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RATIONALIZABLE CONJECTURAL EQUILIBRIUM: BETWEEN NASH AND RATIONALIZABILITY

by

Ariel Rubinstein*
and
Asher Wolinsky**

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* Department of Economics, Tel Aviv University, Tel Aviv.
** Department of Economics, Northwestern University, Evanston, Illinois 60208.

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Abstract

For a steady state to be a Nash equilibrium the agents have to perfectly observe the actions of others. This paper suggests a solution concept for cases where players observe only an imperfect signal of what the others' actions are. The model is enriched by specifying the signal that each player has about the actions taken by the others. The solution, which we call <u>rationalizable conjectural equilibrium (RCE)</u>, is a profile of actions such that each player's action is optimal, given the assumption that it is common knowledge that all players maximize their expected utility given their knowledge. The RCE occupies an intermediary position between Nash equilibrium on the one hand and Rationalizability style Bernheim–Pearce on the other hand. The concept is demonstrated by several examples in which it refines the rationalizability concept and still is not equivalent to Nash equilibrium.

1. Introduction

Games (in normal form) are used to model two very different scenarios. In some cases they aim at modeling situations which occur only once and in which players have to choose their actions simultaneously and independently from each other. In other cases they intend to capture behavior in a continuing situation. The game is then an episode that recurs often enough, but with no strategic links among the repetitions.

The discussion in this paper is in reference to scenario of the second type. Players derive their expectations about others' actions from past experience with this situation, and the steady state behavior is viewed as an equilibrium. However, such a steady state will be a always Nash equilibrium only if agents perfectly observe the actions of others. But agents need not have perfect knowledge of the others' actions - they may observe only an imperfect signal of what the others' actions are. For example, a player may observe only his own payoff, but not the other players' actions. In such a case there are outcomes which satisfy the condition that each player's action is optimal, given the (less than perfect) knowledge he has about the actions of others, but are not Nash equilibria.

To formalize these ideas we have to enrich the model by specifying the signal that each player has about the actions taken by the others. The solution we present in this paper, which we call rationalizable conjectural equilibrium (RCE), is a profile of actions such that each player's action is optimal, given that it is common knowledge that all players maximize their expected utility relative to their information. In other words each player chooses an action which maximizes his payoff given a conjecture regarding the actions of the others; each agent's conjectures are consistent with his signal and his own choice; the conjectures are also consistent with the understanding that everybody rationalizes his action in this manner. If players do not get any information, the RCE coincides with Rationalizability, and if they get exact information about their opponents' actions, it coincides with Nash equilibrium. Thus, the RCE occupies an intermediary position between Nash equilibrium on the one hand and Rationalizability style Bernheim–Pearce on the other hand.

There are two reasons for why modelers may be interested in RCE. First, as a solution concept in its own right which refines the rationalizability concept in a non-trivial way and still is not equivalent to Nash equilibrium. Secondly, in games in which, given some natural signal, the RCE coincides with Nash equilibrium, the Nash equilibrium concept is more compelling because the equilibrium requires less information on the part of the players.

Some related concepts were suggested in the literature. The closest concept is Battigalli and Guaitoli's (1988)'s notion of conjectural equilibrium. This concept requires an action to be a best response to some conjecture which is consistent with the signal he gets but does not impose any rationality conditions on the conjecture assignment about the other players' actions. Fudenberg and Kreps (1989) feature a particular example of conjectural equilibrium. They pointed out that, if a player just observes the outcome of an extensive form game, then the RCE does not have to be a Nash equilibrium.

2. Notation and Definitions

We consider games in normal form. The set of players is $N=\{1,2,\ldots,n\}$; the set of player i's actions is A_i; the set of outcomes is $A=A_1\times A_2\times\ldots\times A_n$; and i's payoff function is $u_i:A\longrightarrow R$. The new ingredients that we add to the description of the game are the signal functions $g_i:A\longrightarrow S_i$. The element $s_i=g_i(a)$ is the signal that player i privately observes when all players choose the actions that make up the profile a. It is assumed that the functions g_i (as the other components of the model) are common knowledge (but the actual signals are not). In what follows we refer to $\langle N, (A_i)_{i\in N}, (u_i)_{i\in N}, (g_i)_{i\in N}\rangle$ as a game.

The solution concept that we are about to present is such that each player's action is optimal, given this player's conjecture about what the other players' actions will be. The conjecture of the player has to be consistent with the signal that he has on the actions the other intend to take, with the knowledge about his action and with the knowledge that the other players follow similar reasoning.

Definition 1: The action-signal pair (a_i,s_i) is g-rationalized by a probability measure μ on $A_{-i}=\times_{j\neq i}A_j$ if

(i) for all $a_{-i}=(a_1,\ldots,a_{i-1},a_{i+1},\ldots,a_n)$ in the support of μ, $g_i(a_i,a_{-i})=s_i$

(ii) a_i is best response, given μ.

Definition 2: The sets of signal–action pairs $B_1, B_2, \ldots, B_n$ are g–rationalizable if, for all i, every (a_i, s_i) in B_i is g–rationalized by a probability measure μ on A_{-i} such that for all a_{-i} in the support of μ and for all j the pair $(a_j, g_j(a_i, a_{-i})) \in B_j$.

Definition 3: A Rationalizable Conjectural Equilibrium (RCE) is an n–tuple of actions $a^* = (a_1^*, \ldots, a_n^*)$ such that there are g–rationalizable sets $B_1, B_2, \ldots, B_n$ such that for all i, $(a_i^*, g_i(a^*)) \in B_i$.

Note that, when $g_i(a) = a$ for all i, the concept of RCE coincides with a pure strategy Nash equilibrium. When $g_i(a)$ is independent of a_{-i}, this concept coincides with the correlated-conjectures version of Bernheim–Pearce's rationalizability.

Definition 2 requires that, for each pair $(a_i, s_i) \in B_i$, the action a_i is best response against a belief on vectors a_{-i} which are consistent with player i's action a_i and the signal s_i, and for each pair $(a_j, g_j(a_{-i}, a_i))$ the action a_j is j's best response against a belief on vectors b_{-j} which are consistent with player j's action a_j and the signal $g_j(a_{-i}, a_i)$ and so on.

The closest concept to the RCE is Battigalli and Guaitoli's (1988) conjectural equilibrium. The CE requires that a player behaves rationally in the sense that his action is a best response given the knowledge of the player about his action, the signal and the game but it does not require common knowledge of rationality. Every RCE is a CE.

Definition 4: A Conjectural Equilibrium (CE) is an n-tuple of actions $a^*=(a_1^*,...,a_n^*)$ such that for all i the pair $(a_i^*,g_i(a^*))$ is g-rationalized.

The rest of the paper is mainly devoted to examples of games which clarify the RCE concept and compare its predictions with those of other solution concepts.

3. Examples

The case of public signals is such that each player can infer from his own signal what signals are held by the other players (i.e, $g_i(a)=g_i(a')$ if and only if $g_j(a)=g_j(a')$). In the next three examples the signals are public.

(1) The distance game

Two players choose locations in the unit interval. Their payoffs are decreasing in the distance between their locations, so there is no conflict of interest and both want to coordinate their location at the same point. The signal is the distance. Thus, $N=\{1,2\}$, $A_i=[0,1]$, $g_i(a_1,a_2)=d(a_1,a_2)=|a_1-a_2|$ and $u_i(a_1,a_2)=L(d(a_1,a_2))$, where L is any decreasing function. It is immediately verifiable that the set of Nash equilibria includes all pairs (a_1,a_2) such that $a_1=a_2$ and that all possible pairs of locations are consistent with Rationalizability. The set of Conjectural equilibria is also very big and includes all (a_1,a_2) such that neither of the locations is in a distance of less than $d(a_1,a_2)$ from the edges of the interval.

Claim: The RCE are all pairs (a_1,a_2) such that $a_1=a_2$.

Proof: Suppose there is a RCE (a_1,a_2) such that $d(a_1,a_2)=d>0$. This means that there are g-rationalizable signal-action sets B_1 and B_2, as described in the definition, such that (a_i,d) belongs to B_i. The pair (a_i,d) can be g-rationalized only by a conjecture which puts positive probabilities on both a_i-d and a_i+d. This is because the action a_i is not a best response against a conjecture which puts probability zero on one of the actions a_i-d or a_i+d. Thus, since $(a_i,d)\in B_i$, both (a_i-d,d) and (a_i+d,d) must be in B_j. Define $m_i=\max\{a_i: (a_i,d)\in B_i\}$. (If there is no maximum, follow the argument with sup). From the above, (m_i+d,d) must belong to B_j and therefore (m_i+2d,d) must belong to B_i, in contradiction to the maximality of m_i. ■

Thus, the RCE coincide here with the Nash equilibria, despite the fact that players have less information than is required by the Nash equilibrium.

(2) Two-characteristic quality competition game

Two firms compete on the business of a fixed population of buyers by selecting for their products two characteristics (x_1,x_2) in $\{1,2,3\}\times\{1,2,3\}$. The action of firm i is denoted by (x_1^i,x_2^i). The total measure of the buyer population is 1. The population is evenly split so that half of it relates lexicographic priority to the first characteristic and the other to the second characteristic. Thus, if for example, $x_1^i<x_1^j$ and $x_2^i\leq x_2^j$ then firm j gets the whole market while if $x_1^i<x_1^j$ and $x_2^i>x_2^j$ both firms split the market equally. Firm i's signal is its share of the market. When choosing (x_1,x_2), a firm incurs a fixed cost of x_1+x_2. A firm's gross profit (before subtracting the fixed cost) as a function of its market share is as follows:

Market share	Gross profit	Marginal profit
1	5.5	1.5
1/2	4	4
0	0	

This pattern of profit will obtain, for example, if the prices of the products are fixed and the marginal cost is increasing in quantity.

The only Nash equilibrium of this game is $(x_1^1,x_2^1)=(x_1^2,x_2^2)=(3,3)$. To see this first note that in Nash equilibrium there may not be some characteristic k such that $x_k^i-x_k^j=2$, since then player i can profit by reducing his characteristic k by a unit. Next, note that in Nash equilibrium the firms split the market. This is because, if firm i's market share is 1, then it must be that, for some $k\neq\ell$, $x_k^i-x_k^j=1$ and $x_\ell^i-x_\ell^j=0$. Otherwise, firm i can reduce its cost without diminishing its market share. But then firm j can gain 4-1 by deviating to $(x_1^j,x_2^j)=(x_1^i,x_2^i)$. It follows that either for some $k\neq\ell$, $x_k^i-x_k^j=1$ and $x_\ell^j-x_\ell^i=1$ or $(x_1^j,x_2^j)=(x_1^i,x_2^i)$. In both cases, with the exception of the configuration in which both choose (3,3), player j can increase its revenue by 1.5 by increasing k by one.

No other configuration in which the firms split the market is a Nash equilibrium, since there is in which case i can profit by reducing his characteristic k by a unit, or $x_k^i-x_k^j=1$ so that j

Every action (x_1,x_2) is rationalizable. To verify this, notice that (1,1) is best response to (3,3) and any other (x_1,x_2) is best response against either (x_1-1,x_2) or (x_1,x_2-1).

In contrast, the RCE outcome must be an equal split of the market.

<u>Claim</u>: All RCE are the action-signal pairs $([(x_1^1,x_2^1),1/2],[(x_1^2,x_2^2),1/2])$ such that (x_1^1,x_2^1) and (x_1^2,x_2^2)

(i) split the market equally and

(ii) belong to {(2,1),(1,2),(3,1),(2,2),(1,3),(3,3)}.

<u>Proof</u>: There is no RCE in which one of the firms has market share 0. If there is a RCE in which firm i's share is 0, then the only action-share pair in B_i in which the share is 0 must be [(1,1),0]. Otherwise, firm i can profit by deviating to (1,1). Therefore, the only action-share pairs in B_j in which firm j's share is 1 are [(1,2),1] and [(2,1),1]. But then [(1,1),0] is not g-rationalizable by any belief on B_j since a deviation to (2,2) is profitable.

Consider next a RCE with equal split of the market. There is no such RCE in which firm i chooses (1,1), since equal shares imply that player i believes that j's action is (1,1) and then i can profitably deviate by increasing any coordinate by 1. As well, there is no such RCE in which firm i chooses (3,2) or (2,3), since then the equal share implies that i believes with probability 1 that j's action is not (3,3) by deviating to (3,3) firm i increases its share from 1/2 to 1 and profit. Finally, B_1=B_2=

{[(2,1),1/2],[(1,2),1/2],[(3,1),1/2],[(2,2),1/2],[(1,3),1/2],[(3,3),1/2]} are g-rationalized:

[(3,3),1/2] is g-rationalized by the belief concentrated on (3,3).

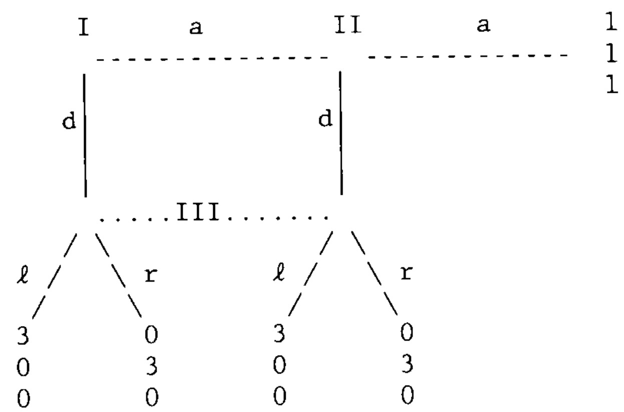
[(2,1),1/2] is g-rationalized by (1,3) and [(1,3),1/2] is g-rationalized by an equal mixture of (1,3) and (3,1) which also g-rationalizes [(2,2),1/2]. Analogously, [(1,2),1/2] and [(3,1),1/2] are also g-rationalized. ■

Notice that this is a Bertrand-like game in which the incentives for "undercutting" lead to a unique Nash equilibrium in which the joint profit is minimized. The RCE relaxes the "Bertrand Paradox" by not insisting that each player knows perfectly his opponent's action, yet it rules out some of the rationalizable outcomes by allowing a player to have the information that can be derived from observing its own payoff.

Note that the set of CE is quite large and includes the configuration where one firm chooses (1,1) and the other chooses any $(x_1,x_2)\neq(1,1)$. The additional requirement of the RCE concept, that the signal-action pairs which g-rationalize an action should be g-rationalizable themselves, is used in the first part of the proof to rule out configurations in which one of the firms dominates the market.

(3) <u>Fudenberg-Kreps' example</u>

Fudenberg and Kreps (1988) bring the following example of an extensive form game.



The pure strategy Nash equilibria are (d,a,ℓ), (d,d,ℓ), (d,d,r) and (a,d,r). That is, in all Nash equilibria at least one of the players I or II chooses d, and so there is no Nash equilibrium in which both I and II play a. Also, every configuration is rationalizable.

If, for all i, the signal g_i is the actual outcome of the game, then in the case that players I and II play a they do not see player III's strategy. Let aa be the outcome of the game when players I and II choose the action a. Then, $B_1=\{(a,aa)\}$ $B_2=\{(a,aa)\}$ and $B_3=\{(r,aa),(\ell,aa)\}$ are g-rationalizable. Player I holds the belief that III plays r and player II holds the belief that player I plays ℓ.

In the next examples the signals are not public and each player forms conjectures regarding the other's signal as well as the other's action.

(4) <u>A Discrete Nash's Demand Game</u>

The players simultaneously announce numbers between 1 and K. If the sum is less than or equal to K+1, they get their respective demands. Otherwise, they get zero. The signal function g_i is i's payoff. All (a_1,a_2) such that $a_1+a_2=K+1$ are Nash Equilibria and all entries are consistent with Rationalizability. Any pair (a_1,a_2) with $a_1+a_2 \leq K+1$ is a RCE since a_i is best response to $a_j=K+1-a_i$. Thus, unlike the Nash equilibrium outcomes, the RCE outcomes include inefficient ones such as (1,1). But the RCE, (and even the CE) rules out the no-trade outcome of both players submitting incompatible demands, since then player i can profitably deviate to demand 1.

(5) Location choice game

Each of N players chooses a location k=1,...,K from among K locations (suppose that K divides N). Let n_k be the number of players in location k. Each player's payoff is a decreasing and convex function of the number of players in his location. For example, suppose that the players are firms and in each location there is a unit of profit potential to be split equally among those locating there. The payoff to a firm locating at k is in this case $1/n_k$.

The Nash equilibria are such that there are exactly N/K players in each location. All the possible configurations of players in locations are rationalizable.

Let player i's signal, g_i, be the number of players in his location. Observe that all RCE as well as all CE are such that there are exactly N/K players in each location. This is because a player who is at a location with more than N/K knows that the expected number of players at any other location is less than N/K. Since the payoff is convex in the number of neighbors, such a player profits by deviating. Thus, the RCE and the CE coincide with Nash equilibrium, but they require less demanding assumptions on what the players know.

(6) The Aggression Game

Player 0 assigns a distinct natural number k_i to each of the players $1,\dots,n$. Then each of these players learns only his own number and chooses one of two possible actions from $A_i=\{\text{attack, defend}\}$. Their payoffs are

$$u_i(\text{defend};\ k_1,\dots,k_n)=0, \text{ and}$$

$$u_i(\text{attack};\ k_1,\dots,k_n) = \begin{cases} 1 & \text{if } k_i=\min\{k_j : j=1,\dots,n\} \\ -1 & \text{otherwise.} \end{cases}$$

Player 0 is indifferent among all outcomes. He is just a device to model deterministic but unknown information, without giving the players common beliefs about it.

$$\text{Define } F(a) = \begin{cases} 1 & \text{if there is an } i \text{ s.t. } a_i=\text{attack} \\ 0 & \text{otherwise.} \end{cases}$$

Let $g_i(a,k_1,\dots,k_n) = (k_i,\ u_i(a,k_1,\dots,k_n),\ F(a))$. That is, each player knows his number, his payoff and if there is any player who chooses to attack.

An interpretation of this model is that the player with the lowest number is the strongest party. He can take advantage of this strength only if he attacks. However, if he attacks when he is not the strongest party, he is beaten.

In any Nash equilibrium only the player with the lowest number chooses to attack. The set of rationalizable outcomes includes almost all possible configurations — the only configurations which are ruled out are those in which some player is assigned the number 1 and chooses to defend. The set of CE includes the outcome that all players get numbers of at least 2 and choose to defend.

Claim: In all RCE the player with the smallest number is the only player who chooses to attack.

Proof: There is no CE in which a player other than the lowest numbered one chooses to attack. This is because each player's signal includes his payoff which would be negative if he attacks when his number is not the lowest. To see that there is no RCE in which no player attacks, note first that there is no such RCE in which one of the players is assigned the number 1, since that player always benefits from attacking. Therefore, there is no RCE in which the lowest number is 2 and nobody attacks, since a player numbered 2 cannot g-rationalizes his decision to defend by a belief that puts positive probability on the existence of a player numbered 1 who does not attack. By induction, for any n, there is no such RCE in which the lowest number is n and hence there is no RCE in which nobody attacks. ■

4. Comments

(a) Many Players

Recall definitions 1 and 2. The fact that the measure μ which g-rationalizes a pair (a_i, s_i) is defined over $\times_{j\neq i}A_j$ means that we allow for correlated conjectures. That is, for example, in a three person game, player 3 may hold a belief which assigns probability 1/2 to each of two pairs of actions (a_1, a_2) and (b_1, b_2). The following two reasons may give rise to correlated beliefs.

(i) Player 3 learns from his signal that players 1 and 2 definitely do not play (a_1, b_2) or (b_1, a_2).

(ii) Player 3 knows excludes (a_1, b_2) from the support of his belief because one of the action-signal pairs $(a_1,\ g_1(a_1, b_2, a_3))$ or $(b_2,\ g_1(a_1, b_2, a_3))$ does not belong to B_1 or B_2 accordingly.

In the absence of these reasons, we may sometimes be interested only in beliefs that do not exhibit any additional correlation. Then the proper modification of definitions 1 and 2 is to require that player i g-rationalizes a pair $(a_i, s_i) \in B_i$ by a conjecture μ satisfying that, if there are sets $(C_j)_{j\neq i}$ such that for all $c_{-i} \in \times_{j\neq i}C_j$, $g_i(a_i, c_{-i}) = s_i$ and for all j, $(c_j, g_j(c_{-i}, a_i)) \in B_j$, then the belief μ conditional on $\times_{j\neq i}C_j$ is a product measure.

(b) Random Signal and Mixed Strategies

So far we confined attention to deterministic signal and pure strategies while the conjectures of the players could be probability distributions.

Assume first that the signal g_i is random. Since we have in mind a steady state scenario in which information is accumulated through repetition we may identify S_i with the set of probability measures on the original space of signals. Our basic definitions are valid without any modifications.

The above case of random signal can be used to consider games with mixed strategies by letting $g_i(m_1,m_2)$ be the distribution of signals which occur when the two players use the mixed strategies m_1 and m_2. Treating $g_i(m_1,m_2)$ as the random signal (as discussed above) is appropriate for the case in which a player chooses the mixed strategy without observing the particular action that he ends up playing. It is inappropriate for the case in which the player observes the realization of his own mixed strategy. Recall the understanding that the signal reflects information accumulated through repetition. In the former case, player i will be able to learn only the distribution $g_i(m_i,m_j)$, while in the latter case he can learn $g_i(a_i,m_j)$ for any a_i in the support of m_i. In the latter case the proper definition is as follows:

Let M_i denote the set of mixed strategies of player i and let K_i be the set of probability measures on S_i. The random signal $g_i(a_i,m_j)$ is an element of K_i interpreted as the distribution of signals when players i and j choose a_i and m_j respectively. Denote by L_i the set of all pairs $(m_i,(k_i(a_i)_{a_i\in Supp(m_i)}))$ where $m_i\in M_i$ and $k_i(a_i)\in K_i$.

<u>Definition 1</u>: The pair $(m_i,(k_i(a_i)_{a_i\in Supp(m_i)}))$ in L_i is <u>g-rationalized</u> by a conjecture (probability measure) μ on M_j if

(i) for all a_i in the support of m_i and m_j in the support of μ, $k_i(a_i)=g(a_i,m_j)$

(ii) m_i is i's best response, given μ.

Condition (i) requires that, for any action a_i that i takes, i's conjecture be consistent with the distribution of signals, $k_i(a_i)$, that he receives. The fact that this requirement is made for <u>any</u> m_j in the support of μ means that the signal distribution that i receives does not allow him to distinguish between the alternative patterns of behavior that he conjectures as possible for j.

<u>Definition 2</u>: The sets B_1, B_2 where B_i is a subset of L_i are <u>g-rationalizable</u> if, for all i, every $(m_i,(k_i(a_i)_{a_i\in Supp(m_i)})$ in B_i is g-rationalized by a probability measure μ on M_j such that for all m_j in the support of μ the pair $(m_j,(g_j(m_i,a_j)_{a_j\in Supp(m_j)}))\in B_j$.

<u>Definition 3</u>: A <u>RCE</u> is a pair of mixed strategies $m^*=(m^*_1,m^*_2)$ satisfying that there are g-rationalizable sets B_1 and B_2, such that $(m_i^*,(k_i(a_i)_{a_i\in Supp(m_i^*)})\in B_i$ for both i.

Notice that, under the last definitions, it follows from the existence of Nash equilibrium that for any signal functions g_i, there exists a RCE.

(c) <u>Experimentation</u>

Recall that we view an equilibrium as a sort of steady state in a recurring situation. When thought upon along these lines, the RCE concept can be criticized on the grounds that RCE points which are not Nash equilibria are "unstable" in the sense that a player will presumably want to experiment in order to learn more about the strategies of his opponents. Alternatively, such information will be revealed if players tremble in their choice of action or otherwise deviate, even if quite infrequently. Fudenberg and Kreps (1988) analyze the experimentation issue. It should be noted that such experimentation technology has to be of a rather special form, if it is to expose completely the strategy profile being played. Experimentation technologies which are short of this may suggest some restrictions on the form of the signal function $g_i(a)$, but we do not think they disqualify the entire concept.

(d) Equivalence to Nash Equilibrium

In three of the examples considered above (the distance, location and aggression games) the RCE coincided with Nash equilibrium. In games with this property the Nash equilibrium concept is more compelling, because in a sense the equilibrium requires less information on the part of the players. It may therefore be of interest to identify conditions under which, for some natural signal function such as one's own payoff, RCE and Nash equilibria are equivalent. Bernheim (1984) analyzes the analogous question for rationalizability -- he presents necessary and sufficient conditions for such equivalence between rationalizable and Nash equilibrium. As a corollary of his analysis he shows that, in two-player Cournot game, the unique pair of rationalizable strategies is the unique Nash equilibrium. This equivalence fails for $n \geq 3$, where any outputs between zero and the monopoly output are rationalizable. In contrast, the RCE lends some support to the Cournot equilibrium for $n \geq 3$ as well: when each firm's signal is its own profit, the unique RCE coincides with the Nash equilibrium for any n. Notice that in this example the CE already coincides with the Nash equilibrium so that the power of RCE is not used. In the distance and aggression games the RCE coincides with the Nash equilibrium while the CE is not. This research direction provides well defined analytic questions.

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