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PRICE REGULATION AND QUALITY OF SERVICE

by

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ABSTRACT

Public concern has been rising about whether market forces are sufficient to ensure the optimal choice of quality of services. We examine a model in which unregulated competition leads to an underprovision of quality from a social perspective and then study the effects of price regulation. Although price floors sometimes have the expected effect of increasing the incentives to raise the quality of services, their imposition changes the nature of competitive behaviour in subtle ways and often changes the relative positions of firms in the market. We show that no price floor exists which will achieve the socially optimal quality choice and the only price floors that result in symmetric firm behaviour are those which involving an overinvestment in quality. We also examine the effects of imperfect consumer information about quality under various consumer learning processes and find that slow but plausible learning behaviour can lead to an increase in quality. We also show that firms typically benefit from consumers' uncertainty about the quality of both their own good their rival's. Informative advertising about the firm's or its rival's product is not in the firm's interest.
1. **Introduction**

Airlines frustrate us. Cancelled flights, uncomfortable seats, execrable food all form part of the bundle included in an airline ticket. Some observers claim the alleged decline in quality of airline service started essentially in step with the deregulation of the industry in 1978. While increased competition has almost certainly led to a lower price for air travel, it often seems difficult to avoid the conclusion that consumers are paying less for an inferior product.¹

Why might competition fail to ensure the provision of a high quality product in the market place? We explore the argument that restrictions on price competition forces airlines to compete along other dimensions of service including higher safety standards, more reliable scheduling, and better in-flight service. With price floors removed, (the argument goes) airlines are driven to engage in intense price competition necessitating a trade-off, providing lower quality for lower prices to maintain profitability. We analyze a simple model of differentiated duopoly in which firms compete for customers through both price and quality. The firms are ex ante different in a manner similar to different locations in a Hotelling model of competition along the line. In our version, each firm's "location" is fixed, but each may choose to offer (at a cost) higher or lower quality of their product to the market. With a quality choice determined, firms then compete in prices for a determinate number of periods. Consumers are located along the line and, depending on their location exhibit a preference

¹ For a litany of the decline in quality of airline service, especially safety, see Nance (1987). For a more rigorous attempt to measure the change in the standards of safety before and after deregulation, see Kennet, (1990).
for one firm or the other. However, their preference may be directed to one rival by offering a more attractive price-quality combination. We then examine the effects of various forms of price regulation in this environment.

Other models of competitive and imperfectly competitive markets have shown how price regulation can induce higher choices of quality. In all of these models, price regulation takes the form of price fixing by the regulatory agency. Douglas and Miller (1974), Panzar (1985) and Schmalensee (1977) examine imperfectly competitive markets and show that as long as the market demand curves are not too elastic, increases in the regulated price induce increases in the level of quality provided.

In the symmetric pure strategy subgame perfect Nash equilibrium of our model, we find that, in the absence of price regulation, firms choose to offer a quality of service strictly below the level which maximizes the sum of consumer plus producer surplus. However, the introduction of price regulation such as price floors does not lead, as is commonly argued, to a simple redirection of competition to the quality dimension. If too low a price floor is chosen, for example a price equal to the unregulated equilibrium price, no symmetric equilibrium in quality choice exists. In fact, the price floor’s initial impact is to provide incentives to decrease quality. The rivals end up offering sharply different types of services to consumers as one firm specializes in high quality and the other in a low quality, low price product. If a higher price floor is imposed, then symmetric equilibria with high quality choices exist but the quality level exceeds the social optimum. This result suggests support for the argument
that airlines may have over-provided service under regulation.\footnote{For a more formal analysis of this phenomenon, see Kennet (1990) and Panzar and Savage (1987).}

The reason that a simple price floor has some not so simple consequences is instructive. If the rival firm can still respond to a fall in the other firm's quality by raising its price, the resulting loss in the quality-reducing firm's market share may be offset by this expected price response. Furthermore, since the lower quality firm can now credibly commit not to lower its price, it may not suffer any loss in revenue and can save on the cost of quality. This intuition suggests that to funnel competition into the quality dimension regulators may have to set not just price floors but price ceilings as well (as was practiced through most of the era of airline regulation). Indeed in our model, if the price is set by regulators at exactly the price which would have emerged from the unregulated equilibrium, the consequences are to induce the firms to choose the socially optimal quality level. Any level above that price, though, results in an overinvestment in quality.

The model also allows us to examine the effects of consumer ignorance about true quality choices of firms. While intuition suggests that if consumers are uninformed about the producers' quality choice then quality will suffer, it is often argued that either competitive pressure or rational consumer learning will force firms to operate the same whether or not consumers are informed. We model the learning behaviour of consumers explicitly. If consumers are uninformed but learn the true quality very quickly, for example after only one period, then, not surprisingly, consumers' ignorance induces firms to choose a lower quality level than they
otherwise would. This effect is less important the longer the firms’ time horizon. More interestingly, if consumers’ learning process is slower, the effects on the firms’ quality choice is ambiguous. Firms may, in fact, choose to increase their quality level. The reason for this counterintuitive result is that firms’ subsequent profits are convex in prices which are linear in consumers’ beliefs. Firms’ expected profits therefore rise with a greater variance in consumers’ beliefs. Higher quality levels may quite easily be associated with greater variance in performance and firms may wish to exploit this effect. Furthermore, firms’ expected profits rise with greater consumer uncertainty about their rival’s quality choice. So there may be no incentive for firms to reveal information about either themselves or their opponents.

2. The Model

The basic model consists of a simple differentiated products duopoly. A unit mass of consumers are located on the line.\footnote{The pricing version of the game is taken directly from Tirole, pp. 279-282.} Two firms, $i = 0,1$, are located at either end and produce similar goods at zero marginal cost.\footnote{Setting marginal costs to zero saves on notation. What is required is that the service be provided at constant marginal cost.} A consumer at point $x$ purchases at most a single indivisible unit of the product in any period and gains expected utility

$$u(p_j; \mu_j, x) = s + (\mu_j - p_j)s - t(d_j(x))^2$$

if he purchases from firm $j$ where $p_j$ is the price charged, $\mu_j$ is a Bernoulli
parameter denoting the consumers' perception of the probability of success of a particular characteristic of the good\(^5\) and \(d_j(x)\) denotes a transportation cost which depends on the location of the consumer. \(d_j(x) = x\) if \(i = 0\), \(d_j(x) = (1 - x)\) if \(j = 1\). The consumer gains utility zero if no purchase is made. Throughout, it is assumed that \(t\) is "small"--more specifically, that \(3t < s\). This assumption ensures that if there were only a single firm operating, it would choose to serve the entire market.

The order of moves is as follows. In the initial stage, companies simultaneously choose the true probability of success parameter, \(\theta_j\), at a cost \(b\theta_j^2\). This "quality" decision can be interpreted as a choice of maintenance levels--for example, airlines deciding about scheduling or plane servicing which determine the probability of a flight delay. It will be apparent that much of this analysis remains valid if \(\theta_j\) and \(\mu_j\) are treated as continuous quality parameters that enter directly into consumers' utilities, and not simply as a Bernoulli parameter; but we focus on the latter interpretation. In the next stage, consisting of \(N + 1\) periods, the firms compete as Bertrand competitors to maximize the discounted sum of expected profits (revenues) at a common discount factor, \(\delta\). For future reference, define \(k = (1 - \delta^{N+1})/(1 - \delta)\). Much of the analysis is concerned with the consequences of different assumptions regarding the information consumers have about the firms' true quality choice. If there is no asymmetric information, of course, \(\mu_j = \theta_j\). The remainder of this section and the two following sections maintain this assumption.

As a base case for analysis, it is useful to note that the choice of

\(^5\) The parameter \(S\) which is a measure of the consumer's valuation of this characteristic is set to one to save on notation. There are no qualitative effects of this normalization.
the $\theta_j$'s which maximizes the sum of consumer plus producer surplus is given by the solution to

$$\max_\theta \int_0^5 k(s + \theta - tx^2)dx - b\theta^2$$

or $\theta_w = \theta_0 - \theta_1 = k/(4b).$ (For the case in which $\theta$ is a Bernoulli parameter, it will be assumed that the slope of the marginal cost function, $2b$, is sufficiently steep so that $k/(4b) < 1$).

It is also useful to characterize the general behaviour of the pricing phase of the game. Fix $\theta_0, \theta_1,$ and fix $p_0$ and $p_1$. The quantity demanded of firm $j$'s product is

$$q_j = [t + (\theta_j - \theta_i) + (p_i - p_j)]/(2t).$$

Thus, demand for a firm's product depends both on the perceived quality differential between its and the rival's product and the price differential. Consumers are willing to pay more for a higher quality product. For any given period, then, firm $j$'s best response function in prices is given by

$$p_j(p_i; \theta_j, \theta_i) = \max(0, [t + (\theta_j - \theta_i) + p_i]/2), \ j = 0,1. \quad (1)$$

Solving the two best response functions yields the unique Bertrand equilibrium in prices which in any given period, given consumers' beliefs, $\mu_0 = \theta_0, \mu_1 = \theta_1,$ is a profile of prices $^6$

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$^6$ The best response functions shift upward by $c$ in the case in which the marginal cost of providing the service is $c > 0$. 
If \(|\theta_0 - \theta_1| < 3t\),
\[ p_0 = t + (\theta_0 - \theta_1)/3, \]
\[ p_1 = t - (\theta_0 - \theta_1)/3. \]  \hspace{1cm} (2)

If \(\theta_j - \theta_i > 3t\),
\[ p_j = (t + \theta_j - \theta_i)/2, \]
\[ p_i = 0. \]

Two types of equilibria are possible in this game's pricing. When the two firms' quality levels are "close," each firm charges a positive price; the higher quality firm the higher price. However, if the quality levels are far apart, the low quality firm's price is zero while the high quality firm sets a price at which it serves the entire market. The presence of potential competition from the low quality provider prevents the high quality producer from realizing the full monopoly price.

The per period profit functions are, therefore,

\[ \pi_j(\theta_j, \theta_i) = (t + (\theta_j - \theta_i)/3)^2/(2t), \text{ if } |\theta_j - \theta_i| < 3t \]
\[ = 0, \text{ if } \theta_i - \theta_j > 3t \]
\[ = (t + \theta_j - \theta_i)/2, \text{ if } \theta_j - \theta_i > 3t, \ j = 0,1. \] \hspace{1cm} (3)

Notice that given (2), if \(|\theta_j - \theta_i| < 3t\), (3) can be written as

\[ \pi_j(\theta_j, \theta_i) = (p_j(\theta_j, \theta_i))^2/(2t). \] \hspace{1cm} (4)

That is, per period profits are convex in the induced equilibrium prices.

Observe, also, that with a finite period game, we only need to invoke
subgame perfection to ensure that the per period Bertrand equilibria form
the equilibrium profile of prices for the N+1 period game.

3. Equilibrium Choice of Quality With Full Information

When consumers know the true values of the two firms’ quality choices, the
equilibrium payoffs of the N+1 period pricing game are determined by the
discounted sum of equation (3). That is, for any given pair, \( \theta_0, \theta_1 \), firm
j’s discounted profits is given by \( V_j(\theta_j, \theta_i) = k\pi_j(\theta_j, \theta_i) - b\theta_j^2 \). For a
fixed \( \theta_i \), firm j’s best response in terms of \( \theta_j \) is the solution to

\[
\max_{\theta_j} V_j(\theta_j, \theta_i)
\]  

(5)

This yields a best response function\(^7\)

\[
\theta_j(\theta_i) = \begin{cases} 
0 & \text{if } \theta_i > 3t, \\
1 & \text{if } \theta_i < 3t + (1 - 18bt/k), \\
[k/(18bt - k)](3t - \theta_i) & \text{otherwise.}
\end{cases}
\]  

(6)

This system yields our first result.

Proposition 1: The unique quality choice symmetric equilibrium is

\(^7\) If \( k > 18bt \), then the objective function in (5) is convex in \( \theta \) and
the set of pure strategy best responses is \( \{0, 1\} \). In this case, there exist
(in general) two asymmetric equilibria, \( \{(\theta_0 = 0, \theta_1 = 1), (\theta_0 = 1, \theta_1 = 0)\} \) and
a mixed strategy equilibrium. That is, if the number of the pricing periods
is large (N big) and the cost of providing quality is low (b small), firms
will tend to specialize in either high or low quality service. In what
follows, this possibility is ruled out, that is, we assume \( k < 18bt \) so that
(5) is a concave problem.
\[ \theta_j - \theta_i - k/(6b) < \theta_w. \]  

(7)

If this equilibrium obtains, the model results in a Cournot type of competition in quality and hence an underinvestment in \( \theta \) from a social point of view. Observe that since lowers \( t \)'s correspond to greater price competition and since the equilibrium choice of the \( \theta \)'s is independent of \( t \), the result that firms will underinvest in quality is, in a sense, robust to the degree of imperfection of competition.  

However, the interpretation of \( \theta \) as a Bernoulli parameter requires \( \theta \) to lie in the interval [0,1] and introduces the possibility of other, asymmetric equilibria. These equilibria are characterized in Table 1. They can be summarized as follows. If transportation costs are small (\( 3t < 1 \)), then in the asymmetric equilibrium, only one firm invests in \( \theta \) and it alone operates in the market. Nevertheless, potential entry by the low-quality (\( \theta_i = 0 \)) firm, remains as a discipline on the incumbent's pricing behaviour. A type of contestable behaviour results. The incumbent firm does not charge the monopoly price but, instead, a 'limit' price that just keeps the low-quality firm from entering (\( p = (t + \theta_j)/2 \)). If transportation costs are sufficiently large (\( 3t > (18bt - k)/k > 1 \)), then the low-quality firm provides some quality and serves some of the market. The resulting prices are \( p_H = t + (\theta_H - \theta_i)/3 \), \( p_L = t - (\theta_H - \theta_i)/3 \). Both low and high quality firms co-exist in the market. While these equilibria are of interest in their own right, the ex ante symmetry of the firms, argues (weakly) in

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8 It is only "in a sense" because the restriction that \( k < 18bt \) must be maintained for the firms' problem to be concave. This restriction is more likely to bind as \( t \) goes to zero.
favour of the symmetric equilibrium and it is this outcome which we focus on in the remainder of the paper.

Table One

<table>
<thead>
<tr>
<th>CASE</th>
<th>Symmetric Equilibrium</th>
<th>Asymmetric Equilibria</th>
</tr>
</thead>
<tbody>
<tr>
<td>I) $k &lt; 9bt$</td>
<td>$\theta_0 = \theta_1 = k/(6b)$</td>
<td>None</td>
</tr>
<tr>
<td>IIa) $k \in [9bt, 12bt]$, $3t &lt; 1$</td>
<td>$\theta_0 = \theta_1 = k/(6b)$</td>
<td>$\theta_i = 0$, $\theta_j = 1$</td>
</tr>
<tr>
<td>IIb) $k \in [9bt, 12bt]$, $3t &gt; 1$</td>
<td>$\theta_0 = \theta_1 = k/(6b)$</td>
<td>$\theta_i = k(3t - 1)/(18bt - k)$, $\theta_j = 1$</td>
</tr>
<tr>
<td>III) $k \in [12bt, 18bt]$, $3t &gt; 1$</td>
<td>$\theta_0 = \theta_1 = k/(6b)$</td>
<td>$\theta_i = 0$, $\theta_j = k/(4b)$</td>
</tr>
</tbody>
</table>

4. The Effects of Price Regulation.

The concern that fierce price competition can lead to a sacrifice in quality is borne out by the above result. It is also possible to show that the regulation of price competition can be used to generate higher investment in $\theta$. Price regulation may take many forms. Regulators may fix prices, they may set price floors, price ceilings, or both. The Civil Aeronautics Board (CAB) generally followed the first policy for passenger air traffic, however, the use of both ceiling and floor was adopted at the end of its regime (Bailey, et al 1985). The regulation of air cargo employed a ceiling and a floor (Carron 1981). Perhaps the best known example of the use of a price floor alone was the regulation of interest rates on time deposits under Regulation Q administered by the Interagency Coordinating
Committee (Mahoney, et al 1987). In this section, we examine the effects of various types of price regulation on quality choice. Throughout, it is assumed that the regulated price is set no lower than the unregulated equilibrium price level of \( t \).

We begin by examining the effects of price floors and focus on the equilibrium behaviour in the pricing game. Let \( \theta_0 \geq \theta_1 \). From the perspective of firm 0, its profit from a price \( p_0 \) is

\[
\pi_0 = p_0(t + (\theta_0 - \theta_1) + p_1 - p_0)/(2t)
\]

and

\[
\pi'_0 = (t + (\theta_0 - \theta_1) + p_1 - 2p_0)/(2t).
\]

(8)

where \( \pi_0' \) denotes the derivative with respect to \( p_0 \). Consider first the case where \( t + (\theta_0 - \theta_1) < p^* \). If \( p_1 < 2p^* - (t + (\theta_0 - \theta_1)) \) then \( p_0 = p^* \), since \( \pi_0 < 0 \) for all \( p_0 \), otherwise, \( p_0 = (t + (\theta_0 - \theta_1) + p_1)/2 \). Thus, the best response of firm 0 is a constant at \( p^* \) until \( p_1 = 2p^* - (t + (\theta_0 - \theta_1)) \).

Since \( t + (\theta_0 - \theta_1) < p^* \), flat portions of both best response functions cross the 45 degree line, so the equilibrium prices are both \( p^* \).

Suppose, instead, that \( p^* < t + (\theta_0 - \theta_1) \). Now the kink in the best response function of firm 1 occurs before the 45 degree line. It meets the best response function of firm 1 before its kink, so solving the system with \( p_1 = p^* \) yields \( p_0 = (t + (\theta_0 - \theta_1) + p^*)/2, p_1 = p^* \) as the equilibrium profile. The two types of best response functions in prices are illustrated in Figures 1A and 1B.

Not surprisingly, the price floor has the effect of making price competition less responsive to quality differentials. Substitution of the
price equilibria gives as payoffs in the quality-setting game

\[
V_j(\theta_j, \theta_i) = \frac{kp^*(t + (\theta_j - \theta_i))/(2t) - b\theta_j^2}{k(t + (\theta_j - \theta_i) - p^*)^2/(4t)} - \frac{k(t + (\theta_j - \theta_i) - p^*)^2/(4t)}{b\theta_j^2}, \quad \text{if } |\theta_j - \theta_i| \leq p^* - t,
\]

\[
= \frac{k(t + (\theta_j - \theta_i) + p^*)^2/(8t) - b\theta_j^2}{k(t + (\theta_j - \theta_i) + p^*)^2/(8t)} - \frac{k(t + (\theta_j - \theta_i) + p^*)^2/(8t)}{b\theta_j^2}, \quad \text{if } \theta_j - \theta_i \geq p^* - t,
\]

\[
= \frac{kp^*(.5 + ((\theta_j - \theta_i) - (p^* - t))/(4t)) - b\theta_j^2}{kp^*(.5 + ((\theta_j - \theta_i) - (p^* - t))/(4t)) - b\theta_j^2}, \quad \text{if } \theta_i - \theta_j \geq p^* - t. \quad (9)
\]

If the \((\theta_0, \theta_1)\) space is divided into the three regions corresponding to the differing profit functions in (9), we can derive "best response" functions conditional on remaining in the region as

\[
\theta_j(\theta_i) = \frac{kp^*/(8bt)}{kp^*/(8bt)} \quad \text{if } \theta_i - \theta_j \geq p^* - t
\]

\[
= \frac{kp^*/(4bt)}{kp^*/(4bt)}, \quad \text{if } |\theta_j - \theta_i| \leq p^* - t
\]

\[
- \frac{k(t + p^* - \theta_i)/(8tb - k)}{k(t + p^* - \theta_i)/(8tb - k)} \quad \text{otherwise.} \quad (10)
\]

This system is not the actual best response correspondence for firm \(j\), however, since the three functions overlap for certain values of \(\theta_1\). In these cases, it is necessary to evaluate the relevant maximized profit functions holding \(\theta_1\) fixed to see which yields the higher profit. The characteristic of the resulting equilibria depends on how high above the unregulated equilibrium price the floor is set.

Table Two describes the set of equilibria. The best response functions which generate these equilibria in the various cases are shown in Figures 2A, 2B and 2C. It is notable that a symmetric equilibrium exists only if the price floor is set high enough above \(t\), that is \(p^* - t \geq kp^*/(16bt)\). Such a price leads the firms to choose higher quality levels but forces them to
overinvest in quality from a social point of view. The resulting choice of
the \( \theta \)'s is \( kp^*/(4bt) \geq k/(4b) + (k/(4bt)) (kp^*/(16bt)) \geq k/(4b) \).

<table>
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<th>Case</th>
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</tr>
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<tbody>
<tr>
<td>I)</td>
<td>( p^* - t &lt; kp^*/(16bt) )</td>
<td>none</td>
</tr>
<tr>
<td></td>
<td>( \theta_i = kp^<em>/(8bt), \theta_j^+ = kp^</em>/(8bt) + tk/(8bt - k) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \theta_i - \theta_j = kp^*/(4bt) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \theta_j = kp^*/(8bt) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \theta_j^+ = kp^*/(8bt) + tk/(8bt - k) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( kp^*/(16bt)(1 + k/(8bt)) )</td>
<td></td>
</tr>
<tr>
<td>II)</td>
<td>( p^* - t \geq kp^<em>/(16bt), kp^</em>/(16bt)(1 + k/(8bt)) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \theta_i = \theta_j = kp^*/(4bt), ) none</td>
<td></td>
</tr>
</tbody>
</table>

However, if a regulating agency attempts to induce a lower investment
in quality by choosing \( p^* - t \leq kp^*/(16bt) \), equilibrium requires a striking
asymmetry between the firms. One firm chooses a higher quality than the
social optimum.

\[
\theta_j = \frac{kp^*}{(4bt)} + \frac{kt}{(8bt - k)}
\]
\[
= \frac{kp^*}{(4bt)} + \frac{k(p^* - t)(p^*k - 8bt(p^* - t))}{(8bt - k)(8bt)}
\geq \frac{kp^*}{(4bt)} + \frac{k(p^*k - p^*k/2)}{(8bt - k)(8bt)}
\geq \frac{k}{(4b)}, \text{(where use is made of the fact that } p^* - t \leq kp^*/(16bt))
\]

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\( ^9 \) As was the case in the non-regulated quality game, it is possible
that other equilibria exist in which one of the quality choices is at a
boundary, \( \theta_i = 1 \). We ignore these here by assuming that all of the
equilibrium values given in Table Two are less than one.
The other firm chooses a quality level generally below the level it would have chosen without the regulation. For example, if \( p^* - t \), then \( \theta_i = k/(8b) < k/(6b) \). The resulting prices are also asymmetric: \( p_j = (p^* + t(8bt)/(8bt - k))/2, \ p_i = p^* \). The low quality firm prices at the price floor and the high quality firm chooses a price strictly above the price floor. The following Propositions are drawn from Table Two.

**Proposition 2:** If, in the pricing stage a firm may not set its price below a level, \( p^* \leq t + kp^*/(16bt) \), \( p^* \geq t \), then the only (interior) equilibria of the quality game are asymmetric.

**Proposition 3:** If, in the pricing stage a firm may not set its price below a level, \( p^* \geq t + kp^*/(16bt) \), then there exists a symmetric equilibrium choice of \( \theta^i \)'s that exceeds the socially optimal quality choice.

The fact that for relatively low price floors a firm might lower its quality choice is, at first glance, surprising. A justification often given for the price regulation of airlines, for example, was that a reduction in the ability to compete in prices would force competition in other dimensions such as quality. The result that only one firm specializes in high quality while the other offers a low quality service runs counter to this expectation. The weakness of this intuition lies in the failure to recognize that price floors can be instruments for collusion rather than competition. In what follows, assume that the price floor is set equal to the non-regulated equilibrium price, \( p^* = t \) and consider the perspective of firm one. From (5), the unregulated equilibrium choice of \( \theta_1 \) must satisfy
\[ dV = (k/2t)[dp_1(q_1^{2t}) + p_1(d\theta_1 + dp_0 - dp_1) - 2b\theta_1 d\theta_1] = 0. \]  

(12)

That is, the effects of a small change in \( \theta_1 \) -- either direct effects on costs and demand or indirect effects via equilibrium prices -- must be zero. However, the price best response functions in (1) indicate that the indirect effect of a change in \( \theta_1 \) on \( p_0 \) is \( dp_0 = -d\theta_1 + dp_1/2 \). If firm 1 lowers \( \theta \), the effect on the equilibrium price of firm 0 is a one for one increase in its price because of the quality differential minus a decrease in response to an expected fall in firm 1's price. Substituting in (12) yields

\[ dV = (k/2t)[(q_1^{2t}) - p_1/2]dp_1 - 2b\theta_1 d\theta_1 = 0. \]  

(13)

If firm 1 cannot lower its price because of regulation, the first term is zero and it enjoys a direct saving in the cost of quality. The fall in quality yields an advantage to the rival firm but that advantage is fully exploited by a resulting rise in its price. With the higher price, firm 1 keeps its original share of the market (at the same, regulated price) and cuts its costs. One way for a regulator to induce a symmetric choice of quality is by setting a high enough price floor to make the option of stealing market share via higher quality attractive to both firms. However, such a price floor results in a higher than socially optimal equilibrium quality choice.

It is possible to show that if the price floor is set equal to \( t \), the average quality choice rises compared to the unregulated system, but the full welfare effects are not clear since the asymmetry of the firms' choices
leads to a higher transportation cost and the convexity of the quality cost function induces an inefficient distribution of quality costs.

Another way for regulation to induce a desired quality choice is to fix a price completely in the market -- to prevent either higher or lower prices. This type of intervention perhaps more accurately characterizes the behaviour of the CAB for most of the period of airline regulation and was the focus of the earlier literature on price regulation and quality choice. The effects of this policy can also be seen in our model.

Proposition 4: Suppose that firms must offer their product at a fixed price, $p^*$. The equilibrium choice of quality is $\theta_0 = \theta_1 = kp^*/(4bt)$.

It is immediate from the Proposition, that if regulators set a fixed price equal to the price which would have emerged in the unregulated game, the firms would choose to provide the socially optimal quality level. This result along with the earlier discussion emphasizes that in order to induce greater quality competition, it is necessary not only to prevent price-cutting but to prevent rival's price hikes which can also dilute the incentives to raise quality. If too high a price is fixed, then firms provide a quality level above the socially optimal level.

The case of price ceilings is straightforward. If the ceiling is above the unregulated equilibrium price, then the regulation has no effect on the equilibrium. If the ceiling is at or below the unregulated equilibrium

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10 Within the context of this model, at least, this result is fairly robust in that as long as consumer's utility functions are separable and concave in quality, setting the regulated price equal to $t$ will ensure that firms choose the level of quality which maximizes the sum of producer and consumer surplus.
price, then the market behaves exactly as in the case of the fixed price.

5. The Choice of Quality When $\theta$ Cannot Be Observed Directly

It is generally assumed that if the true choice of quality cannot be directly observed, firms will have an incentive to underinvest in quality. In this section we illustrate that whether or not this is true depends on how consumers form their beliefs about the duopolists' true choice of quality. We return to the unregulated model of Section Three and focus on the interior symmetric equilibria from that analysis. Attention is restricted to the case in which $3t > 1$ so that for all possible consumer beliefs, the pricing game has an interior solution.

The exact way that consumers may form beliefs following the acquisition of information about the performance of the product is the source of both theoretical and empirical debate. In the airline industry, if consumers have Bayesian priors about the probability of an accident, say, which are concentrated on a single point, or if they know with certainty the true probability, then no updating of beliefs will occur even in the event of an accident. On the other hand, if their beliefs are diffuse, an accident will generally lead to non-trivial updating of beliefs. Tests of these responses have been made in the case of the airline industry. Borenstein and Zimmerman (1988) attempt to determine if an airline's average demand changes significantly following a fatal accident involving one of its planes and conclude that in both the regulated and deregulated periods, no significant effect can be found. However, Chalk (1987) examines the effect that manufacturer's fault crashes had on manufacturer's equity and Mitchell and Maloney (1990) conduct a similar study of the effect of airline-fault
crashes on the airline's stock value. In both cases, a significantly
negative effect was found and the authors argue that much of the fall is due
to a subsequent decline in consumer demand. We sit firmly on the fence in
this matter and examine the consequences of consumer learning on quality
choice under both characterizations of learning behaviour.

Consider first the case in which consumers begin with prior beliefs
\( \mu_0, \mu_1 \), and never revise them. The quantity demanded from firm \( j \), then,
depends only on the prices and the \( \mu \)'s and is independent of the choice of
\( \theta \). Firms, therefore, have only an incentive to minimize the cost of
investment in quality, and the equilibrium choice of \( \theta \) is thus zero.
Consumer ignorance and no learning results in no quality.

As a more interesting extension, consider next the case in which
consumers begin with beliefs \( \mu_{00}, \mu_{10} \), in period 0 but learn the true value
of the \( \theta \)'s immediately. This situation can be interpreted as the case in
which each consumer uses a unit of the service and their total experience is
made public. The mass of consumers and the law of large numbers ensures
that the true value of the \( \theta \)'s is determined after one period of experience.
With this behaviour, the problem of firm \( j \) (from (5) above) becomes

\[
\max_{\theta_j} v_j^0(\theta_j, \theta_i) = v_j(\theta_j, \theta_i) + \left[ (t + (\mu_j - \mu_i)/3)^2 \right] - \left[ (t + (\theta_j - \theta_i)/3)^2 \right]/(2t) \tag{14}
\]

and the best response functions change to

\[
\theta_j(\theta_i) = 0 \text{ if } \theta_i > 3t, \\
= 1 \text{ if } \theta_i < 3t + (1 - 18bt/(k - 1)), \\
= [(k - 1)/(18bt - (k - 1))](3t - \theta_i) \text{ otherwise.} \tag{15}
\]
This system yields the next result.

**Proposition 5:** Let \( k > 1 \). If consumers learn after one period the true choice of both \( \theta \)'s, the unique interior solution is

\[
\theta_j = \theta_i = \frac{(k - 1)}{\delta}.
\]

That is, the ability to hide the true level of quality in the first period leads each firm to cheat on the actual choice. Furthermore, the temptation to cheat falls as \( k \) or \( \delta \) rises. The more important become the subsequent periods, the less firms are tempted to take advantage of consumers' ignorance.

A more interesting case of asymmetry of information arises when the consumers' learning process becomes more drawn out. Suppose that, rather than consuming a unit of the commodity individually, consumers share in the consumption of the same service and therefore any given period yields only one observation on the success or failure of a firm's product. This interpretation matches the model in which the product is, for example, an airplane flight and the quality parameter, \( \theta \), concerns the probability of a service breakdown, perhaps resulting in a delayed flight or even an accident. Consumers, now, must form conjectures about the true value of the firms' choices of the \( \theta \)'s. The information available to them in any period \( \tau \) is the past history of successes of the product, denoted by the \( \tau \)-vector, \( y_\tau \). Consumers observe this information and must then form beliefs about the firms' quality choices. We assume that consumers act as statisticians and
form their conjectures via the maximum likelihood estimator

$$\mu_j(y_\tau) = \mu_j, \text{ if } \tau = 0,$$

$$= (\Sigma_{s=0}^{\tau-1} y_{js})/\tau, \text{ for } \tau > 0.$$  

(A1)

Observe that \(\mu_j(\cdot)\) is an unbiased, consistent estimator of \(\theta_j\).

Given this learning process, fix \(\theta_0\) and \(\theta_1\) and consider firm \(j\)'s expected revenues in any period \(\tau\) (with the expectation taken from period \(0\)).

$$E[\pi_j(\tau) \mid \theta_j, \theta_1] = E((t + (\Sigma y_{js}^0 - \Sigma y_{is}^0)/(3\tau))^2)/(2t)$$

(16)

where the sums are taken from 0 to \(\tau-1\). A few lines of computations using the independence of \(y_j\)'s across time and across firms yields an alternative expression

$$E[\pi_j(\tau) \mid \theta_j, \theta_1] = ((t + (\theta_j - \theta_1)/(3\tau))^2)/(2t)$$

$$+ (\theta_j (1 - \theta_j) + \theta_1 (1 - \theta_1))/(9\tau)/(2t),$$

$$= \pi_j(\theta_j, \theta_1) + (\theta_j (1 - \theta_j) + \theta_1 (1 - \theta_1))/(2t9\tau).$$

(16')

where \(\pi\) is the same as in equation (3). For given \(\theta\)'s, then, firm \(j\)'s discounted expected profits are

$$V_j^1(\theta_j, \theta_1) = V_j^0(\theta_j, \theta_1) + \Sigma \delta^\tau(\theta_j (1 - \theta_j) + \theta_1 (1 - \theta_1))/(2t9\tau),$$

(17)
where the summation is taken from \( r = 1 \) to \( N \). Define

\[
M = \sum_{r=1}^{N} \delta^r /(2t^r)
\]

The computation of the firms' best response functions in \( \theta \) proceed exactly as before and yield

\[
\theta_j(\theta_i) =\begin{cases} 
0 & \text{if } \theta_i > 3t(k - 1 + 3M)/(k - 1), \\
1 & \text{if } 3t(1 + 3M) + (1 - 18t(b+M)/(k - 1)) > \theta_i, \\
[3t(k - 1 + 3M) - (k - 1)\theta_i]/(18t(b + M) \\
- (k - 1))] & \text{else.}
\end{cases}
\] (18)

The objective function of firm \( j \) in (15) is the same as when consumers learn immediately plus a term which depends on the variances of the two random variables. The second term is a function whose maximum is \( \theta_j = 1/2 \). Thus, the equilibrium choice of the \( \theta \)'s in the case in which consumers form MLE estimates over time may be above or below those in which consumers learn immediately. If the solution to (15) exceeds one-half, the lack of consumer information draws \( \theta \) even further down. However, if the solution to (15) is less than one-half, the effect of poor consumer information raises the quality level. We can see this directly by solving the system in (18) to get

**Proposition 6:** Let \( k > 1 \). If consumers form their quality beliefs in common by the maximum likelihood estimator conditional on the observed realizations of the products performance, the unique symmetric equilibrium
choice of $\theta$'s is

$$\hat{\theta}_j(\theta_i) = (k - 1 + 3M)/6(b + M)$$

**Proposition 7:** This value is below the equilibrium choice of $\theta$ under immediate learning if and only if $k - 1 > 3b$, or $\theta > 1/2$.

The intuition for why the effects of consumer learning on the equilibrium choice of $\theta$ are ambiguous can be seen in the expression for per period profits in equation (4). Since per period profits are convex in prices, a firm can raise its expected revenues when subsequent prices are stochastic. Of course, if consumers learn the true value of $\theta_j$ immediately, the subsequent realization of prices is deterministic. The higher the variance of the random variable, the higher the variance of the beliefs of the consumers, the greater the variance of subsequent prices, and, so, the greater the firm's expected revenue.

A closer examination of equation (17) is of interest since it illustrates the return to the two firms from the consumers' lack of information. The last term in (17) is a positive function of the two variances of the random variable of the successes of the two goods. It represents a gain to firm $j$ from consumers' ignorance not only about its own quality choice but also about its rival's choice.

**Proposition 8:** Firm $j$ does better in an environment in which consumers are ignorant about one or both of the firms' choices of $\theta$ and form their beliefs as maximum likelihood estimates than in one in which they learn the true
values after the first period.

That is, firms benefit from consumer’s lack of information about either good. This result runs somewhat counter to other models which suggest that consumers’ tendencies to assume the worst (Grossman (1981)) induces firms to ensure that their quality choice is either made known or ensured via a mechanism of warranties. It also suggests that even if rival firms know one another’s quality choices, it may not be in their interest (ex ante) to reveal them. The general reluctance of firms to engage in negative advertising may in part be explained by this incentive.

Of course, the consumers’ learning process need not be that of the classical statistician proposed above. We could consider consumers as sophisticated Bayesians. If consumers knew the full model and recognized that, given their learning procedure, firms would optimize against it, the internally consistent solution to this problem should generate a consumer belief that corresponded exactly to the equilibrium choice of quality by the firms. This analysis would yield the Bayesian prior. Proposition Nine shows that in this case, the only pure strategy equilibrium choice of quality is to set $\theta = 0$ or 1.

Proposition 9: In an environment with sophisticated Bayesian consumers, a $\theta$ strictly in the interval $(0,1)$ cannot be an equilibrium (pure strategy) choice of quality.

Proof: Suppose that the firm’s equilibrium strategy was a pure strategy choice of $\theta$ in the open interval, $(0,1)$. Consumers would have a degenerate Bayesian prior putting full weight equal to the equilibrium value of $\theta$. Now, though, any observation of $y_r$'s is consistent with the prior and
consumers have no reason to update over time. But we have already seen that if \( \mu_j(*) \) is unaffected by \( \theta \), then firm j's best response is to choose \( \theta_j = 0 \).

6. Conclusion

When an industry can choose quality levels as well as quantity, attempts to restrict price competition will generally have effects on the choice of quality. Our analysis illustrates, though, that the form of price regulation matters in unexpected ways. If only a price floor is set, the possibility that one firm will set a higher price can reverse the other firm's incentive to maintain a high quality level and lead it to respond with a lower than otherwise level. Fixed prices can generate desired quality levels but only if regulators set the price equal to the unregulated equilibrium price. Too high a price results in an overinvestment in quality.

The differentiated duopolist model also provides a simple environment in which to address the effects of consumer ignorance on quality. In an unregulated environment, whether or not consumer ignorance about actual quality levels leads to higher or lower choices by firms depends on how consumers ultimately learn. If consumers do not learn at all, then, not surprisingly, quality suffers. However, if consumers are slow learners, firms might be induced to raise their quality relative to the perfectly informed case. If consumers perfectly forecast the firms' behaviour as Bayesian game players, the effect may be to induce firms not to invest in quality at all.
References


