conformist's best response and R is the Nonconformist's best response) to (λ_t,λ_t^*), then $(\dot{\lambda}_t,\dot{\lambda}_t^*) = \alpha\{(1,0)-(\lambda_t,\lambda_t^*)\} = (\alpha(1-\lambda_t),-\alpha\lambda_t^*)$ for a constant $\alpha > 0$, where the upper dot denotes the (right hand) derivative with respect to time. More generally,

$$\text{(3a)} \qquad \dot{\lambda}_t \in \begin{cases} \{\alpha(1-\lambda_t)\} & \text{if} \quad (\lambda_t-1/2) + m(\lambda_t^*-1/2) > 0 \ , \\ [-\alpha\lambda_t,\alpha(1-\lambda_t)] & \text{if} \quad (\lambda_t-1/2) + m(\lambda_t^*-1/2) = 0 \ , \\ \{-\alpha\lambda_t\} & \text{if} \quad (\lambda_t-1/2) + m(\lambda_t^*-1/2) < 0 \ , \end{cases}$$

$$\text{(3b)} \qquad \dot{\lambda}_t^* \in \begin{cases} \{-\alpha\lambda_t^*\} & \text{if} \quad m^*(\lambda_t-1/2) + (\lambda_t^*-1/2) > 0 \ , \\ [-\alpha\lambda_t^*,\alpha(1-\lambda_t^*)] & \text{if} \quad m^*(\lambda_t-1/2) + (\lambda_t^*-1/2) = 0 \ , \\ \{\alpha(1-\lambda_t^*)\} & \text{if} \quad m^*(\lambda_t-1/2) + (\lambda_t^*-1/2) < 0 \ . \end{cases}$$

A dynamic path of behavior patterns is given by an absolutely continuous function, $(\lambda_t,\lambda_t^*):[0,\infty) \to [0,1]^2$, which satisfies (3a) and (3b) for all $t \in [0,\infty)$ and the initial condition, (λ_0,λ_0^*), which is determined by "history." Note that the set of th stationary points of dynamical system (3) is equal to the set of the Nash equilibria of the static game given in Proposition 1. Instead of the Nash equilibria, I will focus on the <u>socially stable behavior patterns</u>, defined by the attractor of (3): that is, the set of ω-limit points of (3) for an open set of initial conditions, (λ_0, λ_0^*).[5]

The best response dynamics have some attractive features. First, it leads to a piecewise linear dynamical system, which is simple but rich enough to allow for a variety of dynamic behaviors. It permits a simple geometrical analysis. Figure 2 depicts the vector field defined by (3) for each of the six generic cases. As shown, the gradient of the vector field points to the vertex of the unit square that corresponds to the best response in any of the four subspaces, in which the best response is unique. When the best response is not unique, the gradient belongs to the cone formed by the best response direction. Second, it does not rule out revival of "extinct" strategies, unlike the standard formulation of evolutionary dynamics.[6] Third, socially stable behavior patterns due to the best response dynamics are independent of α, or the degree of inertia assumed. Fourth, it can be interpreted as the limiting case of the perfect foresight dynamics introduced by Matsuyama (1991a, 1991b).

According to this perfect foresight dynamics, each agent has to make a

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[5]Strictly speaking, this is different from the way Gilboa and Matsui defined the best response dynamics and the associated notions of accessibility, cyclically stable sets and social stability. In order to formalize perturbations, or mutations, they allow agents to use any distribution when randomizing, and they introduce trembling by considering the best response to the ε-neighborhood of the current behavior patterns. See Gilboa and Matsui (1991) for more detail.

[6]For example, in the replicator dynamics, adapted from the literature of population biology, the growth (reproduction) <u>rate</u> of strategies (species) depends continuously on their relative payoffs (fitness): see Friedman (1991), Nachbar (1990), Samuelson and Zhang (1990), and van Damme (1987, Ch. 9.4). One implication of this specification is that any extinct strategy cannot be revived, except the possibility of mutations. It seems hard to defend this implication in our context. For example, even if all Conformists choose B at the beginning, they might get the idea of choosing R for a variety of reasons, such as watching Nonconformists choose R. And if they start switching from B to R, there is no reason to suppose that the pace of switching should be slower initially when a relatively small fraction of Conformists chooses R.

commitment to a particular action in the short run. The opportunity to change one's action follows the Poisson process with α being the mean arrival rate. In other words, the duration of the commitment, x, is a random variable, whose distribution function is exponential, $1 - \exp(-\alpha x)$. It is independent across agents and thus there is no aggregate uncertainty. When the commitment is over, agents choose the action which results in a higher expected discounted payoff over the next duration of commitment, knowing the future path of behavior patterns.[7] Under this formulation, the behavior patterns evolve according to

$$(4a) \qquad \dot{\lambda}_t \in \begin{cases} \{\alpha(1-\lambda_t)\} & \text{if} \quad \int_t^\infty \{\Pi_{Bs} - \Pi_{Rs}\} e^{(\alpha+\delta)(t-s)} ds > 0 , \\ [-\alpha\lambda_t, \alpha(1-\lambda_t)] & \text{if} \quad \int_t^\infty \{\Pi_{Bs} - \Pi_{Rs}\} e^{(\alpha+\delta)(t-s)} ds = 0 , \\ \{-\alpha\lambda_t\} & \text{if} \quad \int_t^\infty \{\Pi_{Bs} - \Pi_{Rs}\} e^{(\alpha+\delta)(t-s)} ds < 0 , \end{cases}$$

$$(4b) \qquad \dot{\lambda}^*_t \in \begin{cases} \{-\alpha\lambda^*_t\} & \text{if} \quad \int_t^\infty \{\Pi^*_{Bs} - \Pi^*_{Rs}\} e^{(\alpha+\delta)(t-s)} ds < 0 , \\ [-\alpha\lambda^*_t, \alpha(1-\lambda^*_t)] & \text{if} \quad \int_t^\infty \{\Pi^*_{Bs} - \Pi^*_{Rs}\} e^{(\alpha+\delta)(t-s)} ds = 0 , \\ \{\alpha(1-\lambda^*_t)\} & \text{if} \quad \int_t^\infty \{\Pi^*_{Bs} - \Pi^*_{Rs}\} e^{(\alpha+\delta)(t-s)} ds > 0 , \end{cases}$$

where $\delta > 0$ is the discount rate. It can be easily verified that (3) is the limit case of (4), where δ goes to infinity. Furthermore, the phase portrait of (3) is identical to that of the limit case of (4), where α goes to zero, or

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[7] Alternatively, this dynamics can be interpreted as follows. The society is populated by overlapping generations of agents. At each point in time, Conformists of measure equal to $\alpha\theta$, and Nonconformists of measure equal to $\alpha(1-\theta)$, are born. All agents alive face an instantaneous probability of death, α. The risk of death is independent across agents and hence there is no aggregate uncertainty; Conformists of measure, $\alpha\theta$, and Nonconformists, $\alpha(1-\theta)$, die at any point. The population thus remains constant over time. Each agent has to choose his/her strategy at the time of birth, and is restricted to stay with the strategy of his/her choice during his/her lifetime. This interpretation, while unrealistic in the context of fashions in dress, may be reasonable in other contexts, such as fashions in art, lifestyle, political ideology, and so on.

the expected duration of commitment, $1/\alpha$, goes to infinity. Thus, the best response dynamics can be used to identify the long run behavior patterns in the perfect foresight dynamics when adjustment is sufficiently slow.[8]

The main results can now be put forward.

Proposition 2.

The socially stable behavior patterns of (3) are given by:

Cases 1 and 2 ($m^* > 1 > m$): $(\lambda,\lambda^*) = (0,1)$ and $(1,0)$,

Cases 3 and 4 ($m^* < 1$, $m < 1/m^*$): $(\lambda,\lambda^*) = (1,(1-m^*)/2)$ and $(0,(1+m^*)/2)$,

Cases 5 and 6a ($m \geq m^* > 1/m$): $(\lambda,\lambda^*) = (1/2, 1/2)$,

Case 6b ($m^* > m > 1$): the limit cycle .

Proof. Figure 2 would be sufficient for Cases 1 through 5. In Case 6, dynamic paths circle around (1/2,1/2) clockwise. To examine the limit property of a path, define sequences $\{X_n\}_{n=0}^{\infty}$ and $\{Y_n\}_{n=0}^{\infty}$, as follows:

(5a) $P = ((1+X_{2k}/m^*)/2,\ (1-X_{2k})/2)$,

(5b) $Q = ((1-Y_{2k})/2,\ (1+Y_{2k}/m)/2)$,

(5c) $P' = ((1-X_{2k+1}/m^*)/2,\ (1+X_{2k+1})/2)$,

(5d) $Q' = ((1+Y_{2k+1})/2,\ (1-Y_{2k+1}/m)/2)$,

where P (P′) is the point at which the path crosses the locus of $p_B^* = p_R^*$ to the southeast (northwest) of (1/2,1/2) for the k-th time and Q (Q′) is the point at which the path crosses the locus of $p_B = p_R$ to the northwest (southwest) of (1/2,1/2) for the k-th time, as depicted in the diagram for Case 6 given in Figure 2. Since PQ points to (0,1), and QP′ points to (1,1),

[8] To quote Young (1937, p.5), the process "must be sufficiently rapid to outmode previous fashions every year, but it must be sufficiently slow to prevent the leaders from outdistancing their followers."

these sequences satisfy

$$Y_n = \frac{m^*(1+m)X_n}{m(1+m^*) + (mm^*-1)X_n}, \quad X_{n+1} = \frac{m(m^*-1)Y_n}{m^*(m-1) + (mm^*-1)Y_n},$$

or

$$(6) \qquad X_{n+1} = \frac{(m^*-1)(1+m)X_n}{(m-1)(1+m^*) + 2(mm^*-1)X_n}.$$

Equation (6) implies that, if $m \geq m^* > 1$, then $\{X_n\}_{n=0}^{\infty}$ converges to 0 for any X_0, so that all paths converge to (1/2,1/2) in Case 6a. If $m^* > m > 1$, $\{X_n\}_{n=0}^{\infty}$ converges to

$$(7) \qquad X_{\infty} = (m^*-m)/(mm^*-1) = (2\theta-1)\beta(1-\beta)/(2\beta-1)\theta(1-\theta),$$

for any $X_0 > 0$, and so does $\{Y_n\}_{n=0}^{\infty}$. All paths thus converge to a unique limit cycle in Case 6b. This completes the proof of Proposition 2.

4. Discussion

Figure 3 summarizes how asymptotic behaviors of the society depend on (m, m^*) or (θ, β). In Case 1 and Case 2, socially stable behavior patterns are $(\lambda,\lambda^*) = (0,1)$ and $(1,0)$. All Nash equilibria that involve mixed strategies are dynamically unstable. In these cases, the ratio of Conformists to Nonconformists is sufficiently large ($\theta > \beta$, $1-\beta$), so that all agents are matched more frequently with a Conformist, rather than with a Nonconformist ($m^* > 1 > m$). Thus, Conformists effectively play the game among themselves, while Nonconformists play the game against Conformists. In these two stable equilibria, the Conformist, the majority, sets the social custom and the Nonconformist, the minority, revolt against it and acts like a social misfit.

Multiplicity of stable outcomes is not surprising because Conformists form the majority and because the two actions are strategic complements from a Conformist's point of view. Which of the two stable equilibria the society would converge to depend on the initial conditions (or in other words, whether B or R becomes the social custom is determined by history.) Thus, the dynamic process exhibits hysteresis phenomena. As shown in Figure 2, if the Conformist's initial best response is B (that is, (λ_0, λ_0^*) is such that $p_{B0} > p_{R0}$), B emerges as the social custom and the society converges to (1,0). If $p_{B0} < p_{R0}$, on the other hand, R becomes the social custom and the society converges to (0,1).

In Case 3 and Case 4, socially stable behavior patterns are $(\lambda, \lambda^*) = (1,(1-m^*)/2)$ and $(0,(1+m^*)/2)$, while $(\lambda, \lambda^*) = (1/2,1/2)$ is unstable. The mixed strategies by Conformists are ruled out by the dynamic stability, but not those by Nonconformists. Note that $\beta < 1 - \theta$, or equivalently $m^* < 1$, in these cases. Because the ratio of Nonconformists to Conformists is sufficiently large and the matching process is sufficiently biased toward intragroup matchings, a Nonconformist is matched with another Nonconformist more frequently than with a Conformist. This implies mixed strategies by Nonconformists. On the other hand, a Conformist meets another Conformist more frequently than a Nonconformist does ($\beta < 1/2$, or $m < 1/m^*$). This implies that, for any behavior patterns to which Nonconformists are indifferent between the two actions, a Conformist follows what the majority of Conformists does. As a result, λ_t converges to either 0 or 1 along the locus of $p_B^* = p_R^*$, depending on the initial condition. In Case 3, it converges to $(1,(1-m^*)/2)$, if $p_{B0} > p_{R0}$, and to $(0,(1+m^*)/2)$, if $p_{B0} < p_{R0}$. In Case 4, it converges to $(1,(1-m^*)/2)$, if $\lambda_0 + \lambda_0^* > 1$, and to $(0,(1+m^*)/2)$, if $\lambda_0 + \lambda_0^*$

< 1.

In Case 5, the dynamics is globally stable and converges to $(\lambda,\lambda^*) = (1/2,1/2)$. As in Cases 3 and 4, $m^* < 1$ so that Nonconformists effectively play among themselves, which imply mixing. Unlike Cases 3 and 4, however, the matching process is sufficiently biased toward intergroup matching ($\beta > 1/2$) so that a Conformist runs into Nonconformists more often than a Nonconformist does ($m > 1/m^*$). This implies that, for any behavior patterns to which Nonconformists are indifferent between the two actions, a Conformist follows what the majority of Nonconformists does. As a result, λ_t converges to 1/2 along the locus of $p_B^* = p_R^*$.

In Case 6, the best response dynamics generate a spiral path around $(\lambda,\lambda^*) = (1/2,1/2)$, as shown in Figure 2. For any Conformists-Nonconformists ratio, this case occurs if the matching process becomes sufficiently biased toward intergroup matchings ($\beta > \theta$, $1-\theta$ or m, $m^* > 1$). Whether the fluctuation persists forever or eventually settles down, however, depends on the ratio. If there are more Conformists than Nonconformists ($\theta > 1/2$), so that Nonconformists are more concerned with intergroup matchings than Conformists ($m^* > m > 1$), then socially stable behavior patterns become cyclical. Along the cycle, a Nonconformist, wishing to differentiate herself from the masses, changes her action, before it becomes too conventional. A Conformist, whose matchings are more often intragroup than a Nonconformist's, follows the continuing trend for a while.[9] Then he switches his action only after sufficiently many Nonconformists switch their actions. Nonconformists

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[9]One can show that sgn $[\theta\dot{\lambda}_t+(1-\theta)\dot{\lambda}_t^*]$ = sgn $\dot{\lambda}_t$ along the limit cycle: that is, the fraction of the population that chooses B goes together with that of Conformists.

act as fashion leaders and Conformists act as followers.[10] On the other hand, if there are more Nonconformists than Conformists ($\theta \leq 1/2$), then Conformists are more concerned with intergroup matchings than Nonconformists ($m \geq m^* > 1$). Conformists are much quicker to follow Nonconformists in this case, so that Nonconformists cannot maintain the lead forever. The distribution of strategies eventually settles down to $(\lambda, \lambda^*) = (1/2, 1/2)$. The best response dynamics converges globally to the unique Nash equilibrium.

To understand the emergence of the limit cycle further, it would be useful to consider the following two thought experiments. First, starting from the case, $1-\beta < \theta < 1/2$ (or $m > m^* > 1$), where $(\lambda, \lambda^*) = (1/2, 1/2)$ is the unique, globally stable Nash equilibrium, let θ increase. As the society crosses the line $\theta = 1/2$ (or $m = m^*$), $(\lambda, \lambda^*) = (1/2, 1/2)$ loses its stability and bifurcates into a limit cycle. Although the Conformist is still concerned with intergroup matchings more than intragroup ones, he becomes less so than the Nonconformist is, which makes it possible for the Nonconformist to take the lead in switching actions. The regular patterns of fashion cycles thus emerge out of the disorder, as the force of conformity increase. Second, starting from Case 1 ($\theta > \beta > 1/2$, or $mm^* > 1 > m$), where $(\lambda, \lambda^*) = (0, 1)$ and $(\lambda, \lambda^*) = (1, 0)$ are two locally stable Nash equilibria, let θ decline or β increase. As the society crosses the line $\theta = \beta$ (or $m = 1$), the two Nash equilibria first lose their stability and then disappear. A Conformist becomes more concerned with a Nonconformist rather than another Conformist,

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[10] Although the matching process needs to be biased toward intergroup for the existence of cycles, there also needs to be some intragroup matchings. When β approaches the unity, the cycle eventually shrinks to the Nash equilibrium, as can be verified from Equation (7). If there is no matching between a pair of Conformists, the game would be one of strategic substitutes and the mixed equilibrium would become globally stable.

and begin to imitate her. This bifurcation creates a limit cycle. Fashion cycles thus emerge as departure from custom in this case, as the forces of nonconformity increase.

As easily seen from the two Propositions, the Nash equilibrium and the socially stable behavior patterns coincide only in Case 5 and Case 6a, when the best response dynamics are globally stable. In Cases 1 through 4, only a subset of Nash equilibria are selected so that the best response dynamics serve as an equilibrium refinement. In Case 6b, a globally stable cycle emerges, which is not captured by the Nash equilibrium of the static game. This is not to be interpreted as a flaw of the best response dynamics; rather it seems to suggest that the Nash prediction of the static game is unrobust with respect to a natural perturbation of the game into a dynamic setting.

5. Related Work in the Economic Literature

Both positive and negative consumption externalities, built into the payoffs of Conformists and Nonconformists, have been the important subjects in the theory of consumption. For example, in a classic article, Leibenstein (1950) discussed the implications of what he called the "bandwagon effect" and "snob effect" on the static market demand curve. By the bandwagon (snob) effect he referred to the extent to which the demand for a commodity is increased (decreased) because others are consuming the same commodity. In the game presented above, Conformists personify the bandwagon effect and Nonconformists the snob effect. Leibenstein's analysis is static and thus he did not discuss any dynamic implication of combining these two effects, although he stated "[i]n all probability, the most interesting parts of the problem, and also those most relevant to real problems, are its dynamic aspects (p.187)."

Although payoff externalites are simply assumed in the present game, it would be worthwhile to model the mechanisms that could generate conformity and nonconformity in human behaviors. In this respect, recent work by Banerjee (1989) and Bikhchandani, Hirshleifer, and Welch (1990) deserve special mention. They demonstrated how informational externalities could lead to conformist behaviors. In particular, Bikhchandani, Hirshleifer, and Welch explored the role of informational externalities in the emergence of fads and discussed their fragility at the arrival of new information.

The paper by Karni and Schmeidler (1990) is most closely related to the present paper, both in its approach and in its spirit. They have constructed a (discrete time) dynamic game played by two classes of agents, α and β, who choose among three different colors every three periods. The crucial feature of their model is that the preferences of α-agents for a given color decrease with the fraction of β-agents that use the same color, while the preferences of β-agents for a given color increase with the fraction of α-agents that use the same color. Thus, fashion is driven by an emulation of the elite class, α, by the rest of the society, β. The elite class seeks to set itself apart from the rest of the society by adopting different colors, which in turn leads to a new wave of emulation. The class affiliation of an agent plays a significant role in their model, and thus captures the dominant sociological view of fashion, usually associated with Simmel ([1904]1957). Due to the complexity of the model, however, they were only able to generate a numerical example of fashion cycle equilibria. One advantage of the present model is that its simple structure permits characterization of different types of behavior patterns on the two dimensional parameter space and thus enables one to explain both custom and fashion in a unified framework.

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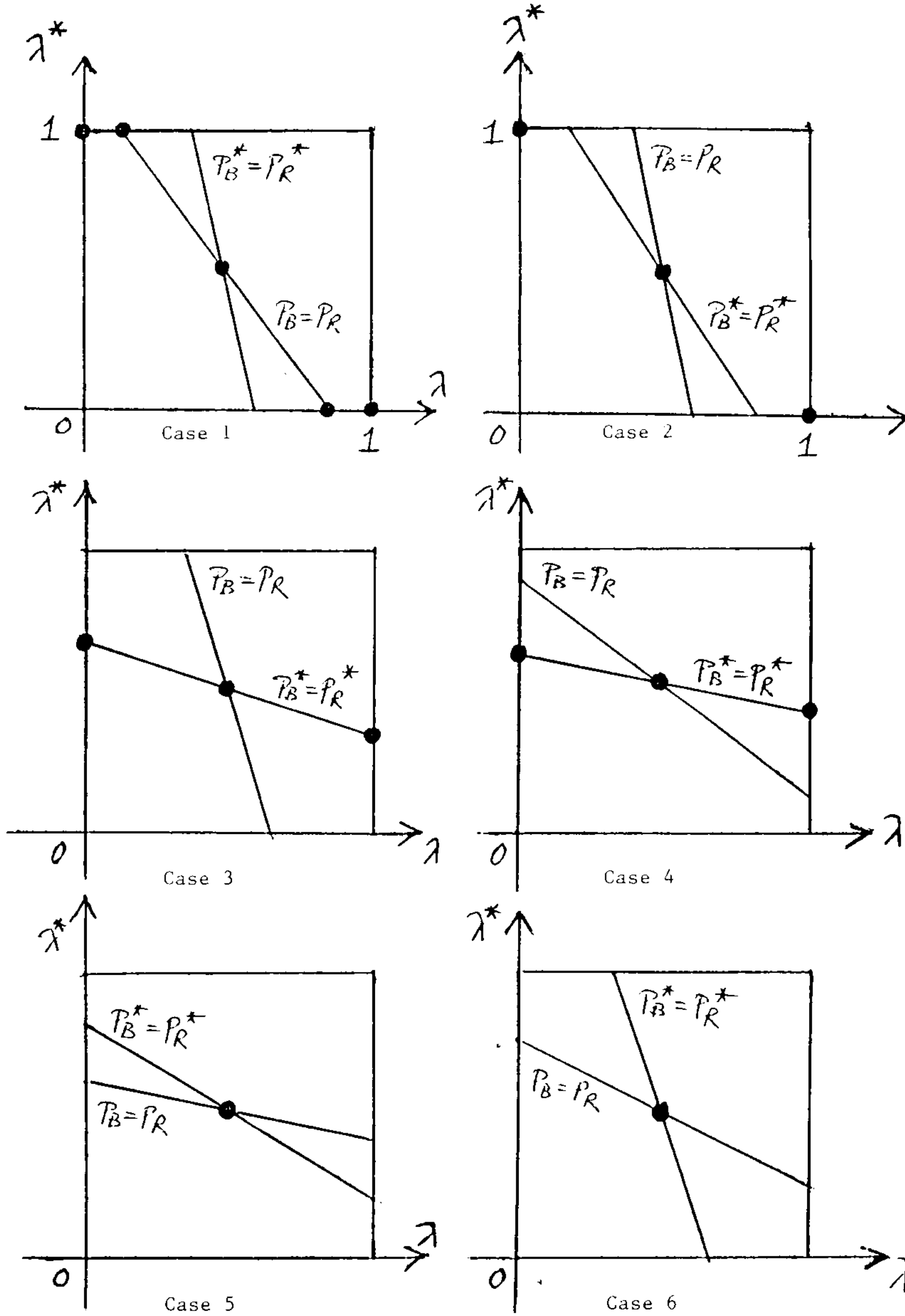

Figure 1: The Nash Equilibria

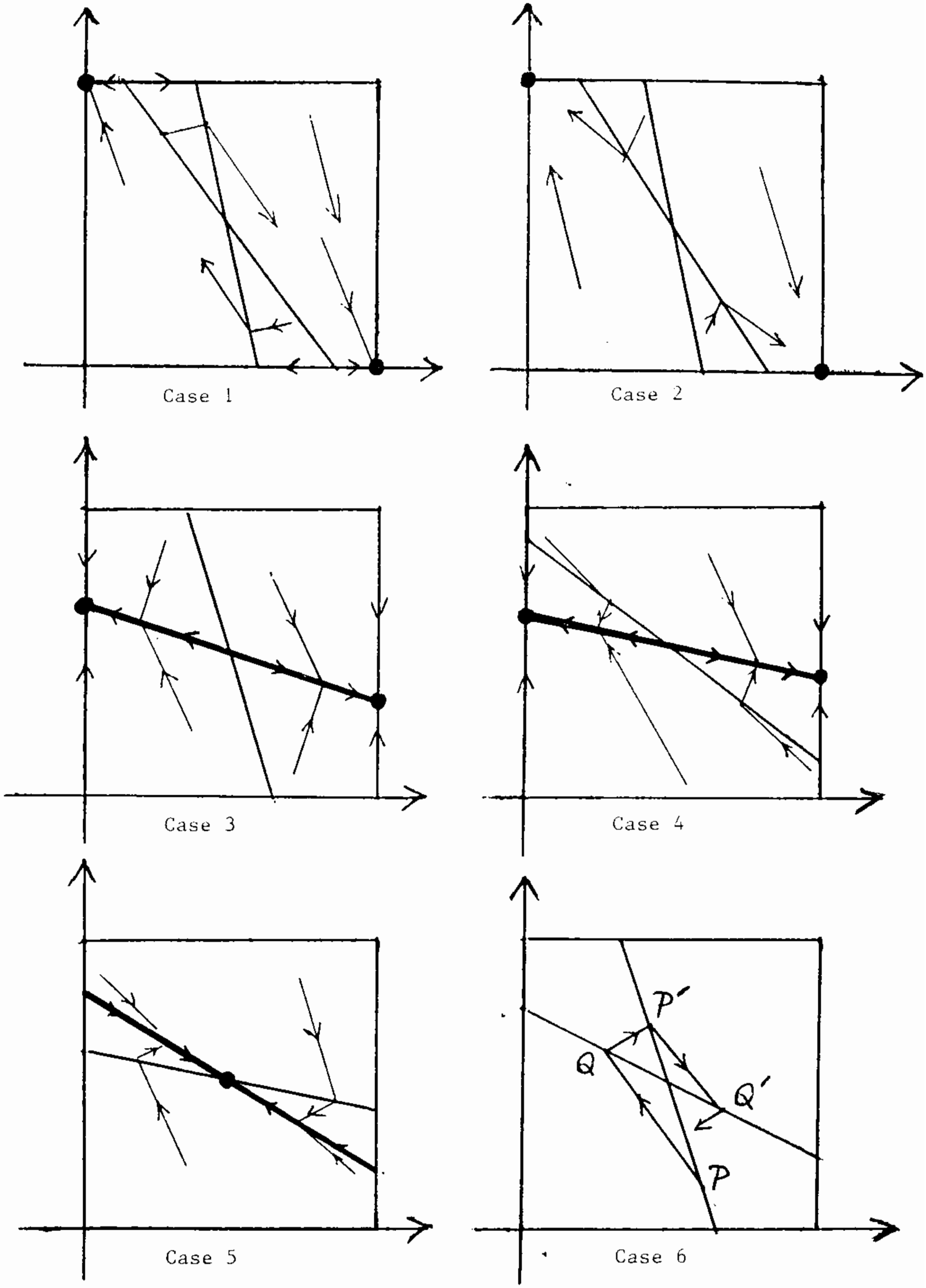

Figure 2: The Best Response Dynamics