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AUCTIONS FOR OIL AND GAS LEASES WITH AN INFORMED BIDDER AND A RANDOM RESERVATION PRICE*

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ABSTRACT

The paper analyzes a first price, sealed bid auction with a random reservation price where the object has an unknown common value, but one buyer has better information than the others. We permit the reservation price to be correlated with the information of the informed buyer, which reflects both his assessment of the value of the object and probability of rejection at any bid. Assuming all random variables are affiliated, we establish the following results. (1) The rate of increase in the distribution of the uninformed bid is never greater than the rate of increase in the distribution of the informed bid. (2) The distributions are identical at bids above the support of the reservation price. (3) The informed buyer is more likely to submit low bids. We demonstrate that these restrictions are satisfied by bid data from the federal sales of offshore drainage leases.

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1. Introduction

In this paper we analyze a model of a first price, sealed bid auction to study the federal sales of offshore oil and gas leases on drainage tracts.[1] Kenneth Hendricks and Robert Porter (1988) (HP henceforth) provide strong evidence that the value of these leases, while uncertain at the time of the sale, is approximately the same for all participants, and that one bidder has superior information. Furthermore, Hendricks, Porter and Richard Spady (1989) (HPS henceforth) argue that, from the point of view of buyers, the reservation price of the seller is effectively a random variable, correlated with the bid of the informed bidder. We extend a model of Richard Engelbrecht-Wiggans, Paul Milgrom, and Robert Weber (1983) (EMW henceforth) to incorporate these features and establish some restrictions on the equilibrium distributions of bids. We then test these implications with bid data for 295 drainage tracts off the coasts of Louisiana and Texas which were sold between 1959 and 1980.

Previous studies of auctions with asymmetric information by Robert Wilson (1967), M. Weverbergh (1979), and EMW were also motivated by the drainage auctions. The basic model is a sealed bid, first price auction with a single informed buyer and one or more uninformed buyers bidding for an object of unknown but common value with a fixed reservation price. Under these assumptions, EMW demonstrate that the distributions of the high uninformed bid and the informed bid are identical above the reservation price with some informed bids possibly concentrated at the reservation price.

We extend the basic model to allow for an exogenous random reservation price which may be correlated with the value of the object and/or the information of the informed buyer. Consequently, to incorporate both his

[1] Drainage tracts are adjacent to tracts on which deposits have already been discovered.

estimate of the value of the object, V, and his estimate of the distribution of the reservation price, R, the signal of the informed buyer, X, must generally be multidimensional. This extension considerably complicates the analysis. To guarantee sufficient regularity conditions on the joint distribution of (V,R,X), we require these random variables to be affiliated, a concept first introduced in the bidding literature by Milgrom and Weber (1982). This assumption implies that the conditional expected value of V is nondecreasing in the realizations of both variables. It also implies that the distribution of R, conditional on X, satisfies the monotone likelihood ratio property with respect to the realizations of X.

With these restrictions, we establish that the equilibrium informed bid is a nondecreasing function of his signal and, consequently, also affiliated with V and R. This relation in turn allows us to establish the following restrictions on the distribution of bids. First, the rate of increase in the distribution of the high uninformed bid is never greater than the rate of increase in the distribution of the informed bid. Second, above the range of R, the two distributions are identical. Third, near the lower bound of the range of R, only the informed buyer bids with positive probability.

Working with data on tracts sold before 1970, HP established that neighbor firms, those which previously purchased leases on adjacent tracts, possessed considerable inside information about the value of the drainage tracts offered for sale. The average return to neighbor firms submitting a winning bid was roughly 180% of their bid, while the average return of nonneighbor firms was approximately equal to their bid. Also, after conditioning over relevant variables which were public information, the <u>ex</u>

post discounted returns on drainage tracts sold before 1970 were highly correlated with the highest neighbor bid and essentially uncorrelated with the high nonneighbor bid. They also provide evidence of collusion among the neighbor firms, so that there was effectively only one informed bid. For example, the average high neighbor bid did not increase with the number of neighbor firms, nor was there a significant effect on nonneighbor bidding.

Working with a superset of this data set that includes tracts sold between 1970 and 1979, HPS argue that the reservation price of the government also had a significant exogenous random component. The official minimum price was typically $25 per acre. Nevertheless, the government rejected a higher bid on 58 of the 295 drainage tracts. In addition, the rejection decision appears to be nonstrategic. Only about a third of the tracts in which the high bid was rejected were offered again at a later date, generally with a lag of 12 to 18 months. Also, the rejection option was rarely used in those instances when the government had the most to gain. Bids over five million dollars on drainage tracts were almost always accepted. HPS conclude that "the purpose of the government's rejection policy was to reduce the incentive that firms might have had to bid the preannounced minimum price on tracts that, on the basis of public information, were regarded as low value tracts."

On the basis of these analyses we treat all neighbor firms as a single bidder and treat the highest neighbor bid as the informed bid. The nonneighbor firms are assumed to be uninformed bidders. Figure 1 illustrates the empirical distributions of the highest neighbor and nonneighbor bid on each tract in our sample (scaled by the log of the bid). Our data includes only those tracts on which at least one firm submitted a bid. Consequently, the height of a distribution at 0 represent the proportion of tracts on which the firms submitted no bid among those tracts which received at least one bid.

Notice that the three restrictions mentioned above appear to be satisfied. The rate of increase in the distribution of the high neighbor bid appears to be everywhere at least as great as the rate of increase in the distribution of the high nonneighbor bid. The distributions are roughly equal above 4 million dollars, where the high bid was rejected on only 6 of 122 tracts (as opposed to 58 rejections on the full sample of 295 tracts). Finally, neighbor firms were much more likely to submit relatively low bids than were nonneighbor firms. For example, more than 15% of the high neighbor bids lie between 0 and .25 million dollars, whereas less than 5% of the high nonneighbor bids lie in this range. We provide formal tests of these restrictions in Section 6.

The paper is organized as follows. The basic model is developed in Section 2. In Section 3, we introduce the concept of affiliated random variables and use this restriction to establish that the bid function of the informed buyer is monotonic. This proof may be of some independent interest. It is apparently the first such result which allows for a multidimensional signal that may be correlated with both the value of the object and the seller's reservation price. In Section 4, we calculate two specific examples with an independent random reservation price and indicate some key features of the equilibrium distribution of bids. Section 5 contains the main theoretical results of the paper, and, in Section 6, we report our statistical tests. Section 7 summarizes our conclusions.

2. The Model

An indivisible object with unknown value V is to be sold in a sealed bid, first price auction. The participants consist of an informed seller, who observes a private signal and sets a reservation price R determined before the bids are revealed, an informed buyer, who observes a private signal X, and an uninformed buyer who observes only a public signal which we hold constant throughout.[2] The buyers submit sealed bids without knowing the reservation price. A buyer wins the object with certainty if his bid is strictly greater than any other bid and no less than the reservation price. If both buyers make the same bid and it is not less than the reservation price, the object is allocated according to some rationing rule which is independent of the reservation price. The winner of the object receives the object in exchange for his bid.

We suppose that V is a real valued random variable and R is a positive random variable, both with finite expectation. The realization of X lies in an n-dimensional Euclidian space. The joint distribution of (V,R,X) is common knowledge. Throughout the paper, random variables and their associated probability distributions are denoted by upper case letters, real numbers by lower case letters, and functions by lower case Greek letters.

In equilibrium, the uninformed buyer must generally randomize his bid. Therefore, we suppose that he observes the realization of a random variable U which is uniformly distributed on [0,1] and independent of (V,R,X). A strategy for the uninformed buyer is a real valued function α of the realizations of U which, without loss of generality, we restrict to be nondecreasing. The probability induced by $\alpha(U)$ is denoted by P_α and the associated distribution function by G_α. Similarly, a strategy for the

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[2] We restrict attention to a single uninformed buyer only to simplify notation. As we argue below, the equilibrium restrictions on the informed bidding strategy and the distribution of the highest uninformed bid are independent of the number of uninformed buyers.

informed buyer is a real valued function β of the realizations of X.[3] The probability induced by $\beta(X)$ is denoted by P_β and the associated distribution function by G_β.

If the seller conditions his reservation price on private payoff relevant information, the informed buyer will also condition his valuation of the object on the event of acceptance. Furthermore, the realization of X will affect the informed buyer's estimate of the probability of acceptance. Let $J(\cdot|x)$ denote the distribution function of R conditional on a realization x of X. Then, in the absence of an uninformed buyer, $E[V-b|R\leq b,X=x]J(b|x)$ is the expected profit to an informed buyer with information x from bidding b. Assuming for the moment that the distribution function of the uninformed bid, G_α, is continuous, $E[V-b|R\leq b,X=x]J(b|x)G_\alpha(b)$ is the expected payoff to the informed buyer with information x from bidding b when facing the strategy α.

Let $K(\cdot|b)$ denote the distribution function of R conditional on an informed bid of b, induced by J and β. Then $K(b'|b)$ is the probability that the reservation price is not greater than b', given that the informed bid is b. If the uninformed buyer wins the object with a bid of b, then he knows that b is not less than the reservation price and that the informed bid is no greater than b. Both events are informative in assessing the value of the object. If G_β is continuous, $\int_{(-\infty,b]}E[V-b|R\leq b,\beta=t]K(b|t)P_\beta(dt)$ is the expected payoff to an uninformed buyer who bids b when facing the strategy β.

An equilibrium is a pair (α,β) which maximizes the expected payoff to each buyer given the strategy employed by the other buyer. Since we have

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[3] If X contains mass points, it may also be necessary for the the informed bidder to randomize his bid. In this case, we may add a continuously distributed independent component to his signal. To simplify the notation, we avoid this complication. See Milgrom and Weber (1985) for further detail.

defined R to be positive, it follows that $J(0|x) = 0$ a.s.-P_X[4] so that a bid of 0 guarantees a buyer zero profits. Standard arguments then establish the following properties of an equilibrium.

LEMMA 1: Suppose (α,β) is an equilibrium. Then, for any bid b,

(a) $E[V-\beta(x)|R\leq\beta(x),X=x]J(\beta(x)|x)G_\alpha(\beta(x)) \geq E[V-b|R\leq b,X=x]J(b|x)G_\alpha(b)$ a.s.-P_X.

(b) $0 = \int_{(-\infty,b']}E[V-b'|R\leq b',\beta=t]K(b'|t)P_\beta(t) \geq \int_{(-\infty,b]}E[V-b|R\leq b,\beta=t]K(b|t)P_\beta(dt)$ a.s.-P_α.

(c) $E[V-b|R\leq b,\beta=b]K(b|b)\ P_\alpha(\{b\})\ P_\beta(\{b\}) = 0$.

Condition (c) implies that the rationing rule in the event of ties is irrelevant. The informed buyer never makes a bid which earns positive profits and has a positive probability of being matched by the uninformed buyer. By increasing his bid slightly, one of the buyers could obtain a positive gain at essentially zero cost. The inequalities in conditions (a) and (b) are then just the statement that each buyer chooses a best response given his information.

Condition (b) also implies that the uninformed buyer earns zero expected profits. The argument, which essentially follows EMW, goes as follows. Let b_α be the lower bound of the support of G_α and suppose that $G_\alpha(b_\alpha) = 0$. If the expected profit of the uninformed buyer is positive, then his probability of winning the object with a bid slightly above b_α must also

[4] For any random vector Z, let P_Z denote the probability induced on $\mathbb{R}^p$. Then, for $\alpha:\mathbb{R}^p \to \mathbb{R}$, $\alpha(z) = 0$ a.s.-P_Z if $\int|\alpha(z)|P_Z(dz) = 0$. In general, a restriction on Z is satisfied a.s.-P_Z if it is satisfied with probability 1 with respect to P_Z.

be positive. This implies that the informed buyer bids b_α or less with positive probability. Moreover, the expected value of the object to an uninformed buyer who wins at a bid slightly above b_α is the expected value of the object when the informed bid is b_α or less, conditional on the reservation value being no greater than b_α. Therefore, there must be a realization x of X for which $\beta(x) \leq b_\alpha$ and which, in the absence of the uninformed buyer, earns a positive return with a bid of b_α. But since the informed bidder never wins the object with a bid of b_α or less, his optimal bid must be greater than b_α. This contradiction establishes the result. A slight modification of the argument also can be applied in the case where $G_\alpha(b_\alpha) > 0$.

Lemma 1 reveals that the role of the uninformed buyer is essentially to impose a constraint on the bid function of the informed buyer. Conditional on realizations of R and the bid by the informed buyer which are no greater than b, the expected value of the object cannot exceed b. Also notice that the restrictions of Lemma 1 remain essentially unchanged if there is more than one uninformed buyer and G_α represents the distribution of the maximum uninformed bid. It is only to simplify notation, therefore, that we restrict attention to the case of a single uninformed buyer.

3. Affiliation and the Monotonicity of the Informed Bid Function

To derive the restrictions on the equilibrium distribution of bids, we must first determine the relation between the bidding strategies and (V,R). However, without some restrictions on (V,R,X), little can be said about (V,R,β). One possibility is to assume that R is independent of (V,X). However, for our application, the evidence strongly suggests that R is correlated with (V,X). To allow for such correlation, we exploit the concept of affiliated random variables, first introduced in the bidding literature by Milgrom and Weber (1982).

A random vector Z is **affiliated** if any two nondecreasing functions over the range of Z are nonnegatively correlated. The precise definition and some general properties are provided in the Appendix (see also Milgrom and Weber). The key restriction on (V,R,X) is provided by Assumption 1.

ASSUMPTION 1: (V,R,X) is affiliated.

This assumption guarantees that the realizations of the signal can be ordered so that higher values of X imply (on average) higher values of both V and R. We allow R to be correlated with V and, in addition, allow it to contain information about V not contained in X. Notice that in general, we cannot reduce X to a one dimensional signal.

Lemma 2 summarizes the critical implications of Assumption 1. The proof is presented in the Appendix.

LEMMA 2: (a) $E[V|R=r',X=x'] \leq E[V|R=r,X=x]$ for $(r,x) \geq (r',x')$ a.s.-$P_{R\times X}\times P_{R\times X}$. (b) $J(r'|x')J(r|x) \geq J(r'|x)J(r|x')$ for $r \geq r'$, $x \geq x'$ a.s.-$P_X\times P_X$.

Part (a) states that the conditional expected value of the object is nondecreasing both in the reservation price and in the information of the informed buyer. If $J(\cdot|x)$ is differentiable with density $j(\cdot|x)$, Part (b) is equivalent to the statement that the survival rate $j(\cdot|x)/J(\cdot|x)$ is nondecreasing in x. Note that, by setting $r = \infty$, Part (b) also implies that $J(r'|x) \leq J(r'|x')$ for $x \geq x'$.

To establish that the informed bid function is monotonic, we require a slight strengthening of Lemma 2a.

ASSUMPTION 2: $E[V|R=r',X=x'] < E[V|R=r,X=x]$ for $(r,x) \geq (r',x')$, $x \neq x'$, a.s.-$P_{R\times X}\times P_{R\times X}$.[5]

If the reservation price is not stochastic, then, using standard arguments, Assumption 1 implies that expected profit must be nondecreasing in x. However, if the distribution of the reservation value shifts too much with an increase in x, expected profit may decline, possibly to zero. To avoid this complication and other technical points, we assume a condition that is only slightly stronger than the requirement that the lower bound of the support of the reservation price is independent of the realization of X. To equate a zero bid with no bid, we also restate our convention that the reservation price always exceed zero.

ASSUMPTION 3: $J(r|x) > 0$ implies $J(r|x') > 0$ a.s.-$P_X\times P_X$.

Our analysis requires no additional regularity conditions. Using Assumptions 1 to 3, we may establish that the equilibrium bid of the informed buyer is a nondecreasing function of the realization of X. The proof extends

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[5] Inspection of the proof of Lemma 3 below reveals that Assumption 1 is sufficient to guarantee that a monotonic β may be selected as a best response to α. However, without Assumption 2, there may be another best response with a different distribution of bids in which β is not monotonic. We have not investigated the conditions under which a continuity argument, combined with Lemma 3 below, could be used to establish the existence of an equilibrium with β monotonic in the absence of Assumption 2.

the standard self-selection argument used by EMW for the case where R is constant.

LEMMA 3: If (α,β) forms an equilibrium, then $x \leq x'$ and $E[V-\beta(x)|R\leq\beta(x),X=x]J(\beta(x)|x)G_{\alpha}(\beta(x)) > 0$ imply $\beta(x) \leq \beta(x')$ a.s.-$P_X \times P_X$.

PROOF: For any pair (x_1,x_2), let $b_i = \beta(x_i)$, $G_i = G_{\alpha}(b_i)$, $J_{ij} = J(b_i|x_j)$, $v_{ij} = E[V|R\leq b_i,X=x_j]$, and $v_{\Delta j} = E[V|b_1<R\leq b_2,X=x_j]$, $i,j = 1,2$. Suppose the lemma is false. Then there is a subset $L \subset R^n \times R^n$, $P_X \times P_X(L) > 0$, such that for $(x_1,x_2) \in L$, $x_1 \geq x_2$, $x_1 \neq x_2$, $b_1 < b_2$, and

(1) $$(v_{22}-b_2)J_{22}G_2 > 0.$$

For $(x_1,x_2) \in L$, the best response property for β (Lemma 1a) implies

(2) $$(v_{11}-b_1)J_{11}G_1 \geq (v_{21}-b_2)J_{21}G_2 \quad \text{a.s.-}P_X \times P_X$$

and

(3) $$(v_{12}-b_1)J_{12}G_1 \leq (v_{22}-b_2)J_{22}G_2 \quad \text{a.s.-}P_X \times P_X.$$

Multiplying equation (2) by J_{12}, equation (3) by J_{11}, and subtracting implies that the following relation holds a.s.-$P_X \times P_X$.

(4) $$[v_{21}J_{21}G_2 - v_{11}J_{11}G_1]J_{12} \leq [v_{22}J_{22}G_2 - v_{12}J_{12}G_1]J_{11} + b_2G_2(J_{12}J_{21} - J_{11}J_{22})$$

By definition, $v_{2j}J_{2j} = v_{1j}J_{1j} + v_{\Delta j}(J_{2j}-J_{1j})$. Therefore, (4) may be written as

$$(5) \quad (v_{12}-v_{11})J_{12}J_{11}(G_2-G_1) + (v_{\Delta 2}-b_2)(J_{11}J_{22}-J_{12}J_{21})G_2 + (v_{\Delta 2}-v_{\Delta 1})(J_{21}-J_{11})J_{12}G_2 \geq 0.$$

Lemma 2 and Assumptions 2 and 3 imply that all three terms in equation (5) are nonpositive. (Note that $v_{\Delta 2} \geq v_{22}$ by Lemma 2a, and $v_{22} > b_2$ from equation (1).) All that remains to be shown is that at least one term is strictly negative. Note first that relation (1) implies that $J_{22} > 0$. It follows from Assumption 3 that $J_{22} \geq J_{21} > 0$. Lemma 2a, which dictates that $v_{21} \geq v_{22}$, and equation (1), which implies $G_2 > 0$, together with relation (2) then imply that $J_{11} > 0$.

By definition, $J_{12} = J_{22}$ implies $v_{12} = v_{22}$. Therefore, relations (1) and (3) imply that either $G_2 > G_1$ or $J_{22} > J_{12}$. First suppose that $G_2 > G_1$. Since $J_{21} > 0$ and $J_{11} > 0$, and because Assumption 2 implies that $v_{12} > v_{11}$, the first term in equation (5) is then negative.

Alternatively, suppose that $J_{22} > J_{12}$. Manipulation of Lemma 2b then implies that $J_{21} > J_{11}$, which, combined with Assumption 2, implies that the third term is negative. Q.E.D.

To understand the role which Lemma 2 plays in the proof of Lemma 3, notice that Lemma 2a ensures that the first and the third terms in equation (5) are nonpositive. If R is independent of (V,X), then the second term of equation (5) is zero, since the distribution of R does not depend upon X. The contradiction then follows immediately from the fact that the change in the probability of winning can be positive only if b_2 is smaller than b_1. When (V,R,X) is affiliated, Lemma 2b ensures that the second term in equation (5) is nonpositive.

To facilitate the statement of our results, we adopt the following convention for zero profit bids.

ASSUMPTION 4: Suppose (α,β) forms an equilibrium. Then

(a) $\int_{(-\infty,b]} K(b|t)P_\beta(dt) = 0$ implies $G_\alpha(b) = G_\alpha(0)$, and

(b) $E[V-b|R\leq b,\beta=b]K(b|b)G_\alpha(b) = 0$ implies $b = 0$, a.s.-P_β.

Assumption 4 requires a nonzero uninformed bid to win the object with positive probability. Similarly, whenever the informed buyer cannot earn positive profit, he submits a bid of zero.[6] Given Assumption 2 and Lemma 3, this convention guarantees that β is nondecreasing a.s.-P_β, but does not restrict the equilibrium bid distributions of bids above the lower bound of R.

4. Two Examples

In this section, we illustrate some of the properties of the bid distributions by way of two simple examples. These examples will serve as a guide to our analysis of the general case.

In both examples, we assume that R is independent of V and X. In this case, $J(b|x)$ and $K(b|b')$ reduce to $K(b)$, and $E[V|X,R]$ reduces to $E[V|X]$. Consequently, the expected payoff to the informed buyer with signal x who bids b may be written as $E[V-b|X=x]K(b)G_\alpha(b)$. As noted by EMW, one immediate consequence of this simplification is that the optimal bid of an informed buyer with signal x depends only on $E[V|X=x]$. Consequently, regardless of the

[6] Condition (b) is consistent with the best response condition for β (Lemma 1a) since we have assumed that $J(0|x) = 0$ a.s.-P_X. Consequently, $b = \beta(x)$ and $E[V-b|R\leq b,\beta=b]K(b|b)G_\alpha(b) = 0$ implies $E[V-b|R\leq b,\beta=b]J(b|x)G_\alpha(b) = 0$.

dimensionality of X, the information of the informed buyer can be indexed by the realization of the conditional expected value of the object, $E[V|X]$, which for simplicity we will identify with V. In this case, the strategy of the informed buyer, β, is simply a function of the realizations of V. Consequently, if $\beta(v) = b$, the expected payoff to the uninformed buyer from bidding b may be expressed as $E[V-b|\beta\leq b]K(b)G_{\beta}(b)$.

Let β^0 denote an optimal bid function for the informed buyer in the absence of an uninformed buyer, and let β denote the equilibrium bid function of the informed buyer when competing against an uninformed buyer.

Example 1: R is a Uniformly Distributed Random Variable

Suppose R is distributed uniformly on [1,3] and V is exponentially distributed with mean 6. In the absence of competition from an uninformed buyer, the problem of an informed buyer with valuation v is to choose b to maximize $(v-b)K(b)$. Solving for b then yields the bidding strategy

$$\beta^0(v) = \begin{cases} 0 & v < 1; \\ (v+1)/2 & v \in [1,5]; \\ 3 & v > 5. \end{cases}$$

The optimal bid function is illustrated in Figure 2a, with the value of v represented on the vertical axis and the bid on the horizontal axis. If the expected value of the object is less than 1, the buyer does not bid. For values of v between 1 and 5, the bid function is increasing with slope 1/2 and range [1,3]. For values of v which exceed 5, the buyer bids 3, which is just sufficient to win the object with certainty.

Example 2: R is a Bernoulli Random Variable

Alternatively, suppose that R has a Bernoulli distribution with equal mass concentrated at 1 and 3, and again assume that V is exponentially distributed with mean 6. In the absence of competition from any uninformed buyer, the optimal bid function for the informed buyer is

$$\beta^0(v) = \begin{cases} 0 & v < 1 \\ 1 & v \in [1,5] \\ 3 & v > 5. \end{cases}$$

The optimal bid function for Example 2 is illustrated in Figure 3a. If the expected value of the object is less than 1, the buyer does not bid. For values of v between 1 and 5, expected profit is maximized by a bid of 1 which wins the object with probability 1/2. For values of v greater than 5, the buyer bids 3 to win the object with certainty.

Now consider the effect on the equilibrium bid of the informed buyer when an uninformed buyer enters the auction. Since we have assumed that X is identical to V, it follows trivially that (V,R,X) is affiliated. Therefore, from Lemma 3, we know that $\beta(v)$ is nondecreasing in v. The nonpositive profit condition for uninformed buyer (Lemma 1b) then implies that, for $\beta(v) > 0$,

(6) $\beta(v) \geq E[V|V \leq v]$.

Lemma 1b also implies that the uninformed buyer never bids in the range where $\beta(v)$ exceeds $E[V|V \leq v]$. Consequently, $G_\alpha(\beta(v)) = G_\alpha(E[V|V \leq v])$. Since the best response condition for the informed buyer (Lemma 1a) implies that $(v-\beta(v))K(\beta(v))G_\alpha(\beta(v)) \geq (v-b)K(b)G_\alpha(b)$ for all b, and since $G_\alpha(b)$ is nondecreasing, it follows that $\beta(v)$ must also satisfy, for all $b \geq E[V|V\leq v]$,

(7) $$(v-\beta(v))K(\beta(v)) \geq (v-b)K(b).$$

We conclude from relations (6) and (7) that $\beta(v)$ is the maximizer of $(v-b)K(b)$ subject to $b \geq E[V|V\leq v]$. Since $\beta^0(v)$ is the maximizer of $(v-b)K(b)$, it follows immediately that $\beta(v) \geq \beta^0(v)$. This relation does not depend on the monotonicity of β, and, in the Appendix, it is established for any distribution of (V,X,R).

Define $\hat{v}$ to satisfy $E[V|V\leq\hat{v}] = 3$. In Example 1, $\beta^0(v) > E[V|V\leq v]$ for $1 < v < \hat{v}$. Therefore, given β^0, the only bids at which the uninformed buyer earns positive profit are between 3 and 6, where the expected return is 6-b. (A bid above 3 wins the object for sure against β^0, and 6 is the unconditional expectation of V.) However, if the uninformed buyer bids in this interval with certainty, the informed buyer no longer earns positive profits. To restore equilibrium, the informed buyer with $v > \hat{v}$ must increase his bid to $\beta(v) = E[V|V\leq v]$. Faced with this revised strategy, the uninformed buyer is now indifferent between submitting a bid between 3 and 6 and staying out. Consequently, his strategy can be adjusted so that the informed buyer with a value of v less than $\hat{v}$ has no incentive to change his bid from $\beta^0(v)$.

In Example 2, $\beta^0(v) = 1$ for $1 < v < 5$. However, as indicated in Figure 3a, $E[V|V\leq 5] > 1$. Therefore, by bidding slightly above 1, an uninformed buyer earns positive expected profits. Also, as in Example 1, a bid slightly above 3 yields positive profits as well. In this case, therefore, the strategy of the informed buyer must be adjusted over two intervals of v, $[\tilde{v},5]$ and $[\hat{v},\infty)$, where $\tilde{v}$ and $\hat{v}$ satisfy $E[V|V\le\tilde{v}] = 1$ and $E[V|V\le\hat{v}] = 3$. To compute the equilibrium strategy of the informed buyer, relations (6) and (7) imply that when the value of the object lies between $\tilde{v}$ and 5, he must choose between a bid of $E[V|V\le v]$, or a bid of 3. This choice arises because the informed buyer's profit function is discontinuous at a bid of 3, where the probability of beating the reservation price jumps from 1/2 to 1. Let v^* denote the value of v at which the informed buyer is just indifferent between a bid of $E[V|V\le v]$ and 3. It is easy to show that $\tilde{v} < v^* < 5$. Therefore, for $v < \tilde{v}$, $\beta(v) = \beta^0(v)$, and for $\tilde{v} < v < v^*$, $\beta(v) = E[V|V\le v]$. For higher realizations of V, the construction follows Example 1 with $\beta(v) = 3$ for $v^* < v < \hat{v}$, and $\beta(v) = E[V|V\le v]$ for $v > \hat{v}$. The function is illustrated in Figure 3a.

The distribution of the informed and uninformed bids for our two examples are illustrated in Figures 2b and 3b. Although it is not possible to compute a closed form solution for the bid distributions in either of these examples, it is a direct consequence of a result in EMW that they are identical for bids above the range of the reservation price. In fact, it is a implication of Theorem 1 below, which also implies that the rates of increase in the two distributions in Example 2 are equal over the interval $[1,b^*]$.

In general, the introduction of a random reservation price may result in an irregular distribution of bids. For instance, in Example 2, the supports of the distribution function of both buyers are not connected. Also, in both examples, there is at least one mass point at a positive bid in the distribution function of the informed bid. Notice, however, that in both cases the support of the informed bid contains the support of the uninformed bid. This property follows from the observation that the expected value of the object to a winning uninformed bid is the same at any price in an interval (b',b) in which the informed agent never bids. Therefore, if the expected profit to the uninformed buyer is zero at b', it must be negative at any higher bid in (b',b).

Also note that in both cases there is a bid b such that $G_\alpha(b) = G_\alpha(0)$ and $G_\beta(b) > G_\beta(0)$. In example 1, G_β is strictly increasing over [1,3], but $G_\alpha(3) = G_\alpha(0)$. In example 2, G_α is strictly increasing over $[1,b^*]$, but G_β is discontinuous at 1. This property is a consequence of the fact that the expected value of the object to a winning uninformed bid is its average value to an informed buyer who bids less. Therefore, to earn nonnegative profit, the bid of the uninformed buyer must exceed some profitable informed bids which would earn positive profits.

In the next section, we extend these arguments to the more general case where (V,R,X) is affiliated.

5. **The Bid Distributions**

Given that the informed bid is nondecreasing in X, it follows from Assumption 1 that (V,R,β) is also affiliated. In this section, we exploit this relation to establish the main theoretical results of the paper. Theorem 1 establishes two results. First, the rate of increase in G_β is never less than the rate of increase in G_α. From this property it follows immediately that (i) the support of β contains the support of α, and (ii) β stochastically dominates α. Second, when R is independent of (V,X), the rate of increase in G_β <u>equals</u> the rate of increase in KG_α. It follows immediately from this property that G_α and G_β are identical above the support of R. Theorem 2 establishes that the informed buyer is more likely to submit a low bid than is the uninformed buyer.

For the remainder of the analysis, we assume that (α,β) forms an equilibrium. Lemma 3 and Assumption 4 then yield the following restatement of Lemma 2 and Assumption 2 in terms of bid of the informed buyer rather than his information signal.

LEMMA 4: (a) $E[V|R=r',\beta=b'] \le E[V|R=r,\beta=b]$, $(r,b) \ge (r',b')$, (with strict inequality if $b > b'$) a.s.-$P_{R\times\beta}\times P_{R\times\beta}$.

(b) $K(r'|b')K(r|b) \ge K(r'|b)K(r|b')$, $r \ge r'$, $b \ge b'$ a.s.-$P_\beta\times P_\beta$.

Lemma 4 combined with the equilibrium conditions of Lemma 1 provide the restrictions necessary to establish our main result on the equilibrium distribution of bids. The details of the argument are complicated by the fact that G_β need not be continuous, piecewise differentiable, nor strictly increasing over the convex hull of its support. Therefore, we confine the text to a heuristic demonstration of the result and provide a complete proof in the Appendix.

THEOREM 1: (a) $b_0 \le b_1$ implies $G_\alpha(b_0)G_\beta(b_1) \ge G_\alpha(b_1)G_\beta(b_0)$.

(b) Suppose R is independent of (V,X). If G_α is strictly increasing over $[b_0,b_1]$, then $K(b_0)G_\alpha(b_0)G_\beta(b_1) = K(b_1)G_\alpha(b_1)G_\beta(b_0)$.

SKETCH OF PROOF: Suppose G_α and G_β are both differentiable and increasing at bid $b > 0$. Denote the densities of all distribution functions by their lower case letters and let $w = E[V|R\le b,\beta=b]$, $v = E[V|R=b,\beta=b]$, $\bar{v} = \int_{-\infty}^{b} E[V|R=b,\beta=t]\,dG_\beta(t)/G_\beta(b)$, $K = K(b|b)$, and $\bar{K} = \int_{-\infty}^{b} K(b|t)\,dG_\beta(t)/G_\beta(b)$.

Suppose the informed buyer considers a unit increase in his bid. Since he wins the object with probability KG_α at bid b, his expected cost conditional on winning at bid b rises by KG_α. On the other hand, the gain in his expected profit from the additional chance of winning the object can be decomposed into two components. The first is his gain when the uninformed buyer bids just above b, $(w-b)Kg_\alpha$. The second is his gain when the reservation price is just above b, $(v-b)kG_\alpha$. Consequently, the first order condition for profit maximization is

$$(8) \qquad (w-b)Kg_\alpha + (v-b)kG_\alpha = KG_\alpha .$$

Similarly, the first order condition for profit maximization by the uninformed buyer is

$$(9) \qquad (w-b)Kg_\beta + (\bar{v}-b)\bar{k}G_\beta = \bar{K}G_\beta .$$

By Assumption 4, the informed buyer makes positive profit at bid b. Therefore, $w-b > 0$.

To prove part (a), note first that Lemma 2b implies that $K \leq \bar{K}$, and Lemma 1b and Lemma 2a imply $\bar{v} \geq b$. Therefore, we may combine equations (8) and (9) to yield

$$(10) \qquad (w-b)\left(\frac{g_\beta}{G_\beta} - \frac{g_\alpha}{G_\alpha}\right) \geq (v-b)\,\frac{k}{K} - (\bar{v}-b)\,\frac{\bar{k}}{\bar{K}} .$$

Lemma 4a implies $v \geq \bar{v}$, and Lemma 4b implies $k/K \geq \bar{k}/\bar{K}$. It follows from (10), therefore, that $g_\beta/G_\beta \geq g_\alpha/G_\alpha$. Integrating over [b',b] establishes part (a).

To prove part (b), note that if R is independent of (V,X), then $K = \bar{K}$, $k = \bar{k}$, and $w = v$. Furthermore, the zero profit condition (Lemma 1b) implies that $\bar{v} = b$. Combining equations (9) and (10) then imply

$$(11) \qquad \frac{g_\beta}{G_\beta} = \frac{g_\alpha}{G_\alpha} + \frac{k}{K} .$$

Integrating over [b',b) establishes part (b). Q.E.D.

Theorem 1a implies the following corollary.

COROLLARY 1: (a) $G_\alpha(b_0) < G_\alpha(b_1)$ implies $G_\beta(b_0) < G_\beta(b_1)$. (b) $G_\alpha \geq G_\beta$.

Part (a) states that the support of G_β contains the support of G_α. Part (b) states that G_β stochastically dominates G_α.

Notice also that, wherever G_α is increasing, Theorem 1b implies that the distribution of $\max(\{R,\alpha\})$ grows at the same rate as the distribution of β. This generalizes a result of EMW which established that G_α and G_β are equal whenever the reservation price is not random. More generally, since at any bid above the support of R, $\max(\{R,\alpha\}) = \alpha$, it follows that the distribution functions are identical over this range.

COROLLARY 2: If $K(b|b) = 1$, for $b \geq b_0$, then $G_\alpha(b) = G_\beta(b)$ for $b \geq b_0$.

Besides the assumption that all distribution functions are differentiable, the sketch of the proof of Theorem 1 provided above also supposes that the two distribution functions have the same support. As we saw in Example 1 of Section 4, this restriction need not be satisfied when the reservation price is random. The more general proof provided in the Appendix handles these cases as well.

We turn next to the restrictions on the distribution functions at low bids. These results require the following assumption.

ASSUMPTION 5: $P_X(\{x \in R^n\colon J(b|x) > 0$ implies $E[V-b|R\leq b, X=x] < 0\}) > 0$.

Assumption 5 states that the informed buyer sometimes receives a signal for which his expected profit is negative at any bid which exceeds the reservation price with positive probability.

Recall that the expected value of the object to the uninformed buyer when he wins the object with a bid of b is the average value of the object to the informed buyer when his equilibrium bid is no greater than b. Therefore, if the informed buyer sometimes expects negative profit at any winning bid, any zero profit, positive uninformed bid must exceed some profitable informed bids. This observation motivates Theorem 2. A precise proof requires a careful limiting argument and is presented in the Appendix.

THEOREM 2: There is a $b > 0$ such that $G_\beta(b) > G_\beta(0)$ and $G_\alpha(b) = G_\alpha(0)$.

Theorem 2 implies that the informed buyer is more likely to submit a low bid than is the uninformed buyer. Either the informed buyer will submit his lowest bid with positive probability while the uninformed bid distribution is continuous, or there is an interval around the lowest informed bid in which the uninformed buyer never submits a bid. The size of this interval depends on the distribution of (V,R,X). Roughly, the interval will be larger as X becomes a more accurate predictor of V and as the distribution of R becomes less concentrated. As X becomes more informative, the distribution of $E[V|X]$ becomes more risky and hence the expected losses, conditional on low realizations of X, increase. As the distribution of R becomes more diffuse, the informed buyer has less incentive to shade his bid over a wider range of signals, reducing the expected profit to a winning uninformed bid over that range.

6. Tests of the Theory

In this section, we analyze the bidding data from a superset of the drainage tracts used by HP in their study of auctions for offshore oil leases. Our data set contains the number and characteristics of the firms who submitted bids, the value of their bids, and the rejection decision on all drainage tracts offered for sale between 1959 and 1979 on which at least one firm submitted a bid.

As we noted in the introduction, the structure of our model is based on the analysis of HP. Working with data from tracts offered for sale during the period 1959 to 1969, they estimated the _ex post_ values of tracts offered for sale and the adjacent tracts owned by firms participating in the auctions. With these estimates, they investigated the relation between tract values and the bidding behavior of the firms.[7] Their work provides strong evidence that neighbor firms had better information about the value of a tract than did

[7] For tracts sold after 1970, we were unable to generate reliable estimates of tract value, both because the production histories are truncated and because expectations of firms about future oil and gas prices are difficult to measure or infer.

nonneighbor firms and that the neighbor firms coordinated their bidding so that there was effectively only one informed bidder. Therefore, we will ignore all but the highest neighbor bid on any tract and call it the **neighbor** bid, B_I. We will call the highest bid among all nonneighbor firms, B_U, the **high nonneighbor** bid.[8]

Our assumption that the reservation price is an exogenous random variable is based on the HPS study of the same data set we employ here. Although the rejection decision is made after the bids are submitted, HPS found no evidence of strategic behavior on the part of the government.[9] Also, based on their estimates of a probit equation of the rejection decision, they conclude that the probability of rejection was higher if the bid was submitted by a neighbor firm, even after conditioning on the value of the bid. While neighbor firms submitted the highest bid on 62% of the tracts, they submitted 82% of the rejected bids. Of tracts on which the high bid was less than .5 million dollars, 27 of the 54 bids submitted by neighbor firms were rejected while only 4 of 15 bids submitted by nonneighbor firms were rejected.[10]

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[8] There were two or more bids by neighbor firms on only 59 of the 257 tracts on which at least one neighbor firm submitted a bid. There were two or more bids by nonneighbor firms on 92 of the 168 tracts on which at least one nonneighbor firm submitted a bid.

[9] According to Darius Gaskins (1976, p. 241), "the primary factor used in evaluating bids is the government's evaluation," which is determined prior to the sale date.

[10] HPS do not report the relation between the rejection decision and the number of bids. While 169 of 295 tracts received two or more bids, those tracts accounted for only 11 of the 58 rejected bids. (Of the 130 tracts that received at least one neighbor and one nonneighbor bid, only 3 were rejected.) Although these figures suggest that the number of bids may have influenced their rejection decision, HPS found that the number of bids is sufficiently correlated with the maximum bid and the neighbor dummy so that its coeffecient is not significantly different from 0. In any event, we have not investigated the theoretical implications of this assumption.

Any test of the theory must incorporate the fact that the properties derived in Section 5 are valid only after conditioning on all relevant information, S, possessed by the nonneighbor firms. Let $G_U(\cdot,s)$ denote the distribution function of B_U, conditional on S=s, $G_I(\cdot,s)$ the distribution function of B_I, conditional on S=s, and $\bar{r}(s)$ the upper bound of the support of R, conditional on S=s. Theorems 1 and 2 then imply the following relations for each realization s of S.

(R1) $[G_U(b,s)-G_U(b-\epsilon,s)]/G_U(b,s) \le [G_I(b,s)-G_I(b-\epsilon,s)]/G_I(b,s)$ for $G_I(b,s),G_U(b,s) > 0$ and $\epsilon > 0$.

(R2) $G_U(b,s) = G_I(b,s)$, $b \ge \bar{r}(s)$.

(R3) For some $\underline{b}(s)$, $G_I(\underline{b}(s),s) > G_I(0,s)$ and $G_U(\underline{b}(s),s) = G_U(0,s)$.

Ignoring for the moment the complications introduced by the information variable S, the histograms presented in Figures 4 and 5 illustrate two aspects of the empirical distributions which directly test these relations. Figure 4 illustrates the relation between $\Delta G_I/G_I$ and $\Delta G_U/G_U$. The ratios were constructed by partitioning the set of all positive neighbor and high nonneighbor bids into 8 equally sized subsets according to their rank. The interval of bids for each subset is indicated on the horizontal axis. For each interval of bids we then divided the number of neighbor bids in that interval by the number of tracts for which the neighbor bid was in that interval or below plus the number of tracts for which no neighbor firm submitted a bid to obtain $\Delta G_I/G_I$. We computed $\Delta G_U/G_U$ similarly.

Figure 4 lends strong support to all three relations. First, $\Delta G_I/G_I$ is as least as large as $\Delta G_U/G_U$ over each range of bids, as required by (R1). Second, the two ratios are roughly equal in the upper four intervals as is required if the distributions are equal over that range. Finally, $\Delta G_I/G_I$ exceeds $\Delta G_U/G_U$ by more than a factor of 5 over the first two intervals lending support to (R3).

Figure 5 provides additional evidence regarding the relative frequency of low bids by the neighbor and nonneighbor firms. Relation (R3) implies that over intervals of low bids, there should be relatively more neighbor bids than high nonneighbor bids. In fact, the theory implies that regardless of which nonneighbor bid is selected on any tract, there should be more neighbor than nonneighbor bids. Figure 5 illustrates the ratio of the number of bids in any interval to the total number of positive bids for the following four criteria: (1) the high positive nonneighbor bid on each tract, (2) all positive nonneighbor bids, (3) the low positive nonneighbor bid on each tract, and (4) all positive (high) neighbor bids. Notice that in the lowest interval, the relative number of neighbor bids is twice the relative number both of high nonneighbor bids and of all nonneighbor bids. The relative number of low nonneighbor bids in this interval is roughly equal to the relative number of neighbor bids. Although a bit weaker, these relations are also satisfied in the second interval. We note also that since relatively more neighbor firms submitted at least one positive bid, these figures actually understate the predominance of neighbor bids in the lower interval when we include the zero bids of the firms.

6.1 The Wilcoxon Test Statistic

We turn now to the question of whether or not the relationships illustrated in Figures 4 and 5 are statistically significant. All of our tests are based on an application of the Wilcoxon rank sum statistic, extended to accommodate mass points in the distributions as outlined by Lehman (1975, pp. 5-23). The test works as follows. Let $\{x_t,\ t = 1,\ldots,T_1\}$, and $\{y_t,\ t = 1,\ldots,T_2\}$ denote the observations from two samples, and define

$$h_{st} = \begin{cases} 1 & \text{if} \quad x_s < y_t, \\ 1/2 & \text{if} \quad x_s = y_t, \\ 0 & \text{otherwise.} \end{cases}$$

The Wilcoxon rank sum test statistic is then defined as $U = \sum_{s=1}^{T_1} \sum_{t=1}^{T_2} h_{st}$. When all T_1T_2 possible pairs of observations are considered, U is the total number of pairs in which the observation from the first sample is less than the observation from the second sample, plus one-half of the pairs in which they are equal.

Let $\{w_1,\ldots,w_n\}$ denote the values at which at least one pair of observations are equal. Let $T = T_1+T_2$ denote the total number of observations, and let K_i denote the total number of realizations from both samples at each value w_i, $i = 1,\ldots,n$. Then, for samples of independent observations generated by the same distribution, the distribution of U is approximately normal with $E[U] = T_1T_2/2$, and $Var[U] = [T_1T_2/12T(T-1)] \times [(T+1)T(T-1) - \sum_{i=1}^{n} K_i(K_i^2-1)]$. The approximation is close even for relatively small sample sizes (e.g. $T_1 = T_2 = 10$). Let $Z = (U-E[U])/Var[U]^{1/2}$. Then we may reject the null hypothesis that the two samples are generated by the same distribution if the realization of Z is significantly different from 0, under the assumption that Z has a standard normal distribution.

6.2 Test for Independence of the Bid Distributions

Given the enormous variation in the winning bids (from $28 Thousand to $114 Million), it is reasonable to suppose that nonneighbor firms were able to differentiate among some of the tracts in our sample and that this fact was common knowledge.[11] If nonneighbor firms do possess a substantial amount of information, one implication is that B_I and B_U should not be independently distributed. To test for the dependence of these distributions, we partition a subset of tracts into two subsets according to the value of the high nonneighbor bid on that tract. Assuming the neighbor bid is independent of the high nonneighbor bid, the empirical distributions of the neighbor bid from the two subsamples should not be significantly different.[12] We consider three variations of this test.

In the first variation, we divide the set of neighbor bids into two subsamples depending on whether or not a nonneighbor firm bid on that tract. In the second variation, we assign a value of zero to the neighbor bid on those tracts in which no neighbor bid was actually submitted. We then consider only those tracts on which a nonneighbor bid was submitted and divide the corresponding neighbor bids into two subsamples depending on whether or not the high nonneighbor bid exceeds the median nonneighbor bid, $\hat{b}_U$. The

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[11] In their earlier study, HP established that, conditioning on the number of neighbor firms, the tract acreage, and the _ex post_ value of the adjacent tract (the most significant of these variables), there is no significant relation between the high nonneighbor bid and the _ex post_ value of the tract. From this, they conclude that these variables contain most of the information possessed by nonneighbor firms in assessing the value of the tract. As noted above, however, we are unable to construct this information for most of the tracts in our sample.

[12] Alternatively, this can be viewed as a test of whether the pair (B_I, B_U) is identically distributed across all tracts.

third variation is similar except that we consider only those tracts on which both a neighbor and nonneighbor bid were submitted.[13] As reported in Table 1, each of these tests strongly rejects the hypothesis that B_U and B_I are independent. In all three cases, the conditional distribution of B_I depends significantly on the range of B_U.

Table 1. Test For Independence

Sample X:	$B_U = 0,\ B_I > 0$	$0 \leq B_U < \hat{b}_U$	$0 \leq B_U < \hat{b}_U,\ B_I > 0$
Sample Y:	$B_U > 0,\ B_I > 0$	$\hat{b}_U < B_U$	$\hat{b}_U < B_U,\ B_I > 0$
# of X Bids	127	84	65
# of Y Bids	130	84	65
# of 0 Bids	0	38	0
z	4.17 (0.0000)	5.57 (0.0000)	4.76 (0.0000)

The number in the parenthesis is the probability that $|Z| > z$. $\hat{b}_u$ is the median positive nonneighbor bid for the given sample of tracts.

Given the strong dependence between B_I and B_U, we conclude that any tests of our theory must be based on the presumption that $G_U(\cdot,s)$ and $G_I(\cdot,s)$ depend nontrivially on additional characteristics of the tract offered for sale. Let μ denote the measure on the nonneighbor information, s, for those tracts used in our sample, and let $G_U(b) = \int G_U(b,s)\mu(ds)$ and $G_I(b) = \int G_I(b,s)\mu(ds)$ denote the respective marginal distributions of B_U and B_I. Rather than test relations (R1) to (R3) directly, we will test their implications for G_U and G_I.

[13] We cannot examine the distribution of neighbor bids over all tracts, with and without nonneighbor bids, because the tracts on which no bids were submitted are not included in our data set. Consequently, of the tracts with no nonneigbor bid, we can include only those with positive neighbor bids.

The dependence of B_I and B_U also introduces a complication for other tests based on the Wilcoxon test statistic. Since the calculation of Var[U] presumes that all observations of B_I and B_U are i.i.d., any test comparing the distributions of G_U and G_I will properly require that we draw our observations of B_I and B_U randomly from different tracts.[14] Unfortunately, this considerably reduces the power of our tests. We will therefore adopt the following strategy. We have assigned each tract to one of two equally sized subsets based on the realizations of a sequence of random numbers. For all of our remaining tests, we then report three statistics. The first statistic uses the neighbor and nonneighbor bids from all tracts. The second and third statistics are computed by alternatively drawing the neighbor bids from the first subsample of tracts and the nonneighbor bids from the second, and visa-versa.

6.3 Test of the Monotone Likelihood Ratio Property

Relation (R1) implies a monotone likelihood ratio property between $G_U(\cdot,s)$ and $G_I(\cdot,s)$. This relation is preserved even after we integrate over s. To test relation (R1), we consider n equally sized subsets of the positive neighbor and high nonneighbor bids partitioned according to rank. Let $b_0 = 0$ and let b_i denote largest bid in the ith subset, $i = 1,\ldots,n$. For each value of $i = 1,\ldots,n$, we restrict attention to those bids which are less than or equal to b_i. We then define two simple step functions as follows. For

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[14] Although we know of no general results, the expression for Var[U] appears to typically overstate the variance of U for random variables which are highly correlated. Therefore, if we consider samples which contain both the neighbor bid and the high nonneighbor bid from the same tracts, the variance of Z is typically less than 1 so that the tests based on these samples are biased in favor of the null hypothesis.

$j = I,U$, let $X_j^i = 1$ if $b_{i-1} < B_j \leq b_i$, and $X_j^i = 0$, otherwise. If relation (R1) is satisfied for all s, then X_I^i should stochastically dominate X_U^i for all i. That is, the ratio of the number of neighbor bids in interval i to the number of all neighbor bids (including zero bids) in interval i or below should exceed the corresponding ratio for the high nonneighbor bids.

Unfortunately, since our data includes only those tracts on which at least one bid was submitted, we cannot directly test this relation. Let H_U^i and H_I^i denote the respective distribution functions of X_U^i and X_I^i. As a substitute, we test the relationship between $H_U^i(\cdot|B_I>0)$ and $H_I^i(\cdot|B_U>0)$.[15] Notice that, if the distribution of S conditional on $B_I > 0$ is equal to the distribution of S conditional on $B_U > 0$, then $H_U^i(b|B_I>0) - H_I^i(b|B_U>0) = H_U^i(b) - H_I^i(b)$, so that a test of a relation between $H_U^i(b|B_I>0)$ and $H_I^i(b|B_U>0)$ is equivalent to a test of the same relation between H_U^i and H_I^i. There appears to be no obvious way to test this equality, however.

Table 2 reports the value of our test statistic under the hypothesis that $H_U^i(b|B_I>0) = H_I^i(b|B_U>0)$ for partitions of 8 sets of bids, and of 4 sets of bids, each for the three subsamples described above. Let Z denote the Wilcoxon test statistic defined above where X_U^i generates the $\{x_t\}$ sample and X_I^i generates the $\{y_t\}$ sample. Table 2 uses the following definitions.

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[15] An alternative is to simply guess the number of tracts which were offered for sale but received no bids and add that number to the number of zero bids for both neighbor and nonnneighbor firms. Tests based on the assumption that neighbor and nonneighbor zero bids were independent choices generated similar results. Figure 4 is constructed under the assumption that at least one bid was offered on every tract offered for sale. Since there were 127 tracts on which only neighbor firms submitted a bid but only 38 tracts on which only nonneighbor firms submitted a bid, this figure probably overstates slightly the ratio $\Delta G_I/G_I$ relative to $\Delta G_U/G_U$.

ΔN - Number of neighbor bids and high nonneighbor bids in $(b_{i-1}, b_i]$.

ΔN_H - Number of tracts on which the highest bid is in $(b_{i-1}, b_i]$.

ΔN_R - Number of rejected high bids in $(b_{i-1}, b_i]$.

N_I - Number of neighbor bids in $[0, b_i]$ on tracts with $B_U > 0$.

N_{HU} - Number of high nonneighbor bids in $[0, b_i]$ on tracts with $B_I > 0$.

ΔN_I - Number of neighbor bids in $(b_{i-1}, b_i]$ on tracts with $B_U > 0$.

ΔN_{HU} - Number of high nonneighbor bids in $(b_{i-1}, b_i]$ on tracts with $B_I > 0$.

z - Realization of Z using the entire sample.

z_j - Realization of Z when neighbor bids are taken from the jth random subset of tracts and the nonneighbor bids are taken from its complement.

Table 2. Test For The Monotone Likelihood Ratio Property

	1	2	3	4	5	6	7	8
b_i	.217	.564	1.370	2.447	4.221	6.671	14.133	114.128
ΔN	53	53	53	53	54	53	53	53
ΔN_H	37	37	33	30	40	40	37	41
ΔN_R	19	15	6	8	4	4	2	0
ΔN_I	13	17	13	16	15	14	19	23
N_I	51	68	81	97	112	126	145	168
ΔN_{HU}	8	5	14	17	19	23	23	21
N_{HU}	135	140	154	171	190	213	236	257
z	3.75 (.0001)	4.70 (.0000)	1.59 (.0559)	1.57 (.0582)	0.90 (.1841)	0.09 (.4641)	1.01 (.1562)	1.82 (.0344)
z_1	2.68 (.0037)	3.23 (.0006)	0.91 (.1814)	1.24 (.1075)	0.45 (.3264)	-0.31 (.6217)	-0.01 (.5040)	2.21 (.0136)
z_2	2.72 (.0033)	3.49 (.0002)	1.42 (.0427)	.96 (.1685)	0.82 (.2061)	0.37 (.3557)	1.36 (.0869)	0.25 (.4013)
z	5.81 (.0000)		2.22 (.0132)		0.72 (.2358)		1.97 (.0244)	
z_1	4.11 (.0000)		1.49 (.0681)		0.11 (.4562)		1.57 (.0582)	
z_2	4.10 (.0000)		1.65 (.0495)		0.85 (.1977)		1.21 (.1131)	

The number in parentheses is the probability that $Z > z$ for a standard normal distribution.

The z statistics reported in Table 2 test the equality of $\Delta N_I/N_I$ and $\Delta N_{HU}/N_{HU}$ for each of the relevant intervals. For bids less than .56 million dollars, we obtain a clear rejection of the equality of $\Delta G_U/G_U$ and $\Delta G_I/G_I$ in favor of the one sided alternative that $\Delta G_I/G_I$ exceeds $\Delta G_U/G_U$. The evidence for rejecting the null hypothesis for bids in the range .56 to 2.54 Million is much weaker. For higher bids where the probability of rejection is 10% or less, our tests provide no evidence that equality of the two rates can be rejected, except possibly in the highest octile. Furthermore, for no interval of bids would we reject the hypothesis that the rates are equal in favor of the one sided alternative that $\Delta G_U/G_U$ exceeds $\Delta G_I/G_I$. We conclude that these tests confirm the evidence provided in Figure 4 in providing strong support for relation (R1).

6.4 Tests for Equality of Bid Distributions

Relation (R2) implies that G_I and G_U should be identical above the support of the reservation price. As indicated in Table 2, the proportion of high bids which were rejected is not more than 10% at any octile of bids above 2.45 Million. However, it is difficult to obtain a precise estimate of the rate of change in the probability of rejection. First, the rejection ratios do not correspond to a single probability distribution, but rather to the points $\int K(b|b,s)\mu(ds)$ in the one parameter family of probability distributions $\int K(\cdot|b,s)\mu(ds)$. Second, estimates of $K(b|b)$ require roughly the square of the number of data points required to estimate G_α or G_β with the same precision. Nevertheless, it is clear that changes in the rejection probability are nearly zero at extremely high bids. We should therefore expect to accept the hypothesis that $G_I(b) = G_U(b)$ at the upper tails of the distributions.

When applied to B_I and B_U and appropriately normalized, the Wilcoxon test statistic measures $\int[G_U(b)dG_I(b)-G_I(b)dG_U(b)]$. Consequently, if positive differences in one interval are offset by negative differences in another, we may accept the null hypothesis that $G_I = G_U$, even if these differences are quite large. To be reasonably confident that the distributions are roughly identical, therefore, it is important to compare the distributions from as many angles as possible.

If the distributions are identical, one implication is that the rates of change of G_I and G_U are identical. From this perspective, the results reported in Table 2 support (R2), since the hypothesis of equality of rates of change cannot be rejected for bids in the upper tails of these distributions. More to the point, if the distributions are equal, the proportion of bids in each sample should be equal over any interval. In addition, the distributions, conditional on bids in an arbitrary interval, should also be equal.

The test statistic reported in Table 3 is effectively a weighted average of these two tests for equality.[16] For each interval among the octiles, quartiles, halves, as well as the complete range of bids, we define $X_j = B_j$ if B_j is in the interval, and $X_j = 0$, otherwise, for $j = I,U$. We then test the equality of the distributions of X_I and X_U.[17] Let Z denote the Wilcoxon test statistic where X_U generates the $\{x_t\}$ sample and X_I generates the $\{y_t\}$ sample. The following definitions are used in Table 3.

N_H = Number of tracts on which the highest bid is in (b_{i-1},∞).

N_R = Number of rejected high bids in (b_{i-1},∞).

[16] We also performed the two tests separately over the same intervals with similar results.

[17] In this case, the absence of the tracts on which no bids were submitted does not present a sample selection problem. Letting q denote the probability that a tract receives no bid, our sample consists of draws from $[G_U(b)-q]/(1-q)$ and $[G_I(b)-q]/(1-q)$. Since $0 \leq q < 1$, it follows that $G_I(b) = G_U(b)$ if and only if $[G_I(b)-q]/(1-q) = [G_U(b)-q]/(1-q)$.

ΔN_I - Number of neighbor bids in $(b_{i-1},b_i]$.

ΔN_{HU} - Number of high nonneighbor bids in $(b_{i-1},b_i]$.

Table 3. Test of Equality of Bid Distributions

<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th></tr>
<tr><td>b_i</td><td>.217</td><td>.564</td><td>1.370</td><td>2.447</td><td>4.221</td><td>6.671</td><td>14.133</td><td>114.128</td></tr>
<tr><td>N_R</td><td>58</td><td>39</td><td>24</td><td>18</td><td>10</td><td>6</td><td>2</td><td>0</td></tr>
<tr><td>N_H</td><td>295</td><td>258</td><td>221</td><td>188</td><td>158</td><td>118</td><td>78</td><td>41</td></tr>
<tr><td>ΔN_I</td><td>39</td><td>42</td><td>32</td><td>31</td><td>27</td><td>27</td><td>28</td><td>31</td></tr>
<tr><td>ΔN_{HU}</td><td>14</td><td>11</td><td>21</td><td>22</td><td>27</td><td>26</td><td>25</td><td>22</td></tr>
<tr><td>z</td><td>3.60
(.0002)</td><td>4.95
(.0000)</td><td>1.56
(.0606)</td><td>1.21
(.1131)</td><td>0.00
(.5000)</td><td>0.24
(.5948)</td><td>0.41
(.3409)</td><td>1.37
(.0853)</td></tr>
<tr><td>z_1</td><td>2.38
(.0087)</td><td>3.15
(.0008)</td><td>0.92
(.1814)</td><td>1.34
(.0901)</td><td>-0.20
(.5793)</td><td>-0.42
(.6628)</td><td>-0.43
(.6664)</td><td>2.08
(.0188)</td></tr>
<tr><td>z_2</td><td>2.70
(.0035)</td><td>3.05
(.0011)</td><td>1.37
(.0853)</td><td>0.37
(.3557)</td><td>0.17
(.4325)</td><td>0.65
(.2578)</td><td>0.95
(.1711)</td><td>-0.29
(.6141)</td></tr>
<tr><td>z</td><td colspan="2">5.98
(.0000)</td><td colspan="2">2.04
(.0207)</td><td colspan="2">0.18
(.4286)</td><td colspan="2">1.38
(.0838)</td></tr>
<tr><td>z_1</td><td colspan="2">4.14
(.0000)</td><td colspan="2">1.72
(.0427)</td><td colspan="2">-0.47
(.6808)</td><td colspan="2">1.49
(.0681)</td></tr>
<tr><td>z_2</td><td colspan="2">4.30
(.0000)</td><td colspan="2">1.17
(.1210)</td><td colspan="2">0.65
(.2578)</td><td colspan="2">0.48
(.3156)</td></tr>
<tr><td>z</td><td colspan="4">5.71
(.0000)</td><td colspan="4">1.38
(.0838)</td></tr>
<tr><td>z_1</td><td colspan="4">4.13
(.0000)</td><td colspan="4">1.09
(.1379)</td></tr>
<tr><td>z_2</td><td colspan="4">3.97
(.0000)</td><td colspan="4">0.92
(.1788)</td></tr>
<tr><td>z</td><td colspan="8">4.52
(.0000)</td></tr>
</table>

The number in the parenthesis is the probability that $Z > z$ for a standard normal distribution.

At conventional significance levels, we may certainly reject equality of the distributions over the entire range of bids. However, our results largely support equality of the distributions above 2.447M. Notice that, for this range of bids, less than 7% of the high bids were rejected (although Table 2 indicates that the marginal rejection rate is closer to 10%).

6.5 Test for Predominance of Neighbor Bids at Low Values

Relation (R3) states that, given the nonneighbor information s, there is a threshold $\underline{b}(s)$ at or below which only the neighbor firm bids with positive probability. Integrating over s then implies a $\underline{b}$ such that $[G_\beta(\underline{b})-G_\beta(0)]/G_\beta(0) > [G_\alpha(\underline{b})-G_\alpha(0)]/G_\alpha(0)$.[18] This observation leads us to the following tests of (R3).

Restrict attention to the set of positive bids. For each b_i, let $X_j^i = 1$ if $0 < B_j \leq b_i$, and $X_j^i = 0$ otherwise, for $j = I,U$. We then test if the distribution of X_U^i (weakly) stochastically dominates the distribution of X_I^i. Relation (R3) implies that we should reject this hypothesis for low values of i. In fact, such a relation should be satisfied not only for the **high** nonneighbor bid, but for any selection of nonneighbor bids on each tract. Accordingly, we also construct X_U^i from the samples of **all** positive nonneighbor bids and all **low** nonneighbor bids (i.e. the lowest positive nonneighbor bid on each tract with a positive nonneighbor bid). Our results are reported in Table 4. Let

[18] Since there are more tracts without nonneighbhor bids than tracts without neighbor bids, rejection of $[G_\beta(\underline{b})-G_\beta(0)]/G_\beta(0) \leq [G_\alpha(\underline{b})-G_\alpha(0)]/G_\alpha(0)$ in favor of its alternative is less likely than rejection of $G_\beta(\underline{b})-G_\beta(0) \leq G_\alpha(\underline{b})-G_\alpha(0)$.

ΔN_I = Number of neighbor bids in $(0,b_i]$.

ΔN_{HU} = Number of high nonneighbor bids in $(0,b_i]$.

ΔN_{AU} = Number of all nonneighbor bids in $(0,b_i]$.

ΔN_{LU} = Number of low nonneighbor bids in $(0,b_i]$.

Table 4. Test For Predominance of Neighbor Bids in the Lower Intervals

	1	2	3	4	5	6	7	8
b_i	.217	.564	1.370	2.447	4.221	6.671	14.133	114.128
ΔN_I	39	81	113	144	171	198	226	257
ΔN_{HU}	14	25	46	68	95	121	146	168
z	2.08 (.0188)	3.87 (.0001)	3.45 (.0003)	3.13 (.0009)	2.08 (.0188)	1.17 (.1210)	1.25 (.1056)	*
z_1	1.38 (.0838)	2.67 (.0038)	2.05 (.0202)	2.20 (.0139)	1.48 (.0694)	0.61 (.2709)	-0.63 (.7357)	*
z_2	1.56 (.0594)	2.80 (.0026)	2.83 (.0023)	2.23 (.0129)	1.45 (.0735)	1.05 (.1469)	1.25 (.1056)	*
ΔN_{AU}	24	70	129	180	244	297	252	388
z	3.76 (.0001)	3.95 (.0000)	2.75 (.0030)	2.40 (.0082)	0.95 (.1711)	0.15 (.4404)	-1.13 (.8708)	*
z_1	2.13 (.0166)	2.64 (.0041)	1.36 (.0869)	1.93 (.0268)	0.66 (.2546)	0.15 (.4404)	-2.54 (.9945)	*
z_2	3.16 (.0008)	2.94 (.0016)	2.53 (.0062)	1.45 (.0735)	0.67 (.2514)	0.92 (.1788)	1.01 (.1587)	*
ΔN_{LU}	23	55	91	109	136	146	158	168
z	0.42 (.3372)	-0.26 (.6026)	-2.05 (.9798)	-1.82 (.9656)	-3.24 (.9994)	-2.53 (.9943)	-2.08 (.9812)	*
z_1	0.02 (.4920)	-0.26 (.6026)	-1.59 (.9441)	-1.82 (.9656)	-3.24 (.9994)	-2.53 (.9943)	-2.08 (.9812)	*
z_2	0.56 (.2877)	-0.25 (.5987)	-1.32 (.9066)	-1.64 (.9495)	-2.52 (.9941)	-1.25 (.8944)	-0.52 (.6985)	*

The number in parentheses is the probability that $Z > z$ for a standard normal distribution.

For the sample of high nonneighbor bids and the sample of all nonneighbor bids, Table 4 establishes a clear rejection of the null hypothesis over the lowest quartile of bids. For the sample of low nonneighbor bids, our tests do not lead to rejection of the null hypothesis over any of the intervals. However, it should be noted that they do not lead to rejection of its alternative over the first quartile either. We conclude that the data provides reasonably strong support for relation (R3).

7. Conclusion

This paper examines how asymmetries in the distribution of information among agents affect their behavior in a strategic setting. We study this issue in the context of a first price auction with a random reservation price, in which one buyer has superior private information and all other buyers have access only to public information. Equilibrium bidding behavior under this information structure requires that uninformed buyers collectively bid less frequently than the informed buyer but, if they bid, they submit high rather than low bids. Specifically, we show that the distribution of the informed bid stochastically dominates the distribution of the high uninformed bid in the range of the reservation price, and the two distributions are identical above this range. These theoretical implications are strongly borne out by our data from the auctions for offshore oil drainage leases.

As Ashenfelter's (1989) discussion of wine auctions demonstrates, a random reservation price strategy is not unique to offshore oil and gas lease auctions, but can also be found in the private sector. Therefore, the implications of a random reservation price for bidding behavior under different information structures may be of more general interest than suggested by our work here.

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APPENDIX

A1. The Effect of Introducing Uninformed Buyers on the Informed Bid

In this section, we establish that the introduction of unininformed buyers never lowers the equilibrium bid of the informed buyer. We suppose that bids must nonnegative.

THEOREM A1: If the auction with no uninformed buyers has an equilibrium, then it has an equilibrium β^0 such that, for any equilibrium (β,α) with an uninformed buyer, $G_\alpha(\beta(x)) > 0$ implies $\beta^0(x) \le \beta(x)$ a.s.-P_X.

PROOF: Let (β,α) be an equilibrium for an auction with an uninformed buyer and β^0 an equilibrium for an auction with no uninformed buyer. Then, since $E[V-b|R\le b,X=x]J(b|x)$ is right continuous in b a.s.-P_X, we may define $\beta^0(x)$ to be the smallest maximizer of $E[V-b|R\le b,X=x]J(b|x)$ a.s.-P_X. Let $L = \{x \in R^p: \beta(x) < \beta^0(x)$ and $G_\alpha(\beta(x)) > 0\}$. Suppose $P_X(L) > 0$. Then, the definition of β^0 implies $\int_{\{x\in L\}}E[V-\beta(X)|R\le\beta(x),X=x]J(\beta(x)|x)G_\alpha(\beta(x))P_X(dx) <$ $\int_{\{x\in L\}}E[V-b|R\le\beta^0(x),X=x]J(\beta^0(x)|x)G_\alpha(\beta(x))P_X(dx)$. But since $G_\alpha(\beta(x)) \le$ $G_\alpha(\beta^0(x))$ for $x \in L$, the best response property for β (Lemma 1a) requires $\int_{\{x\in L\}}E[V-\beta(X)|R\le\beta(x),X=x]J(\beta(x)|x)G_\alpha(\beta(x))P_X(dx) \ge$ $\int_{\{x\in L\}}E[V-\beta^0(X)|R\le\beta^0(x),X=x]J(\beta^0(x)|x)G_\alpha(\beta^0(x))P_X(dx) \ge$ $\int_{\{x\in L\}}E[V-\beta^0(X)|R\le\beta^0(x),X=x]J(\beta^0(x)|x)G_\alpha(\beta(x))P_X(dx)$. A contradiction. Q.E.D.

A2. Affiliation

In this section, we define the concept of affiliated random variables and establish Lemma 2 of the text. If Z is a random vector taking values in $\mathbb{R}^p$, then the probability measure on $\mathbb{R}^p$ induced by Z is denoted by P_Z.

For $x,y \in \mathbb{R}^p$, let $x \wedge y$ denote the pointwise minimum of x and y, and let $x \vee y$ denote the pointwise maximum of x and y. A function $f:\mathbb{R}^p \to R$ is **affiliated** if $x,y \in \mathbb{R}^p$ implies $f(x\wedge y)f(x\vee y) \geq f(x)f(y)$. A set $S \in \mathbb{R}^p$ is a **sublattice** if ι_S is affiliated.

For any set $A \subset \mathbb{R}^p$, let ι_A denote the indicator function for A on $\mathbb{R}^p$. A set $A \in \mathbb{R}^p$ is **increasing** if ι_A is nondecreasing. Given two sets A and B, let AB denote their intersection.. A random variable Z is **affiliated** if, for all increasing subsets A_1,A_2 of $\mathbb{R}^p$ and every sublattice S of $\mathbb{R}^p$, $P_Z(A_1A_2S)P_Z(S) \geq P_Z(A_1S)P_Z(A_2S)$. Milgrom and Weber (1982) establish the following result.

LEMMA A1: Z is affiliated if and only if for any nondecreasing functions α_1 and α_2 on a sublattice $S \subset \mathbb{R}^p$, $E[\alpha_1(Z)\alpha_2(Z)\iota_S(Z)]\ E[\iota_S(Z)] \geq E[\alpha_1(Z)\iota_S(Z)]\ E[\alpha_2(Z)\iota_S(Z)]$.

PROOF OF LEMMA 2: (a) Let I and J be two disjoint intervals[19] of $\mathbb{R}\times\mathbb{R}^n$ such that $(x',r') \in I$ and $(x,r) \in J$ implies $x' \leq x$ and $r' \leq r$. Let T be be the minimal sublattice containing $I\cup J$, let $S = \mathbb{R}\times T$, and let $Z = (V,R,X)$. Then, since T is a lattice, letting $\alpha_1(Z) = V$, and $\alpha_2(Z) = \iota_J(R,X)$, Assumption 1 and Lemma A1 imply that

$$\text{(A1)} \quad E[V\iota_J(R,X)]\ E[\iota_T(R,X)] \geq E[V\iota_T(R,X)]\ E[\iota_J(R,X)].$$

[19] A subset A is an **interval** of a subset $S \subset \mathbb{R}^p$ if there are p-tuples $(a_1,\ldots,a_p)$ and $(b_1,\ldots,b_p)$ such that $A = \{x \in S: a_i \leq (<)\ x_i \leq (<)\ b_i,\ i = 1,\ldots,p\}$, where the symbol $\leq (<)$ indicates that we are allowing arbitrary open, half-open, and closed intervals.

Similarly, letting $\alpha_1(Z) = V$ and $\alpha_2(Z) = \iota_{(T-I)}(X)$, the application of Lemma A1 implies, after cancelling terms,

(A2) $$E[V\iota_T(R,X)]\ E[\iota_I(R,X)] \geq E[V\iota_I(R,X)]\ E[\iota_T(R,X)].$$

Mulitiplying (A1) and (A2) and cancelling terms, then yields

(A3) $$E[V\iota_J(R,X)]\ E[\iota_I(R,X)] \geq E[\iota_J(R,X)]\ E[V\iota_I(R,X)].$$

(b) Let I and J be two disjoint intervals of R^n such that $x' \in I$ and $x \in J$ implies $x' \leq x$ and let T be be the minimal sublattice containing $I \cup J$. Let $S = \Re \times (-\infty, r] \times T$, $Z = (V,R,X)$, and $\alpha_1(Z) = \iota_{(r',r]}(R)$. Suppose first that $\alpha_2(Z) = \iota_J(X)$. Then Assumption 1 and Lemma A1 imply

(A4) $$E[\iota_{(r',r]}(R)\iota_J(X)]\ E[\iota_{(-\infty,r]}(R)\iota_T(X)]$$
$$\geq E[\iota_{(r',r]}(R)\iota_S(X)]\ E[\iota_{(-\infty,r]}(R)\iota_J(X)]$$

Suppose next that $\alpha_2(Z) = -\iota_I(X)$. Then a similar application of Lemma A1 implies, after cancelling terms,

(A5) $$E[\iota_{(r',r]}(R)\iota_T(X)]\ E[\iota_{(-\infty,r]}(R)\iota_I(X)],$$
$$\geq E[\iota_{(r',r]}(R)\iota_I(X)]\ E[\iota_{(-\infty,r]}(R)\iota_T(X)]$$

Multiplying (A4) and (A5) and simplifying yields

(A6) $$E[\iota_{(-\infty,r']}(R)\iota_I(X)]\ E[\iota_{(-\infty,r]}(R)\iota_J(X)] \leq E[\iota_{(-\infty,r']}(R)\iota_J(X)]\ E[\iota_{(-\infty,r]}(R)\iota_I(X)].$$

Since the intervals of R^n generate its Borel sets, Parts (a) and (b) follow from relations (A3) and (A6). Q.E.D.

A3. Proofs of Theorems 1 and 2

If f is a monotonic function, let $f^-(r) = \lim_{s\uparrow r} f(s)$.

LEMMA A2: Suppose f and g are nondecreasing, nonnegative, right continuous functions. Then $f(r_0)g(r_1) < f(r_1)g(r_0)$, $r_0 < r_1$ implies an $\eta < 1$ and $r_2 > r_0$ such that $[g^-(r_2)-g(r)]f(r) < \eta[f(r_2)-f(r)]g(r)$, $r \in (r_0,r_2)$.

PROOF: Suppose $f(r_0)g(r_1) < f(r_1)g(r_0)$ for some $r_0 < r_1$. Then we may choose $\eta_0 < 1$ so that $[g(r_1)-\eta_0 g(r_0)]/[f(r_1)-f(r_0)] < \eta_0 g(r_0)/f(r_0)$. Since $\eta_0 < 1$ and f and g are right continuous and nondecreasing, $[g^-(r)-\eta_0 g(r_0)]/[f(r)-f(r_0)]$ attains a minimum over $(r_0,r_1]$ at some r_2 for which $f(r) < f(r_2)$ for $r < r_2$. Therefore, for $r \in (r_0,r_2)$, $[g^-(r)-\eta_0 g(r_0)][f(r_2)-f(r_0)] \geq [g^-(r_2)-\eta_0 g(r_0)][f(r)-f(r_0)] =$ $[g^-(r_2)-\eta_0 g(r_0)][f(r_2)-f(r_0)] - [g^-(r_2)-\eta_0 g(r_0)][f(r_2)-f(r)]$ which implies

(A7) $$[g^-(r_2)-\eta_0 g(r_0)][f(r_2)-f(r)] \geq [g^-(r_2)-g(r)][f(r_2)-f(r_0)].$$

By definition, $[g^-(r_2)-\eta_0 g(r_0)]/[f(r_2)-f(r_0)] < \eta_0 g(r_0)/f(r_0)$. Multiplying both sides by $[f(r_2)-f(r_0)]f(r_0)$, adding $g^-(r_2)f(r_2) + \eta_0 g(r_0)f(r_0)$, and simplifying implies an $\eta < 1$ such that

(A8) $$[g^-(r_2)-\eta_0 g(r_0)]f(r_2) < \eta g^-(r_2)[f(r_2)-f(r_0)].$$

Multiplying relation (A7) by $f(r_2)$, substituting relation (A8), and dividing by $[f(r_2)-f(r_0)]$ then yields $\eta g^{-}(r_2)[f(r_2)-f(r)] > f(r_2)[g^{-}(r_2)-g(r)]$ which may be rewritten as $(\eta-1)[g^{-}(r_2)-g(r)][f(r_2)-f(r)] + \eta g(r)[f(r_2)-f(r)] > f(r)[g^{-}(r_2)-g(r)]$. Q.E.D.

PROOF OF THEOREM 1: Suppose part (a) is false. Then $G_\alpha(b_0)G_\beta(b_1) < G_\alpha(b_1)G_\beta(b_0)$ for some $b_0 < b_1$. It then follows from Lemma A2 and the right continuity of G_α and G_β that b_1 may be chosen so that $G_\beta(b_1) > G_\beta(b_0) > 0$, and, for some $\eta < 1$,

$$\text{(A9)} \qquad [G_\beta^{-}(b_1)-G_\beta(b)]G_\alpha(b) < \eta[G_\alpha(b_1)-G_\alpha(b)]G_\beta(b), \quad b \in (b_0,b_1).$$

We will use the best response properties of Lemma 1 to establish a contradiction to (A9).

Since (A9) implies that $G_\alpha(b) < G_\alpha(b_1)$ for $b < b_1$, the zero profit condition (Lemma 1b) implies a nondecreasing sequence $b^k \to b_1$ such that $\lim_{k\to\infty}\int_{(-\infty,b^k]}E[V-b^k|R\le b^k,\beta=b]K(b^k|b)P_\beta(db) = 0$. Assumption 4 also implies that (b^k) may be chosen so that $K(b^k|b)G_\beta(b^k) > 0$ a.s.-P_β, and hence $K(b_1|b) > 0$ a.s.-P_β. Also, Lemma 4a implies $E[V|R\le b^k,\beta=b] \le E[V|R\le b_1,\beta=b]$ a.s.-P_β, and Assumptions 3 and 4 imply that $E[V|R\le b_1,\beta=b]$ is strictly increasing in b a.s.-P_β. Then, since the nonpositive profit condition (Lemma 1b) implies $E[V-b_1|R\le b_1,\beta=b_1]K(b_1|b_1)P_\beta(\{b_1\}) = \int_{(-\infty,b_1]}E[V-b_1|R\le b_1,\beta=b]K(b_1|b)P_\beta(db) - \lim_{k\to\infty}\int_{(-\infty,b^k]}E[V-b^k|R\le b^k,\beta=b]K(b^k|b)P_\beta(db) \le 0$, it follows that $P_\beta(\{b_1\}) = 0$, and

(A10) $\int_{(-\infty,b_1]} E[V-b_1|R\leq b_1,\beta=b]K(b_1|b)P_\beta(db) = 0.$

Lemma 4 implies that $E[V-b_1|R\leq b_1,\beta=b]$ is strictly increasing and $K(b_1|b)$ is nondecreasing in b a.s.-P_β. Then, since $K(b_1|b) > 0$ a.s.-P_β, G_β is continuous at b_1 and $G_\beta(b_1) > G_\beta(0)$, relations (A9), (A10), the best response property for β (Lemma 1a), and the nonpositive profit condition for α (Lemma 1b) imply that b_0 may chosen to satisfy the following relations.

(A11) $[G_\beta(b_1)-G_\beta(b)]G_\alpha(b) < \eta[G_\alpha(b_1)-G_\alpha(b)]G_\beta(b), \quad b \in (b_0,b_1),$

(A12) $E[V-b_1|R\leq b_1,\beta=b_0]K(b_1|b_0)[G_\beta(b_1)-G_\beta(b_0)]$

$\geq \eta \int_{(b_0,b_1]} E[V-b_1|R\leq b_1,\beta=b]K(b_1|b)P_\beta(db) > 0.$

(A13) $E[V-b_0|R\leq b_0,\beta=b_0]K(b_0|b_0)G_\alpha(b_0) \geq E[V-b_1|R\leq b_1,\beta=b_0]K(b_1|b_0)G_\alpha(b_1).$

(A14) $\int_{(-\infty,b_1]} E[V-b_1|R\leq b_1,\beta=b]K(b_1|b)P_\beta(db)$

$\geq \int_{(-\infty,b_0]} E[V-b_0|R\leq b_0,\beta=b]K(b_0|b)P_\beta(db).$

For $b \geq 0$ and $i = 1,2$, let $K_i(b) = K(b_i|b)$, $w_i(b) = E[V|R\leq b_i,\beta=b]$, and $w_\Delta(b) = E[V|b_0<R\leq b_1,\beta=b]$. Then relation (A13) implies

(A15) $[w_1(b_0)-b_1]K_1(b_0)[G_\alpha(b_1)-G_\alpha(b_0)]$

$+ [[w_\Delta(b_0)-b_0][K_1(b_0)-K_0(b_0)] - [b_1-b_0]K_1(b_0)] \; G_\alpha(b_0) \leq 0.$

Similarly, substituting (A12) into (A14) and rearranging terms yields

$$\text{(A16)}\quad [w_1(b_0)-b_1]K_1(b_0)[G_\beta(b_1)-G_\beta(b_0)] + \eta\int_{(-\infty,b_0]}[[w_\Delta(b)-b_0][K_1(b)-K_0(b)] - [b_1-b_0]K_1(b)]\ P_\beta(db) \geq 0.$$

We will show that relations (A15) and (A16) and the restrictions of Lemma 4 are inconsistent with relation (A11).

Since Lemma 4a implies $w_\Delta(b) \leq w_\Delta(b_0)$, $b \leq b_0$ a.s.-P_β, relation (A16) implies

$$\text{(A17)}\quad [w_1(b_0)-b_1]K_1(b_0)[G_\beta(b_1)-G_\beta(b_0)] + \eta\int_{(-\infty,b_0]}[[w_\Delta(b_0)-b_0][K_1(b)-K_0(b)] - [b_1-b_0]K_1(b)]\ P_\beta(db) \geq 0.$$

Also, since Lemma 4a and relation (A11) imply $w_\Delta(b_0) > b_0$, and Lemma 4b implies $[K_1(b)-K_0(b)]K_1(b_0) \leq [K_1(b_0)-K_0(b_0)]K_1(b)$, $b \leq b_0$ a.s.-P_β, relation (A17) implies

$$\text{(A18)}\quad [w_1(b_0)-b_1]K_1(b_0)[G_\beta(b_1)-G_\beta(b_0)]\ K_1(b_0) + \eta[[w_\Delta(b_0)-b_0][K_1(b_0)-K_0(b_0)] - [b_1-b_0]K_1(b_0)]\int_{(-\infty,b_0]}K_1(b)\ P_\beta(db) \geq 0.$$

Finally, since Lemma 4b implies $K_1(b) \geq K_1(b_0)$, $b \leq b_0$ a.s. P_β, relation (A12) implies $[w_1(b_0)-b_1]K_1(b_0) > 0$, it follows from (A18) that

$$\text{(A19)}\quad [w_1(b_0)-b_1]K_1(b_0)[G_\beta(b_1)-G_\beta(b_0)] + \eta[[w_\Delta(b_0)-b_0][K_1(b_0)-K_0(b_0)] - [b_1-b_0]K_1(b_0)]\ G_\beta(b_0) \geq 0.$$

Multiplying (A16) by $G_\beta(b_0)$, (A18) by $G_\alpha(b_0)/\eta$ and subtracting then implies $[G_\beta(b_1)-G_\beta(b_0)]G_\alpha(b_0) \geq \eta[G_\alpha(b_1)-G_\alpha(b_0)]G_\beta(b_0)$, contradicting relation (A11). This proves part (a).

(b) If R is independent of (V,X), then $K(b|b') = K(b)$. Let $v(b) = E[V|\beta=b]$ and $H(b) = G_\alpha(b)K(b)$. Suppose G_α is strictly increasing on (b',b") with $b' > 0$. Then Part (a) implies that G_β is also strictly increasing on (b',b") so that Assumption 4 guarantees that $v(b) > b$, $b \in (b',b")$ a.s.P_β.

We establish first that G_β and H are continuous on (b',b"). For $b, b+\epsilon \in (b',b")$, the zero profit condition (Lemma 1b) implies $(b+\epsilon)G_\beta(b+\epsilon) - bG_\beta(b) = \int_{(b,b+\epsilon]} v(t)P_\beta(dt)$. Letting $\epsilon \downarrow 0$ then yields $P_\beta(\{b\}) = 0$. Similarly the best response property for β (Lemma 1a) implies $[v(b-\epsilon)-b]H(b) \leq [v(b-\epsilon)-b+\epsilon]H(b-\epsilon)$ or, equivalently, $[v(b-\epsilon)-b+\epsilon][H(b)-H(b\epsilon)] \leq \epsilon H(b)$. Letting $\epsilon \downarrow 0$ then yields $H(b-\epsilon) \uparrow H(b)$.

Given the continuity of G_β, we may express the zero profit condition for α in terms of the Stieltjes integral, $bG_\beta(b) = b_0G_\beta(b_0) + \int_{b_0}^{b} v(t)dG_\beta(t)$, $b \in [b',b")$. Let $\pi(b) = v(b)-b$. Then, applying the formula for integration by parts yields

$$\text{(A20)} \qquad \int_{b'}^{b''} \pi(b)dG_\beta(b) = \int_{b'}^{b''} G_\beta(b)db.$$

Now consider an arbitrary increasing sequence $b' = b_0 < b_1 < \ldots < b_{n-1} < b_n = b"$. The best response condition for β implies, for $i = 1,\ldots,n$,

$$\text{(A21)} \qquad \pi(b_i)[H(b_i)-H(b_{i-1})] \geq [b_i-b_{i-1}]H(b_{i-1})$$

and

$$\text{(A22)} \qquad \pi(b_{i-1})[H(b_i)-H(b_{i-1})] \leq [b_i-b_{i-1}]H(b_i)$$

Since π is of bounded variation (the difference between two monotone functions) and H is continuous, (A21) and (A22) imply

(A23) $\int_{b'}^{b''}\pi(t)dH(t) = \int_{b'}^{b''}H(t)dt, \quad b \in [b',b'']$.

Furthermore, since G_β and H are continuous and positive on [b',b"], it follows from (A20) and (A23) that, for $b \in [b',b'']$,

(A24) $\int_{b'}^{b''}(t)d\log(G_\beta(t)) = b - b'$,

and

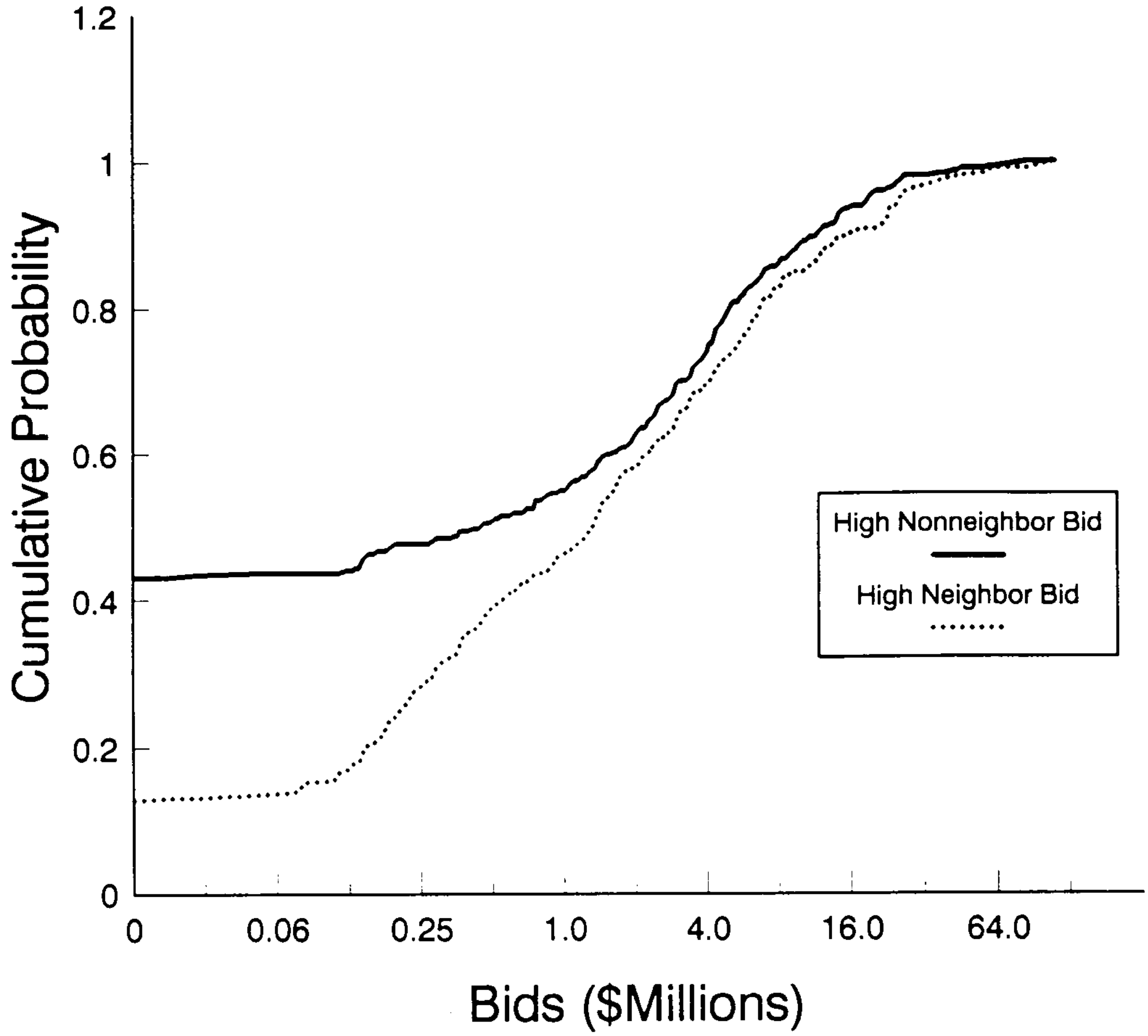
(A25) $\int_{b'}^{b''}\pi(t)d\log(H(t)) = b - b'$.

Subtracting (A25) from (A24) and recalling that $\pi(b) > 0$, $b \in [b',b'']$, it follows that $\int_{b'}^{b}d\log(H(t)) = \int_{b'}^{b}d\log(G_\beta(t))$, $b \in [b',b'']$. Upon evaluating this expression and subsituting for H, we obtain part (b). Q.E.D.

PROOF OF THEOREM 2: We will show that $G_\beta(b^*) = G_\beta(0)$ implies $G_\alpha(b') = G_\alpha(0)$, for some $b' > b^*$. Fix $b_1 > b^*$. Since Assumptions 4 and 5 imply that $G_\beta(0) > 0$, it follows from Assumptions 3 and 4 that $K(b_1|b)G_\alpha(b_1) > 0$ a.s.-P_β. The best response property for β implies $E[V-b_1|R\le b_1,\beta=b]K(b_1|b)G_\alpha(b_1) \le 0$ for $b \le b^*$ a.s.-P_β. Therefore, it follows from Assumption 5 that $\int_{(-\infty,b^*]}E[V-b_1|R\le b_1,\beta=b]K(b_1|b)P_\beta(db) < 0$. The right continuity of G_β then implies a $b' > b^*$ such that, for $b_2 \le b'$, $\int_{(-\infty,b_2]}E[V-b_1|R\le b_1,\beta=b]K(b_1|b)P_\beta(db) < 0$ and hence from Lemma 4a that

$\int_{(-\infty,b_2]} E[V-b_2|R\leq b_2,\beta=b]K(b_2|b)P_\beta(db) < 0$. The result then follows from the zero profit condition for α (Lemma 1b). Q.E.D.

Figure 1. Distribution of Bids



All bids are represented in 1972 dollars.

Figure 2a. Bid Functions for Example 1.

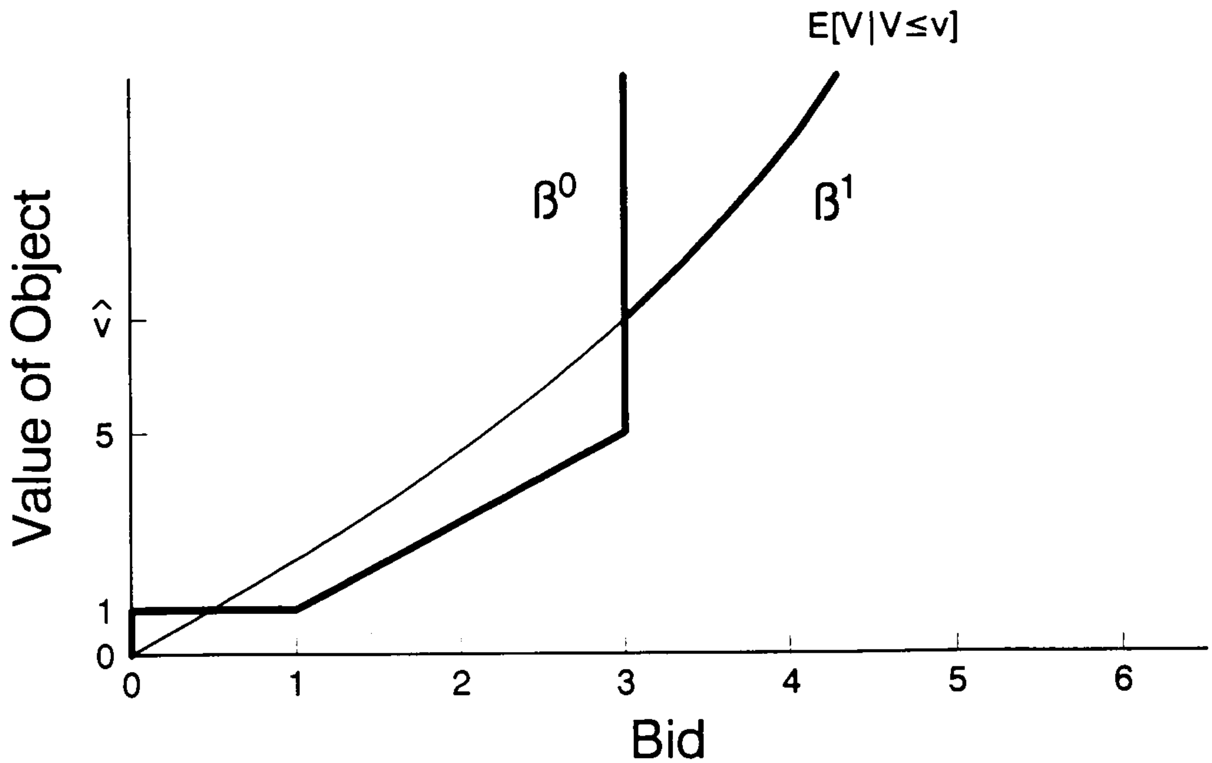


Figure 2b. Distribution Functions for Example 1.

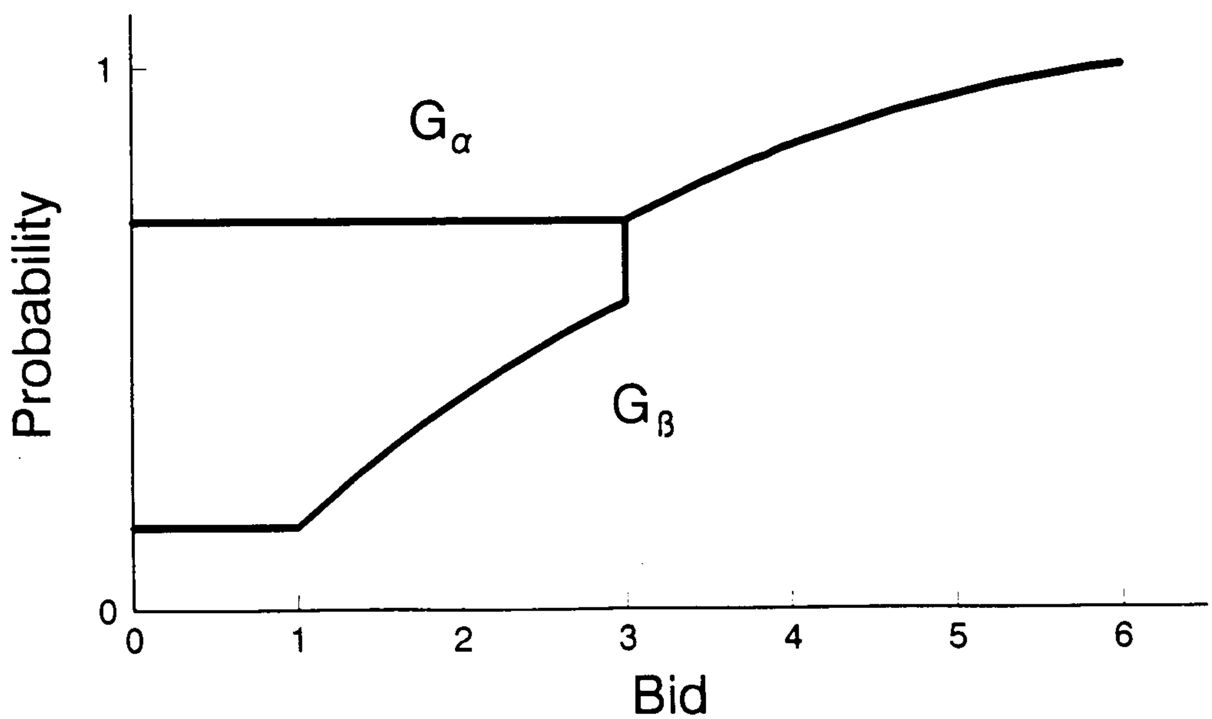


Figure 3a. Bid Functions for Example 2.

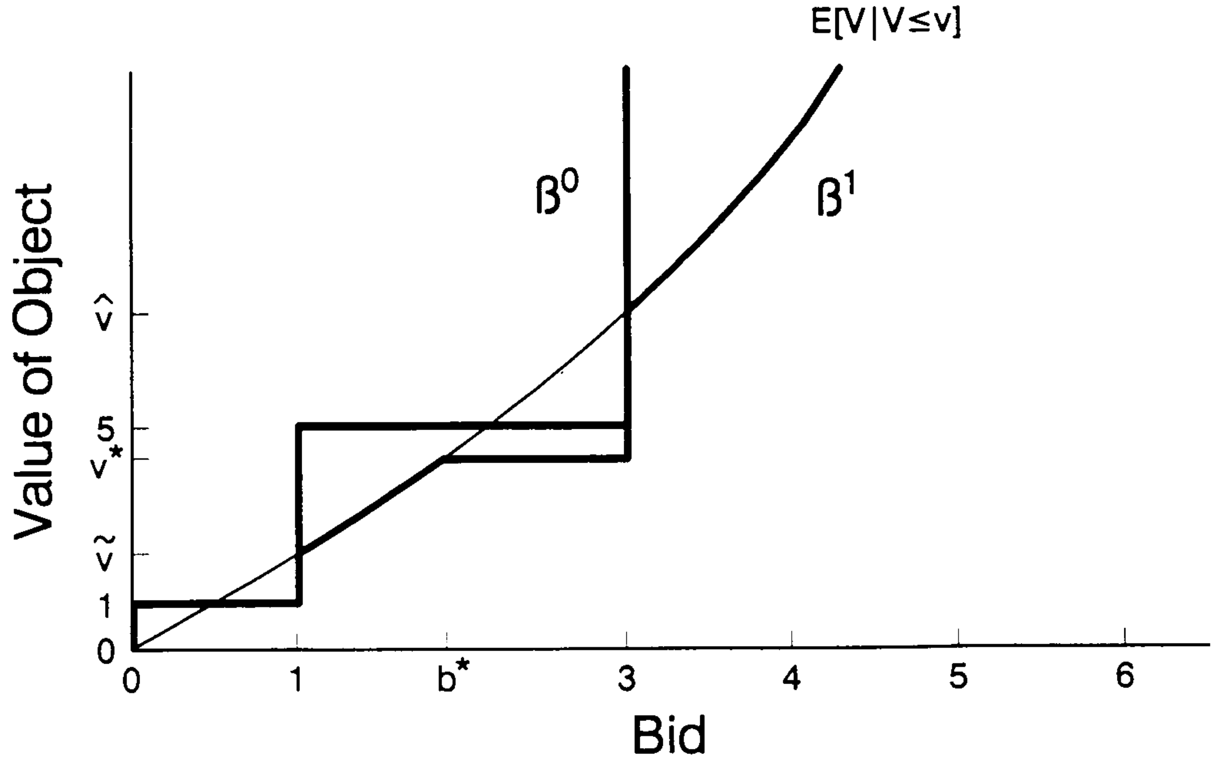


Figure 3b. Distribution Functions for Example 2.

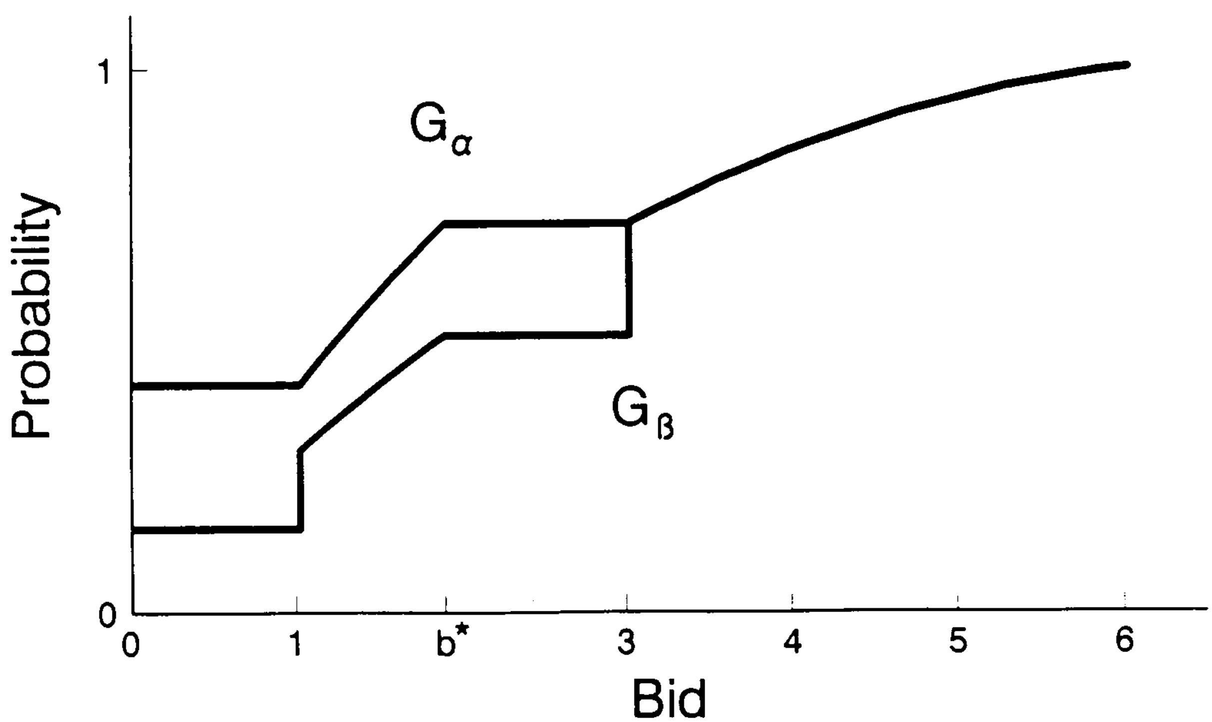


Figure 4. Rates of Change in the Distributions

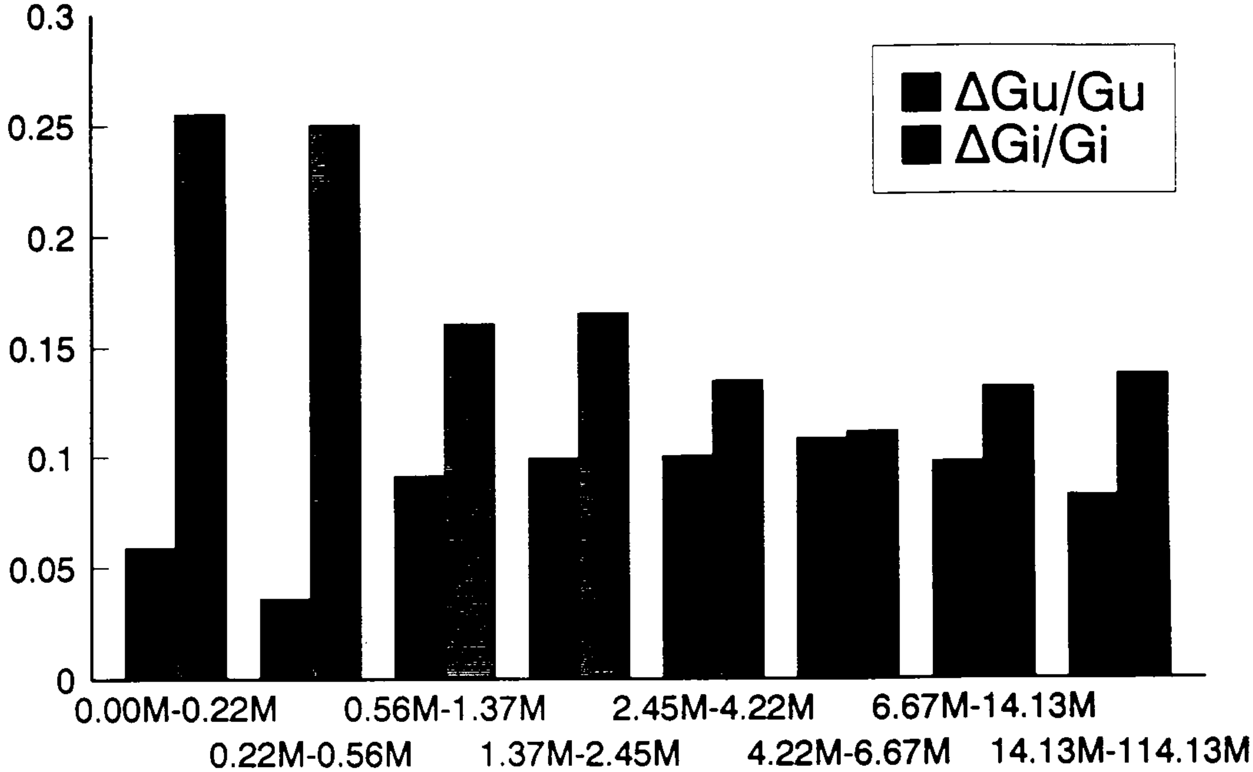


Figure 5. Relative Number of Low Bids