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# THE TIMING OF ENTRY INTO NEW MARKETS

by

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## Abstract

Under what circumstances will a successful incumbent in a related market be the first to enter a new market? We present a model in which the order of entry into new markets has long run effects on the firms' profits. We assume that a firm that is successfully producing in a related market has valuable information about the demand in the new market. By his choice of location in product space in the new market the incumbent reveals information about the demand to the potential entrant. Thus, the incumbent would like to enter after the newcomer in order to prevent the rival from free riding on his proprietary information; however, the rival would also like to enter second so that he can benefit from the other's information.

When both firms want to enter last, the order of entry is modeled as a timing game in continuous time. Using a refinement of Nash equilibrium known as "risk-dominance" we show that when the informational advantage of the incumbent is very great, his implicit threat to wait out his rival is less powerful than the equivalent threat by the potential entrant, and the incumbent will enter first. On the other hand, the incumbent's lower incentive to enter the new market due to the "cannibalization" effect of entering a related market is a weapon that the incumbent can use to "force" the rival to enter first, in equilibrium. We also find that incumbent entrants into new markets are more likely to succeed in the new market, in equilibrium, than are newcomers, regardless of order of entry. On the other hand, looking cross sectionally across markets, incumbents are more likely to succeed when they are early rather than late entrants, but newcomers are more likely to succeed when they are late entrants rather than early.

## I. Introduction

It is a rule of thumb in bridge that, whenever possible, one should not break a new suit. If you can force the opponents to break the new suits it puts your team at a competitive advantage. In this paper we analyze the incentives of and the competitive advantages and disadvantages to firms of being the first to enter new markets. In particular, we ask: under what circumstances will a successful incumbent in a related market choose to be the first to pioneer a new market rather than entering after a newcomer has entered? We show that in our model, as in bridge, both firms would prefer to be the later entrant. Which firm actually enters first is derived as the equilibrium to a sequential game in continuous time, and we present conditions under which each firm is, in equilibrium, more likely to enter first.

There is evidence from industry studies that pioneering entry by newcomer firms is an important phenomenon, even in the presence of the threat of responsive entry by established incumbents in related markets. Schmalensee's (1978) study of the ready-to-eat cereal industry is a clear example, as is the automated teller machine market studied by Lane (1989). In both of these industries the first entrants into the new market were not the firms that were already successful in related markets, but rather were start-up firms or firms operating in unrelated industries. Nevertheless, in both cases it was the incumbents from related markets that were the eventual survivors, though their late arrival suggested apparent reluctance to enter the market. Why would the incumbents (who, when viewed ex post, apparently had an ex ante higher probability of success in the new market) be so reluctant to pioneer the new market, while newcomers (who ultimately failed) showed greater enthusiasm? A feature of my model is that reluctance to enter a new market does not signal that the incumbent views the market as unprofitable; rather, it means that the incumbent views it <u>more</u> profitable if he can induce the rival to enter first.

Other authors (Kamien and Schwartz 1978; Conner 1987; Ghemawat 1986; and Reinganum 1983) have pointed out that incumbents tend to have less incentive to enter a new related market than do new entrants. The incumbent's reward for entry must net out the decreased profits in the old market due to the new substitute product, while from the perspective of a new entrant, the decrease in profits in the old market are an externality that he does not internalize. Kamien and Schwartz show formally that monopoly incumbents are less likely to innovate than newcomers to a market. Reinganum shows that, in the Nash equilibrium of an innovation race, the incumbent invests less than the rival, and therefore the rival is most likely to innovate first. Conner shows that if the innovator need not introduce the innovation upon discovery but can wait until a more desirable introduction time, then the incumbent will invest more in innovation but wait until the rival introduces the new product to introduce its own.

Our paper takes a different approach. We do not analyze the R&D problem, but rather focus on the strategic positioning decision faced by the firms. We assume that a firm that is successfully producing in a related market has proprietary information about the new market that the rival does not have. Specifically, the incumbent has some information about which types of new products are more likely to be successful. The order of entry affects the firms' long run expected profits because it determines the transmission of information.[1] If the informed firm enters the new market first, the uninformed potential entrant can learn about the market just by observing the type of product that the incumbent plans to produce (even without waiting to see whether the incumbent's new product turns out to be a success). If the uninformed firm enters first it must commit to a product type without benefit of the other firm's knowledge. The advantage to late entry into a market by a new entrant is that it permits the firm to learn from the informed rival before committing to a product type. Of course, there is a corresponding advantage to late entry by the incumbent: late entry prevents the uninformed rival from benefitting from the incumbent's information. In addition, whichever firm is the later entrant enjoys the power to position itself <u>relative</u> to the observed strategy of the other. We will show that, although both firms would like the strategic advantage of going last, there is no actual delay in equilibrium. However, when the informational advantage of the incumbent is very great, his implicit "threat" to wait out his rival is <u>weak</u>; that is, it is <u>less</u> powerful (in a sense to be made formal later) than the equivalent threat by the potential entrant. In this case, contrary to the results in the innovation-race literature, late entry is so valuable to the entrant that he can "force" the incumbent to enter first. On the other hand, if by entering the new market

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[1] In interesting related work, Ramey (1988) analyzes a model is which late entrants benefit from information transmission, but in a model with ex ante symmetric firms. Thus, the natural focus of his paper is on issues of delay of entry, rather than the order of entry of firms that are identifiably different ex ante. McGahan (1990) considers whether a firm with an informational advantage will find it optimal to deter entry through large capacity commitment. In that model the informed firm is assumed to enter first; the model focuses on the firm's capacity choice at the time of entry.

the incumbent will strongly cannibalize his original market, this decreased incentive to enter the new market is a weapon the incumbent can use against the rival to induce him to enter first.

We also find that incumbent entrants into new markets are more likely to succeed in the new market, in equilibrium, than are newcomers, regardless of order of entry. Looking cross sectionally across markets, incumbents are more likely to succeed when they are early rather than late entrants, but newcomers are more likely to succeed when they are late entrants rather than early.

## II. The Model

Let firm A be a (successful) incumbent producing a product, x. Firm A is considering introducing to the market a never-before produced product, y. At the same time, a potential entrant, B, is considering entering the y market. It is assumed that x and y are substitutes in consumption. In deciding whether and when to enter, the positions of the two firms are different for two reasons: first, the decision problem for A must account for the effect of y on A's profits in the x market, while B's decision problem involves no such considerations; second, A, being an experienced producer in x, has expertise and knowledge that may be valuable in y.

Because y is a new product, no one knows whether it will be successful, which particular features of the product will be popular, what the optimal production process is, and so forth. Suppose that there are many possible technologies for producing the product. We will think of these technologies as determining a location in product space, and to avoid endpoint problems we will take the product space to be the circumference of a circle (see

Salop 1979). Denote a position on the circle w. One could alternatively interpret the choice of location w on the circle to determine the firm's marginal cost, though we will adopt the former interpretation in our discussion.

Some comments are in order regarding our meaning of "entry." What is important in our model is that, at the time of "entry," a firm commits to a particular technology, or location in product space. We are not so much predicting when a firm will begin to sell output as the order in which firms will take a position in the market. For example, this may mean investing in development of a particular type of product. If the time it takes for the rival to discern the other's product strategy is long, then being first may confer an important "head start." (See Lieberman (1987) for a survey of the literature on first mover advantages.) We abstract from these issues by assuming that committing to a technology and producing the product are simultaneous events.

A firm must commit to a technology before learning what the success of the chosen product will be. For example, before the introduction of personal computers, IBM and Apple had to commit to a particular operating system before knowing either the relative costs of various possible technologies, or their relative desirability in the eyes of consumers. We assume that rivals can learn or observe the other's chosen location in product space before nature has revealed the level of demand associated with that technology. Continuing our example, this means that Apple knew that IBM was developing a disk operating system (DOS) before either firm knew how successful that system would be. This confers a potential advantage on the firm that goes <u>second</u>: that firm can make its product choice having observed the strategy of the first entrant.

We assume that the level of demand for each firm's product is unknown ex ante, and by choice of "technology" (i.e., product characteristics) the firms determine a probability of a successful product and an ex ante correlation between the likelihood of success of their products.

Let inverse demand be given by

$$
\begin{aligned}
(1) \qquad & p_x = m - bx - c(y_A + y_B) \\
& p_y^i = \tilde{a}_i - b(y_A + y_B) - cx, \quad i = A,B, \; b > 0, \; c > 0,
\end{aligned}
$$

where $y_i$ is the output of firm i in the y market[2]. Marginal costs are constant at $k_j$, $j = x,y$. Thus, flow profits to firm A are

$$
\begin{aligned}
\pi^A = {} & (m - bx - c(y_A + y_B))x - k_x x \\
& + (\tilde{a}_A - b(y_A + y_B) - cx)y_A - k_y y_A .
\end{aligned}
$$

Flow profits to firm B, if it enters the y market, are

$$
\pi^B = (\tilde{a}_B - b(y_A + y_B) - cx)y_B - k_y y_B .
$$

The intercept $\tilde{a}_i$ is a random variable for each firm. Its distribution is determined by the location the firm chooses in product space, and how "close" (in a sense to be discussed presently) the locations of the two

[2]The demand structure implies that the uncompensated cross-price effect, b, is symmetric. Of course, this would only be strictly true if income effects were zero; nevertheless, we adopt this structure for tractability. We do not believe the assumption materially affects the results of the paper.

firms are to each other.

This specification captures the idea that one product may turn out to be greatly preferred to the other, because one firm was better able to guess accurately which product features will appeal to many people. "Greatly preferred" means that, if $a_i > a_j$, $p_i = a_i - a_j + p_j$, regardless of the quantities sold. That is, in any equilibrium the price charged for $y_i$ will exceed that for $y_j$. Further, by observing his rival's product specifications, a firm can design a product whose success is likely to be highly correlated by imitating the rival's product; or uncorrelated (or perhaps negatively correlated) by innovating relative to the rival's product.

To further simplify, we assume that $a_i$ can take only two values, $a^H$ and $a^L$, $a^H > a^L > k_y$. If A locates at $w_A$ and B at $w_B$, then the joint distribution of the a's can be fully described by the function P(w), where $P(w_k) \equiv P_k = \text{prob}(a_k = a^H | w = w_k)$, k = A,B, and the function $Z(w_A, w_B) \equiv Z_{AB} = \text{prob}(a_A = a^H,\ a_B = a^L)$. These parameters determine the covariance as follows:

$$\sigma_{ij} = \text{cov}(a_i, a_j) = [P_i(1 - P_j) - Z_{ij}]$$

and the following probabilities:

$$\text{prob}(a_i = a^H, a_j = a^H) = P_i P_j + \sigma_{ij}$$
$$\text{prob}(a_i = a^L, a_j = a^L) = (1 - P_i)(1 - P_j) + \sigma_{ij}$$
$$\text{prob}(a_i = a^L, a_j = a^H) = (1 - P_i)P_j - \sigma_{ij}.$$

Now suppose that the firms had no prior knowledge that generated distinct priors over the technologies. In other words, suppose the firms placed the same probability of success, P, on every location in product space. Although firms may not know which product type is likely to succeed, we assume that they can easily determine which are "similar" and which are "dissimilar" to each other. If a firm adopts a similar product location, or "imitates," he is choosing to cast his lot with his rival and they succeed or fail together. If he decides to strongly differentiate his product from his rival's (or "innovate"), then he benefits from the increased probability of succeeding when his rival fails, but also risks the increased probability of failing when his rival succeeds.

Output decisions are made by each firm after their $a_i$ have been revealed. More realistically, one could assume that it takes some period of uninformed competition before the $a_i$ are revealed. However, for a long enough horizon, a short enough period, or a sufficiently low discount rate, this period of information gathering will be an unimportant component of the present discounted value of the future profit stream. As our purpose is not to study this information gathering (see Grossman, et al. 1977), and there does not appear to be an interaction between it and the problem at hand, we ignore it in this paper.

Finally, we assume that once a technology is adopted the choice is irreversible for the foreseeable future. It is crucial to our model that, in particular, the pioneer firm is locked into its technology choice for an economically meaningful period of time. However, our reduced-form profit functions permit the possibility of exit from the market entirely (see Judd 1985).

Once one firm has entered a market and chosen a technology, the rival firm has the option of imitating or innovating relative to the rival. Specifically, by its choice of technology the <u>later</u> entrant chooses the <u>covariance</u> in success rates between the two firms. If firm A is operating in both the x and y markets, and firm B is competing in the y market, each firm i's ex ante expected profits are

$$
\begin{aligned}
(2) \qquad E\pi^i = {} & (P_A, P_B + \sigma_{AB})\pi^i(a_A^H, a_B^H) + ((1 - P_A)(1 - P_B) + \sigma_{AB})\pi^i(a_A^L, a_B^L) \\
& + (P_A(1 - P_B) - \sigma_{AB})\pi^i(A_A^H, a_B^L) + (P_B(1 - P_A) - \sigma_{AB})\pi^i(a_A^L, a_B^H)
\end{aligned}
$$

where

$$\pi^A(a_A^j, a_B^k) = \max_{x, y_A} \pi^A(x, y_A; a_A = a^j, a_B = a^k), \; j,k \in \{H,L\},$$

and

$$\pi^B(a_A^j, a_B^k) = \max_{y_B} \pi^B(y^B; a_A = a^j, a_B = a^k),$$

that is, $\pi^i(a_A^j, a_B^k)$ is the reduced form profit function. Maximizing expected profits with respect to the covariance:

$$(3) \qquad dE\pi^i/d\sigma_{AB} = \pi^i(a_A^H, a_B^H) + \pi^i(a_A^L, a_B^L) - \pi^i(a_A^H, a_B^L) - \pi^i(a_A^L, a_B^H).$$

Denote this derivative $\Delta^2 E\pi^i$, and note that

$$(4) \qquad \mathrm{sign}(\Delta^2 E\pi^i) = \mathrm{sign}(d^2\pi^i(a_A, a_B)/da_A da_B)$$

when the cross derivative on the right takes a constant sign.

<u>Lemma 1</u>: Under the specification given by (1), $\Delta^2 E\pi^i < 0$, i = A,B.

Proof: Straightforward but tedious calculations show that

$$d^2\pi^A(a_A,a_B)/da_Ada_B = d^2\pi^B(a_A,a_B)/da_Ada_B = -4/9b. \qquad \text{Q.E.D.}$$

This result means that both firms would choose the corner solution in which the covariance is as low as possible.

We have assumed that the firms' choice of product characteristics determines a distribution over their demand conditions. We could instead assume that demand is fixed and the technology choice determines the firms' marginal costs. Then the choice of correlation would lead to a condition analogous to (3),

$$dE\pi^i/dt = \Delta^2E\pi^i,$$

and, analogously,

$$\text{sign } \Delta^2E\pi^i = \text{sign}(d^2\pi^i(k_A,k_B)/dk_Adk_B),$$

where $\pi^i(k_A,k_B)$ is the reduced form profit function for firm i, whose arguments are the firms' marginal costs. With linear demand, straightforward calculations show that

$$d^2\pi^i(k_A,k_B)/dk_Adk_B = d^2\pi^i(a_A,a_B)/da_Ada_B \text{ for both } i = A,B.$$

Some remarks are in order regarding the expression $d^2\pi^i(k_A,k_B)/dk_Adk_B$, which we denote expression (5). A similar expression shows up in other contexts

(Glazer 1989; Reinganum 1983; Bagwell and Staiger 1990) but, as these other authors have noted, it does not appear to be possible to characterize general conditions on demand that determine its sign.[3] Investigating the sign on a case-by-case basis for different functional forms appears the only tractable route, but one quickly exhausts the set of functional forms that are amenable to brute force calculation. Given these difficulties, it is nevertheless interesting to note that neither we nor other authors have succeeded in finding demand specifications in which the sign of (5) is positive (unless one firm is assumed to have a large expected cost advantage). This is true under Bertrand or Cournot assumptions. We and others have verified that for demand functions of the form $p = a - bx$, $p = a - bx - cx^2$, and $p = a - b\ell n\ x$, (5) is negative under Cournot competition. The expression is also negative under linear demand with increasing marginal costs of the form $k_i + \alpha y$, $i = A,B$. In addition, (4) is negative for linear demand under Bertrand competition, and (5) is negative under Bertrand competition with linear demand as well as Cournot. Therefore, at least for changes in costs (resp. demand levels) that are small enough that the linear model is a good approximation, we feel confident in assuming more generally that (5) (resp. (4)) is negative.[4]

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[3]The difference between our expression (5) and the expression that arises in the papers cited is the presence of the x market in our model. Both firms must take into account A's operations in x when determining their output in y. Consequently, (5) has terms that would not otherwise appear. Of course, the added complication makes an already intractable expression even more so, but our investigation on the sign of (5) on a case-by-case basis does not reveal an example in which the added market changes the sign of (5).

[4]It is also worth nothing that (5) is not related to Bulow, Geanakoplos and Klemperer's (1985) definition of strategic substitutes and complements. Whether two markets are strategic complements or substitutes depends on the sign of the cross derivative of the direct profit function with respect to

The implication of Lemma 1 is that, absent other considerations, the firms have congruent interests regarding the covariance in demand (or cost) parameters; namely, both would prefer to have as low a covariance as is feasible. The high profits that a firm accrues when it "succeeds" (gets $a^H$) and his rival fails (gets $a^L$) more than makes up for the low profits when he fails and the rival succeeds <u>relative</u> to the profits he gets when both succeed or both fail together. If both firms were completely ignorant as to which technologies were more likely, ex ante, to succeed, then they would be indifferent as to who enters first; the latter entrant would innovate as much as possible (i.e., choose the technology with the lowest covariance) relative to the first. (Note, however, that both prefer sequential entry to simultaneous entry if the simultaneity impedes the firms' ability to knowingly differentiate themselves from each other.)

Now suppose that due to his successful operations in the x market, firm A has better information or expertise that allows him to put higher probability of success on some locations $w_i$ than others. Firm B, on the other hand, views all product locations as equally likely to succeed. When B has inferior information it may be in his interest to wait until A has chosen a position to choose his own. A's product choice reveals valuable information about the market to B. Roughly speaking, if B's

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the firms' outputs; in our notation:

$$d^2\pi^A(x,y_A,y_B)/dy_A dy_B,$$

while (5) is the cross derivative of the reduced-form profit function with respect to the cost parameters. The Bulow-Geanokoplos-Klemperer concept describes the way reaction curves shift for a given set of parameters, whereas (4) and (5) summarize the effect on profits the of change in the actual equilibrium when parameters change. These are conceptually quite different effects and the two need not take the same sign.

objective were to maximize the probability of success, he would want to observe A's choice and then imitate it. However, by imitating A's product (or cost) technology, B also maximizes the covariance between his likelihood of success and his rival's, which lowers his expected profits. Thus, his optimal product choice involves a tradeoff between the advantages of imitation (higher probability of success) and the advantages of innovation (lower correlation of success rates).

Suppose A chooses location $w_j$. What information does this reveal? Suppose the important features of the product can be measured on an ordinal scale--for example, an ice cream product may be ranked on how much butterfat it contains; a computer may be judged on how fast it is and how user-friendly it is. In some cases the characteristics themselves are clearly desirable but involve tradeoffs (a faster or more user friendly computer is more costly to build); in other cases the optimal level of the trait itself (such as sweetness) may be in question. In any case, A's choice locates its product in characteristic space, and we assume that products that are "closer" to A's choice are more highly correlated with its success, and that the closer is i to j in this product space, the closer is $P_i$ to $P_j$. More formally, we assume a function P(w) with the properties

(A.1) $\exists$ a unique $\bar{w}$ such that $\bar{w} = \operatorname{argmax} P(w)$

(A.2) $P(\bar{w} + k) = P(\bar{w} - k)$ for $k \leq$ one-half the circumference of the circle.

(A.3) $dP(w)/d(|w - \bar{w}|) \leq 0$ for $|w_i - w_j| \leq$ one-half the circumference of the circle.

(A.4) $dP^2(w)/d(|w - \bar{w}|^2) \leq 0$ for $|w_i - w_j| \leq$ one-half the circumference of the circle.

A.1-A.4 mean that the function P(w) has a unique maximum at $\bar{w}$, and is symmetric, decreasing, and concave around $\bar{w}$. Let $\bar{P} = P(\bar{w})$.

We specify the covariance function as follows. Without loss of generality, let $w_i < w_j$. If $\sigma(w_i, w_j)$ is the covariance in the joint distribution of $a_i$ and $a_j$, then

(A.5) $\quad d\sigma(w_i, w_j)/d(w_j) < 0.$

(A.6) $\quad d^2\sigma(w_i, w_j)/d(w_j)^2 > 0.$

In other words, as the locations get farther apart the covariance in success rates decreases at a decreasing rate.

We assume that B knows the functions P(w) and $\sigma(w_i, w_j)$, but not the location of $\bar{w}$. B's prior on the location of $\bar{w}$ is uniform. Thus, if A is expected to choose a product location with a high probability of success, B's posterior after observing A's choice will assign a higher probability of success to products located "close" to $w_A$.

Now suppose that given A's location $w_A$, the location that maximizes B's expected probability of success is also $w_A$. Then B's optimal location choice, if he observes A's choice is

$$\max_{\sigma_{AB}} (P_B P_A + \sigma_{AB})\pi^B(a_A^H, a_B^H) + [(1 - P_B)(1 - P_A) + \sigma_{AB}]\pi^B(A_A^L, A_B^L)$$
$$+ (P_A(1 - P_B) - \sigma_{AB})\pi^B(a_A^H, a_B^L) + [P_B(1 - P_A^L) - \sigma_{AB}]\pi^B(a_A^L, a_B^H),$$

given that $P_B = P_B(\sigma_{AB})$. Thus, B's optimal location decision must satisfy

$$\Delta^2\pi^B + [P_A(\pi^B(a_A^H, a_B^H) - \pi^B(a_A^H, a_B^L)) + (1 - P_A)(\pi^B(a;_A^L, a_B^H) - \pi^B(a_A^L, a_B^L))](dP_B/d\sigma_{AB})$$

$$\equiv \Delta^2\pi^B + [P_A\Delta\pi^B(a_A^H, \cdot) + (1 - P_A)\Delta\pi^B(a_A^L, \cdot)](dP_B/d\sigma_{AB}) = 0.$$

Thus, B's optimal location, given B's estimate of $P_A$, satisfies

(6) $$dP_B(\sigma_{AB}^*)/d\sigma_{AB} = -\Delta^2\pi^B/[P_A\Delta\pi^B(a_A^H, \cdot) + (1 - P_A)\Delta\pi^B(a_A^L, \cdot)],$$

where the numerator is the marginal value to B of decreasing the covariance, and the denominator is the expected marginal value to B of increasing his probability of success. Call the correlation that satisfies (6), $\sigma_B^*$.

The solution is illustrated in Figure 1.

[FIGURE 1 ABOUT HERE]

Let $\bar{\sigma}$ be the highest correlation achievable--that is, it is the correlation if both firms locate together. $C_1$ is B's opportunity frontier in this space when B locates at $w_A$. A.1-A.6 determine the increasing concave shape of the frontier. B's isoprofit curves are linear, with profit increasing to the northwest. Thus, B's optimal location results in $(P_B^*, \sigma_B^*)$ when A locates at $w_A$.

Now consider A's decision problem if A enters first. A must choose $P_A$ to maximize

(7) $$(P_BP_A + \sigma_{AB})\pi^A(a_A^H, a_B^L) + [(1 - P_B)(1 - P_A) + \sigma_{AB}]\pi^A(a_A^L, a_B^L)$$

$$+ (P_A(1 - P_B) - \sigma_{AB})\pi^A(a_A^H, a_B^L) + (P_B(1 - P_A) - \sigma_{AB}\pi^A(a_A^L, a_B^H),$$

keeping in mind that $P_B = P_B(P_A)$ via (6). Thus, $P_A^*$ solves

$$[P_A\pi^A(a_A^H, a_B^H) - (1 - P_A)\pi^A(a_A^L, a_B^L) - P_A\pi^A(a_A^H, a_B^L) + (1 - P_A)\pi^A(a_A^L, a_A^H)]$$
$$+ P_B\pi^A(a_A^H, a_B^H) - (1 - P_B)\pi^A(a_A^L, a_B^L) + (1 - P_B)\pi^A(a_A^H, a_B^L) - P_B\pi^A(a_A^L, a_B^H)$$

$$(8) \quad = [P_A^*\Delta\pi^A(a_A^H, \cdot) + (1 - P_A^*)\Delta\pi^A(a_A^L, \cdot)](dP_B(P_A^*)/dP_A)$$
$$+ [P_B(P_A^*)\Delta\pi^A(\cdot, a_B^H) + (1 - P_B(P_A^*))\Delta\pi^A(\cdot, A_B^L)] = 0,$$

where the first term in brackets is negative and the second is positive. The first term is the loss that A incurs if increasing $P_A$ also increases $P_B$ at the margin by $dP_B/dP_A$; the second term is the direct benefit to A of increasing his probability of success.

In the region at and around $\overline{P}$, $dP_B/dP_A > 0$. To see this, note that $P_B$ is really a random variable. By (6) and A.1-A.6, B's positioning strategy once A enters is to choose a distance $\Delta w$ (determined by $\sigma$) from A, which is independent of A's location. However, B could locate $\Delta w$ to the "right" or "left" of A in product space. Assume B chooses left or right with probability 0.5 each. Then if A.4 holds with strict inequality, B's expected probability of success <u>falls</u> as A moves small distances away from $\overline{P}$. Thus, in this region, $dEP_B/dP_A < 0$ and A will choose an interior solution at which $P_A < \overline{P}$. An interesting result, then, is that when A.4 holds with strict inequality, A will choose a location that he knows does not maximize his probability of success, in order to decrease B's probability of success. Of course, B knows that this is A's optimal strategy, and because B knows the functions $P(\cdot)$ and $\sigma(\cdot)$ B even knows what A's probability of success is at $w_A$. Nevertheless, since B's prior is uniform over the circle and the function $P(\cdot)$ is symmetric and concave, the location that maximizes B's expected probability of success is exactly $w_A$. Thus, the strategies in (6) and (8) are consistent.

If A.4 holds with equality then locating at $\bar{w}$ will maximize A's profits. In order to simplify the analysis we will assume this to be the case for the remainder of the paper.

Now suppose that B moves first. Since B has no information about the relative desirability of the locations, B can be assumed to choose at random according to a uniform distribution on the circle. Let B's choice be $w_B$. What will be A's optimal location in response? A's tradeoff is to move close to $\bar{w}$ (to maximize $P_A$), and also move away from $w_B$ (to minimize $\sigma_{AB}$). Thus, given $w_B$, $\sigma^*_{AB}$ solves

$$[P_B \Delta\pi^A(\cdot, a_B^H) + (1 - P_B)\Delta\pi^A(\cdot, a_B^L)] dP_A(\sigma^*_{AB})/d\sigma_{AB} + \Delta^2\pi^A = 0,$$

for an interior solution. Thus, $\sigma^*_{AB}$ must satisfy

$$dP_A(\sigma^*_{AB})/d\sigma_{AB} = -\Delta^2\pi^A \;/\; [P_B \Delta\pi^A(\cdot, a_B^H) + (1 - P_B)\Delta\pi^A(\cdot, a_B^L)], \tag{9}$$

where the right side is a constant. Figure 2 illustrates Condition (9).

[FIGURE 2 ABOUT HERE]

A's opportunity frontier depends on B's location, which is random. If by his good fortune B locates at $\bar{w}$ then A's opportunity set is denoted $C_2$.

If B locates at $w_1$, with $P(w_1) < \bar{P}$, then A's frontier will look like $C_3$. It is downward sloping over the region in product space in which A can move away from $w_B$ and toward $\bar{w}$. Beyond $\bar{w}$ there is a positive tradeoff between P and $\sigma$. Since an optimum will never be located in the negatively sloped region, it is represented in broken lines. The solid-line frontiers that are higher to the northwest correspond to <u>lower</u> probability of success locations for B.

Let $-\Delta\pi^A/P_B\pi^A(\cdot,a_B^H) + (1 - P_B)\Delta\pi^A(\cdot,a_B^L) = K$. Hence, A's isoprofit curves are linear with slope K. For some range of locations for B with low probability of success, A may be at the corner solution at which $P = \bar{P}$. ($dP/d\sigma$ need not be 0 at $\bar{P}$.) As B locates closer to the optimal product, A will choose an interior solution. Thus, there will be a locus of tangencies characterizing A's optimal location choice such as L.

In equilibrium, the presence of B induces A to sacrifice some probability of success in order to differentiate himself from his rival, if his rival chooses a product location that is sufficiently close to $\bar{w}$. It is interesting that, at the margin, the higher the probability that B's product succeeds, the <u>lower</u> the probability of A's success, and this is by A's choice.

Denote the optimal correlation for A if B chooses $\bar{w}$ to be $\hat{\sigma}_A$. Comparing Figures 1 and 2, $\sigma_B^*$ may be greater or less than $\hat{\sigma}_A$. Thus, although it appears likely, we cannot unambiguously claim that the ex ante expected value of the correlation chosen by A if B enters first will be less than $\sigma_B^*$, the correlation B will choose if A enters first.

On the other hand, one can conclude that the extremes on the low (or negative) end of the covariance are likely to have resulted from the incumbent entering after the new firm. Thus, in markets with highly differentiated products, it is likely that previous incumbents in related products were late rather than early entrants.

Additionally, we can conclude the following:

<u>Proposition 1</u>: Assume that A.4 holds with equality. Then the expected probability of success for an incumbent is higher when he is the first rather than second entrant; the expected probability of success for a newcomer is higher if he is the second rather than first entrant.

<u>Proof</u>: By A.4, when the incumbent enters first he chooses $\bar{w}$, at which the probability of success is $\bar{P}$. When he enters second he faces the tradeoff given by (9), illustrated in figure 2. Since B's entry decision is uniform on the product circle, there will certainly be a set of positive probability on the product circle such that, if B locates there, A will locate strictly away from $\bar{w}$. Thus, A's <u>expected</u> probability of success for A is certainly less than $\bar{P}$.

For B, when B enters first he chooses a location blindly. His expected probability of success is simply the expected probability of success over the entire product space. When he enters second he will, on average, locate closer to $\bar{w}$, by (6). Thus, the result holds by A.1 and A.3. Q.E.D.

We now ask whether the firms would prefer to enter first or last. Consider first firm A. The advantage to firm A of moving last is that by doing so B is forced to choose a location in ignorance. The disadvantage is that by going first, B may, by chance, enter near the optimal location and "stake it out" for himself. B's entry near $\bar{w}$ does not preclude A from entering at $\bar{w}$ also, but B's presence there will render it no longer profit maximizing for A to do so -- the correlation with B's product will be higher than desirable to A. By entering first, A insures his position at or near $\bar{w}$, and insures that B will locate some distance away.

Thus, whether the incumbent prefers to enter first or last depends in part on how likely it is that B will randomly locate at a near-optimal position in product space.

Firm B faces a similar tradeoff. By entering first he enters blindly, but has some chance of locating near the optimal product. If he waits for A to enter he learns which products are more likely to succeed, but A will have already staked out the best product. Once again, the intuition is that it is more likely that B will want to enter last if the probability of blindly locating near the optimal product is low.

As the discussion suggests, the tradeoff could tip either way for each firm -- it might be optimal for either or both firms to enter first if the stakeout effect is important. However, intuitively one would expect that as A's informational advantage becomes greater, it becomes more likely that both firms would prefer to enter second.

We formalize this notion as follows. Let the closed interval $[w_1, w_2]$ be a strict subset of the product circle such that for all $w \in [w_1, w_2]$, $P(w) > 0$, and for all $w \notin [w_1, w_2]$, $P(w) = 0$. That is, the interval contains all locations that have positive probability of success. Now hold constant the function $P(\sigma)$ over this domain. Then we say that information is <u>more valuable</u> as the size of the circle grows, holding constant the size of $[w_1, w_2]$. This leads to the following intuitive result:

Proposition 2: For sufficiently valuable information, both firms prefer to enter second rather than first.

Proof: See Appendix A.

Because we are interested in new markets here, the spirit of this paper is that A's proprietary information is of great potential value. That is, left to guess on his own, we think it is reasonable to expect that in a new market B is likely to choose a product mix with very low probability of success. We will assume that by their very nature, new markets are characterized by great uncertainty about the product mix that consumers will find desirable. Thus, for the remainder of the paper we will assume that by definition of a new market, information is sufficiently valuable that, by Proposition 2, both firms prefer to enter last.

## III. The Timing Game

Our analysis thus far indicates that, when the informational asymmetry is great both firms would prefer to enter the new market second rather than first, but the payoffs to doing so differ between the firms. Both firms would like to be able to credibly "threaten" the other to wait him out. Intuitively, one would expect that the firm that bears the least cost in waiting, and/or has the most to lose in entering first, would be able to make the more credible threat to wait until the other has entered, and would be the firm to succeed in entering last. We formalize that intuition here by modeling the players' choice of entry time as a game in continuous time.

We will assume that the firms face an infinite horizon over which they would enter, and have discount rate r. They choose their entry time noncooperatively, but each can instantaneously observe the other's action. Once a firm enters it locates and produces according to the profit maximizing strategies derived in the previous section. That is, we assume that each firm locates optimally in product space conditional on entry.

In order to describe the payoffs it will be convenient to have some notation that keeps track of the status of entry into the new market. Let E denote entry, N denote no entry. Then $(i,j)$, $i,j \in \{E,N\}$ describes the state in which firm A is in state i (has or has not entered), and firm B is in state j. If i and j = E (that is, both have entered) we must keep track of the order in which they entered. Then $(E,E;k)$, $k = A,B,T$ will denote the case in which both firms have entered and firm k entered last ("won" the game). If the firms entered simultaneously, then $k = T$ denotes a tie. Using this notation, $\pi^A(i,j;k)$ will denote firm A's (maximum) ex ante expected flow profits in the state described by $(i,j;k)$, given the optimal locations in that state as derived in Section 2. For firm A the expectation is taken over the a's, given the actual probabilities of success and correlation at the two locations. $\pi^B(E,E;A)$ is B's expected profit given his prior over the locations and (9); $\pi^B(E,E;B)$ is B's expected profit given his prior over the locations and his expectation of the value of information that will be revealed by A's entry (as determined by (8)). For what follows we note the following:

<u>Lemma 2</u>:

(10) $\quad \pi^B(E,E;B) > \pi^B(E,N) = 0$

and

(11) $\quad \pi^B(E,E;A) > \pi^B(N,N) = 0$

Proof: Both (10) and (11) follow directly from the assumption that $a^H > a^L > k_y$ and Cournot competition. Q.E.D.

Additionally, we assume

(A.7) $\pi^A(E,E;A) > \pi^A(N,E)$

and

(A.8) $r > 0$.

By (A.7), the incumbent would prefer to enter the new market once the rival is in there (but before learning whether or not he is successful) rather than not entering at all. This is the only interesting case for the model. The idea is that the two markets are sufficiently related in demand that if B enters the new market, the incumbent is threatened in his own market because success by the newcomer would be very costly. This makes it worthwhile for the incumbent to compete in the new market. (Of course, it is certainly not necessary that the two markets even be related for A.7 to hold.)

Assumption A.8 implies that it is optimal to enter immediately, rather than later, once the rival has entered.

A formal model of the order of entry in continuous time requires that we define strategies that specify each player i's action at each time t, including times <u>after</u> the rival's entry if i has not yet entered (that is, off the equilibrium path) for each possible previous entry history of the rival. In addition, issues arise in the continuous time game that do not arise in the discrete time analog. For example, one must formalize the continuous time meaning of entry "immediately after but not simultaneous to" the opponent.

These difficulties are not intractable (see, especially, Simon and Stinchcombe (1989) and Fudenberg and Tirole (1985)). However, we are able to simplify the game significantly without affecting its richness. We assume that, once one firm as entered, the rival follows immediately. That is, the formal game that we will model ends once one firm enters. This simplifies the game because it restricts the possible game histories to one--the history in which no firm has entered. No other histories are feasible since, if one firm has entered, the game is over. Thus, the firms' strategies need only specify what each firm will do at each time $t \in [0,\infty)$ if neither has entered before t.

The interesting part of the game is in determining the timing of first entry and, certainly, in equilibrium, the second entrant will follow immediately, by Lemma 2, A.7 and A.8. Our modification of the formal structure of the game retains the interesting strategic interaction, and does not affect the equilibrium outcome, but significantly simplifies the analysis.

We define a history at time t as the interval $[0,t)$, during which no entry has occurred. A subgame at any time t (before which no entry has occurred) is the interval $[t,\infty)$.

There are four possible play paths:

1. No entry ever.
2. A enters at some finite time $t \geq 0$.
3. B enters at some finite time $t \geq 0$.
4. Both A and B enter simultaneously at some finite time $t \geq 0$.

At any time t each player can choose from only two possible actions: enter (1) or do not enter (0). A (pure) strategy for player i is an index function $I^i(t)$, $I^i\colon \mathbb{R}^+ \to \{0,1\}$, where $t \in \mathbb{R}^+$ is the current time, before which no entry has occurred. We require that strategies satisfy

(C.1) $I^i(t)$ must be continuous from the right with respect to time.

Condition (C.1) means that the "first time" of entry is well defined in the continuum for any strategy, which will be important in the proof of Propositions 3 and 4. Condition (C.1) is analogous to Condition F3 in Simon and Stinchcombe requiring right-continuity with respect to histories. Our simplification reduces possible history types to one, and as a result continuity with respect to time is equivalent to continuity with respect to histories, but we are spared the task of defining a metric over histories. In addition, the problems that can arise when (C.1) is violated, in which well-defined strategies do not uniquely determine game outcomes, are not an issue in the simplified game because (for example) strategies of the type "enter at each time t before which my rival has entered" do not arise.

One market structure of particular interest in this setting is the case in which

(12) $\pi^A(E,E;B) < \pi^A(N,N)$.

We view this to be an important case for the following reasons. In our model the presence of a potential entrant is known and certain. One might ask: How did the market get into this situation? That is, if entry occurs in equilibrium at $t = 0$, as we have shown, how did the market arrive at a time before which no entry had occurred in y and A was an unthreatened monopolist long enough to develop its expertise? One answer is that, for some time before "t = 0," the presence of a rival was not certain. During that time, firm A might imagine that its entry into y would reveal the presence of a market that others had not yet discovered. In such a situation, firm A faced the real choice of remaining a monopolist in x with no rival or presence in y, and operating in y but risking imitation in the new market, once revealed. In such a situation, firm A would certainly not enter the new market if (12) held and the firm put a sufficiently high probability on imitation once it entered. Conversely, if (12) failed, firm A might enter the new market even though doing so invited entry, rather than wait for others to discover it and enter first.

We do not model this larger game but only note that this story appears to us a compelling reason to consider the case in which (12) holds. We can immediately state the following result:

<u>Proposition 3</u>: Assume (12). Then the pair of strategies

$$(S2)\quad \begin{cases} I^A(t) = 0 & \forall\ t \in [0,\infty) \\ I^B(t) = 1 & \forall\ t \in [0,\infty) \end{cases}$$

is the only subgame perfect Nash equilibrium in pure strategies to the timing game. That is, firm B will enter immediately with probability one, (and A will follow immediately thereafter).

<u>Proof</u>: The proof is subsumed in the proof of Proposition 4.

The effect of (12) is precisely what one would intuitively expect. When (12) holds, firm A's implicit "threat" to wait until firm B enters or not enter at all is completely credible. Indeed, it is a dominant strategy for firm A to wait until firm B enters.

Now suppose that (12) does not hold. That is, assume

$$(13) \qquad \pi^A(E,E;B) > \pi^A(N,N).$$

This is the most interesting case form a theoretical standpoint, since in this case both firms would prefer late entry over early entry, but both prefer early entry to none at all. Violation of (12) implies that the new market is so attractive and/or competition in the new market has sufficiently small effect on the incumbent's market, that A prefers to compete in y as an early entrant to remaining a monopolist in x with no competition or presence in y. We can now show the following result:

Proposition 4: Assume (13) holds. Then the pair of strategies

$$(S1) \quad \begin{cases} I^A(t) = 0 & \forall\ t \in [0,\infty) \\ I^B(t) = 1 & \forall\ t \in [0,\infty) \end{cases}$$

and the pair

$$
(S2) \quad \begin{cases} I^A(t) = 1 & \forall\ t \in [0,\infty) \\ \\ I^B(t) = 0 & \forall\ t \in [0,\infty) \end{cases}
$$

are the only subgame perfect Nash equilibria in pure strategies to the timing game.

Before proceeding to the proof, let us comment on the proposition. If strategies (S2) are played, then A will enter the market immediately, the formal game ends, and B follows right away. If (S1) are played, B enters immediately and A follows. In either case, there is no strategic waiting (which, socially, would be pure deadweight loss). More important, both are equilibria even if one firm has a much stronger incentive to hold out than the other. For example, in our setting it may be much more costly for B to wait than for A, because B gives up the entire flow profits for the duration of the wait, but A gives up only the net profits since he continues to operate in x while waiting. Intuitively, one might be suspect of an equilibrium concept in which such considerations are irrelevant. More intuitively appealing would be an equilibrium concept that allows us to predict which firm will enter first, depending on their relative costs and benefits of waiting. Such considerations will lead us to a refinement of equilibrium concept that will allow us to make such predictions.

For the proof of Propositions 3 and 4 we need to specify formally what the players' payoffs are in a "tie." The natural interpretation of a tie is that each player i chooses his location without observing firm j's choice, and knowing that firm j will choose without first observing firm i's location. Since firm A has private information about the probability of success at each location, the appropriate equilibrium concept is Bayesian-Nash. However, an exhaustive analysis of this simultaneous-entry game would, in our opinion, be a digression here. Rather, we assume that, if the firms enter simultaneously, they adopt the strategy they would choose if they entered first. That is, A locates at $\bar{w}$, and B randomizes according to the uniform distribution.

It is easily verified that these strategies do form a Bayesian equilibrium to the simultaneous entry game. Although there are many other equilibria in this game, others would require some sort of communication to implement. Thus, we view our focus on these particular strategies as natural, since they require no communication, and justifiable since they form a Bayesian equilibrium.

Then, by Proposition 2,

(14) $$\pi^{i}(E,E;T) < \pi^{i}(E,E;i).$$

Proof of Proposition 4: First we show that (S1) and (S2) are subgame perfect equilibria. Then we show that no other subgame perfect equilibria exist in pure strategies.

Equilibria

If i's ($i = A$ or B) strategy is "not enter" at every time t ($I^{i}(t) = 0$) then j's ($j = A,B$, $j \neq i$) best response at time 0 is to choose the first entry time $\tau$ that maximizes

$$\int_0^{\tau} \pi^j(N,N)e^{-rt}dt + \int_{\tau}^{\infty} \pi^j(E,E;i)e^{-rt}dt.$$

By (13) and Lemma 2, $\pi^j(N,N) < \pi^j(E,E;i)$ for both $j = A,B$. Thus, $\tau^* = 0$.

After any history (at any time $s > 0$), if i's strategy is to not enter at every time $t \geq s$, then j's best response is to choose the first entry time $\tau$ to maximize

$$\int_s^{\tau} \pi^j(N,N)e^{-rt}dt + \int_{\tau}^{\infty} \pi^j(E,E;i)e^{-rt}dt.$$

Again, by (13) and Lemma 2, $\tau^* = t$.

Thus, strategy $I^j(t) = t\ \forall t$ is the unique best response to $I^i(t) = 0\ \forall t$, and by the above arguments, it is also subgame perfect.

<u>Uniqueness</u>

Let $L_i = L(I^i(t))$ be the smallest t such that $I^i(t) = 1$ for $i = A,B$. By (C.1), $L_i$ is well defined, and $I^i(L_i) = 1$.

Then either

a. $L_i = L_j$; or

b. (without loss of generality) $L_i < L_j$.

<u>Case (a)</u>: If, at $L_i$, player j enters, his payoff from then on is

$$\int_{\tau}^{\infty} \pi^j(E,E;T)e^{-rt}dt.$$

If he does not enter his payoff is

$$\int_{\tau}^{\infty} \pi^{j}(E,E;j)e^{-rt}dt.$$

Since $\pi^{j}(E,E;T) < \pi^{j}(E,E;j)$ by (14), j can do better if $I^{j}(L_{i}) = 0$. Thus, (a) is not an equilibrium.[5]

Case (b): First we show that if $L_i < L_j$ then $L_i = 0$. Define $K_i$ as the time such that, if player i's strategy were to not enter (first) until $K_i$, player j would be indifferent between entering at time $t = 0$ and waiting until i has entered. That is, $K_i$ solves

$$\int_{0}^{\infty} \pi^{j}(E,E;i)e^{-rt}dt = \int_{0}^{K_i} \pi^{j}(N,N)e^{-rt}dt + \int_{K_i}^{\infty} \pi^{j}(E,E;j)e^{-rt}dt.$$

By Proposition 2 ($\pi^{j}(E,E;i) < \pi^{j}(E,E;j)$),

$$K_i > 0. \tag{15}$$

If at $t = 0$ it is optimal for i that $L_i < L_j$, then it must be the case that $L_j \geq K_j$. Then i's optimal strategy at $t = 0$ is to choose $\tau$ to maximize

$$\int_{0}^{\tau} \pi^{i}(N,N)e^{-rt}dt + \int_{\tau}^{\infty} \pi^{i}(E,E;j)e^{-rt}dt.$$

By (13) and Lemma 2, $\tau^* = 0$. thus, i's optimal strategy at $t = 0$ is to

[5]The payoffs defined above reflect the idea that, in continuous time, entry can occur sequentially, but at the "same instant in time." See Simon and Stinchcombe for elaboration.

enter immediately.

For any t such that $0 \leq t \leq L_j - K_j$, i's optimal strategy is to enter immediately, by the same argument. For all t such that $L_j - K_j < t \leq L_j$, i's optimal strategy is to wait, by definition of $K_j$ and the stationarity of the game.

Let $S_i = S(I^i(t))$ be the <u>next</u> time i enters after $L_j - K_j$. Since $I^i(t) = 0$ for $L_j - K_j < t \leq L_j$, $S_i$ must be strictly greater than $L_j$. Now, if it is optimal for j to enter at $L_j < S_i$, then (by the same argument as above), $L_j = L_j - K_j$, which is a contradiction by (15). Q.E.D.

The reader will note that we have considered only pure strategies. In Appendix B we show that there is a unique subgame perfect equilibrium in behavior strategies. However, we do not find this equilibrium to be intuitively compelling. In an equilibrium in behavior strategies, roughly speaking, each firm's instantaneous hazard rate of entry must be the one that makes his opponent indifferent between entering now or waiting a short time. If player 1 bears a very high cost of waiting, then player 2's "probability" of entry would have to be very high in order for player 1 to find it worthwhile to wait to see if player 2 enters. Conversely, if player 2 bears a low cost of entry then player 1's equilibrium entry probability must be low to leave player 2 indifferent between entering and waiting. Thus, two features of such an equilibrium are disturbing. First, each player's equilibrium strategy is a function only of the opponent's payoffs rather than his own. Although neither player can do better by deviating, he does no worse either, and it is difficult to motivate how the players would reach such an equilibrium. Second, the equilibrium always has the feature that the player with the higher cost of waiting or lower value of winning has a lower probability of entry. In other words, the behavior-strategies equilibrium would predict that, on average, the player who can more credibly threaten to wait out his rival, because his waiting costs are low or benefit to winning is high, will enter first.

We view this as an artifact of the way randomized strategy equilibria are calculated, rather than as a serious predictor of economic behavior. Therefore, we reject the notion of equilibrium in behavior strategies in favor of an attempt to choose between the pure strategies equilibria. Of course, readers who find the notion of behavior strategies sufficiently compelling that they are willing to accept the strong counter-intuitive implications will reach conclusions about order of entry opposite to ours. Put differently, any attempt to validate our theory empirically would necessarily be testing the joint hypothesis that the theory presented in Section II is correct, and that our refinement of pure strategies equilibria is a better predictor of economic behavior than behavior strategies equilibria.

We adopt the refinement known as "risk dominance." To motivate this idea, consider the following thought experiment.

Suppose firm A expected the firms to play equilibrium S1. Firm A might wonder if, indeed, firm B planned on playing that equilibrium. If in reality firm B planned on playing S2, then firm A's strategy would not be optimal. So, to the extent that firm A thinks it possible that firm B will play S2, firm A bears some risk. The risk is measured by what firm A would lose by playing S1 when he could do better by playing S2 along with firm B.

Now suppose that firm A's return in S1 is so high that, even if firm A thought that firm B might play S2 with probability .9, firm A would still prefer to take his chances and play S1. This would not only make it sensible to expect firm A to play S1 but, if we think of the players as forming Bayesian priors over the other's behavior, it would also make it rational for firm B to expect firm A to play S1. Of course, one must also consider firm B's risk associated with S1. Harsanyi and Selten (1988) developed a formal refinement algorithm, known as "risk dominance," that allows us to choose between two equilibria precisely along the lines described above. Although Harsanyi and Selten define the concept for games in normal form, we adopt it in the most straightforward way for selecting between two subgame perfect equilibria.

A formal definition of risk dominance is as follows:[6]

Consider two equilibria, $\alpha$ and $\beta$. We must first define the resistance of $\alpha$ against $\beta$. Let $\lambda_i$ be the largest probability such that, if player j played his $\beta$ strategy with probability $\lambda_i$ and his $\alpha$ strategy with probability $(1 - \lambda_i)$, then i would be indifferent between playing $\alpha$ or $\beta$. Define $\lambda_j$ analogously. Then the resistance of $\alpha$ against $\beta$ is $\min(\lambda_i, \lambda_j) = R(\alpha, \beta)$.

We say that $\alpha$ risk-dominates $\beta$ if $R(\alpha,\beta) > R(\beta,\alpha)$.

Note that the payoff to firm A under S1 is

$$\int_0^\infty \pi^A(E,E;B)e^{-rt}dt = \pi^A(E,E;B)/r.$$

The payoff to B under S1 is

---

[6] This definition follows Myerson (1991).

$$\int_0^\infty \pi^B(E,E;B)e^{-rt}dt = \pi^B(E,E;B)/r.$$

Similarly, the payoffs to firms A and B, respectively, under S2 are $\{\pi^A(E,E;A)/r,\ \pi^B(E,E;A)/r\}$. Using these payoffs to calculate the resistance it is straightforward to show:

Proposition 5: S1 risk-dominates S2 (that is, the incumbent enters last) if and only if

$$(16) \qquad \frac{\pi^A(E,E;B) - \pi^A(N,N)}{\pi^A(E,E;A) - \pi^A(E,E;T)} < \frac{\pi^B(E,E;A)}{\pi^B(E,E;B) - \pi^B(E,E;T)}$$

Proof: See Appendix A.

The denominator on the left side is the incremental gain to firm A of winning relative to a tie; the numerator is the incremental value of being in the market as a loser relative to no entry into the new market at all. The terms on the right are the corresponding values for firm B, keeping in mind that $\pi^B(N,N) = 0$. Thus, Proposition 5 implies the following:

Corollary 5.1: Firm A is more likely to be the later entrant, ceteris paribus:

a. The higher is the incremental value to firm A of later entry relative to a tie;

b. the lower is the incremental value to firm A of being in the new market as first entrant over being out of the market;

c. the lower is the incremental value to firm B of late entry over a tie;

d. the higher is the value to firm B of being in the new market as a first entrant.

We can identify a-d with the costs and benefits of late entry analyzed in Section II. Rewriting (16), S1 risk dominates S2 iff

$$(17) \qquad \frac{\pi^B(E,E;A)}{\pi^A(E,E;B) - \pi^A(N,N)} > \frac{\pi^B(E,E;B) - \pi^B(E,E;T)}{\pi^A(E,E;A) - \pi^A(E,E;T)}$$

The two sides of expression (17) separate the two countervailing effects we identified in Section II. First, firm A bears a lower waiting cost than firm B in that the incremental profits of entry to firm A net out the loss in profits in its original market, while firm B counts the entire profit in Y as a gain. This is the "cannibalization" effect studied and formalized by Kamien and Schwartz, Reinganum, Conner, and others. The greater this effect is, the larger the left side of expression (17) will be, and the more likely that S2 dominates, i.e., the incumbent will be the late entrant. This result is consistent with the above-mentioned studies.

However, we also identify an additional benefit to late entry that may favor late entry by firm B. If firm B is the later entrant, it can choose product characteristics after learning from firm A's product choice. If firm A is the late entrant, he learns nothing from firm B, but he prevents firm B from profiting from firm A's expertise at firm A's expense.

How profitable it is for firm B, and how costly for firm A, for firm B to enter second depends, then, on how important the learning effect is. To understand the effect, consider two extremes.

Suppose each location on the circle has the same probability of success and that this is common knowledge. Then the value of going last is only to choose the largest feasible covariance, and firms A and B would be indifferent as to who was the later entrant on these grounds. Since the chosen correlation would be the same regardless of which firm entered last, and since (by Lemma 1) $d\pi^A/d\sigma = d\pi^B/d\sigma$, the right side of (17) would be unity. In such case, the cannibalization effect would dominate, and the incumbent would enter second.

At the other extreme, suppose $P_i = 0$ at all $w_i$ except one, $\bar{w}$, at which $P = \bar{P} > 0$. In this case, firm A will always locate at $\bar{w}$, regardless of where firm B locates or the order of entry. If firm B enters first, or in a tie, the probability is zero that it will locate at $\bar{w}$. However, if firm B enters second, it will imitate firm A exactly. Thus, $\pi^A(E,E;A) = \pi^A(E,E;T)$, but $\pi^B(E,E;B) - \pi^B(E,E;T) > 0$. Thus, the right side of (17) is zero, and S2 will risk dominate S1. In other words, the incumbent will enter first.

The discussion suggests that as information becomes more important, the probability of the incumbent entering first increases. We cannot show this to be everywhere true, but it is certainly true as information becomes extremely valuable:

<u>Proposition 6</u>: As information becomes more valuable (in the sense defined earlier) near the limit, the incumbent is more likely to be the first entrant.

<u>Proof</u>: The proof follows from the observation that as information becomes extremely valuable, $\pi^B(E,E;A) \to 0$, but $\pi^A(E,E;A)$ approaches a finite limit, and the other terms in (14) are unaffected. Q.E.D.

Finally, consider the effect of the correlation choice on the order of entry. As an extreme, suppose that the probability of success increased as one located closer to $\bar{w}$, but success <u>distributions</u> were independent for the two firms, regardless of location. If firm A had no superior information, then we would again be in the world in which the only difference between the firms' entry incentives is that firm A would lose some profits in its current market if it entered the new market. In this situation the firms would again be indifferent as to which entered first, <u>and</u> indifferent between simultaneous and sequential entry. Thus, the model suggests that the cannibalization effect in itself has no implications on order of entry. The reason is that, although the incumbent has less incentive to enter the new market than the rival, it has no reason to stay out if it knows entry is inevitable.

## IV. <u>Conclusions</u>

The model suggests a number of avenues for future research. A full understanding of order of entry into new markets would require a taxonomy of market conditions under which both firms would prefer early entry, both would prefer late entry entry, and so forth. When both prefer early entry, another game of timing emerges that may merit study. A full analysis of the Bayesian game of simultaneous entry with one-sided incomplete information may yield interesting results. Certainly, it would be important to know how the presence of more than two firms affects the outcome. Of course, all such studies would be enriched by continued progress in empirical research that explicitly studies order of entry, incumbency effects, and success rates in new markets.

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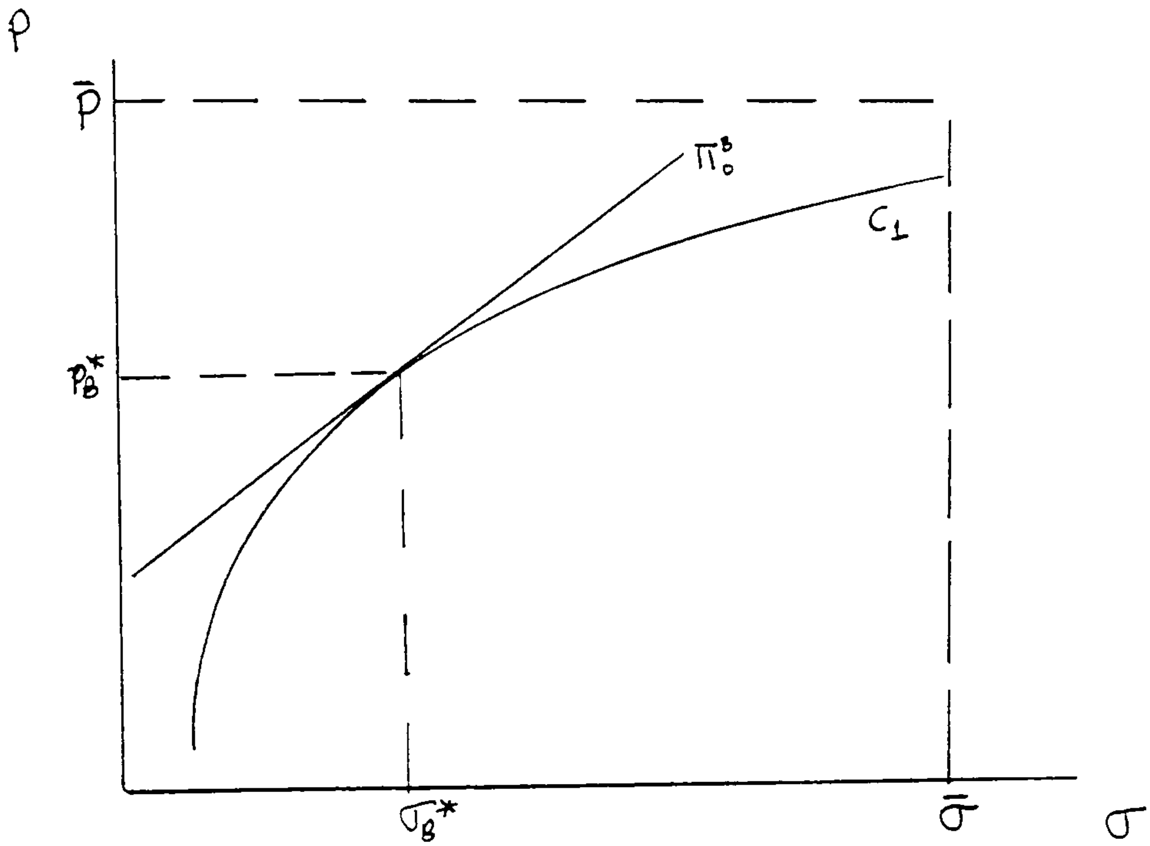


Figure 1

## Appendix A

Proof of Proposition 2: As information gets extremely valuable the probability that B will choose a location that has a positive probability of success goes to zero when B enters first. When A enters first B will certainly choose a location with positive probability of success, and that probability of success is independent of the value of the information (since we hold $P(\sigma)$ constant). Thus, as information gets more valuable, (using the notation of Section III)

$$\pi^B(E,E;A) \to 0$$

$$\pi^B(E,E;B) \text{ is unaffected}$$

$$\pi^A(E,E;A) \to \pi^A(E,N)$$

$$w_A \to \bar{w},$$

and

$$\pi^A(E,E;B) \text{ is unaffected.}$$

Clearly, under the assumption of Cournot competition

$$\pi^A(E,N) > \pi^A(E,E;B) \text{ for}$$

$$\pi^B(E,E;B) \text{ evaluated at } \bar{w} \text{ and}$$

$$\pi^A(E,N) \text{ evaluated arbitrarily close to } \bar{w}.$$

By Lemma 2

$$\pi^B(E,E;B) > 0. \qquad \text{Q.E.D.}$$

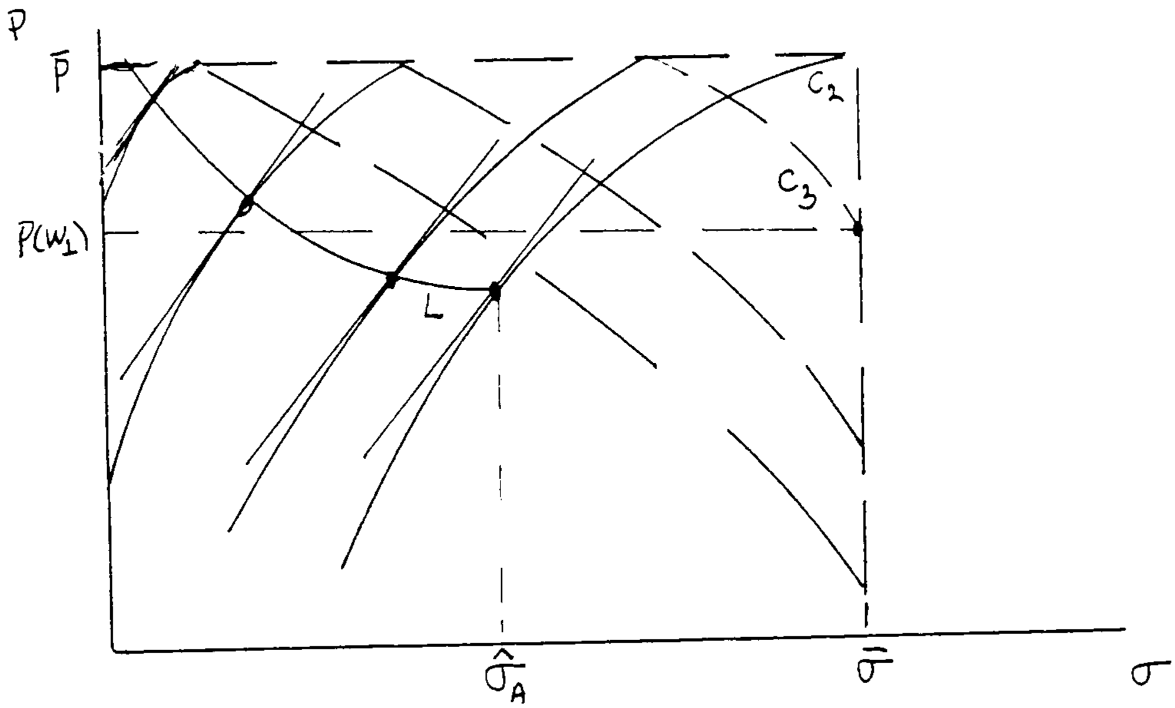


Figure 2

Proof of Proposition 5: First we calculate the resistance of S1 against S2.

For firm A, we want to find the maximum probability $\lambda$ that B can put on S2 such that A still (weakly) prefers to play S1. If B deviates to S2 with probability $\lambda$, A's payoff if A plays S1 is

$$\text{B.1} \qquad (1/r)[\lambda\pi^A(N,N) + (1-\lambda)\pi^A(E,E;A)]$$

If A plays S2 his payoff is

$$\text{B.2} \qquad (1/r)[\lambda\pi^A(E,E;B) + (1-\lambda)\pi^A(E,E;T)]$$

Equating B.1 and B.2 and solving for $\lambda$

$$\lambda = \frac{\pi^A(E,E;A) - \pi^A(E,E;T)}{\pi^A(E,E;A) - \pi^A(E,E;T) + \pi^A(E,E;B) - \pi^A(N,N)}$$

For firm B we want to find the maximum probability $\gamma$ that A can put on deviating to S2 such that B still (weakly) prefers S1.

If B plays S1 his payoff is

$$\text{B.3} \qquad (1/r)[\gamma\pi^B(E,E;T) + (1-\gamma)\pi^B(E,E;A)]$$

If B plays S2 his payoff is

$$\text{B.4} \qquad (1/r)[\gamma\pi^B(E,E;B)]$$

Equating B.3 and B.4 and solving for $\gamma$:

$$\gamma = \frac{\pi^B(E,E;A)}{\pi^B(E,E;A) + \pi^B(E,E;B) - \pi^B(E,E;T)}$$

The resistance of S1 against S2 is $\min(\lambda,\gamma)$.

Similar calculations show that the resistance of S2 against S1 is

$$\min(1 - \lambda,\ 1 - \gamma).$$

Now if $\lambda < \gamma$ then $R(S1,S2) = \lambda$ and $R(S2,S1) = (1 - \gamma)$. Thus, S1 risk dominates S2 iff

B.5 $\qquad \lambda > 1 - \gamma$

If $\lambda > \gamma$ then $R(S1,S2) = \gamma$ and $R(S2,S1) = 1 - \lambda$ and S1 risk dominates S2 iff $\gamma > 1 - \lambda$, which is identical to B.5.

Thus, S1 risk dominates S2 if

$$\frac{\pi^A(E,E;A) - \pi^A(E,E;T)}{\pi^A(E,E;A) - \pi^A(E,E;T) + \pi^A(E,E;B) - \pi^A(N,N)}$$

$$> \frac{\pi^B(E,E;B) - \pi^B(E,E;T)}{\pi^B(E,E;A) + \pi^B(E,E;B) - \pi^B(E,E;T)}$$

Since the numerator and denominator of both the left and right side terms are positive by (13), (14), and Lemma 2, the result holds. Q.E.D.

## Appendix B

We define a randomized strategy for player i to be a function $F^i: \mathfrak{R}^+ \rightarrow \mathfrak{R}^+$ such that the probability of entry before time t is F(t). We require:

1. $\lim_{t\infty}\rightarrow F(t) \leq 1$;

2. F(t) is right continuous.

At any time t, for player i to play a randomized strategy during the interval $[t, t + \Delta t]$ he must be indifferent between entering for sure at t and waiting until $t + \Delta t$ to enter. For this to be optimal at time zero, player i must be indifferent between entering at t and waiting until $t + \Delta t$, conditional on no entry before t. We first find the probability of entry during $\Delta t$ for player j that makes i indifferent between the two pure strategies:

Let the probability that j enters during $[t, t+\Delta t]$ be $\lambda^j \Delta t$. If i enters at t (which we will interpret to mean during $\Delta t$) then his payoff is

$$(3) \qquad (1/r)e^{-rt}[\lambda^j \Delta t \pi^i(E,E;T) + (1 - \lambda^j \Delta t)\pi^i(E,E;j)].$$

If i waits $\Delta t$ and then enters, his payoff is

$$(4) \qquad (1/r)e^{-rt}[\lambda^j \Delta t \pi^i(E,E;i) + (1 - \lambda^i \Delta t)(\pi^i(N,N)r\Delta t + \pi^i(E,E;j)e^{-r\Delta t}].$$

Given $\Delta t$, $\lambda^i \Delta t$ must equate (3) and (4). Thus,

$$\lambda^{j}\Delta t = \frac{\pi^{i}(E,E;j)(1 - e^{-r\Delta t}) - r\pi^{i}(N,N)\Delta t}{\pi^{i}(E,E;j)(1 - e^{-r\Delta t}) + \pi^{i}(E,E;i) - \pi^{i}(E,E;T) - \pi^{i}(N,N)\Delta t}.$$

Now, to find $\lambda^{i}(t)$ we divide by $\Delta t$ and take the limit as $\Delta t \rightarrow 0$. Thus, using L'Hopital's rule,

$$(5) \qquad \lambda^{i}(t) = [r(\pi^{i}(E,E;j) - \pi^{i}(N,N))]/[\pi^{i}(E,E;i) - \pi^{i}(E,E;T)],$$

which is positive by Lemma 2, (3), and (14) in the text. Notice that $\lambda^{i}(t)$ is not a function of t.

By construction, $\lambda^{i}(t) = f^{i}(t)/(1 - F^{i}(t))$, where $f^{i}$ is the derivative of $F^{i}(t)$.

Now, it is well-known that the only function that satisfies (1), (2) and (6) is $F(t) = e^{-\lambda^{i}t}$ (see Thompson, Ch. 4). Thus, during any period $\Delta t$ that the players randomize, the function describing the distribution must be exponential, with parameter given by (5).

The pair of strategies $F^{i}(t) = 1 - e^{-\lambda^{i}t}$ form a subgame perfect equilibrium because of the "no memory" feature of the exponential distribution: for any times t and s,

$$pr(T \leq s) = pr(t \leq T \leq t + s|T > t).$$

Now we note that the probability that player i enters during any interval $[t,t + \Delta t]$ conditional on reaching t, is $1 - e^{\lambda^{i}\Delta t}$, which is increasing in $\lambda^{i}$. The conditional probability of i entering during the interval $[t,t + \Delta t]$ exceeds the condition probability for j if and only if $\lambda^{i} > \lambda^{j}$; or

$$\frac{\pi^i(E,E;j) - \pi^i(N,N)}{\pi^i(E,E;i) - \pi^i(E,E;T)} < \frac{\pi^j(E,E;i) - \pi^j(N,N)}{\pi^j(E,E;j) - \pi^j(E,E;T)}$$

Thus, i has a higher probability of entry than j if the value to i of entering last (rather than trying) is higher than that for j, and if the value to i of entering first relative to no entry is <u>lower</u> for i than for j.