Discussion Paper No. 900

LIQUIDITY, LOANABLE FUNDS, AND REAL ACTIVITY*

by

Timothy S. Fuerst

J. L. Kellogg Graduate School of Management

Northwestern University

April 17, 1990

Revised: August 1990

Abstract: This paper develops a general equilibrium model of two traditional explanations of the monetary "black box" linking money and real activity: the liquidity effect and the loanable funds effect. These effects are modeled with a monetary production economy in which central bank injections of cash are funnelled into the economy though the credit market. As a result, only borrowers have direct access to the newly injected cash. The model has several interesting implications: 1) monetary injections cause fluctuations in asset prices for non-Fisherian reasons, 2) monetary injections increase current and future real activity, and, 3) the central bank has the ability to dampen or magnify fluctuations in real activity.

* This paper is drawn from my doctoral dissertation at The University of Chicago. I have received helpful comments from Andrew Atkeson, John Cochrane, Kyoo Kim, Robert E. Lucas, Jr., Nancy Stokey, and Michael Woodford.

1. INTRODUCTION.

This paper develops a general equilibrium model that tries to capture two traditional explanations of the monetary "black box" linking money and real activity: the liquidity effect and the loanable funds effect. I use these two terms in exactly the same fashion as Friedman and Schwartz (1982):

> As excess cash accumulates [after an unexpected increase in the monetary growth rate], holders of cash, finding the composition of their portfolios disturbed, will try to adjust the portfolios by replacing cash with other assets (including both securities and physical assets). In the process they will bid up the price of other assets and force down the rate of interest. This is the pure liquidity effect....
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> Where does the extra money come from? How is it put in circulation? In the present financial system, it is natural to view the increased rate of monetary growth as coming through the banking system, via larger open market purchases by the central bank, which add to reserves of commercial banks, inducing them to expand more rapidly their loans and investments. In the first instance, therefore, the higher rate of monetary growth appears to take the form of an increase in the flow supply of loanable funds. Taken by itself,...the increase in the supply of loanable funds would produce a once-for-all drop in the interest rate....
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> As time passes,...[the liquidity and loanable funds effects]...will be superseded by more basic effects. Even though unanticipated, the rising money balances and initially lower interest rates will stimulate spending, [and] the spending will affect both prices and output [pp. 482-483, 485-486].

The model developed here is directly motivated by a question of Friedman and Schwartz: "where does the extra money come from, how is it put in circulation?" It is an institutional fact that monetary injections occur through financial intermediaries. These intermediaries thus serve two functions: they intermediate between savers and borrowers, and they act as the funnel through which central bank cash injections occur. This paper explores how this second function may effect the first. The loanable funds available to intermediaries consists of previously deposited balances by savers, and the newly injected currency. Assume that it is prohibitively costly for savers to change their investment decisions after each monetary injection. Then the supply of loanable funds will be determined by the current injection, and only borrowers will have direct access to this new cash.[1] In a world with no uncertainty, this arrangement is unimportant: savers will base their investment decisions on the perfectly foreseen monetary injection and the perfectly foreseen loan demand of borrowers. However, in a world with uncertain monetary injections and uncertain loan demand, the savers' inability to alter their investment decisions ex post will have real consequences. Cash injections larger than average (or loan demand smaller than average) will produce a "loose" financial market, while cash injections smaller than average (or loan demand larger than average) will produce a "tight" financial market, where "loose" and "tight" are measured relative to a world where savers can ex post alter their savings decisions.

Consider the real effects of a larger-than-anticipated monetary injection. Such an injection will leave the borrowers more liquid than anticipated, and redistribute purchasing power in their favor. This

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[1]This arbitrary division of the population between savers and borrowers is made for simplicity, not realism. What I have in mind is not a division of people, but a division of goods: one class of goods is purchased with the help of intermediaries, while a second class of goods is not. With this interpretation, I am assuming that central bank cash injections can be used only for the purchase of a certain class of goods.

redistribution will have consequences for both asset prices and real activity. As for the former, the borrowers will bid up asset prices and decrease nominal interest rates. As for real activity, since the asymmetric injection leaves the borrowers relatively cash rich, it stimulates real demand for the goods and services they purchase. Just the opposite is true for the non-borrowers. Hence, the injection will alter the <u>composition</u> of current output, shifting it towards the goods and services the borrowers consume. These compositional effects can produce a wide variety of results depending on the preferences of the borrowers and the non-borrowers (or, following the interpretation given in footnote 1, depending on which goods are purchased with the help of financial intermediaries). The goal of this paper is to illustrate one of these possibilities in a standard real business cycle model modified to include a particular financial structure.

The paper proceeds as follows. Section 2 compares the paper to other work and discusses the general methodology used here. The model used throughout the remainder of the paper is developed in Section 3. The economic environment is described, and an equilibrium is characterized. This Section sets the stage for Sections 5, 6, and 7, that explore the real effects of monetary injections. I take a slight detour in Section 4 to explore asset pricing in this economy with liquidity and loanable funds effects. Section 8 concludes the paper, and provides a discussion of optimal monetary policy.

2. LITERATURE REVIEW AND GENERAL METHODOLOGY.

The liquidity and loanable funds effects have a fairly long history in monetary economics. However, only recently have these effects been captured in a model of economic equilibrium. This is the seminal contribution of Grossman and Weiss (1983) and Rotemberg (1984). These authors analyzed cash-in-advance models in which agents return to the bank to replenish their cash balances every two periods. The trips are staggered such that half of the population is at the bank at any one time. A central bank injection distributes new currency only to those agents at the bank. This asymmetry produces real effects. In Grossman and Weiss's pure endowment economy, the cash injection results in a lower real rate of interest and a change in the share of output eaten by the two halves of the population. Rotemberg analyzes a production economy with capital. Here, the monetary injection lowers the real rate of interest, and increases next period's capital stock and level of output.

The models developed in Scheinkman and Weiss (1986), and Kehoe, Levine, and Woodford (1988), capture similar effects of monetary injections. In these models, all agents receive the new cash injection. However, agents differ in their pre-injection cash balances so that the new cash redistributes wealth from cash-rich agents to cash-poor agents through inflation tax effects.

The key to the transmission mechanism in all of these models is the asymmetry of the monetary injection. Unfortunately, this asymmetry also greatly complicates the analysis. Each asymmetric injection produces a wealth redistribution that will persist forever. To keep track of the evolving wealth distribution within the economy is an impossible task. I view these wealth movements as a nuisance. They are likely empirically unimportant (consider the relative sizes of a typical open-market operation and the stock of aggregate wealth), and are much different from the short term liquidity effects that I am trying to model. One way around this "wealth problem" is to restrict the analysis to a single unexpected monetary injection that perturbs the initial equilibrium. The injection will produce a one time unexpected wealth redistribution, but thereafter the distribution of wealth will be unaffected by the deterministic monetary policy. This is the route taken in the models developed by Grossman and Weiss (1983), Rotemberg (1984), and Scheinkman and Weiss (1986). A second way to deal with this problem is to restrict the discussion to a special class of equilibria in which these wealth effects are tractable. This is the route taken by Kehoe, Levine, and Woodford (1988). These authors confine their analysis to two-state Markov equilibria in which the two states are defined by the cash position of the two types of agents within the economy. A shortcoming of this approach is that the set of monetary policies considered must be seriously restricted in order for a two-state equilibrium to exist.

The common problem in both of these earlier approaches is the small set of monetary policies that can be analyzed. One would like to analyze the welfare consequences of arbitrary stochastic monetary policies. To allow for this generality, I eliminate these problematic wealth effects entirely by using a methodology suggested by Lucas (1990). The trick is to lump all the sectors of the economy into one gigantic household. Each sector uses cash to carry out transactions in spatially separated markets, and is subject to shifts in within-period liquidity resulting from monetary injections. However, at the end of each period, this large household reunites and pools all cash so that no wealth innovations result from monetary injections. The usefulness of the Lucas (1990) methodology is that we can model cash injections that are asymmetric within the family, but symmetric across households. Hence, we can capture the liquidity and loanable funds effects in an identical household environment that can be subjected to a rich class of stochastic monetary policies.

This methodology is not without cost. By entirely eliminating these wealth effects, the model loses the persistent and lingering effects of a monetary injection captured, for example, in Grossman and Weiss (1983). The persistence in these models derives from the private economy's inability to "undo" the effects of a monetary injection - its inability to ensure itself against the wealth risk of monetary injections. By using this methodology, I am implicitly assuming that the private economy can eventually "undo" any central bank action - that this wealth risk is diversifiable. There are at least two distinct advantages to this methodology. First, and foremost, we will be able to analyze the welfare consequences of a wide variety of stochastic monetary policies. Secondly, this methodology allows us to isolate the liquidity and loanable funds effects: the non-neutralities discussed in this paper have absolutely nothing to do with wealth effects - all are a result of what I call liquidity or loanable funds effects.

3. THE BASIC MODEL.

The economy consists of infinitely-lived identical families with preferences over uncertain consumption and leisure streams given by

$$E\left\{ \sum_{t=0}^{\infty} \beta^{t} \left(U(c_t) + V(1-L_t)\right) \right\}$$

where $U(x) = \log(x)$, $V(x) = x$, c_t and L_t are time t consumption and work effort, respectively, and the leisure endowment is normalized to one. The expectation is over realizations of the economy-wide shocks. The k-vector $s_t \in S \subset R^k$ denotes the state of the world at time t, where S is a compact set. These states are iid across time with a distribution function given by Φ.

Along with these families, the economy contains numerous identical firms. Each is initially endowed with $k_0 > 0$ goods and has access to a stochastic production technology. Specifically, if the firm "plants" k_t goods during period t-1, and hires H_t units of labor during period t, $f(k_t,H_t,s_t)$ homogeneous goods are harvested and available for sale during period t. Capital and consumption goods are identical, so, for simplicity, I assume 100% depreciation of the capital stock. The production function will take the standard Cobb-Douglas form: $f(k_t,H_t,s_t) = \theta(s_t)k_t^{1-\alpha}H_t^{\alpha}$, where $\alpha \in (0,1)$, and $\theta:S\rightarrow R$ is continuous and strictly positive.

To complete a description of the economy, I need to specify the manner in which trade is carried out. Each family consists of a worker/shopper pair. Workers sell their labor services to firms, and shoppers purchase consumption goods from the firms. Firms alone have access to the production technology. Using their stock of capital and the labor services they purchase from workers, the firms produce contemporaneous output. This output is then sold in a goods market to shoppers (for consumption) and other firms (for investment). To introduce money into the economy, I will impose cash-in-advance constraints on all of these transactions, ie., shoppers and firms must use cash to finance all their purchases. To help finance the wage bill and capital accumulation, firms can borrow cash from financial intermediaries. These intermediaries have two sources of cash: cash deposited by families and currency injected by the central bank. To make an analysis of this financial structure tractable, I will follow the methodology outlined in the previous section, and lump the entire economy into gigantic households consisting of four members: a shopper, a worker, a firm, and a financial intermediary. In effect, each household is an entire economy. To introduce notation, I will trace the behavior of one of these gigantic households through a period. The household begins a period with M_t dollars. N_t dollars are deposited in the intermediary. After this deposit is made, the household separates. The shopper travels to the goods market, the worker travels to the labor market, while the intermediary and firm both travel to a financial market. Once separated, the state of the world (s_t) is revealed to all. This state determines the current productivity shock affecting the firm (θ_t), and the current monetary injection (X_t) given to each of the financial intermediaries. The representative intermediary now has $N_t + X_t$ dollars to lend out. The only borrower in the financial market is the representative firm which is seeking cash to finance current activity. With a moment's reflection, it is clear that only one trade will occur in equilibrium: the intermediary will loan all of its cash out to the firm. These loans will come due at the end of the current period. With its newly acquired cash, the firm travels to the labor market, and hire workers at the wage W_t. The firm then ships current production to the goods market for sale at the price P_t. After the goods are shipped, the firm travels to the goods market with the remainder of its borrowed cash to purchase capital goods for next period's production (k_{t+1}). This discussion motivates the following cash-in-advance constraints:

$$M_t - N_t \geq P_t c_t \tag{3.1}$$

$$N_t + X_t \geq P_t k_{t+1} + W_t H_t \tag{3.2}$$

The first constraint applies to the shopper, while the second applies to the firm. Notice that the firm is required to purchase capital from other firms, and cannot use contemporaneous revenues to fund contemporaneous operations. Since output is either consumed or invested, (3.1) and (3.2) imply that the firm and the shopper are competing for their share of contemporaneous output. Before the household is reunited, the firm places its purchases into the production technology - they are not available for current consumption. At the end of the period, the household is reunited, loan repayments are made, all remaining cash is pooled, and the goods purchased by the shopper are consumed. The cash balances that the household ends the period with are therefore given by[2]

$$M_{t+1} = M_t + X_t + W_t L_t + P_t f(k_t, H_t, s_t) - W_t H_t - P_t k_{t+1} - P_t c_t$$

I will restrict my discussion to stationary rational expectations equilibria in which the prices and decision rules are fixed functions of the state of the system. With a growing money supply, nominal prices will clearly not be stationary. To avoid this problem, I will measure all nominal variables relative to the beginning of period per capita money stock, and let lower case letters represent these rescaled quantities. In particular, this implies that x is the money growth rate. I will assume that $x: S \to R$ is continuous and non-negative.

We can now formulate the representative household's problem. Let (m, k, κ) denote the household's beginning of period cash balances, the household's beginning of period capital stock, and the economy-wide per capita capital stock. Primes will denote next period values. Let $J(m, k, \kappa)$ represent the value function corresponding to the household's problem. Then J satisfies the functional equation

$$J(m,k,\kappa) = \max_{n \in [0,m]} \int \max_{c,L,H,k'} \left\{ U(c) + V(1-L) + \beta J(m', k', \kappa') \right\} \Phi(ds')$$

[2]Note that in equilibrium, the loan repayment completely drops out of this condition because the household is in effect borrowing from itself. Rather than complicating the model by adding notation for out-of-equilibrium borrowing and lending, I will instead directly impose financial market equilibrium at this point. In Section 4, I introduce notation of this type to calculate the equilibrium prices of arbitrary financial securities.

where the maximization is subject to the following revised versions of (3.1) and (3.2),

$$m - n \geq pc \tag{3.3}$$

$$n + x \geq pk' + wH \tag{3.4}$$

and m' is given by

$$m' = \frac{m + x + wL + pf(k,H,s') - wH - pk' - pc}{1 + x}$$

In equilibrium, $L = H$, $m' = 1$, and $k = \kappa$. Hence, I will drop the m's and κ's from all subsequent notation. Let λ_1 and λ_2 denote the multipliers associated with (3.3) and (3.4), respectively. Then, the first order conditions for the consumer's problem, evaluated at equilibrium, are[3]

$$\int \lambda_1(k,s')\ \Phi(ds') = \int \lambda_2(k,s')\ \Phi(ds') \tag{3.5}$$

$$U'(c(k,s')) = p(k,s')\left\{ \lambda_1(k,s') + \beta J_m(k')/[1+x(s')] \right\} \tag{3.6}$$

$$V'(1-L(k,s')) = \beta J_m(k')w(k,s')/[1+x(s')] \tag{3.7}$$

$$p(k,s')f_L(k,L,s')\beta J_m(k')/[1+x(s')] = w(k,s')\left\{\lambda_2(k,s') + \beta J_m(k')/[1+x(s')]\right\} \tag{3.8}$$

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[3] I assume that the choices of n and L are interior. This will be satisfied for all the equilibria considered below.

$$\beta J_k(k') = p(k,s')\left\{ \lambda_2(k,s') + \beta J_m(k')/[1+x(s')] \right\} \tag{3.9}$$

$$1 - n(k) \geq p(k,s')c(k,s'), \text{ with equality if } \lambda_1(k,s') > 0 \tag{3.10}$$

$$n(k) + x(s') \geq p(k,s')k'(k,s') + w(k,s')L(k,s'),$$
$$\text{with equality if } \lambda_2(k,s') > 0 \tag{3.11}$$

The envelope conditions are

$$J_m(k) = \int \frac{U'(c(k,s'))}{p(k,s')} \Phi(ds') \tag{3.12}$$

$$J_k(k) = \beta \int \left\{ p(k,s')f_k(k,L,s')J_m(k')/[1+x(s')] \right\} \Phi(ds') \tag{3.13}$$

Combining (3.9) and (3.13) we have

$$\beta^2 \iint \left\{ p(k',s'')f_k(k',L,s'')J_m(k'')/[1+x(s'')] \right\} \Phi(ds'') =$$
$$p(k,s')\left\{ \lambda_2(k,s') + \beta J_m(k')/[1+x(s')] \right\} \tag{3.14}$$

Equations (3.5)-(3.8), (3.10)-(3.12), and (3.14), represent eight restrictions on the nine unknown functions n, c, L, k', J_m, λ_1, λ_2, p and w. The ninth restriction comes from the goods market clearing condition:

$$c(k,s') + k'(k,s') = f(k,L,s') \tag{3.15}$$

An equilibrium is defined by a set of nine non-negative continuous functions, with p and w strictly positive, and n and L bounded between zero and one, that solve this system of equations.

I will follow three routes in analyzing the equilibria of this economy. First, I will consider an economy in which the capital stock is fixed (Section 5). Secondly, I will turn to the opposite extreme and assume that labor is supplied inelastically (Section 6). With either of these assumptions, one can directly construct a competitive equilibrium, and analyze its properties. For the more general case of a variable capital stock and non-trivial labor decision, I will resort to numerical methods (Section 7). I will numerically calculate a competitive equilibrium of this economy, and see how this equilibrium changes under different productivity parameters. Before beginning any of these routes, I will take a slight detour and consider the behavior of asset prices in this loanable funds setting.

4. ASSET PRICING.

In this identical household world, the only asset traded in equilibrium is the one period bond issued by the firm to the intermediary. In particular, resource allocation is unaffected by the introduction of other financial securities. However, the equilibrium allocations implicitly determine the prices of all these securities. I will use this fact to derive an expression for the equilibrium price of any asset.

As argued earlier, in equilibrium the typical firm manager will raise $N_t + X_t$ units of currency by issuing one period bonds to the financial intermediary. The firm uses this cash to finance current production and capital acquisition. Suppose that the firm can also use this cash to purchase (or sell) a wide variety of securities in the financial market. These securities can only be traded in the financial market, but dividends and coupon payments are distributed directly to the household at the end of each period. For now, imagine that only one such security exists and let e denote the number of shares of this security that the firm owns when it enters the financial market. Let R_t denote the security's time t dollar price, Π_t the security's nominal payoff at the end of the period, and e′ the new position in the security. r and π will represent the rescaled versions of R_t and Π_t, ie., r and π are measured relative to the beginning of period per capita money stock.[4] The constraints facing the typical household can now be expressed as:

$$m - n \geq pc$$

$$n + x + er \geq wH + pk' + e'r$$

$$m' = \frac{m + x + wL + pf(k,H,s') - wH - pk' - pc + r[e-e'] + e'\pi}{1 + x}$$

Let $J(m,k,\kappa,e,s)$ denote the value function for a household beginning a

[4]By not subscripting r and π, I am implicitly restricting my discussion to securities with normalized payoff streams that are stationary. (For example, bonds with finite maturity are omitted.) However, it should be clear that the pricing equation derived below (equation (4.5)) also applies to these non-stationary assets.

period with cash balances m, capital stock k, and shares e, when the per capita capital stock is κ, and the state of the world is s.[5] In equilibrium, $m = m' = 1$, $k = \kappa$, and $e = e' = 0$. For simplicity, I omit all these variables in what follows. The first order condition for the choice of e', evaluated at equilibrium, is:

$$\beta\left\{\frac{J_m(s')}{1+x(s')}\pi(s') + J_e(s')\right\} = r(s,s')\left\{\lambda_2(s,s') + \beta\frac{J_m(s')}{1+x(s')}\right\} \tag{4.1}$$

The envelope condition for e is:

$$J_e(s) = \int r(s,s')\left\{\lambda_2(s,s') + \beta\frac{J_m(s')}{1+x(s')}\right\}\Phi(s,ds') \tag{4.2}$$

Combining (4.1) and (4.2) we have:

$$\beta\frac{J_m(s')}{1+x(s')}\pi(s') + \beta\int r(s',s'')\left\{\lambda_2(s',s'') + \beta\frac{J_m(s'')}{1+x(s'')}\right\}\Phi(s',ds'')$$
$$= r(s,s')\left\{\lambda_2(s,s') + \beta\frac{J_m(s')}{1+x(s')}\right\} \tag{4.3}$$

This is a difference equation in r. Reintroducing time subscripts, and solving (4.3) forward, we obtain:

$$r_t = \frac{1}{\lambda_{2t} + \beta J_m(s_t)/[1+x_t]} E\left\{\sum_{\tau=0}^{\infty}\beta^{\tau}\pi_{t+\tau}\frac{\beta J_m(s_{t+\tau})}{1 + x_{t+\tau}} \,|s_t\right\} \tag{4.4}$$

where $E(\cdot|s_t)$ denotes the expectation conditional on s_t. We will now

[5] It is fairly simple to incorporate serially correlated shocks into this Section, so I have done so.

simplify this expression so that it is comparable to the standard asset pricing framework.[6] First, recall that the envelope condition for m is $J_m(s_t)=E\left\{U'(c_{t+1})/p_{t+1} | s_t\right\}$. Next note that $\pi_{t+\tau}$ and $[1+x_{t+\tau}]p_{t+\tau+1}$ are both measured relative to the beginning of period $t+\tau$ per capita money stock, ie., $\left\{\pi_{t+\tau} \;/\; [1+x_{t+\tau}]p_{t+\tau+1}\right\} = \Pi_{t+\tau}/P_{t+\tau+1}$. Finally, the first order condition for the choice of c (equation (3.6)) implies that $U'(c)/p=\lambda_1+\beta J_m/[1+x]$. Using these facts, we have:

$$R_t = \frac{1}{\Lambda_t + U'(c_t)/P_t} E\left\{ \sum_{\tau=0}^{\infty} \beta^{\tau+1} \Pi_{t+\tau} U'(c_{t+\tau+1})/P_{t+\tau+1} | s_t \right\} \quad (4.5)$$

where $\Lambda_t \equiv [\lambda_{2t}-\lambda_{1t}]/M_t^s$, and M_t^s denotes the beginning of time t per capita money stock. Equation (4.5) is the major result of this section. Absent the term Λ_t, this expression is the monetary version of the standard asset pricing formula.

What is Λ_t? I call it the "liquidity effect". Liquidity premiums arise when cash in one market is more valuable than cash in another market. The value of cash in the goods market is measured by λ_1, while λ_2 measures the value of cash in the financial market. Hence, one would expect that when liquidity effects are absent (ie., when $\lambda_{1t} = \lambda_{2t}$), the asset price should be determined by the Fisherian fundamentals. This is exactly what occurs as is obvious from (4.5). Similarly, when the financial market is relatively liquid (ie., $\lambda_{2t} < \lambda_{1t}$ and $\Lambda_t < 0$), the

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[6] See, for example, Sargent (1987), Chapter 3.

asset price is high relative to Fisherian fundamentals, and when the financial market is relatively tight (ie., $\lambda_{2t} > \lambda_{1t}$ and $\Lambda_t > 0$), the asset price is low relative to the Fisherian fundamentals. Note that <u>all</u> assets traded in the financial market are influenced by these liquidity effects.

One asset of particular interest is a one period nominal bond. Let Q_t denote the time t price of a claim to one dollar at the end of time t. (4.5) yields the following expression for this bond price:[7]

$$Q_t = \frac{\beta E\left\{U'(c_{t+1})/P_{t+1} \mid s_t\right\}}{\Lambda_t + U'(c_t)/P_t} \tag{4.6}$$

Do monetary injections raise bond prices (and lower interest rates) via a liquidity effect, or lower bond prices (and raise interest rates) via an inflation effect? This is an empirical question that cannot be answered a priori. One criticism of standard Fisherian interest rate determination is that this ambiguity is entirely absent: the inflation effect lowers bond prices. In the present model, the ambiguity remains as is evident from (4.6): injections tend to lower Λ_t, but also tend to increase P_{t+1}. As a final comment, note that we can use (4.6) to derive an alternative expression for Λ_t:

$$\Lambda_t = [1/Q_t]\ \beta E\left\{U'(c_{t+1})/P_{t+1} \mid s_t\right\} - U'(c_t)/P_t \tag{4.7}$$

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[7]Note the relationship between Q_t and λ_{2t}. From (4.4), $Q_t < 1$ if and only if $\lambda_{2t} > 0$: Bonds yield a positive return if and only if cash is scarce in the financial market.

An additional dollar in the financial market will buy $1/Q_t$ bonds. Each bond will yield one dollar that will be available for next period's purchases. Hence, the value of a marginal dollar in the financial market is equal to the first term in (4.7). Similarly, the second term is the value of a marginal dollar in the goods market for it can immediately yield $1/P_t$ goods.

5. THE MODEL WITH A FIXED CAPITAL STOCK.

We begin our analysis of the model developed in Section 3 by demonstrating the existence of an equilibrium in the case of a fixed capital stock:

Proposition 5.1: Assume that the capital stock is fixed at unity, and that the money growth process satisfies the following two conditions:

$$1)\quad \int \left\{ \alpha\beta/(x(s')[1+x(s')]) \right\} \Phi(ds') > 1, \text{ and,}$$

$$2)\quad \int \left\{ \frac{\alpha\beta[1+\bar{x}]}{[1+x(s')](\alpha[1+x(s')] + x - \bar{x})} \right\} \Phi(ds') < 1,$$

where $\bar{x} \equiv \max(x(s))$. Then there exists a competitive equilibrium where $\lambda_1 > 0$ and $\lambda_2 > 0$.

Proof: The proof will be by construction. Assume that $\lambda_1 > 0$ and that $\lambda_2 > 0$. Then (3.7), (3.10), (3.11), and (3.12) imply that

$$J_m = 1/[1-n]$$

$$p(s') = [1-n]/f(L,s')$$

$$w(s') = [1+x(s')][1-n]/\beta$$

$$L(s') = \beta[n+x(s')]/\{[1+x(s')][1-n]\}$$

Using these results, we can solve (3.6) and (3.8) for the multipliers:

$$\lambda_1(s') = \{1 - \beta/[1+x(s')]\}/[1-n] \tag{5.1}$$

$$\lambda_2(s') = \beta\left\{\{\alpha[1-n]/[n+x]\} - 1\right\}/\{[1-n][1+x(s')]\} \tag{5.2}$$

Substituting (5.1)-(5.2) into (3.5), we have an implicit expression for n:

$$1 = \int \frac{\alpha\beta[1-n]}{[1+x(s')][n+x(s')]} \Phi(ds') \tag{5.3}$$

The right-hand side of (5.3) is strictly decreasing in n, and is equal to zero at $n = 1$. By assumption 1, it is greater than one at $n = 0$. Hence, there exists a unique $n \in (0,1)$ that solves (5.3). Our construction of an equilibrium is now complete. All that remains is to show that in this equilibrium the implied multipliers are strictly positive. From (5.1), $\lambda_1 > 0$ since $x \geq 0$. As for λ_2, (5.2) implies that $\lambda_2 > 0$ if and only if $\alpha[1-n] > [n+x(s')]$, for all $s' \in S$. This is equivalent to $n < [\alpha-\bar{x}]/[1+\alpha]$. From (5.3), this is exactly assumption 2. ■

What is the economic intuition for the Proposition's two restrictions on the money process? The first condition is needed to ensure that $n > 0$. Anticipating the one period bond price (Q) given below in (5.6), the first assumption implies that if $n = 0$, then the return on bonds will be so high that the representative shopper will want to increase n. Hence, $n = 0$ cannot be an equilibrium. The second restriction constrains the volatility of the money growth process. Recall that the Proposition considers equilibria in which the cash-in-advance constraints bind in each state of the world. To make the firm willing to always exhaust its cash balances, the monetary injection must not vary too greatly from state to state, for otherwise the firm would not spend all of its cash in high injection states.

The model's equilibrium prices and quantities are thus given by

$$p(s') = [1-n]/\{\theta(s')L^{\alpha}\} \tag{5.4}$$

$$w(s') = [1+x(s')][1-n]/\beta \tag{5.5}$$

$$Q(s') = [n+x(s')]/\{\alpha[1-n]\} \tag{5.6}$$

$$L(s') = \beta[n+x(s')]/\{[1+x(s')][1-n]\} \tag{5.7}$$

$$c(s') = \theta(s')L(s')^{\alpha} \tag{5.8}$$

where $n \in (0,1)$ uniquely satisfies (5.3), and Q is the one period bond price calculated using the results in Section 4. For a given distribution of x, (5.6) implies that the bond price is increasing in the monetary injection (the nominal interest rate is decreasing in x). The lower interest rate decreases the opportunity cost of using cash to finance current production. As a result, the firm manager expands current production: equilibrium work effort in (5.7) is increasing in x. To expand employment, the firm must offer a higher wage. Hence, the equilibrium nominal wage given in (5.5) is increasing in x. Finally, since the household's shopper has a fixed amount of cash with which to buy goods, the <u>current</u> price level (equation (5.4)) is decreasing in x. (Next period's price level is obviously increasing in x.)

One curiosity in the above results is that the equilibrium bond price is unaffected by the current productivity shock. This may seem counterintuitive: periods of high productivity should generate increased labor demand and thus push up nominal interest rates. This intuition is in general correct. However, in the present case, the multiplicative nature of the productivity shock implies that it will have no effect on labor demand: the increased productivity is exactly offset by the decreased equilibrium good price (the marginal revenue product of labor is invariant to θ). For other productivity disturbances, the former intuition is valid. For example, if α is stochastic, then (5.6) implies that the nominal interest rate is increasing in α.

Returning to (5.7)-(5.8), note that the central bank has the ability to alter the variability of output. Taking logs of (5.8), and using (5.7), we have

$$\mathrm{Var}(\log(c)) = \mathrm{Var}(\log(\theta)) + \alpha^2\mathrm{Var}(\log([n+x]/[1+x])) + 2\alpha\mathrm{Cov}(\log(\theta),\log([n+x]/[1+x])) \quad (5.9)$$

The second and third terms in (5.9) are determined by the central bank.

Countercyclical policy can be thought of as $\mathrm{Cov}(\theta, x) < 0$ and will tend to decrease output variability. Procyclical policy can be thought of as $\mathrm{Cov}(\theta, x) > 0$, and will tend to increase output variability. Which if either does the representative household prefer? I will discuss this question in Section 8.

A final issue concerns the variability of monetary policy. (5.3) implies that the equilibrium n will be increasing in the variance of x (consider a mean preserving spread around the distribution of the initial x). What effect does this have on the potency of a given monetary injection? (5.7) implies that there is no effect: $\frac{dL}{dx} = \beta/[(1+x)^2]$, which is unaffected by n. However, this result is not at all robust. For example, one can show that if V is logarithmic, then

$$\frac{dL}{dx} = \beta\left\{[1-n]/[(1+x)(1-n) + \beta(n+x)]\right\}^2$$

which is strictly decreasing in n - the slope of the money-labor relation is decreasing in the variability of monetary policy. What is the intuition for this result? Greater monetary volatility leads the household to try to ensure itself against "liquidity risk" by increasing the liquidity it directly provides to the financial market. Hence, the supply of pre-injection loanable funds increases with the variance of the injection. The injection thus becomes a smaller share of the post-injection loanable funds. As a result, a given injection will have a smaller effect on real activity.[8]

6. THE MODEL WITH A FIXED LABOR SUPPLY.

The existence of an equilibrium in the case of an inelastic labor supply is demonstrated in the following proposition:

Proposition 6.1: Assume that labor is supplied inelastically at unity, and that the money growth process satisfies the following two conditions:

$$1)\quad \int [(1-\alpha)\beta^2/x(s)]\Phi(ds) > 1, \text{ and,}$$

$$2)\quad \int \frac{(1-\alpha)\beta^2}{(1-\alpha)\beta - \bar{x}(1+\alpha\beta-\beta) + x(s)}\,\Phi(ds) < 1,$$

where $\bar{x} \equiv \max\{x(s)\}$. Then there exists a competitive equilibrium where n is a fixed number between zero and one, and $\lambda_1 > 0$, $\lambda_2 > 0$.[9,10]

Proof: The proof will be by construction. Assume that n is constant, and that $\lambda_1 > 0$, $\lambda_2 > 0$. Under these assumptions, we will

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[8] Note that the incomplete information model of Lucas (1973) has the same implication for the slope of the Phillips curve but for an entirely different reason.

[9] This Theorem exploits the well known fact that for log preferences and Cobb-Douglas production, the optimal savings rate is a constant. This is why n being constant is a good guess at an equilibrium.

[10] As in Proposition 5.1, the two restrictions on the money process are needed to ensure that n > 0, and that the cash-in-advance constraints bind in each state of the world.

calculate expressions for p, c, k', J_m, and n. (3.10), (3.11), and (3.15) imply that

$$p(k,s') = \frac{1 + x(s')}{f(k,s')} \tag{6.1}$$

$$c(k,s') = f(k,s')\left\{\frac{1 - n}{1 + x(s')}\right\} \tag{6.2}$$

$$k'(k,s') = f(k,s')\left\{\frac{n + x(s')}{1 + x(s')}\right\} \tag{6.3}$$

The continuity of x implies that all of these functions are continuous. As for J_m, (3.12) implies that $J_m \equiv 1/[1-n]$. Using these results, solve (3.6) and (3.14) for λ_1 and λ_2. This yields:

$$\lambda_1(k,s') = \left\{1 - \beta/[1+x(s')]\right\} / [1-n] \tag{6.4}$$

$$\lambda_2(k,s') = \beta\left\{((1-\alpha)\beta/[n+x]) - 1/[1+x]\right\}/[1-n] \tag{6.5}$$

We can now calculate an implicit expression for n. Substituting (6.4) and (6.5) into (3.8) yields

$$1 = \int \frac{(1-\alpha)\beta^2}{n + x(s')} \Phi(ds') \tag{6.6}$$

The right-hand side of (6.6) is strictly decreasing in n, is less than one for n = 1, and, by assumption 1, is greater than one at n = 0. Hence, there exists a unique $n \in (0,1)$ that satisfies (6.6). Our construction of an equilibrium is now complete. All that remains is to verify that in this equilibrium $\lambda_1 > 0$ and $\lambda_2 > 0$. Since $x \geq 0$, (6.4) implies that $\lambda_1 > 0$. As for λ_2, (6.5) implies that $\lambda_2 > 0$ if and only if $(1-\alpha)\beta/[n+x] > 1/[1+x]$. This will be satisfied if $n < (1-\alpha)\beta - \bar{x}[1+\alpha\beta-\beta]$. From (6.6), this is exactly assumption 2. ∎

The model's equilibrium prices and quantities are given by

$$p(k,s') = \frac{1 + x(s')}{f(k,s')} \tag{6.7}$$

$$Q(s') = \frac{n + x(s')}{(1-\alpha)\beta[1+x(s')]} \tag{6.8}$$

$$k'(k,s') = \delta(s')f(k,s') \tag{6.9}$$

$$c(k,s') = [1-\delta(s')]f(k,s') \tag{6.10}$$

where $\delta(s') \equiv [n+x(s')]/[1+x(s')]$ denotes the fraction of output that is saved, and Q is the one period bond price calculated using the results of Section 4. An injection of reserves by the monetary authority pushes up the price level (6.7) and the bond price (6.8). Consumers have a fixed amount of cash available for current purchases. A higher price level forces households to lower current consumption: from (6.10), c is decreasing in x. However, firms have received an additional amount of cash that more than compensates for the higher price of capital goods. Hence, as the size of the monetary injection rises, the share of output that is consumed $(1-\delta)$ shrinks, while the share of output that is saved

(δ) rises. Note that monetary injections have no effect on the <u>level</u> of contemporaneous output, because it is a function of the current (pre-determined) capital stock. These injections only alter the <u>composition</u> of current output. However, a higher level of savings improves the distribution of output next period, ie., unexpectedly large injections of reserves this period tend to increase output next period.

Increased monetary variability in this fixed-labor model has effects similar to those discussed in Section 5. (6.6) implies that n is increasing in the variance of x, and $\frac{d\delta}{dx} = [1-n]/\{(1+x)^2\}$, which is strictly decreasing in n. Hence, increased monetary variability decreases the effect of a given monetary injection.

7. THE MODEL WITH VARIABLE LABOR AND CAPITAL.

Up to this point in the paper, we have analyzed two extreme cases of the loanable funds model outlined in Section 3. In Section 5, we held the capital stock fixed and examined the effects of monetary injections on current real activity. In Section 6, we held the labor supply fixed and examined the effects of monetary injections on future real activity. These two models have exact opposite predictions for the behavior of one variable: the current price level. In the labor-only model, monetary injections decrease the current price level, while in the capital-only model, monetary injections increase the current price level. What is the reason for these opposite conclusions? We normally think of monetary injections as affecting only the demand side of the economy (what textbooks often call "aggregate demand"), and thus pushing up the price level. However, the monetary injections in the labor-only model affect the supply side of the economy by affecting the production decisions of firms. Only the firms get the new cash, and, by assumption, the only "good" they purchase is leisure. Hence, the new cash has no direct impact on the demand for current production - it only affects the supply. Eventually, this cash finds its way into the coffers of households where it will affect their demand for goods - monetary injections increase next period's price level. The transmission mechanism is much different in the capital-only model. In this variant of the model, cash injections are used by the firms to purchase current production - the new cash immediately "leaks out" into the goods market and affects the current price level. Here, monetary injections immediately affect the demand side of the economy, and thus increase the current price level.

This discussion leads naturally to the following question: what will be the affect of monetary injections in a world with variable labor and capital? One would conjecture that injections would increase current and future real activity, while having an ambiguous effect on the price level. In the remainder of this Section, I will confirm this conjecture.

The strategy is to numerically calculate an equilibrium of the economy defined by equations (3.5)-(3.8), (3.10)-(3.12), and (3.14)-(3.15). To make this task simpler, I will begin by collapsing these nine equations in nine unknowns into two equations in two unknowns. Assume that λ_1 and λ_2 are both strictly positive, and that the equilibrium n does not depend on k. As above, let δ denote the fraction of output that is saved, and assume that it does not depend on

k. Then we have

$$c(k,s') = [1-\delta(s')]f(k,L,s') \tag{7.1}$$

$$k'(k,s') = \delta(s')f(k,L,s') \tag{7.2}$$

Using this expression for equilibrium consumption, (3.10) implies that the equilibrium goods price is given by

$$p(k,s') = [1-n]/\{[1-\delta(s')]f(k,L,s')\} \tag{7.3}$$

We will now work towards two equations in the constant $n \in (0,1)$ and the function $\delta:S\to(0,1)$. Under our assumptions, (3.12), (3.7), and (3.6), yield the following expressions for J_m, w, and λ_1, respectively:

$$J_m = 1/[1-n] \tag{7.4}$$

$$w(s') = [1-n][1+x(s')]/\beta \tag{7.5}$$

$$\lambda_1(s') = \{1-\beta/[1+x(s')]\}/[1-n] \tag{7.6}$$

Summing (3.10)-(3.11), and using (3.15), (7.3), and (7.5), we can solve for the equilibrium L:

$$L(s') = \{\beta/[1-n]\} - \beta/\{[1+x(s')][1-\delta(s')]\} \tag{7.7}$$

Substituting these results into (3.8), we can calculate an expression for λ_2 solely in terms of n and δ:

$$\lambda_2(s') = \frac{\alpha\beta}{[1+x(s')]\{[1-\delta(s')][1+x(s')]-[1-n]\}} - \beta/\{[1-n][1+x(s')]\} \tag{7.8}$$

We are now ready to collapse this system into two equations in two unknowns. Before doing so, a quick review of what we have done up to now will be useful. We have used seven of the equilibrium conditions (namely, conditions (3.6)-(3.8), (3.10)-(3.12), and (3.15)) to express eight of the equilibrium functions (c, k', L, J_m, λ_1, λ_2, p, and w) solely in terms of the unknown constant n and the unknown function $\delta:S\to(0,1)$. All that remains is to find these unknowns. To do so, we will use the two remaining equilibrium conditions: (3.5) and (3.14). Substituting the former eight equilibrium functions into (3.5) and (3.14) yields two equations in n and δ:

$$\int\left\{\frac{\alpha\beta[1-n]}{[1+x(s')]\{[1-\delta(s')][1+x(s')]-[1-n]\}}\right\}\Phi(ds') = 1 \tag{7.9}$$

$$\left\{\frac{\alpha\beta[1-n]}{[1+x(s')]\{[1-\delta(s')][1+x(s')]-[1-n]\}}\right\}\left\{\frac{\delta(s')}{1-\delta(s')}\right\} = \int\left\{\frac{\beta^2[1-\alpha]}{[1+x(s)][1-\delta(s)]}\right\}\Phi(ds) \tag{7.10}$$

A constant $n \in (0,1)$ and a function $\delta:S\to(0,1)$, that solve these two equations will, <u>by construction</u>, define a candidate equilibrium for this economy. I use the term "candidate" because we began this construction by assuming that the cash-in-advance constraints hold with equality. To verify that the n and δ that solve (7.9)-(7.10) are in fact a competitive equilibrium, we must use (7.6) and (7.8) to verify that the implied multipliers are strictly positive (this condition is satisfied in all the simulations considered below).

To solve (7.9)-(7.10) for n and δ, I must provide values for the givens in these equations: α, β, x, and Φ. I consider three different values for α: .25, .75, and .9. β is set equal to .95. The money growth rate takes on thirteen different values, varying from 2% to 8% in increments of .5%. The money growth rate defines the current state, and each of these states is equally probable.

With the computed n and δ,[11] all the equilibrium prices and quantities can be calculated. To do so, I need to provide values for the productivity shocks and the initial capital stock. To isolate the monetary non-neutralities, I will consider a world in which the real economy is deterministic: $\theta \equiv 1$. As for the initial capital stock, I set it equal to the stationary state of the Pareto problem: $k_0 = \{\alpha/[1-\beta(1-\alpha)]\}\ [\beta(1-\alpha)]^{1/\alpha}$.

Figures 1-5 graph the equilibrium output, investment, consumption, price level, and one period bond price, for three different values of α. (All are graphed relative to their mean values for easy comparison.) The $\alpha = .25$ equilibrium is denoted with X's, squares denote the $\alpha = .75$ equilibrium, and triangles denote the $\alpha = .9$ equilibrium. The Figures

[11] The nonlinear system solver of GAUSS, Version 2.0, was used to carry out all the simulations in this paper.

reveal that in all three cases monetary injections stimulate output, investment spending, and asset prices. A comparison of Figures 1 and 2 reveals that in each case δ, the portion of output invested, is also increasing in the injection. The behavior of the price level is a little more complicated. For $\alpha = .25$, the price level is increasing in the monetary injection. The reason is that a large portion of the loanable funds are used to finance investment expenditures as opposed to increased employment: the "demand" effects of monetary injections dominate the "supply" effects. For $\alpha = .90$, the opposite is true: "supply" effects dominate "demand" effects, and the price level is decreasing in the monetary injection. Of particular interest, is the economy's behavior for $\alpha = .75$. In this case, the "supply" and "demand" effects of monetary injections almost entirely cancel. Monetary injections have little effect on the current price level, but stimulate output, investment spending, and asset prices.

8. CONCLUSION.

One enlightening way of viewing the model developed in this paper is to see it as a real business cycle model in which a particular financial structure has been imposed. The nature of the financial structure produces two particular frictions, otherwise absent from the economy. The first requires that all transactions be carried out in a decentralized fashion with the use of cash. This restriction yields the cash-in-advance constraints on purchases, and produces inflation tax distortions: since labor services and capital must be purchased with cash, higher inflation rates discourage work effort and capital accumulation.[12] The second friction is the closing of one particular insurance market. Households and firms would like to insure themselves against the "liquidity risk" of dealing with financial intermediaries, ie., households and firms would prefer to directly negotiate loan agreements after productivity shocks and monetary injections are realized. (In the terminology of the model, the household would like to choose n after all uncertainty is resolved.) These direct negotiations are ruled out by assumption. I make no apologies for closing this market, for this is <u>precisely</u> the market that must be closed to capture the liquidity and loanable funds effects. In any event, closing this market will cause the model to further depart from Arrow-Debreu outcomes. In particular, monetary shocks will increase the liquidity of firms and thus increase the demand for labor and capital goods.

As shown in Fuerst (1990), optimal monetary policy in this environment is in general activist - involving either procyclical or countercyclical policy. What is the intuition for this activism? Any optimal policy must eliminate the two frictions outlined above: the inflation tax, and the closing of the "liquidity risk" insurance market. The former friction can be easily eliminated with a deflation. The second friction is a little more complicated. The representative household would like to be able to quickly and costlessly transfer liquidity or loanable funds between the two markets. These transfers

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[12] Cooley and Hansen (1989) capture similar inflation tax effects in their cash-in-advance production economy. However, in their model, capital is a "credit" good so that the inflation tax affects the capital stock only through its effect on work effort. The current model is more like the one in Stockman (1981), where cash is needed to purchase capital.

are ruled out by assumption. However, by altering the size of its injection, the central bank can carry out these rapid movements of liquidity. In effect, the central bank can provide insurance against this "liquidity risk" and open the closed market. To provide this insurance, the optimal policy will in general need to be quite activist, responding to contemporaneous productivity shocks.

One unique empirical prediction of the current model is that monetary injections will have compositional effects - large injections will shift current real activity in the direction of the sectors most closely linked to the financial sector, and away from sectors less closely linked. Equivalently, the model predicts that money-income correlations should differ across industries in a predictable way. If one could find a metric for "closeness to the financial sector", we could test this hypothesis. Industries most closely linked should have large positive responses to unanticipated injections, while other industries should have smaller, or possibly even negative, responses.[13] Evidence of this type is presented by Ahmed (1987) and Kretzmer (1989). Both of these papers look at the cross-industry responses of real activity to monetary shocks, and find significant differences across industries. Are their results consistent with the model developed in this paper? To answer this question, we need the financial metric mentioned above. I leave the development of such a metric as a future task, and instead hope that these rankings are at least suggestive.

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[13] A related prediction concerns the behavior of nominal prices within an industry that is closely linked to intermediaries. The crucial question is which side of the industry is linked: the supply side, the demand side, or both? The results of Section 7 imply that the answer to this question will determine the effect of a monetary injection on contemporaneous nominal prices within the industry.

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Figure 1

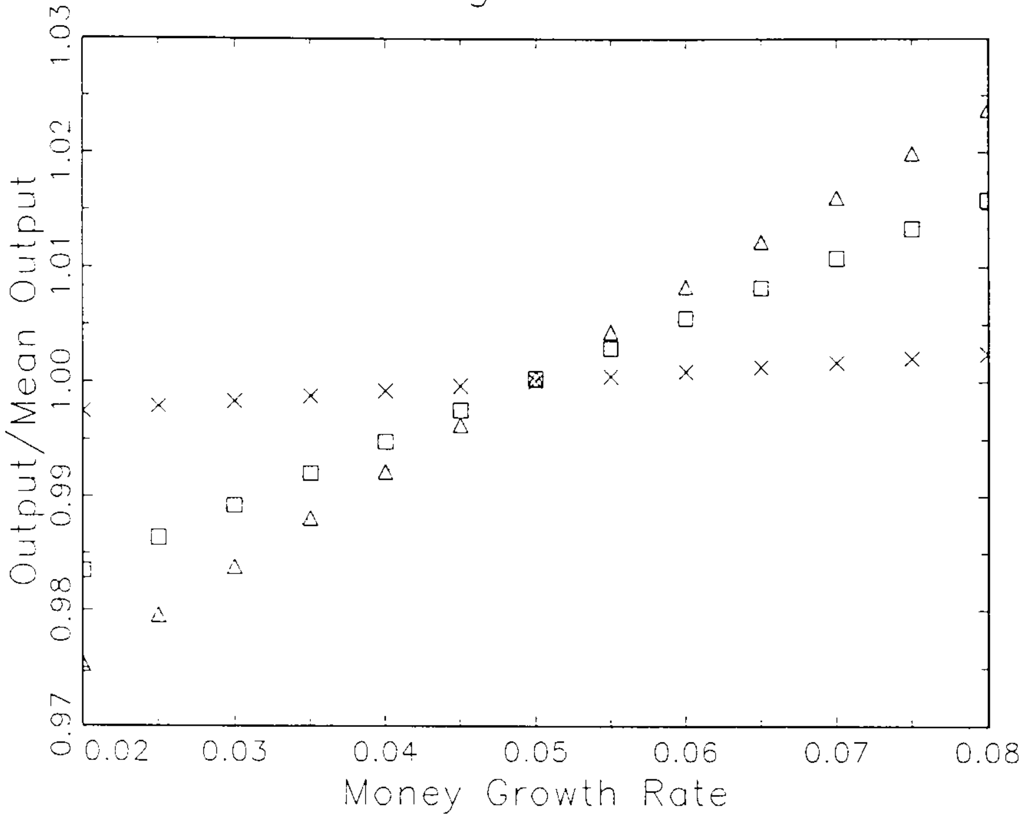


Figure 2

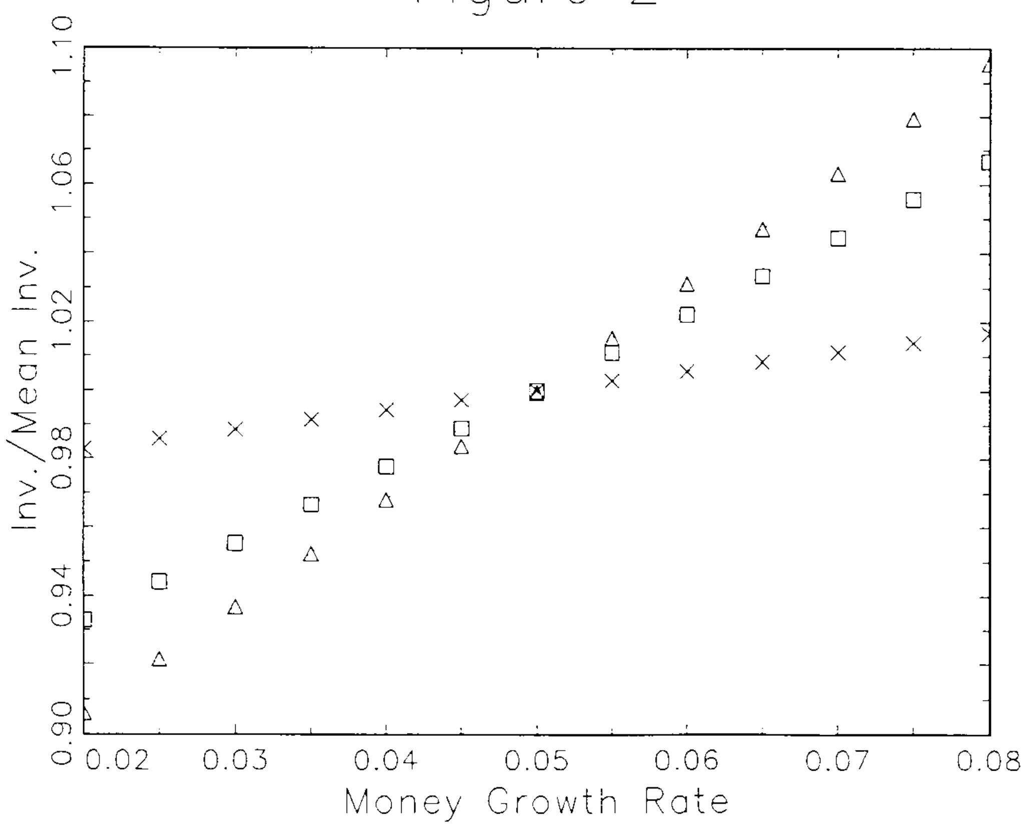


Figure 3

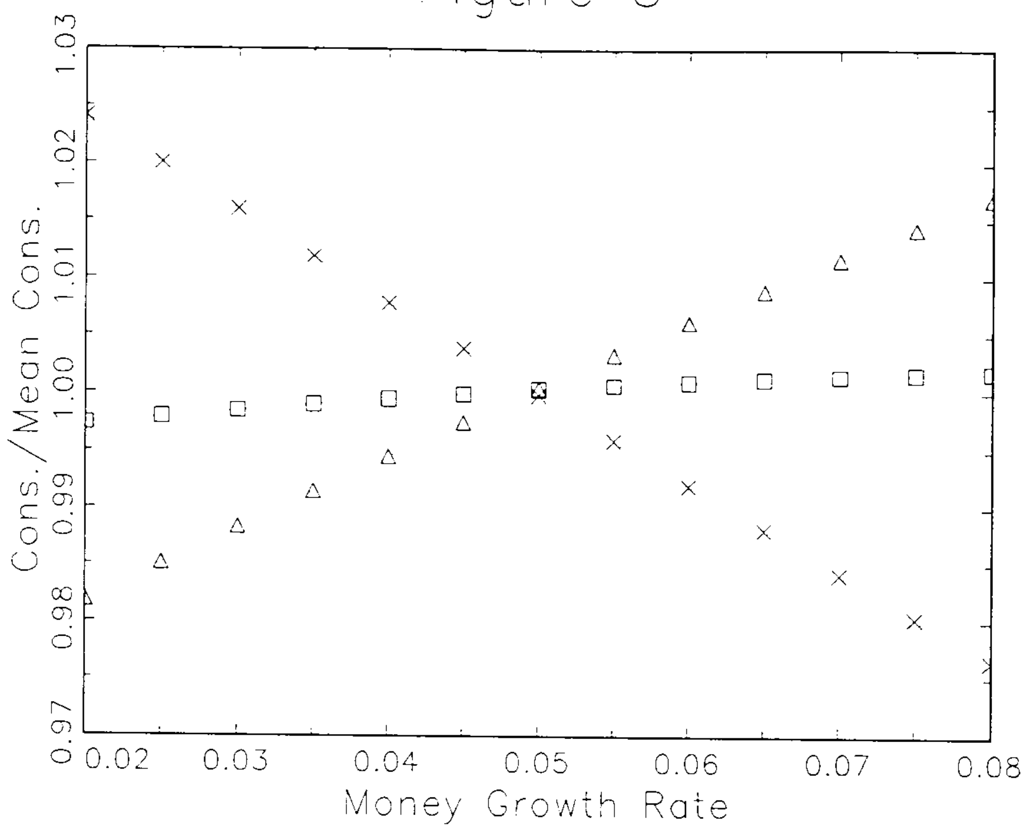


Figure 4

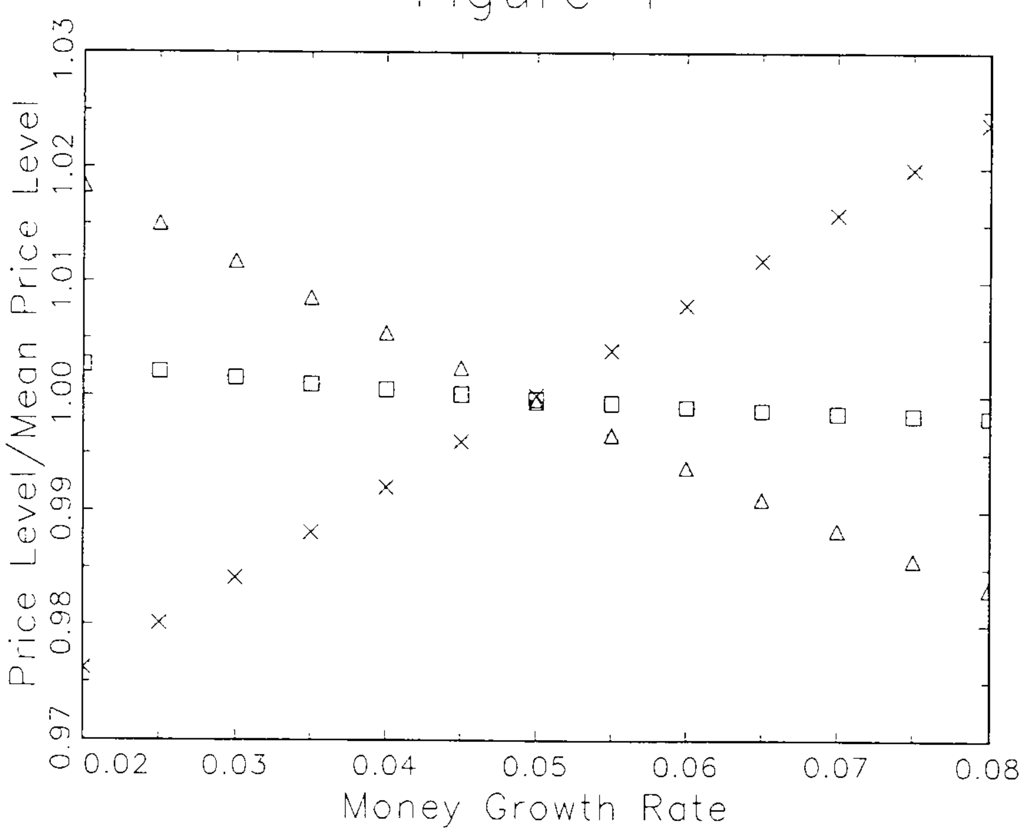
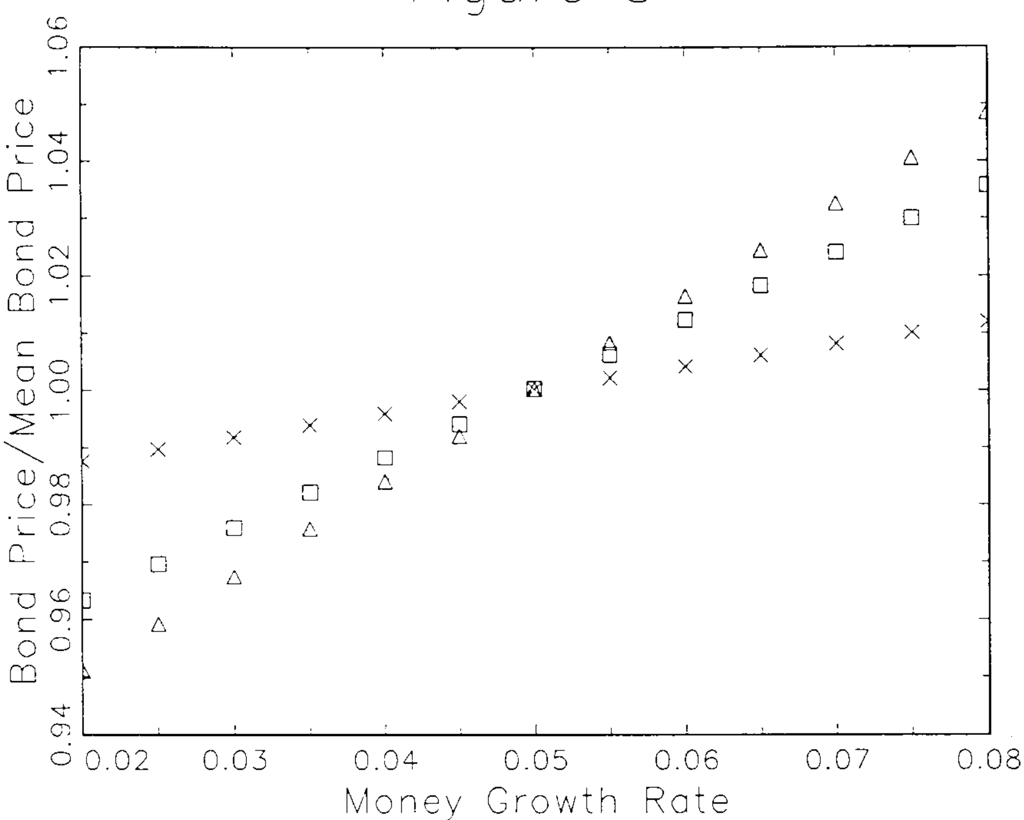


Figure 5



Bond Price/Mean Bond Price
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0.02
0.03
0.04
0.05
0.06
0.07
0.08
Money Growth Rate