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BONUSES AND PENALTIES AS EQUILIBRIUM INCENTIVE DEVICES,
WITH APPLICATION TO MANUFACTURING SYSTEMS

by

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Abstract

Although psychologists view bonuses and penalties as very different means of providing incentives for workers, economists have had less success at making sense of the distinction. A rational worker should be indifferent as to whether a payment scheme is called a bonus or a penalty plan if the actual contingent pay stream is identical in the two cases. In this paper we provide a framework for understanding the difference between payment plans that are deemed to be penalty or bonus schemes, and derive implications for when such plans should be implemented as a function of observable features of the manufacturing and monitoring systems.

We call a payment plan a "bonus" scheme if the high payment occurs infrequently in equilibrium; a payment scheme entails a possible "penalty" if the low wage occurs infrequently. The frequency of high and low payments is derived in equilibrium in a model with moral hazard and probabilistic monitoring. We focus on the role of commitment and the possibility of false positives in the monitoring technology. It is shown that when the firm can commit to a monitoring intensity the workers will (almost) always be diligent and a penalty scheme will be observed. When commitment is infeasible the optimal payment structure depends on whether the monitoring technology permits false positives. In the absence of false positives the workers will be observed to face a penalty scheme if found shirking, but when false positives are possible there will be considerable shirking by workers in equilibrium, and a bonus scheme will be observed.

We then analyze the crucial features of our theoretical monitoring technology in the context of actual employment situations. We find that middle-management and other non-production jobs are appropriate for bonus-type incentives, whereas in unskilled jobs or aspects of highly skilled jobs that require diligence but no skill, such as arriving to the job on time, we predict penalty incentives. We argue that the observed scarcity of penalty-type schemes can be explained by our model, without resorting to psychological justifications. In addition, we interpret the Japanese manufacturing systems as having a particular, built-in monitoring system that can be analyzed in our framework and shown to implement a high level of diligence from factory workers.
I. Introduction

Psychologists and personnel managers seem to believe that rewards and punishments are very different devices for eliciting diligence from workers. Kahneman and Tversky (1979) argue that the "framing" of a risky prospect--that is, which outcome one takes to be the status quo--affects one's evaluations of the project. The popular psychology of "carrots" and "sticks" translates in firms into a prescription for bonuses for diligence and penalties for malfeasance.

Economists, however, have been unable to make sense of the distinction.\(^1\) To see why, suppose a worker is paid a base wage \( b \) plus \$5 for each unit of output. This might be called a bonus scheme, since the worker is rewarded for each unit produced. However, if we rewrite this wage function \( w = b + 5q \) as \( w = 600 - 5(100 - q) \) it might now be called a penalty scheme, since it penalizes low levels of output. Indeed, any payment scheme that rewards high productivity could be rewritten as an equivalent payment scheme that "punishes" low productivity. Since the payoff functions are identical, a rational agent should be indifferent to whether it is proposed, or "framed," in the form of a bonus or penalty, and his behavior under the two should be identical.

This line of reasoning suggests that the bonus versus penalty distinction is an empty one, or at best a psychological device with no economic content. Of course, this conclusion is not very satisfying since, given the ubiquity of "bonus" and "penalty" features of real labor contracts, and the commonly held belief that they are meaningfully different, one would prefer an economic model that allowed us to predict when one or the other is more likely to be observed. In this paper we define bonuses and penalties in

\(^1\)This point is perhaps first made by Lazear (1989).
what we think is both an economically meaningful and an intuitive way and
present a model in which either bonuses or penalties emerge as an equilibrium
incentive device. The model allows us to predict the sorts of jobs, the
components of jobs, and the characteristics of firms in which bonuses
(penalties) are likely to be observed. In addition, the model leads in a
natural way to a consideration of manufacturing systems. We argue that the
design of a manufacturing system determines in great part the kind of
compensation system to be implemented and the probability of shirking in
equilibrium. Our main conclusions can be summarized as follows:

• Workers in jobs that tend to be evaluated subjectively, such as those in
which a main component involves discretion and judgement, are likely to
have bonuses rather than penalties as part of their compensation.

• Workers in jobs in which output is easily measured, such as digging
ditches, or in components of jobs that are easily measured, such as
punctuality, are likely to be subject to penalties in their compensation.

• Identifiable features of the manufacturing technology determine whether
the factory workers will be paid bonuses or assessed penalties, and will
determine the workers' level of diligence. In particular, technical
features of the Japanese manufacturing system induce a high level of
diligence by the factory workers.

In addition, we argue that our model can explain the relative scarcity
of observed penalty schemes, and ubiquity of bonus schemes, in business
settings.

The organization of the paper is as follows. Section 2 provides the
framework for our analysis, including our definition of bonus and penalty
schemes and some discussion of related literature. In Section 3 we present
the formal model, and Sections 4 and 5 contain our results under the two
different generic monitoring technologies we consider. Our interpretation of
the model in the context of real business firms is contained in Section 6, and
we apply the model to Japanese manufacturing systems in Section 7. Section 8
contains concluding comments.

2. The Framework

Webster's Dictionary defines the word bonus as "anything given in
addition to the customary or required amount." The word "customary" is what
motivates our definition. To make the point we will introduce some notation.

Let a worker's compensation contract be \( w = (h, l) \), with \( h > l \), that
specifies the payment of \( h \) dollars upon observation of the realization of a
random variable \( Y \in H \), where \( H \) is a set of real numbers, and the payment of \( l \)
upon observation of \( Y \in L, L \cap H = \emptyset \). We define the "customary wage" to be
the one that in equilibrium is observed more often. Thus, the contract \( w \) is
said to be a penalty scheme if, in the equilibrium of the game in which that
contract is offered to the agent, the probability that \( h \) is paid is larger
than the probability that \( l \) is paid. Similarly, if the wage that is paid with
a larger probability is \( l \), the contract is said to be a bonus scheme.\(^2\)

Our approach reverses the traditional view of cause and effect in
bonus/penalty schemes: in our view one does not see \( h \) more often than \( l \)
because a penalty scheme is at work, but rather we define the contract to be a

\(^2\)It is important to note that the issue is not one of timing. If, say,
the signal \( Y \) is observed monthly, then either \( h \) or \( l \) is paid once a month.
The contract is considered a bonus scheme if, over several months, \( l \) is more
often observed than \( h \).
penalty scheme when we see h more often than l. It follows that the question 
"Do penalties and bonuses have different effects on behavior?" with which 
psychologists have been concerned, is meaningless in our framework; the 
payment scheme and behavior are determined jointly as an equilibrium.\(^3\) On 
the other hand, our definition is entirely consistent with the notion of 
"framing," if in fact workers take their frame of reference to be their 
customary level of compensation.

Whether the high or low payment is more likely to be observed depends on 
the probability that the worker is diligent, which in turn depends on the 
compensation contract between the worker and the firm. Drawing on the work of 
Olivella (1989), we analyze this contracting problem as a game, and find the 
equilibrium under different monitoring technologies. In all the models that 
we analyze a unique equilibrium exists, given any wage contract. Thus, we can 
identify that equilibrium with the given contract and determine whether the 
contract is a bonus or a penalty scheme according to our definition.

Consider the following example. Suppose that most workers are usually 
punctual. If all workers who are punctual during 30 days receive "extra pay," 
we will very often see the extra pay being awarded. The customary 
compensation is therefore the base wage plus the "extra pay." Only from time 
to time does one observe a wage below that level. Thus, in our framework, 
regardless of how this pay structure is represented to the workers, we would

\(^3\)Other attempts at dealing explicitly with the bonuses versus penalties 
question have quite a different slant. Fenoaltea (1984) and Chwe (1990) 
analyze the trade-off between financial incentives and physical punishment. 
These models were intended to explain physical punishment as an incentive 
device, rather than to explain the modern use of financial rewards and 
penalties in labor contracts. To the extent that they are relevant to modern 
labor markets, it is probably only with respect to impoverished workers, for 
whom dismissal may be tantamount to a threat of hunger.
consider it a penalty scheme. When an incentive scheme is successful, it becomes a penalty scheme.

Note that we cannot predict that the firm will necessarily call this a penalty scheme. However, we do assert that workers would in fact view it as such, precisely because they would be "accustomed" to the high level of pay. Further, we believe that it is consistent with the notion of framing to designate this payment plan as a penalty scheme, since workers would arguably take the high level of pay as their frame.

Suppose now that workers are often late despite the incentive scheme. Then we will see the extra pay being awarded very seldom, and whether the firm advertises it as a punctuality bonus or a tardiness penalty, we consider it a bonus scheme. If an incentive scheme is ineffective, it becomes a bonus scheme. Again, we view this as consistent with common usage as well as with framing theory.

In order for the firm to determine the level of performance of a particular worker, some means of evaluating his productivity must be implemented. We now describe this monitoring technology.

In the standard agency literature there is a view that a trade off exists between incentive contracting and monitoring. The assumption in agency models is that, although input is not directly observable, some measure of output provides an imperfect signal of the worker's input. It is further assumed that output is observed at zero cost. In this context, monitoring has been thought of as the ability to observe inputs directly, at some cost, and therefore may serve as a substitute for or supplement to agency contracting.⁴

⁴See Holmstrom (1979), Shapiro and Stiglitz (1982), Singh (1985), Sparks (1986) for models of optimal monitoring in principal-agent models, under the assumption that the owner can commit beforehand to any monitoring strategy.
In our view monitoring and contracting are inherently inseparable because, in reality, output is not observed at zero cost, and input may be unobservable at any cost. We model monitoring as a costly means of measuring or evaluating a worker's output. Thus, monitoring may be thought of as an unavoidable cost of implementing an incentive contract, where the costly monitoring effort is aimed at providing some measure of output.

The idea that output is costly to observe (even in the absence of "team" or joint production problems) is particularly important when applied to middle managers and production workers. For example, if a factory worker fabricates metal fittings for later assembly, his productivity is not known unless someone (or something) counts the number of parts he produces. This counting is costly. If the care or skill of the worker determines the parts' quality, then measuring the worker's output requires, in addition, measuring or estimating the quality of the parts he produces. This is a more complex and rich problem, which we will address further in Sections 6 and 7. Suffice it to say for the moment that the standard American solution to this problem is statistical sampling, in which the higher the percentage of parts sampled, the better the estimate of a worker's true output, but the higher the cost of monitoring. In our framework, sampling is an example of what we mean by a monitoring technology, and in what follows it will perhaps be useful for the reader to keep this example in mind.

One feature of monitoring that we focus on in particular is the role of commitment. We explore a situation in which the owner of a firm cannot commit ex ante to the intensity with which he monitors his workers. To see the importance of commitment, suppose it is the policy of a general contractor carefully to inspect each job performed by his employee, an electrician. The
certain expectation of having his work inspected may be sufficient to induce enough diligence by the electrician that no errors are made or codes violated. However, the contractor, knowing this, has no ex post incentive actually to perform the inspection; but if the electrician anticipates that the inspection will not be performed he will not be so diligent. This suggests that when the contractor cannot credibly commit to performing the promised inspection there may be a problem with existence of an equilibrium. In fact we will show that when commitment is precluded an equilibrium does not exist in pure strategies, but does exist in mixed strategies. This implies that if the employer cannot commit to a monitoring strategy he cannot implement diligence with probability one; there will, in equilibrium, be some positive probability of shirking by the employee.

When is commitment likely to be an issue in real firms? First, when the monitoring is performed by a human being, the choice of monitoring intensity may actually be made ex post by the monitor. This is likely to be the case when monitoring takes the form of a supervisor proof reading a document for errors, walking the shop floor to oversee workers' effort, examining the work of a subcontractor, or writing a subjective performance evaluation of a subordinate. It is likely that mechanical monitoring is a means of inducing commitment; for example, automatic checking of typists through a word processing system may effectively commit the principal ex ante to a particular monitoring intensity. So-called "autonomous machines" that automatically check for defectives in manufacturing processes are another example. However, whether commitment is indeed induced in any particular case depends on the marginal cost of the mechanical monitor and whether the system can be disabled. If the mechanical monitoring system can be disabled, and if the
marginal cost of operating the monitoring system is positive, the mechanical monitor may have no better commitment features than a human monitor.

Second, the lack of ability to commit to a monitoring intensity is only important when the actual ex post monitoring is unobservable or unverifiable. Even if the firm cannot commit to monitoring the worker, if the two parties can write a contract as a function of the actual ex post monitoring effort the full commitment case is replicated. However, enforcing such a contract requires the monitoring to be verifiable to a third party. In the examples suggested above of a proof reader, general contractor, middle manager, or shop floor supervisor the care with which the proof reader reads, the supervisor watches, and so forth, may be inherently unverifiable.

Finally, in a broader sense, commitment requires not only the monitoring itself, but also a commitment to carrying out the contract as a function of the observed signal. The inability of the government to commit to ex post enforcement leads Besanko and Spulber (1988) to study a no-commitment model in the context of law enforcement, and Melumad and Mookherjee (1987) and Reinganum and Wilde (1986) study a no-commitment model in the context of tax evasion.

3. The Model

The firm is composed of a risk neutral owner and a risk averse worker. The owner is the residual claimant. The worker exerts unobservable effort $a \in \{s,d\}$ where $s$ stands for "shirk" and $d$ for "diligent." This effort generates cash flow $\pi > 0$ when $a = d$ and zero when $a = s$. The worker has a von Neumann-Morgenstern utility function that is additively separable in money
income and effort: $U(m,a) = u(m) - V(a)$, where

**Assumption 1:** The function $u(\cdot)$ satisfies: $u(0) = 0$, $u'(\cdot) > 0$, $u''(\cdot) \leq 0$; and $V$ is the disutility of effort. Let $V(d) = v > 0$ and $V(a) = 0$. The worker obtains utility $T \geq 0$ (for sure) if he leaves the firm.

Cash flow is not observable, so contracts on cash flow cannot be signed. We also assume that, for organizational reasons exogenous to the model, the firm cannot be sold to the worker. However, a monitoring technology that provides society with a publicly observable and verifiable signal $Y$ is available. Contracts contingent on this signal, and on this signal alone, can be written. The owner will spend resources on monitoring according to a cost function $C(\theta): [\theta^{\text{min}}, \theta^{\text{max}}] \rightarrow \mathbb{R}^+$, where $\theta$ is a measure of monitoring intensity, and $C(\cdot)$ satisfies

**Assumption 2:** $C'(\theta) > 0$ for all $\theta > \theta$, $C'(\theta^{\text{min}}) = C(\theta^{\text{min}}) = 0$, $C'(\theta^{\text{max}}) = \infty$, and $C''(\cdot) > 0$.

We assume the signal can take only two values: $H$ (for "high") or $L$ (for "low"). The probabilities of these outcomes depends on the monitoring intensity, $\theta$, and the effort, $a$, exerted by the worker. The realization of $H$ and $L$ will have the natural interpretation, $H$ indicating that the worker was diligent, $L$ indicating that the worker shirked, because the monitoring technologies specified below satisfy the monotone likelihood ratio condition (MLRC).\(^5\)

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Monitoring systems are not perfect, and can fail in two obvious ways. The monitor may conclude that the worker was diligent when he in fact shirked, or may conclude that the worker shirked when he was indeed diligent. Accordingly, we consider two different monitoring technologies. The first permits only the possibility of false negatives, i.e., that workers may mistakenly be determined to be diligent. In this case the signal may be H even if the worker actually shirked. The second case allows also for the possibility of false positives, that is, that the worker may be falsely accused of shirking. The two monitoring technologies are formalized in the following way.

**Case 1: No False Positives (NFP) Monitoring Technology**

**NFP 1a:** \([g^{\min}, g^{\max}] = [0, 1].\)

**NFP 1b:** \(\Pr(Y = L | a = d) = 0; \Pr(Y = L | a = s) = \theta.\)

Thus, the probability of a false positive is 0 for all \(\theta.\)

Under the NFP monitoring technology, the value to the owner of increasing monitoring intensity is that it decreases the probability of falsely rewarding diligence, without increasing the probability of detecting actual diligence.

As an example of a NFP technology, consider a worker whose job it is to stamp metal parts used in a later stage of production. If the worker does not carefully calibrate his machine he may make parts that do not properly fit together. If they do not fit the worker certainly shirked, but if they do it
may be because he relied on the calibration performed by the machine's previous operator. Thus, he may be falsely found diligent but not falsely accused of shirking.

On the other hand, if the machine can introduce errors on its own, regardless of the care by the worker, false positives will be possible. We call this the FP case, formalized as follows:

Case 2: False Positives (FP) Monitoring Technology

**FP 2a:** \[ \theta^{\text{min}}, \theta^{\text{max}} = (1/2), 1 \].

**FP 2b:** \[ \Pr(Y = H|a = d) - \Pr(Y = L|a = s) = \theta \].

Under the FP technology, not only can a shirking worker be found diligent, but, in contrast to the NFP technology, a diligent worker may be accused of shirking. As \( \theta \) increases, both the probability of false positives and the probability of false negatives decreases; that is, the signal becomes more informative. These assumptions are summarized in Table 1.

[INSERT TABLE 1 ABOUT HERE]

The reader will note that we do not consider the case in which only false positives can occur. In such a case a worker could be falsely accused of shirking but never falsely accused of diligence, and increasing monitoring intensity would only increase the amount of diligence detected. It is easy to see that, without commitment, the owner will never choose to monitor ex post under such a monitoring technology, since increased monitoring only increases
the probability of paying a higher wage. In consequence, the worker shirks with probability one if hired, and the worker is never hired in equilibrium if T is positive.6

The following assumptions are maintained throughout:

Assumption 3: u(x) > v + T.

Assumption 4: h, l ≥ 0

Assumption 3 implies that in the first-best world without moral hazard, the equilibrium would entail hiring the worker, inducing diligence with probability one, and paying him a wage u⁻¹(v + T). This is a necessary but not sufficient condition for the existence of equilibria in which the worker is hired.

Assumption 4 ensures that our monitoring problem has a well-defined solution. In the absence of a lower bound on the penalties that can be imposed on workers, the principal can approach the full information-first best actions arbitrarily closely by following the "boiling in oil" policy of combining an arbitrarily large punishment with an arbitrarily low probability of detection.7 The fact that real business firms devote a substantial amount of resources to detection of malfeasance (see Dickens, et. al., 1989) suggests either that there are limits to firms' ability to implement a boiling in oil scheme, or that other features of the environment preclude its optimality (Polinsky and Shavell, 1979). We make the simplest assumption that allows us

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6A similar result is obtained if the owner can choose the probability of false positives and false negatives independently. No matter what the worker's strategy is, the owner has an incentive ex-post to maximize the probability that a false positive occurs. The implication of this is that the signal becomes uninformative about the worker's effort. The worker therefore shirks with probability one, and the worker is never hired if T is positive. (See Olivella (1990).)

7See Becker (1968), Mirrlees (1974), and Gjesdal (1976)).
to avoid the non-existence problem, and also for simplicity we set the lower bound on penalties to be zero, that is, $h, l \geq 0$.

Whenever the problems are separable, we will solve the games only for the equilibrium monitoring and diligence strategies, but for brevity, not the equilibrium wage structure. Because these problems are solved recursively, beginning with the worker's strategy and the monitoring strategy taking the wage structure as fixed, any statement about the solution for the subgame that we show true given any wage is also true about the solution to the entire game. For analysis of the equilibrium wage structure the reader is referred to Olivella (1989).

4. **The Monitoring Game with Full Commitment**

We begin by analyzing the full commitment (FC) game. In this game the owner's choice of $\theta$ is observed by the worker before he chooses his effort level. The owner can therefore commit to a level of monitoring intensity before production starts. The order of moves is the following: at the beginning of the game, the owner offers the worker a contract $w = (h, l)$, in which wage $h$ is to be paid if the realization of the signal is high, and wage $l$ will be paid if the signal is low. A generic element of $w$ is denoted by $w_i$. Then the worker decides whether to sign the contract. If the worker rejects the contract, he receives his opportunity utility $T$, and the owner receives zero. The game ends if the worker rejects the contract. If the worker accepts the contract, then the owner chooses $\theta$. Next, the worker observes $\theta$ and then chooses whether to shirk or be diligent. Finally, at the end of the period,

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8Note that false positives are possible in one of our monetary technologies. This, as Nalebuff and Sharfstein (1987) show, is not enough to avoid the "boiling in oil" paradox.
cash flow and the signal are realized, and the wage is paid.

We allow the worker to adopt a mixed strategy. Denote by \( p \) the probability that the worker is diligent, i.e., \( p \) is the probability that the worker sets \( a = d \). Then the payoffs of the players are the following:

a. The owner's payoff, provided that the worker accepts the contract, is:

\[
U^0(p, \theta, w) = \pi p - E(w_1|\theta, p) - C(\theta)
\]

where \( E(\cdot|\theta, p) \) denotes expectation conditional on \( p \) and \( \theta \).

b. The worker's payoff is:

\[
U^w(p, \theta, w) = E(u(w_1)|\theta, p) - pv.
\]

**Proposition 1.1:** Under either the FP or NFP monitoring technology, if the owner can commit ex ante to any monitoring strategy the worker is diligent in equilibrium with probability one.

**Proof:** See the Appendix.

This result takes the monitoring intensity to be a choice variable for the firm. This is certainly natural when commitment is infeasible, but it may be the case that commitment devices or the manufacturing technology have inherent in them a monitoring intensity. We will argue later that this is the case in the Japanese manufacturing system. Thus, we would also like to solve
the model in the case of commitment when the level of monitoring intensity is exogenous. In this case the only choice variable for the firm is the wage structure.

**Proposition 1.2:** If the monitoring intensity $\theta$ is exogenous and high, the owner prefers a NFP technology to a FP monitoring technology.

**Proof:** See the Appendix.

From Propositions 1.1 and 1.2 directly follow the following implications:

**Corollary 1:** In jobs or aspects of jobs for which the owner can commit to a monitoring intensity, incentives will be provided via penalties (where the penalties are never actually imposed.)

**Corollary 2:** When owners can commit to a monitoring technology with a high level of inherent monitoring they would prefer one that cannot falsely accuse workers of shirking.

5. **Monitoring When Commitment is Infeasible**

In the no-commitment (NC) setting, where $\theta$ is assumed to be unobservable to the worker, the game proceeds as follows. As before, at the beginning of the period the owner offers a contract to the worker, which he accepts or rejects. If the worker accepts, the worker's effort decision and the owner's monitoring intensity decision are made, in effect, simultaneously. These
Simultaneous decisions are made in the "second stage of the game." When
the owner chooses a wage schedule in the first period, he is, in effect,
choosing a second stage equilibrium, subject to the worker's voluntary
participation constraint.

In this section we prove that, if the owner is unable to commit to any
monitoring strategy, then:

1. under the FP monitoring technology, the worker shirks with
   probability larger than $1/2$ (Proposition 2.1 below), and that this
   implies that a bonus scheme will be observed in equilibrium
   (Corollary 3.1); and

2. under the NFP monitoring technology, the worker shirks with a
   probability that tends to zero as $C'(v/(v + T))$ tends to zero
   (Proposition 2.2), and that this implies that we observe a penalty
   scheme in equilibrium as long as $C'(v/(v + T))$ is small enough
   (Corollary 3.2).

The intuition behind these results is the following. Under the FP
monitoring technology, lowering the monitoring intensity, $\theta$, is beneficial to
the owner when the worker is diligent: the probability of having to pay $(h -
\ell)$ decreases as $\theta$ decreases. Lowering $\theta$ harms the owner when the worker
shirks: he has to pay $(h - \ell)$ with a higher probability. If $p > 1/2$ (i.e.,
if the probability that the worker shirks is smaller than the probability that

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9 Recall that we assumed in the FC games of the previous section that the
owner chooses $\theta$ after the worker accepts the contract (rather than choosing $\theta$
and $w$ simultaneously). This makes the strategy sets in the FC and NC games as
similar as possible. This is done to highlight the fact that the differences
between the equilibria in the FC and NC games come exclusively from the owner
not being able to commit to $\theta$ before the worker chooses his strategy.
the worker is diligent), the benefits of lowering $\theta$ outweigh the costs, due to
the symmetry of the monitoring technology. Thus, the owner lowers $\theta$ down to
$g_{\min}$ if $p > 1/2$, to which the best response of the worker is to set $p = 0$.
Note that this argument is completely independent of the monitoring cost
function. Even if the monitoring costs were zero for all $\theta$, the result would
still hold.

This result disappears if we assume that false positives are impossible.
Even if $p > 1/2$, the owner still has an incentive to monitor intensely as long
as the marginal costs of monitoring are not too high, because increasing the
monitoring intensity only increases the probability of detecting shirking.
Note, however, that the lack of commitment is still causing some problems: if
the worker were diligent with probability 1 the incentives of the owner would
disappear.

From Proposition 2, it is clear that for $C'(v/(v+T))$ small enough, there
is more shirking under the FP monitoring technology. We should therefore see
business firms trying to avoid monitoring technologies capable of false
positives.

5.1. The FP Monitoring Technology When Commitment Is Precluded

The equilibrium is found recursively, starting from the last
(simultaneous) moves. The owner and the worker face the decision of
simultaneously choosing $\theta$ and $p$, respectively. To find the Nash equilibria of
this subgame, the wage contract inherited from the first stage is taken as
fixed. We find the reaction function of the worker to the owner's choice of
$\theta$, and the reaction function of the owner to the worker's choice of $p$. The
equilibria are found where these two reaction functions meet in $[1/2, 1] \times$
We concentrate on equilibria where the owner’s payoff is positive, since he could always obtain zero by offering the worker an unacceptable contract.

For given \( w \), the worker solves

\[
\max_{p \in [0,1]} \, p[\theta u(h) + (1 - \theta)u(l)] + (1 - p)[\theta u(l) + (1 - \theta)u(h)] - pv,
\]

which yields the following reaction function:

\[
\rho^*(\theta, w) = \begin{cases} 
0 & \text{if } (2\theta - 1)[u(h) - u(l)] < v \\
[0,1] & \text{if } (2\theta - 1)[u(h) - u(l)] = v \\
1 & \text{if } (2\theta - 1)[u(h) - u(l)] > v
\end{cases}
\]

From (1) we see that, for any \( w \) satisfying \( u(h) - u(l) \geq v \), the worker is indifferent between shirking and not shirking if \( \theta \) is set equal to

\[
\theta^c(w) = \frac{1}{2} \left[ 1 + \frac{v}{u(h) - u(l)} \right].
\]

The owner’s reaction function, \( \theta^c(w, p) \), is found by solving:

\[
\max_{\theta \in [1/2,1]} \, p \cdot \left( \theta u + (1 - \theta)l \right) - (1 - p) \cdot (l + (1 - \theta)h + \theta l) - C(\theta).
\]

The first order condition is:

\[
(1 - 2p)(h - l) - C'(\theta) \begin{cases} 
> 0 & \text{then } \theta = 1 \\
eq 0 & \text{then } \theta \in [1/2,1] \\
< 0 & \text{then } \theta = 1/2.
\end{cases}
\]

Hence, using Assumption 2, we have that

\[\text{Assumption 2 is sufficient to ensure that a unique equilibrium } <p, \theta> \text{ exists given any wage schedule } w.\]
\begin{equation}
\theta(w,p) = \begin{cases} 
\sqrt{\frac{1}{2}} & \text{if } (1-2p)(h-\theta) \leq 0 \\
\frac{(1-2p)(h-\theta)}{[C']^{-1}[(1-2p)(h-\theta)]} & \text{if } (1-2p)(h-\theta) > 0.
\end{cases}
\end{equation}

The next Lemma, which holds for both monitoring technologies, will be useful later in our analysis.

Lemma 1: If the owner's payoff is positive, it must be the case that $p > 0$.

Proof: The lemma follows directly from the assumptions
   a) the owner obtains zero if the worker rejects the contract;
   b) $T \geq 0$;
   c) $u(0) = 0, u'(\cdot) > 0, u''(\cdot) < 0, C(\cdot) > 0$.

Then at $p = 0$, $U(0,\theta,w) = -E(w_i|\theta,0) - C(\theta) \leq 0$, where the inequality follows from applying Jensen's inequality to the individual rationality constraint.

Q.E.D.

Proposition 2.1: For any subgame perfect equilibrium of the entire NC-FP game, the probability of shirking, $1 - p^*(w^*)$, is larger than $1/2$.

Proof: Suppose $p \geq 1/2$. Then (1) implies $\theta > 1/2$ and $h > \ell$. Therefore, $(1 - 2p)(h - \ell) < 0$. This implies, by (6), $\theta = 1/2$, a contradiction. Q.E.D.

Corollary 3.1: In the NC-FP game, for any equilibrium (of the entire game) in which the owner's payoff is positive, a bonus scheme is observed.

Proof: If the owner's payoff is positive, it means that $p > 0$ by Lemma 1, This implies, by (1), that $\theta > 1/2$. The probability $Q$ that wage $h > \ell$ is paid
is \( Q = p\theta + (1-p)(1-\theta) = (2\theta-1)p + 1 - \theta < (2\theta-1)1/2 + 1 - \theta = 1/2 \) where the inequality holds because \( \theta > 1/2 \) and \( p < 1/2 \) by Proposition 2.1. Q.E.D.

5.2 The NFP Monitoring Technology When Commitment is Precluded

Under this monitoring technology, \( \theta \in [0,1] \) instead of \([1/2,1] \).

Proceeding as in Section 5.1, fix \( w \) and find the worker's best response \( p^r(w,\theta) \) to \( \theta \) by solving

\[
\max_{p \in [0,1]} pu(h) + (1-p)[(1-\theta)u(h) + \theta u(l)] - pv
\]

which yields:

\[
(5) \quad p^r(w,\theta) = \begin{cases} 
0 & \text{if } \theta(u(h)-u(l)) < v \\
[0,1] & \text{if } \theta(u(h)-u(l)) = v \\
1 & \text{if } \theta(u(h)-u(l)) > v.
\end{cases}
\]

For any \( w \) satisfying \( u(h) - u(l) \geq v \), the worker is indifferent between shirking and not shirking if \( \theta \) is set equal to

\[
(6) \quad \theta^c(w) = \frac{v}{u(h) - u(l)}.
\]

The owner's reaction function \( \theta^r(p,w) \) is found by maximizing \( U^o(p,\theta,w) \) with respect to \( \theta \). This yields

\[
(7) \quad (h-t)(1-p) - c'(\theta) = \begin{cases} 
0 & \text{if } 0 \text{ then } \theta = 1 \\
\geq 0 & \text{if } \theta \in [0,1] \\
< 0 & \text{if } \theta < 0.
\end{cases}
\]

Hence, using Assumption 2 we have that

\[
(8) \quad \theta^r(w,p) = \begin{cases} 
0 & \text{if } (1-p)(h-t) < 0 \\
\lceil c' \rceil^{-1}[(1-p)(h-t)] & \text{if } (1-p)(h-t) \geq 0.
\end{cases}
\]

The owner's first stage problem is to

\[
(9) \quad \max_{w \in \mathbb{R}, \theta \in [0,1], p \in [0,1]} p\theta - ph - (1-p)[(1-\theta)h + \alpha t] - c(\theta)
\]
subject to

(i) \( p \in p^c(\omega, \theta) \)

(ii) \( \theta \in \theta^c(\omega, p) \)

(iii) \( l \geq 0, h \geq 0 \)

(iv) \( pu(h) + (1 - p)[(1 - \theta)u(h) + \theta u(l)] \geq pv + T \).

**Proposition 2.2:** If the equilibrium payoff for the owner is positive, the constraint \( l \geq 0 \) is binding, and the equilibrium probability of the worker being diligent is \( p = 1 - C'(\theta^c(\omega))/h \), where \( \theta^c(\omega) = v/u(h) \).

**Proof:** See the Appendix.

**Proposition 2.3:** As \( C'(v/(v+T)) \) tends to zero, the equilibrium probability that the worker is diligent tends to one.

**Proof:** See the Appendix.

**Corollary 3.2:** If \( C'(v/(v+T)) \) is small enough, in the NC-NFP game we observe penalties in equilibrium.

**Proof:** The probability \( Q \) that wage \( h \) is paid is \( Q = p + (1-p)(1-\theta) \geq p \). Since \( p \) tends to 1 as \( C'(v/(v+T)) \) tends to 0, \( Q > 1/2 \) for small enough \( C'(v/(v+T)) \). Q.E.D.

6. **Applications of the Model**

The personnel management literature makes a distinction between
performance measurement and performance appraisal. Performance is measured by
the sorts of methods we alluded to in Section 2--proof reading, statistical
sampling, and so forth. However, in jobs in which the worker's output is
thought to be inherently unmeasurable, such as middle management, performance
is evaluated by "appraisal." This is a subjective technique that typically
involves a supervisor rating the employee on various dimensions of job
performance including effectiveness as a leader, organizational ability, and
diplomacy. Because appraisal techniques are inherently at risk of bias and
inaccuracy, and because their accuracy depends on the level of monitoring
effort chosen by the supervisor (in a way that is not contractible), our
interpretation is that appraisal methods fit in the NC-FP category.

That appraisals are not perfect is well accepted; Whisler and Harper
(1962) suggest that at least part of the reason is precisely a problem of
commitment:

"Probably anyone who directs a group or workers of such size
that he can observe their performance daily and intimately
can spot outstanding individuals as distinguished from
employees who merely do enough to get by. That is, he can
do so if he really takes the trouble to follow up each
worker and make a continuing study of his performance on the
job. However, anyone who is thoroughly familiar with rating
procedure in its current state of development knows that the
average supervisor does no such thing."\(^{11}\)

In addition, Hamner (1975) finds that workers tend to evaluate their own
performance more highly than management is likely to. If workers know their
true effort, this study also suggests that there is at least a possibility of
false positives arising from employee appraisals.

These arguments, together with Corollary 3.1, suggest that workers in

\(^{11}\)Whisler and Harper, p. 59.
jobs that are evaluated by appraisal will be observed to have bonus, rather than penalty, contracts; that is, the "base" or low level of pay will be the more common, with higher compensation paid infrequently. Concomitant is the implication that the probability of diligence on these dimensions will be low (by Proposition 2.1).

The evaluation of a CEO's performance through the stock market value of the firm also fits in this category. The stock market value constitutes a publicly observable signal that is clearly capable of producing false positives, since it is undoubtedly possible that a manager who has consistently been diligent in studying his product market, input markets, and internal organization, suffer bad results in the stock market because, say, his diligence does not compensate for an unobserved lack of talent for the job, or because of an unobserved negative demand shock. Thus, the model predicts that the nature of the job induces a bonus scheme for top management.

Nevertheless, even for jobs in which output is difficult to measure, not all components of job performance are evaluated by appraisal. Components that are objectively verifiable, such as attendance and punctuality, are typically measured rather than appraised. Conversely, in jobs in which output is more easily measured there are often unmeasurable aspects of productivity. Quality of output is an important example. Quality control experts argue that paying workers by a sort of piece rate, even if possible, would distort workers' incentives so much in favor of quantity rather than quality that a system of no explicit incentives, or an appraisal system, is superior.\(^{12}\)

In general, employees in jobs that require a great deal of employee discretion and judgement tend to be evaluated by appraisal, and the model

\(^{12}\)See also Holmstrom and Milgrom (1990).
therefore predicts that such employees will be observed to receive bonuses. Conversely, in jobs that are purely mechanical (such as digging ditches) or components of jobs that can be measured without risk of false punishment (such as tardiness and absence) we would expect (by Corollary 3.2) to see penalties in equilibrium.

For job components that are measured rather than appraised the predictions of our model depend on whether the measurement technology is capable of producing false positives, and whether it serves as a commitment device. Assuming that a punctuality clock cannot falsely accuse a worker of tardiness or absence and/or that the clock is a commitment mechanism, the model predicts that we will observe penalties and a high level of diligence on these dimensions in equilibrium.

In this context, our result that middle and top managers are likely to engage in a high degree of shirking deserves some comment, since it is clearly contrary to the view in the business press that managers tend to be "workaholics." It is important to note that the job components on which managers are apparently highly diligent are those that are quite easily measured, such as hours at the office and amount of travel. Our model does, indeed, predict a high degree of diligence on these dimensions for any job. However, the nature of managerial shirking predicted in the model is in components of the job that are measurable only with (type one and type two) error, such as how deeply the manager thought about a problem, and how creatively he was able to solve it. It is not at all clear that American business firms have had a high degree of success eliciting diligence on these dimensions.

The model further predicts (compare Proposition 1.1 to Propositions 2.2
and 2.3 for \( C(v/(v + T)) \) small) that employers would like to devise means of committing to a monitoring intensity. As suggested earlier, mechanical monitoring devices may be seen in this model as a commitment device. This suggests that, for any total cost of monitoring, owners would prefer monitoring devices with low (or zero) marginal cost and high fixed cost: to the extent that the marginal cost is high, the owner has an incentive to disable to device ex post, and the commitment is undermined. Indeed, a monitoring device with a higher total cost may be preferable to a less costly one if it has a lower marginal cost, since it is the low marginal cost that implements the commitment. Thus, the model has empirically different implications for the types of monitoring devices that are optimal for a firm from the standard model of cost minimization.

Finally, our model is consistent with the common observation that, in business settings, penalties are rare relative to bonus schemes. This is generally attributed to the hypothesized psychological effect of calling a payment plan a bonus scheme rather than a penalty contract. Our model offers an alternative explanation. We have shown that, under monitoring technologies with a significant risk of false positives,\(^{13}\) bonus schemes are observed. It is indeed difficult to produce many good examples of monitoring technologies in which the risk of false positives is negligible, and although

\(^{13}\)The reader might note that our conclusions are based only on two different possibilities: a monitoring technology with a .5 risk of false positives when no monitoring is made (the FP monitoring technology) and a monitoring technology with a zero risk of false positives when no monitoring is made (the NFP monitoring technology). However, in an analysis not reported here, we have verified that for intermediate monitoring technologies, the qualitative result that a bonus scheme is more likely the greater is the risk of false positives, is supported.
this may simply be due to the authors' lack of imagination, we believe that it reflects a real phenomenon; that in real jobs there are exogenous factors affecting productivity such that, if monitoring were not performed, a worker would face a non-negligible risk of being falsely accused of shirking. But if real monitoring systems are, for the most part, prone to producing false positives, bonus systems should be the norm and penalties the exception.

7. The Japanese Manufacturing System

Much has been made in the press of the "quality crisis" in American industry. In response, practitioners and scholars have taken to studying Japanese manufacturing techniques in order to understand Japan's great success in quality control. On the one hand, technical features of Japanese manufacturing are studied with the focus on features such as inventory control, product mix capabilities, and start-up times. At the same time there is talk of a cultural difference that accounts for the alleged fact that Japanese workers are more diligent than Americans. We argue here that technical features of the Japanese manufacturing system are important precisely because of their built-in monitoring features; put differently, the equilibrium level of diligence in Japan is higher not because of culture but because of the manufacturing system. In this section we describe briefly the Japanese system and argue that by its very design it implements commitment for at least some components of monitoring, and it prevents false positives.

The American system of manufacturing is largely based on the idea of keeping defects below a minimum acceptable level by detecting defects through

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statistical sampling. The Japanese system is designed to (ideally) eliminate defects by preventing them, rather than detecting them after they have occurred. The set of techniques, the factory design, and the management system to implement this is referred to as Total Quality Control (TQC). Part of the means of implementing TQC is the inventory management system known as Just-In-Time (JIT). Another part is a practice of servicing capital equipment known as Total Preventive Maintenance (TPM). These systems have evolved together and tend to be adopted together (although it is not logically necessary to do so), so we will refer to them jointly as the Japanese Manufacturing System.\(^{15}\)

The explicit orientation in the Japanese Manufacturing System is to eliminate quality control supervisors in favor of the workers performing this task themselves. In the JIT system production proceeds in extremely small lots in an assembly line fashion. This is true not only for actual assembly but for all stages of production, including fabrication of parts. When intermediate steps of production are performed in large batches, typically many defective parts will be produced before the error is detected. For example, it is common in an American factory to have a two to four week lead time in parts manufacture. If an error is being made in one function, the operator at the next function will throw the defectives into a rework bin to be handled by a rework team. If the error caused 10 percent of the parts to be defective, 10 percent of, say, a two week supply will have to be reworked or scrapped. More important, it will be virtually impossible to trace the

\(^{15}\)The JIT system is not employed universally in Japan. Indeed, Hay (1988) claims that it is not even Japan's dominant manufacturing system. At the same time, JIT, TQC and TPM are being adopted in the U.S. on an important scale. Thus, we refer to these systems as "Japanese" only to be consistent with common usage.
cause of the error, since it will be unknown when exactly the defectives were produced, by whom, on which particular machine, and so forth. An important feature of the JIT system is that defectives are detected almost immediately, before many are produced and when the cause can be traced.

In a JIT factory, if a worker cannot perform his task because he received a faulty part he cannot throw the part into a bin and continue with another, since there are no inventories for him to draw upon. He must stop the line and confer with his supplier, who must solve the problem before work can continue. Workers have a disincentive to pass on workable but faulty parts to the next station since the next worker may find that the defect prevents him from performing his task and requires him to stop the line, fixing responsibility on the worker who passed the part on. Thus, each operator necessarily serves as a monitor of his parts supplier or the previous assembler, and by backwards induction, each worker has an incentive to be diligent. In this way, the system itself induces commitment to monitoring the quality of work by each worker.

The backwards induction is supported by the consumer, who is the ultimate monitor. Customers serve as monitors in Western manufacturing as well, but there the backwards induction fails because the American system of large-lot production makes accountability difficult by obscuring the information about the source of errors within the firm.

When the firm's customer is another firm rather than the final consumer, one might expect this monitoring role to be more noisy; however, the Japanese system puts a great deal of emphasis on careful control of incoming intermediate products. This takes several forms, but most important, downstream firms insist on very small and frequent deliveries, so as to mimic
the JIT system as closely as feasible with the supplier. Thus, the benefits of the JIT system for monitoring of quality and, in particular, the commitment feature, are preserved between vertically related firms.

Thus far we have argued that the JIT inventory system combined with small, frequent deliveries between vertically integrated firms serves as a commitment device for monitoring the quality of output. Indeed, our description should make clear that the system itself dictates the level of monitoring. Ideally, the goal of TQC is 100 percent detection of defects. Although this is not achieved, the level of monitoring built into the system is very high. Further, it appears that, given the system, the level of monitoring is exogenous to the firm; one cannot tamper with the level of inherent monitoring and preserve the JIT feature, because it is part of the JIT system. Thus, by Corollary 2 in Section 4, the firm would like to operate in a NFP setting. We argue that one role of TPM is to eliminate at least one source of false positives.

When an error is detected, attention is focused on the previous worker. For purposes of accountability as well as correcting the mistake, it must be certain that the previous worker was responsible for the error. One potential source of false positives is the possibility that the defect was the result of faulty equipment maintenance rather than due to the operator's error. TPM transfers responsibility for equipment problems from a separate maintenance team to the operator himself.\footnote{While we argue that transferring responsibility for maintenance to workers mitigates the incentive problem, Holmstrom and Milgrom (1990) argue that it exacerbates it. The difference is that we view capital maintenance as an input into production, while they view it as a substitute task that competes for workers' time.} The thrust of TPM is that, relative to
Western management systems, a great deal more time is spent preventing breakdowns rather than repairing them. The operator is trained in the maintenance, care, and repair of the machine, and may be involved in its design. The result is that breakdowns are rare and when defects occur due to faulty equipment, the responsibility nevertheless rests with the operator. The obvious cost of such a system is that it loses the benefits of specialization between the caretaker of the machine and its operator; in our view the value is that false positives that would otherwise be possible due to a difficulty in distinguishing between the input of the two individuals are eliminated. The production of a defective item when the worker was diligent when using the machine is a signal of the worker shirking when caring for the machine.

These arguments, taken together, lead us to the following conclusion: The Japanese Manufacturing System implements commitment to a high level of monitoring with no false positives. By Proposition 1.1, this system induces extremely high diligence by workers. Accordingly, the predicted compensation structure is a penalty scheme in which the penalties are never actually observed.

One could argue that a backward induction system similar to the one inherent in the Japanese system could be implemented in the western manufacturing system. This would require that, in each step of production, the employee that works on a given item attaches his name to the item, so that any defect can be traced back to the worker who was responsible for it. Indeed, any consideration of incentive provision for factory workers would suggest that some sort of tracing device should be implemented whereby managers can determine the productivity of individual workers and their error
rates; conversely, to the extent that such systems are not implemented, it is a puzzle not only for our model but any model of incentive provision.

Some difficulties with integrating a tracing system into a western-style manufacturing factory suggest themselves. First, workers would have an incentive to omit their name tag from faulty items, or attach the name or code of another worker. In other words, they would have an incentive to undermine the tracing system. Addressing this problem would require some sort of monitoring system to implement the tracing system itself. The fact that tracing the source of faulty parts is difficult in western manufacturing systems makes a tracing system important, but may in turn make monitoring the tracing system equally difficult and again subject to false positives and false negatives.

Second, union or other organizational constraints may make it difficult in practice to introduce explicit incentive devices, for reasons outside the model, in any firm (Japanese or western). An advantage of the JIT system, then, is that it was designed and is implemented as an inventory control technique rather than as an incentive mechanism; the incentive features are a built in, implicit side benefit of the system. We do not argue here that the Japanese manufacturing system was designed explicitly to improve monitoring capabilities, but only that the system in fact has that effect.

8. Conclusions

The theme of this paper is that the form incentive provision takes in firms depends crucially on the structure of the firm itself. We argue that middle managers, who are typically evaluated by a subjective appraisal system, will be compensated according to a wage contract that is characterized by
bonuses rather than penalties. Our model also predicts that the overall performance of personnel compensated in this way will be low. We also show that compensation contracts for factory workers will be testably different as a function of observable features of the manufacturing system.

A fundamental implication of the model is that the design of a manufacturing system should take into account not only the engineering features that enhance productivity, but the implied monitoring features as well. It is possible that manufacturing systems that are highly touted are efficient not only because of the engineering features but because of the indirect effect they have on diligence in equilibrium.

Finally, we should note that this paper is an attempt to make sense of the distinction between bonuses and penalties in incentive schemes and draw implications regarding the diligence of workers; we do not explain why the payment scheme itself is only two-tiered. In our model the optimality of only two levels of pay arises directly from our assumption that the signal that monitoring produces takes only two values. An important open question is whether the fact that two-tiered contracts are frequently observed is due to a technological constraint in monitoring, as in our model, or whether other features of the incentive problem endogenously give rise to these simple contracts. We view our model as complementary research addressing the latter question (see, for example, Holmstrom and Milgrom, 1987), but we view that question as outside the scope of this paper.
References


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Proof of Proposition 1.1

Define \( \theta^c(w) \) as the monitoring intensity that leaves the worker indifferent between shirking and diligence for a given \( w \). This \( \theta^c(w) \) is obtained by equating \( E(u(w_1) | \theta, 1) \) to \( E(u(w_1) | \theta, 0) \) and solving for \( \theta \). It is easy to check that \( \theta^c(w) \) increases if \( u(h) - u(l) \) decreases. Of course,

\[
\begin{align*}
    p = 1 & \text{ if } \theta > \theta^c(w) \\
    p \in [0, 1] & \text{ if } \theta = \theta^c(w) \\
    p = 0 & \text{ if } \theta < \theta^c(w)
\end{align*}
\]

Since it is feasible for the owner to commit to the monitoring intensity \( \theta^c(w) \), for large enough \( \pi \) (given \( u(*) \), \( C(*) \), \( T \), and \( v \)) the equilibrium involves the worker setting \( p = 1 \) and the owner setting \( \theta = \theta^c(w) \): once the owner and the worker have signed the contract \( w \), if \( p \) was less than 1 (say, \( 1 - p = \delta > 0 \)), the owner would gain by deviating to \( \theta^c(w) + \epsilon \) where \( \epsilon \) is arbitrarily small but positive. The gains from such deviation are \( \delta \pi \) whereas the costs are \( C[\theta^c(w) + \epsilon] - C[\theta^c(w)] \) (which can be made arbitrarily small) plus \( E(w_1 | \theta^c(w)+\epsilon, 1) - E(w_1 | \theta^c(w), 1-\delta) \). The gains are larger than the costs for \( \pi \) large enough. On the other hand, if \( \theta = \theta^c(w) + \epsilon \), the owner gains by deviating to \( \theta' = \theta^c(w) + \epsilon/2 \), since the worker still sets \( p = 1 \), but the monitoring costs and expected wage outlay are smaller. Finally, the owner would never set \( \theta < \theta^c(w) \), because by raising \( \theta \) to \( \theta^c(w) \), the owner gains \( \pi \), which is larger than the increase in monitoring costs and in the expected wage outlay for \( \pi \) large enough.

Q.E.D.
Proof of Proposition 1.2

We prove first that for $\pi$ large enough, the equilibrium has $p^* = 1$. Suppose that in equilibrium the firm offers a contract $w = (h, \ell)$ such that $1 - p = \delta > 0$. This implies that $w$ is such that $\theta^c(w) = \delta$ (see the proof of Proposition 1.1). Then the owner would gain by deviating to $w' = (h + \epsilon, \ell)$, where $\epsilon > 0$ (so that $\delta > \theta^c(w)$ and $p = 1$) is arbitrarily small. The gains are a fixed positive amount $\pi\delta$, while the costs $E(w'|\delta, 1) - E(w|\delta, 1 - \delta)$ are less than $\delta\pi$ for $\pi$ large enough.

Thus, for the FP case the firm's problem is

$$\min_{h, \ell} \delta h + (1 - \delta)\ell$$

s.t.

(i) $u(h) + u(\ell) - \nu \geq 2T$

and

(ii) $(2\delta - 1)(u(h) - u(\ell)) \geq \nu$

where $\delta$ is the exogenous level of monitoring in the FP technology. Constraint (ii) ensures that the worker, with payoff given by $p[\hat{\lambda}u(h) + (1 - \delta)u(\ell)] + (1 - p)[(1 - \delta)u(h) + \delta u(\ell)] - pv$, (weakly) prefers to be diligent. Constraint (i) is obtained by solving for $\delta$ in constraint (ii) and substituting this into the individual rationality constraint. It is straightforward to show that both constraints will bind at an optimum. Thus, the constraints form two equations in two unknowns, with solutions:
\[
\begin{aligned}
   u(h) &= T + (\hat{\theta}/(2\hat{\theta} - 1))v, \\
   u(\ell) &= T - ((1-\hat{\theta})/(2\hat{\theta} - 1))v,
\end{aligned}
\]

assuming that \(u^{-1}(T - ((1-\hat{\theta})/(2\hat{\theta} - 1))v) \geq 0\). We will assume \(T\) sufficiently large (or \(\hat{\theta}\) close enough to one) that this condition is satisfied.

In the NFP case the firm’s problem is

\[
\begin{align*}
\min h \\
\text{s.t.} \\
(i) & \quad u(h) - v \geq T \\
(ii) & \quad \hat{\theta}(u(h) - u(\ell)) \geq v \\
(iii) & \quad h, \ell \geq 0.
\end{align*}
\]

It is again straightforward to show that \(\ell^* = 0\). Thus, the solution to the problem is

\[
\begin{align*}
u(h) &= \max[T + v, v/\hat{\theta}].
\end{align*}
\]

For \(\hat{\theta}\) close to 1 and/or \(T\) large, \(u(h) = T + v\). In this case we can compare the cost to the principal of the FP and NFP case. The principal’s expected wage outlay under the NFP technology is

\[
W(\text{NFP}) = u^{-1}(T + v).
\]

In the FP case it is
\[ W(\text{FP}) = \hat{\alpha}u^{-1}[T + (\hat{\theta}/(2\hat{\theta} - 1))\nu] + (1 - \hat{\theta})u^{-1}[T - ((1 - \hat{\theta})/(2\hat{\theta} - 1))\nu]. \]

For risk averse workers, by Jensen's inequality,

\[ W(\text{NFP}) < W(\text{FP}), \]

for \( \hat{\theta} < 1. \) For \( \hat{\theta} = 1, \) \( W(\text{NFP}) = W(\text{FP}). \) Q.E.D.

Proof of Proposition 2.2

Step 1: Show that the worker is left indifferent between shirking and not shirking. Since the owner's payoff is positive, \( p > 0 \) must hold in equilibrium, by Lemma 1. This implies, by (5), that \([u(h) - u(l)]\theta \geq \nu, \) that is, \( h > l, \) and \( \theta > 0. \) Now, \( \theta > 0 \) implies \((1 - p)(h - l) > 0\) in (8), and hence \( p < 1. \) Again by (5), \( p < 1 \) implies \([u(h) - u(l)]\theta = \nu. \)

Step 2: Show that if \( \varepsilon \) is large enough, in equilibrium \([\partial U^0/\partial p](p^*, \theta^*, \omega^*) = \varepsilon - (h^* - l^*)\theta^* > 0. \) The owner's payoff is

\[ U^0 = p\varepsilon - p(h^* - l^*)(1 - p)(1 - \theta^*)(h^* + \theta^*l^*) - C(\theta^*) = \varepsilon - \theta^*(h^* - l^*) - [1 - \theta^*h^* + \theta^*l^*] - C(\theta^*). \] Therefore, \([\partial U^0/\partial p] = \theta^*(h^* - l^*). \) By contradiction, assume \( \varepsilon - \theta^*(h^* - l^*) \leq 0. \) Then

\[ U^0 \leq -(1 - \theta^*)h^* + \theta^*l^*) - C(\theta^*) - E(\omega_T^*|\theta^*, p = 0) - C(\theta^*), \] since \( p^* \geq 0. \) By step 1 the worker is left indifferent between shirking and being diligent, and therefore \( 0 \leq T \leq U^0(p^*, \theta^*, \omega^*) = E(u(\omega_T^*|\theta^*, p = 0). \) By concavity of \( u \) and \( u(0) = 0, \) this implies \( E(u(\omega_T^*|\theta^*, p = 0) \geq u^{-1}(T) \geq 0, \) and therefore \( U^0 \leq 0, \) a contradiction.

Step 3: In step 1 we have shown that \( p > 0 \) and \( \theta > 0. \) This implies that we can replace constraints (1) and (11) in (9) by \( \theta[u(h) - u(l)] = \nu \) and \( C'(\theta) = \)
(h - \ell)(1 - p), respectively. Moreover, solving for \theta in \theta[u(h) - u(\ell)] = \nu and substituting into constraint (iv) yields u(h) - \nu \geq T. Therefore, we can rewrite (9) as

\[ \max_{\theta \in [0,1], p \in [0,1]} \{ px - ph - (1 - p)[(1 - \theta)h + \theta \ell] - C(\theta) \} \]

s.t.

(i) \theta[u(h) - u(\ell)] = \nu
(ii) C'(\theta) = (h - \ell)(1 - p)
(iii) h, \ell \geq 0
(iv) u(h) \geq \nu + T.

Now suppose that \ell^* > 0 in the solution (p^*, \theta^*, h^*, \ell^*) of this maximization problem. We reach a contradiction by finding (p, \theta, h, \ell) \neq (p^*, \theta^*, h^*, \ell^*), which satisfies all constraints and increases the objective function.

Set \ell = \ell^* - \epsilon, where \epsilon > 0 is small; set \theta = \theta^* - \epsilon^\theta, \epsilon^\theta > 0, to maintain (i), which we can do because \theta^* = \nu/(u(h^*) - u(\ell^*)) > 0; set p = p^* + \epsilon_p, \epsilon_p > 0, to restore (ii), and we know that increasing p is the right direction since C' is increasing. Finally, let h = h^* so that (iv) is satisfied.

Show now that U^0(p, \theta, h, \ell) > U^0(p^*, \theta^*, h^*, \ell^*). The decrease in \theta is a first order negligible change since [\partial U^0/\partial \theta](p^*, \theta^*, h^*, \ell^*) = (1 - p^*)(h^* - \ell^*) - C'(\theta^*) = 0 by (ii). Raising p results in a first order increase in the objective function since [\partial U^0/\partial \theta](p^*, \theta^*, h^*, \ell^*) = \pi - \theta^*(h^* - \ell^*) > 0 by Step 1. Finally, lowering \ell results in another first order increase in the objective function: [\partial U^0/\partial \ell](p^*, \theta^*, h^*, \ell^*) = -(1 - p^*)\theta^* < 0. (Recall that we are lowering \ell.) This shows \ell^* = 0.
Step 4: Using $f^* = 0$ (recall that $u(0) = 0$), solve for $\theta$ in constraint (i) to get $\theta = v/u(h) = \theta^c(w)$, and solve for $p$ in constraint (ii) to get $p = 1 - C'(\theta^c(w))/h$. Q.E.D.

Proof of Proposition 2.3: In the proof of Proposition 2.2 we showed that $u(h) \geq v + T$, $\theta = v/u(h)$, and $p = 1 - C'(v/u(h))/h$. Since $C'(\cdot)$ is increasing, these expressions imply $p \geq 1 - C'(v/(v + T))/(u^{-1}(v + T))$. Hence, $p$ tends to one as $C'(v/(v + T))$ tends to zero. Q.E.D.
Table I: Conditional Probabilities for the Realization of the Signal

<table>
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<tr>
<th>Pr</th>
<th>a = d</th>
<th>a = s</th>
</tr>
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<tbody>
<tr>
<td>H</td>
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<td>1 - δ</td>
</tr>
<tr>
<td>L</td>
<td>0</td>
<td>δ</td>
</tr>
</tbody>
</table>

No false positive M.T.  \( δ \in [0, 1] \)

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<td>δ</td>
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</tbody>
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False positive M.T.  \( δ \in [1/2, 1] \)