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MONOPOLY PRICING STRATEGIES

by

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ABSTRACT

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The design of monopoly pricing strategies is examined in a general framework with an unknown population distribution of consumer characteristics, downward-sloping, multi-unit consumer demand, and increasing marginal cost. Reference point pricing is introduced and is shown to implement the profit-maximizing allocation. The design of generalized priority service is extended to the unknown demand setting. Nonlinear pricing is shown to be approximately optimal for the monopolist as the number of consumers gets large.

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1. Introduction

The pricing policy of a monopolist is very sensitive to the information available about aggregate demand, the characteristics of individual consumer demands, and the firm's production cost function. This paper examines optimal selling strategies for a monopolist in a model with the following features. First, the distribution of tastes across the set of the firm's customers is not known. Consumer tastes are a finite sample from a known global population distribution. Second, consumers have general downward-sloping demand functions. Third, increasing marginal costs of production are allowed. Thus, a general pricing mechanism must elicit information from the firm's customers not only to determine each person's bundle but also to observe aggregate demand. This significantly widens the class of monopoly pricing strategies.

The model generalizes the analysis of Harris and Raviv [6] who assume unit demands and a capacity constraint (see also [9]). They show that if the monopolist does not face a binding capacity constraint, it is optimal to offer a fixed per unit price. If there is a binding capacity constraint, the monopolist will employ either a priority pricing scheme or, under some conditions, a modified Vickrey auction. With general downward-sloping demands and increasing marginal cost these pricing strategies are no longer optimal.

The nonlinear pricing framework of Spence [15], Mussa and Rosen [12], Spulber [16], and Maskin and Riley [8] generally assumes that the population distribution of consumer types is known to the monopolist. Nonlinear pricing also is optimal with unknown aggregate demand if marginal costs are constant since consumption levels are independent, but it is no longer optimal with increasing marginal costs.

The present analysis extends the existing literature in two ways. First, it is shown that with unknown aggregate demand, and increasing marginal cost, as the size of the sample of consumers grows, nonlinear pricing is approximately efficient. Second, two pricing policies are examined that implement the monopolist's optimal selling continuously differentiable with bounded second derivatives in Q and θ. Also, u is strictly decreasing in Q, $u_1 < 0$, for u positive. Further, u is strictly increasing and concave in θ, $u_2 > 0$ and $u_{22} \leq 0$ for u positive. Consumer absolute risk aversion in output, $(-u_1/u)$, is nonincreasing in θ, $\partial(-u_1/u)/\partial\theta \leq 0$. That is, higher demand types have a lower level of absolute risk aversion (see Maskin and Riley [8]). Assume that there exists $\bar{Q} > 0$ that solves $u(\bar{Q},1) = c$, so that consumption is bounded above.

The consumer's taste parameter, θ_i, is private information, unknown to the monopolist or to other consumers. The values of θ_i are viewed as draws from a cumulative distribution function $F(\theta)$ on $[0,1]$ with positive density $f(\theta)$ everwhere differentiable, which is common knowledge. Let $\tilde{\theta} \equiv (\theta_1,\ldots,\theta_n)$, $\theta_{(i)} \equiv (\theta_1,\ldots,\theta_{i-1},\theta_{i+1},\ldots,\theta_n)$, $d\tilde{F} \equiv dF(\theta_1)dF(\theta_2) \ldots dF(\theta_n)$, and $dF_{(i)} \equiv dF(\theta_1) \ldots dF(\theta_{i-1})dF(\theta_{i+1}) \ldots dF(\theta_n)$. Also, let $I^m \equiv [0,1]^m$ for any integer m, $1 \leq m \leq n$. Assume that the hazard rate $v(\theta) \equiv f(\theta)/(1 - F(\theta))$ is nondecreasing. This is satisfied, for example, by the uniform and the exponential family of distributions.

The monopolist must design a profit-maximizing pricing and allocation strategy with incomplete information about customer demand. The monopolist does not know the distribution of its customer's tastes. The distribution of tastes, $\tilde{\theta}$, is viewed as a draw from the distribution $\tilde{F} = F(\theta_1)F(\theta_2) \ldots F(\theta_n)$. The distribution $F(\theta)$ represents the overall population distribution of consumers. The monopolist's customers are then a sample from the population distribution. This is a realistic assumption, as the firm may have accurate statistical data about the population distribution of income levels, age, education, or location, but may lack information about the characterization of the subgroups that purchase from the firm.

trade, $V(\theta_i) \geq 0$.

3. The Profit Maximizing Strategy

The monopolist's program is to choose the direct mechanism (Q,p) from the set of feasible mechanisms to maximize expected profit,

$$(2) \qquad \Pi(Q,p) = \int_{I^n} [\textstyle\sum_{i=1}^{n} p(\theta_i) - C(\sum_{i=1}^{n} Q(\theta_i,\theta_{(i)}))]d\tilde{F}.$$

The first order necessary conditions for an interior maximum of $\Pi(Q,p)$, <u>without the feasibility constraints</u>, are

$$(3) \qquad u(Q^*(\theta_i,\theta_{(i)}),\theta_i) - (1/v(\theta_i))u_2(Q^*(\theta_i,\theta_{(i)}),\theta_i) - c(\textstyle\sum_{j=1}^{n} Q^*(\theta_j,\theta_{(j)})) = 0,$$

for all $i = 1,\ldots,n$, for all $\tilde{\theta} \in I^n$. The following is proved in the Appendix.

PROPOSITION 1: (a) <u>The profit-maximizing mechanism</u>, (Q^*,p^*), <u>that solves the monopolist's program</u> (2), <u>satisfies the conditions for an interior maximum</u> (3). (b) <u>The output schedule</u> $Q^*(\theta_i,\theta_{(i)})$ <u>is strictly increasing in</u> θ_i, <u>for all</u> $\tilde{\theta}$. (c) <u>The profit-maximizing outlay schedule is</u>

$$(4) \qquad p^*(\theta_i) = \int_{I^{n-1}} [U(Q^*(\theta_i,\theta_{(i)}),\theta_i) - \int_0^{\theta_i} U_2(Q^*(\theta,\theta_{(i)}),\theta)d\theta]dF_{(i)}.$$

Increasing marginal costs imply that a higher value of θ_i for a given consumer "crowds out" the output allocation to other consumers. Thus, the output allocation to any consumer is strictly decreasing in other consumer taste parameters, of samples from the cumulative distribution of consumer preference parameters.[5]

The proof of the approximate efficiency of nonlinear pricing is related to the standard proof of the weak law of large numbers but differs in two important respects. First, the first order conditions (3) give an optimal quantity assignment that depends on the (normalized) size of the sample. The proof involves replication of consumer types. Second, concavity of the virtual utility, convexity of costs, and Jensen's inequality play a role in developing the upper bound on the efficiency loss.

PROPOSITION 3: <u>As the number of consumers becomes large, profit from nonlinear pricing approximates the maximum profit</u>.

PROOF: Replicate the number of consumers m times, so that there are m consumers of each type θ_i, $i = 1,\ldots,n$. Let $m = 1/n$ so that the population size is normalized to one, without loss of generality. Then, for any n, the monopolist's profit evaluated at the profit-maximizing mechanism $(Q^*(\theta_i,\theta_{(i)},n), p^*(\theta_i,n))$ is

$$\Pi^*(n) = \int_{I^n} [\Sigma_{i=1}^{n} (1/n)[U(Q^*(\theta_i,\theta_{(i)},n),\theta_i) - (1/v(\theta_i))U_2(Q^*(\theta_i,\theta_{(i)},n),\theta_i)] - C(\Sigma_{\ell=1}^{n} (1/n)Q^*(\theta_\ell,\theta_{(\ell)},n)]d\tilde{F}.$$

Note that $\int_{I^n} [\Sigma_{\ell=1}^{n} (1/n)Q^*(\theta_\ell,\theta_{(\ell)},n)]d\tilde{F} = \int_I q^*(\theta,n)dF$. Given concavity of $[U(Q,\theta) - (1/v(\theta))U_2(Q,\theta)]$ in Q, and convexity of C, Jensen's Inequality implies that

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[5] Arrow and Radner ([1], p. 362) show in a model of the allocation of resources within a large team that "limited communication yields almost as high a return as full communication."

By Chebyshev's Inequality, for $0 < \varepsilon < \infty$, it can be shown that $\Pr\{A(\varepsilon,n)\} \leq \mathrm{var}[q^*(\theta,n)]/\varepsilon^2 n$. Since $c(Q) \geq c$ for all Q, it follows from (3) that $u(Q^*,\theta_i) \geq c$ for all i. Thus, $Q^*(\theta_i,\theta_{(i)},n) \leq \bar{Q}$ for all $\theta_i,\theta_{(i)}$, and n, where $\bar{Q}$ solves $u(\bar{Q},1) = c$. Therefore, $q^*(\theta,n) \leq Q$ for all n, so that $\mathrm{var}[q^*(\theta,n)]/\varepsilon^2 n \leq \bar{Q}^2/\varepsilon^2 n$, which goes to zero as n increases to infinity. Thus, $(1/n)\,\Sigma_{i=1}^n\, q^*(\theta_i,n)$ converges in probability to $\int_I q^*(\theta,n)dF$. So, for any given ε^*, it is possible to select n^* such that for all $n \geq n^*$, the first term on the r.h.s. of (7) is less than $\delta/2$. For any given $\delta > 0$, by the continuity of C, one can select $\varepsilon^* > 0$ such that for all x,y such that $|x - y| < \varepsilon^*$, $|C(x) - C(y)| < \delta/2$. Therefore, for all $n \geq n^*$, it follows that $\Pi^*(n) - \tilde{\Pi}(n) \leq \delta/2 + \Pr[\sim A(\varepsilon^*,n)]\ \delta/2 < \delta$. Q.E.D.

Therefore, with a sufficiently high number of consumers, the monopolist's information about the actual distribution of types improves to the point where a dominant-strategy mechanism is approximately optimal.

The proposition has an important implication regarding the standard model of a monopolist that posts a constant per-unit price. Suppose that consumer utility is $U = \theta_i \min\{Q_i,q\}$, so that consumers have unit demands. Then, posted pricing is approximately optimal for the monopolist.[6]

COROLLARY 1: <u>Given unit demands, as the number of consumers becomes large, profit from a posted price approaches the maximum profit</u>.

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[6]The sketch of the proof is as follows. Given unit demands, and a posted price ρ, the consumer purchases the good if and only if $\theta \geq \rho$. thus, the expected revenue from a single consumer equals $\rho\int_\rho^1 dF(\theta) = \rho(1 - F(\rho))$. By the definition of integration, $\rho(1 - F(\rho)) = \int_\rho^1 [\theta - (1 - F(\theta))/f(\theta)]dF(\theta) = \int_0^1 [\theta - (1/v(\theta))]\tilde{q}(\theta)dF(\theta)$, where $\tilde{q}(\theta) = 1$ for $\theta \geq \rho$ and $\tilde{q}(\theta) = 0$ otherwise. So, maximum profit from posted prices $\tilde{\Pi}(n)$ can be defined as in the proof of Proposition 3.

facilities are rationed based on availability at any given time.

In the expected demand case,[9] define $\theta^*(q) = \min(\theta: q^*(\theta) \geq q)$ since $q^*(\theta_i)$ is nondecreasing. Define an output allocation rule $S^*(q_i, q_{(i)})$ by

$$S^*(q_i, q_{(i)}) = Q^*(\theta^*(q_i), (\theta^*(q_1), \ldots, \theta^*(q_{i-1}), \theta^*(q_{i+1}), \ldots, \theta^*(q_n))).$$

Let $q = \psi(Q, q_{(i)})$ solve $Q = S(q, q_{(i)})$. By a change of variables, the following holds.

PROPOSITION 4: <u>The monopoly profit maximum is attained by nonlinear reference point pricing</u>

$$(8) \qquad R^*(q_i) = \int_{I^{n-1}} \int_0^{S(q_i, q^*_{(i)})} u(Q, \theta^*(\psi(Q, q^*_{(i)})))dQ\, dF_{(i)},$$

<u>where</u> $q^*_{(i)} = q^*(\theta_1), \ldots, q^*(\theta_{i-1}), q^*(\theta_{i+1}), \ldots, q^*(\theta_n))$.

Thus, given (S^*, R^*), consumers follow strategies $q^*(\theta_i)$ at a Bayes-Nash equilibrium.

The properties of the reference point price schedule $R(q)$ differ from those generally obtained in a nonlinear pricing model. With unit demands, the schedule is $R^*(q) = \theta^*(q)q - \int_0^{\theta^*(q)} q^*(\theta)d\theta$, which is convex since $q^*(\theta)$ is increasing in θ.

PROPOSITION 5: <u>The reference point pricing schedule with unit demands exhibits quality premia, that is</u> $R^*(q)/q$ <u>is increasing in</u> q.

6. <u>Generalized Priority Service</u>

Priority service is introduced in a unit demand framework by Harris and Raviv [6]

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[9] Maximum demand, $Q^*(\theta_i, 0)$, and minimum demand $Q^*(\theta_i, 1)$ are similar.

PROPOSITION 8: <u>The monopoly profit maximum is attained by the output-reliability price schedule</u>

(30) $$R^*(x,y) = U(x,\theta^*(x,y)) - \int_0^{\theta^*(x,y)} U_2(x^*(y,\theta),\theta)d\theta$$

<u>where</u> $\theta^*(x,y)$ <u>solves</u> $x = x^*(y,\theta)$.

An interesting property of the schedule is that output and reliability are <u>revenue complements</u> $R^*_{12}(x,y) = u_2(x,\theta^*(x,y))\theta^*_2(x,y) > 0$ since $u_2 > 0$ and $\theta^*_2 > 0$.

7. Conclusion

The pricing strategies can be extended to accommodate random capacity. Let k be a parameter in the monopolist's cost function, $C(Q,k)$, with distribution $G(k)$ on $[k_0,k_1]$. The optimal direct mechanism is then state-contingent, $Q_i = Q^*(\theta_i,\theta_{(i)},k)$. Reference point pricing $R(q)$ implements the optimal allocation given an allocation rule $S^*(q_i,q_{(i)},k)$. Generalized priority pricing $R(x,y)$ implements the optimal allocation with the reliability schedule $y^*(x,\theta_i) = (G \times F_{(i)})\{(\theta_{(i)},k): x \leq Q^*(\theta_i,\theta_{(i)},k)\}$. An interesting issue arises if the level of aggregate demand and available capacity are not observable ex post. Then, it is necessary to design contingent contracts such that the monopolist complies with announced allocation rules.

A general characterization of monopoly pricing strategies under uncertainty has been presented. The theory shows the crucial dependence of the pricing strategy on information, consumer demand, and firm cost. Unobservable aggregate demand and random capacity requires the design of pricing strategies and allocation rules that are more complex than nonlinear pricing.

$$\text{(A1)} \quad \Sigma_{\ell=1}^{n} (\partial Q_\ell/\partial\theta_i) = [\frac{-u_2(Q_i,\theta_i)(1 - d(1/v(\theta_i))/d\theta_i)) + (1/v(\theta_i))u_{22}(Q_i,\theta_i)}{u_1(Q_i,\theta_i) - (1/v(\theta_i))u_{21}(Q_i,\theta_i)}]$$

$$\div [1 - c' \Sigma_{\ell=1}^{n} [u_1(Q_\ell,\theta_\ell) - (1/v(\theta_\ell))u_{21}(Q_\ell,\theta_\ell)]^{-1}].$$

Note that, given $u_2 > 0$, $u_{22} < 0$ and $dv(\theta)/d\theta \geq 0$, $\Sigma_{\ell=1}^{n} \partial Q_\ell/\partial\theta_i > 0$. Furthermore,

$$\text{(A2)} \quad (\partial Q_i/\partial\theta_i) = [\frac{-u_2(Q_i,\theta_i)(1 - d(1/v(\theta_i))/d\theta_i)) + (1/v(\theta_i))u_{22}(Q_i,\theta_i)}{u_1(Q_i,\theta_i) - (1/v(\theta_i))u_{21}(Q_i,\theta_i)}]$$

$$\times [1 - [\frac{-c'[u_1(Q_i,\theta_i) - (1/v(\theta_i))u_{21}(Q_i,\theta_i)]^{-1}}{1 - c' \Sigma_{\ell=1}^{n} [u_1(Q_\ell,\theta_\ell) - (1/v(\theta_\ell))u_{21}(Q_\ell,\theta_\ell)]^{-1}}]].$$

The first and second bracketed terms are positive. So, $\partial Q_i/\partial\theta_i \geq 0$, for all $\tilde{\theta} \in I^n$, for all $i = 1,\dots,n$. It can be shown that this is a sufficient condition for the mechanism (Q^*,p^*) to be feasible. The mechanism (Q^*,p^*) that solves the monopolist's relaxed problem (without the feasibility constraints) thus solves the monopolist's program (2).

Q.E.D.

Holland.

[12] Mussa, M. and S. Rosen (1978), "Monopoly and Product Quality," Journal of Economic Theory, 18, 310-317.

[13] Myerson, R. B. (1979), "Incentive Compatibility and the Bargaining Problem," Econometrica, 47, 61-73.

[14] Panzar, J. C. and D. S. Sibley (1978), "Public Utility Pricing Under Risk: The Case of Self-Rationing," American Economic Review, 68, 888-895.

[15] Spence, A. M. (1977), "Nonlinear Prices and Welfare," Journal of Public Economics, 8 (August), 1-18.

[16] Spulber, D. F. (1981), "Spatial Nonlinear Pricing," American Economic Review, 71, 923-933.

[17] Spulber, D. F. (1990), "Optimal Nonlinear Pricing and Contingent Contracts," Strategy Discussion Paper No. 90-26, Kellogg Graduate School of Management, Northwestern University.

[18] Spulber, D. F. (1990), "Capacity-Contingent Nonlinear Pricing by Regulated Firms," Strategy Discussion Paper No. 90-27, Kellogg Graduate School of Management, Northwestern University.

[19] Viswanathan, N. and E. T. S. Tse (1989), "Monopolistic Provision of Congested Service with Incentive-Based Allocation of Priorities," International Economic Review, 30 (February), 153-174.

[20] Wilson, R. (1989), "Efficient and Competitive Rationing," Econometrica, 57 (January), 1-40.

[21] Wilson, R. (1989), "Ramsey Pricing of Priority Service," Journal of Regulatory Economics, 1, 189-202.

[22] Woo, C.-K. (1990), "Efficient Electricity Pricing with Self-Rationing," Journal of Regulation, 2, 69-81.