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AN EXPERIMENTAL STUDY OF VOTING RULES AND POLLS IN THREE-WAY ELECTIONS*

by

Robert Forsythe**
Roger Myerson***
Thomas Rietz***
and
Robert Weber***

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ABSTRACT

This paper reports experiments designed to help resolve longstanding controversies about the comparison of voting rules for multi-candidate elections. By paying subjects conditionally on election outcomes, we create electorates with (publicly) known preferences. Each electorate then votes, with or without pre-election polls, under one of three voting rules: plurality, approval and Borda. We find that Condorcet losers occasionally win regardless of the voting rule or presence of polls. Duverger's law (the predominance of two candidates) appears to hold under plurality voting, but close three-way races often arise under approval voting and Borda rule. Voters do not generally cast votes that match their poll responses, but they do so more often under plurality voting. Polls predict outcomes better under plurality voting and may serve as equilibrium selection signals. Voters usually cast votes that are consistent with some strategic equilibrium. By the end of an election series, most votes are consistent with a single equilibrium, although that equilibrium varies by group and rule.

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**Department of Economics, University of Iowa, Iowa City, Iowa 52242.

***Kellogg Graduate School of Management, Northwestern University, Evanston, Illinois 60208.

An Experimental Study of Voting Rules and Polls in Three-Way Elections

I. Introduction

There is a long tradition of experimental research studying elections (see McKelvey and Ordeshook [1987]). Some of this research has examined plurality voting in three-candidate elections (Plott [1977]). Some of it has looked at the impact of polls (Plott [1977] and McKelvey and Ordeshook [1985a, 1985b]). All of it has focused on environments with incomplete information. Either candidates do not know voter preferences, or voters do not know each candidate's position and hence, the candidate's value to them. In this paper, we consider a much simpler, complete information setting for studying voter behavior in elections. In our setting, candidates are exogenous and voter preferences over each candidate are commonly known.

We compare how voters behave under plurality voting, approval voting, and Borda rule.[1] Under plurality voting, voters each cast one vote for one candidate. Approval voting allows voters to cast one vote each for as many candidates as they wish. Under Borda rule, if there are n candidates, voters cast n-1 votes for one candidate, n-2 for a second, etc., casting 0 votes for the last candidate. Under each rule, the candidate with the highest vote total wins the election. We examine differences among the rules in three-candidate elections since, in two-candidate elections, non-abstention votes coincide for these rules.[2]

We examine election outcomes, asking whether they correspond to "proper voting equilibria".[3] In such equilibria, voters form expectations about the probable outcomes of an election. Given these expectations, their optimal actions result in an outcome which justifies the expectations. Since some history may be necessary to achieve such an equilibrium, voters in our experiment participated in repeated elections with the same candidates and electorate. Some sessions also include non-binding, pre-election polls. Poll and election history gives voters the opportunity to form perceptions of each candidate's chances of winning, and hence can facilitate equilibrium behavior. History may also play a role in the equilibrium selection process when multiple equilibria exist.[4] We find that equilibria explain the data quite well and that, at least under plurality voting, history appears to be an important factor in determining which candidates win later elections.

We also study the frequency with which elections are won by Condorcet losers--candidates who would lose in a two-way race against *any* one of the other candidates.[5] For example, consider an electorate in which 40% of the voters prefer one candidate (a conservative) and the remaining 60% prefer one of the two other (liberal) candidates. The liberal voters strongly prefer either of the two liberal candidates to the conservative,

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[1]Riker [1982a] describes these voting rules in depth.

[2]Under approval voting, casting votes for both candidates is equivalent to abstaining.

[3]We use Myerson and Weber's [1989] definition of proper voting equilibria.

[4]Multiple equilibria exist under approval and plurality voting for our experimental parameters.

[5]See Riker [1982a] for definitions of the Condorcet Criterion, Condorcet Winners and Condorcet Losers.

but are split by their slight preference for one of the liberal candidates over the other.[6] In this case, the conservative, who would lose the two way election against either liberal, is a Condorcet loser. However, under plurality voting, if the liberal candidates split the liberal vote fairly evenly, the conservative (Condorcet loser) wins. We find that this occurs most often under plurality voting and least often under approval voting and Borda rule without polls. However, repeated elections and the presence of pre-election polls tends to equalize the frequency of Condorcet losers winning across the three voting rules.

We also examine the proposition known as Duverger's Law.[7] Roughly put, this proposition states that only two candidates in a many-candidate field will receive substantial vote totals under plurality voting. The justification is that voters will identify two front-runners, and then--expecting that a vote for any other candidate will be "wasted"--will ignore all other candidates when deciding how to cast their votes.[8] A simple scenario in the previous example under plurality voting illustrates the principle behind Duverger's law. Suppose that, recognizing the danger of a split vote, the liberal voters seek out some coordinating signal to help select one liberal, whom they will all support. Coordination in this manner leaves two candidates with substantial vote totals: one liberal and the conservative. We find that Duverger's Law holds more strongly under plurality voting than does its analog under either approval voting or Borda rule. Rather, we find that close, three-way races are the norm under approval voting and Borda rule.

In the sessions with polls, we study poll accuracy, and how poll results affect individual behavior and election outcomes. Individual poll responses affect poll results, and poll results may affect voter behavior. Hence, voters may choose to change their responses in light of poll outcomes or strategically distort their poll responses. Thus, poll results may be quite different from election outcomes. Nonetheless, election outcomes may be predictable from poll results. Indeed, this may be an equilibrium phenomenon since, in equilibrium, polls do not need to forecast elections accurately; they only need to induce voter *expectations* that are realized in the following election. Returning once again to our example, suppose a poll shows the conservative winning, but with one of the liberals ahead of the other. This may always result in the leading liberal winning the election. In this case, the poll serves to coordinate the liberal vote around the leading liberal. While the poll results and election outcomes differ, the election outcome will still be perfectly predictable from the poll. We often observed this phenomenon under plurality voting. However, poll results under approval voting and Borda rule seem to have little impact on the outcome in the following election.

We also analyze individual voting behavior. We classify voter behavior as sincere, strategic (but not sincere), or dominated. A large body of research, beginning with Black [1958], has assumed voters sincerely "vote their preferences". That is, they cast vote vectors that weakly rank the candidates the same as their

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[6]As we discuss later, this example and the experimental parameters we use are based loosely on the 1970 U.S. Senatorial contest in New York.

[7]For descriptions of Duverger's Law, Riker [1982a and 1982b].

[8]Note that, if Duverger's Law does hold and the same candidate is eliminated under all three voting rules, then there should be no differences between outcomes under each rule. They would each, in essence, be conducted on the same two-candidate race.

preference ordering. Alternatively, voters may respond to their perceptions of candidate viability as well as their preferences. This can result in sincere voting, but may also result in insincere voter behavior that is, nevertheless, optimal. Finally, voters may (for whatever reason) choose to cast strategically dominated vote vectors. (Dominated vote vectors are ones that would never be cast by rational voters who believed that their own vote could change the election outcome.) Returning to our example, under plurality voting, voting for the conservative (respectively, favored liberal) is the unique sincere strategy for conservative (resp., liberal) voters. (If everyone votes sincerely, the Condorcet loser will win.) Voting for their less-preferred liberal candidate is strategic behavior for liberal voters if they perceive that candidate has the best chance of winning. (Then, the more "viable" liberal might win.) Voting for a liberal (respectively, the conservative) is a dominated strategy for a conservative (resp., liberal) voter.[9] We find that, while voters seldom cast dominated votes, they often cast in-sincere, but strategic, votes when they had the opportunity to do so.

Finally, we study whether individuals responded truthfully to polls in the sense that their election vote corresponded to their response in the preceding poll. While, pollsters rely on truthful poll responses for poll accuracy, there are no obvious incentives for individuals to respond truthfully to polls. As discussed above, there may be incentives for voters to mis-represent themselves in the poll or change their vote after seeing poll results. Under the payoffs we used, we find that the majority of voters *did not* cast votes that matched their poll responses when there were clear incentives for mis-representation in the poll.[10]

In the next section we present our experimental design and give the proper voting equilibria for the individual elections (i.e. the stage-game equilibria). In Section III, we state our hypotheses and present our results. In the final section, we provide conclusions and discuss future research directions.[11]

II. The Experiment

The experiment consisted of six sessions with twenty-eight different voters in each cohort. We conducted the sessions using approval voting at Northwestern University and the sessions using Borda rule and plurality voting at the University of Iowa. Subjects were drawn from subject pools recruited by university-wide advertising or directly from M.B.A. and undergraduate classes throughout the university.

Upon arrival, subjects were seated in a large classroom and given copies of the instructions for the session. (Appendix I contains these instructions.) The instructions were read aloud and questions were answered in public in order to make all instructional information common knowledge.

Each subject was given a voter identification number and assigned to an initial voting group consisting of 14 of the 28 subjects. At all times, there were two distinct voting groups in the room. Each voting group

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[9] In any situation in which this vote breaks a tie, it breaks it in favor of the voter's less preferred candidate. In any situation in which this vote creates a tie, it stops the voter's more preferred candidate from winning outright.

[10] Under the payoffs we used, obvious incentives for mis-representation existed under approval voting and Borda rule.

[11] The appendices contain the instructions and a more formal hypothesis and testing section. A data and statistical supplement, available from the authors, contains detailed summaries of election outcomes and the raw data and tables of critical values for test statistics we use.

was divided into voters of three "types", differing by their payoffs conditional on the winning candidate. The composition of each voting group was unchanged for eight voting periods. This allowed voters to form expectations and develop voting strategies based on the group's common history. After eight periods, voters were randomly re-assigned to new groups and new types. Voters then used new payoff schedules with randomly re-arranged and re-labeled rows and columns. This allowed us to observe several different groups in each cohort while minimizing any repeated-game effects that might carry over from one group to the next. In each of the six sessions, we conducted three series of eight elections each. Thus, a total of six voting groups were formed per session. Each subject participated in three voting groups sequentially and in a total of twenty-four voting periods. This gives 288 elections and a total of 4032 voter responses in elections.

All groups in a session used the same voting rule. Also, in sessions with polls, each group participated in a poll before each election. Three of the six sessions involved pre-election polls in every period, yielding 144 polls and 2016 voter responses to polls.

At the beginning of a session, each voter's folder contained the payoff schedules for each of the three voting groups in which he or she would participate. Each group used a payoff schedule equivalent to the "symmetric" payoff schedule given in Figure 1.[12] For each voting group, rows and columns of this payoff schedule were randomly shuffled and re-labeled as discussed above.[13] Within a group, each individual payoff schedule was identical except for a box placed around that individual's voter type. In this way, each voter knew his or her own payoffs, the payoffs to the other voter types in the group, and the number of voters of each type. However, voters did not know the specific assignment of types to others in the room. Furthermore, since poll and election responses were collected from both groups simultaneously, and outcomes for both groups were posted publicly, the voters did not know the specific identities of others in their groups.

Notice that, under the payoff schedule used, the Blue candidate is a Condorcet loser (would lose two-way races with either Orange or Green). However, if all voters vote sincerely, Blue will win plurality voting elections. While the majority of the electorate prefers either Orange or Green to Blue, they are evenly split. *A priori*, neither candidate appears to be more likely to win. We modeled this payoff structure loosely on the 1970 race for the U.S. Senate seat from New York in which an apparent Condorcet loser won. While over 60% of the votes were cast for liberal candidates, Richard L. Ottinger (Orange here) and Charles E. Goodell (Green here) split the liberal vote, allowing the conservative, James R. Buckley ("Blue" here), to win the election with 38.6% of the vote. Neither of the liberal candidates appeared to be a better challenger. While Ottinger was the Democratic party nominee, Goodell was the incumbent and was nominated by both the Republican and Liberal parties. Neither could muster enough support to overtake Buckley in pre-election straw

[12] Voter types are designated by their most preferred candidate here. They were designated only by number in the actual payoff tables.

[13] Given the structure of payoffs, subjects presumably could identify which candidate was the same as the Blue candidate in this payoff schedule and which voter type was the same as Voter Type 3 (B). However, they should not be able to identify the other two candidates or voter types from one group to the next. Throughout this paper, we will refer to this payoff schedule. The actual voter types and responses have been transformed so they match this schedule for reporting purposes.

Figure 1: "Symmetric" Payoff Schedule

Payoff Schedule Group: __

Voter Type	Election Winner: Orange	Green	Blue	Total Number of Each Type
1 (O)	$1.60	$1.20	$0.30	4
2 (G)	$1.20	$1.60	$0.30	4
3 (B)	$0.60	$0.60	$1.90	6

votes and polls.[14]

A. Implementing Voting Rules

Each voter's folder contained a set of election ballots. Voters were told that they could choose to abstain in any election (or poll), by turning in blank ballots. If they did vote, then they had to vote according to a precise rule. The wording from the instructions for implementing each voting rule was as follows:

Plurality: "If you do not abstain, you may vote for at most one candidate. To do this, place a check next to the candidate for whom you are voting."

Approval: "If you do not abstain, you may cast one vote each for as many candidates as you wish. To do this, place a check next to each candidate for whom you are voting."

Borda Rule: "If you do not abstain, you must give two votes to one candidate, and one vote to one of the other candidates. To do this, write "2" next to the candidate to whom you are giving two votes and write "1" next to the candidate to whom you are giving one vote."

In practice, the admissible vote vectors under plurality voting were (1,0,0), (0,1,0), (0,0,1) and (0,0,0). Under approval voting, the vectors (1,1,1), (1,1,0), (1,0,1) and (0,1,1) were also admissible. Finally, under Borda rule, (2,1,0), (2,0,1), (1,2,0), (0,2,1), (1,0,2), (0,1,2) and (0,0,0) were admissible.

If a tie occurred between two or more candidates, we selected the winner randomly. To do this, we placed colored balls corresponding to the names of the tied candidates in a box and asked one of the subjects to draw a ball from the box. The candidate whose name was the same as the color of the selected ball was declared the winner.

[14] We assume that our subjects were unaware of the name/color correspondence, and by using color coded candidates we avoided connotations such as those attached to "liberal" and "conservative" labels.

B. Implementing Polling Rules

In sessions with polls, the instructions informed voters that polls were non-binding. Voters were told that they could vote in the election even if they abstained from the poll, and further, that their vote need not match their poll response. In these sessions, each voter's folder also contained a set of polling forms. Before each period's election, we asked each voter to submit a polling form. (Voters who abstained submitted blank polling forms.) Polls were conducted according to the same voting rule as the election. Before conducting the election, we announced the total number of poll-votes for each candidate and recorded these totals on the blackboard in the front of the room.

C. Stage-Game Voting Equilibria

In the discussion of results we will focus on the proper voting equilibria which may occur in each single-period election (i.e. on the stage-game equilibria) according to Myerson and Weber's [1989] definitions. Here, without going into detailed computations, we discuss each equilibrium and how voters expectations and responses serve to support that equilibrium.

In the equilibrium model, voters recognize that their vote matters only because it might change the election outcome. The original Myerson and Weber [1989] model makes two critical assumptions. First, they assume that voter perceptions of relative candidate viability can be represented by a vector of "win" probabilities. Second, conditional on *some* two candidates being close enough in the election that an individual voter can be pivotal, the relative likelihood of a *particular* pair of candidates being in this close race is proportional to the product of their win probabilities. (While slightly *ad hoc*, this assumption facilitates equilibrium calculations. A later version of their model disposes with "win" probabilities, and takes the "conditional-tie" probabilities as the fundamental element.) Under these assumptions, any voter's objective function can be written as:

$$\max_{v \in V} \sum_{i=1}^{3} p_i \left[u_i - \bar{u}(p) \right] v_i \tag{1}$$

where v is a vote vector from the voting rule's admissible set V, p_i is the probability that candidate i wins, u_i is the utility that the voter derives from i's winning, v_i represents the votes given to candidate i, and $\bar{u}(p) := \sum_{j=1}^{3} p_j u_j$ is the voter's expected utility from the election. (In essence, this simply maximizes the covariance between a voter's expected utility from each candidate and the number of votes that the voter gives to each candidate.)

Table I describes the proper equilibria that may arise from our payoffs under each voting rule. Equilibria are defined by optimal vote vectors for each voter type and by each candidate's probability of winning. For each candidate ranking which can arise in equilibrium, we write "=" to represent a "close" race, and ">" to represent a "strict ranking". In close races, voters place a positive (though not necessarily equal) probability on either candidate finishing ahead of the other. In strict rankings, voters view the probability that the losing candidate will win as being small enough that they ignore it.[15] For example B>O=G denotes the candidate ranking at an equilibrium in which Blue has the highest vote total, followed by Orange and Green, who are close to each other but each unlikely to beat Blue. While voters place positive probabilities on each candidate winning in O=G=B equilibria, the win probabilities are not equal in any of the equilibria here. In each, the Condorcet loser, Blue, is least likely to win even though we call these equilibria close, three-way races.[16]

Table I: Equilibrium Win Probabilities for Each Candidate and Consistent Individual Vote Vectors (Probability and Vote Vectors are in the Order: Orange, Green and Blue)

		Vote Vectors Consistent with the Equilibrium:			
Voting Rule	Voter Type	O=G=B	O>B>G	G>B>O	B>O=G
Approval Voting	O	(1,0,0)* & (1,1,0)*	(1,0,0)*	(1,1,0)*	--
	G	(0,1,0)* & (1,1,0)*	(1,1,0)*	(0,1,0)*	--
	B	(0,0,1)*	(0,0,1)*	(0,0,1)*	--
	Win Probabilities:	(9/22,9/22,4/22)	(1,0,0)	(0,1,0)	--
Borda Rule	O	(2,1,0)* & (2,0,1)†	--	--	--
	G	(1,2,0)* & (0,2,1)†	--	--	--
	B	(1,0,2)* & (0,1,2)*	--	--	--
	Win Probabilities:	(31/66,31/66,4/66)	--	--	--
Plurality Voting	O	--	(1,0,0)*	(0,1,0)†	(1,0,0)*
	G	--	(1,0,0)†	(0,1,0)*	(0,1,0)*
	B	--	(0,0,1)*	(0,0,1)*	(0,0,1)*
	Win Probabilities:	--	(1,0,0)	(0,1,0)	(0,0,1)

*The vote vector is sincere in that it weakly ranks the candidates the same as the voter's preferences.
†The vote vector is not sincere, but it is strategic since it is the voter's best response given the equilibrium.

Under plurality voting, the symmetric payoff matrix results in three possible equilibria. If all voters vote sincerely, Blue wins the election followed by Orange and Green, who are in a close race for second. We denote this equilibrium by B>O=G. Since neither Orange nor Green appears to have any advantage as a challenger to Blue, no one has any incentive to change their vote. The expected vote totals under this equilibrium are 4, 4 and 6 for Orange, Green and Blue respectively. In this equilibrium, we expect the Condorcet loser (Blue) to win. The other two equilibria are "coordinated" in the sense that "O" or "G" voters form a coalition and all vote for either Orange or Green. These equilibria are O>B>G and G>B>O. Here,

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[15] In the actual rankings listed in the data and statistical supplement, ">" implies that the candidates differed by two or more votes, "≥" implies that the candidates differed by one vote and "=" implies that the candidates were tied. In the former case, no single voter could change the winner by switching between un-dominated vote vectors. In the latter two cases, a single voter could change the outcome.

[16] To understand the unequal probabilities, consider the O=B=G equilibrium under our payoffs and approval voting. In the equilibrium, some of type "O" voters must be willing to vote for both Orange and Green, while others vote only for Orange. In a large electorate, this can only be the case if "O" voters are indifferent between the two actions. This implies that $p_G \times (1.2\text{-}1.6p_O\text{-}1.2p_G\text{-}0.3p_B)=0$. A similar requirement for "G" voters implies that $p_O=p_G$. Together with $p_O+p_G+p_B=1$, this yields $p_O=p_G=9/22$ and $p_B=4/22$. Strictly speaking, we are assuming risk neutrality here, so that the payoffs can be substituted for utilities.

the "strategic" type "O" and "G" voters ("O" voters who vote for Green and vice versa) have no incentive to change. Voting for their favorite candidate will be perceived (at equilibrium) to increase the chance that Blue wins much more than the chance that their favored candidate wins. The vectors of expected vote totals for these equilibria are (8,0,6) and (0,8,6), respectively.

According to (1), under approval voting a voter should vote for candidate *i* if $u_i > \overline{u}(p)$ and not vote for candidate *i* if $u_i < \overline{u}(p)$. (A voter is indifferent if $u_i = \overline{u}(p)$.) This voter response pattern gives three possible equilibria: O>B>G, G>B>O and O=G=B. The first two equilibria result when either "O" or "G" voters cast approval votes for their second-favorite candidate, making that candidate the winner. They do not have an incentive to withdraw this vote because this would increase the chances that Blue would win. We expect the leading candidate to win with expected vote totals of (8,4,6) and (4,8,6), respectively. The third equilibrium results when "O" and "G" voters are indifferent between casting 0 and 1 votes for their second favorite candidate. This results in a close three-way race with candidate win probabilities of (9/22,9/22,4/22). No one withdraws votes from their second favorite candidate because this would increase the probability that Blue would win. No one else casts votes for their second favorite candidate because this would increase the chance that their favorite candidate would lose.

Borda rule yields a unique equilibrium: O=G=B. If "O" and "G" voters perceived Blue as no threat, they would "dump" their 1-votes on Blue in a effort elect their favorite candidate. However, this would make Blue the winner. Hence, at equilibrium Blue *must* be perceived to have some chance of winning. Type "B" voters scatter their 1-votes evenly between Orange and Green. Candidate win probabilities of (31/66,31/66,4/66) support this equilibrium, making all voter types indifferent between casting their 1-vote for their second-favorite or least-favorite candidate.[17]

III. Results

Here, we briefly discuss our results in terms of election outcomes, poll/election results and individual voter behavior.[18] We label the sessions AWOPS1 (approval voting without polls), AWPS1 (approval voting with polls), BWOPS1 (Borda rule without polls), BWPS1 (Borda rule with polls), PWOPS1 (plurality voting without polls) and PWPS1 (plurality voting with polls).

First, we will discuss election results. We will look at the frequency with which each candidate won the elections. The frequency with which Blue won the elections tells us how often a Condorcet loser can win under each voting rule. We will see whether coalitions formed and whether Duverger's Law held under each voting rule. We will also examine further evidence for or against specific equilibria having occurred.

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[17] Again, strictly speaking, we are assuming risk neutrality to calculate these probabilities.

[18] Appendix II contains a more formal hypothesis testing section. The data and statistical supplement (available from the authors) contains more detailed summaries of the elections and the raw data.

Second, we will discuss the interaction between polls and elections. We will study the effects of polls and the accuracy of the polls in predicting the outcome or rankings in the following election. In particular, we will ask whether polls served as coordinating signals that determined which, of multiple, equilibria occurred.

Third, we will discuss individual behavior. We will look at the extent to which voters voted sincerely; insincerely, but strategically; or according to a dominated strategy. We will also look at the extent to which voters voted in a manner consistent with their poll responses or changed their votes after poll information was revealed. Finally, we will ask whether voters responded to poll and preceding election information in predicted ways.

A. Election Hypotheses and Results

1. Election Winners and the Frequency of Condorcet Losers Winning

All voting rules are subject to "paradoxes" in which the system does not aggregate voter preferences nicely (see Arrow [1963] and Saari [1989]). Here in particular, plurality voting is subject to the possibility that the Condorcet loser (Blue) will win the election. Arguments for approval voting and Borda rule include the possibility that they will reduce the frequency of this occurrence (see Riker [1982a] and Saari [1985 and 1989]).

Here, we ask how often each candidate won? In particular, how often did Blue win? Also, did different voting rules, repeated elections or polls appear to change the winners? While sincere voting implies Blue will lose under Borda rule, will likely lose under approval voting and will win under plurality voting, strategic voting may lead to significantly different results. The coordinated equilibria under approval and plurality voting, imply Blue loses. The O=G=B equilibria under approval voting and Borda rule imply that Blue can win.

Table II gives the fraction of elections that each candidate won in each session. With one exception, Blue wins fewer elections than *either* Orange or Green. That exception is plurality voting without polls.[19] Table III shows the number of times Blue won the elections in each period of each session.[20] It also shows whether or not Blue won significantly more or less than we would predict if voters cast random votes.[21]

Blue won significantly more often that predicted by random voting only in the first two periods under plurality voting without polls. Blue did not win any of the elections in the first period under approval voting

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[19]As explained later, this could result from coalitions of "O" and "G" voters coordinating on Green with no way to successfully switch to coordination on Orange. Polls may provide a means for switching between candidates. Thus, they may mitigate this effect in the plurality elections with polls.

[20]Appendix II contains a similar table showing the number of wins by group.

[21]Many of the statistical tests are based on Monte Carlo simulations using 1000 groups of 6 or 8 elections with 28 voters in each. In the simulated elections, voters are assumed to randomly choose between the non-abstention vote vectors, assigning equal probability to each. Non-abstention vote vectors under approval voting are: (1,0,0), (0,1,0), (0,0,1), (0,1,1), (1,0,1) and (1,1,0). Under Borda rule, they are (2,1,0), (2,0,1), (1,2,0), (0,2,1), (1,0,2) and (0,1,2). Under plurality voting, they are (1,0,0), (0,1,0) and (0,0,1). The data and statistical supplement, available from the authors, contains the cumulative density functions from the simulations as well as some analytic density functions under various nulls for specific hypotheses we test.

Table II: Summary of Election Results

Session (Voting Rule)	Fractions of Election Won By Each Candidate*			Number of Ties for First Place		Average Normalized Winning Margin† (Std. Dev.)	Average Normalized Total Spread† (Std. Dev.)	Average Normalized Low Vote Total‡ (Std. Dev.)
	Orange	Green	Blue	2-Way	3-Way			
AWOPS1 (Approval)	0.4132	0.4965	0.0903	17	4	0.8750 (0.9812)	1.6875 (1.3552)	0.2920 (0.0321)
AWPS1 (Approval)	0.4653	0.3194	0.2153	13	4	0.8125 (0.7043)	1.7083 (1.0711)	0.2867 (0.0358)
BWOPS1 (Borda)	0.2847	0.6181	0.0972	4	2	1.0938 (1.0245)	2.0938 (1.3825)	0.2835 (0.0367)
BWPS1 (Borda)	0.4340	0.4549	0.1111	5	4	0.9167 (0.7741)	1.6458 (0.0104)	0.2955 (0.0245)
PWOPS1 (Plurality)	0.1771	0.5625	0.2604	10	0	2.0208 (1.6947)	5.8333 (2.3550)	0.1031 (0.1039)
PWPS1 (Plurality)	0.3542	0.4479	0.1979	7	0	1.6250 (1.2138)	5.6875 (2.0017)	0.1012 (0.0831)

*Total wins over number of elections. Wins are scored as follows: Outright Wins = 1, Two-Way Ties = 1/2 and Three-Way Ties = 1/3.
†The winning margin is the number of votes separating the 1st and 2nd place candidates. The total spread is the number of votes separating the 1st and 3rd place candidates. Borda rule margins and spreads were divided by two since, by changing a his or her vote, a single voter can change the vote totals by twice as much as possible under plurality or approval voting.
‡Normalized by dividing by the total number of votes cast.

and Borda rule without polls. Thus, the initial period results support the hypotheses that, without polls, plurality voting encourages wins by Condorcet losers and approval voting and Borda rule discourage them. However, the presence of polls and repeated elections change these results. Overall, Blue won a relatively small fraction of elections in all sessions. Under approval voting and Borda Rule, Blue won more often with polls than without. Under plurality voting, Blue won less often with polls than without.

2. Duverger's Law Versus Close, Three-Way Races

Under plurality voting, a version of Duverger's law will hold if "O" and "G" voters coordinate by forming a coalition to elect either the Orange or Green candidate. Since successful coalitions require at least 7 "O" and "G" voters, the other of these candidates will receive 1 or no votes. Further, voters may use some information from previous elections or polls to determine the candidate on whom they coordinate.[22] In contrast to plurality voting, no equilibrium under approval voting and Borda rule predicts a Duverger's Law type of outcome. The only equilibrium under Borda rule is a close, three-way race. One of the equilibria under approval voting is also a close, three-way race. The other two equilibria (O>B>G and G>B>O) still predict closer races than the equilibrium under plurality voting. So, under approval voting and Borda rule, we expect closer elections with no candidate being driven out.

[22]As discussed earlier, from the payoff schedule and number of voters of each type, there is no *a priori* reason to expect the coalition to form around a particular candidate. Other than polls and election outcomes, no other communication is allowed.

Table III: Number of Times the Condorcet Loser Won‡ the Elections
(Out of 6 Elections Per Period)

Period	AWOPS1	AWPS1	BWOPS1	BWPS1	PWOPS1	PWPS1
	Session					
1	0.00*	0.83	0.00*	0.33*	4.00†	3.00
2	0.00*	1.50	0.00*	1.00	3.50†	2.50
3	1.33	2.33	0.33*	1.33	1.50	0.00*
4	0.33*	0.83	1.00	1.33	0.50*	2.00
5	0.50*	1.83	0.00*	0.00*	0.50*	0.50*
6	0.00*	0.00*	3.00	0.33*	1.00	0.00*
7	1.33	0.50*	0.00*	0.00*	1.00	1.50
8	0.83	2.50	0.33*	1.00	0.50*	0.00*
Total	4.33	10.33	4.67	5.33	12.50	9.50

*According to simulation results (given in Appendix III), reject the hypothesis that voters voted randomly in favor of the alternative that voters voted in a manner that made the Condorcet Loser less likely to win (at the 10% level of confidence).
†According to simulation results (given in Appendix III), reject the hypothesis that voters voted randomly in favor of the alternative that voters voted in a manner that made the Condorcet Loser more likely to win (at the 10% level of confidence).
‡Scored as follows: Outright Wins = 1; Two-Way Ties = 1/2; and Three-Way Ties = 1/3.

Here, we ask how often Duverger's law appeared to have held relative to the number of times the races appeared very close. Duverger's law predicts that either Orange or Green will always come in last with a smaller than expected vote total. Alternatively, one may think of this as a higher than expected spread between the second and third place candidates, or a lower than expected percentage of the vote taken by the third place candidate. The O = G = B equilibrium may be interpreted as a positive probability of each candidate winning, a lower than expected spread between the second and third place candidates and a higher than expected percentage of the vote taken by the third place candidate.

Table IV shows the number of times Orange or Green finished last out of the six elections per period in each session.[23] It also shows whether this loss rate significantly exceeds the rate we would predict if voters cast random votes. Orange or Green finished last most often under plurality voting and more often in sessions with polls than without. Under plurality voting, this loss rate *always* exceeds the rate predicted by random voting significantly.

We also test whether the Orange or Green vote totals were significantly smaller than expected assuming random voting. Table V shows the elections in which this occurred. Under approval voting and Borda rule, we never reject the hypothesis that voters voted randomly in favor of the alternative that Orange or Green vote totals were too small. We find significantly smaller vote totals for Orange or Green 33 times in PWOPS1 and 38 times in PWPS1. In addition, *whenever* Orange or Green received significantly fewer votes than predicted by random voting, that candidate was *always* the one who was behind in the preceding election or poll. Thus, if a candidate is driven out of the race, it appears that the preceding election or poll may

[23] Appendix II contains a similar table showing the number of last place finishes by group.

Table IV: Number of Times the Orange or Green Finished Last†
(Out of 6 Elections Per Period)

Period	AWOPS1 (Approval)	AWPS1 (Approval)	BWOPS1 (Borda)	BWPS1 (Borda)	PWOPS1 (Plurality)	PWPS1 (Plurality)
1	0.00	3.17	0.50	2.67	5.50*	6.00*
2	2.00	5.00	2.00	2.00	6.00*	6.00*
3	2.17	3.67	2.67	1.67	6.00*	6.00*
4	2.17	2.67	3.00	4.17	6.00*	6.00*
5	3.50	4.17	1.00	1.00	5.00*	5.50*
6	3.00	2.50	3.00	2.17	5.50*	6.00*
7	3.17	4.00	0.50	1.50	5.50*	6.00*
8	4.67	5.50*	2.67	4.00	5.00*	6.00*
Total	17.50	30.67	15.33	19.17	44.50	47.50

Session (Voting Rule)

*According to simulation results (given in Appendix III), reject the hypothesis that voter voted randomly in favor of the alternative that voters voted in a manner that made Orange or Green <u>more</u> likely to finish last (at the 10% level of significance).
†Scored as follows: Outright Loses by Orange or Green = 1;
Orange or Green Tied with Blue for Last = 1/2;
and Three-Way Ties = 2/3.

determine the candidate.

Figure 2 shows the average spread between the second and third place candidates in each session. It also shows the average percentage of the vote taken by the third place candidate. Under plurality voting the average spreads between the second and third place candidates were 3.8125 and 4.0625. In all approval voting and Borda rule sessions, the second place candidate beat the third place candidate by an average of less than one vote (when normalized by dividing by 2 for Borda rule). Under plurality voting, the fraction taken by third place was close to zero. Under approval voting and Borda rule, it was close to 1/3. In Appendix II, we show that these effects were significant. In 15 of 16 periods (across groups) and all groups (across periods) we reject random voting in favor of a Duverger's law effect under plurality voting. We reject random voting and in favor of close, three-way races in all groups and periods under approval voting and in 13 of 16 periods and 11 of 12 groups under Borda rule.

A final measure of whether we observed close, three way races is the number of ties or the number of races in which one voter could have changed the outcome. Table II show the number of ties in each session. There were more ties under approval voting than under plurality voting. While there were fewer ties under Borda rule than plurality voting, this may be misleading since there were three times the number of votes cast under Borda rule. To control for this, we look at the number of elections in which a single voter could have changed the outcome by enough to bring the last place finisher into at least a tie for the lead. We will call this a close, three-way finish. Close, three-way races occurred 38 times in AWOPS1, 39 times in AWPS1, 30 times in BWOPS1, 35 times in BWPS1, 5 times in PWOPS1 and 3 times in PWPS1. Thus, there is more

Table V: Elections in Which We Reject the Hypothesis that Voters Voted Randomly in Favor of the Hypothesis that Voters Tend to Vote in a Manner that Made the Orange or Green Candidate's Vote Total Smaller

PWOPS1 (Plurality)			PWPS1 (Plurality)		
Group	Period(s)	Candidate	Group	Period(s)	Candidate
A	2-8 --	Orange Green	A	2,6,7 3,4	Orange Green
B	2 5-8	Orange Green	B	1 2,4,7,8	Green Orange
C	2-8 --	Orange Green	C	1,3,5,7 2,4,6,8	Green Orange
D	-- 2-8	Orange Green	D	1,3,7 5,6,8	Green Orange
E	3,4,7,8 --	Orange Green	E	1,3,4,6-8 5	Green Orange
F	3-5	Orange	F	2,4,6,8 3,5,7	Orange Green

evidence for equilibria with tight-three races in approval voting and Borda rule.

3. Other Equilibrium Evidence

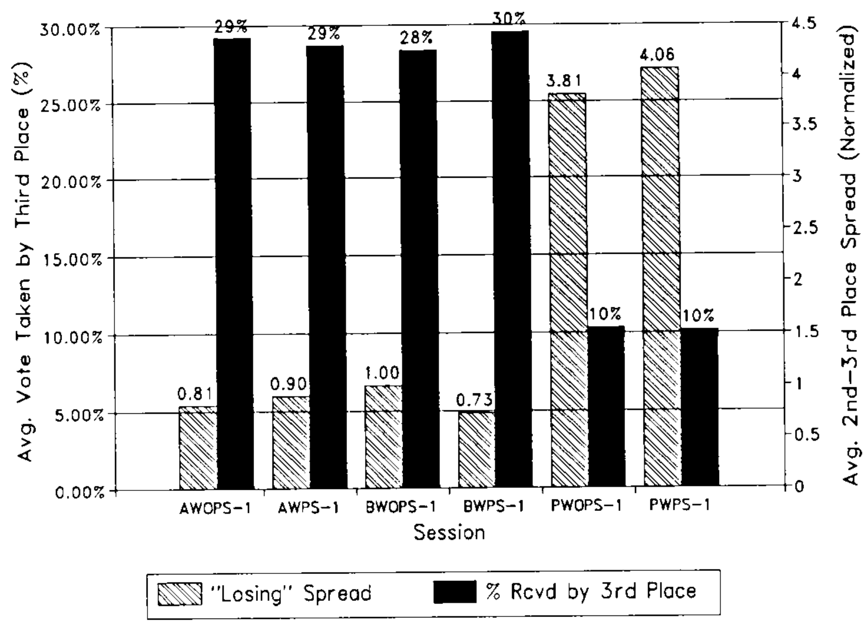
In the last two sections, we discussed some evidence for specific equilibria occurring (O = G = B under approval voting and Borda rule, and O>B>G or G>B>O under plurality voting). Here we discuss the difficulties of determining which equilibria occurred and some additional evidence.

Determining whether an equilibrium occurred is difficult at best. We can only examine which equilibrium best helps to organize the data. This cannot be done by simply looking at the win probabilities and the vote vectors in Table I. First, many vote vectors are consistent with more than one equilibrium. Further, some equilibria involve mixed strategies. In these cases, votes cast and, hence, election rankings are randomly determined. However, recognizing this, we can present some evidence on which equilibria best organized the data.

First, while voters did not always vote consistently with an equilibrium, they did so 92.2% of the time overall (out of 4032 votes cast) and 95.4% of the time in the final periods of voting groups (of 504 votes cast).[24] Further, we can reject the hypothesis of random voting in favor of the alternative that voters tended to vote consistently with an equilibrium in all elections except four: Period 1, Group A and Period 5, Group E in PWOPS1; and Period 1, Groups A and B in PWPS1. Under Borda Rule, this implies that we accept the

[24]Because of the repeated nature of the game, there may be some incentives to cast votes that are inconsistent with the stage game equilibria in early elections in a voting group. These incentives disappear by the final period with a group.

Figure 2: Average Percentage of the Vote Taken by Third Place and the Average Spread Between Second and Third Place



hypothesis that voters generally voted consistently with the O = G = B equilibrium. Under approval voting, we can seldom reject the hypothesis that "O" and "G" voters mixed with the 50/50 mix required for equal expected vote totals.[26] In both cases, the winning frequencies of each candidate are not significantly different from those predicted in Table I.

Under plurality voting, we can try to distinguish between the equilibria by looking at the number of "O" or "G" voters that cast votes consistently with each equilibrium. We begin with the null that "O" and "G" voters cast (1,0,0) and (0,1,0) votes with equal probability. Under this null, the probability of six or more voters voting consistently with a single equilibrium is less than 10%. Thus, we reject the null in favor of a specific alternative equilibrium if six or more of the "O" or "G" voters voted consistently with it. Table VI shows the specific alternatives we accept. After initial periods, equilibria usually seem to have been established. (We accept equilibria in all but two elections after period 2.) Further, without polls, once a coordinated equilibrium was established, it seldom changed. With polls, the equilibria appear to have changed frequently. Further, as discussed below, the winner was usually the Orange or Green candidate who was ahead in the poll. Thus, polls may have served as an coordinating signal for switching between equilibria.

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[26]In 82.3% of the 96 elections, "O" and "G" voters apparently voted consistently with the O = G = B equilibrium prediction. In the other elections, they appeared to vote in a manner consistent with one of the coordinated equilibria. Results are given in Appendix II.

Table VI: Specific Alternative Equilibria Accepted in Favor of the Null of Random Voting Under Plurality Voting

		Group					
Session	Period	A	B	C	D	E	F
PWOPS1	1	None	None	None	B>O=G	B>O=G	B>O=G
	2	G>B>O*	B>O=G	G>B>O	None	None	B>O=G
	3	G>B>O	B>O=G	G>B>O	O>B>G	G>B>O	G>B>O
	4	G>B>O	None	G>B>O	O>B>G	G>B>O	G>B>O
	5	G>B>O	O>B>G	G>B>O	O>B>G	G>B>O	G>B>O
	6	G>B>O	O>B>G	G>B>O	O>B>G	B>O=G	G>B>O
	7	G>B>O	O>B>G	G>B>O	O>B>G	G>B>O*	G>B>O
	8	G>B>O	O>B>G	G>B>O	O>B>G	G>B>O	None
PWPS1	1	None	None	O>B>G	O>B>G	O>B>G	B>O=G
	2	G>B>O	G>B>O*	G>B>O	B>O=G	B>O=G	G>B>O
	3	O>B>G	G>B>O	O>B>G	O>B>G	O>B>G	O>B>G
	4	O>B>G*	G>B>O	G>B>O*	B>O=G	O>B>G*	G>B>O
	5	B>O=G	G>B>O*	O>B>G	G>B>O	G>B>O	O>B>G
	6	G>B>O	G>B>O*	G>B>O	G>B>O	O>B>G	G>B>O
	7	G>B>O	G>B>O*	O>B>G	O>B>G	O>B>G*	O>B>G
	8	B>O=G	G>B>O	G>B>O	G>B>O	O>B>G	G>B>O

*Also accept B>O=G because exactly 2 "O" and "G" voters cast votes for their second most preferred candidate.

B. Polls and Poll/Election Interaction

In this section, we ask about polls. How did they compare to election results? How accurate were they? Did they appear to affect election outcomes? Table VII gives a summary of poll results.

Except under plurality voting, polls had little apparent impact on the frequencies with which each candidate won. Again, except under plurality voting, Blue (the Condorcet loser) won fewer elections than polls. There were always more ties in elections and, again except under plurality voting, elections appear closer than polls in terms of average winning margin, average total spread and highest low vote total. Under plurality voting, the winning margin fell, the overall spread increased and low vote totals fell between the poll and the election. Thus, while the candidate who finished last in the poll became less viable, the races still tightened between the two poll leaders.

1. Poll Reliability and Correlation with Election Outcomes

Individual poll responses and poll rankings may not correspond to individual behavior or election rankings for several reasons. First, even if voters intend to vote the way they indicate they will at the time of the poll, they may react to poll results and change their votes. That is, by measuring intended voting behavior and revealing that information, we change the very behavior we are trying to measure. Second, because voters know other voters may react to poll results, they may deliberately misrepresent their response in polls. Finally, since there are no direct payoffs to polls, voters may see the link to payoffs in the following election as tenuous. Alternatively, they may view polls as completely uninformative because they are non-

Table VII: Summary of Poll and Election Results for Sessions with Polls

Session (Voting Rule) & Events Summarized	Fractions of Polls/Elections Won By Each Candidate*			Number of Ties for First Place		Average Normalized Winning Margin† (Std. Dev.)	Average Normalized Total Spread† (Std. Dev.)	Normalized Low Total‡ (Std. Dev.)
	Orange	Green	Blue	2-Way	3-Way			
AWPS1 (Approval) Polls	0.2604	0.3542	0.3854	9	3	1.6250 (1.5523)	3.2708 (2.2000)	0.2405 (0.0696)
Elections	0.4653	0.3194	0.2153	13	4	0.8125 (0.7043)	1.7083 (1.0711)	0.2867 (0.0358)
BWPS1 (Borda) Polls	0.3160	0.4514	0.2326	4	2	1.2708 (0.9728)	2.8229 (1.5659)	0.2344 (0.0561)
Elections	0.4340	0.4549	0.1111	5	4	0.9167 (0.7741)	1.6458 (0.0104)	0.2955 (0.0245)
PWPS1 (Plurality) Polls	0.3542	0.4688	0.1771	4	0	1.8125 (1.2489)	3.3333 (1.7300)	0.2108 (0.0693)
Elections	0.3542	0.4479	0.1979	7	0	1.6250 (1.2138)	5.6875 (2.0017)	0.1012 (0.0831)

*Total wins over number of elections where wins are scored as follows: Outright Wins = 1, Two-Way Ties = 1/2 and Three-Way Ties = 1/3.
†The winning margin is the number of votes separating 1st and 2nd place candidates. The total spread is the number of votes separating the 1st and 3rd place candidates. Borda rule margins and spreads were divided by two since, by changing a his or her vote, a single voter can change the vote totals by twice as much as possible under plurality or approval voting.
‡Normalized by dividing by the total number of votes cast.

binding. Thus, voters may not feel sufficiently motivated to respond to polls accurately, strategically, or at all.

Simple measures of poll accuracy include how often they predicted the exact election ranking (including ties), weakly predicted the election ranking, predicted the winner or predicted the loser. Table VIII gives these frequencies. By all of these measures, polls did a better job of predicting the election outcomes under plurality voting than under approval voting or Borda rule. Table VIII also gives the average number of voters who abstained in the poll, but voted in the following election. This rate is highest under Borda rule (31.8%) and lowest under plurality voting (6.0%). Under approval voting and Borda rule, the rates for "G" voters were 24.3% and 41.0%. These were significantly higher than rates for "O" and "G" voters (14.1% under approval voting and 25.0% under Borda rule). Finally, Table VIII shows how often we reject the hypothesis that polls make the correct prediction 90% of the time. Except under plurality voting, polls were always significantly less than 90% accurate.

Table VIII: Summary of the Ability of Polls to Predict Election Results

Session	Voting Rule	Obs.	Number of Times the Poll Predicted: Exact Ranking	Weak Ranking	Winner(s)	Loser(s)	Average Number of Voters Who Abstained' from the Poll but Voted in the Election (of 14 Voters per Election)
AWPS1	Approval	48	0*	18*	24*	26*	2.5833
BWPS1	Borda	48	7*	20*	22*	26*	4.4583
PWPS1	Plurality	48	25*	38	43	42	0.8333

*According to a Binomial Test (assuming we have one independent observation per voting group), reject the hypothesis that the probability that the poll correctly forecasts the election outcome 90% of the time. Goldberg [1960] describes the test.
'Both (0,0,0) and (1,1,1) were counted as abstentions under Approval Voting.

Another measure of poll accuracy is the correlation between candidate vote totals in the poll and the elections. Alternatively, we can examine the correlation between candidate rankings in the poll and in the following elections. In each poll and election, we assign ranks of 1, 2 and 3 to candidates who finish first, second and third. (Tied candidates are given the average of the two or three ranks.) Table IX gives the correlations between poll and election vote totals and rankings. Again, according to these measures, polls appear most accurate under plurality voting, where we find the only significantly positive correlation coefficients.

Table IX: Correlations of Poll and Election Vote Totals and Rankings

Session	Voting Rule	Obs.	Correlation Coefficients between Poll and Election Vote Totals for Candidates: Orange	Green	Blue	Correlation Coefficients between Poll and Election Rankings for Candidates: Orange	Green	Blue
AWPS1	Approval	48	-0.1049	-0.1032	-0.0996	0.0714	0.0153	-0.3127*
BWPS1	Borda	48	-0.0676	0.1417	-0.0252	-0.0571	-0.1848	-0.0060
PWPS1	Plurality	48	0.8235*	0.8711*	0.5996*	0.8553*	0.9407*	0.2676*

*Reject the null of independence (at the 10% level of confidence) according to the test statistic $z := \frac{1}{2} \times \sqrt{n-3} \times \ln\left[\frac{1+r}{1-r}\right]$, where n is the number of observations and r is the sample correlation coefficient which should be asymptotically distributed N(0,1). See Hoel [1971].

2. Poll Effects on Election Outcomes

Even if polls do not reflect election outcomes accurately and are not highly correlated to election outcomes, polls may affect election outcomes. Individuals reacting to the poll may create both this inaccuracy and the poll's effects on the election outcomes. To test whether the poll rankings were independent of the election outcomes, we run a series of χ^2 tests.[26] Table X gives these statistics. Only under plurality voting do we find a significant relationship between the poll ranking and the election ranking or the probability of particular candidates winning. The results under approval voting and Borda rule are consistent with the O = G = B equilibrium occurring regardless of the poll outcome. The results under plurality voting would arise if voters used poll results to coordinate on one of the ranked equilibria. As discussed earlier, when either the Orange or Green candidate received significantly fewer votes than predicted by random voting, it was *always* the candidate who was behind in the preceding poll. Similarly, Orange or Green candidates who received significantly more votes were those who lead in the poll.[27]

Table X: χ^2 Tests for the Effects of the Poll Ranking on the Following Election Outcome

Testing for Independence of:	Session	Voting Rule	χ^2 Statistic	dof.	Prob>χ^2
The Poll Ranking and The Election Ranking	AWPS1	Approval	116.8660	120	0.564
	BWPS1	Borda	94.5915	108	0.818
	PWPS1	Plurality	144.3678*	70	0.000
The Poll Ranking and Whether Orange Won the Election	AWPS1	Approval	24.3822	30	0.754
	BWPS1	Borda	26.6634	36	0.871
	PWPS1	Plurality	47.5552*	20	0.000
The Poll Ranking and Whether Green Won the Election	AWPS1	Approval	30.6193	30	0.434
	BWPS1	Borda	27.2830	36	0.852
	PWPS1	Plurality	74.8089*	20	0.000
The Poll Ranking and Whether Blue Won the Election	AWPS1	Approval	27.4922	30	0.597
	BWPS1	Borda	23.7862	24	0.474
	PWPS1	Plurality	43.5111*	20	0.002

*Reject the hypothesis of independence at the 10% level of significance.

C. Individual Behavior

1. <u>Sincere and Strategic Voting Behavior</u>

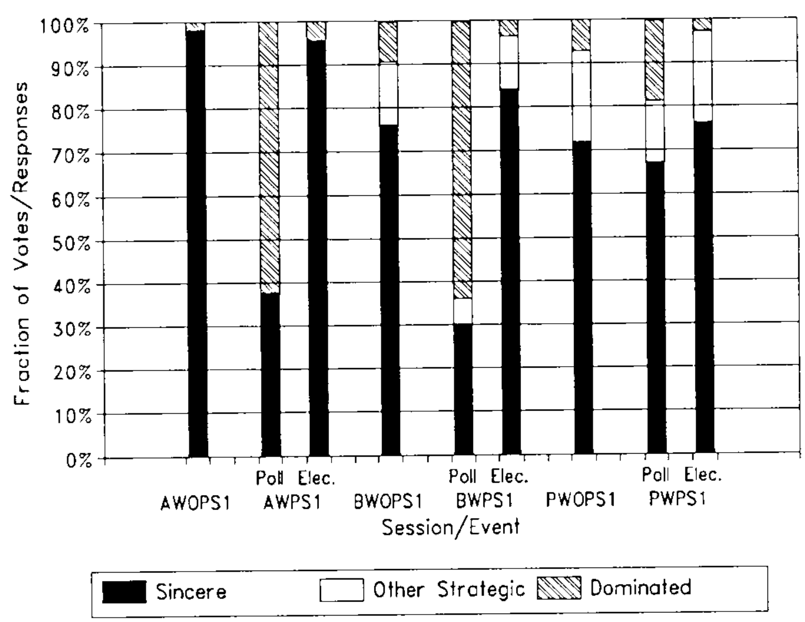
Recall, we can classify vote vectors into three categories: sincere (the vector weakly ranks candidates the same as the voter's preferences do), other strategic (votes that may be cast to prevent a voter's least favorite candidate from winning) and strategically dominated (there are no conditions under which this voter's vector is pivotal <u>and</u> in which this vote vector would swing the election in favor of the voter's more preferred candidate.) We first ask how often voters vote sincerely or strategically and whether this changes with voting rules, polls and/or voter types.

[26]Including ties, there are 13 possible rankings in each poll and election, though not all occurred. See Hoel [1971] for a description of χ^2 tests.

[27]Results are given in Appendix II.

Under our payoffs with approval voting, the following vote vectors are sincere: (1,0,0) or (1,1,0) cast by "O" voters; (0,1,0) or (1,1,0) cast by "G" voters; and (0,0,1), (0,1,1) or (1,0,1) cast by "B" voters. All other vectors are dominated. Under Borda rule, sincere vote vectors rank candidates the same as preferences would. Other strategic vote vectors are: (1,2,0) or (2,0,1) cast by "O" voters and (0,2,1) or (2,1,0) cast by "G" voters. Other vectors are dominated. Under plurality voting, sincere vectors cast one vote for the favorite candidate. Other strategic vote vectors are (0,1,0) cast by "O" voters and (1,0,0) cast by "G" voters. Other vote vectors are dominated. Figure 3 shows the aggregate fraction of votes cast that were sincere, other strategic and dominated. It also shows the aggregate fraction of poll responses that correspond to sincere, other strategic and dominated vote vectors. Table XI breaks the poll responses and vote vectors down by voter type.

Figure 3: Aggregate Fractions of the Vote Vectors Cast that were Sincere, Other Strategic and Dominated



We find that, while individuals sometimes cast dominated vote vectors (4.9% of the 4032 vote vectors cast), all voter types in each session, group and period cast significantly fewer dominated votes than predicted by random voting.[28] While most voters cast sincere votes, a fraction under both plurality voting and Borda

[28]The incidence of dominated vote vectors falls to 3.5% (of 504 votes cast) in the final periods of the voting groups. Results are given in Appendix II.

Table XI: Sincere and Strategic Voting Behavior and Corresponding Poll Responses

Session (Voting Rule)	Voters	Poll Responses Corresponding to Vote Vectors that were: Sincere	Other Strategic	Dominated	Vote Vectors that were: Sincere	Other Strategic	Dominated	Total
AWOPS1 (Approval)	O	--	--	--	189	--	3	192
	G				191		1	192
	B				279		9	288
	All				659		13	672
AWPS1 (Approval)	O	85	--	107	185	--	7	192
	G	90		102	177		15	192
	B	88		200	280		8	288
	All	263		409	642		30	672
BWOPS1 (Borda)	O	--	--	--	142	43	7	192
	G				134	56	2	192
	B				234	--	54	288
	All				510	99	63	672
BWPS1 (Borda)	O	63	19	110	145	42	5	192
	G	60	21	111	146	40	6	192
	B	80	--	208	274	--	14	288
	All	203	40	429	565	82	25	672
PWOPS1 (Plurality)	O	--	--	--	92	95	5	192
	G				139	45	8	192
	B				252	--	36	288
	All				483	140	49	672
PWPS1 (Plurality)	O	114	54	24	114	76	2	192
	G	142	40	10	125	65	2	192
	B	196	--	92	273	--	15	288
	All	452	94	126	512	141	19	672

rule cast other strategic votes. (Under plurality voting, it takes at least three voters voting strategically to attain one of the coordinated equilibria.) Overall, 92.2% of the 4032 votes cast were consistent with some equilibrium.

Poll responses differed considerably from this. Under approval voting, 60.9% of the poll responses corresponded to dominated vote vectors and 36.3% were consistent with an equilibrium.[29] Under Borda rule, 63.8% corresponded to dominated vote vectors and 33.0% were consistent with an equilibrium.[29] Under plurality voting, 18.7% corresponded to dominated vote vectors and 81.3 were consistent with an equilibrium. Thus, under approval voting and Borda rule, all voter types, especially "B" voters, *usually* claimed they would cast dominated votes, but seldom did (68.6% of the time for "B" voters and 53.1% of the time for "O" and

[29]The remaining vote vectors, while not dominated for all possible ties, were not optimal in any of the equilibria.

"G" voters). While "B" voters sometimes did this under plurality voting (27.8% of the time), "O" and "G" voters seldom did (8.3% of the time).

2. Truthful Versus Deceptive Polling Responses

As mentioned above, individuals may cast votes that differ from their poll responses for several reasons, some of them strategic. Table XI shows that individuals often did this. Here, we ask whether particular individuals always responded truthfully to polls in the sense that their poll responses matched their vote. We do this by asking whether individuals appeared to cast vote vectors that matched their poll responses significantly more (or less) often than predicted by random voting.[30] In the next section we ask whether these changes between the poll and the election corresponded to rational responses as predicted by Myerson and Weber [1989].

Table XII shows the number of voters who cast vote vectors that matched their poll responses more often than predicted by random voting. According to this measure, only a minority of voters under approval voting and Borda rule tended to poll truthfully. Further, this fraction dropped in later groups. Conversely, most voters under plurality voting tended to poll truthfully. This fraction dropped only slightly in later groups. Thus, while approval voting and Borda rule may promote sincere voting, they apparently do not promote truthful poll responses. And, while plurality voting appears to promote strategic, but insincere, voting, it does promote truthful polling.

3. Individual Responses to Preceding Elections and Polls

We have seen that most of the elections we conducted were consistent with some equilibrium--generally the O=G=B equilibrium under approval voting and Borda rule and one of the coordinated equilibria under plurality voting. However, poll outcomes usually differed considerably from the election outcomes. Not only did individual poll responses and poll outcomes often not predict election responses and outcomes, but poll responses and outcomes often did not correspond to any election equilibrium. However, individuals may have responded systematically to polls and previous elections, leading them to select particular equilibria in the following election. If true, this implies polls and previous elections played an important roll in the equilibrium selection process.

To test this, we begin with Myerson and Weber's [1989] assumption that voters strategically cast ballots to break the ties that they perceive to be most likely. We test whether voters responded as if they perceived the most likely tied candidates were the top two finishers in the preceding poll or election (top three candidates if there was a three way tie). This leads to several predictions about how individual will react to previous election and poll rankings. We will test whether we observed these particular individual responses

[30]Under approval voting and Borda rule, the probability of casting a vote vector that matches a non-abstention poll response is 1/6 if voters randomly choose between non-abstention vote vectors. Under plurality voting, the probability is 1/3.

Table XII: Number Voters Who Gave Truthful* Poll Responses Significantly More or Less Often than Predicted by Random Behavior

Session (Voting Rule)	Voter Type	Number of Voters	Groups A & B		Groups C & D		Groups E & F	
			More† Often	Less‡ Often	More† Often	Less‡ Often	More† Often	Less‡ Often
AWPS1 (Approval)	"O"	8	3	--	1	--	1	--
	"G"	8	8	--	7	--	2	--
	"B"	12	4	--	0	--	2	--
	All	28	15	--	8	--	5	--
BWPS1 (Borda)	"O"	8	4	--	1	--	2	--
	"G"	8	4	--	4	--	1	--
	"B"	12	2	--	2	--	3	--
	All	28	10	--	7	--	6	--
PWPS1 (Plurality)	"O"	8	7	0	8	0	8	0
	"G"	8	7	0	8	0	7	1
	"B"	12	10	0	7	0	6	2
	All	28	24	0	23	0	21	3

*"Truthful" means the poll response matched the vote cast in a given period.
†Number of voters for whom we reject the hypothesis that voters voted randomly in favor of the alternative that they were more likely to cast votes that matched their poll response at the 10% level of significance.
‡Number of voters for whom we reject the hypothesis that voters voted randomly in favor of the alternative that they were less likely to cast votes that matched their poll response at the 10% level of significance.
Note, for this test, only plurality voting has a meaningful (zero or larger) critical value.

using χ^2 tests for independence between the particular rankings and frequency of predicted responses.[31] Table XIII gives χ^2 statistics for the following ranking/response relationships.

Under approval voting, "O" and "G" voters should cast a vote for their second favorite candidate if Blue was strictly ahead of their favorite candidate in the preceding election or poll. However, the frequency of "second" approval votes should never change if individuals always expect the O = G = B equilibrium.[32] We find that "O" and "G" voters did respond by casting more "second" approval votes if Blue was ahead of their favorite candidate in the preceding poll. However, we cannot conclude that this effect held across elections in either session. Rather, voter responses across periods were consistent with voters always expecting close three-way races.

Under Borda rule, "O" and "G" voters should cast 2 votes for their favorite candidate unless that candidate came in last in the preceding election or poll. They should cast 0 votes for Blue unless Blue came in last in the poll. Type "B" voters should cast 1 vote for whomever of Orange or Green was behind in the

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[31]The χ^2 tests we conduct use nxm contingency tables where the columns represent the outcomes of the previous poll or election and the rows represent the voter responses of the appropriate voter type(s). We test for independence of the rows (previous poll or election outcomes) and columns (current voter responses).

[32]In the coordinated equilibria, the χ^2 tests will reject independence since all "O" and "G" voters should vote for the leading candidate from the previous poll or election. If the O = G = B equilibria always holds, the χ^2 test will not reject independence since, regardless of the ranking in the previous poll or election, voters respond with the same mix of votes for their favorite and second favorite candidates.

poll. Again, voting frequencies should be unaffected if voters always expect the O = G = B equilibrium and vote consistently with that expectation.[33] We do find that "B" voters responded to the poll results as predicted. However, "O" and "G" voter behavior was apparently independent of whether their favorite candidate or Blue was last in the poll. (Recall that, under Borda rule, the type "B" voters abstained 68.3% of the time in polls. Due to this, "O" and "G" voters may have found poll results uninformative and chosen to ignore them.) Again, consistent with always expecting the O = G = B equilibrium, there were no significant effects across elections in either session.

Under plurality voting, "O" and "G" voters should cast their vote for their second favorite candidate if their favorite candidate came in last in the preceding election or poll. We find this effect was significant with polls. We also find it held across elections in the session without polls. However, the opposite effect appeared across elections in the session with polls. In this session, "O" and "G" voters tended to poll for the candidate who lost the preceding election. If that candidate garnered sufficient support in the poll, the voters then elected that candidate. Thus, voters responded to poll results instead of the preceding election results. Further, polls seemed to facilitate alternating between the two coordinated equilibria. Finally, the Condorcet loser won less often in sessions with polls. Thus, while voters attained coordinated equilibria in repeated elections without polls, they attained them with greater frequency when polls served as coordinating signals.

Table XIII: χ^2 Tests for the Effects of Previous Election and Poll Rankings on the Individual Responses in Elections

Testing for Independence of:	Session	Voting Rule	Ranking Event	χ^2 Statistic	d.f.	Prob>χ^2
Favorite Cand./Blue Ranking	AWOPS1		Prev. Elec.	1.0337	2	0.596
and	AWPS1	Approval	Prev. Elec.	1.6710	2	0.434
Votes Cast by "O" and "G" Voters	AWPS1		Poll	8.6301*	2	0.013
Poll or Previous Election Loser	BWOPS1		Prev. Elec.	2.4297	4	0.657
and	BWPS1	Borda	Prev. Elec.	1.4924	4	0.828
Votes Cast by "O" and "G" Voters	BWPS1		Poll	1.0964	4	0.895
Orange/Green Ranking	BWOPS1		Prev. Elec.	2.7558	2	0.252
and	BWPS1	Borda	Prev. Elec.	1.7966	2	0.407
Votes Cast by "B" Voters	BWPS1		Poll	9.0907*	2	0.011
Favorite Cand./Blue Ranking	PWOPS1		Prev. Elec.	208.5223*	4	0.000
and	PWPS1	Plurality	Prev. Elec	11.6137*	2	0.000
Votes Cast by "O" and "G" Voters	PWPS1		Poll	184.6684*	4	0.000

*Reject the hypothesis of independence at the 10% level of significance.

[33]Again, in the mixed strategy equilibrium, the previous poll or election rankings should not affect voter responses and the χ^2 tests should not reject independence.

IV. Conclusions and Discussion of Further Research

We reported the results of a series of three candidate experimental elections. We argue the (single) parameter set we use is a particularly interesting one that creates electorates based loosely on the 1970 U.S. Senatorial contest in New York. Using these elections, we studied a variety of issues both at the aggregate outcome and individual behavior levels. Under three different voting rules, we asked how often Condorcet losers won elections, how often Duverger's law appeared to hold, whether particular game-theoretic equilibria were helpful when characterizing observed behavior, how polls and repeated elections affected outcomes and whether individuals responded to polls and previous elections in predicted ways. Here, we summarize our results by voting rule starting with the most familiar, plurality voting.

Under plurality voting, we found support for Duverger's law creating two-candidate races in which the Condorcet loser lost the elections. This result requires a significant amount of voter coordination, which was not immediate. It often took a few periods for a cohort arrive at equilibria in which the Condorcet loser did lose. However, support for these coordinated equilibria became stronger in later periods of each repeated election series with fixed electorates and candidates. (Of the 14 elections which do not support a coordinated equilibrium, only 3 were in the last 4 periods of a repeated election series.) These equilibria require that either "O" or "G" voters cast strategic, but insincere, votes. Consistent with this, we find that the amount of strategic voting was higher under plurality voting than under the other two rules. Further, most "O" and "G" voters used the previous poll or election ranking to decide who to vote for in the next election. Polls assisted with the coordination, seeming to provide "O" and "G" voters with clear signals about the candidates for whom they should vote. In contrast to poll accuracy under the other two rules, polls correctly forecast the even the exact ranking of the candidates the majority of the time under plurality voting. Also unlike the other two rules, plurality voting did not provide voters with apparent incentives to misrepresent their intentions in the polls. This resulted in a higher rate of truthful polling.

Under approval voting, outcomes usually were most consistent with close three-way races in which the Condorcet loser, Blue, won less often than the other two candidates. In the session without polls, voters appeared to act as if they generally expected this equilibrium to arise. Voters also seemed to respond to polls in a manner that supports this equilibrium. In particular, "O" and "G" voters tended to cast votes for their second favorite candidate if Blue was strictly ahead of their favorite candidate in the previous poll or election. Voters seemed aware of this response pattern. It implies that "B" voters would like Blue to finish last while "O" and "G" voters would like Blue to finish second or better. The first case leads "O" and "G" voters to cast votes only for their favorite candidates, allowing Blue to win. The latter case will result in more votes for "O" and "G", reducing Blue's chances of winning. Thus, voters of each type have incentives to misrepresent their preferences in the poll or abstain completely from it. Further, after being behind in the poll, Blue should rise in the standings and vice versa. The data support these results with voters abstaining or casting poll responses for their least favored candidate over 60% of the time. Further, out of 16 times that Blue won the poll, Blue won one election, tied with Orange once and was in a three-way tie twice. Out of the 26 times that Blue lost the poll, Blue never lost the election by more than two votes, lost by only one vote 10 times and won or tied for first thirteen times.

Under Borda rule, outcomes were generally consistent with the equilibrium prediction of close three-way races in which the Condorcet loser, Blue, won less often than Orange or Green. We argued that simple best response behavior required that "O" and "G" voters would cast their "1" vote for Blue if Blue finished last in the previous balloting (either election or poll). As with approval voting, this behavior creates an incentive for "B" voters to vote so that their most preferred candidate finishes last in the previous poll. Type "B" voters tried to accomplish this by abstaining from polls an extraordinary amount (more than 2/3) of the time. This abstention strategy was apparently transparent to "O" and "G" voters who seemed to disregard poll results when voting. In the polls, Blue finished last in 26 of the 48 polls conducted, but was only able to win or tie for first in 4 of these instances.

We view this as an initial study of three candidate elections rather than a in-depth study of particular issues that arise in these elections. Future research will examine the robustness of these results by examining different sets of preferences and different voting group sizes and a wider range of voting rules.

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Appendix I: Instruction Sets

In this appendix, we give the instructions sets we used and sample payoff tables, ballots, polling forms and record sheets for session with polls . Each subject had a copy of the instructions. They were read aloud and all questions were answered before the sessions began.

INSTRUCTIONS

GENERAL

This experiment is part of a study of voting procedures. The instructions are simple and if you follow them carefully and make good decisions, you can make a considerable amount of money which will be paid to you in cash at the end of the experiment.

The experiment will consist of a series of separate decision making periods. [**In sessions with polls:** In each period you will have the opportunity to participate in a poll and then vote in an election with three candidates.] [**In sessions without polls:** In each period you will have the opportunity to vote in an election with three candidates.] The candidates are named Orange, Green and Blue. You must vote according to the rules discussed below. The votes cast will determine the winning candidate in each election. In the next period, the process will be repeated, with the exception that the identities of some of the members in your voting group may change. Your payoff in each period will depend upon your payoff schedule -- described below -- and on which candidate wins the election in which you voted.

VOTING GROUPS

Initially, each participant will be assigned randomly to one of two groups of voters. Then, in each period, two separate and totally independent elections will take place, each involving one of the two groups of voters. Your payoff will depend only on your decisions and those of the others in your group. The decisions made by the other group of voters will have no effect on your payoffs.

Each voting group will remain unchanged for ___ periods. After ___ periods, we will change the voting groups. When this happens, all participants will again be randomly assigned to one of two new groups. After each re-assignment, the members of the group you are in and the individual payoff schedules will generally not be the same as they were previously.

VOTING RULES

You will find a set of voting "ballots" in your folder. [**In sessions with polls:** (Ballots are printed in black on white.)] Each period, when an election is held, you must fill out one of these ballots. On each ballot, the three candidates are listed separately. There are places for you to record the period number and your voting group. After filling in the period number and your voting group, you must decide how you will cast your vote in your group's election. You may always choose not to vote for any candidate. If you do decide to vote, you must do so according to the following rule (which applies to all voters in your group):

[For sessions using approval voting:

If you do not abstain, you may cast one vote each for as many candidates as you wish. To do this, place a check next to each candidate for whom you are voting.]

[For sessions using Borda rule:

If you do not abstain, you must give two votes to one candidate, and one vote to one of the other candidates. To do this, write "2" next to the candidate to whom you are giving two votes and write "1" next to the candidate to whom you are giving one vote.]

[For sessions using plurality voting:

If you do not abstain, you may vote for at most one candidate. To do this, place a check next to the candidate for whom you are voting.]

After filling out your ballot, simply hand it to the experimenter who will be collecting them. Even if you choose to abstain, you must turn in a ballot (with only the period number and your voting group written on it). If your ballot is incorrectly filled out, it will be returned to you so that you may correct it.

After all the ballots have been collected, we will total them, and announce the outcome of your group's election by telling you the number of votes each candidate received. The candidate with the highest number of votes in an election will be declared the winner of that election. The votes cast by one voting group will have no effect on the other group's election.

If two or more of the candidates tie with equal (highest) vote totals, we will randomly determine a winner. Specifically, we have a tie-breaking box and we have three colored balls: an orange one, a green one and a blue one. Balls corresponding to the tied candidates will be put in the box and one of you will be asked to randomly draw a ball from the box. The candidate whose name is the same as the color of the selected ball will be declared the winner. After the winner of the election has been announced, you will be able to determine your payoffs. This is discussed next.

PAYOFF RULES

In each period, the payoff you receive will be determined by which candidate wins your voting group's election. In your folder you will find a payoff schedule that applies to your initial voting group. (Your initial group is either Group "A" or Group "B". There are also schedules for your future voting groups: Groups "C" or "D" and Groups "E" or F".) There are three types of voters in each group. Voter types differ by their payoffs. The payoff schedule shows your voter type, how payoffs will be determined for your voter type, how payoffs are determined for other voter types and the number of voters of each type. As an example, suppose that you are initially assigned to Group Q and your payoffs are as follows:

PAYOFF SCHEDULE FOR VOTING GROUP: Q

Voter Type	Election Winner: Orange	Election Winner: Green	Election Winner: Blue	Total Number of Each Type.
1	$0.15	$0.35	$0.10	6
2	$0.35	$0.05	$0.20	6
3	$0.10	$0.00	$0.50	4

This schedule tells you that you are initially in Voting Group Q and are a voter of Type 2. Your payoffs are the ones that have been outlined. These tell you what your payoffs will be for **every** election that is held as long as you are a member of group Q. Thus, for each election you participate in with group Q, you will receive $0.35 if the orange candidate wins, $0.05 if the green candidate wins and $0.20 if the blue candidate wins. You can also see that there are 5 other voters besides yourself with the same payoffs as you. There are 6 voters with payoffs corresponding to the first row of the payoff schedule and 4 voters with payoffs corresponding to the third row of the payoff schedule. Remember that this is only an example and does not correspond to the actual payoff schedules used in the experiment.

[For sessions with polls:

POLLING RULES

Prior to each election, we will be conducting non-binding polls (separately for each of the two voting groups). In each poll, we will ask all participants how they would vote if the election were held at the time of the poll. Polls will be conducted according to the same voting rules as the elections.

You will find a set of polling forms in your folder. (These are printed in black on grey and have "POLLING FORM" written at the top. Otherwise, they are identical to ballots.) During each poll, you must fill out one of these forms. After filling in the period number and your voting group, you must decide whether to participate in the poll. (You can always choose not to participate in the poll.) If you do decide to participate, you should vote according to the above voting rule. In this way, the poll is a "straw" vote.

After filling out your polling form, hand it to the experimenter who will be collecting them. Even if you choose not to participate in a poll, you must turn in a polling form with only the period number and your voting group written on it. If your polling form is incorrectly filled out, it will be returned to you so that you may correct it.

After all the polling forms have been collected, we will total them, and announce the results for your voting group by telling you the number of "straw" votes each candidate received in the poll. After the polls, we will give you time to consider the results. Thus, you will know how the voters in your group responded in the poll before you are asked to vote. The poll responses of one group will have no effect on the results of the other group's poll.

When all participants are ready, we will conduct the actual election for each group. You may participate in your group's election even if you did not participate in the poll. Furthermore, in the election, you may vote in a manner that differs from what you stated in the poll.]

RECORDING RULES

You have been given several Record Sheets. During each period you should record your group, **[For sessions with polls:** outcome of the poll,] how you voted in the election, and the outcome of the election in the spaces provided. After the winner of the election is announced, you should record your payoff in the row labeled "YOUR PERIOD EARNINGS" for that period. Each sheet provides you with a record of your group's results from earlier periods.

At the end of an experimental session, add "YOUR PERIOD EARNINGS" from each of the periods on each record sheet and record the total in the row corresponding to "SUB-TOTAL EARNINGS." Add the sub-totals from each record sheet together and place this amount on your receipt. The experimenter will pay you this amount in cash.

Also, at the end of the experiment, we would appreciate it if you would take a few moments to fill out a brief questionnaire which we will pass out. While your answers on the questionnaire will help us with our research, filling it out is purely voluntary. Your payment will not depend upon whether you fill out the questionnaire or your answers to specific questions.

If you have any questions during the experiment, ask the experimenter and he or she will answer them for you. Other than these questions, you must keep silent until the experiment is completed. If you break silence while the experiment is in progress, you will be given one warning. If you break silence again, you will be asked to leave the experiment and you will forfeit your earnings.

Are there any questions?

ELECTION BALLOT

Participant Number: 1

Group ___

Period ___

The votes you fill in below will be counted in this period's election.

___ Orange Candidate

___ Green Candidate

___ Blue Candidate

POLLING FORM

Participant Number: 2

Group ___

Period ___

If this period's election were held at this time, how would you vote?

___ Orange Candidate

___ Green Candidate

___ Blue Candidate

PAYOFF SCHEDULE

You are a voter of Type _1_ in Voting Group _A_.

Voting Rule: In each election, you may vote for at most one candidate.

The following payoff schedule applies to your group. Your specific voter type and payoffs have been outlined.

Payoff Schedule Group: _A_

Voter Type	Election Winner: Orange	Green	Blue	Total Number of Each Type
1	$1.60	$1.20	$0.30	4
2	$1.20	$1.60	$0.30	4
3	$0.60	$0.60	$1.90	6

Record Sheet

Player Number: ___ Record Sheet Number: ___

Period	Group	Event	Votes Received: Orange	Votes Received: Green	Votes Received: Blue	Payoffs
1		Your Poll Resp.				
		Poll Results				
		Your Vote				
		Election				
					YOUR PERIOD EARNINGS	
2		Your Poll Resp.				
		Poll Results				
		Your Vote				
		Election				
					YOUR PERIOD EARNINGS	
:	:	:	:	:	:	:
:	:	:	:	:	:	:
:	:	:	:	:	:	:
8		Your Poll Resp.				
		Poll Results				
		Your Vote				
		Election				
					YOUR PERIOD EARNINGS	
					SUB-TOTAL EARNINGS	

Appendix II: Hypotheses, Tests and Results

In this appendix, we give tests and results that support the propositions in the text. For many tests, we use Monte Carlo distributions of 1000 groups of 6 or 8 elections with 14 voters voting randomly in each election. When they can be conveniently calculated, we also use the analytic distributions under the null hypotheses. These distributions are in the data and statistical supplement available from the authors. Generally, all tests are one-sided at the 10% level of significance.

I. Election Propositions

A. Condorcet Propositions

P1 The Condorcet loser (Blue) tended to lose under approval voting and Borda rule and win under plurality voting.

Null: Voters cast random votes.
Alt: Voters voted in a manner that made the Condorcet loser more or less likely to win.

We count the number of times that the Condorcet loser (Blue) won out of the six elections per period in a session. (Two-way ties count 1/2 and three-way ties count 1/3.) Under the null, we use Monte Carlo simulations to find the cumulative density function of wins by a single candidate out of six elections. Table III in the text gives the number of times that the Condorcet loser won across the six elections in each period of each session. We reject the null in favor of the alternative that the Condorcet loser was more likely to win in Periods 1 and 2 of PWOPS1. We reject the null in favor of the alternative that the Condorcet loser was less likely to win in Periods 1, 2, 4, 5 and 6 of AWOPS1; Periods 6 and 7 of AWPS1; Periods 1, 2, 3, 5, 7 and 8 of BWOPS1; Periods 1, 5, 6 and 7 of BWPS1; Periods 4, 5 and 8 of PWOPS1 and Periods 3, 5 6 and 8 of PWPS1.

We also count the number of times the Condorcet Loser (Blue) won out of the eight elections per group in a session. Again, we use Monte Carlo simulations to find the cumulative density function. Table A.II.I gives the actual number of times that the Condorcet loser won across the eight elections in each group of each session. We reject the null in favor of the alternative that the Condorcet loser was more likely to win only in Group D of PWOPS1. We reject the null in favor of the alternative that the Condorcet loser was less likely to win in Groups A, C, E and F of AWOPS1; Group D of AWPS1; Groups C, D and F of BWOPS1; Groups B and D of BWPS1; Groups A and F of PWOPS1; and Group C of PWPS1. ■

B. Duverger Propositions

P2 Plurality voting promotes Duverger convergence with Orange or Green becoming inviable.

Null: Voters cast random votes.
Alt: Voters tended to vote in a manner that made Orange or Green more likely to finish last than random voting.

We count the number of times Orange or Green finished last out of the six elections per period in a session. (If Orange, Green or both lost outright, we count one. If Orange or Green tied with Blue for last, we count 1/2. Three way ties are counted as 2/3.) We use Monte Carlo simulations to find the cumulative

Table A.II.I: Number of Times the Condorcet Loser Won‡ the Elections
(Out of 8 Elections Per Group)

	Session (Voting Rule)					
Group	AWOPS1 (Approval)	AWPS1 (Approval)	BWOPS1 (Borda)	BWPS1 (Borda)	PWOPS1 (Plurality)	PWPS1 (Plurality)
A	0.33*	1.50	1.00	1.00	0.50*	2.00
B	1.33	2.50	2.00	0.67*	3.00	2.00
C	0.83*	1.00	0.00*	1.00	1.00	0.00*
D	1.00	0.50*	0.67*	0.00*	5.00†	2.50
E	0.00*	2.33	1.00	1.33	3.00	2.00
F	0.83*	2.50	0.00*	1.33	0.00*	1.00

*Reject the hypothesis that voters voted randomly in favor of the alternative that voters voted in a manner that made the Condorcet loser less likely to win (at the 10% level of significance).

†Reject the hypothesis that voters voted randomly in favor of the alternative that voters voted in a manner that made the Condorcet loser more likely to win (at the 10% level of significance).

‡Scored as follows: Outright Wins = 1; Two-Way Ties = 1/2; and Three-Way Ties = 1/3.

density function. Table IV in the text gives the actual number of times that Orange or Green finished last across the six elections in each period of each session. Table A.II.II gives the actual number of times that Orange or Green finished last across the eight elections in each group of each session. We reject the null in favor of the alternative that Orange and Green were more likely to finish last in Period 8 of AWPS1, (across groups) and all groups and periods except Group F in PWOPS1 under plurality voting. ■

Table A.II.II: Number of Times Orange or Green Finished Last‡
(Out of 8 Elections Per Group)

	Session (Voting Rule)					
Group	AWOPS1 (Approval)	AWPS1 (Approval)	BWOPS1 (Borda)	BWPS1 (Borda)	PWOPS1 (Plurality)	PWPS1 (Plurality)
A	0.67	6.00	1.00	1.00	7.50*	8.00*
B	4.67	6.50	4.00	4.80	8.00*	7.50*
C	5.17	4.00	1.50	3.00	8.00*	8.00*
D	4.50	2.50	2.83	2.00	8.00*	8.00*
E	2.00	6.17	3.00	6.17	7.00*	8.00*
F	3.67	5.50	3.00	2.17	6.00	8.00*

*Reject the hypothesis that voters voted randomly in favor of the alternative that voters voted in a manner that made Orange or Green more likely to lose (at the 10% level of significance).

‡Scored as follows: Outright Loses by Orange or Green = 1; Orange or Green Tied with Blue for Last = 1/2; and Three-Way Ties = 2/3.

P3 Plurality voting promotes Duverger convergence with Orange or Green becoming inviable. The candidate who becomes inviable is the one who was behind in the preceding election or poll.

Null: Voters cast random votes.
Alt: Voters tended to vote in a manner that made Orange or Green vote totals smaller.

We use the analytic cumulative density function of vote totals received by a single candidate under the null that voters vote randomly. Table V in the text shows the elections in which the Orange or Green candidate received significantly fewer votes that predicted by random voting. We never reject the null in favor of the alternative that the Orange or Green candidate received significantly fewer votes than expected under approval voting or Borda rule. We reject it 33 times in PWOPS1 and 38 times in PWPS1. In all of these cases, the Orange or Green candidate who received a significantly low vote total was behind in the preceding election or poll. ■

P4	Approval voting and Borda rule promote close three-way races.
	Null: Voters cast random votes.
	Alt: Voters tended to vote in a manner that decreased the overall spread in the elections.

We look at the average normalized spread in the six elections per period and eight elections per group in a session. (Under approval and plurality voting, the spread is the high vote total minus the low vote total. Under Borda rule, we divide this figure by 2, since a single voter can affect totals by twice as much as under the other rules.) Again, we use the Monte Carlo simulations to find the cumulative density functions. Table A.II.III and Table A.II.IV give the actual average normalized spreads. We generally reject the null in favor of close, three-way races for approval voting and Borda rule and in favor of large spreads (Duverger's law) under plurality voting.

Table A.II.III: Average Normalized[1] Spreads in Elections (Out of 6 Elections Per Period)

	Session (Voting Rule)					
Period	AWOPS1 (Approval)	AWPS1 (Approval)	BWOPS1 (Borda)	BWPS1 (Borda)	PWOPS1 (Plurality)	PWPS1 (Plurality)
1	2.50*	1.83*	3.08	2.83	3.17	5.67†
2	1.83*	2.33*	3.25	1.75*	5.00†	5.33†
3	1.67*	2.00*	1.92*	1.42*	5.67†	7.17†
4	1.33*	1.67*	1.92*	1.17*	6.33†	4.67†
5	1.83*	1.00*	1.58*	1.75*	6.83†	5.00†
6	2.33*	1.67*	1.75*	1.50*	6.17†	6.17†
7	0.83*	1.50*	2.00*	1.17*	6.50†	5.33†
8	1.17*	1.67*	1.25*	1.58*	7.00†	6.17†

*Reject the hypothesis that voters voted randomly in favor of the alternative that voters voted in a manner that <u>decreased</u> the election spread (at the 10% level of significance).

†Reject the hypothesis that voters voted randomly in favor of the alternative that voters voted in a manner that <u>increased</u> the election spread (at the 10% level of significance).

[1] Spread normalized by dividing by 2 for Borda rule.

An alternative means of testing P4 is to see whether the percentage of the vote received by the third place candidate was more that would be predicted under random voting. Again, we use the Monte Carlo simulations for critical values. Table A.II.V and Table A.II.VI give the actual average percentages of the vote taken by the third place candidates. Again, we generally reject the null in favor of close, three-way races for approval voting and Borda rule and in favor of large spreads (Duverger's law) under plurality voting. ■

Table A.II.IV: Average Normalized‡ Spreads in Elections
(Out of 8 Elections Per Group)

Group	Session (Voting Rule)					
	AWOPS1 (Approval)	AWPS1 (Approval)	BWOPS1 (Borda)	BWPS1 (Borda)	PWOPS1 (Plurality)	PWPS1 (Plurality)
A	1.75*	1.88*	3.31	2.06*	6.13†	5.88†
B	1.25*	2.75*	2.00*	1.63*	6.25†	5.00†
C	1.38*	1.75*	1.75*	1.88*	6.25†	6.00†
D	2.50*	1.25*	1.75*	1.56*	6.50†	5.13†
E	2.00*	0.75*	1.50*	1.63*	4.88†	5.88†
F	1.25*	1.88*	2.25*	1.13*	5.00†	6.25†

*Reject the hypothesis that voters voted randomly in favor of the alternative that voters voted in a manner that decreased the election spread (at the 10% level of significance).

†Reject the hypothesis that voters voted randomly in favor of the alternative that voters voted in a manner that increased the election spread (at the 10% level of significance).

‡Spread normalized by dividing by 2 for Borda rule.

Table A.II.V: Average Low Vote Percentage in Elections
(Out of 6 Elections Per Period)

Period	Session (Voting Rule)					
	AWOPS1 (Approval)	AWPS1 (Approval)	BWOPS1 (Borda)	BWPS1 (Borda)	PWOPS1 (Plurality)	PWPS1 (Plurality)
1	0.268	0.285†	0.264	0.266	0.238	0.119*
2	0.290†	0.265	0.255	0.285†	0.131*	0.119*
3	0.295†	0.273†	0.286†	0.302†	0.099*	0.036*
4	0.304†	0.287†	0.282†	0.310†	0.071*	0.143*
5	0.292†	0.307†	0.294†	0.294†	0.071*	0.131*
6	0.277†	0.295†	0.298†	0.302†	0.095*	0.083*
7	0.306†	0.301†	0.285†	0.305†	0.071*	0.107*
8	0.302†	0.281†	0.305†	0.302†	0.048*	0.071*

*Reject the hypothesis that voters voted randomly in favor of the alternative that voters voted in a manner that decreased the percentage of the vote received by the third place candidate (at the 10% level of significance).

†Reject the hypothesis that voters voted randomly in favor of the alternative that voters voted in a manner that increased the percentage of the vote received by the third place candidate (at the 10% level of significance).

C. Equilibria Propositions

P5 We observe one of the stage game equilibria.

Null: Voters vote randomly.
Alt: Voters vote in a manner consistent with some stage game equilibrium more often than random voting predicts.

Table A.II.VI: Average Low Vote Percentage in Elections
(Out of 8 Elections Per Period)

Group	AWOPS1 (Approval)	AWPS1 (Approval)	BWOPS1 (Approval)	BWPS1 (Approval)	PWOPS1 (Plurality)	PWPS1 (Plurality)
	Session (Voting Rule)					
A	0.288†	0.288†	0.242	0.283†	0.080*	0.116*
B	0.300†	0.251	0.282†	0.301†	0.128*	0.134*
C	0.300†	0.295†	0.294†	0.288†	0.071*	0.080*
D	0.283†	0.301†	0.300†	0.297†	0.036*	0.116*
E	0.280†	0.313†	0.301†	0.298†	0.134*	0.080*
F	0.301†	0.276†	0.282†	0.307†	0.170*	0.080*

*Reject the hypothesis that voters voted randomly in favor of the alternative that voters voted in a manner that decreased the percentage of the vote received by the third place candidate (at the 10% level of significance).

†Reject the hypothesis that voters voted randomly in favor of the alternative that voters voted in a manner that increased the percentage of the vote received by the third place candidate (at the 10% level of significance).

P6 Distinguishing between the stage game equilibria.

Approval Voting:		Notes: B voters vote (0,0,1) in all equilibria.
Null:	Eq is O=G=B	O voters use a 50/50 Mix of (1,0,0) and (1,1,0).
		G voters use a 50/50 Mix of (0,1,0) and (1,1,0).
Alts:	Eq is O>B>G	O Voters Vote (1,0,0) and G Voters Vote (1,1,0).
	Eq is G>B>O	O Voters Vote (1,1,0) and G Voters Vote (1,0,0).

Borda Rule: All voters mix or the population consists of a mix of pure strategy voters.

Plurality Voting:		Notes: B Voters Vote (0,0,1) in all equilibria.
Null:	O and G voters are casting random, undominated ballots (50/50 mix of (1,0,0) and (0,1,0)).	
Alts:	Eq is B>O=G	O Voters Vote (1,0,0) and G Voters Vote (0,1,0).
	Eq is O>B>G	O Voters Vote (1,0,0) and G Voters Vote (1,0,0).
	Eq is G>B>O	O Voters Vote (0,1,0) and G Voters Vote (0,1,0).

Approval Voting:

P5: For "O" and "G" voters, 1/3 of the meaningful vote vectors are consistent with an equilibrium. For "B" voters, 1/6 are consistent. Under the null of random voting, we can compute the probability of observing at least the number of votes that we observed consistent with an equilibrium in each election. We reject the null at the 10% level of significance if 2 "B" voters and 5 or more "O" and "G" voters, or 3 or more "B" voters and 4 or more "O" and "G" voters cast consistent votes. We reject the null in favor of voters tending to vote consistent with some equilibrium in all elections.

P6: The analytic distribution of number of O and G voters who cast votes consistent with either of the ranked equilibrium can be computed. The probability of casting a random vote consistently with one or the other equilibrium is 1/2 given that "O" and "G" voters cast random vote vectors that are consistent with an equilibrium. According to this distribution, we reject the null in favor of the appropriate alternative if 6 or more of the "O" and "G" voters vote consistently with the alternative. Table A.II.VII gives the equilibria we accept. We reject the null in favor of one of the ranked equilibria in 12 of 48 elections without polls (10 times in Groups C, D, E and F) and 9 of 48 elections with polls

(4 times in Groups C, D, E and F). Thus, we seldom can reject the null of a close, three-way race. We reject it less often with polls, especially in later groups.

Table A.II.VII: Specific Equilibria Accepted Under Approval Voting According to Votes Cast by "O" and "G" Voters (The Null is O=G=B)

		Group					
Session	Period	A	B	C	D	E	F
AWOPS1	1	O=G=B	O=G=B	O=G=B	G>B>O	O=G=B	O=G=B
	2	O=G=B	O=G=B	O=G=B	G>B>O	O>B>G	O=G=B
	3	O=G=B	O=G=B	O=G=B	O=G=B	O=G=B	O>B>G
	4	O=G=B	G>B>O	O=G=B	O=G=B	O=G=B	O=G=B
	5	O=G=B	O=G=B	G>B>O	G>B>O	O=G=B	O=G=B
	6	O=G=B	O=G=B	G>B>O	G>B>O	O=G=B	G>B>O
	7	O=G=B	O=G=B	O=G=B	O=G=B	O=G=B	O=G=B
	8	O=G=B	G>B>O	O=G=B	O=G=B	O>B>G	O=G=B
AWPS1	1	O=G=B	O=G=B	O>B>G	O=G=B	O=G=B	O=G=B
	2	O=G=B	O=G=B	O=G=B	O=G=B	O=G=B	O>B>G
	3	G>B>O	O=G=B	O=G=B	O=G=B	O=G=B	O=G=B
	4	G>B>O	O=G=B	O=G=B	O=G=B	O=G=B	O=G=B
	5	O=G=B	O=G=B	O=G=B	O=G=B	O=G=B	O=G=B
	6	G>B>O	O=G=B	O=G=B	O=G=B	O=G=B	O=G=B
	7	G>B>O	O=G=B	O>B>G	G>B>O	O=G=B	O=G=B
	8	O=G=B	O>B>G	O=G=B	O=G=B	O=G=B	O=G=B

Borda Rule:

Both: For all voter types, 1/3 of the meaningful vote vectors are consistent with the equilibrium. Thus, under the null, we can compute the analytic distribution of the number of voters who cast votes consistent with the equilibrium. We reject the null in favor of voters tending to vote consistently with some equilibrium if 7 or more voters are voting consistently with the equilibrium. We reject the null in all elections.

Plurality Voting:

P5: For "O" and "G" voters, 2/3 of the meaningful vote vectors are consistent with an equilibrium. For "B" voters, 1/3 are consistent. Under the null of random voting, we can compute the probability of observing at least the number of votes that we observed consistent with an equilibrium in each election. We reject the null at the 10% level of significance if 3 "B" voters and 8 "O" and "G" voters, or 4 or more "B" voters and 7 or more "O" and "G" voters cast consistent votes. We reject the null in favor of voters tending to vote consistently with some equilibrium in all elections except in Election 1, Group A and Election 5, Group E in PWOPS1 and Election 1 in Groups A and B in PWPS1.

P6: The analytic distribution of the number of "O" and "G" voters who cast votes consistent with each alternative equilibrium can be derived under the null of a 50/50 mix of (1,0,0) and (0,1,0) votes. According to this distribution, we reject the null in favor of the appropriate alternative if 6 or more of the "O" and "G" voters vote consistently with the alternative. Table VI in the text shows when we accept the null or reject in favor of a specific alternative. We can usually reject the null in favor of some equilibrium, usually in favor of one of the coordinated equilibria: O>B>G or G>B>O. There was a great deal of switching between equilibria in the session with polls (especially in later groups) and very little switching in the session without polls. ■

II. Poll/Election Propositions

A. Poll Reliability

> **P7** Polls usually give the same exact rankings, weak rankings, winners and/or losers as in the following elections.
>
> Null: The probability that the poll will forecast the election correctly (according to the appropriate ranking) is 90%. (Assume we have 6 independent observations (1 per group) in each experiment.)
>
> Alt: The probability that the poll will forecast the election correctly is less than 90%.

Table VIII in the text shows the number of times that the polls accurately predicted the exact election ranking, the weak election ranking, the election winner(s) and the election loser(s). It also shows that we reject the null that polls correctly predict the election outcome 90% of the time by any of the measures under approval voting and Borda rule. Under plurality voting, we only reject the ability of the polls to predict the exact election ranking. ■

> **P8** Poll vote totals and rankings are significantly correlated to election vote totals and rankings.
>
> Null: The correlation coefficient between poll vote totals (rankings) is zero.
>
> Alt: The correlation coefficient between poll vote totals (rankings) is significantly above or below zero.

Table IX in the text shows the correlation coefficients between the poll and election rankings as well as the poll and election totals for the sessions with polls. We reject the null that the poll and election vote totals have a zero correlation coefficient in favor of negative correlation for the Blue candidate under approval voting. We reject the null in favor of positive correlation for both totals and rankings for all candidates under plurality voting. ■

> **P9** Individuals respond truthfully to polls in the sense that they cast the vote vector they claimed they would in the poll.
>
> Null: Voters cast random, non-abstention vote vectors.
>
> Alt: Voters voted in a manner that made it more or less likely their vote would match their poll response.

Table A.II.VIII, shows the number of times each voter cast a vote vector that was the same as their poll response. Voters 16 and 19 in PWPS1 always cast vote vectors that matched their poll response. All other voters sometimes cast votes that did not match their poll responses. We test whether a voter's vote was random or was more likely to match that voter's poll response, conditional on the poll response. The probability that a random vote will match a given (non-abstention) poll response under approval voting and Borda rule is 1/6. Under plurality voting, it is 1/3. Under plurality voting, we find that voters were usually more likely to cast votes that matched their poll responses regardless of the group. Under approval voting and Borda rule, we find that while voters in early groups appeared more likely to cast votes that matched their poll responses, voters in later groups did not regularly cast votes that matched their poll responses. ■

Table A.II.VIII: Fraction of Truthful* Poll Responses

	Groups A & B				Groups C & D				Groups E & F			
Voter(s)	Voter Type	AWPS1 (Approval)	BWPS1 (Borda)	PWPS1 (Plurality)	Voter Type	AWPS1 (Approval)	BWPS1 (Borda)	PWPS1 (Plurality)	Voter Type	AWPS1 (Approval)	BWPS1 (Borda)	PWPS1 (Plurality)
1	"B"	0/8	0/8	5/8†	"G"	0/8	0/8	5/8†	"B"	0/8	0/8	0/8‡
2	"B"	4/8†	0/8	8/8†	"O"	1/8	0/8	6/8†	"O"	1/8	0/8	7/8†
3	"O"	2/8	2/8	7/8†	"B"	6/8†	1/8	3/8	"B"	7/8†	0/8	0/8‡
4	"O"	6/8†	6/8†	8/8†	"B"	8/8†	3/8†	8/8†	"G"	7/8†	1/8	4/8†
5	"G"	6/8†	6/8†	7/8†	"B"	3/8†	2/8	8/8†	"O"	0/8	3/8†	7/8†
6	"O"	3/8†	2/8	7/8†	"B"	4/8†	2/8	3/8	"G"	1/8	1/8	2/8
7	"B"	2/8	2/8	6/8†	"B"	0/8	3/8†	5/8†	"G"	0/8	1/8	5/8†
8	"O"	2/8	3/8†	6/8†	"O"	0/8	0/8	5/8†	"B"	0/8	0/8	8/8†
9	"B"	4/8†	1/8	6/8†	"O"	0/8	0/8	5/8†	"O"	0/8	2/8	6/8†
10	"G"	8/8†	6/8†	5/8†	"B"	8/8†	3/8†	3/8	"O"	6/8†	5/8†	2/8
11	"G"	3/8†	0/8	7/8†	"B"	0/8	0/8	1/8	"O"	1/8	0/8	7/8†
12	"B"	1/8	6/8†	5/8†	"O"	1/8	2/8	3/8	"O"	1/8	0/8	4/8†
13	"G"	5/8†	2/8	7/8†	"G"	6/8†	6/8†	5/8†	"B"	0/8	0/8	6/8†
14	"G"	3/8†	2/8	8/8†	"B"	1/8	0/8	8/8†	"G"	2/8	1/8	5/8†
15	"B"	4/8†	0/8	8/8†	"G"	2/8	0/8	5/8†	"B"	0/8	0/8	8/8†
16	"B"	1/8	1/8	8/8†	"O"	0/8	3/8†	8/8†	"B"	0/8	0/8	8/8†
17	"G"	4/8†	2/8	3/8	"O"	2/8	1/8	4/8†	"B"	1/8	0/8	2/8
18	"O"	2/8	4/8†	6/8†	"B"	1/8	6/8†	8/8†	"G"	3/8†	7/8†	4/8†
19	"B"	2/8	1/8	8/8†	"G"	0/8	1/8	8/8†	"G"	0/8	0/8	8/8†
20	"B"	2/8	0/8	6/8†	"G"	1/8	0/8	8/8†	"B"	0/8	2/8	8/8†
21	"O"	4/8†	5/8†	3/8	"B"	3/8†	2/8	8/8†	"B"	2/8	1/8	7/8†
22	"G"	4/8†	3/8†	5/8†	"B"	0/8	1/8	7/8†	"B"	1/8	0/8	8/8†
23	"B"	2/8	0/8	2/8	"G"	1/8	2/8	7/8†	"G"	0/8	3/8†	6/8†
24	"B"	1/8	1/8	2/8	"O"	2/8	0/8	4/8†	"O"	6/8†	0/8	6/8†
25	"O"	1/8	2/8	8/8†	"G"	0/8	2/8	5/8†	"B"	0/8	0/8	3/8
26	"B"	1/8	1/8	8/8†	"G"	1/8	0/8	4/8†	"O"	0/8	0/8	4/8†
27	"O"	5/8†	5/8†	7/8†	"B"	7/8†	1/8	7/8†	"G"	0/8	3/8†	5/8†
28	"G"	5/8†	3/8†	6/8†	"O"	1/8	6/8†	5/8†	"B"	0/8	4/8†	0/8‡
All Type "O"		25/64†	29/64†	52/64†		7/64	12/64	40/64†		15/64	10/64	43/64†
All Type "G"		38/64†	24/64†	48/64†		11/64	11/64	47/64†		13/64	17/64†	39/64†
All Type "B"		24/96†	13/96	72/96†		41/96†	24/96†	69/96†		11/96‡	7/96‡	58/96†
All		87/224†	66/224†	172/224†		59/224†	47/224†	156/224†		39/224	34/224	140/224†

*"Truthful" means the poll response matched the vote cast in a given period.

†Reject the null that voters voted randomly in favor the alternative that voters were more likely to cast votes that matched their poll responses at the 10% level of significance according to a binomial test (see Goldberg [1961]).

‡Reject the null that voters voted randomly in favor the alternative that voters were less likely to cast votes that matched their poll responses at the 10% level of significance according to a binomial test (see Goldberg [1961]). For this test, only plurality voting has a meaningful (zero or larger) critical value.

B. Polls as Election Predictors

P10	The poll rankings affect the rankings in the following election. Null: Poll rankings had no impact on the following election rankings. Alt: Poll rankings affected the probability that any particular election ranking will result.

Table X in the text gives a series of χ^2 statistics for testing the independence of the poll rankings and the following election outcomes. There are 13 possible poll or election rankings (including ties). We run (two-sided) χ^2 tests to determine whether there was a significant relationship between the poll ranking and the election ranking in a period. We find there was a significant relationship under plurality voting. The χ^2 statistics for approval voting and Borda rule are not significant. We also ask whether the poll ranking affected whether a given candidate won the following election. Again, there are 13 possible poll rankings, but only 4 possible election outcomes (the candidate wins outright, is in a two way tie, is in a three way tie or loses outright). Again, the χ^2 statistics under approval voting and Borda rule are not significant. The statistics for all three candidates are significant under Plurality Voting. ■

P11	Under Plurality voting with polls, whichever of Orange or Green was ahead in the poll won the following election. Null: Voters voted randomly conditional on the poll. Alt: Conditional on the poll, voters voted in a manner that increased the vote total of the candidate (of Orange or Green) who was ahead in the poll and, thus, increased this candidate's chances of wining the election.

Table A.II.IX shows the elections in which one candidate received significantly more votes that predicted by random voting according to the analytic density function of votes received by one candidate under the null of random voting. It also gives the candidate and tells whether that candidate was ahead in the preceding election or poll. We reject the null of random voting in favor of the alternative that Orange or Green got more votes 32 times under plurality voting with polls. The winning candidate was *always* the candidate (of Orange and Green) who was ahead in the poll. Further, Table X in the text shows that Orange being ahead of Green in the poll increases Orange's chances of winning the election and vice versa. ■

P12	Under Plurality voting with polls, whichever of Orange or Green was behind in the poll lost the following election. Null: Voters voted randomly conditional on the poll. Alt: Conditional on the poll, voters voted in a manner that decreased the vote total of the candidate (of Orange or Green) who was behind in the poll and, thus, decreased this candidate's chances of wining the election.

Table V in the text shows that we reject the null of random voting in favor of the alternative that Orange or Green got fewer votes 38 times in PWPS1. The losing candidate was *always* the candidate (of Orange and Green) who was behind in the poll. Further, Table X in the text shows that Orange being ahead of Green in the poll decreases Green's chances of winning the election and vice versa. ■

Table A.II.IX: Elections in Which We Reject* the Hypothesis that Voters Voted Randomly in Favor of the Alternative that Voters Voted in a Manner that Made Either the Orange or Green Candidate's Vote Total Higher

Session (Voting Rule)	Group	Period(s)	Candidate	Was this Candidate Behind in the Preceding Election or Poll?
AWOPS1 (Approval)	A	2,5	Green	Candidates Tied
		3	Green	Yes
	C	6	Green	Yes
	D	1	Green	N.A.
		5	Green	Candidates Tied
		2,6	Green	Yes
	E	1	Orange & Green	N.A.
	F	3	Orange	Candidates Tied
AWPS1 (Approval)	B	1	Orange	Yes
BWOPS1 (Borda)	A	1	Green	N.A.
		2	Green	Yes
	D	1	Green	N.A.
		2	Green	Yes
	F	6	Green	No
BWPS1 (Borda)	A	1	Green	Yes
	B	4	Green	No
	E	1	Orange	Candidates Tied
PWOPS1 (Plurality)	A	3-8	Green	Always
		5-8	Orange	Always
	B	2-8	Green	Always
	C	3-8	Orange	Always
	D	3-5,8	Green	Always
	E	1	Green	N.A.
	F	3-7	Green	Always
PWPS1 (Plurality)	A	3	Orange	Yes
		2,6,7	Green	Always
	B	3,4,8	Green	Always
		1,3,5,7	Orange	Always
	C	2,4,6,8	Green	Always
	D	1,3	Orange	Always
		5,6,8	Green	Always
	E	1,3,6,8	Orange	Always
		5	Green	Yes
	F	3,5,7	Orange	Always
		2,4,6,8	Green	Always

*One sided test at the 10% level of significance.

C. Polls as Individual Behavior Predictors

Table A.II.X shows that the favorite candidate/Blue poll rankings and "O" and "G" voter responses from AWPS1. We reject the null in favor of the alternative that "O" and "G" voters were more likely to cast a vote for their second favorite candidate if their favorite candidate was behind Blue in the poll. ■

P13 Under approval voting, "O" and "G" voters cast fewer approval votes for their second favorite candidate if their favorite candidate was ahead of Blue in the poll and more if their favorite candidate was behind Blue in the poll.

Null: The fraction of votes that "O" and "G" voters cast for their second favorite candidate was independent of the poll ranking between their favorite candidate and Blue.

Alt: "O" and "G" voters were more likely to cast votes for their second favorite candidate if their favorite candidate was behind Blue in the poll.

Table A.II.X: Favorite Candidate/Blue Poll Rank and Subsequent Votes Cast by "O" and "G" Voters in AWPS1

Votes Cast for Second Favorite Candidate	Favorite Candidate/Blue Poll Rank			χ^2 Stat. 2 d.f. (Prob $>\chi^2$)
	Blue Ahead	Blue Tied	Blue Behind	
0	54	26	99	8.6301
1	90	30	85	(0.013)

P14 Under Borda rule, "O" and "G" voters cast two votes for their second favorite candidate if their favorite candidate came in last in the poll and cast zero votes for their second favorite candidate if that candidate came in last in the poll.

Null: The fraction of times that "O" and "G" voters cast 0, 1 and 2 votes for their second favorite candidate was independent of whether their favorite candidate or Blue came in last in the poll.

Alt: "O" and "B" voters cast more 2 votes for their second favorite candidate if their favorite candidate came in last in the poll and more 0 votes for their second favorite candidate if Blue came in last in the poll.

Table A.II.XI shows whether a voter's favorite candidate or Blue came in absolutely last in the poll the voter's response from BWPS1. Thus, we cannot reject the null. ■

Table A.II.XI: Last Place Poll Finisher and Subsequent Votes Cast by "O" and "G" Voters in BWPS1

Votes Cast for Second Favorite Candidate	Poll Loser			χ^2 Stat. 4 d.f. (Prob $>\chi^2$)
	Blue	Neither	Favorite Candidate	
0	25	20	11	1.0964
1	118	105	69	(0.895)
2	17	11	8	

P15	Under Borda Rule, "B" voters cast more positive votes for which ever of Orange or Green was behind in the poll. Null: The fraction of positive votes that "B" voters cast for Orange and Green was independent of the Orange/Green poll ranking. Alt: "B" voters were more likely to cast positive votes for whichever of Orange or Green was behind in the poll.

Table A.II.XII shows the Orange/Green poll rankings and fraction of positive votes cast for Orange and Green by "B" voters in BWPS1. Thus, we reject the null in favor of the alternative that "B" voters were more likely to cast positive votes for whomever of Orange or Green that was behind in the poll. ■

Table A.II.XII: Orange/Green Poll Rank and Subsequent Votes Cast by "B" Voters in BWPS1

	Orange/Green Poll Rank			χ^2 Stat. 2 d.f. (Prob > χ^2)
Positive Votes Cast for:	Orange > Green	Orange = Green	Orange < Green	
Orange	52	12	97	9.0907
Green	60	12	53	(0.011)

P16	Under plurality voting, "O" and "G" voters cast their vote for their favorite candidate unless their favorite candidate came in absolutely last or tied with Green for last. Null: The fraction of votes that "O" and "G" voters cast for Orange and Green was independent of whether their favorite candidate came in last or tied with Blue for last in the poll. Alt: "O" and "G" voters was more likely to cast their vote for their second favorite candidate if their favorite candidate came in last or tied with Blue for last in the poll.

Table A.II.XIII shows the Favorite Candidate/Blue poll rankings and "O" and "G" voter responses from PWPS1. Thus, we reject the null in favor of the alternative that "O" and "G" voters are more likely to cast their vote for their second favorite candidate if their favorite candidate is last or tied with Blue for last in the poll. ■

Table A.II.XIII: Favorite Candidate Poll Rank and Subsequent Votes Cast by "O" and "G" Voters in PWPS1

	Favorite Candidate Poll Ranking			χ^2 Stat. 4 d.f. (Prob > χ^2)
Vote Cast	Favorite Cand. Last in Poll	Favorite Cand. Tied w/ Blue for Last in Poll	Other Rankings	
Favorite Cand.	26	8	205	184.6684
2nd Favorite Cand.	106	12	23	(0.000)
Blue	0	0	4	
Abstain	0	0	0	

III. Individual Behavior Propositions

Table A.II.XIV gives the number of vote vectors that voters cast which were consistent with sincere voting, other strategic voting and dominated vote vectors. It also shows when we accept or reject propositions about the frequency of these vote vectors under the null that voters cast random ballots.

A. Sincere Voting

P17 Voters always vote sincerely.

Null: Voters always cast vote vectors that ranked the candidates in the same weak order as their preferences.
Alt: Voters sometimes cast insincere votes.

Table A.II.XV shows the number of times we reject the null for individual voters. ■

Table A.II.XV: Number of Rejections of the Hypothesis that Voters Always Voted Sincerely (of 28 Voters)

Session	Voting Rule	All Periods/Groups	Period ≥ 3	Period ≥ 5	Groups C, D, E and F
AWOPS1	Approval	8	4	2	5
AWPS1	Approval	14	9	7	8
BWOPS1	Borda	26	23	18	20
BWPS1	Borda	22	17	15	13
PWOPS1	Plurality	28	26	24	20
PWPS1	Plurality	26	26	26	23

P18 Voters voted sincerely more often than predicted by random behavior.

Null: Voters cast random vote vectors.
Alt: Voters tended to cast more sincere votes than predicted by random voting.

Using the analytic distribution of votes under the null, we reject the null according to a binomial test for all voter types and all periods under approval voting and Borda rule. We reject the null for all "B" voters under plurality voting. We also reject the null for "O" voters in Periods 1, 2, 5 and 6 of PWOPS1; "G" voters in all periods of PWOPS1; "O" voters in Periods 1, 3, 4, 5 and 7 of PWPS1 and "G" voters in Periods 2, 4, 5, 6, 7 and 8 of PWPS1. See Table A.II.XIV. ■

Table A.II.XIV: Number of Sincere, Other Strategic and Dominated Voted Vectors′ Cast in Each Period (Out of 24 Vote Vectors Cast by Type "O" Voters, 24 Cast by Type "G" Voters and 36 cast by Type "B" Voters Each Period)

Session (Voting Rule)	Voter Type	Type of Vote	Qualifying Vote Vectors	Period 1	2	3	4	5	6	7	8
AWOPS1 (Approval)	"O"	Sincere	(1,0,0) (1,1,0)	24*	24*	24*	23*	23*	24*	24*	23*
		Dominated	Others	0†	0†	0†	1†	1†	0†	0†	1†
	"G"	Sincere	(0,1,0) (1,1,0)	23*	24*	24*	24*	24*	24*	24*	24*
		Dominated	Others	1†	0†	0†	0†	0†	0†	0†	0†
	"B"	Sincere	(0,0,1) (1,0,1) (0,1,1)	32*	36*	34*	35*	35*	35*	36*	36*
		Dominated	Others	4†	0†	2†	1†	1†	1†	0†	0†
AWPS1 (Approval)	"O"	Sincere	(1,0,0) (1,1,0)	21*	22*	23*	24*	23*	24*	24*	24*
		Dominated	Others	3†	2†	1†	0†	1†	0†	0†	0†
	"G"	Sincere	(0,1,0) (1,1,0)	22*	22*	22*	22*	22*	23*	23*	21*
		Dominated	Others	2†	2†	2†	2†	2†	1†	1†	3†
	"B"	Sincere	(0,0,1) (1,0,1) (0,1,1)	33*	34*	36*	36*	35*	35*	35*	36*
		Dominated	Others	3†	2†	0†	0†	1†	1†	1†	0†
BWOPS1 (Borda)	"O"	Sincere	(2,1,0)	19*	18*	14*	17*	20*	17*	20*	17*
		Strategic	(2,0,1) (1,2,0)	4	5	9	6	3	6	3	7
		Dominated	Others	1†	1†	1†	1†	1†	1†	1†	0†
	"G"	Sincere	(1,2,0)	20*	18*	14*	17*	16*	16*	16*	17*
		Strategic	(0,2,1) (2,1,0)	4	6	10	7	7	7	8	7
		Dominated	Others	0†	0†	0†	0†	1†	1†	0†	0†
	"B"	Sincere	(1,0,2) (0,1,2)	29*	26*	28*	30*	30*	31*	29*	31*
		Dominated	Others	7†	10†	8†	6†	6†	5†	7†	5†
BWPS1 (Borda)	"O"	Sincere	(2,1,0)	16*	18*	20*	19*	18*	20*	16*	18*
		Strategic	(2,0,1) (1,2,0)	7	6	4	4	6	3	7	5
		Dominated	Others	1†	0†	0†	1†	0†	1†	1†	1†
	"G"	Sincere	(1,2,0)	18*	17*	18*	15*	21*	20*	20*	17*
		Strategic	(0,2,1) (2,1,0)	5	5	5	8	3	3	4	7
		Dominated	Others	1†	2†	1†	1†	0†	1†	0†	0†
	"B"	Sincere	(1,0,2) (0,1,2)	33*	33*	35*	36*	34*	34*	34*	35*
		Dominated	Others	3†	3†	1†	0†	2†	2†	2†	1†
PWOPS1 (Plurality)	"O"	Sincere	(1,0,0)	15*	13*	10	9	12*	12*	11	10
		Strategic	(0,1,0)	7	9	14‡	15‡	12‡	12‡	12‡	14‡
		Dominated	(0,0,1)	2†	2†	0†	0†	0†	0†	1†	0†
	"G"	Sincere	(0,1,0)	17*	20*	19*	18*	16*	17*	17*	15*
		Strategic	(1,0,0)	6	3	4	5	7	6	6	8
		Dominated	(0,0,1)	1†	1†	1†	1†	1†	1†	1†	1†
	"B"	Sincere	(0,0,1)	31*	35*	33*	33*	27*	30*	31*	32*
		Dominated	(1,0,0) (0,1,0)	5†	1†	3†	3†	9†	6†	5†	4†
PWPS1 (Plurality)	"O"	Sincere	(1,0,0)	20*	10	19*	15*	14*	8	17*	11
		Strategic	(0,1,0)	2	14‡	5	9	10	16‡	7	13‡
		Dominated	(0,0,1)	2†	0†	0†	0†	0†	0†	0†	0†
	"G"	Sincere	(0,1,0)	8	22*	6	20*	16*	21*	13*	19*
		Strategic	(1,0,0)	14‡	2	18‡	4	8	3	11	5
		Dominated	(0,0,1)	2†	0†	0†	0†	0†	0†	0†	0†
	"B"	Sincere	(0,0,1)	36*	34*	35*	34*	32*	33*	34*	35*
		Dominated	(1,0,0) (0,1,0)	0†	2†	1†	2†	4†	3†	2†	1†

*Reject the hypothesis that voters voted randomly in favor of the alternative that voters tended to vote sincerely (according to a binomial test at the 10% level of significance). †Reject the hypothesis that voters voted randomly in favor of the alternative that voters tended not to cast dominated vote vectors (according to a binomial test at the 10% level of significance). ‡Reject the hypothesis that voters voted randomly in favor of the hypothesis that voters tend to vote using strategic, but not sincere, vote vectors (according to a binomial test at the 10% level of significance).

B. Dominated Voting

P19 Voters never cast dominated ballots.

Null: Voters never cast vote vectors that were strategically dominated for the stage game.
Alt: Voters sometimes cast dominated vote vectors.

Table A.II.XVI shows the number of times we reject the null for individual voters. ■

Table A.II.XVI: Number of Times We Reject the Hypothesis that Voters Never Cast Vote Vectors that Would be Strategically Dominated in the Stage Game (of 28 Voters)

Session	Voting Rule	All Periods/Groups	Period ≥ 3	Period ≥ 5	Groups C, D, E and F
AWOPS1	Approval	8	4	2	5
AWPS1	Approval	14	9	7	8
BWOPS1	Borda	15	10	8	9
BWPS1	Borda	11	6	5	4
PWOPS1	Plurality	17	13	12	11
PWPS1	Plurality	7	3	3	2

P20 Voters cast dominated ballots less often that predicted by random behavior.

Null: Voters cast random vote vectors.
Alt: Voters tended to cast fewer dominated votes than predicted by random voting.

Again, using the analytic distribution of vote vectors under the null, we reject the null according to binomial tests for all voter types in all periods of all sessions. ■

C. Strategic Voting

P21 When they had the opportunity to do so (Borda rule and plurality voting here), voters never cast strategic, but not sincere, vote vectors.

Null: Voters never cast vote vectors that were strategic, but not sincere.
Alt: Voters sometimes cast vote vectors that were strategic, but not sincere.

Table A.II.XVII shows the number of times we reject the null for individual voters under both Borda rule and plurality voting. ■

Table A.II.XVII: Number of Times We Reject the Hypothesis that Voters Never Cast Strategic, but Insincere, Vote Vectors

Session	Voting Rule	All Periods/Groups	Period ≥ 3	Period ≥ 5	Groups C, D, E and F
BWOPS1	Borda	24	22	17	16
BWPS1	Borda	19	17	15	13
PWOPS1	Plurality	24	21	19	15
PWPS1	Plurality	26	26	26	23

P22 Voters cast strategic, but not sincere, vote vectors more often than predicted by random behavior.

Null: Voters cast random vote vectors.
Alt: Voters tended to cast more strategic, but not sincere, votes than predicted by random voting.

Using the analytic distribution of vote vectors under the null, we reject the null according to binomial tests for "O" voters in Periods 3-8 of PWOPS1; "O" voters in Periods 2, 6 and 8 of PWPS1; and "G" voters in Periods 1 and 3 of PWPS1. ■

D. Equilibria Propositions

P23 Under approval voting, "O" and "G" voters cast fewer approval votes for their second favorite candidate if their favorite candidate was ahead of Blue in the previous election and more if their favorite candidate was behind Blue in the previous election.

Null: The fraction of votes that "O" and "G" voters cast for their second favorite candidate was independent of the previous election ranking of their favorite candidate and Blue.
Alt: "O" and "G" voters were more likely to cast a vote for their second favorite candidate if their favorite candidate was behind Blue in the previous election.

Table A.II.XVIII shows that Favorite Candidate/Blue rankings in the preceding election and "O" and "G" voter responses from AWOPS1 and AWPS1. We do not reject the null. Instead, the data are consistent with voters always thinking there will be a close three way race and casting votes for their second favorite candidate about 1/2 the time. ■

Table A.II.XVIII: Favorite Candidate/Blue Ranking in the Preceding Election and Votes Cast by "O" and "G" Voters in AWOPS1 and AWPS1

Session	Votes Cast for Second Favorite Candidate	Favorite Candidate/Blue Ranking in the Preceding Election			χ^2 Stat. 2 d.f. (Prob $>\chi^2$)
		Blue Ahead	Blue Tied	Blue Behind	
AWOPS1	0	17	38	96	1.0337
	1	23	38	124	(0.596)
AWPS1	0	34	49	78	1.6710
	1	46	55	74	(0.434)

P24 Under Borda rule, "O" and "G" voters cast two votes for their second favorite candidate if their favorite candidate came in last in the previous election and cast zero votes for their second favorite candidate if that candidate came in last in the previous election.

Null: The fraction of times that "O" and "G" voters cast 0, 1 and 2 votes for their second favorite candidate was independent of whether their favorite candidate or Blue came in last in the previous election.

Alt: "O" and "G" voters cast more 2 votes for their second favorite candidate if their favorite candidate came in last in the previous election and more 0 votes for their second favorite candidate if that candidate came in last in the previous election.

Table A.II.XIX shows whether a voter's favorite candidate or Blue came in absolutely last in the preceding election the voter's response from BWOPS1 and BWPS1. We cannot reject the null. ■

Table A.II.XIX: Last Place Finishers in the Preceding Election and Votes Cast by "O" and "G" Voters in BWOPS1 and BWPS1

Session	Votes Cast for Second Favorite Candidate	Preceding Election Loser: Blue	Neither	Favorite Candidate	χ^2 Stat. 4 d.f. (Prob $>\chi^2$)
BWOPS1	0	54	15	7	2.4297
	1	149	58	30	(0.657)
	2	13	7	3	
BWPS1	0	30	14	6	1.4924
	1	141	87	30	(0.828)
	2	13	11	4	

P25 Under Borda Rule, "B" voters cast more positive votes for which ever of Orange or Green was behind in the previous election.

Null: The fraction of positive votes that "B" voters cast for Orange and Green was independent of the Orange/Green ranking in the previous election.

Alt: "B" voters were more likely to cast positive votes for whichever of Orange or Green was behind in the previous election.

Table A.II.XX shows the Orange/Green rankings in the preceding election and fraction of positive votes cast for Orange and Green by "B" voters in BWOPS1 and BWPS1. We cannot reject the null. Instead, it appears that "B" voters may always assume a close, three-way race and cast positive votes for Orange and Green about one half of the time each. ■

Table A.II.XX: Orange/Green Ranking in the Preceding Election and Votes Cast by "B" Voters in BWOPS1 and BWPS1

Session	Votes Cast for:	Orange/Green Ranking in the Preceding Election: Orange > Green	Orange = Green	Orange < Green	χ^2 Stat. 2 d.f. (Prob $>\chi^2$)
BWOPS1	Orange	36	12	84	2.7558
	Green	42	12	62	(0.252)
BWPS1	Orange	45	27	63	1.7966
	Green	44	27	44	(0.407)

P26 Under plurality voting, "O" and "G" voters cast their vote for their favorite candidate unless their favorite candidate came in absolutely last or tied with Blue for last in the previous election.

Null: The fraction of votes that "O" and "G" voters cast for Orange and Green was independent of whether their favorite candidate came in last or tied with Blue for last in the previous election.

Alt: "O" and "G" voters were more likely to cast their vote for their second favorite candidate if their candidate came in last or tied with Blue for last in the previous election.

Table A.II.XXI shows the Orange/Green rankings in the preceding election and "O" and "G" voter responses from PWOPS1 and PWPS1. In PWOPS1, we reject the null in favor of the alternative that "O" and "G" voters were more likely to cast their vote for their second favorite candidate if their favorite candidate was last or tied with Blue for last in the preceding election. While we reject the null for PWPS1, it appears that voters were more likely to cast their vote for their favorite candidate if that candidate was last in the preceding election. This reflects the "alternating" phenomenon we observed. Recall from above that voters usually cast their vote for their second favorite candidate if their favorite candidate was last in the poll. ■

Table A.II.XXI: Favorite Candidate Rank in the Preceding Election and Subsequent Votes Cast by "O" and "G" Voters in PWOPS1 and PWPS1

		Favorite Candidate Ranking in Preceding Election			
Session	Vote Cast	Favorite Cand. Last	Favorite Cand. Tied w/ Blue for Last	Other Rankings	χ^2 Stat. (Prob $>\chi^2$)
PWOPS1	Favorite Cand.	23	5	171	208.5223
	2nd Favorite Cand.	111	7	9	4 d.f.
	Blue	9	0	0	(0.000)
	Abstain	0	0	0	
PWPS1	Favorite Cand.	113	2	96	11.6137
	2nd Favorite Cand.	43	2	80	2 d.f.
	Blue	0	0	0	(0.000)
	Abstain	0	0	0	