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PRICING TO SIGNAL PRODUCT LINE QUALITY*

by

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INTRODUCTION

Most firms offer a variety of products, that is, a product line. Further, these products seem to share a "quality" basis: while some firms offer high-quality products, others specialize in lower-quality goods. This basic dichotomy may reflect, for example, the decision to use a high- versus-low-quality input, for some input that is common to all products.

Eventually, consumers acquire experience with the firm's products and properly assess the general quality of the product line. This learning process, however, may take considerable time. While the product line is relatively young, consumers will be uninformed of product line quality and will make their purchase decisions based upon signals of quality that they observe. In particular, consumers will attempt to infer product line quality from the product line price schedule. This in turn implies that a firm with a high-quality product line must "distort" its prices so as to ensure that consumers correctly infer the quality of the product line.

The presence of pricing distortions raises several issues: Are the prices of all products distorted? Are prices distorted up or down? For which products are prices distorted most? A convincing resolution to these questions would have both practical and predictive content. The purpose here is to provide an equilibrium analysis of these issues for the case of a monopolist whose product line quality is either high or low.

The principle contribution of the paper is the identification of a simple and practical rule that characterizes the optimal high-quality product prices. The rule entails carrying out the following thought experiment. Suppose a high-quality monopolist "pretends" that its quality of products is known to be high but that its costs of production are higher than they truly are. In particular, let there be some constant, common for all products, such that the unit cost of producing a high-quality product for any product i rises by that constant times the difference between the true unit cost of high-and-low-quality production in market i. This is a simple notion of "higher costs" in the multiproduct context. Now, the described hypothetical problem is a standard, complete-information monopoly pricing problem. The key finding of this paper is that the prices which solve this standard problem are also the prices that a high-quality monopolist selects in the focal equilibrium of the original, incomplete-information game.

The implications of this rule are most sharp under the assumption that the demand for any product is not directly dependent upon the prices of other products in the product line. This assumption offers a useful benchmark, being appropriate if cross elasticities of demand are small. It is found that supra-monopoly prices are initially charged for all products in the high-quality product line. Thus, the price of every product is distorted upward. The price distortion is greatest for goods with inelastic demands. If prices eventually settle at the true monopoly prices, it follows that all prices should decline in a high-quality product line and that the absolute decline should be greatest for inelastic goods. The percentage decline in price, however, is independent of elasticities. Finally, an economy of scale to signaling is isolated, with the corresponding prediction that a high-quality monopolist will advertise its entire price schedule and/or "target" its products to particular consumer groups.

The conclusions developed here are consistent with various evidence that the marketing literature has reported on price-quality relationships. For example, a number of experimental studies indicate that consumers infer a higher quality from a higher price (Monroe [1973]). This inference is corroborated in several case studies. Products such as fountain-pen ink and car wax (Gabor and Granger [1965]), as well as vodka, skis, and television sets (Buzzell, Nourse, Matthews, and Levitt [1972]) have been successfully introduced at high prices to denote high quality. Empirical data is also supportive. Examining *Consumer Reports* data, researchers have documented positive price-quality rank-order correlations for many products, and especially for consumer durables (Gerstner [1985], Tellis and Wernerfelt [1987]). Finally, Curry and Riesz [1988] provide a recent longitudinal analysis of *Consumer Reports* data for consumer durables which indicates declining trends in real prices and the correlation between price and quality. These results are consistent with the hypothesis that a new good is introduced at a high, supra-monopoly price. Later, as the good matures and information about its product quality is more widely available, price drops to a more profitable level and the price-quality correlation weakens. Thus, as Bagwell and Riordan [forthcoming] discuss further, the marketing literature provides evidence that is broadly supportive of the prediction of high and declining prices for high-quality products.

This paper is also related to a large economics literature. The idea that cost effects can cause high prices to signal high quality has been developed previously by Bagwell [1990], Bagwell and Riordan, Davis [1990], Fertig [1988], Milgrom and Roberts [1986], and Ramey [1986], among others. All of these papers, however, focus on the somewhat unlikely case of a single-product monopoly. The issue of which prices to distort most, for example, does not arise in this context. It is worth noting in this regard that the marketing data discussed above also does not compare pricing patterns within a particular firm's product line.

The notion of solving multi-dimensional signaling problems by pretending that information is complete and costs are distorted has been explored previously in the context of a limit pricing game with two signals by Bagwell and Ramey [1987]. This approach has since been used in a variety of other games with two signals (Bagwell and Ramey [1990], Chu [1989], and Rogoff [1990]). Ramey [1988] develops this technique further for an abstract game with a continuum of types and any finite number of signals. Recent work by Srinivasan [1990] is also related. He examines the limit pricing schedule that is selected by a multimarket incumbent.

The paper is organized in five sections. Section 2 defines the basic model, and section 3 offers a formal characterization of equilibria. The technique of eliminating dominated strategies (Milgrom and Roberts) and equilibrium dominated strategies (Cho and Kreps [1987]) is used to select a focal equilibrium. Empirical implications are described in section 4, and concluding thoughts are found in section 5. Most of the proofs are contained in an appendix section.

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THE MODEL

Consider a monopolist introducing a new product line of N goods. Product line quality is represented by a single dimensional parameter t, where t is either low (L) or high (H) with $H > L > 0$. Thus, the monopolist either introduces a high-quality or a low-quality product line.

Let $c^i(t)$ represent the constant unit cost of producing product i, $i=1,\dots,N$, when quality is type t, $t \in \{L,H\}$. Assume $c^i(H) > c^i(L) \geq 0$ in order to capture the fact that high-quality goods are typically more costly to produce. It is sometimes useful to denote the vector of unit costs, which is given by $c(t) \equiv (c^1(t),\dots,c^N(t))$.

The monopolist chooses a price, P^i, for each product i. These choices are summarized by the vector $P \equiv (P^1,\dots,P^N)$. Consumers observe the entire price vector and form some belief as to the probability that product line quality is high. Let $b^\circ \in (0,1)$ be the prior belief and $b(P) \in [0,1]$ be the posterior belief that $t = H$. Having seen the vector of prices and formed some belief b, consumers next bring forth a demand for each product i. Let $D^i(P,b)$ be the demand for product i, and assume that $D^i(P,b)$ is continuous, nonnegative, and decreasing in P^i. Observe that the demand for product i may depend on the prices of all products, reflecting the fact that some of the monopolist's products may be substitutes or complements.

The firm's profit function is now easily expressed. The profit in market i is given by

$$\Pi^i(P,c^i(t),b) \equiv (P^i - c^i(t))D^i(P,b) \quad .$$

The total profit to the monopolist is thus

$$\Pi(P,c(t),b) \equiv \sum_{i=1}^{N} \Pi^i(P,c^i(t),b) \quad .$$

This function is continuous and is assumed to possess a unique maximizer, $P(c(t),b)$, for every t and b. Assume $P(c(H),b) \neq P(c(L),b)$ and $\Pi(P(c(t),b),c(t),b) > 0$ for every t and b. Since $P(c(t),b)$ is a vector, $P^i(c(t),b)$ is used to represent the maximizing price in market i.

To interpret the framework, one can imagine a monopolist that uses either a high-quality or a low-quality input. This input is common to all the monopolist's products. For example, the input may correspond to the quality of paint for a line of automobiles, the quality of fabric for a clothing line, or the quality of transit refrigeration for a distributed line of food or beverage products. Other examples are easy to imagine. Consumers are assumed unable to observe input quality. Thus, consumers must experience the good in order to know quality with certainty; quality cannot be determined by inspection.

Consumers do have indirect information about product line quality, however. In particular, they have prior and posterior beliefs about quality. Following the Harsanyi tradition, it is assumed that the prior belief corresponds to the probability that "nature" picks t = H.[1] The consumers' posterior belief depends upon the observed price vector P. It is because of this process of belief formation that the high-quality monopolist may practice distortionary pricing.

The next task is to define a sequential equilibrium (Kreps and Wilson [1982]) for the game. The triplet $\{\hat{P}(L), \hat{P}(H), \hat{b}(P)\}$ forms an *equilibrium* if and only if

(E1) Sequential Rationality

$$\hat{P}(t) \in \underset{P}{argmax} \; \Pi(P, c(t), \hat{b}(P)), \; t = L, H$$

(E2) Bayes-Consistency

$$\hat{P}(L) = \hat{P}(H) \text{ implies } \hat{b}(\hat{P}(L)) = b^{\circ}$$

$$\hat{P}(L) \neq \hat{P}(H) \text{ implies } \hat{b}(\hat{P}(L)) = 0, \hat{b}(\hat{P}(H)) = 1 \quad .$$

Thus, whatever its type, the monopolist selects its vector of prices as a best response to the belief function of consumers. Consumers' beliefs are not completely arbitrary, however, since they must agree with Bayes' rule on the equilibrium path (i.e., for equilibrium price vectors). For prices off the equilibrium path, where $P \notin \{\hat{P}(L), \hat{P}(H)\}$, no restriction on beliefs is placed. As is well known, this freedom in specifying off-equilibrium-path beliefs is a source of multiple equilibria.

Following Milgrom and Roberts, structure is placed on out-of-equilibrium beliefs with the requirement that consumers never believe that a dominated strategy has been selected. In the present context, a price vector P is said to be *dominated* for a monopolist of type t by an alternative vector $\tilde{P}$ if and only if

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[1] As discussed by Bagwell [1990], the prior can be endogenized with the assumption that the monopolist has private information about the relative future profits of high- and low-quality product lines, respectively. From this perspective, b^{o} is the probability that the monopolist receives information that causes a high-quality choice to be optimal.

$$\max_b \Pi(P, c(t), b) < \min_b \Pi(\tilde{P}, c(t), b) \quad .$$

It seems reasonable that a consumer should never believe that a monopolist of type t would select a vector such as P, since the alternative selection $\tilde{P}$ is *always* superior.

To this end, an *undominated equilibrium* is defined by a triplet, $\{\hat{P}(L), \hat{P}(H), \hat{b}(P)\}$, satisfying (E1), (E2), and

(E3) Elimination of Dominated Strategies

$$\hat{b}(P) = 1(0) \text{ if } P \text{ is dominated for } L(H) \text{ and not } H(L) \quad .$$

It is understood here that P is dominated for a monopolist of type t if and only if there exists some $\tilde{P}$ which dominates P for the monopolist of type t.

It is useful at this point to introduce some additional structure to the model so that the set of dominated strategies may be easily characterized. In particular, assume that $\Pi(P, c(t), b)$ increases in b whenever $\Pi(P, c(t), b) > 0$. This captures the simple notion that more optimistic beliefs should increase demand and thus profits, provided prices are "on average" above costs.[2] It is now direct to show the following:

LEMMA 1: P is dominated for a monopolist of type t if and only if

$$\Pi(P, c(t), 1) < \Pi(P(c(t), 0), c(t), 0) \quad .$$

[2]This condition holds, e.g., if b enters into the demand function in an exponential fashion. The condition is also satisfied when costs are fairly similar across markets and the increase in demand in any market i when b increases is greater the higher is P^i.

Thus, a price vector P is dominated for a monopolist of type t precisely when it is dominated by the particular price vector $P(c(t), 0)$. A formal proof is offered in the Appendix.

An alternative refinement of off-equilibrium beliefs has been suggested by Cho and Kreps. Fix a particular equilibrium satisfying (E1) and (E2). Let $\hat{\Pi}(t)$ denote the equilibrium profit to the monopolist of type t, that is, $\hat{\Pi}(t) \equiv \Pi(\hat{P}(t), c(t), \hat{b}(\hat{P}(t)))$. Consider now some deviant price vector P, where $P \neq \hat{P}(t)$ for any t. This deviation is said to be *equilibrium dominated* for monopolist of type t if and only if

$$\max_{b} \Pi(P, c(t), b) < \hat{\Pi}(t) \quad .$$

The idea is that a monopolist of type t would never select a price vector such as P, since by following the equilibrium (i.e., selecting $\hat{P}(t)$) the monopolist would have certainly done better.

This gives rise to what Cho and Kreps refer to as an *intuitive equilibrium*, which in the present context is a triplet, $\{\hat{P}(L), \hat{P}(H), \hat{b}(P)\}$ satisfying (E1), (E2), and

(E4) <u>Elimination of Equilibrium Dominated Strategies</u>

$\hat{b}(P) = 1(0)$ if P is equilibrium dominated for L(H) but not for H(L).

A simple characterization of equilibrium dominated strategies now follows:

LEMMA 2: P is equilibrium dominated for a monopolist of type t if and only if

$$\Pi(P, c(t), 1) < \hat{\Pi}(t) \quad .$$

Again, a formal proof is direct and offered in the Appendix.

3
CHARACTERIZATION OF EQUILIBRIA

A. *Characterization of Undominated Separating Equilibria*

A *separating equilibrium* occurs if $\hat{P}(H) \neq \hat{P}(L)$. In this event, the product line price vector signals quality, in the sense that consumers infer quality from the product line prices.

Consider now the possibility that $\Pi(P(c(H), 1), c(L), 1) < \Pi(P(c(L), 0), c(L), 0)$. Using Lemma 1, it follows that the full-information price for the high-quality monopolist is dominated for the low-quality monopolist. Thus, (E3) gives $\hat{b}(P(c(H), 1)) = 1$, and so it must be that $\hat{P}(H) = P(c(H), 1)$. In other words, when this inequality holds, the high-quality monopolist is able to signal its quality without distorting its price. To avoid this rather trivial case, it is assumed henceforth that $\Pi(P(c(H), 1), c(L), 1) > \Pi(P(c(L), 0), c(L), 0)$.

The characterization of the low-quality monopolist's pricing strategy in a separating equilibrium is straightforward. In particular, suppose $\hat{P}(L) \neq P(c(L), 0)$. Then

$$\Pi\big(P(c(L), 0), c(L), \hat{b}(P(c(L), 0))\big) \geq \Pi(P(c(L), 0), c(L), 0) > \Pi(\hat{P}(L), c(L), 0)$$

contradicting the original supposition. This gives:

LEMMA 3: In any separating equilibrium, $\hat{P}(L) = P(c(L), 0)$.

That is, in any separating equilibrium, the low-quality monopolist simply charges its complete-information monopoly price vector.

In characterizing the high-quality monopolist's pricing, it will be useful to have the high-quality monopolist "pretend" its costs are higher than they truly are. But, since a vector of costs exists, there are many ways in which "higher costs" might be defined. A particularly simple family of cost vectors can be defined by an index γ, where γ ranges over positive real numbers, and any continuous, increasing function $k(\gamma)$ that satisfies $k(H) = 0 > k(L) = -1$.[3] The family is then represented by

$$c^i(\gamma) \equiv c^i(H) + k(\gamma)(c^i(H) - c^i(L))$$

The vector of costs is thus written $c(\gamma)$. Hence, when $\gamma > H$, the cost in any market i rises above $c^i(H)$ by an amount which is proportional to the "cost of quality improvement" in market i. This family offers a simple way of measuring the notion of higher costs, precisely because the multiplying constant, $k(\gamma)$, is the same for every market i.

Given this notation, the following definitions are straightforward to interpret:

$$\Pi^i(P, c^i(\gamma), b) \equiv (P^i - c^i(\gamma))D^i(P,b), \qquad \Pi(P, c(\gamma), b) \equiv \sum_{i=1}^{N} \Pi^i(P, c^i(\gamma), b)$$

Similarly, $P(c(\gamma), b)$ is defined as the unique maximizer of $\Pi(P, c(\gamma), b)$, with $P^i(c(\gamma), b)$ being the ith component of $P(c(\gamma), b)$. Assume that $P(c(\gamma^1), b) \neq P(c(\gamma^2), b)$ whenever $\gamma^1 \neq \gamma^2$ and that $P(c(\gamma), b)$ is continuous in γ and b. Finally, it is convenient to assume that $\Pi(P(c(\gamma),1), c(\gamma), 1) > 0$ over the relevant range of γ's.[4]

The next lemma provides a connection between prices and profits.

LEMMA 4: $\Pi(P(c(\gamma), 1),\ c(t), 1)$ decreases in γ for $\gamma > t$.

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[3]As an example, consider $k(\gamma) = (\gamma - H)/(H-L)$.

[4]Formally, this condition need hold only for $\gamma \leq \gamma^\circ$, where $\gamma^\circ > H$ is defined below in (1).

Lemma 4 is proved in the Appendix. The lemma simply indicates that the profit when quality is believed to be high to a monopolist with true costs c(t) of pricing as if costs were c(γ) diminishes as γ increases above t (i.e., as "fictional" costs rise).

One last assumption is needed before the characterization of the high-quality monopolist's separating price vector can be presented. In particular, assume there exists $\gamma' > H$ such that $\Pi(P(c(\gamma'), 1), c(L), 1) < \Pi(P(c(L), 0), c(L), 0)$. This assumption ensures that there exists some γ' sufficiently high such that the low-quality firm would not choose to select the price vector $P(c(\gamma'), 1)$ even if in so doing it were thought to have a high-quality product line. That is, this assumption guarantees a γ' such that $P(c(\gamma), 1)$ is dominated for the low-quality monopolist for all $\gamma \geq \gamma'$.

With the structure in place, the following theorem may now be stated.

THEOREM 1: There exists at most one undominated separating equilibrium outcome, and in it $\hat{P}(H) = P(c(\gamma^{\circ}), 1)$ **for some** $\gamma^{\circ} > H$.

Before proving the theorem, it is well to remark on its implications. The theorem indicates that the high-quality price vector can be found using an easy thought experiment. In particular, suppose it were known that the high-quality monopolist sold a high-quality product line and suppose also that the high-quality monopolist's costs increased to some new cost vector $c(\gamma^{\circ}, 1)$ where $\gamma^{\circ} > H$. How should the monopolist price its product line? This hypothetical problem is easily solved, as it simply entails finding a complete-information monopoly price vector.

Remarkably, the answer to this hypothetical problem is also the answer to the significantly more complex problem of pricing to signal product line quality.

The proof of Theorem 1 can be executed in three steps. First, the existence of γ° must be established. By assumption,

$$\Pi(P(c(H),1),\ c(L),1) > \Pi(P(c(L),0),\ c(L),0)$$

and

$$\Pi(P(c(\gamma'),1),\ c(L),1) < \Pi(P(c(L),0),\ c(L),0)$$

for some $\gamma' > H$. Since all functions are continuous, Lemma 4 implies the existence and uniqueness of $\gamma^\circ > H$ satisfying

(1) $\quad \Pi(P(c(\gamma^\circ),1),\ c(L),\ 1) = \Pi(P(c(L),0),\ c(L),0)$

This completes the first step.

The second step is to show that $P(c(\gamma^\circ),1)$ uniquely solves the following program:

$$\max_P \Pi(P,c(H),1)$$

$$\text{subject to } \Pi(P,c(L),1) \le \Pi(P(c(L),0),\ c(L),0) \quad .$$

For assume to the contrary that P_1 exists with $P_1 \neq P_0$, where $P_0 \equiv P(c(\gamma^\circ),1)$, such that:

(2) $\quad \Pi(P_0,c(L),1) = \Pi(P(c(L),0),\ c(L),0) \ge \Pi(P_1,c(L),1)$

(3) $\quad \Pi(P_1,c(H),1) \ge \Pi(P_0,c(H),1)$

Recall also that

(4) $\quad \Pi(P_0,c(\gamma^\circ),1) > \Pi(P_1,c(\gamma^\circ),1)$

must hold. Adding (2) and (3) gives

(5) $$\sum_{i=1}^{N}\left(c^i(H) - c^i(L)\right)\left(D^i(P_0,1) - D^i(P_1,1)\right) \geq 0 \quad .$$

Note that (3) may be rewritten as

(6) $$\sum_{i=1}^{N}\left[P_1^i D^i(P_1,1) - P_0^i D^i(P_0,1) + c^i(H)\left(D^i(P_0,1) - D^i(P_1,1)\right)\right] \geq 0 \quad .$$

Next, using $\gamma^\circ > H$ and (5), it follows that

$$\sum_{i=1}^{N}\left(c^i(\gamma^\circ) - c^i(H)\right)\left(D^i(P_0,1) - D^i(P_1,1)\right) = k(\gamma^\circ)\sum_{i=1}^{N}\left(c^i(H) - c^i(L)\right)\left(D^i(P_0,1) - D^i(P_1,1)\right) \geq 0 \quad .$$

Thus, using (6), it must be that

$$\sum_{i=1}^{N}\left[P_1^i D^i(P_1,1) - P_0^i D^i(P_0,1) + c^i(\gamma^\circ)\left(D^i(P_0,1) - D^i(P_1,1)\right)\right] \geq 0 \quad ,$$

or equivalently

$$\Pi\left(P_1, c(\gamma^\circ), 1\right) \geq \Pi\left(P_0, c(\gamma^\circ), 1\right) \quad .$$

But this contradicts (4).

The final step is to show that $\hat{P}(H) = P_0$ in an undominated separating equilibrium. For suppose not. Then, since $\hat{P}(H)$ must satisfy the program constraint (lest the price be mimicked), the second step above implies that

$$\Pi\left(P_0, c(H), 1\right) > \Pi\left(\hat{P}(H), c(H), 1\right) \quad .$$

But by continuity and Lemma 4, for ϵ small and positive,

(7) $$\Pi\left(P\left(c(\gamma^\circ+\epsilon), 1\right),\ c(H), 1\right) > \Pi\left(\hat{P}(H), c(H), 1\right)$$

and

$$\Pi\left(P\left(c(\gamma^\circ+\epsilon), 1\right),\ c(L), 1\right) < \Pi\left(P_0, c(L), 1\right) = \Pi\left(P(c(L),0),\ c(L), 0\right) \quad .$$

The latter inequality implies that $P(c(\gamma^\circ+\epsilon),1)$ is dominated for the low-quality monopolist. Next, since $\hat{P}(H)$ is the high-quality equilibrium price vector,

$$\Pi(\hat{P}(H),c(H),1) \geq \min_{b} \Pi(P(c(H),0),\ c(H),b) = \Pi(P(c(H),0),\ c(H),0)$$

is necessary for equilibrium (lest a deviation occur to $P(c(H),0)$). This means that $\hat{P}(H)$ is not dominated for the high-quality monopolist by Lemma 1. Using (7), $P(c(\gamma^\circ+\epsilon),1)$ is also not dominated for the high-quality firm. (E3) then implies that $\hat{b}(P(c(\gamma^\circ+\epsilon),1)) = 1$. But by (7) the high-quality monopolist then deviates to $P(c(\gamma^\circ+\epsilon),1)$, which completes the proof.

B. Existence of the Undominated Separating Equilibrium

The next task is to determine conditions under which the unique undominated separating equilibrium outcome described above exists. While the proof makes clear that the price vector $P(c(\gamma^\circ),1)$ is the "most efficient" or profitable separating vector for the high-quality monopolist, there remains the issue of whether the high-quality monopolist is in fact willing to distort its prices sufficiently to separate. In particular, it must be established that

(8) $\Pi(P(c(\gamma^\circ),1),c(H),1) > \Pi(P(c(H),0),c(H),0)$

lest the high-quality monopolist deviate to the price vector P(c(H),0) (even in the case where beliefs are as pessimistic as possible).

The traditional approach in signaling models is to impose a "single crossing property" assumption which requires that higher prices are more desirable for a high-quality monopolist when traded against belief changes than for a low-quality monopolist. Such an assumption ensures that the "strong" type, the high-quality monopolist in the current context, is willing to perform the necessary signaling distortion; i.e., the assumption implies that an inequality such as represented in (8) is satisfied. Fortunately, as shown in the Appendix, (8) is satisfied in the current model without imposing any additional structure on the model. In fact, it is proved in the Appendix that the following theorem holds.

THEOREM 2: There exists a unique undominated separating equilibrium outcome.

Together, Theorems 1 and 2 characterize the unique undominated separating equilibrium outcome.

C. *Existence of Pooling Equilibria*

A *pooling equilibrium* arises when $\hat{P}(H) = \hat{P}(L)$; in this case, prices are "pooled" and consumers are unable to infer the product line quality. It is established below, however, that pooling equilibria fail to be refined equilibria in the current model.

A first issue is whether pooling equilibria exist which are also undominated.

THEOREM 3: There exists $b^* \in (0,1)$ such that, if $b^\circ < b^*$, then there does not exist an undominated pooling equilibrium.

The proof of this theorem is found in the Appendix. The simple intuition is that a high-quality monopolist will not accept a pooling equilibrium if pessimistic beliefs reduce pooling profits sufficiently; the firm would be better off to ensure separation with a deviation that would never be attractive to a low-quality monopolist. Specifically, the high quality firm will deviate to $P(c(\gamma^\circ + \epsilon), 1)$, a price vector which is dominated for a low-quality monopolist, unless pooling profits are sufficiently large.

Pooling equilibria are ruled out more generally using the stronger refinement suggested by Cho and Kreps. In fact, the following theorem is proved in the Appendix.

THEOREM 4: There does not exist an intuitive pooling equilibrium.

The idea is that, were a pooling equilibrium to exist, the high-quality monopolist could always find a deviant price vector which could not possibly improve upon equilibrium profit for the low-quality monopolist, even though the deviant price vector need not represent a dominated strategy for this firm, and which could improve upon equilibrium profit for the high-quality monopolist, if consumers believe that the product line is high quality upon observing the deviation. As demonstrated in the Appendix, the deviant price vector is characterized in terms of a "cost-increasing" price distortion, being represented by $P(c(\tilde{\gamma}), 1)$ for some $\tilde{\gamma} \in (L, \gamma^\circ]$.

Together, Theorems 3 and 4 suggest that pooling equilibria are not focal. These equilibria fail to be undominated unless the prior probability of a high-quality product line is large; further, pooling equilibria are never intuitive. The focal equilibrium is the separating equilibrium described in Theorem 1. As discussed in Theorem 2, this equilibrium exists and corresponds to the unique undominated separating equilibrium outcome. In addition, as the next theorem reports, this equilibrium also gives the unique intuitive separating equilibrium outcome.

THEOREM 5: There exists a unique intuitive separating equilibrium outcome, and in it $\hat{P}(H) = P(c(\gamma^\circ), 1)$, where $\gamma^\circ > H$ is defined by (1).

This theorem is proved in the Appendix. Along with Theorem 4, Theorem 5 establishes that the featured separating equilibrium is indeed focal, as it gives the only intuitive equilibrium outcome.

4
IMPLICATIONS-OF OPTIMAL PRICING

The previous section suggests that the plausible equilibrium is a separating equilibrium in which the high-quality monopolist acts as if its quality were known but its costs were higher. Moreover, the sense in which costs are higher is extremely simple—for any product i, the high-quality monopolist should act as if costs increase by some multiple of the cost of quality improvement in market i, where this multiplying constant is the same for all i. This provides a potentially practical algorithm with which a firm could select optimal prices for a new product line.

In this section, some restrictions are placed on the demand functions so that empirical predictions can be made. In particular, it is assumed that demands are independent, in that the demand for product i depends only on the price of product i and the belief which consumers hold as to product line quality. Thus, the price of other products in the product line enters into the demand for product i only indirectly, through the belief function. This setup allows strong predictions as to the nature of price distortions, and may be thought of as an approximation to the more general scenario in which the monopolist sells products with cross elasticities that are nonzero but small in magnitude. This framework is also useful as a means of illustrating a new motivation for price advertising, which is discussed at the end of this section.

A. Distort All or Some Prices?

A first question that arises is whether the high-quality monopolist should distort the price of every product or only some products. Let the profits now be written

$$\Pi^i(P^i, c^i(\gamma), b) \equiv (P^i - c^i(\gamma))D^i(P^i, b), \quad \Pi(P, c(\gamma), b) \equiv \sum_{i=1}^{N} \Pi^i(P^i, c^i(\gamma), b)$$

Since demands are separable, the monopoly price in market i may be written $P^i(c^i(\gamma), b)$. Using Theorem 1, it then follows that $\hat{P}^i(H) = P^i(c^i(\gamma^\circ), 1)$, where $\gamma^\circ > H$ is defined by (1). Thus, to determine if the price in market i is distorted, $P^i(c^i(H), 1)$ and $P^i(c^i(\gamma^\circ), 1)$ must be compared.

But it is a direct matter to show:

Lemma 5 When demands are independent, $P^i(c^i(\gamma), 1)$ increases in γ.

The proof of this lemma is in the Appendix. It follows that $P^i(c^i(\gamma^\circ), 1) > P^i(c^i(H), 1)$ since $\gamma^\circ > H$. Thus, the monopolist with a high-quality product line distorts *each* price. Furthermore, high prices are used to signal high-quality; that is, supra-monopoly prices are selected for each product.

The fact that high prices are used to signal high quality is similar to the findings of Bagwell and Riordan, who investigate a model with N=1 and a linear demand structure. The key intuition is that higher prices restrict demand and are consequently more attractive to higher-cost, higher-quality firms.

There is also a simple logic underlying the result that each price is distorted.[5] Suppose to the contrary that the price in some market i is distorted $(P^i(c^i(\gamma^\circ), 1) > P^i(c^i(H), 1))$ but that the price in some market j is not $(P^j(c^j(\gamma^\circ), 1) = P^j(c^j(H), 1))$. Now, the second step in the proof of

[5]A related result is found by Srinivasan [1990], in the context of a multimarket limit pricing game.

Theorem 1 indicates that the focal separating equilibrium entails the high-quality monopolist maximizing its profits over price vectors which the low-quality monopolist would not choose to mimic. Consider, then, a deviation that leaves the profits associated with mimicry constant. In particular, there will exist small positive numbers, σ^i and σ^j, such that the low-quality monopolist is indifferent between mimicking the original price vector and the deviant price vector in which the price in market i is $P^i(c^i(\gamma^\circ-\sigma^i),1)$ and the price in market j is $P^j(c^j(H+\sigma^j),1)$, with all other prices unchanged. Reasoning as in Lemma 4, the price change in market i increases the profit from mimicry while the price change in market j has an opposite and offsetting effect. The high-quality monopolist also views the change in price in market i favorably, but, for the high-quality firm, the price change in market j has no first order effect on profits since its profits are maximized in market j at $P^j(c^j(H),1)$. It follows that high-quality profits are higher at the deviant price vector and thus that all prices must be distorted.

This result generates the prediction that the (real) price of *all* products in the product line should eventually decline. To see why, suppose that the fraction of consumers who know quality, perhaps from previous experience or published quality reviews, increases as the markets mature. The cost to a low-quality monopolist of mimicking high prices then increases through time, since there are fewer uninformed consumers to "fool" and many informed consumers who could be more profitably sold to at the price vector $P(c(L),0)$. As Bagwell and Riordan demonstrate, once the fraction of informed consumers get sufficiently large, the price vector $P(c(H),1)$ will not be mimicked. Thus, while the short-run, high-quality price vector is $P(c(\gamma^\circ),1)$, the long-run, high-quality price vector is $P(c(H),1)$, implying that the prices of all products eventually decline.

B. *Price Distortions and Price Elasticities*

A next question is whether more or less elastic products are distorted most. This will indicate whether the eventual decline in price is greatest for elastic or inelastic goods.

To deal with this question, it is assumed for the moment that demands are CES, where $\epsilon^i > 1$ is the price elasticity of demand in market i when quality is believed to be high. As is well known, in this case

$$P^i(c^i(\gamma),1) = \frac{c^i(\gamma)\epsilon^i}{\epsilon^i - 1}$$

Several observations are now at hand. Observe first that the distortion in market i is given by

$$P^i(c^i(\gamma^o),1) - P^i(c^i(H),1) = k(\gamma^o)(c^i(H) - c^i(L))\frac{\epsilon^i}{\epsilon^i - 1} \quad .$$

Thus, for a given ϵ^i, the absolute distortion is greatest in markets for which the cost of quality improvement is greatest.[6] Equivalently, when the cost of quality improvement is great, the eventual decline in price is great.

Next, observe that

$$\frac{d}{d\epsilon^i}\left[P^i(c^i(\gamma^o),1) - P^i(c^i(H),1)\right] = k(\gamma^o)(c^i(H) - c^i(L))\frac{(-1)}{(\epsilon^i - 1)^2} < 0 \quad .$$

It is the inelastic goods which are distorted most. Consequently, the eventual reduction in price should be greatest for inelastic goods.

Finally, note that

[6] Note that γ^o is held fixed throughout the comparative statics exercises. The idea is that, for some given demands and costs, there will exist some γ^o. Distortions can then be compared for the given γ^o across the different demands and costs. Thus, the results presented here do not capture the effect of a *change* in one product's costs or elasticities, since such a change in primitives might also change γ^o.

$$\frac{P^i(c^i(\gamma^o), 1) - P^i(c^i(H), 1)}{P^i(c^i(H), 1)} = \frac{k(\gamma^o)(c^i(H) - c^i(L))}{c^i(H)}$$

Thus, the eventual percentage decline in price is independent of price elasticities. Rather, the percentage price distortion in market i is a constant multiple of the percentage cost reduction associated with low-quality production in market i. As long as low-quality production represents a percentage savings that is similar across products, the percentage price reduction should be the same for all products.

In summary, the model admits a variety of predictions when demands are independent. The general premise that high-quality products are sold initially at supra-monopoly prices and then later at monopoly prices is consistent with the empirical findings of Curry and Riesz. Interesting future work might examine firm-specific data to follow the product line pricing dynamics of a particular firm. Do all prices decline? It also would be interesting to categorize products by price elasticity and then to compare the sizes of price reduction.

C. *A Signaling Role for Price Advertising and "Consumer Targeting"*

This section concludes with a brief demonstration of a profit-enhancing role for extensive price observability. Consider two scenarios. In scenario A, consumers of product i observe only the price of product i. Thus, in this case, the price of product j does not affect the demand for product i in any direct or indirect fashion. In scenario B, consumers have independent demands and observe the full product line price vector. Here, as above, the price of product j has an indirect effect on the demand for product i. Scenario A is associated with an absence of general price information, while scenario B corresponds to a situation in which the monopolist ensures that its entire product line price schedule is known to consumers in each market i.

In scenario A, the signaling problem i is completely separable across markets. Thus, in any one market i, Theorem 1 with N=1 indicates that the high-quality separating price in scenario A is

$$\hat{P}_A^i(H) = P^i(c^i(\gamma^i), 1) \quad ,$$

where $\gamma^i > H$ solves

$$\Pi^i(P^i(c^i(\gamma^i),1),\ c^i(L),1) = \Pi^i(P^i(c^i(L),0),\ c^i(L),0)$$

Now, in general, γ^i will depend on $c^i(L)$ and other parameters (e.g., ϵ^i if demand is CES). It follows that γ^i typically will not equal γ^j, except in the case of identical markets. Also, the second step of the proof to Theorem 1 ensures that $\hat{P}_A(H)$, the scenario A high-quality price vector, uniquely solves the following program:

$$(9) \qquad \max_{P} \sum_{i=1}^{N} \Pi^i(P^i, c^i(H), 1)$$

subject to $\Pi^i(P^i, c^i(L), 1) \leq \Pi^i(P^i(c^i(L),0), c^i(L), 0)$

for all i

Thus, $\hat{P}_A(H)$ is the efficient mode of separation when prices are only "locally" observable and separation must occur on a market-by-market basis.

Next, consider scenario B. This is simply the problem analyzed above, and so Theorem 1 indicates that

$$\hat{P}_B^i(H) = P^i(c^i(\gamma^{\circ}), 1)$$

where $\gamma^{\circ} > H$ solves

$$\sum_{i=1}^{N} \Pi^i(P^i(c^i(\gamma^{\circ}),1),\ c^i(L),1) = \sum_{i=1}^{N} \Pi^i(P^i(c^i(L),0),\ c^i(L),0) \quad .$$

Further, using the second step once more, the vector $\hat{P}_B(H)$ must uniquely solve:

$$(10) \qquad \max_{P} \sum_{i=1}^{N} \Pi^i(P^i, c^i(H), 1)$$

$$\text{subject to } \sum_{i=1}^{N} \Pi^i(P^i, c^i(L), 1) \leq \sum_{i=1}^{N} \Pi^i(P^i(c^i(L),0),\ c^i(L), 0)$$

That is, $\hat{P}_B(H)$ is the most efficient means of separating when prices are "globally" observable.

The two scenarios may now be compared. Low-quality profits are of course the same in each case, since $\hat{P}^i_A(L) = \hat{P}^i_B(L) = P^i(c^i(L),0)$. As for high-quality profits, it is immediate upon comparing (9) and (10) that high-quality profits can not be lower in scenario B than in scenario A. This is because any P satisfying the constraints in (9) also satisfies the constraint in (10), but the reverse is not true. Simply put, incentives are "pooled" when prices are well known, meaning that a low-quality monopolist must mimic *all* high-quality prices in order to fool consumers.[7] This in turn enables the high-quality monopolist to signal "hard" in some markets and "soft" in others, a strategy which is not feasible when consumers observe only "local" prices and mimicry can be undertaken on a market-by-market basis.

In fact, it is easy to see that the high-quality monopolist will typically do strictly better using scenario B than scenario A. Provided markets are not identical, γ^i will normally differ from γ° for some i. This implies $c^i(\gamma^\circ) \neq c^i(\gamma^i)$ and, by Lemma 5, $\hat{P}^i_A(H) \neq \hat{P}^i_B(H)$ for some i. Thus, in general, $\hat{P}_A(H) \neq \hat{P}_B(H)$ and, since $\hat{P}_B(H)$ uniquely maximizes (10), it follows that the high-quality monopolist does strictly better when its product line prices are well known to all buyers.

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[7]The notion of "incentive pooling" is developed in a repeated game context by Bernheim and Whinston [1990]. Bagwell and Riordan discuss incentive pooling in a dynamic signaling game where constraints are pooled through time.

One implication of the result is that firms should price advertise in an extensive fashion to ensure that consumers in any market i are acquainted with the entire product line price schedule. In this way, an economy of scale in signaling costs can be achieved.[8] A second implication is that a firm should "target" a particular group of consumers that might be potential buyers of all of the firm's products. In this way, each consumer will see many prices as a consequence of ordinary shopping activity. Thus, the theory predicts that multiproduct firms have incentive to sell products that are targeted to particular consumer groups, where consumer groups may be defined by income, age, sex, etc.[9]

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CONCLUSION

This paper offers two principle contributions. First, a simple and practical rule is developed that generates the optimal prices for a monopolist introducing a new, high-quality product line. This rule entails "pretending" that consumers know quality before purchase but that the unit cost of producing high quality in any market i is higher. Moreover, the prices implied by the rule emerge in the focal sequential equilibrium outcome. Second, for the case of independent demands, some empirical predictions are offered. Supra-monopoly prices should be charged initially for all products in the product line. All prices should then decline, but the absolute price decline should be steepest for goods which face relatively inelastic demands. Furthermore, an

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[8]The theory also suggests a potential explanation for branding. The role of branding may be to inform consumers that separate products draw from a common quality base so that signaling economies can be realized. Of course, to formalize such a notion, the decision to brand and the beliefs of consumers when branding does not occur would need to be endogenized.

[9]For example, the Ralph Lauren corporation sells clothes, sheets, towels, and even furniture, all of which is designed for a particular group of consumers.

economy of scale to signaling is identified which suggests that a monopolist will advertise its entire price schedule and/or "target" its products to particular consumer groups.[10]

Several extensions appear promising. It would be interesting to allow for a continuum of quality types. Work by Ramey [1988] suggests that the general rule described above might extend to this setting, although a less intuitive and more complex refinement (viz, refinement "D1" as defined by Cho and Kreps) may be needed for equilibrium selection. It also would be intriguing to allow the monopolist to choose some low-quality goods and some high-quality goods. This represents a considerable increase in the dimensionality of the type space, and clean results may be difficult to obtain. In addition, future work might consider more general cost technologies. While the arguments made above are robust to the introduction of economies of scale in the form of fixed cost terms, further analysis might consider the nature of optimal pricing when cost linkages exist across products. Finally, and most importantly, an empirical analysis of the time path of prices for a new product line of experience goods would be quite valuable.

[10]The model also can be extended to examine the welfare implications of export subsidies, as discussed in Bagwell [1990]. If demands are independent, an export subsidy can be shown to raise the welfare of the exporting country, by enabling a high-quality product line to be signaled with a less-distorted, high price vector.

APPENDIX

PROOF OF LEMMA 1: By assumption, $\Pi(P(c(t),0), c(t),0) > 0$. Thus

(A1) $$\min_b \Pi(P(c(t),0), c(t), b) = \Pi(P(c(t),0), c(t), 0) > 0 \quad .$$

Next, it is direct that

(A2) $$\max_b \Pi(P,c(t),b) = \Pi(P,c(t),1)$$

if $\Pi(P,c(t),1) \geq 0$ and

(A3) $$\max_b \Pi(P,c(t),b) \leq 0$$

otherwise. Now, suppose the inequality in Lemma 1 holds. Then clearly P is dominated by $P(c(t),0)$. Finally, suppose P is dominated. Then there exists $\tilde{P}$ such that

$$\min_b \Pi(\tilde{P},c(t),b) > \max_b \Pi(P,c(t),b).$$

The inequality in Lemma 1 certainly holds if $\Pi(P,c(t),1) < 0$, so suppose $\Pi(P,c(t),1) \geq 0$. Then

$$\min_b \Pi(\tilde{P},c(t),b) = \Pi(\tilde{P},c(t),0) > 0.$$

But

$$\Pi(P(c(t),0),c(t),0) \geq \Pi(\tilde{P},c(t),0),$$

so the desired inequality holds.

PROOF OF LEMMA 2: Observe first that $\hat{\Pi}(t) > 0$, since positive profit always can be made with a deviation to P(c(t),0), as shown in (A1). Suppose now that the inequality in Lemma 2 holds. Using (A2) and (A3), it is immediate that P is equilibrium dominated. Next, suppose P is equilibrium dominated for the monopolist of type t. Then the inequality in Lemma 2 follows, since $\Pi(P,c(t),1) \le \max_b \Pi(P,c(t),b) < \hat{\Pi}(t)$..

Proof of Lemma 4: Let $\gamma^1 > \gamma^2 > t$ and $P_1 \equiv P(c(\gamma^1),1)$ and $P_2 \equiv P(c(\gamma^2),1)$. Then

(A4) $\quad \Pi(P_1, c(\gamma^1),1) - \Pi(P_2, c(\gamma^1),1) > 0$

(A5) $\quad \Pi(P_2, c(\gamma^2),1) - \Pi(P_1, c(\gamma^2),1) > 0$

Adding (A4) and (A5) gives

$$\sum_{i=1}^{N} (c^i(\gamma^2) - c^i(\gamma^1))(D^i(P_1,1) - D^i(P_2,1)) > 0 \quad .$$

Since k is increasing, this gives

(A6) $$\sum_{i=1}^{N} (c^i(H) - c^i(L))(D^i(P_2,1) - D^i(P_1,1)) > 0 \quad .$$

Next, note that (A5) may be rewritten as

(A7) $$\sum_{i=1}^{N} \left[P_2^i D^i(P_2,1) - P_1^i D^i(P_1,1)\right] + \sum_{i=1}^{N} c^i(\gamma^2)\left[D^i(P_1,1) - D^i(P_2,1)\right] > 0 \quad .$$

But observe that

(A8) $$\sum_{i=1}^{N} (c^i(t) - c^i(\gamma^2))(D^i(P_1,1) - D^i(P_2,1)) = (k(\gamma^2)-k(t))\sum_{i=1}^{N} (c^i(H)-c^i(L))(D^i(P_2,1)-D^i(P_1,1)) > 0$$

by (A6) and $\gamma^2 > t$. Using (A7) and (A8) gives

$$\sum_{i=1}^{N} \left[P_2^i D^i(P_2,1) - P_1^i D^i(P_1,1)\right] + \sum_{i=1}^{N} c^i(t)\left[D^i(P_1,1) - D^i(P_2,1)\right] > 0$$

or equivalently

$$\Pi\left(P_2,c(t),1\right) > \Pi\left(P_1,c(t),1\right) \quad .$$

Proof of Theorem 2: The theorem is proved in two steps. First, it is shown that the theorem holds if (8) holds. Then it is shown that (8) does indeed hold. To execute the first step, put $\hat{P}(H) = P(c(\gamma^{\circ}),1)$ and $\hat{P}(L) = P(c(L),0)$. Let $\hat{b}(P) = 0$ for all $P \neq \hat{P}(H)$ such that $\Pi(P,c(L),1) \geq \Pi(P(c(L),0), c(L),0)$. These prices are not dominated for the low-quality monopolist. Let $\hat{b}(P) = 1$ for all P such that $\Pi(P,c(L),1) < \Pi(P(c(L),0), c(L),0)$. These prices are dominated for the low-quality monopolist. Finally, put $\hat{b}(\hat{P}(H)) = 1$. Now, observe that $\hat{P}(H)$ is not dominated for the high-quality monopolist. This follows directly from (8). Thus, the described beliefs satisfy (E2) and (E3).

Consider now (E1). A low-quality monopolist's optimal price vector given these beliefs is $\hat{P}(L) = P(c(L),0)$. Clearly, this price vector is preferred to any other P such that $\hat{b}(P) = 0$. By construction, P's for which $\hat{b}(P) = 1$ are also nonimproving. For the high-quality monopolist, any P for which $\hat{b}(P) = 0$ yields profit no higher than $\Pi(P(c(H),0), c(H),0)$. But this profit is nonimproving under (8). Finally, any P for which $\hat{b}(P) = 1$ is also nonimproving, as is shown in the second step of the proof of Theorem 1. Thus, (E1) also holds. Further, Theorem 1 indicates that these strategies give the unique undominated separating equilibrium outcome when (8) holds.

The second step of the proof is to show that (8) holds. Let $P_0 \equiv P\left(c(\gamma^{\circ}),1\right)$. Assume to the contrary that

(A9) $\Pi(P(c(H),0),c(H),0) - \Pi(P_0,c(H),1) \geq 0$.

Then, since $\Pi(P_0,c(\gamma°),1) > 0$, it is easily established that

$$\Pi(P_0,c(\gamma°),1) - \Pi(P(c(H),0),c(\gamma°),1) \geq 0 \quad .$$

implies

(A10) $\Pi(P_0,c(\gamma°),1) - \Pi(P(c(H),0),c(\gamma°),0) > 0$.

Adding (A9) and (A10) gives

(A11) $\sum_{i=1}^{n} (c^i(H) - c^i(L))(D^i(P(c(H),0),0) - D^i(P_0,1)) > 0$,

since $\gamma° > H$.

Using (1) and the fact that $P(c(H),0) \neq P(c(L)0)$ gives

(A12) $\Pi(P_0,c(L),1) - \Pi(P(c(H),0),c(L),0) > 0$.

Adding (A9) and (A12) yields

$$\sum_{i=1}^{N} (c^i(H) - c^i(L))(D^i(P_0,1) - D^i(P(c(H),0),0)) > 0 \quad ,$$

which is in contradiction with (A11).

Proof of Theorem 3: To establish this theorem, it is first useful to show that $\Pi(P(c(H),b),c(H),b)$ is strictly increasing in b. This follows because

$$\Pi(P(c(H),b_1),c(H),b_1) \geq \Pi(P(c(H),b_2),c(H),b_1)$$
$$> \Pi(P(c(H),b_2),c(H),b_2)$$

for any $b_1 > b_2$. Next, using (8),

$$\Pi(P(c(H),1),c(H),1) > \Pi(P(c(\gamma°),1),c(H),1)$$
$$> \Pi(P(c(H),0),c(H),0)$$

It now follows from continuity that there exists a unique $b^* \in (0,1)$ such that

$$\Pi(P(c(H),b^*),c(H),b^*) = \Pi(P(c(\gamma^\circ),1),c(H),1) \quad .$$

Further, for all $b < b^*$,

(A13) $$\Pi(P(c(H),b),c(H),b) < \Pi(P(c(\gamma^\circ),1),c(H),1) \quad .$$

Suppose now that an undominated pooling equilibrium exists. Letting P_p represent the pooling price, observe that the high-quality monopolist equilibrium profit is easily bounded, as

(A14) $$\Pi(P_p,c(H),b^\circ) \le \Pi(P(c(H),b^\circ),c(H),b^\circ) \quad .$$

Thus, if $b^\circ < b^*$, then (A13) and (A14) give

(A15) $$\Pi(P_p,c(H),b^\circ) < \Pi(P(c(\gamma^\circ),1),c(H),1) \quad .$$

Next, recall from (7) and (8) that $P(c(\gamma^\circ + \epsilon),1)$ is dominated only for the low-quality monopolist. Thus (E3) requires that $\hat{b}(P(c(\gamma^\circ + \epsilon),1)) = 1$. But then, using continuity, (A15) indicates that the high-quality monopolist should deviate, completing the proof.

PROOF OF THEOREM 4: Let $\hat{P}(H) = \hat{P}(L) = P_p$ correspond to a pooling equilibrium. Note first that

$$\hat{\Pi}(L) \equiv \Pi(P_p,c(L),b^0) \ge \min_b \Pi(P(c(L),0),c(L),b)$$
$$= \Pi(P(c(L),0),c(L),0) > 0 \quad ,$$

and so, if $P_0 \equiv P(c(\gamma^\circ),1)$, using (1) gives

(A16) $$\hat{\Pi}(L) \ge \Pi(P_0,c(L),1) > 0 \quad .$$

Observe also that

(A17) $$\hat{\Pi}(L) < \Pi(P_p,c(L),1) \le \Pi(P(c(L),1),c(\;\;),1)$$

Using (A16), (A17), and Lemma 4, it follows that there exists a unique $\tilde{\gamma} \in (L, \gamma^\circ]$, with $\tilde{P} \equiv P(c(\tilde{\gamma}),1)$, such that

$$\text{(A18)} \qquad \Pi\left(P_p, c(L), b^\circ\right) - \Pi\left(\tilde{P}, c(L), 1\right) = 0 \quad .$$

Next, since $\Pi\left(\tilde{P}, c(\tilde{\gamma}), 1\right) > 0$, it follows that

$$\Pi\left(\tilde{P}, c(\tilde{\gamma}), 1\right) - \Pi\left(P_p, c(\tilde{\gamma}), 1\right) \geq 0$$

implies

$$\text{(A19)} \qquad \Pi\left(\tilde{P}, c(\tilde{\gamma}), 1\right) - \Pi\left(P_p, c(\tilde{\gamma}), b^\circ\right) > 0 \quad .$$

Adding (A18) and (A19) gives

$$\text{(A20)} \qquad \sum_{i=1}^{N} \left(c^i(H) - c^i(L)\right)\left(D^i\left(P_p, b^\circ\right) - D^i\left(\tilde{P}, 1\right)\right) > 0$$

since $\tilde{\gamma} > L$. Thus, (A16)-(A20) are necessary if a pooling equilibrium exists.

It is now established that

$$\text{(A21)} \qquad \Pi\left(\tilde{P}, c(H), 1\right) - \Pi\left(P_p, c(H), b^\circ\right) > 0$$

is also necessary for a pooling equilibrium. Otherwise, the converse of (A21) may be added to (A18) to give

$$\text{(A22)} \qquad \sum_{i=1}^{N} \left(c^i(H) - c^i(L)\right)\left(D^i\left(P_p, b^\circ\right) - D^i\left(\tilde{P}, 1\right)\right) \leq 0$$

which contradicts (A20).

Thus, using Lemma 4, the fact that $\tilde{\gamma} > L$, (A18), and (A21), it is clear for ϵ small and positive that

$$\text{(A23)} \qquad \begin{aligned} &\Pi(P(c(\tilde{\gamma}+\epsilon),1),c(H),1) > \hat{\Pi}(H) \equiv \Pi(P_p,c(H),b^{\circ}) \\ &\Pi(P(c(\tilde{\gamma}+\epsilon),1),c(L),1) < \hat{\Pi}(L) \equiv \Pi(P_p,c(L),b^{\circ}) \ . \end{aligned}$$

It follows from (E4) then that $\hat{b}(P(c(\tilde{\gamma}+\epsilon),1)) = 1$, which by (A23) causes the high-quality firm to deviate.

Proof of Theorem 5: Let $P_0 \equiv P(c(\gamma^{\circ}),1)$ and suppose $\hat{P}(H) \neq P_0$ in an intuitive separating equilibrium outcome. Then it is a straightforward matter to mimic the third step of the proof of Theorem 1 to reach a contradiction. Next, it must be established that the described outcome corresponds to an intuitive equilibrium. Put $\hat{P}(H) = P_0$ and $\hat{P}(L) = P(c(L),0)$. Observe that P is equilibrium dominated if and only if it is dominated for a low-quality monopolist. Thus, beliefs may be specified as in the proof of Theorem 2 so as to satisfy (E2) and (E4). Further, it is argued in this same proof that (E1) is satisfied when beliefs are defined in this way.

Proof of Lemma 5: Let $\gamma^1 > \gamma^2$ *and* $P_1^i \equiv P^i(c^i(\gamma^1),1)$ and $P_2^i \equiv P^i(c^i(\gamma^2),1)$. Then:

$$\begin{aligned} &\Pi^i\left(P_1^i,c^i(\gamma^1),1\right) - \Pi^i\left(P_2^i,c^i(\gamma^1),1\right) > 0 \\ &\Pi^i\left(P_2^i,c^i(\gamma^2),1\right) - \Pi^i\left(P_1^i,c^i(\gamma^2),1\right) > 0 \end{aligned}$$

Adding gives

$$\left(c^i(\gamma^1)-c^i(\gamma^2)\right)\left(D^i\left(P_2^i,1\right)-D^i\left(P_1^i,1\right)\right) > 0 \quad ,$$

whence $D^i\left(P_2^i,1\right) > D^i\left(P_1^i,1\right)$. But since $D^i(P^i,1)$ decreases in P^i, $P_1^i > P_2^i$ is required.

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