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# ADVERTISING AND COORDINATION

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* Northwestern University and University of California, San Diego.
This paper is a revised and extended version of our earlier paper, "Advertising, Coordination and Signaling." Helpful comments from Laurie Bagwell, Steve Matthews, John Moore, Mike Riordan, Bill Rogerson, and two anonymous referees are gratefully acknowledged. Bagwell wishes to thank the NSF for support under grant SES-8909586.

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We show that when relevant market information such as price is difficult to communicate, advertising plays a key role in bringing about optimal coordination of purchase behavior: an efficient firm uses advertising expenditures in place of price to inform sophisticated consumers that it offers a better deal. This provides a theoretical explanation for Benham's (1972) empirical association of the ability to advertise with lower prices and larger scale. We find that advertising improves welfare unambiguously when firms' price choices are the only source of uncertainty. When advertising must also signal the identity of the efficient firm, however, a welfare tradeoff arises between advertising and coordination. Our results extend readily to situations of partial price observability and product quality uncertainty.

## 1. Introduction

Sellers often find it difficult to transmit relevant information to their prospective customers. Satisfactory communication of price information, in particular, is hampered by the need to communicate a large number of prices (e.g., multi-product retail stores) or a complex pricing structure (e.g., long-distance telephone service). Price advertising may even be illegal (historic examples in the U.S. include retail eyeglass, liquor and prescription drug markets). Similarly, it is seldom easy to provide useful hard information about product qualities or the variety of product offerings (e.g., automobile or rug dealers).

In their efforts to communicate such information, sellers frequently resort to advertisments featuring vague and unverifiable claims to "low prices, high quality and great selection," as opposed to hard information. The price information that is available typically consists of "loss leader" items making up a small fraction of a seller's total product line, which leaves buyers to wonder about the pricing policy for the remaining items. Assuming that firms

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seek to maximize profits in placing these kinds of ads, it becomes a challenge for economists to reconcile the prevalence of such advertising with the hypothesis of rational consumer behavior.

Empirical studies have uncovered a deeper puzzle. In his study of the retail eyeglass industry, Benham (1972) found that eyeglass prices in the U.S. varied systematically from state to state based on whether state laws allowed advertising in any form, but did not depend on whether advertisements were allowed to mention price. In other words, the effect of advertising hinged not on the specific information conveyed by sellers' claims, but on the mere ability to make claims! Benham also showed that prices tended to be lower, and eyeglass retailers tended to operate at a larger scale, in states that permitted advertising; he found this to be consistent with Stigler's (1961) notion that advertising allows a seller better to exploit scale economies. Other studies have found a similar relationship.[1]

In this paper we propose a theoretical explanation for these phenomena, based on the fundamental idea that buyers and sellers can mutually benefit by exploiting scale economies. Our concept of scale economies differs, however, from the traditional notion of declining average costs. We posit instead the following two properties. The better profit property states simply that a firm profits from an expansion of its market share. The centerpiece of our analysis is a less familiar effect, called the better deal property, which asserts that a firm offers consumers a better deal when it expects more business. The latter property hinges on the cost and demand conditions facing sellers, and it may or may not be related to the presence of declining average costs. We argue below that conditions giving rise to the better deal property are ubiquitous, particularly in retail markets.

In the presence of these two properties, buyers and active sellers both prefer business to be concentrated among fewer firms, giving rise to the potential for coordination gains from reallocation of market shares. Coordination gains may go unrealized, however, if relevant information on price or quality cannot be communicated. In this setting, apparently noninformative advertising can play a key role: by expending resources on advertising, a firm communicates that it intends to capture large market share, and sophisticated consumers infer from this that they will obtain a better deal from the advertising firm. The advertising expenditures themselves, rather than the particular claims that are made, serve to communicate indirectly the information that cannot be conveyed directly.

To develop this idea, we propose a simple model wherein two firms compete for the business of a large number of consumers, but price information cannot be communicated. Firms are instead able to send advertising messages consisting simply of observable expenditures of resources, i.e. "public money burning." One of the firms is known to be more efficient, and because of the better profit and better deal properties it is optimal for this firm to capture the market. The optimal outcome is not assured, however, if consumers are unsophisticated in interpreting advertising messages, in the sense that they take no account of firms' incentives to send them.

We require consumers to be more sophisticated, by imposing the following restriction on their inferences: consumers never believe that a firm has chosen an advertising-price combination that is equilibrium dominated, i.e. which is incapable of increasing the profits that the firm earns in a conjectured equilibrium. Given this restriction, consumers recognize that a firm advertises only if it expects to increase its profits by doing so. Thus, when the efficient firm advertises, consumers know that it plans to increase its market share and, based on the better deal property, to offer a lower price. The better profit property then gives the efficient firm an incentive to use this advertising strategy. In essence, under our inference restriction the observable advertising level becomes a signal of the unobservable price choice. Advertising then makes possible an "implicit price competition" that leads to optimal coordination. This result implies a direct association between the ability to advertise, lower prices and larger scale, consistent with Benham's findings.

We show, in fact, that there is a unique equilibrium outcome that satisfies the inference restriction, in which the efficient firm captures the market and chooses zero advertising; allowing advertising is then unambiguously welfare-enhancing. We extend the model to incorporate uncertainty as to which firm is more efficient, so that the efficient firm must choose a strictly positive advertising level to establish its identity. In this case two equilibrium outcomes survive the inference restriction: in one outcome optimal coordination is achieved, but resources are dissipated in the form of advertising, while in the other there is zero advertising but suboptimal coordination. The added uncertainty therefore leads to a welfare tradeoff between advertising and coordination, and it will be undesirable to achieve optimal coordination if advertising costs exceed coordination gains. There is no tradeoff from the consumer point of view, however, since utility is necessarily higher in the positive-advertising outcome.

While our model is simple and stylized, our basic conclusions extend readily to a much broader class of environments, incorporating multiple-product firms, observable prices, product quality choice, and various permutations of the timing of decisions; the key unifying assumptions are that some relevant information is difficult to communicate, coordination gains exist and consumers are sufficiently sophisticated in interpreting advertising messages. Moveover, although in our model the communicative power of advertising hinges on advertising expenditures, we feel that sellers' ability to incorporate specific messages into their advertising makes our theory especially compelling: the familiar vague claims may well serve the function of alerting consumers to the potential for coordination gains.

This paper relates most closely to two branches of literature. First, in a variety of models, Kihlstrom and Riordan (1984), Klein and Leffler (1981), Matthews and Fertig (1990), Milgrom and Roberts (1986), Nelson (1970, 1974), G. Ramey (1987) and Rogerson (1986) have explored the role of advertising in signaling product quality. These papers model quality as an experience attribute of the product, i.e. an attribute that consumers can directly observe only through product purchase and use. In these models advertising becomes a useful signal where high quality is associated with low marginal production cost or high returns from repeat business, and the papers' findings are related chiefly to advertising associated with particular brands (e.g., Coke). By contrast, we emphasize the role of advertising in communicating search attributes such as price. Our analysis links the usefulness of advertising to scale economies, and gives a theory of advertising behavior by retail firms. Further, we extend our model to consider price and advertising as signals of product quality, where quality is a search attribute, and we demonstrate that it may be impossible to signal quality through price alone.[2]

Second, our work relates closely to recent papers in which the observable portion of a player's strategy may signal the unobservable portion. Ben-Porath and Dekel (1988), Glazer and Weiss (forthcoming), Kohlberg and Mertens (1986) and van Damme (1989) base their analyses on forward induction-type refinements, as we do in this paper. A different approach is taken by Kihlstrom and Riordan (1984) and Wolinsky (1984), who consider advertising and price, respectively, as signals of firms' hidden strategy choices. In these papers observing one component of a firm's strategy tells consumers what the other component has to be in order to satisfy a constant-profit condition.

The plan of the paper is as follows. Section 2 explains the better profit and better deal properties in greater detail and discusses motivating examples. The coordination role of advertising in a model of imperfect price information is developed in Section 3, and in Section 4 the model is extended to incorporate incomplete information. In Section 5 the model is further extended to allow for observable prices and product quality choice, and Section 6 concludes.

## 2. Better Profit and Better Deal Properties

Our point of departure is the hypothesis that a firm and consumers may collectively benefit from coordination of purchase decisions. Consider a firm selling a homogeneous good, which incurs costs $C(q)$ when q units are sold; we assume $C' > 0$. There are many consumers, each demanding $Q(p)$ units when the price is p. If the firm sets a price of p and m consumers visit the firm, then total profits are:

$$\Pi(p,m) = pmQ(p) - C(mQ(p))$$

Let $p^*(m)$ give the maximizer of this function, which we shall assume is uniquely defined and continuous for $m > 0$. Let $\Pi^*(m)$ give the maximixed value of profits. It is easy to see that $\Pi^*$ must be strictly increasing in m, for the firm cannot fail to benefit from an increase in its total demand mQ. We call this unsurprising observation the better profit property.

Coordination benefits will arise when this property is combined with a less familiar phenomenon: in many cases it will be true that the firm will offer consumers a lower price if more consumers visit the firm, that is, $p^*$ will be strictly decreasing in m. In particular we have:

$$\frac{dp^*}{dm} = \frac{C''QmQ'}{\Delta_p}$$

where $\Delta_p$ represents the second derivative of $\Pi$ with respect to p, evaluated at $p^*$; this we assume is strictly negative. It follows that $dp^*/dm < 0$ if and only if $C'' < 0$. For larger m to generate a lower profit-maximizing price, it is necessary and sufficient for marginal cost to be decreasing in total output. We will refer to this as the better deal property, since consumers get a better deal if more of them visit the seller.

While it is customarily assumed that costs are convex, there are many situations in which decreasing marginal cost is actually the more appropriate hypothesis. Consider the following examples.

1. Manufacturer quantity discounts will lead to decreasing marginal costs for retail sellers: as more units are sold, the retailer qualifies for lower wholesale prices, which translate into lower marginal costs.

2. Learning effects are often important: as a seller expands its output, personnel become better acquainted with operating procedures, and coordination of managers and workers

improves.

3. Technology choice might give rise to declining marginal costs, as expanding scale leads to adoption of low-marginal cost technologies. Consider the cost function $\tilde{C}(q,t)$ with technology parameter t; let $\tilde{C}_{qt} < 0$ and $\tilde{C}_{tt} > 0$. $\tilde{C}$ is then minimized by $t^*(q)$, which is strictly increasing in q. The cost function $C(q) = \tilde{C}(q,t^*(q))$ will be concave when the effect of $t^*$ is strong, even if $\tilde{C}_{qq} > 0$ for each given technology.[3]

Since declining marginal costs imply declining average costs, the better deal property might be regarded as a strengthened version of the usual notion of scale economies. This is not correct, however, as one can readily conceive of situations in which the better deal property holds even when marginal costs are nondecreasing. We mention two examples that are particularly relevant for retail establishments.

4. Consumer heterogeneity. The above framework may be modified by allowing consumers to differ in their preferences for the firm's product. Suppose that consumers having weaker preference for the product also have more elastic demand. As the firm's market share expands, these weaker-preference consumers enter the customer base, and the firm's total demand curve becomes more elastic. This leads the firm to reduce its price as market share grows, even if it has constant unit costs.

5. Product variety. Suppose that the firm stocks a range of products, and can expand the range by incurring a greater stocking cost. Moreover, consumer utility is increasing in the range of products stocked. If marginal stocking costs are increasing, then the firm will stock a greater range of products when it expects more customers, and consumers thereby obtain a better deal. If unit sales costs are constant for each product stocked, then the better deal property holds despite the fact that the price of each stocked product does not depend on the firm's market share.

The last two examples are developed in more detail in Appendix A. Still other motivations for the better deal property may be given, including fixed markup pricing behavior, loss-leader strategies and product quality choice; the latter two are considered in

Section 5.[4] These examples demonstrate that the better profit and better deal properties encompass many market environments that might not fit the conventional definition of scale economies, and that these environments would seem to be quite common, particularly on the retail level.

## 3. The Coordination Role of Advertising

In this section we demonstrate that when relevant information is difficult to communicate and the better profit and better deal properties are present, a role arises for advertising in bringing about optimal coordination of consumer purchases among potential sellers. The key idea is that advertising allows the seller to communicate indirectly the information that cannot be directly communicated. To make the analysis as simple as possible, we focus on the case of identical consumers, single-product firms and inability to communicate price information; however, it will become apparent that our ideas extend to much richer settings, including those described in the preceding section.

### A. A Model with Imperfect Price Information

Let there be two firms, Firm 1 and Firm 2, and a continuum of identical consumers. The consumers are uniformly distributed on [0,1] with unit mass. Each firm sells a single product, and a consumer purchasing from Firm i demands $Q(p_i)$ units, where $p_i$ is Firm i's price. The operating profits of Firm i are given by $\Pi_i(p_i,m_i) - A_i$, where $m_i$ is the mass of consumers purchasing from Firm i and $A_i$ is Firm i's level of advertising; we assume that consumers gain no direct utility from advertising. As in Section 2 we assume that $\Pi_i$ is continuous and has a unique continuous maximizer $p_i^*(m_i)$ for $m_i > 0$, and we denote maximized profits by $\Pi_i^*(m_i)$. We add the assumption that $\Pi_i$ is strictly quasiconcave in $p_i$ for $m_i > 0$, so that operating profits are strictly increasing in $p_i$ for $p_i < p_i^*(m_i)$ and strictly decreasing for $p_i > p_i^*(m_i)$.[5] Operating profits are $-A_i$ if $m_i = 0$.

Most importantly, we assume:

<u>Better Profit Property</u>: $\Pi_i^*(m_i)$ is strictly increasing in $m_i$.

<u>Better Deal Property</u>: $p_i^*(m_i)$ is strictly decreasing in $m_i$.

Moreover, we will take Firm 1 to be the more efficient seller of the good, so that it will choose to offer a lower price than would Firm 2, given the same number of customers:

<u>Differential Efficiency</u>: $p_1^*(m) < p_2^*(m)$ for every $m > 0$.

This assumption is illustrated in Figure 1.

The firms and consumers play the following multi-stage game.

<u>Stage 1</u>: The firms simultaneously choose advertising and price levels, where the option is available of staying out of the market completely. If a firm chooses to enter, a setup cost of $F > 0$ must be deducted from operating profits. Assume further that $F < \Pi_i^*(1)$ for both i, which makes either firm viable as a monopolist. The payoff from staying out of the market is zero.

<u>Stage 2</u>: Consumers observe whether or not firms have chosen to operate, and they observe the firms' advertising levels, but they do not observe the firms' prices. Consumers then make price search decisions. Let $K_1$ be the cost to each consumer of searching one firm, and $K_2$ the cost of searching two firms. To simplify the analysis we assume that $K_1$ is zero and $K_2$ is infinite, that is, each consumer is able to visit one and only one firm. Thus at this stage the consumers decide only on which firm to visit, conditional on the information they have observed.[6]

<u>Stage 3</u>: Consumers observe the price chosen by the firm they have visited, and purchase the desired number of units at this price.

We analyze the Bayes-Nash equilibria of this game that satisfy sequential rationality and a weakened notion of consistent beliefs (Harsanyi (1967-8), Kreps and Wilson (1982)). Sequential rationality requires that at each decision point, an agent's decisions are optimal (firms maximize profits, consumers prefer a lower price) given some beliefs about the decision rules of the other agents; where possible these beliefs must be Bayes consistent with the equilibrium decision rules used by the other agents; and at each decision point an agent conjectures that the future decisions of other agents will be made according to their equilibrium decision rules.

We impose a further condition related to Kreps and Wilson's consistency criterion: consumers must form independent price conjectures, meaning that conjectures of a firm's pricing strategy can depend only on that firm's advertising level, and not on the advertising level of the other firm. This restriction is motivated most directly by the fact that firms do not know each others' prices when they choose advertising.[7]

## B. Advertising and Sophisticated Inferences

To see how advertising brings about coordination, we must first consider the equilibria which arise when there is no advertising. Such equilibria provide a useful benchmark, as they correspond to an environment in which all advertising is illegal. Assume therefore that firms and consumers play the above multi-stage game with the exception that firms choose only prices, rather than advertising and prices. Let $\hat{p}_i$ give the equilibrium price choice of Firm i. It is easy to see that there are three possible equilibrium outcomes in this case.

Outcome 1: Firm 1 captures the market. Here Firm 1 chooses $\hat{p}_1 = p_1^*(1)$ and Firm 2 stays out of the market. Consumers have only one firm to visit, so Firm 1 captures all of them. Should Firm 2 unexpectedly enter the market, consumers would continue to visit Firm 1 based on the conjecture that Firm 2 must be charging some price above $p_1^*(1)$.[8] Thus Firm 2 optimally stays out to save the setup cost F.

Outcome 2: Firm 2 captures the market. This is like Outcome 1, except the roles of the firms are reversed. Now the more efficient firm is unable to capture any market share from the less efficient firm, as consumers believe that the more efficient firm charges a high price if it enters. This contrasts with our usual intuition concerning natural monopoly situations such as this, in which Firm 1 would be expected to prevail through the threat of aggressive pricing. With imperfect price information, monopoly misallocation is exacerbated by the prospect that production may be concentrated at the less efficient firm.

Outcome 3: The firms split the market. Here the equilibrium market shares $\hat{m}_1$ and $\hat{m}_2$ are determined by $p_1^*(\hat{m}_1) = p_2^*(\hat{m}_2)$ and $\hat{m}_1 + \hat{m}_2 = 1$, which is satisfied by a unique pair of market shares. Given that $\hat{m}_1$ and $\hat{m}_2$ are anticipated, the firms maximize profits by choosing these prices. Consumers are then indifferent about which firm to visit, and $\hat{m}_1$ and $\hat{m}_2$ are thus consistent with utility-maximizing visitation behavior. Existence of this kind of equilibrium requires that (a) the differential efficiency effect does not overwhelm the better deal property, i.e. we have $p_1^*(m_1) > p_2^*(1)$ for sufficiently small $m_1$ (as in Figure 1); and (b) F is small enough to sustain both firms in the market, i.e. $\Pi_i^*(\hat{m}_i) \geq F$. This outcome is the least efficient of the three, as it makes the least use of available coordination economies. Moreover, the better deal and differential efficiency properties imply $\hat{m}_2 > \hat{m}_1$, so that the less efficient firm has the greater market share.

Now consider the possibilities for equilibria in which there is advertising. Let us denote Firm i's equilibrium strategy by $(\hat{A}_i, \hat{p}_i)$ in the event that Firm i chooses to operate. If we suppose that consumers ignore advertising in making their visitation decisions, then an active firm will optimally choose $\hat{A}_i = 0$. The three equilibria derived above thus continue to be equilibria under this specification of consumer behavior. Moreover, there will exist additional equilibria that give the prices and market shares of Outcome 3, in which the firms choose strictly positive advertising levels. In these new equilibria, firms do not cut their advertising because their customers would react by defecting to the rival firm, based on the conjecture that advertising cuts are associated with price increases.[9] Therefore, without futher restrictions on consumer behavior, advertising can only increase the potential for inefficiency.

This brings us to the final ingredient in our theory of advertising: for advertising to play a coordination role, there must be some degree of sophistication in the way consumers draw inferences from observed advertising. In particular, consumers should recognize the implications of the better profit and better deal properties for a firm's incentives to choose advertising and price. Thus, when a firm deviates from an equilibrium by choosing a large advertising level, consumers implicitly receive the message: "I am expending these resources only because I anticipate capturing a large market share. Given this, you know I will be offering a better deal, so you should visit me." If consumers understand this message and accept its logic, then advertising becomes a tool through which the efficient firm can bring about optimal coordination.

To formalize this restriction on consumer inferences, we must introduce a bit more notation. Let $W_i$ denote the equilibrium payoff of Firm i in a given equilibrium; this is defined by:

$$W_i = \begin{cases} \Pi_i(\hat{p}_i,\hat{m}_i) - \hat{A}_i - F, & \text{if the firm operates} \\ 0, & \text{if the firm stays out} \end{cases}$$

An advertising-price pair $(A_i,p_i)$ is said to be <u>equilibrium</u> <u>dominated</u> relative to a given equilibrium if:

$$\max_{m_i} \Pi_i(p_i,m_i) - A_i - F < W_i$$

That is, no matter what market share the firm obtains as a result of choosing $(A_i,p_i)$, Firm i would do strictly better by sticking with its equilibrium strategy. We now impose the following restriction on consumer beliefs:

<u>No Equilibrium Dominated Conjectures</u>: Consumers never conjecture that the firm has played an equilibrium dominated strategy, if it is possible to conjecture that some non-equilibrium dominated strategy was played.[10]

When consumers draw inferences subject to the no equilibrium dominated conjectures restriction, the efficient firm is able to use advertising as a communication channel, through which it brings about optimal coordination of consumer purchase behavior. To demonstrate this, let us first consider Outcome 2, in which $(\hat{A}_2,\hat{p}_2) = (0,p_2^*(1))$ and Firm 1 stays out. Choose a price $p_1' \in (p_1^*(1),p_2^*(1))$, and a market share $\tilde{m}_1 < 1$ satisfying $p_1^*(\tilde{m}_1) < p_1'$. Using the better profit property we have, for all $p_1$ and all $m_1 \leq \tilde{m}_1$:

$$\Pi_1(p_1,m_1) \leq \Pi_1^*(m_1) \leq \Pi_1^*(\tilde{m}_1) < \Pi_1^*(1)$$

For $m_1 > \tilde{m}_1$, the following holds for all $p_1 \geq p_1'$:

$$\Pi_1(p_1,m_1) \leq \Pi_1(p_1',m_1) \leq \max_{m_1 \in [\tilde{m}_1,1]} \Pi_1(p_1',m_1) < \Pi_1^*(1)$$

where the first inequality follows from the quasiconcavity of $\Pi_1$ in price and the fact that $p_1 \geq p_1' > p_1^*(\tilde{m}_1) > p_1^*(m_1)$, and the final inequality derives from the better profit property (when the maximizing $m_1$ is less than unity) and the requirement that $p_1' > p_1^*(1)$ (when the maximizing $m_1$ equals unity). Thus by choosing $m_1'$ sufficiently close to unity, we can make $\Pi_1^*(m_1')$ close enough to $\Pi_1^*(1)$ to give, for all $p_1 \geq p_1'$:

(1) $$\max_{m_1} \Pi_1(p_1,m_1) < \Pi_1^*(m_1') < \Pi_1^*(1)$$

Now let the advertising level $A_1'$ be defined by:

(2) $$A_1' = \Pi_1^*(m_1') - F$$

We can be sure that $A_1' > 0$ if $m_1'$ is chosen sufficiently close to unity. Consider an advertising-price pair $(A_1', p_1)$ with $p_1 \geq p_1'$. Using (1) and (2) we have:

$$\max_{m_1} \Pi_1(p_1, m_1) - A_1' - F < \Pi_1^*(m_1') - A_1' - F = 0$$

and since $W_1 = 0$, it follows that any $(A_1', p_1)$ with $p_1 \geq p_1'$ is equilibrium dominated.

Our inference restriction then requires that should consumers unexpectedly observe Firm 1 commencing operations and choosing $A_1'$, they must not infer $p_1 \geq p_1'$ so long as there is some other inference that is not equilibrium dominated. For the latter we note that $(A_1', p_1^*(1))$ is not equilibrium dominated:

$$\Pi_1^*(1) - A_1' - F > \Pi_1^*(m_1') - A_1' - F = 0$$

This demonstrates in fact that Firm 1 strictly prefers choosing $(A_1', p_1^*(1))$ and capturing the market to its equilibrium payoff $W_1 = 0$.

It is now easy to see that Outcome 2 can no longer be supported as an equilibrium when consumer inferences are restricted. For suppose Firm 1 deviates from the equilibrium by commencing operations with advertising level $A_1'$. This convinces consumers that $p_1 < p_1'$. Moreover, by independent price conjectures consumers must continue to conjecture $p_2 = p_2^*(1)$ upon observing Firm 2's equilibrium advertising level $\hat{A}_2 = 0$. Since $p_1' < p_2^*(1)$, sequential rationality requires that consumers defect to Firm 1 when the advertising profile $(A_1, A_2) = (A_1', 0)$ is observed. It then follows that Firm 1's equilibrium strategy of staying out is no longer a best response, as $(A_1', p_1^*(1))$ gives a strictly larger payoff. In this way advertising allows mutually beneficial coordination among the efficient firm and consumers.

Next, we observe that the inference restriction also rules out Outcome 3, whether or not the supporting equilibria require positive advertising. For suppose we have $W_1 > 0$ in one of these equilibria. We may apply the above arguments directly by replacing (2) with:

$$A_1' = \Pi_1^*(m_1') - F - W_1$$

where $A_1' > \hat{A}_1 \geq 0$ is implied by $p_1' < p_2^*(1)$.[11] It is at once apparent that $(A_1', p_1)$ is equilibrium dominated for all $p_1 \geq p_1'$, and capturing the market with $(A_1', p_1^*(1))$ gives Firm 1 a payoff strictly larger than $W_1$. Since $p_1' < \hat{p}_2$, it follows that consumers will defect to Firm 1 when $(A_1', \hat{A}_2)$ is observed, and Firm 1 consequently deviates. This establishes that the inference restrictions eliminate all of the outcomes save Outcome 1.[12]

As an aside, observe that the arguments that eliminate Outcome 2 show also that $(A_1', p_1)$ for $p_1 \geq p_1'$ is strictly dominated by staying out, as the latter gives a strictly greater payoff no matter what the consumer visitation decisions are. It is therefore reasonable to ask whether we might instead require consumer inferences to place no weight on strictly dominated, as opposed to equilibrium dominated, strategies, since this is sufficient to eliminate Outcome 2. Such a restriction is not, unfortunately, sufficient to eliminate Outcome 3, for it may well be true that strategies $(A_1, p_1)$ with $p_1$ slightly above $p_1^*(\hat{m}_1)$ are strictly dominated only when $A_1$ is so large that Firm 1 finds it unprofitable to deviate. In particular, market-splitting equilibria survive elimination of strictly dominated strategies when the minimum gain to capturing the entire market is sufficiently small; this also holds true for elimination of weakly dominated strategies.[13,14]

It remains to show that Outcome 1 does survive the inference restriction. Since $W_1 = \Pi_1^*(1) - F$ in this case, it follows that any $(A_1, p_1)$ with $A_1 > 0$ is equilibrium dominated; thus we may specify that consumers conjecture $p_1 = p_1^*(1)$ for any $A_1$ they observe. As for Firm 2, suppose that $(A_2, p_2)$ is <u>not</u> equilibrium dominated. Since:

$$\underset{m_2}{\mathrm{Max}}\, \Pi_2(p_2,m_2) \leq \Pi_2^*(1)$$

it follows that $(A_2,p_2^*(1))$ is not equilibrium dominated. This allows us to specify that consumers conjecture $p_2 = p_2^*(1)$ for any observed $A_2$. These conjectures satisfy the inference restriction, and by sequential rationality all consumers visit Firm 1 for any observed profile $(A_1,A_2)$. Thus it is optimal for Firm 2 to stay out.

With this we have proven:

Proposition 1: Outcome 1 uniquely survives the inference restriction, i.e. in every equilibrium satisfying no equilibrium dominated conjectures, the more efficient firm captures the market and chooses zero advertising.

This proposition establishes an important coordination role for dissipative advertising. We can distinguish two kinds of coordination improvements that advertising makes possible: (a) consumers purchase from a single firm, as opposed to dividing purchases among multiple firms; this allows realization of all available coordination economies; and (b) consumers purchase from the most efficient firm. The fact that advertising is necessary to ensure realization of these coordination gains provides a rationale for Benham's empirical association of non-price advertising with lower market prices and larger-scale firms, as in our model coordination gains are directly associated with lower prices and larger scale.

One usually thinks of prices as playing the role of bringing about such coordination. Indeed, in the present case coordination problems arise as a direct consequence of imperfect price information. What we have shown is that when the better profit and better deal properties hold, advertising leads to optimal coordination precisely because it permits the efficient firm to communicate credibly that its price is low. This allows us to view dissipative advertising as a form of implicit price competition, possessing coordination features typically associated with price competition itself. Of course, the misallocations inherent in monopoly pricing persist; advertising conveys some, but not all, of the benefits of price competition.

Advertising can succeed in communicating low price only to the extent that consumers are sophisticated enough to grasp its implicit message. One may object that our formalization of sophisticated inference is unrealistic, either because consumers are unable to evaluate the implicit message due to computational or informational deficiencies, or because consumers and firms must have very strong prior agreement on an equilibrium before equilibrium dominance can be checked. These considerations should, however, be balanced against the mutual desire of consumers and efficient firms to communicate. Our results establish that these agents will have good reason to look to advertising as a means of resolving their communication problems.

Moreover, advertising seems especially well-suited to this role since it allows explanation of the implicit message as part of the advertisment. This point is illustrated by a recent advertising campaign of the Builders' Square hardware chain, in which the slogan pointed explicitly to coordination benefits: "The more we sell, the lower the price; the lower the price, the more we sell." Direct communication such as this gives advertising a special capability to establish focal outcomes.

Our model is readily extended to the case of multi-good sellers, and Proposition 1 may be derived using arguments similar to those in the single-good case. In this setting, the efficient firm uses advertising to communicate that all of its prices lie close to the levels that are optimal when the firm captures the market; thus, the single advertising variable allows many price choices to be signaled. The details of the extension are developed in Appendix B.[15]

### C. The Efficiency of Advertising as a Coordinating Device

An important aspect of the preceding analysis is that advertising plays its coordination role only by overturning inefficient market outcomes. Once the efficient outcome is established, advertising is no longer needed, and the efficient firm reduces it to zero. We might conclude from this that advertising is a very attractive coordinating device in the sense that only the potential for advertising is necessary for acheiving coordination gains. Successful coordination requires no actual expenditure of resources.

This conclusion follows mainly from a special feature of our model, however: once the efficient outcome is anticipated, the efficient firm may simply ignore the reactions of its inefficient rival, as the rival commits at the outset to stay out of the market. Thus the efficient outcome is always associated with zero advertising. This will not continue to be true if entry of inefficient rivals remains possible after the efficient firm's advertising level is determined, for it may then be necessary to advertise at positive levels to keep consumers from defecting to an entrant. Thus, the need to maintain entry barriers, in the form of positive advertising, may impose a resource cost that outweighs the gains from coordination. Advertising may then be detrimental on balance.

To study this possibility, let us modify the multi-stage game of Section 3A to consider entry of an inefficient firm after the efficient firm has chosen its advertising and price levels. Stage 1 will now be divided into the following substages:

Stage 1A: Firm 1 chooses whether or not to enter the market, and it chooses price and advertising levels if it enters.

Stage 1B: Firm 2 observes whether or not Firm 1 has entered, and also Firm 1's advertising level. Firm 2 then makes its own entry, price and advertising decisions.

We now denote Firm 2's equilibrium strategy by $(\hat{A}_2(A_1),\hat{p}_2(A_1))$, with the convention that $A_1 = -1$ means Firm 1 has stayed out. Stages 2 and 3 remain as above, and all the other assumptions are maintained. We continue to focus on the Bayesian-Nash equilibria that satisfy sequential rationality and independent price conjectures.

It is straightforward to show that the equilibria of the new game generate all of the equilibrium outcomes of the original game, and further that the new game adds no new possibilities for equilibrium advertising when Firm 2 captures the market or the firms split the market. There are, however, new possibilities when Firm 1 captures the market: for any $\hat{A}_1 \in [0,\Pi_1^*(1)-F]$, there is an equilibrium in which Firm 1 chooses $(\hat{A}_1,p_1^*(1))$ and Firm 2 stays out if and only if $A_1 \geq \hat{A}_1$. When consumers observe $A_1 < \hat{A}_1$ in these equilibria, they conjecture that Firm 1's price is higher than that of Firm 2. Based on this conjecture, consumers rationally defect to Firm 2, and by sequentially rationality Firm 2 enters and sets $(\hat{A}_2(A_1),\hat{p}_2(A_1)) = (0,p_2^*(1))$ when $A_1 < \hat{A}_1$.[16] Thus the threat of entry forces Firm 1 to choose positive advertising, even though it captures the market in equilibrium.

The positive-advertising equilibria do not, however, seem reasonable if consumers are sophisticated in interpreting Firm 1's advertising policy. For suppose Firm 1 chooses an advertising level slightly below $\hat{A}_1$. In this case a different sort of implicit message is sent: "I am saving a small amount of money by cutting advertising. But I have no reason to stop offering you a better deal, since by doing so I would end up reducing my own profits." In other words, consumers should recognize that a small decrease in advertising cannot be profitable for Firm 1 unless it also chooses a price close to $p_1^*(1)$. By communicating in this way, Firm 1 should be able to economize on the advertising level required to deter entry.

Our inference restriction allows precisely this kind of communication. To see this, consider an equilibrium in which Firm 1 captures the market and chooses advertising level $\hat{A}_1 > 0$. Fix $p_1' \in (p_1^*(1),p_2^*(1))$, and let $m_1'$ be given by (1). Let $A_1' < \hat{A}_1$ be defined by:

$$\Pi_1^*(m_1') - A_1' - F = \Pi_1^*(1) - \hat{A}_1 - F$$

If this defines $A_1' < 0$, set $A_1' = 0$. One may immediately show, using precisely the same argument as in the preceding subsection, that $(A_1',p_1)$ is equilibrium dominated for every $p_1 \geq$

$p_1'$, and that Firm 1 strictly prefers capturing the market with $(A_1', p_1^*(1))$ to its equilibrium payoff. Moreover, independent price conjectures means in this instance that consumers do not conjecture Firm 2 has departed from its equilibrium strategy $(\hat{A}_2(A_1), \hat{p}_2(A_1))$ when $(A_1', \hat{A}_2(A_1'))$ is observed; in particular, they conjecture $p_2 = \hat{p}_2(A_1') = p_2^*(1)$, where the latter equality follows from the sequential rationality of Firm 2's price strategy. Since $p_1' < p_2^*(1)$, consumers continue to purchase from Firm 1. It follows that none of the equilibria with $\hat{A}_1 > 0$ can satisfy the inference restriction, as Firm 1 would always prefer some downward deviation.

This proves that our inference restriction eliminates all equilibria in which Firm 1 chooses positive advertising and captures the market. One can also show that the inference restriction rules out all equilibria in which Firm 1 does not capture the market. Finally, the equilibrium outcome in which Firm 1 chooses zero advertising and captures the market does satisfy the inference restriction, where the notion of equilibrium dominance is suitably modified to take account of the added complexity of Firm 2's strategy.[17] Thus we obtain an analog to Proposition 1 for the case in which an inefficient firm may react to the efficient firm's advertising decision. On these grounds we may conclude that advertising operates with unambiguous efficiency in carrying out its coordination role, when consumers interpret advertising in a sophisticated way.

## D. Coordination when Prices are More Flexible than Advertising

In the preceding analysis we have assumed that the firms commit to their prices at the same time as advertising levels are chosen. There are many situations, however, in which we might expect prices to be more easily adjusted than advertising. In the latter case the strategic situation is changed, since firms would then be able to alter their prices in response to each others' observed advertising.

We may consider this possibility by again modifying the multi-stage game of Section 3A, dividing Stage 1 into the following substages:

<u>Stage 1A</u>: Firms 1 and 2 simultaneously choose advertising levels, where there is again an option of staying out of the market altogether.

<u>Stage 1B</u>: Each firm, if it had commenced operations in Stage 1A, observes the other firm's Stage 1A decisions and then chooses its price.

Thus the modified game allows the prices to be changed in reaction to advertising decisions. The equilibrium strategy of Firm i is now denoted by $(\hat{A}_i,\hat{p}_i(A_j))$, where the dependence of price on $A_j$ indicates price flexibility; we continue to let $A_i = -1$ denote the decision to stay out. The remaining stages are as before, and the assumptions and equilibrium concept are maintained.

It is straightforward to show that the set of equilibrium outcomes of the modified game is exactly the same as in the original game, and there are no new possibilities for equilibrium advertising. Because of the added price flexibility, however, the ability of the efficient firm to communicate low price through advertising has been weakened. Consider Outcome 2, under which Firm 2 chooses $\hat{A}_2 = 0$, and suppose we have defined $p_1'$ and $A_1'$ as in Section 3B. Consumers might now view the unexpected advertising choice $A_1'$ as communicating the following message: "I am expending these resources <u>because I expect that Firm 2 will choose $A_2' > 0$</u>, and I anticipate capturing a large market share <u>in this event</u>. Given this, you know I will be offering a better deal, so you should visit me." This means consumers can interpret the observation of $A_1'$ as ruling out price reactions $p_1(A_2') \geq p_1'$, but they might still conjecture $p_1(0) > p_2^*(1)$, i.e. Firm 1 responds to $A_2 = 0$ with a high price. Given this rationalization, consumers continue to visit Firm 2 after the advertising profile $(A_1',0)$ is observed.

Communication becomes more difficult because price flexibility leads to added strategic ambiguity: consumers must now infer the advertising strategy which Firm 1 had expected Firm 2 to choose, since this determines the price reaction that Firm 1 was trying to communicate. This new difficulty arises, however, because we have allowed consumers to form implausible conjectures about what Firm 1 expects Firm 2 to do. In Outcome 2, Firm 1 actually expects Firm 2 to choose its equilibrium strategy $\hat{A}_2 = 0$, not some deviation $A_2' > 0$. Thus Firm 1's deviation to $A_1'$ should send the a different implicit message: "I am expending these resources because I anticipate capturing the market in the event of $A_2 = 0$, as this is the only event I can expect in view of Firm 2's equilibrium strategy. Given this,..." The strategic ambiguity thus disappears once consumers take into account the fact that Firm 1 expects Firm 2 to follow its equilibrium strategy.

This suggests that we need to modify our notion of equilibrium dominance. We now say that the strategy $(A_i, p_i(A_j))$ with $A_i > -1$ is equilibrium dominated relative to a given equilibrium if:

$$\underset{m_i}{\text{Max}}\ \Pi_i(p_i(\hat{A}_j), m_i) - A_i - F < W_i$$

where $W_i$ is Firm i's equilibrium payoff and $(\hat{A}_j, \hat{p}_j(A_i))$ is the equilibrium strategy of Firm j.[18] Thus equilibrium dominance applies only to that part of a firm's strategy that is relevant given the other firm's equilibrium strategy.

It now follows that $(A_1', p_1(A_2))$ will be equilibrium dominated relative to Outcome 2 whenever $p_1(0) \geq p_1'$. Combining this with the independent price conjectures condition (which as in Section 3C means that consumers will conjecture $p_2 = \hat{p}_2(A_1')$ when $(A_1', 0)$ is observed) eliminates Outcome 2, and the remaining arguments of Section 3B are extended along similar lines.[19,20]

## 4. Signaling Structure along with Strategy

In the preceding section we showed that advertising fills its coordination role through its mere potential to be employed, so that successful coordination is accomplished with no actual advertising expenditure. In real markets, however, one observes a great deal of advertising expenditure, and we feel it is important to reconcile our theory with this observation. Of course, it is possible that advertising shifts consumers' demand curves directly, by raising their desire for the advertised product. In this case profit maximization would lead firms to choose positive advertising, and coordination would be accomplished via the potential for choosing advertising above profit-maximizing levels.

We propose another possibility that does not require that advertising affect demand directly: when there is a greater degree of uncertainty in the marketplace, it may be necessary to use advertising to communicate more than firms' pricing policies. Suppose, for example, that consumers are also uncertain about firms' costs, so they are not sure which firm is more efficient. Advertising might then play two roles: (1) in its coordination role it would continue to communicate pricing strategies; and (2) advertising would serve to communicate cost structure.

The latter role is analogous to the more familiar signaling approach to credible communication (e.g. Spence's (1974) analysis of job-market signaling). In this section we show that advertising may communicate both pricing strategy and cost structure; strategy is signaled through "off-equilibrium-path" advertising, while signaling of structure requires positive equilibrium advertising.[21] Moreover, optimal coordination now requires structural information to be communicated, and advertising may become an inefficient coordination mechanism due to the cost of the latter communication.

### A. A Model with Uncertain Cost Structure

For simplicity we suppose that there is structural uncertainty concerning only the costs of Firm 2. Thus, Firm 1's costs are known by all, but consumers and Firm 1 are uncertain as to whether Firm 2 is more or less efficient than Firm 1. We say that Firm 2 is of a "high type," or type H, if it is less efficient than Firm 1; this is the situation considered in the preceding section. If Firm 2 is of a "low type," or type L, then Firm 2 is the more efficient firm. Firm 2 knows its true type, while consumers and Firm 1 do not, at least at the outset.

We follow Harsanyi (1967-8) in modeling this incomplete-information situation by replacing uncertainty over a player's type with uncertainty over which player is actually playing the game. Thus there are two players associated with Firm 2, corresponding to Firm 2's types; we call these Firm 2H and Firm 2L. Consumers and Firm 1 believe that with probability $\rho \in (0,1)$ they are playing against Firm 2H, just as in Section 3, but with probability $1-\rho$ they are playing against Firm 2L.

We extend the assumptions of Section 3A, including the better deal and better profit properties, to the three firms i = 1,2H,2L. The differential efficiency condition becomes:

$$p_{2L}^*(m) < p_1^*(m) < p_{2H}^*(m) \text{ for every } m > 0.$$

This is illustrated in Figure 2. The multi-stage game proceeds as in Section 3A, with the addition of a "Stage 0" in which "Nature" determines whether Firm 2's position is to be taken by Firm 2H or Firm 2L. Firms' equilibrium strategies are denoted $(\hat{A}_i,\hat{p}_i)$, i = 1,2H,2L, where the decision to stay out of the market is denoted $\hat{A}_i = -1$. Let $\hat{m}_i(A_1,A_2)$ for i = 1,2 represent the equilibrium visitation decisions of consumers conditional on observed advertising. We continue to analyze Bayes-Nash equilibria that satisfy sequential rationality.

The addition of structural uncertainty makes the game more complex, and for this reason we need to be more explicit about the equilibrium conditions. A profile of strategies gives an equilibrium if and only if:

1. Firm 1 maximizes profits. In this case Firm 1 maximizes the expected value of profits, where expectation is taken with respect to the probability of facing Firm 2H vs. 2L:

$$(\hat{A}_1,\hat{p}_1) \in \underset{(A_1,p_1)}{\arg\max}\ \rho\Pi_1(p_1,\hat{m}_1(A_1,\hat{A}_{2H})) + (1-\rho)\Pi_1(p_1,\hat{m}_1(A_1,\hat{A}_{2L})) - A_1 - F$$

if the maximized value is positive; otherwise $\hat{A}_1 = -1$.

2. Firms 2H and 2L maximize profits. For i = 2H,2L:

$$(\hat{A}_i,\hat{p}_i) \in \underset{(A_i,p_i)}{\arg\max}\ \Pi_i(p_i,\hat{m}_2(\hat{A}_1,A_i)) - A_i - F$$

if the maximized value is positive; otherwise $\hat{A}_i = -1$.

3. Consumers make utility-maximizing visitation decisions. The key new feature is that visitation decisions are made after consumers observe advertising and draw the appropriate inferences as to the identity of Firm 2. In a Bayes-Nash equilibrium, these inferences are based on knowing the equilibrium strategies of Firms 2H and 2L. There are three possibilities:

a. $\hat{A}_{2H} \neq \hat{A}_{2L}$ and $\hat{A}_1 \geq 0$. Observing $A_2$ then reveals the identity of Firm 2, and visitation decisions are made on the basis of the true pricing strategies. For i = 2H,2L, if $\hat{A}_i \geq 0$:

$$\hat{m}_1(\hat{A}_1,\hat{A}_i) \begin{cases} = 1, & \hat{p}_1 < \hat{p}_i \\ \in [0,1], & \hat{p}_1 = \hat{p}_i \\ = 0, & \hat{p}_1 > \hat{p}_i \end{cases}$$

and $\hat{m}_2(\hat{A}_1,\hat{A}_i) = 1 - \hat{m}_1(\hat{A}_1,\hat{A}_i)$. In this case we have a separating equilibrium.

b. $\hat{A}_{2H} = \hat{A}_{2L} \geq 0$ and $\hat{A}_1 \geq 0$. Since observing $A_2$ reveals nothing about the identity of Firm 2, consumers maximize expected utility, taking expectation over Firm 2's possible identities. Let V(p) give the utility of purchasing at price p, with $V' < 0$. Putting $\hat{A}_2 = \hat{A}_{2H} = \hat{A}_{2L}$:

$$\hat{m}_1(\hat{A}_1,\hat{A}_2) \begin{cases} = 1, & V(\hat{p}_1) > \rho V(\hat{p}_{2H}) + (1-\rho)V(\hat{p}_{2L}) \\ \in [0,1], & V(\hat{p}_1) = \rho V(\hat{p}_{2H}) + (1-\rho)V(\hat{p}_{2L}) \\ = 0, & V(\hat{p}_1) < \rho V(\hat{p}_{2H}) + (1-\rho)V(\hat{p}_{2L}) \end{cases}$$

and $\hat{m}_2(\hat{A}_1,\hat{A}_2) = 1 - \hat{m}_1(\hat{A}_1,\hat{A}_2)$. This is called a pooling equilibrium.

c. One firm enters and one stays out. In this case all consumers simply visit the firm that has entered.[22]

When both firms have entered and consumers observe an off-equilibrium-path advertising profile $(A_1,A_2) \neq (\hat{A}_1,\hat{A}_{2H}),(\hat{A}_1,\hat{A}_{2L})$, our equilibrium concept requires that consumers make visitation decisions that maximize expected utility subject to some conjecture of the firms' pricing strategies. Moreover, independent price conjectures implies that consumers conjecture $p_i = \hat{p}_i$ if $A_i = \hat{A}_i$ is observed, i.e. a unilateral deviation by Firm j does not lead consumers to believe that Firm i has deviated. As in Section 3, it continues to be possible to rationalize any visitation decisions as responses to an off-equilibrium-path advertising profile, based on arbitrary price conjectures concerning the deviating firm. Now, however, we have added a new dimension to these conjectures: they may encompass not only the price choices of the three firms (strategy), but also the identity of Firm 2 (structure).

As above, we will consider the implications of imposing our inference restriction as a standard of plausibility for off-equilibrium-path conjectures. Note that in the present context, no equilibrium dominated conjectures implies that consumers cannot conjecture that either version of Firm 2 plays an equilibrium dominated strategy, when it is possible to rationalize a given observed deviation by a non-equilibrium dominated strategy for either type of Firm 2.

Before analyzing the equilibria, we must give conditions under which it is possible for advertising to serve as a credible signal of cost structure:

Sorting Condition: For $m_2' > m_2$:

$$\Pi^*_{2H}(m_2') - \Pi^*_{2H}(m_2) < \Pi^*_{2L}(m_2') - \Pi^*_{2L}(m_2)$$

That is, Firm 2H gains strictly less than does Firm 2L from any given increase in market share.

The sorting condition can be derived from the assumption that Firm 2H has strictly greater marginal costs than does Firm 2L at every output level.[23]

B. Structural Uncertainty and Coordination

The introduction of structural uncertainty raises new possibilities for suboptimal coordination. As in Section 3, if advertising is prohibited, or if consumers are unsophisticated, inefficiency may arise in the form of Firm 2H capturing the market, but there is now the added possibility that Firm 1 captures the market from Firm 2L. Moreover, there are new market-splitting outcomes, associated with the various configurations of relative efficiency and levels of advertising. The following lemma shows, however, that imposing the inference restriction considerably simplifies the set of possible outcomes.

Lemma: In any equilibrium satisfying no equilibrium dominated conjectures, we must have $\hat{m}_2(\hat{A}_1,\hat{A}_{2L}) = 1$, i.e. Firm 2L must capture the market.

Proof: Given in Appendix C.

This result is similar to Proposition 1 in that the inference restriction allows the more efficient firm to communicate that its price is lower. In this case, however, Firm 2L must also communicate that it is the more efficient firm. This is made possible by the sorting condition: since Firm 2L differentially prefers capturing the market, there are profitable advertising deviations for Firm 2L that would never be profitable for Firm 2H. No equilibrium dominated conjectures then requires consumers to eliminate the possibility that Firm 2H would make such a deviation.

Given the fact that Firm 2L must capture the market in any equilibrium satisfying the inference restriction, only three possibilities remain for equilibrium division of market shares. These parallel the outcomes discussed in Section 3B:

Outcome 1: Firm 1 captures the market from Firm 2H. Here $\hat{m}_1(\hat{A}_1,\hat{A}_{2H}) = 1$, and since by the Lemma we have $\hat{m}_1(\hat{A}_1,\hat{A}_{2L}) = 0$, it follows that the equilibrium must be separating. Firm 2H chooses $\hat{A}_{2H} = -1$, and Firm 1 responds with $\hat{A}_1 = 0$. We have an equilibrium as long as $\rho\Pi_1^*(1) - F \geq 0$, for otherwise Firm 1 would prefer to stay out. This is the most efficient outcome from the point of view of coordination economies. But since this outcome is also associated with a range of possible values of $\hat{A}_{2L}$, the possibility arises that coordination gains are dissipated in the form of advertising expenditures by Firm 2L.

Outcome 2: Firm 2H captures the market. Firm 1 now chooses $\hat{A}_1 = -1$, and Firms 2H and 2L respond with $\hat{A}_{2H} = \hat{A}_{2L} = 0$. Thus only pooling equilibria are associated with this outcome. Here the distribution of market shares is suboptimal, but no resources are consumed by advertising.

Outcome 3: Firms 1 and 2H split the market. As with Outcome 1, this outcome can be supported only by a separating equilibrium. Market shares are uniquely determined by the necessary condition $p_1^*(\hat{m}_1(\hat{A}_1,\hat{A}_{2H})) = p_{2H}^*(\hat{m}_2(\hat{A}_1,\hat{A}_{2H}))$. This outcome is least efficient, since it makes the least use of coordination economies and may also require positive advertising by all three firms.

We can invoke the inference restriction once again to further reduce the set of possible outcomes. Consider first the outcomes supported by separating equilibria. The following proposition shows that Outcome 3 is eliminated by the inference restriction, and also that any equilibrium supporting Outcome 1 must specify the minimum level of advertising by Firm 2L that deters Firm 2H from misrepresenting itself as Firm 2L.

Proposition 2: There is at most one separating equilibrium outcome that survives the inference restriction. This outcome is characterized by:

(a) $\hat{m}_2(\hat{A}_1,\hat{A}_{2L}) = \hat{m}_1(\hat{A}_1,\hat{A}_{2H}) = 1$, i.e. the most efficient firm captures the market in each state;

(b) $\hat{A}_{2L} = \Pi_{2H}^*(1) - F > 0$, i.e. Firm 2L chooses a positive level of advertising, but this

level is the smallest that discourages deviation by Firm 2H.
Moreover, such an equilibrium exists if and only if $\rho\Pi_1^*(1) - F \geq 0$.

Proof: Given in Appendix C.

The proof of Proposition 2 first eliminates equilibria with $\hat{A}_{2L}$ above the level given in (b). Once (b) is established, it becomes possible in a market-splitting equilibrium for Firm 2H to deviate and capture the market by advertising at a level slightly below $\hat{A}_{2L}$. Here it is the least efficient firm that uses advertising to bring about coordination benefits, which it is able to provide relative to the market-splitting outcome. When Firm 2H captures the market, however, a pooling equilibrium arises; the only separating equilibria that survive the inference restrictions are those in which the most efficient firm captures the market in each state.

From this we conclude that advertising may communicate both strategic and structural information, and thereby bring about optimal coordination. Credible communication of cost structure requires Firm 2L to choose strictly positive advertising. In effect, Firm 2L's advertising carries the implicit message: "I could never recover this advertising expense if my costs were high. You should therefore infer that I am the efficient firm, and given this you know I will offer a better deal." Note however that no additional advertising is needed for communicating price strategy. The latter occurs only "out-of-equilibrium;" that is, such advertising would be used only to upset inefficient market configurations.

We turn now to pooling equilibria. It has been established that Firm 1 stays out and Firms 2H and 2L choose zero advertising in any pooling equilibrium satisfying the inference restriction. Firm 1 can use positive advertising to communicate that it chooses a price close to $p_1^*(1)$, but consumers will continue to visit Firm 2 if the probability of Firm 2 being the low type is sufficiently high. This proves:

Proposition 3: There is at most one pooling equilibrium outcome that survives the inference restriction. This outcome is characterized by:

a. $\hat{m}_2(\hat{A}_1,\hat{A}_{2L}) = \hat{m}_2(\hat{A}_1,\hat{A}_{2H}) = 1$, i.e. Firm 2 always captures the market;

b. $\hat{A}_1 = -1$ and $\hat{A}_{2H} = \hat{A}_{2L} = 0$.

Moreover, such an equilibrium exists if and only if:

$$\rho V(p^*_{2H}(1)) + (1-\rho)V(p^*_{2L}(1)) \geq V(p^*_1(1)) \tag{3}$$

Combining Propositions 2 and 3, it follows that only two kinds of equilibria can survive the inference restriction: (1) a separating equilibrium in which full coordination efficiency is achieved, but resources are dissipated by advertising; and (2) a pooling equilibrium in which there is no advertising, but some coordination benefits are unrealized. Thus, in the presence of structural uncertainty, advertising can bring about optimal coordination only at a cost. There is a welfare tradeoff between unrealized coordination gains and the costs of transmitting the information needed to achieve optimal coordination.

Clearly there is no tradeoff from the consumer point of view, since in the separating equilibrium consumers benefit from coordination but bear none of the costs of information transfer. To see how the tradeoff operates on the producer side, observe that the firms earn greater expected profits in the separating equilibrium if:

$$\rho\Pi^*_1(1) - F + (1-\rho)[\Pi^*_{2L}(1) - (\Pi^*_{2H}(1) - F) - F]$$

$$\geq \rho\Pi^*_{2H}(1) + (1-\rho)\Pi^*_{2L}(1) - F$$

The left- and right-hand sides give the expected producer surpluses in the separating and pooling equilibria, respectively. This condition is equivalent to:

$$\rho(\Pi^*_1(1) - \Pi^*_{2H}(1)) \geq (1-\rho)\Pi^*_{2H}(1) \tag{4}$$

On the left-hand side of (4) we have the gain in expected profits when Firm 1 captures the market rather than Firm 2H; this will be positive if we extend the sorting condition to allow Firm 1 to gain more from an increase in market share than Firm 2H. The right-hand side is equal to the expected cost increase due to advertising and increased entry in the separating equilibrium. For large enough $\rho$, producers benefit in the separating equilibrium, and thus separation is unambiguously superior from the welfare point of view. For small $\rho$, however, producers prefer the pooling equilibrium, and the welfare comparison is ambiguous.

Propositions 2 and 3 reinforce our result that the possibility of advertising will be associated with lower prices and larger scale, since advertising rules out market-splitting outcomes. Moreover, based on comparison of the two propositions, it follows that prices will be even lower, and scale even larger, when strictly positive advertising occurs. Our results also clarify the welfare tradeoffs associated with advertising prohibitions. For suppose that in the absence of advertising, expected coordination gains are maximized (if (3) holds Firms 2H and 2L capture the market, and otherwise Firm 1 captures the market), while the separating outcome of Proposition 2 obtains when advertising is allowed. It then follows that removal of advertising prohibitions benefits consumers as a consequence of coordination gains, while due to the costs of advertising total welfare may either rise or fall.

Note finally that no equilibrium will satisfy the inference restriction if $\rho$ is too small to induce Firm 1 to enter in the separating equilibrium, but too large to prevent Firm 1 from capturing the market in the pooling equilibrium. The situation here is quite similar to the case of nonsustainable industry structures in the cost-based theory of natural monopoly: there is no specification of market shares and advertising expenditures that allows all firms to at least break even, and that also deters market-capturing advertising strategies. Thus, dissipative advertising, and the implicit price competition that advertising makes possible, may generate instability in the market outcome.[24,25]

C. Altering the Order of Moves

Unlike the complete information case, altering the signaling game by changing the order of moves has important implications for the results under incomplete information. Here we give a brief summary of the key findings, and defer the details of the analysis to Appendix D.

First, the signaling game may be modified by making Firm 2 an entrant, along the lines of Section 3C. The results are now altered in that equilibria supporting Outcome 1 may specify $\hat{A}_1 > 0$ and still satisfy the inference restriction: Firm 1 can conjecture that a downward advertising deviation would capture the market from both Firms 2H and 2L, whereas in equilibrium Firm 1 captures the market only from Firm 2H. This possibility for a discretely higher market share serves to expand the range of equilibrium undominated prices, and it can happen no advertising level below $\hat{A}_1$ will convince consumers that Firm 1 charges a lower price than Firm 2H.

Further, Outcome 2 may survive for a larger range of $\rho$, based on the fact that Firm 1's advertising deviation may induce a separating reaction by Firms 2H and 2L. It turns out that Outcome 2 survives under precisely the same circumstances as Outcome 1 equilibria with $\hat{A}_1 > 0$, and both are eliminated when $\rho$ is sufficiently close to unity.

Next consider the possibility that prices can be adjusted after advertising levels are observed, as in Section 3D. In this case the argument that eliminated Outcome 3 no longer holds: an advertising deviation by Firm 2H can lead Firm 1 to shift its price to $p_1^*(1)$, and since consumers can conjecture $p_2 = p_{2H}^*(1)$, Firm 2H is no longer able to capture the market. Firm 1 deviations can overturn Outcome 3, subject to the same difficulty as in the preceding discussion of entry; if $\rho$ is sufficiently close to unity, Firm 1's deviation will eliminate Outcome 3, but the outcome survives if $\rho$ is close enough to zero.

In sum, the complete information results carry over to the incomplete information case as long as $\rho$ is close enough to unity, i.e. as long as Firm 1 is sufficiently confident that it is playing the game of Section 3A. It is important to note, however, the Lemma continues to hold under either modification of the signaling game, for every value of $\rho$; thus Firm 2L always captures the market under the inference restriction. The key conclusion is that a firm will use advertising to bring about coordination gains if and only if it is sufficiently confident that it is most efficient.[26]

## 5. Advertising and Coordination with Observable Prices

Thus far we have assumed that sellers cannot communicate any price information whatsoever, but our conclusions extend beyond such environments. In this section we augment our model to consider two situations in which price communication is possible: sellers offer a line of products, and are able to communicate some, but not all, of their products' prices; and sellers can communicate the prices but not the quality attributes of their products. In each case, competition in the observable price variables may be insufficient to bring about optimal coordination, and as a consequence there arises a coordination role for advertising.

### A. Loss Leaders

Let us modify the framework of Section 2 by supposing the firm produces two goods, q and s, according to the joint cost function C(q,s). Price communication is possible only for good s, which can be thought of as a "loss leader" item. Demand functions for each consumer are given by Q(p,r) and S(p,r), where p and r are the prices of q and s, respectively. The profit function becomes:

$$\Pi(p,r,m) = pmQ(p,r) + rmS(p.r) - C(mQ(p,r),mS(p,r))$$

Let $p^*(r,m)$ denote the profit-maximizing choice of p when r and m are prespecified, and let $\Pi^*(r,m)$ denote the corresponding maximized profit level.

In this case it is no longer immediate that $\Pi^*$ is increasing in m, as the firm cannot raise r to exploit higher market share. $\Pi^*$ will, however, be strictly increasing in m when C exhibits global scale economies in q and s jointly.[27] As for $p^*$, we now have:

$$\frac{dp^*}{dm} = \frac{[C_{qq}QQ_p + C_{ss}SS_p + C_{qs}(SQ_p + QS_p)]m}{\Delta_p}$$

where once again $\Delta_p < 0$ is assumed. Observe that sufficient conditions for $p^*$ to be a decreasing function of m are that marginal costs decrease in each output separately, sales of q greatly exceed sales of s, and cost complementarities are insignificant (i.e. $C_{qs}$ is small in magnitude). It is also true, however, that even if marginal costs are increasing, the derivative will be negative when there are strong cost complementarities ($C_{qs} << 0$) and the goods are not excessively close substitutes ($S_p < \varepsilon$ for small $\varepsilon > 0$). Thus joint production gives a cost-based source of better deal effects that need not hinge on declining marginal costs.

We now extend the game of Section 3A. Let the firms' profits be $\Pi_i(p_i,r_i,m_i) - A_i$, where $\Pi_i$ is continuous, strictly quasiconcave in $p_i$ and $r_i$, and $\Pi_i(p_i,r_i,0) = 0$. Let $p_i^*(r_i,m_i)$ denote the $p_i$ that maximizes $\Pi_i$, and let $\Pi_i^*(r_i,m_i)$ denote the maximized profit level. Our hypotheses become:

<u>Better Profit Property</u>: $\Pi_i^*(r_i,m_i)$ is strictly increasing in $m_i$ for all $r_i$.
<u>Better Deal Property</u>: $p_i^*(r_i,m_i)$ is strictly decreasing in $m_i$ for all $r_i$.
<u>Differential Efficiency</u>: $p_1^*(r,m) < p_2^*(r,m)$ for all r,m.

To these we add:

<u>Differential Profitability</u>: $\Pi_1^*(r,m) > \Pi_2^*(r,m)$ for all r,m.

The firms now choose $A_i$, $p_i$ and $r_i$ in Stage 1, and in Stage 2 consumers observe $A_i$

and $r_i$, but not $p_i$, prior to making visitation decisions. Multiple equilibrium outcomes are possible if q is sufficiently important to consumers relative to s, since a low $r_i$ will not attract consumers if they conjecture that $p_i$ is high. Further, the use of this threat to punish $r_i$ deviations permits a great variety of possible equilibrium outcomes, based on various $r_i$ levels and divisions of the market.

We now extend Proposition 1 to this augmented game. Suppose first that Firm 2 has captured the market in equilibrium; since Firm 1 stays out, we have $\hat{A}_2 = 0$, $\hat{p}_2 = p_2^*(\hat{r}_2,1)$ and:

$$\hat{r}_2 = \arg\max_{r_2} \Pi_2^*(r_2,1)$$

Following the argument of Section 3B, we may choose $p_1' \in (p_1^*(\hat{r}_2,1), p_2^*(\hat{r}_2,1))$ and $m_1' < 1$ such that for all $p_1 \geq p_1'$:

$$\max_{m_1} \Pi_1(p_1,\hat{r}_2,m_1) < \Pi_1^*(\hat{r}_2,m_1') < \Pi_1^*(\hat{r}_2,1)$$

and $A_1'$ is defined by:

$$A_1' = \Pi_1^*(\hat{r}_2,m_1') - F$$

Differential profitability and the fact that Firm 2 does not prefer to stay out imply $\Pi_1^*(\hat{r}_2,1) - F > 0$, so that we have $A_1' > 0$ if $m_1'$ is taken sufficiently close to unity. It follows that all Firm 1 strategies $(A_1,p_1,r_1)$ with $A_1 = A_1'$, $p_1 \geq p_1'$ and $r_1 = \hat{r}_2$ are equilibrium dominated, while $(A_1',p_1^*(\hat{r}_2,1),\hat{r}_2)$ is not equilibrium dominated. Under the inference restriction consumers must visit Firm 1 when they observe $A_1 = A_1'$, $A_2 = \hat{A}_2$ and $r_1 = r_2 = \hat{r}_2$, and Firm 1's deviation overturns the equilibrium. As for market-splitting equilibria, the inference restriction allows Firm 1 to capture the market by choosing $A_1 > \hat{A}_1$ and $r_1 = \hat{r}_1$, i.e. Firm 1 uses only advertising to convince consumers that $p_1$ is close to $p_1^*(\hat{r}_1,1)$. Thus Proposition 1 continues to hold in this setting.

It is important to note that optimal coordination could not be assured if advertising were not possible, even though deviations in the observable price do have some communicative power. One simple way to see this is as follows. Suppose Firm 2 captures the market in equilibrium and Firm 1 is unable to advertise. For Firm 1 to overturn the equilibrium in this case, it is necessary and sufficient that there exist $p_1'$ and $r_1'$ giving consumers strictly higher utility than $\hat{p}_2$ and $\hat{r}_2$, with $p_1' > p_1^*(r_1',1)$ and:

$$\max_{m_1} \Pi_1(p_1',r_1',m_1) - F = 0 \tag{5}$$

It is easily shown that the left hand side of (5) is strictly decreasing in $p_1'$ for $p_1' > p_1^*(r_1',1)$.[28] Since $W_1 = 0$, it follows that under the inference restriction consumers cannot infer $p_1 > p_1'$ when Firm 1 enters with $r_1 = r_1'$, and this overturns the equilibrium. Deviations of this kind will typically involve low values of $r_1'$, which have a positive direct effect on consumer utility; we may regard such loss leader pricing strategies of as another form of advertising, which communicates the hidden price information without being strictly dissipative.

It is not always possible, however, to communicate the needed information through loss leader pricing alone. Assume first that the firms are constrained to choose $r_i \geq 0$. If F is low, or if the loss leader is an unimportant part of the firm's profits, then (5) may be satisfied for very high levels of $p_1'$ even when $r_1'$ is close to zero; Firm 1 may then be unable to assure consumers a better deal though loss leader pricing. Essentially, entry with $r_1'$ does not "burn" enough profit to convince consumers that $p_1$ is sufficiently low, and advertising becomes necessary. Moreover, entry strategies involving low $r_1$ may be self-defeating if $p_1^*(r_1,m_1)$ is decreasing in $r_1$, as the low loss leader price drives up Firm 1's profit-maximizing hidden price.[29] These effects are amplified to the extent that the loss leader represents an unimportant part of consumers' utility.

It may be most plausible, moreover, to assume that the firms can counteract loss leader prices by limiting availability of the advertised good ("bait and switch" tactics), as opposed to being constrained to meet demand. In this case loss leader pricing represents an even less effective way to dissipate profits credibly. Finally, if $r_i < 0$ is allowed, it becomes possible for firms to mimic any level of dissipative advertising, and in fact to pursue direct market-capturing strategies, by offering consumers sufficiently high cash payments. These tactics will work, however, only to the extent that firms are committed to meet demand for the advertised good, and dissipative advertising expenditures are needed when such commitments cannot be made.

We conclude that the possibility of advertising remains necessary to assure optimal coordination, even when the firms can use loss-leader strategies. The important point is that price competition on a limited subset of goods may be insufficient to bring about coordination gains even when consumers are sophisticated in interpreting the observed prices. Thus it is the inability to communicate sufficient price information, rather than a complete inability to communicate, that gives rise to advertising's coordination role.

It is straightforward to extend our remaining results to this augmented game. One important modification occurs when Firm 2 can delay its entry decision; in this case the threat of entry may impose a restriction on $\hat{r}_1$ that would not arise if the firms made entry decisions simultaneously. Essentially, it may still be necessary to impose a limit pricing entry barrier, even though advertising entry barriers are never needed. Further, in the incomplete information version of the loss leader game, the outcomes surviving the inference restriction involve the same division of market shares as previously (Firm 2L always captures the market; Firm 1 captures the market from Firm 2H in separating equilibria, and vice-versa in pooling equilibria). Equilibrium $r_1$ and $r_2$ are uniquely determined in each of the two surviving outcomes, and moreover the separating outcome will involve strictly positive advertising by Firm 2L if the loss leader represents a small enough proportion of firms' sales.

B. Product Quality

We now return to the assumption that there is a single good q with price p, but the quality of the good may assume a variety of levels indexed by v. Production costs are given by C(q,v), with $C_v > 0$ and $C_{qv} > 0$, and design costs are given by D(v), with $D' > 0$. Let the demand function for each consumer be Q(p,v), satisfying $Q_v > 0$ and $Q(p,0) = 0$. The profit function becomes:

$$\Pi(p,v,m) = pmQ(p,v) - C(mQ(p,v),v) - D(v)$$

For given p and m let $v^*(p,m)$ be the profit maximizing quality level, and let $\Pi^*(p,m)$ denote maximized profits. The latter is a weakly increasing function of m, and it is strictly increasing as long as p is not so low that $v^*(p,m) = 0$. Note that:

$$\frac{dv^*}{dm} = \frac{-(p - C_q)Q_v + C_{qv}Q + C_{qq}QmQ_v}{\Delta_v}$$

where $\Delta_v$ is the second derivative of $\Pi$ with respect to v, evaluated at $v^*$, which is assumed to be strictly negative. From this it may be seen that the effect of m on $v^*$ revolves around two effects. The first is a positive revenue effect: the marginal return to increased quality is proportional to the markup $p - C_q$, and as m rises this marginal return rises. The second is an ambiguous cost effect: higher m makes it more attractive to reduce marginal cost directly via lower quality, and if $C_{qq} > 0$ quality reductions will reduce marginal cost indirectly by cutting total sales; the latter effect is reversed if $C_{qq} < 0$, however. Thus, higher market share will lead the firm to provide higher quality if the revenue effect outweighs the cost effect, or if the cost effect is positive due to decreasing marginal costs and a small direct effect of quality on marginal costs.

The sign or the derivative is clear cut in the case of constant marginal costs, however. If we take $C(q,v) = c(v)q$, with $c > 0$, $c' > 0$, then we have:

$$\frac{dv^*}{dm} = \frac{-D'}{m\Delta_v}$$

which is strictly positive. Here profit-maximizing quality choice causes the revenue effect to exceed the cost effect by precisely $D'/m$.

Product quality choice may be introduced into the game of Section 3A is in a manner similar to the loss leader case, with the better profit and deal properties and differential efficiency and profitability extending in an obvious fashion (e.g., the better deal property now asserts that $v_i^*(p_i,m_i)$ is strictly increasing in $m_i$). The firms choose price, quality and advertising in the first stage, while consumers observe price and advertising, but not quality, prior to making visitation decisions. Consumers do observe quality prior to purchase, so we are assuming that quality is a search attribute (see Nelson (1970)).

It is easy to extend our results to the current setting, along the lines of the preceding section. In contrast to the loss leader case, however, it is never essential for a firm to use advertising to convince consumers that it expects to capture the market; price cuts alone suffice for this. To see why this is true, consider the set of prices that could appear in some strategy that survives equilibrium dominance:

$$\{p_i \mid \Pi_i(p_i,v_i,m_i) - F \geq W_i \text{ for some } v_i,m_i\}$$

Under our assumptions this set is an interval $[\underline{p}_i,\bar{p}_i]$ with $\underline{p}_i > 0$. For each $p_i \in [\underline{p}_i,\bar{p}_i]$ we may let $\underline{v}_i(p_i)$ denote the lowest quality level that may appear together with $p_i$ in a strategy that survives equilibrium dominance:

$$\underline{v}_i(p_i) \equiv \min \{v_i \mid \Pi_i(p_i,v_i,m_i) - F \geq W_i \text{ for some } m_i\}$$

It is easy to see that $\underline{v}_i$ is continuous and $\underline{v}_i(\underline{p}_i) = v_i^*(\underline{p}_i,1)$. Observe that any strategy specifying $p_i$ and $v_i < \underline{v}_i(p_i)$ must be equilibrium dominated, while choosing $p_i$ and $v_i = \underline{v}_i(p_i)$ is not equilibrium dominated. Thus, by deviating to $p_i$ sufficiently close to $\underline{p}_i$, Firm i convinces consumers that $v_i$ must be close to $v_i^*(\underline{p}_i,1)$.

This does not mean that price signals are sufficient to establish an efficient equilibrium, however, since it is not clear that $v_i^*(\underline{p}_i,1)$ gives a better deal once the price is cut. It is in fact most reasonable to assume that $v_i^*$ is strictly increasing in $p_i$,[30] so that consumers will anticipate quality reductions in conjunction with the lower prices. Consider Outcome 2, in which Firm 2's equilibrium quality is $v_2^*(\hat{p}_2,1)$. If $v_1^*(\underline{p}_1,1) < v_2^*(\hat{p}_2,1)$, then Firm 1 may find it impossible to signal via price both that it expects to capture the market and that it offers a better deal, as consumers might be excessively sensitive to the lower quality associated with the needed price cuts. In this case advertising is necessary for bringing about efficient coordination.

This line of thinking extends to the signaling of structural information. Firm 2L of the incomplete information model is the more efficient provider of quality in the present context, but it may find it impossible to use price cuts to separate from Firm 2H while at the same time offering consumers a better deal than Firm 1. Advertising then becomes essential for signaling structural as well as strategic information. In this case Firm 2L must choose strictly positive advertising in the separating equilibrium that satisfies the inference restriction; this contrasts with signaling models in which quality is chosen by "Nature" (e.g., Milgrom and Roberts (1986)), where in the absence of repeat business effects advertising would not be used in equilibria robust to the elimination of equilibrium dominated conjectures.

## 6. Conclusion

We have provided a theory of advertising that explains the prevalence of "vague" retail advertisements as well as Benham's association of the ability to advertise with lower prices and larger scale. The analysis hinges on three key assumptions: sellers find it difficult to communicate relevant information (e.g., price, quality, selection), buyers and active sellers mutually benefit when buyers concentrate their purchases among fewer firms, and consumers are sophisticated in interpreting advertising messages.

Our work establishes a rich set of environments in which consumers gain by coordinating their purchase activities. This set does not, however, include markets for which consumer network externalities are the only source of coordination benefits (e.g., Katz and Shapiro (1986)). This is because network benefits are brought about by consumer choices, whereas advertising allows only the firms' choices to be signaled. Thus an individual consumer may be unwilling to switch to a firm that has used advertising to communicate that it offers favorable terms, since network benefits are lost if other consumers do not also switch. What is needed for efficient coordination is a means of "public money burning" that could allow individual consumers to coordinate adoption decisions; interesting future work might consider mechanisms (e.g., costly adoption preannouncements) that promote consumer coordination in this sense.

Thoughout our analysis we have maintained the assumption that consumers observe perfectly a firm's level of advertising. We intend to relax this assumption in future work. We also plan to extend our association of high advertising, low prices and large scale to settings with free entry of firms and more detailed consumer search strategies.

Finally, an intriguing area for future research concerns the transition path to equilibria. Implicit in our story is an unmodelled dynamic in which positive advertising is used to "break" inefficient equilibria. While our analysis has considered only steady states of such a dynamic, it does suggest a transition path along which advertising is expanded in the short term, and then decreased gradually, in the course of establishing an efficient equilibrium.

# APPENDIX A

## Consumer Heterogeneity and Product Variety

In this appendix we analyze in more detail the consumer heterogeneity and product variety motivations for the better deal property. First consider the following model of consumer heterogeneity. Let the firm's cost function be given by $C(q) = cq$ for $c > 0$, and suppose that there are are a continuum of consumers indexed by $\theta \in [0,1]$. We take the population of consumers to be uniformly distributed on this interval, with total mass one. If consumer $\theta$ purchases q units at price p, his utility is $\theta U(q) - pq$, where $U' > 0$, $U'' < 0$. Demand functions are then given by:

$$Q(p,\theta) = (U')^{-1}\left(\frac{p}{\theta}\right)$$

Let m now represent the mass of consumers who visit the firm, and let these be the consumers having strongest preference for the good, i.e. the subinterval [1-m,1]. As m rises, the marginal consumer's demand elasticity will correspondingly rise if $U'''$ is negative, or at least is small enough in magnitude. This raises the elasticity of total demand, leading to a lower profit-maximizing price. In particular, note that $p^*$ is determined by:

$$\int_{1-m}^{1} Q(p^*,\theta)d\theta + (p^* - c) \int_{1-m}^{1} Q_p(p^*,\theta)d\theta = 0$$

With the aforementioned restriction on the third derivative of U, Q and $Q_p$ are both strictly increasing in $\theta$. Thus, the derivative of this expression with respect to m must satisfy:

$$Q(p^*, 1\text{-}m) + (p^* - c)Q_p(p^*, 1\text{-}m) < 0$$

which gives $dp^*/dm < 0$. Here the better deal property holds because expanding its market requires the firm to attract "more distant" consumers having more elastic demand.

Now consider the following model of product variety. Consumers are once more identical, but now the firm sells a range of products. Let there be a continuum of products indexed by $\theta \in [0,1]$, having uniform density with unit mass. The utility achieved from purchasing $q_\theta$ units of product $\theta$ at price $p_\theta$ is:

$$U_\theta(q_\theta) - p_\theta q_\theta$$

and when the subset $K \subset [0,1]$ of products is offered, total utility is:

$$\int_K (U_\theta(q_\theta) - p_\theta q_\theta) d\theta$$

Let the cost function for product $\theta$ be $C_\theta(q_\theta) = c_\theta q_\theta$. Maximizing profit from product $\theta$ generates per-consumer revenue of $R_\theta$, given by:

$$R_\theta = \max_{p_\theta} (p_\theta - c_\theta)(U'_\theta)^{-1}(p_\theta)$$

Let us index the products so that $R_\theta$ is strictly increasing in $\theta$.

In addition to the costs incurred for each product, the seller must pay a stocking cost $S(k)$ to offer a product line having mass k; assume $S' > 0$, $S'' > 0$. When the firm stocks mass k of products and sells to m consumers, its profits are:

$$m \int_{1-k}^{1} R_{\theta} d\theta - S(k)$$

Let $k^*(m)$ maximize this expression. It is easy to see that $dk^*/dm > 0$ as a result of increasing marginal stocking costs, and thus larger m leads each consumer to have greater utility as a consequence of expanded product availability. Here we have a better deal effect despite the fact that the price of each particular product is independent of m.

# APPENDIX B

## Multi-good Sellers

In this appendix we extend Proposition 1 to the case of multi-good sellers. Suppose that each of the two firms sells n goods. $p_i$ becomes an n-vector of prices, and we may let the continuous function $V(p_i)$ denote the utility of a consumer who visits Firm i having price $p_i$. We assume that $\Pi_i(p_i,m_i)$ is continuous and strictly quasiconcave in $p_i$, with $p_i^*(m_i)$ denoting the profit-maximizing price vector and $\Pi_i^*(m_i)$ the maximized profit level; we also have $\Pi_i(p_i,0) = 0$. The better profit property is as in the text, but for the better deal property we have $V(p_i^*(m_i))$ strictly increasing in $m_i$, and for differential efficiency we have $V(p_1^*(m)) > V(p_2^*(m))$ for all $m > 0$. Finally, the multi-stage game is exactly as in Section 3A, including in particular the assumption that consumers visit one and only one firm.

The extension of Proposition 1 requires four steps.

Step 1: Put $P_i(m,z) \equiv \{p_i | \Pi_i(p_i,m) \geq z\}$. We claim that for all $m > 0$ there exists $z < \Pi_i^*(m)$ such that $P_i(m,z)$ is bounded. If not, then for some $K > 0$ we may choose a sequence $\{z^n\}$, $z^n \uparrow \Pi_i^*(m)$, and a corresponding sequence $\{p_i^n\}$ with $p_i^n \in P_i(m,z^n)$ and $|p_i^n - p_i^*(m)| > K$ for all n. For each n define $\tilde{p}_i^n = \lambda^n p_i^n + (1-\lambda^n)p_i^*(m)$ by $|\tilde{p}_i^n - p_i^*(m)| = K$. Since $p_i^*(m) \in P_i(m,z^n)$ and $\lambda^n \in (0,1)$, the quasiconcavity assumption implies that $\tilde{p}_i^n \in P_i(m,z^n)$ for all n. As $\{\tilde{p}_i^n\}$ is contained in the compact set $\{p_i| \; |p_i - p_i^*(m)| = K\}$, there exists a cluster point $\tilde{p}_i$ of the sequence. Since $z^n \leq \Pi_i(\tilde{p}_i^n,m) < \Pi_i^*(m)$, by continuity of $\Pi_i$ we have $\Pi_i(\tilde{p}_i,m) = \Pi_i^*(m)$. But strict quasiconcavity has been violated because $|\tilde{p}_i - p_i^*(m)| = K > 0$.

Step 2. Put:

$$d_i(m,z) \equiv \max_{P_i(m,z)} |p_i - p_i^*(m)|$$

Note that $d_i(m,z)$ is defined for $z < \Pi_1^*(1)$ sufficiently close to $\Pi_1^*(1)$ and is nonincreasing in z. We claim that for all $m > 0$:

$$\lim_{z \to \Pi_i^*(m)^-} d_i(m,z) = 0$$

If not, then there exists $\{z^n\}$, $z^n \uparrow \Pi_i^*(m)$ and $K > 0$ such that $d_i(m,z^n) \geq K$ for all n. Further, for each n there exists $p_i^n$ satisfying $|p_i^n - p_i^*(m)| = d_i(m,z^n)$ and $\Pi_i(p_i^n,m) \geq z^n$. Thus, $\{p_i^n\} \subset \{p_i | \ K \leq |p_i - p_i^*(m)| \leq d_i(m,z^1)\}$, and since the latter is a compact set the sequence has a cluster point $\tilde{p}_i$. Again continuity of $\Pi_i$ gives $\Pi_i(\tilde{p}_i,m) = \Pi_i^*(m)$, and we obtain a contradiction of strict quasiconcavity.

<u>Step</u> <u>3</u>: Put:

$$s_i(m,z) = \sup_{\tilde{m} \in [m,1]} d_i(\tilde{m},z)$$

taking supremum over the extended reals. We claim that:

$$\lim_{m \to 1} s_i(m,z) = d_i(1,z)$$

wherever $d_i(1,z)$ is defined. If not, then we may choose $K > 0$ and $\{m^n\}$, $m^n \uparrow 1$, such that $s_i(m^n,z) > K > d_i(1,z)$ for all n. For each n we may choose $\tilde{m}^n \in [m^n,1]$ such that $d_i(\tilde{m}^n,z) > K - \varepsilon > d_i(1,z)$, for fixed $\varepsilon > 0$. Using quasiconcavity, it follows that for each n there is some $p_i^n \in P_i(\tilde{m}^n,z)$ such that $|p_i^n - p_i^*(\tilde{m}^n)| = K - \varepsilon$. Put:

$$A \equiv \max_{m \in [m^1,1]} |p_i^*(m) - p_i^*(1)|$$

Then we have $\{p_i^n\} \subset \{p_i | \ K - \varepsilon - A \leq |p_i - p_i^*(1)| \leq K - \varepsilon + A\}$, and since the latter set is compact the sequence has a cluster point $\tilde{p}_i$. We have $|\tilde{p}_i - p_i^*(1)| = K - \varepsilon$, and continuity of $\Pi_i$ gives $\Pi_i(\tilde{p}_i,1) \geq z$. But this implies $\tilde{p}_i \in P_i(1,z)$, so that $d_i(1,z) \geq K - \varepsilon$, in contradiction to the definitions of K and $\varepsilon$.

Step 4: We now reconstruct the arguments of Section 3B for the more general case. Using continuity of V we may fix $\varepsilon > 0$ so that $|p_1 - p_1^*(1)| < \varepsilon$ implies $|V(p_1) - V(p_1^*(1))| < V(p_1^*(1)) - V(p_2^*(1))$. Using Step 2 we may choose $z' < \Pi_1^*(1)$ such that $d_1(1,z') < \varepsilon/3$. Then Step 3 allows us to choose $\tilde{m}_1 < 1$ such that $s_1(\tilde{m}_1,z') - d_1(1,z') < \varepsilon/3$; let $\tilde{m}_1$ be sufficiently close to unity so that $|p_1^*(m_1) - p_1^*(1)| < \varepsilon/3$ for all $m_1 \geq \tilde{m}_1$. For $m_1 \leq \tilde{m}_1$ we have, for all $p_1$:

$$\Pi_1(p_1,m_1) \leq \Pi_1^*(m_1) \leq \Pi_i^*(\tilde{m}_1) < \Pi_1^*(1)$$

For $m_1 > \tilde{m}_1$, $\Pi_1(p_1,m_1) > z'$ implies:

$$|p_1 - p_1^*(m_1)| \leq d_1(m_1,z') \leq s_1(\tilde{m}_1,z') < 2\varepsilon/3$$

Thus, since $|p_1 - p_1^*(1)| \geq \varepsilon$ implies $|p_1 - p_1^*(m_1)| \geq 2\varepsilon/3$ for $m_1 > \tilde{m}_1$, $|p_1 - p_1^*(1)| \geq \varepsilon$ implies $\Pi_1(p_1,m_1) \leq z'$ for all $m_1 > \tilde{m}_1$.

Now choose $m_1'$ such that:

$$\max\{\Pi_1^*(\tilde{m}_1),z'\} < \Pi_1^*(m_1') < \Pi_1^*(1)$$

Defining $A_1'$ as in (2) of the text, it follows that any $(A_1',p_1)$ with $|p_1 - p_1^*(1)| \geq \varepsilon$ satisfies, for all $m_1$:

$$\Pi_1(p_1,m_1) - A_1' - F \leq \max\{\Pi_1^*(\tilde{m}_1),z'\} - A_1' - F < \Pi_1^*(m_1') - A_1' - F = 0$$

and so all such $(A_1', p_1)$ are equilibrium dominated. $(A_1', p_1^*(1))$ is not equilibrium dominated, however. Thus, observing $(A_1', \hat{A}_2)$ leads consumers to infer $|p_1 - p_1^*(1)| < \varepsilon$ under the inference restriction, and the definition of $\varepsilon$ ensures $V(p_1) > V(p_2^*(1))$. The other results of Sections 3 and 4 extend similarly.

## APPENDIX C

### Proofs of Lemma and Proposition 2

Proof of Lemma: Following the approach of Section 3B, for each $p_2' > p_{2L}^*(1)$ we may find $m_{2L}(p_2')$ that satisfies, for all $p_2 \geq p_2'$:

$$\max_{m_2} \Pi_{2L}(p_2,m_2) < \Pi_{2L}^*(m_{2L}(p_2')) < \Pi_{2L}^*(1) \tag{C1}$$

Note that $\Pi_{2L}^*(m_{2L}(p_2')) \rightarrow \Pi_{2L}^*(1)$ as $p_2' \rightarrow p_{2L}^*(1)$. Define the function $A_{2L}(p_2')$ by:

$$A_{2L}(p_2') = \Pi_{2L}^*(m_{2L}(p_2')) - F - W_{2L} \tag{C2}$$

Suppose now that Firm 2L has equilibrium market share $\hat{m}_{2L}$ with $0 < \hat{m}_{2L} < 1$. Using the sorting condition and the better profit property:

$$\Pi_{2L}^*(1) - \Pi_{2L}^*(\hat{m}_{2L}) > \Pi_{2H}^*(1) - \Pi_{2H}^*(\hat{m}_{2L}) > 0 \tag{C3}$$

Choosing $p_2' \in (p_{2L}^*(1), p_1^*(1))$ sufficiently close to $p_{2L}^*(1)$ gives, using (C2) and (C3):

$$\Pi_{2L}^*(m_{2L}(p_2')) - \Pi_{2L}^*(\hat{m}_{2L}) = A_{2L}(p_2') - \hat{A}_{2L} > \Pi_{2H}^*(1) - \Pi_{2H}^*(\hat{m}_{2L}) \tag{C4}$$

From the latter inequality we have:

$$\Pi_{2H}^*(1) - A_{2L}(p_2') - F < \Pi_{2H}^*(\hat{m}_{2L}) - \hat{A}_{2L} - F \leq W_{2H}$$

where the weak inequality follows from the equilibrium conditions (Firm 2H cannot strictly prefer $\hat{A}_{2L}$). Thus, $(A_{2L}(p_2'),p_2)$ is equilibrium dominated for Firm 2H, for all $p_2$. Moreover, from (C1) and the equality in (C4) it follows that $(A_{2L}(p_2'),p_2)$ is equilibrium dominated for Firm 2L whenever $p_2 \geq p_2'$, and it is apparent that Firm 2L strictly prefers capturing the market with $(A_{2L}(p_2'),p^*_{2L}(1))$ to its equilibrium payoff.

Thus, when consumers observe $(\hat{A}_1,A_{2L}(p_2'))$ they must conclude that Firm 2 is actually Firm 2L and that it charges $p_2 < p_2'$. Since $p_2' < p^*_1(1) \leq \hat{p}_1$, we must have $\hat{m}_2(\hat{A}_1,A_{2L}(p_2')) = 1$, which induces Firm 2L to deviate. Finally, related arguments rule out $\hat{m}_{2L} = 0$. Q.E.D.

<u>Proof of Proposition 2</u>: Suppose first that:

$$\hat{A}_{2L} > \Pi^*_{2H}(1) - F - W_{2H}$$

From (C2) we have, substituting the value of $W_{2L}$ implied by the Lemma:

$$\Pi^*_{2L}(m_{2L}(p_2')) - A_{2L}(p_2') - F = \Pi^*_{2L}(1) - \hat{A}_{2L} - F$$

This implies $A_{2L}(p_2') < \hat{A}_{2L}$ for every $p_2' > p^*_{2L}(1)$. Note also that $A_{2L}(p_2') \rightarrow \hat{A}_{2L}$ as $p_2' \rightarrow p^*_{2L}(1)$. Choose $p_2' > p^*_{2L}(1)$ sufficiently close to $p^*_{2L}(1)$ to give:

$$\hat{A}_{2L} > A_{2L}(p_2') > \Pi^*_{2H}(1) - F - W_{2H}$$

from which it follows that $(A_{2L}(p_2'),p_2)$ is equilibrium dominated for Firm 2H for all $p_2$. Using (C1) we have that $(A_{2L}(p_2'),p_2)$ is equilibrium dominated for Firm 2L for all $p_2 \geq p_2'$, and also that $(A_{2L}(p_2'),p^*_{2L}(1))$ is not equilibrium dominated. Thus $\hat{m}_2(\hat{A}_1,A_{2L}(p_2')) = 1$ is implied by the inference restrictions, and Firm 2L strictly prefers $A_{2L}(p_2')$ to its equilibrium advertising choice.

Since the equilibrium conditions rule out:

$$\hat{A}_{2L} < \Pi^*_{2H}(1) - F - W_{2H}$$

we conclude that under the inference restrictions we have, in any equilibrium:

(C5) $$\hat{A}_{2L} = \Pi^*_{2H}(1) - F - W_{2H}$$

In particular, for $W_{2H} = 0$ this gives the unique level of $\hat{A}_{2L}$ in any equilibrium that supports Outcome 1.

Now consider an equilibrium that supports Outcome 3. We may define $m_{2H}(p_2')$ and $A_{2H}(p_2')$ for $p_2' > p^*_{2H}(1)$ by analogy to (C1) and (C2). Further, since $\Pi^*_{2H}(m_{2H}(p_2')) \to \Pi^*_{2H}(1)$ as $p_2' \to p^*_{2H}(1)$, we also have $A_{2H}(p_2') \to \hat{A}_{2L}$ as $p_2' \to p^*_{2H}(1)$, using (C5). Thus there exist $A_2' < \hat{A}_{2L}$, $p_2' \in (p^*_{2H}(1), p^*_{2H}(\hat{m}_2(\hat{A}_1,\hat{A}_{2H})))$ such that $(A_2',p_2)$ is equilibrium dominated for all $p_2 \geq p_2'$ for both Firms 2H and 2L, whereas $(A_2',p^*_{2H}(1))$ is not equilibrium dominated for Firm 2H. Consumers thus infer $p_1 = \hat{p}_1 = p^*_{2H}(\hat{m}_2(\hat{A}_1,\hat{A}_{2H}))$ and $p_2 < p_2'$ upon observing $(\hat{A}_1,A_2')$. Since $\hat{m}_2(\hat{A}_2,A_2') = 1$ is implied, it follows that both Firms 2H and 2L desire to deviate.

It remains to show that an equilibrium with $\hat{A}_{2L}$ given by (C5) and $\hat{m}_1(\hat{A}_1,\hat{A}_{2H}) = \hat{m}_2(\hat{A}_1,\hat{A}_{2L}) = 1$ satisfies the inference restriction. Suppose that $(A_1,p_1)$ with $A_1 > 0$ is not equilibrium dominated for Firm 1. Then for some $m_1,m_1'$:

$$W_1 \leq \rho\Pi_1(p_1,m_1) + (1-\rho)\Pi_1(p_1,m_1') - A_1 - F$$

$$\leq \rho\Pi^*_1(m_1) + (1-\rho)\Pi^*_1(m_1') - A_1 - F \leq \Pi^*_1(1) - A_1 - F$$

so $(A_1, p_1^*(1))$ is not equilibrium dominated. Thus consumers may conjecture $p_1 = p_1^*(1)$ and $p_2 = p_{2L}^*(1)$ upon observing $(A_1, \hat{A}_{2L})$, and the corresponding response $\hat{m}_1(A_1, \hat{A}_{2L}) = 0$ deters deviation by Firm 1. Similarly, consumers may conjecture $p_1 = p_1^*(1)$ and $p_2 = p_{2H}^*(1)$ upon observing $(\hat{A}_1, A_2)$ with $A_2 \in [0, \hat{A}_{2L})$, since $(A_2, p_{2H}^*(1))$ is not equilibrium dominated for Firm 2H. Of course, $A_2 > \hat{A}_{2L}$ is equilibrium dominated in conjunction with any $p_2$ for both types of Firm 2.

Q.E.D.

# APPENDIX D

## Altering the Order of Moves in the Incomplete Information Model

In this appendix we consider in detail the implications of revising the order of moves for the signaling game of Section 4A.

1. Signaling Model with Entry. Suppose first that the game is revised by making Firm 2 an entrant who chooses advertising and price after observing the advertising choice of Firm 1. Firm 2H's equilibrium strategy becomes $(\hat{A}_{2H}(A_1),\hat{p}_{2H}(A_1))$, and the strategy $(A_{2H}(A_1),p_{2H}(A_1))$ is equilibrium dominated according to the modified notion if:

$$\max_{m_2} \Pi_1(p_{2H}(\hat{A}_1),m_2) - A_{2H}(\hat{A}_1) - F < W_{2H}$$

Similarly for Firm 2L. Firm 1's equilibrium strategy is written $(\hat{A}_1,\hat{p}_1)$, and equilibrium dominance continues to be defined as in Section 3B.

Among possible equilibrium outcomes, only Outcome 1 is affected when the signaling game is modified in this way: $\hat{A}_1 > 0$ can now arise in equilibrium, supported by the threat of $\hat{A}_{2H}(A_1) \geq 0$ for $A_1 < \hat{A}_1$. Outcome 2 is unaffected since $\hat{A}_1 = -1$ implies $\hat{A}_{2H}(\hat{A}_1) = \hat{A}_{2L}(\hat{A}_1) = 0$, and Outcome 3 is unaffected since the opportunity for Firms 2H and 2L to react to Firm 1's advertising does not change the scope for punishing Firm 1's deviations, as consumers can in any case punish Firm 1 by conjecturing $p_1 > p_2$.

The Lemma extends directly to this case: there exist $p_2' \in (p_{2L}^*(1),p_1^*(1))$ and $A_2' > 0$ such that all Firm 2H strategies with $A_{2H}(\hat{A}_1) = A_2'$ are equilibrium dominated, Firm 2L strategies with $A_{2L}(\hat{A}_1) = A_2'$, $p_{2L}(\hat{A}_1) \geq p_2'$ are equilibrium dominated, but Firm 2L strategies with $A_{2L}(\hat{A}_1) = A_2'$ and $p_{2L}(\hat{A}_1) = p_{2L}^*(1)$ are not equilibrium dominated. Observing $(\hat{A}_1,A_2')$ then leads consumers to infer $p_1 = \hat{p}_1 \geq p_1^*(1)$ and $p_2 < p_2' < p_1^*(1)$ under the inference restriction. Similar arguments establish that (C5) continues to hold in any equilibrium that satisfies the inference restriction.

Outcome 3 continues to be ruled out by the argument of Proposition 2: there is some $A_2' < \hat{A}_{2L}(\hat{A}_1)$ that pins both $\hat{p}_{2H}(\hat{A}_1)$ and $\hat{p}_{2L}(\hat{A}_1)$ close to $p^*_{2H}(1)$ and $p^*_{2L}(1)$, respectively. Deviating to $A_2'$ allows Firms 2H and 2L to capture the market since $\hat{p}_1 > p^*_{2H}(1)$ in equilibria that support Outcome 3. Further, Outcome 1 continues to survive the inference restriction whenever $\rho$ is large enough to induce entry by Firm 1.

In contrast to the findings of Section 3C, however, we cannot be sure that $\hat{A}_1 = 0$ in equilibria that support Outcome 1. Before further analyzing this case, it will be convenient to establish the following claim: for any $p_1$ and $p_1'$ satisfying $p_1 > p_1' \geq p^*_1(1)$, we have:

(D1) $$\max_{m_1} \Pi_1(p_1,m_1) < \max_{m_1} \Pi_1(p_1',m_1)$$

To see why this is true, note first that if $p_1' \geq p^*_1(0^+)$, then (D1) follows as a simple consequence of quasiconcavity. Suppose that $p_1' < p^*_1(0^+)$, so that $p_1' = p^*_1(m_1')$ for some $m_1'$, and also that (D1) does not hold. For all $m_1 \geq m_1'$ we have:

$$\Pi_1(p_1,m_1) < \Pi_1(p_1',m_1) \leq \max_{m_1} \Pi_1(p_1',m_1) \leq \max_{m_1} \Pi_1(p_1,m_1)$$

where the first inequality follows from quasiconcavity in conjunction with the better deal property, and the last inequality is the negation of (D1). These inequalities imply that there exists $\tilde{m}_1 < m_1'$ such that:

$$\Pi_1(p_1,\tilde{m}_1) = \max_{m_1} \Pi_1(p_1,m_1)$$

but then:

$$\Pi_1^*(\tilde{m}_1) \geq \Pi_1(p_1,\tilde{m}_1) \geq \max_{m_1} \Pi_1(p_1',m_1) \geq \Pi_1^*(m_1')$$

which contradicts the better profit property.

Suppose now that $\hat{A}_1 > 0$ in an equilibrium supporting Outcome 1, and consider an advertising level $A_1' \neq \hat{A}_1$ that induces separation by Firms 2H and 2L, i.e. $\hat{A}_{2H}(A_1') \neq \hat{A}_{2L}(A_1')$ with both nonnegative. In this case, the sorting condition and sequential rationality for Firms 2H and 2L imply $\hat{m}_2(A_1',\hat{A}_{2L}(A_1')) > \hat{m}_2(A_1',\hat{A}_{2H}(A_1'))$, which in turn implies $\hat{m}_2(A_1',\hat{A}_{2H}(A_1')) < 1$ and thus $\hat{p}_{2H}(A_1') > p_{2H}^*(1)$. $\hat{p}_{2H}(A_1')$ may be arbitrarily close to $p_{2H}^*(1)$, however (recall $p_1 = \hat{p}_{2H}(A_1')$ can be conjectured). Thus, to capture the market from Firm 2H, Firm 1 must choose $A_1'$ so as to make prices $p_1 > p_{2H}^*(1)$ equilibrium dominated. The required level of $A_1'$ satisfies:

(D2)
$$A_1' \geq \max_{m_1} \Pi_1(p_{2H}^*(1),m_1) - F - W_1$$

$$= \max_{m_1} \Pi_1(p_{2H}^*(1),m_1) - \rho\Pi_1^*(1) + \hat{A}_1$$

This condition implies that $(A_1',p_1)$ is equilibrium dominated for all $p_1 > p_{2H}^*(1)$, since using the claim we have:

$$\max_{m_1} \Pi_1(p_1,m_1) < \max_{m_1} \Pi_1(p_{2H}^*(1),m_1)$$

If $A_1'$ fails (D2), however, then for some small $\varepsilon > 0$, $(A_1',p_1)$ will not be equilibrium dominated for any $p_1 \in (p_{2H}^*(1),p_{2H}^*(1)+\varepsilon)$. Thus, (D2) is necessary and sufficient for $(A_1',p_1)$ to be equilibrium dominated for every $p_1 > p_{2H}^*(1)$.

When $A_1'$ satisfies (D2), consumers conjecture $p_1 \leq p_{2H}^*(1)$ and $p_2 = \hat{p}_{2H}(A_1') >$

$p^*_{2H}(1)$ upon observing $(A'_1, \hat{A}_{2H}(A'_1))$, so that $\hat{m}_1(A'_1, \hat{A}_{2H}(A'_1)) = 1$ is necessary under the inference restriction. Consumers are always able to conjecture $p_1 = p^*_1(1)$, however, so $\hat{m}_1(A'_1, \hat{A}_{2L}(A'_1)) = 0$ cannot be ruled out. It follows that Firm 1 will desire to deviate if and only if $A'_1 < \hat{A}_1$ for some $A'_1$ satisfying (D2), and thus the following is necessary for elimination of equilibria in which $\hat{A}_1 > 0$:

$$\text{(D3)} \qquad \rho > \frac{\max_{m_1} \Pi_1(p^*_{2H}(1), m_1)}{\Pi^*_1(1)}$$

If (D3) does not hold, then for all $A_1 < \hat{A}_1$ there are strategies $(A_1, p_1)$ with $p_1 > p^*_{2H}(1)$ that are not equilibrium dominated. What happens in this case is that Firm 1 can believe it will capture customers from Firm 2L when it deviates, whereas it does not in the equilibrium, and this expands the range of price choices that can potentially dominate Firm 1's equilibrium payoff.

Next, suppose that $A'_1 \neq \hat{A}_1$ induces pooling by Firms 2H and 2L, i.e. $\hat{A}_{2H}(A'_1) = \hat{A}_{2L}(A'_1) \geq 0$. To capture the market from Firms 2H and 2L, Firm 1 must choose $A'_1$ sufficiently large to make all prices $p_1 \geq p'_1$ equilibrium dominated, where $p'_1 > p^*_1(1)$ and:

$$\rho V(p^*_{2H}(1)) + (1-\rho)V(p^*_{2L}(1)) = V(p'_1)$$

It follows that (3) must fail in order for the required $p'_1$ to exist. The required level of $A'_1$ is now given by:

$$\text{(D4)} \qquad A'_1 > \max_{m_1} \Pi_1(p'_1, m_1) - \rho\Pi^*_1(1) + \hat{A}_1$$

Since deviation now allows Firm 1 to capture the market from both Firms 2H and 2L, Firm 1 will desire to deviate to some $A_1'$ that makes (D4) close enough to equality. Thus, under the inference restriction, equilibria that support Outcome 1 must specify $\hat{A}_1 = 0$ if and only if (D3) holds and (3) is violated. Note that these two conditions are satisfied when $\rho$ is sufficiently close to unity.

Finally, Outcome 2 will survive the inference restriction if Firm 1 cannot profitably capture the market through some positive level of advertising. If $A_1' \geq 0$ leads Firms 2H and 2L to separate, then Firm 1 captures the market from Firm 2H if and only if:

$$\max_{m_1} \Pi_1(p_{2H}^*(1), m_1) - A_1' - F \leq 0$$

The deviation is profitable for Firm 1 if and only if:

$$\rho \Pi_1^*(1) - A_1' - F > 0$$

Again (D3) is necessary to induce deviation by Firm 1. When $A_1'$ leads Firms 2H and 2L to pool, a profitable market-capturing deviation is available to Firm 1 if and only if (3) is violated. It follows that Outcome 2 survives the inference restrictions under the same circumstances as do the Outcome 1 equilibria with $\hat{A}_1 > 0$

2. Signaling Game with Flexible Prices. We now alter the signaling game by allowing the firms to adjust prices after observing one another's advertising levels, as in Section 3D. Equilibrium strategies become $(\hat{A}_i, \hat{p}_i(A_j))$ for i = 1, j = 2 and i = 2H,2L, j = 1. $(A_1, p_1(A_2))$ is equilibrium dominated for Firm 1 if:

$$\rho \max_{m_1} \Pi_1(p_1(\hat{A}_{2H}), m_1) + (1-\rho) \max_{m_1} \Pi_1(p_1(\hat{A}_{2L}), m_1) - A_1 - F < W_1$$

Note that equilibrium dominance in this case reflects both the dependence of Firm 1's price on its rival's advertising level, and the fact that Firm 1 does not know at the outset which type of Firm 2 it is facing. For Firms 2H and 2L, equilibrium dominance continues to be defined as in Section 3D.

Price flexibility adds no new possibilities for equilibrium outcomes, as the scope for consumer punishment of advertising deviations is unaffected. The Lemma continues to hold: Firm 2L can still use its advertising to pin consumers' conjecture of $p_2$ close to $p_{2L}^*(1)$; further, we must have $\hat{m}_2(\hat{A}_1,A_2') < 1$ for the $A_2'$ to which Firm 2L would deviate in the Lemma, else Firm 2L would have deviated there to begin with; thus, by sequential rationality $\hat{p}_1(A_2') \geq p_1^*(1)$ must hold, and $\hat{m}_2(\hat{A}_1,A_2') < 1$ becomes inconsistent with the inference restriction. Similarly (C5) continues to hold.

We can no longer invoke the argument of Proposition 2 to eliminate Outcome 3, however. Consider an equilibrium supporting Outcome 3 in which $\hat{m}_1(\hat{A}_1,A_2) = 1$ for all $A_2 \neq \hat{A}_{2H},\hat{A}_{2L}$, so that $\hat{p}_1(A_2) = p_1^*(1)$ for such $A_2$. Using (C5), it follows that for all $A_2' < \hat{A}_{2L}$, $(A_2',p_{2H}(A_1))$ with $p_{2H}(\hat{A}_1) = p_{2H}^*(1)$ is not equilibrium dominated for Firm 2H, and thus $\hat{m}_1(\hat{A}_1,A_2') = 0$ satisfies the inference restriction. Since neither Firm 2H nor Firm 2L would deviate to $A_2' > \hat{A}_{2L}$, we have that the inference restriction is never sufficient to induce Firms 2H and 2L to overturn equilibria that support Outcome 3.

Now consider the possibility that Firm 1 would overturn the equilibrium by deviating to capture the market from Firm 2H. Let $\hat{m}_1(A_1,\hat{A}_{2H}) = 0$ for $A_1 \neq \hat{A}_1$ in an equilibrium supporting Outcome 3, so that $\hat{p}_{2H}(A_1) = p_{2H}^*(1)$. For Firm 1 to capture the market from Firm 2H, it must choose $A_1'$ so that $(A_1',p_1(A_2))$ is equilibrium dominated whenever $p_1(\hat{A}_{2H}) \geq p_{2H}^*(1)$; a necessary and sufficient condition for this is, using the claim from above:

$$\text{(D5)} \qquad \rho \max_{m_1} \Pi_1(p_{2H}^*(1),m_1) + (1-\rho)\Pi_1^*(1) - A_1' - F < W_1$$

Here it is supposed that Firm 1 makes the most optimistic conjecture of the consumer response should $\hat{A}_{2L}$ be observed. Since Firm 1 will not in fact receive market share when $\hat{A}_{2L}$ is observed, a deviation occurs if:

(D6) $$\rho\Pi_1^*(1) - A_1' - F > W_1$$

Thus for Firm 1's deviation to overturn the equilibrium, it is necessary and sufficient that:

$$\rho \max_{m_1} \Pi_1(p_{2H}^*(1),m_1) + (1-\rho)\Pi_1^*(1) < \rho\Pi_1^*(1)$$

or:

(D7) $$\rho > \frac{\Pi_1^*(1)}{2\Pi_1^*(1) - \max_{m_1} \Pi_1(p_{2H}^*(1),m_1)}$$

We conclude that Outcome 3 is eliminated under the inference restrictions if and only if $\rho$ is sufficiently close to unity.

It is easy to see that Outcome 1 is not ruled out by the inference restriction in the presence of price flexibility, and further that Outcome 2 survives if and only if (3) is satisfied.

3. Signaling Game with Entry and Price Flexibility. We now combine the preceding two alterations of the signaling game: Firm 2 is an entrant that chooses advertising and price after observing $A_1$, and Firm 1 can adjust its price after observing $A_2$. Equilibrium strategies are $(\hat{A}_1,\hat{p}_1(A_2))$ for Firm 1 and $(\hat{A}_i(A_1),\hat{p}_i(A_1))$ for Firm i, i = 2H,2L. Equilibrium dominance is defined as above. The possible equilibrium outcomes are the same as in the case of the signaling model with entry.

As was the case with price flexibility alone, the inference restriction no longer allows us to rule out Outcome 3 based on deviations by Firms 2H and 2L. Further, it becomes more difficult to rule out Outcome 3 based on deviation by Firm 1, since $A_1 \neq \hat{A}_1$ can now lead to pooling by Firms 2H and 2L. Eliminating Outcome 3 now requires that both (D7) holds and (3) fails; these conditions are implied by $\rho$ sufficiently close to unity.

Outcome 1 continues to survive the inference restriction, but it becomes more difficult to eliminate equilibria supporting Outcome 1 that specify $\hat{A}_1 > 0$: when $A_1'$ leads to separation by Firms 2H and 2L, strategies $(A_1', p_1(A_2))$ with $p_1(\hat{A}_{2H}(A_1')) \geq p_{2H}^*(1)$ are equilibrium dominated if and only if (D5) holds with weak inequality, and (D6) is necessary to induce deviation. Thus ruling out $\hat{A}_1 > 0$ requires that (D7) holds and (3) fails, which is harder to satisfy in that (D7) is stronger than (D3). Similarly, Outcome 2 survives the inference restrictions if and only if either (D7) fails or (3) holds.

4. Signaling Game when Firm 1 can Delay its Reaction. Let us now modify the signaling game of Section 4A by giving Firm 1 an added option: it can choose $(A_1, p_1)$ simultaneously with Firms 2H and 2L, or it can elect to delay in order to observe $A_2$ prior to choosing advertising and price. We write Firm 1's equilibrium strategy as $(\hat{A}_1, \hat{p}_1, \hat{A}_1(A_2), \hat{p}_1(A_2))$. $\hat{A}_1 \geq 0$ represents no delay, and in this case $\hat{p}_1 \geq 0$ and $\hat{A}_1(A_2)$ and $\hat{p}_1(A_2)$ are dummy strategies. $\hat{A}_1 = -1$ denotes delay, in which case $\hat{p}_1$ is a dummy strategy and $\hat{A}_1(A_2)$ and $\hat{p}_1(A_2)$ are the advertising and price reactions. Equilibrium strategies are $(\hat{A}_i, \hat{p}_i)$ for Firms i = 2H,2L. The consumers' equilibrium responses are denoted $\hat{m}_i(A_1, A_1', A_2)$, where $A_1$ gives Firm 1's advertising level if it does not delay, and $A_1'$ gives the advertising choice if delay is selected.

The key new feature of this case is that there will always exist an equilibrium supporting Outcome 1, since Firm 1's expected profits become $\rho(\Pi_1^*(1) - F) > 0$ when Firm 1 uses its delay strategy in equilibrium. Note further that Firm 1 will necessarily delay and choose zero advertising in any equilibrium that supports Outcome 1, as $\hat{A}_{2H} = -1$ in these equilibria.

Outcome 2 is affected by the possibility that Firm 1's reaction can be used to punish advertising deviations by Firms 2H and 2L; any advertising levels satisfying $\hat{A}_{2H} = \hat{A}_{2L} \leq$

$\Pi^*_{2H}(1)$ - F can arise in an equilibrium supporting Outcome 2. We also have a strictly larger set of possibilities for equilibria supporting Outcome 3, as Firm 1's delay strategy allows it to incur lower expected entry costs, and thus it can be induced to choose greater levels of advertising in equilibrium.

The Lemma and (C5) continue to hold when the inference restriction is applied to this version of the game, and also Outcome 1 survives the inference restriction. As in the price flexibility case, we can use (C5) to argue that deviations by Firms 2H and 2L cannot be used to overturn Outcome 3 under the inference restriction, since we can specify an equilibrium supporting Outcome 3 in which Firm 1 delays and chooses $\hat{p}_1(A_2) = p^*_1(1)$ for all deviant $A_2$, while $(A_2, p^*_{2H}(1))$ is not equilibrium dominated for Firm 2H for any deviant $A_2$ that might be contemplated. As for Firm 1 deviations, note that Firm 1 can capture the market from Firm 2H by convincing consumers that $p_1 < p^*_{2H}(m_2')$, where $m_2'$ is defined by $p^*_{2H}(m_2') = p^*_1(1-m_2')$. Using the claim, Firm 1 strategies with $A_1 = A_1' \geq 0$ are equilibrium dominated for all $p_1 \geq p^*_{2H}(m_2')$ if and only if:

$$\max_{m_1} \Pi_1(p^*_{2H}(m_2'), m_1) - A_1' - F < W_1$$

and deviation is assured if and only if:

$$\rho\Pi^*_1(1) - A_1' - F > W_1$$

Thus to overturn Outcome 3 it is sufficient that:

(D8) $$\rho > \frac{\max_{m_1} \Pi_1(p^*_{2H}(m_2'), m_1)}{\Pi^*_1(1)}$$

Strategies with $A_1 = -1$ and $A_1(\hat{A}_{2H}) = A_1' \geq 0$ are equilibrium dominated for all $p_1(\hat{A}_{2H}) \geq p^*_{2H}(m_2')$ if and only if:

$$\rho[\max_{m_1} \Pi_1(p^*_{2H}(m_2'),m_1) - A_1'] + (1-\rho)\Pi^*_1(1) - F < W_1$$

where Firm 1 makes the most optimistic conjecture about the consumer response should $\hat{A}_{2L}$ be observed, which is $\hat{m}_1(-1,0,\hat{A}_{2L}) = 1$. Deviation is assured if and only if:

$$\rho(\Pi^*_1(1) - A_1' - F) > W_1$$

Thus Outcome 3 is overturned if:

$$\text{(D9)} \qquad \rho > \frac{\Pi^*_1(1) - F}{2\Pi^*_1(1) - F - \max_{m_1} \Pi_1(p^*_{2H}(m_2'),m_1)}$$

It follows that the inference restriction eliminates Outcome 3 if and only if either (D8) or (D9) hold. Note that (D9) is weaker than (D7) both because of the presence of the entry cost in (D9), which reflects the fact that Firm 1 does not have to incur the entry cost when $\hat{A}_{2L}$ is observed, and because of the higher price which Firm 2H chooses in the market-splitting outcome when it cannot adjust its price in response to Firm 1's advertising. Note further that (D8) is a weaker condition than (D7) if and only if:

$$[\Pi^*_1(1) - \max_{m_1} \Pi_1(p^*_{2H}(m_2'),m_1)]^2 \geq F[\Pi^*_1(1) - \max_{m_1} \Pi_1(p^*_{2H}(m_2'),m_1)]$$

which holds if and only if F is sufficiently small. Thus when F is small, deviations that do not involve delay will overturn Outcome 3 for a larger range of $\rho$ than deviations involving delay, while for large F this ranking is reversed.

Note finally that Outcome 2 survives if and only if (3) holds, as equilibrium dominance for Firm 1 is unaffected by delay when Firms 2H and 2L pool. We can introduce further price flexibility by allowing Firms 2H and 2L to adjust their prices after observing Firm 1's final advertising choice; the only effect of this change is that $p^*_{2H}(m'_2)$ is replaced by $p^*_{2H}(1)$ in (D8) and (D9), which makes it harder to eliminate Outcome 3. We can also allow Firm 1 to adjust its price without delaying its advertising choice. In this case, using strategies with no delay to capture the market from Firm 2H requires a higher level of advertising, as Firm 1 can now choose $\hat{p}_1(\hat{A}_{2L}) = p^*_1(1)$ when it conjectures that it captures the market from Firm 2L. The effect of this is to replace (D8) by the more restrictive (D7) as a sufficient condition for eliminating Outcome 3. In sum, Outcome 3 becomes more difficult to rule out as price flexibility for either firm is increased.

## NOTES

1. See Cady (1976) and Luksetich and Lofgreen (1976) for studies of the retail prescription drug and Minnesota liquor industries, respectively.

2. Among other related papers, Bagwell (1987) and Rogerson (1986) present models in which firms signal low costs in order to convince consumers that a better deal will be provided, and Wolinsky (1984) studies price as a signal of a quality attribute that combines search and experience characteristics

3. Recent empirical studies strongly suggest that declining marginal costs may be quite common in actual industries. V. Ramey (forthcoming) gives empirical evidence of decreasing marginal costs in manufacturing industries, and Brown and Medoff (1990) find that quantity discounts are widespread and important. Further, the U.S. Federal Trade Commission (1980, p. 44) cites evidence that manufacturer quantity discounts are significant in the retail eyeglass industry, and also that management skills and capital borrowing rates improve as scale increases. Finally, Cady (1976) suggests a significant role for quantity discounts in the retail prescription drug industry.

4. If firms follow the naive rule of choosing price as a fixed markup over average cost, then the better deal property emerges at once from the conventional definition of scale economies. Of course, it then becomes important to explain why firms would follow such a naive rule.

5. Our results continue to hold if the quasiconcavity assumption is weakened to the property that for some $z < \Pi_i^*(1)$, the set $\{p_i \mid \Pi_i(p_i,m_i) \geq z\}$ is bounded uniformly in $m_i$.

6. Our results extend immediately to the case of sequential search with $K_2 > 0$ and $K_1$

sufficiently small to induce an initial search. Our model has the feature that consumers direct their search toward the firm that they expect to have the lowest price, which on the equilibrium path is the firm actually charging the lowest price.

7. The consistency criterion requires that observed deviations from the equilibrium strategies are rationalized by positing small "trembles" from the equilibrium strategies, and also that these trembles are independent across players. See Bagwell and Ramey (1989) for a formalization of consistency in games with continuous strategy spaces; this formalization implies the independent price conjectures criterion that we use here.

8. Consumers are able to make such pessimistic conjectures about Firm 2's pricing because the equilibrium concept places no restrictions on the beliefs that an agent may form about another agent's past decisions when the latter agent has been observed to deviate from its equilibrium strategy profile. Such pessimism can also give rise to an equilibrium in which no firm enters; however, since $K_1 = 0$, this equilibrium survives only if consumers play a dominated strategy by visiting no firm when entry occurs. Further, it is straightforward to show that this equilibrium is inconsistent with the inference restriction developed below, even when $K_1 > 0$. We therefore exclude consideration of this equilibrium in the sequel.

9. By contrast, $K_1 = 0$ ensures that the winning firm does not advertise in Outcomes 1 and 2, assuming that consumers do not use dominated strategies. It is straightforward to confirm that this conclusion holds for $K_1 > 0$ when the inference restriction developed below is imposed.

10. This criterion derives from Kohlberg and Mertens' (1986) concept of strategic stability for finite normal form games. Cho and Kreps (1987) have adapted this idea to signaling games with continuous strategy spaces. Our use of equilibrium dominance can be viewed as a direct application of Cho and Kreps' <u>intuitive criterion</u> to a coordination game. Moreover, we follow

Cho and Kreps in motivating our refinement by accompanying the informed player's deviation with a speech.

11. $p_1' < p_2^*(1)$ implies $p_1' < p_1^*(\hat{m}_1)$, since in equilibrium we have $\hat{p}_1 = p_1^*(\hat{m}_1) = \hat{p}_2 = p_2^*(\hat{m}_2) > p_2^*(1)$. Since we also have $p_1' > p_1^*(1)$, there exists $\tilde{m}_1 > \hat{m}_1$ that satisfies $p_1' = p_1^*(\tilde{m}_1)$. Using (1) we have:

$$\Pi_1^*(m_1') - F > \max_{m_1} \Pi_1(p_1', m_1) - F$$

$$\geq \Pi_1^*(\tilde{m}_1) - F > \Pi_1^*(\hat{m}_1) - F = W_1 + \hat{A}_1$$

which gives $A_1' > \hat{A}_1 \geq 0$.

12. It should be noted that Firm 1's entry will be by itself sufficient to eliminate Outcome 2, with no need for positive advertising, if and only if:

$$\max_{m_1} \Pi_1(p_2^*(1), m_1) < \Pi_1^*(m_1')$$

for $m_1' < 1$ satisfying $\Pi_1^*(m_1') = F$; here the setup cost F is sufficiently high that entry automatically "burns" enough profit to convince consumers that $p_1 < p_2^*(1)$. This is no longer possible when F is low, and advertising becomes necessary for overturning Outcome 2. In Outcome 3, however, Firm 1 incurs F under its equilibrium strategy, so that advertising is always necessary for communicating low price.

13. It can be shown that a market-splitting equilibrium survives elimination of strictly dominated strategies if and only if:

$$W_i \geq \Pi_i^*(1) - \underset{m_i}{\mathrm{Max}}\ \Pi_i(p_i^*(\hat{m}_i),m_i)$$

for i = 1,2, while the equilibrium is eliminated if the inequality fails for either i. An identical condition holds with respect to elimination of weakly dominated strategies. Note that some positive-advertising equilibria may be eliminated even if Outcome 3 itself survives.

14. Although elimination of equilibrium dominated strategies is needed for our result, we can do without the requirement of independent price conjectures by eliminating weakly dominated strategies instead. This is because our quasiconcavity assumption means that $(A_2,p_2)$ is weakly dominated by $(A_2,p_2^*(1))$ for every $p_2 < p_2^*(1)$. Thus, under a restriction of no weakly dominated conjectures, consumers would be required to conjecture $p_2 \geq p_2^*(1)$ even if $A_1$ affected their conjectures of $p_2$, and they would continue to visit Firm 1 after observing $(A_1',\hat{A}_2)$.

15. Mixed strategies may be considered if we allow for random $m_i$ in the definition of the inference restriction. To extend Proposition 1 to allow for mixed strategies, note first that under the inference restriction, Firm 1 must capture the market with probability one when it chooses the advertising level at the top of the support of its mixed advertising strategy, else it could profitably capture the market with some upward advertising deviation. Then an advertising level slightly below the top of the support must also capture the market, as a consequence of either the equilibrium conditions or the inference restriction. Thus Firm 1 does not choose a mixed advertising strategy under the inference restriction, and further its equilibrium advertising choice must capture the market with probability one.

16. $A_1 < \hat{A}_1$ might also lead to market-splitting if consumers conjecture $p_1 = p_2^*(m_2')$, where

$m_2'$ is the share of the market that defects to Firm 2 when $A_1$ is observed. Moreover, Firm 2's entry decision may involve $A_2 > 0$ if consumer conjectures make this necessary. The arguments of the next two paragraphs deal equally well with these additional possibilities for off-equilibrium-path behavior.

17. In particular, we evaluate equilibrium dominance based on the assumption that Firm 2 expects Firm 1 to follow its equilibrium strategy $\hat{A}_1 = 0$, i.e. $(A_2(A_1),p_2(A_1))$ is equilibrium dominated if:

$$\underset{m_2}{\mathrm{Max}}\ \Pi_2(p_2(0),m_2) - A_2(0) - F < W_2$$

See Section 3D for discussion of this modified definition. Of course, in this section our result requires only that $(p_2(0),A_2(0)) = (p_2^*(1),A_2)$ is <u>not</u> equilibrium dominated, so that we do not actually make use of the added restrictions implied by the modified concept.

18. Eliminating equilibrium dominated strategies according to this notion of equilibrium dominance is implied by the <u>elimination of never weak best responses</u> criterion (Kohlberg and Mertens (1986), Cho and Kreps (1987)). In our context, a strategy is a never weak best response if it gives the player a payoff strictly lower that his equilibrium payoff in all Nash equilibria that give rise to the same equilibrium outcome. In particular, in all of these equilibria we must have $A_j = \hat{A}_j$, but consumer visitation responses to $(A_i,\hat{A}_j)$ for $A_i \neq \hat{A}_i$ may take on only the subset of [0,1] that could appear in Nash equilibria, i.e. that could deter deviation by Firm 1. Our concept is a bit weaker in that we allow any consumer response to be entertained in assessing equilibrium dominance. Our results continue to hold when we apply elimination of never weak best responses in place of the modified equilibrium dominance, except as indicated in note 19; similar comments apply with respect to our

incomplete information results.

19. Under the modified notion of equilibrium dominance we may also extend the results of Section 3C to allow Firms 1 and 2 to adjust prices simultaneously after Firm 2 has made its entry and advertising decisions. In this case, equilibrium dominance is invoked to rule out strategies $(A_1', p_1(A_2))$ with $p_1(\hat{A}_2(A_1')) \geq p_1'$, i.e. Firm 1 anticipates that Firm 2 will choose its equilibrium advertising reaction to $A_1'$. The case of entry with price flexibility is the one instance where elimination of never weak best responses would not give our results, since the latter criterion fixes only the equilibrium outcome and not the particular response $\hat{A}_2(A_1')$ that Firm 2 would choose off the equilibrium path. We allow Firm 1 to make its message contingent on anticipation of this Firm 2 response, which resolves the strategic ambiguity concerning which price response Firm 1 was trying to communicate, but equilibrium dominance is still the criterion by which the credibility of the message is assessed.

20. The modified notion of equilibrium dominance that we introduce here does not allow the incumbent firm in Bagwell and Ramey (1990) to avoid the entrant's knockout strategy in the reduced game obtained by elimination of weakly dominated strategies, since in the reduced game the entrant can choose only large quantities in conjunction with a large capacity, and all of Firm 1's quantity responses then become equilibrium dominated in any postentry subgame in which knockout is the unique Nash equilibrium. Incidentally, neither does elimination of never weak best responses allow the incumbent to communicate large quantity, for reasons precisely analogous to the incumbent's inability to exploit elimination of weakly dominated strategies, as discussed in that paper.

21. Ben-Porath and Dekel (1988) have developed a related analysis of signaling strategy and structure through payoff dissipation.

22. As above, we exclude from consideration the equilibrium in which no entry occurs.

23. In the framework of Section 2, let the cost function be $C(q,\alpha)$, where $C_{qq} < 0$ and $C_{q\alpha} > 0$. The sorting condition holds when Firm 2's types are identified with values $\alpha_H$ and $\alpha_L$ of $\alpha$, with $\alpha_H > \alpha_L$.

24. The nonexistence problem disappears if Firm 1 is given the opportunity to delay its decisions until after it observes Firm 2's advertising choice, since delaying gives Firm 1 expected profits of $\rho(\Pi_1^*(1) - F) > 0$ in equilibria supporting Outcome 1. A detailed analysis of this case is given in Appendix D.

25. Allowing for mixed strategies requires that the sorting condition be strengthened as follows: for all $p_2$ and all distribution functions $G(m_2)$ that are not degenerate at $m_2 = 1$:

$$\Pi_{2H}^*(1) - \int_0^1 \Pi_{2H}(p_2,m_2)dG(m_2) < \Pi_{2L}^*(1) - \int_0^1 \Pi_{2L}(p_2,m_2)dG(m_2)$$

Then by analogy to note 15, Firm 2L must capture the market with probability one when it chooses the advertising level at the top of the support of its mixed advertising strategy, since otherwise there would be some upward advertising deviation that would be equilibrium dominated for all $p_2$ for Firm 2H, and would allow Firm 2L to profitably capture the market. Thus the Lemma continues to hold, and further Firm 2L must choose a pure strategy under the inference restriction. The condition $\hat{A}_{2L} = \Pi_{2H}^*(1) - F + W_{2H}$ for separating equilibria and Proposition 3 extend directly. The inference restriction does not, however, rule out mixed-strategy equilibria in which Firms 1 and 2H split the market with positive probability. A sufficient condition for eliminating all such equilibria is:

$$\rho > \frac{\Pi_1^*(1)}{2\Pi_1^*(1) - \max_{m_1} \Pi_1(p_{2H}^*(1), m_1)}$$

In this case Firm 1 will deviate based on reasoning similar to that discussed in Section 4C; see Appendix D for explanation of this condition. Essentially, a firm will choose to overturn a mixed equilibrium when it does not capture the market with probability one and when it is sufficiently confident that it is most efficient.

26. Appendix D also considers the combined case of entry and price flexibility, which leads to similar conclusions.

27. In particular, the better profit property holds if C exhibits declining ray average costs; properties of multiple-output cost functions are discussed in Sharkey (1982, ch. 4). If the firm is allowed to ration supplies of s, rather than being required to meet demand, then we have $\Pi^*$ strictly increasing in m without the added assumption on the cost function.

28. Appendix D gives a formal proof.

29. This condition holds if, for example, the goods are substitutes, the marginal cost of q is strongly decreasing in q, and cost complementarities are slight.

30. This holds if $C_{qv} >> 0$, since in this case higher p significantly reduces the marginal cost of quality by reducing sales. Further, $v^*$ is strictly increasing in p if $C(q,v) = c(v)q$ and $Q_{vp} > -\varepsilon$ for small $\varepsilon > 0$. The fact that price cuts reduce the incentive to provide quality when quality is a search attribute has been noted by Wolinsky (1984).

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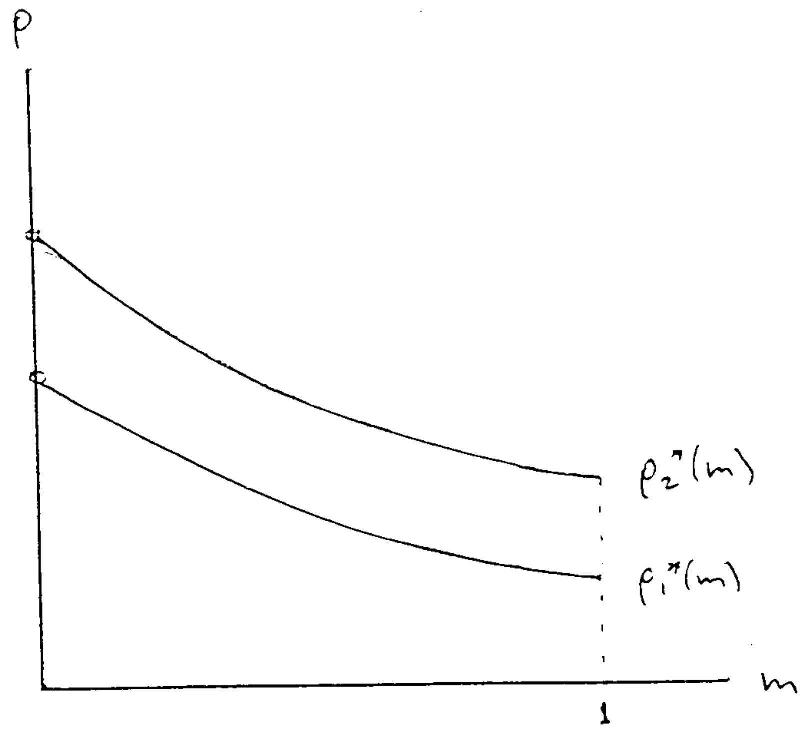


Figure 1

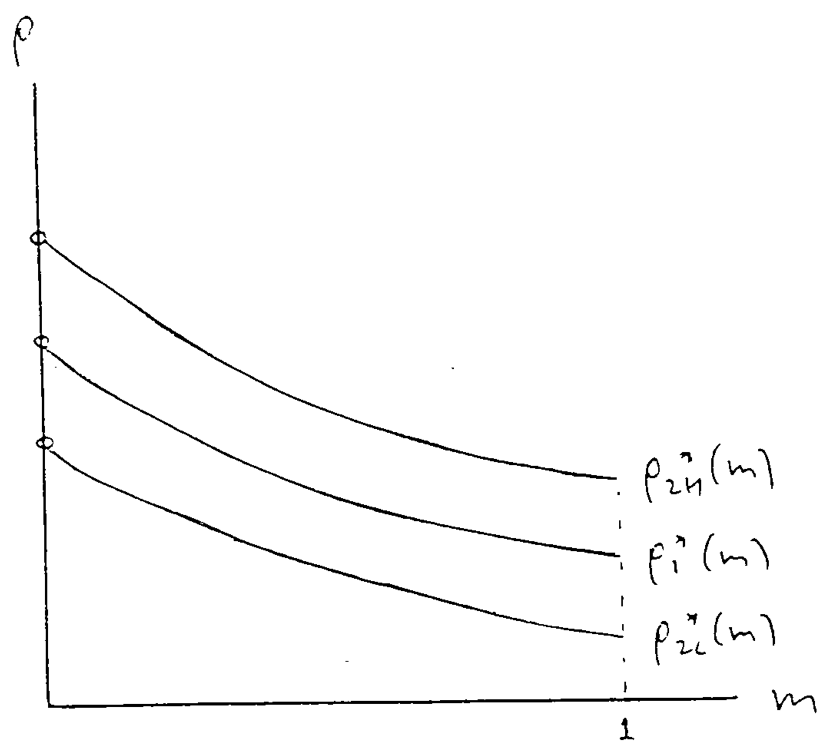


Figure 2