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INNOVATION AND PRODUCT DIFFERENTIATION*

by

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Abstract: Economic theory has primarily viewed an innovation as a single, discontinuous change. Historical and empirical evidence, on the other hand, shows that a major part of economic development has come from a cumulation of small improvements to original technologies and quality additions to early products. We focus analysis on competition in post-discovery phase, emphasising in particular that a key dimension to this competition is the innovations that lead to product differentiation and quality improvement. In a duopoly model with a single adoption choice, we derive endogeneously the level and diversity of product innovations. We demonstrate the existence of equilibria in which firms emerge at different points of the quality spectrum. In such equilibria, no monopoly rent is dissipated and later innovators make more profits. Incumbent firms may well be the early innovators, contrary to the predictions of the "incumbency inertia" hypothesis. The role of underlying factors as consumer diversity, learning and market lock-in, in determining market expectations and hence the innovation outcomes is analysed. Finally, innovative incentives under a cartel and social planner are contrasted with the duopoly outcomes.

1. Introduction

Economic theory, going back to Schumpeter, has viewed an innovation as a single, discontinuous change, a major break with the past which radically alters the production possibilities of an industry or economy. "The historic and irreversible change in the way of doing things we call 'innovation' and we define: innovations are changes in production functions which cannot be decomposed into infinitesimal steps. Add as many mail coaches as you please, you will never get a railroad by so doing" (Schumpeter (1935), p7). The first important point to note is that innovations, in this view, are primarily once and for all changes. Modifications or improvements in a new technology are strictly second order effects. Hence, the overwhelming importance of the first breakthrough and the absolute necessity of patent protection to give competitive firms the right incentives to make that breakthrough. Secondly, innovations represent fundamental discontinuities, radical alterations in products and production technologies. Consequently, effective patent protection should be possible since that which is being protected is wholly different from options that competitors have access to. This viewpoint on innovative activity probably explains why the bulk of economic research has been concentrated on analyses of "patent races."[1]

Historical and empirical evidence, on the other hand, point to the overwhelming importance of ongoing and continuous change, the importance of improvements and modifications after the first breakthrough. A large portion of productivity growth has historically come from a cumulation of small improvements to original technologies. Product variation and quality changes accumulate over time. Individually none of the changes might represent a substantial break with qualities or varieties or technologies

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[1]A few representative contributions are Loury (1979), Dasgupta-Stiglitz (1980), Lee-Wilde (1980), Fudenberg et.al. (1983), Harris-Vickers (1985); see Reinganum (1989) for a survey of this area.

available previously, but together the changes are substantive. To take Schumpeter's railroad example, in order to illustrate the importance of post breakthrough innovations: Fishlow (1966) found in his study of the American railroads that at a time of significant cost reductions, the years between 1870 and 1910, the largest cost saving, by far, was due to a succesion of improvements in the design of locomotives and freight trains. The process included no single major break with the past and yet "its cumulative character and lack of a single impressive innovation should not obscure its rapidity" (ibid, p 35).[2] A second important stylised fact about innovative activity is the great imperfection of patent protection, a fact which suggests that much of what is patented may not in fact represent the kind of discontinuous radical change that Schumpeter envisaged. Mansfield et. al. (1981) found from their survey of 48 product innovations, of which 70% were patented, that about 60% were competed against within four years of the patent. It appears then that the focus of economic analysis should be, not on the race to discover, but on the nature of competition that ensues in the post discovery phase. Further, such competition has at its core, two key dimensions: product differentiation through quality improvements[3], which allows an innovator to distinguish his product from the initial discovery, and repeat innovations, which bring about the ongoing technological changes that we observe.

That an analysis of the development and adoption phase is important, is not an original insight. What is new, however, is our belief that much of the fundamental innovative activity is carried out during the development-adoption phase. In this sense, the classical distinctions between invention-innovation and development-adoption, with the

[2]Rosenberg(1982) makes a very forceful case for the historical importance of continuous changes and also contains numerous other examples. In particular, refer to chapters 1,3 and 5. Curiously, of the early economic thinkers, Karl Marx seems to have been the one who emphasized the importance of evolutionary changes. "A critical history of technology would show how little any of the inventions of the eighteenth century are the work of a single individual", (Marx, (1867), p.406).

[3]We do not deny the importance of ongoing process innovations and indeed much of our analysis can be straightforwardly adapted to this setting. Complete information about a

creative improvements coming in the first stage, is at best misleading. Moreover, the nature of innovation in the second stage is critically different. In particular, at this phase quality improvements and product differentiation are of paramount importance. The costs involved in incorporating higher quality into the new product are often temporal, i.e. it takes longer to advance technology beyond that of the competition. Through such activities is an original breakthrough transformed into a host of differentiated and better products. This suggests the importance of analyzing this issue within a framework which allows product heterogenity and the growth of such differences over time. In this paper we build a first simple model to do precisely this.

To be more precise about our model, consider the following scenario. A technology or idea or quality level is available at date zero.[4] Over time the basic technology or idea grows and matures, on account of firms' efforts in assimilating knowledge, and also possibly from a flow of exogeneous information. At any point of time, any firm, in the industry, can introduce a product into the market incorporating the currently available technology. A firm's profits are increasing in the quality level of its product but better products involve the cost of waiting for technological growth. The questions we are interested in are: how much does the initial idea get matured, the technology get improved before the first introduction into the market, the level of innovation? How diverse are the subsequent products that are innovated? How frequent is innovation?

The answers to the above questions clearly turn on a firm's expectations about future possibilities. The expectations are of two kinds: technological and market expectations. The technological expectations are about the maturation process: how quickly

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rival's process innovation does seem a more demanding assumption than such information about product innovations. In the sequel, we think of product innovation alone

[4]It is unimportant for our purposes whether this initial idea came from a prior breakthrough or not, although the reader might wish to think of the breakthrough as having happened at date zero.

will technology improve, how costly will such improvements be?[5] If the market were a monopoly, such considerations alone would determine the optimal timing and level of the first and subsequent innovations.[6] In a competitive market, the optimal level of innovation is determined, additionally, by market expectations and evaluations about the innovative behavior of competitors. Two principal questions for a potential inovator are: if I do not innovate today, will the competition, i.e. how likely is pre-emption? Further, if I do innovate, how much better would the competitor's product be when they eventually innovate? When repeat innovations are possible, market expectations also incorporate beliefs about all future innovations. Market expectations are the key to understanding innovations in imperfetly competitive markets. In this paper we study the case of a single innovation[7] and a companion paper (Dutta-Rustichini (1990a)), the analysis of repeat innovations is carried out.

For convenience, we study a duopoly model. Each firm chooses the extent to which it lets the initial technology mature before it introduces a product incorporating such technology. In the event that a competitor has already innovated, a firm will optimally decide how much longer it waits, how much better is the technology going to be before it innovates, that is an optimal level of vertical differentiation. If a competitor has not innovated yet, a firm trades off maturation possibilities against initial monopoly rents as a pre-emptor.

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[5]Additionally there are also expectations about the possibility of other major discoveries, which may make the current technology obsolete. We do not explore this dimension of the problem here.

[6]We use the term innovation to denote, interchangably, the maturation process in which money and time is expended to improve quality, and the product introduction itself.

[7]Repeat innovation possibilities introduce the familiar issues: how much cooperation can be sustained through repetition? What class of equilibria in the dynamic game, Markov perfect versus renegotiation proof versus trigger etc., are "reasonable"? How does one pick among the indeterminate number of equilibria within a certain class? All of these questions are interesting (and we address some of them in our companion piece), but they are peripheral to the preemption and maturation issues we explore here. Further, there is an argument for understanding the "one shot" game before going to the dynamic one.

We demonstrate the intuitive result that markets have either of two outcomes. A pre-emption equilibrium may arise in which the possibility of capturing monopoly rents results in a race to be the first innovator. Consequently rents are dissipated and the market may under-innovate relative to a cartel or social optimum. Much more interestingly, there may instead be a maturation equilibrium in which competition results in an even flow of innovations. An early entrant enters unchallenged to exploit the initial monopoly rents and a market niche which is eventually on the low end of the quality spectrum. A technological leader optimally waits to let a product mature before adoption. In such an equilibrium an early innovator earns lower lifetime profits, i.e. rents are not equalized (nor dissipated). Further, in such an equilibrium, a market may over-develop a product, relative to the cartel or social optimum. This outcome should be contrasted with the previous literature on adoption of new technologies, which ignores product innovations and product differentiation at the adoption stage. As Fudenberg-Tirole (1985) argue convincingly, (see also Tirole (1989)), with a homogeneous good, a second entrant can only look forward to a more competitive market. The lure of monopoly rents drive each firm to try to pre-empt the other's adoption. Consequently, the only possible outcome is rent dissipation and under innovation.[8]

A second question we analyse is that of the differential incentives of entrants and incumbents to innovate. It is often argued that the "cannibalisation effect", (the incumbents' loss of current revenues from innovating), would tend to lower incumbent incentives to innovate.[9] We demonstrate the great importance of distinguishing between

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[8]Reinganum (1981) had also shown the possible diffusion of adoptions, even within the context of homogeneous goods. However, her result was critically dependent on the firms being able to pre-commit, and indeed Fudenberg-Tirole (1985) demonstrated that without pre-commitment, the only equilibrium in her model is a pre-emption equilibrium.

[9]For example, Schumpeter (1934) says, "...it is not essential to the matter--though it may happen--that the new combinations should be carried out by the same people who control the productive or commercial process which is to be displaced by the new. On the contrary, new combinations are, as a rule, embodied, as it were, in new firms which generally do not arise out of the old ones but start producing beside them; to keep to the

direct incumbents, i.e. those currently in the very market in which innovations are being analyzed, versus indirect incumbents, i.e. those with better information about the market. For direct incumbents, it is true that incumbents are less likely to innovate first if the current returns are very big. But if the current revenues are not significantly large, a forward induction argument suggests exactly the converse outcome. An entrant can credibly signal early on, an intention to be a high technology firm, by passing up on the best low-tech entry point for itself. On the other hand, precisely on account of the cannibalisation factor, the fact of passing up of early innovation opportunities by the incumbent conveys no credible signal about future behavior. For indirect incumbents, the result is even more striking. We define an indirect incumbent to be a firm which can better exploit a monopolistic position in the market. We show that such advantage works surprisingly to a firm's disadvantage, in that it necessarily forces such a firm to adopt first and earn strictly lower profits in a maturation equilibrium.

To isolate the factors which determine the extent of pre-emption and maturation incentives in a market, we focus on diversity of consumer preferences, "learning from other's mistakes" and market lock-in effects. Although in general the effect of these factors is ambiguous, we do show that in a variety of instances, there is a natural critical level of consumer diversity (which increases payoffs to differentiation) and learning/imitation possibilities (which increases the ability to differentiate). Above this critical level, maturation equilibrium appears and below it, the only possibility is a pre-emption equilibrium. We also show that more technological environments lead to a greater level and diversity of innovations.

We also contrast the market outcomes with those that may arise in a cartel or under a social planner. A cartel is better able to internalise market returns to diversity. This encourages early first innovation since it increases waiting costs. On the other hand, a

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example already chosen, in general it is not the owners of stage-coaches who build railways."

cartel may delay the second innovation more than would a competitive market. This discourages an early first innovation. We do however give robust conditions under which a market may actually over-innovate relative to both the cartel and the social planner, innovating later and maintaining greater diversity in products.

The principal conclusion to be drawn from this analysis is that incorporating heterogenity of products and admitting its central connection to innovations, fundamentally alters the perceptions on the market's incentives to innovate. The pace and diffusion of a new idea corresponds more closely to the observed appearance of a range of differentiated products. Further, monopoly rents may well not be dissipated.

The previous literature on adoption-development includes Reinganum (1981), Fudenberg-Tirole (1985) and Katz-Shapiro (1987) who analyse the timing of adoption of an initial breakthrough, which is publicly available and used to produce a homogeneous good. Benoit (1983) and Ramey (1988) consider the consequences of imitation of a new fixed technology, again producing a homogeneous good, when only one firm has access to this technology.

A further contribution of this paper is that it offers a unified way to think about product differentiation and innovation. Vertical differentiation models (as Shaked-Sutton (1982)) or sequential differentiation models (as Prescott-Visscher (1979)), have firms choosing quality first and then engaging in price competition. All these models carry exogenously imposed structures on the timing and sequence of quality choice. Our framework is the natural paradigm within which such models can be embedded, since we endogeneously model the quality choice as a choice of timing or technology maturation. Similarly, our model naturally contains as a sub-case, the adoption models which do not allow for product differentiation, as Fudenberg-Tirole (1985) and Reinganum (1981).[10]

[10]In these models, the cost of adoption is assumed to decline over time whereas we assume it to be constant. This is however a superficial difference. It is needed in these models to ensure diffusion of technology, which happens straightforwardly in our

Section 2 describes the model. Section 3 conducts the equilibrium analysis. Section 4 examines the differential incentives of entrants and incumbents. Section 5 analyzes the effect of consumer diversity, learning and lock-in. Section 6 contrasts the competitive outcome with those generated by either a cartel or a social planner. Section 7 concludes by discussing some extensions of the basic model.

2. The Innovation and Product Differentiation Problem

An idea for a new product arrives in an industry. The idea is available to all firms in the industry. The question that will concern us here is the pace of innovation and diffusion of this idea. The knowledge embodied in the idea forms the basis for the development of a new product.[11] A firm's own efforts in assimilating this knowledge and the flow of additional, exogeneous information related to the basic idea determine, to a large extent, the attributes or characteristics of a new product a firm introduces into the market. Thus a basic idea generates a host of differentiated products.

To be precise, suppose the payoff relevant attributes of firm i's product can be represented by a variable $x_i \in R_+$. $x_i(t)$ should be thought of as the level of technology or quality, that is available in period t, to firm i. This quality level grows (stochastically) at a (mean) rate, $f_i(e_t)$, where e_t should be thought of as firm i's expenditure or effort in period t. We make the following simplifying assumption on the maturation process.

(A0) i) Temporal costs dominate: e_t is a 0-1 variable, i.e. the choice for a firm is whether or not to keep a laboratory open, and of course the idea grows if and only if the laboratory

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formulation on account of product differentiation possibilities. We could just as easily add this effect on to our formulation.

[11] As noted before, we shall talk exclusively of product innovation, although much of what we say carries over verbatim to the case of process innovations, i.e. cost-reducing technological innovations.

is open, i.e. $f_i(0)=0$. In fact suppose further, that once closed, a laboratory cannot be re-opened.

ii) Symmetric maturation processes: $f_i(1)$ is identical for the two firms.

iii) Deterministic maturation: x grows through a simple deterministic geometric process. The basic idea, represented by x(0), grows deterministically over time at rate ρ^{-1}, $\rho > 0$.

One may equivalently imagine that the basic idea grows in a publicly accessible environment like a government or university laboratory. (A0iii) is chosen as the simplest possible specification of technological expectations, since our interest is in exploring the formation of market expectations.[12] (A0ii) does not force the identity of the first innovator in a market by exogeneous considerations. Both of these can be relaxed with absolutely no change in the analysis. (A0i) is a very useful simplification. Its absence would complicate the analysis, although the reader is invited to check that the results remain unaltered. For much of what follows, we shall normalize and let $\rho = 1$ and $x(0) = 0$, i.e. that the state of technology at period t is precisely the vintage. The more flexible formulation, with ρ possibly different from 1, will be useful in analyzing comparative statics of the technological environment.

Consider a duopolistic market, with firms indexed by a generic index, i = 1,2. j will index the "other" firm. Either firm can introduce a product at any time, incorporating the currently available technology.[13] We shall represent the flow profits of a monopoly selling a product with attribute x, as $\delta R(x)$. If firm i has introduced a product with attribute x_i (at time x_i, and from hereon we will use time and attribute interchangeably), then firm j

[12]The general game structure we study here is useful in analysing a number of other strategic timing issues. In some of these, stochastic growth is essential. Dutta-Rustichini (1990b), develop a general theory of games of entry, which allows, inter alia, stochastic processes generating the "state variables".

[13]An implicit assumption, then, is that technology is irreversible. Even if technology or quality was reversible, i.e. if at quality x a firm could choose to introduce a product incorporating an attribute $s \in [0,x)$, under natural restrictions on returns they would not.

has a choice on its introduction time. Suppose the technology or quality variable grows in the interim, at a rate $\gamma^1, \gamma > 0$. That γ may be different from the growth rate of the underlying technology (i.e. 1), captures the idea that a firm learns from the experience of its competitor. This learning may be about market conditions, consumer preferences, ease of marketing new products, unknown and intangible factors like consumer habits, available substitutes etc. In such cases, $\gamma < 1$. On the other hand, casual empiricism strongly suggests that there are "lock-in" effects, with early innovators keeping more than half the market share, when "me too" brands appear. In this case, of course $\gamma > 1$. As before, we shall discuss mostly the case $\gamma = 1$, except when we examine the efffect on adoption decisions of differing diversity possibilities. If j introduces a product at x_j, then flow profits to the duopolists from that point on are $\delta r_i(x_i, x_j)$ and $\delta r_j(x_i, x_j)$, respectively. These profits could be thought of either as the returns to Cournot or Bertrand competition in the duopoly market. We give examples below.

The principal simplifying assumption that we make is that duopoly returns depend only on the <u>relative qualities</u>, i.e.

(A1) $\qquad r_i(x_i, x_j) = r_i(x_i - x_j),\ i = 1,2\ ,\ i \neq j$

This assumption cleans up the exposition considerably. We indicate in Section 7 how the analysis is complicated (although the results are largely unchanged) by the absence of this assumption. To complete the specification of the model, we have to specify what happens if both firms attempt to introduce a product at the same time.

(A2) If both i and j attempt to enter at any period t, then only one of them succeeds in doing so, and the probability of i entering is $p \in (0,1)$, and j adopts with probability $1 - p$.

Simultaneous adoption is hence ruled out although adoption at any $\tau > t$ is feasible. An ex-ante justification for this assumption is that typically there is an "adoption technology", like advertising, which may have limited capacity. As with (A1), not making this assumption complicates but does not substantially alter the analysis. We return to this theme in Section 3.

2.1 Strategies and Equilibrium

A pure strategy of firm i specifies, at any time t, a decision on "adopt" or "do not adopt", if the firm has not adopted already. This decision is of course conditioned on the available information, which in our model is the knowledge: has j adopted at any $s < t$, and if yes, when. For well-known reasons[14], we confine attention to pure strategies, although in asymmetric versions of our game the equilibria are unique in the class of all strategies, including mixed strategies. The equilibrium concept we employ is that of subgame perfection. This amounts to checking that starting at any possible state (date), the continuation strategies do in fact form best responses to each other.

2.2 Two Examples

We present briefly two examples of duopoly profits that satisfy (A1)

Example 2.1 Cournot Competition

This example is adapted from Kreps (1990) and shows that if demand functions are based on relative differences, then this extends to Cournot profits.

Consider the following demands for products of vintage x_i, $i = 1,2$.

$$P_i = a - Q_i - b(x_i - x_j)\, Q_j \quad i,j = 1,2, \; i \neq j \tag{1}$$

Writing $x_1 - x_2 = \theta$, it is easy to see that if $b(\theta) \in (0,1)$, for all θ, then the products are imperfect substitutes. Further, if $b(\cdot)$ is decreasing, then the demand for a product is increasing the higher its quality relative to the other product.

Taking costs of production to be zero, straightforward computation yields the

[14]See, for example, Rubinstein (1988).

Cournot quantities, given vintages x_1 and x_2, to be

$$Q_1(\theta) = \frac{2 - b(\theta)}{4 - b(\theta)b(-\theta)} a$$

$$Q_2(\theta) = \frac{2 - b(-\theta)}{4 - b(\theta)b(-\theta)} a$$

From that it follows that the Cournot profits are

$$r_1(\theta) = [Q_1(\theta)]^2$$

$$r_2(\theta) = [Q_2(\theta)]^2 \tag{3}$$

Straightforward calculus shows that under the symmetry assumption, $b'(\theta) = b'(-\theta)$, r_1 is increasing in the relative quality advantage of product 1, and r_2 is decreasing.

Example 2.2 Bertrand Competition

This example is adapted from Shaked-Sutton (1982), and gives a construction based on utility functions.

Consumers have preferences on quality, with this preference index ranging over $[a,b]$, $b > 2a > 0$. Each consumer has y units of a numeraire good and uses it to buy a single unit from either producer. The m-th consumer's utility from buying a good of vintage x_i is $mx_i + y - p_i$, $i = 1,2$, $m \in [a,b]$.

So, in a duopoly, with qualities x_i, prices p_i yield market shares of $[a,\bar{m}]$ and $[\bar{m},b]$. Letting, without loss of generality, $x_1 > x_2$ (i.e. $\theta > 0$), the high quality customers buy from firm 1 and the market divides at $\bar{m} = \frac{p_1 - p_2}{x_1 - x_2}$ $(p_1 > p_2)$.

Straightforward computation yields prices in Bertrand equilibrium as

$$p_1(\theta) = \frac{2b - a}{3}\theta$$

$$p_2(\theta) = \frac{b - 2a}{3}\theta \tag{4}$$

It follows that

$$r_1(\theta) = \left(\frac{2b - a}{3}\right)^2 \theta$$

$$r_2(\theta) = \left(\frac{b - 2a}{3}\right)^2 \theta \qquad (5)$$

In this case it is easy to see that both r_1 and r_2 are increasing in θ, i.e. that the more diverse are the products the greater are profits for both the technological or quality leader as well as the laggard. This is of course directly the phenomenon of differentation lessening the severity of Bertrand price competition.

Incidentally, it is also straightforward in this example to show that the monopoly profits for a product of vintage x is

$$R(x) = \frac{b^2}{4} x \qquad (6)$$

It depends only on the upper bound of consumer preference on quality, since a monopolist only services a fraction of the market optimally, and the choice is which of the high quality seeking customers to serve.

2.3 Some Assumptions on Returns

From hereon let us place a symmetry restriction on the duopoly profits, i.e. $r_i = r_j = r$, say. This assumption is of course satisfied in Examples 2.1 and 2.2 and as with (A0 iii) is made so as to not force the identity of who moves first through factors exogeneous to the game. Also, denote $\theta = x_i - x_j$, the generic difference in qualities. The natural monotonicity assumptions are

(A3) i) Monopoly returns, R(x) are increasing in x

ii) In a duopoly, $r(\theta)$ is increasing in θ, whenever $\theta \geq 0$.

(A3) ii) merely says that a technological leader has duopoly profits increasing in his lead. Whether a quality laggard makes more or less as θ increases, depends as we saw in

Examples 2.1 and 2.2 on the market's preferences over diversity We make no assumptions hence on $r(\theta)$, for $\theta < 0$.

A useful technical quasi-concavity assumption is

(A4) i) $e^{-\delta x} [aR(x) + b]$ is strictly quasi-concave on R_+, for $a,b \in R$

ii) $e^{-\delta\theta}r(\theta)$ is strictly quasi-concave on R_+

(A4) is satisfied in Example 2.2 and under obvious conditions on the diversity function $b(\cdot)$, by Example 2.1. Another useful assumption which simplifies exposition is

(A5) r and R are non-negative and differentiable.

3. Pre-emption and Maturation Equilibria

In this section we characterize the subgame perfect equilibria of the (symmetric) innovation game of Section 2. The principal question we analyze is: does (imperfect) competition lead to under-development of technology, is the innovation-adoption process necessarily a race? Is there necessarily an equalization of rent as would be predicted by a "race to pre-empt"? As we argued in the introduction, an important stylized fact of the innovation process is the product differentiation activities which lie at the heart of much innovation. An alternative way of phrasing the question then is: how would imperfectly competitive firms trade-off pre-emption and differentiation incentives? In this section we show that a consequence of such conflicting incentives is that either of two equilibria will result. The first, which we call pre-emption equilibrium arises from an inability by firms to sufficiently differentiate their products (in payoff terms). This inability in turn stems from factors as market lock-in by a first entrant, a slow imitation technology or insufficient diversity of consumer preferences. We focus on these underlying factors in Section 5. Here we show that the consequent attractiveness of having a leading position in the market, leads firms to adopt "too early", equalize rents but achieve that by dissipating potential rents. This is the equilibrium that previous literature (Fudenberg-Tirole 1985, Tirole 1989) had found to be the only outcome consistent with competition. The second equilibrium, which we call maturation equilibrium, is one in which competition results in an even flow of innovations. A potential technological leader optimally waits to develop a differentiated product. The other firm enters earlier to exploit a temporary monopoly position. None of the rent associated with this monopoly position is dissipated. However, an early entrant makes strictly less in equilibrium than the eventual quality leader. That such rent non-equalization can happen in a symmetric game may appear at first somewhat paradoxical. It results of course from the fact that time (or technology or quality development) is unidirectional. A second entrant can pre-empt the first one if the latter earns higher ex-ante profits. However, a first entrant cannot unilaterally decide to be the follower.

3.1 The Follower's Problem

Consider a continuation subgame after firm i adopts the technology at x. j's problem is to pick an optimal state $x + \theta$, at which to follow.[15] In other words, j solves[16]

$$\max_{\theta \geq 0} \; e^{-\delta\theta} r(\theta) \tag{8}$$

By (A4) ii), $e^{-\delta\theta}r(\theta)$ is single peaked on R_+. Let this peak occur at $\theta^* > 0$. It follows that firm j would follow at $x + \theta^*$. In other words, in equilibrium all strategies prescribe: if the other firm has already moved, then move if and only if θ^* has elapsed since the competitor's adoption.

[15]As mentioned in footnote 4, the implicit assumption is that quality or technology is irreversible, i.e. $\theta < 0$ is ruled out. This would follow directly if r was increasing on R_- as well. This last restriction may not be satisfied if there is a market preference for diversity, as in Example 2.2. However, the nature of our analysis would stay fundamentally unchanged if $\theta < 0$ is admissible. We discuss this further in Section 7.

[16]Note that flow returns were normalized to $\delta r(\theta)$, so the infinite horizon discounted returns are $r(\theta)$.

Let F(x) denote the lifetime returns, to firm j, if it follows optimally a first adoption by firm i, at date x.

$$F(x) = e^{-\delta x}\left\{e^{-\delta\theta^*}r(\theta^*)\right\} \qquad (9)$$
$$\equiv e^{-\delta x}\phi$$

ϕ the optimal returns to a follower's differentiation activities, is the direct index of differentiation possibilities in the market. In Section 5 we will relate ϕ to underlying factors as diversity in consumer preferences, imitation or learning possibilities and market lock in.

3.2 The Leader's Problem

Following Fudenberg-Tirole (1985, 1989), we develop the returns to a potential first entrant, or leader. Let L(x) denote the returns to a firm i evaluated at date 0 if it innovates at quality x, anticipating an optimal follow by j at $x + \theta^*$.

$$L(x) = e^{-\delta x}\left\{(1 - e^{-\delta\theta^*})R(x) + e^{-\delta\theta^*}r(-\theta^*)\right\} \qquad (10)$$
$$\equiv e^{-\delta x}\left\{\lambda_1 R(x) + \lambda_2\right\}$$

Any potential leader evaluates returns from two sources: $\lambda_1 R(x)$, the monopoly phase and λ_2, the phase in which it is a technological laggard in a duopoly. The effect of market differentiation possibilities on λ_k, k = 1,2 are more ambiguous, than on ϕ. We shall develop this in Section 5 when we explore the comparative statics of equilibria.

By (A4) i) L(x) is single-peaked. Further, note that $\frac{L(x)}{F(x)} = \frac{\lambda_1 R(x) + \lambda_2}{\phi}$, which is an increasing function of x. Hence, L and F have at most one intersection. Two interesting possibilities arise:[17] (Note that in Figs. 1 and 2, x^I refers to the intersection point of L and F and x^M refers to argmax L.)

(please insert Fig. 1 and Fig. 2 here)

[17]We ignore, for now, the two remaining possibilities: L (resp. F) is forever above F (resp. L). We return to these briefly after Proposition 1.

Proposition 1: *i) In a pre-emption market, the only pure strategy equilibrium is one in which firms i and j try to innovate at all dates after* x^I *if neither has innovated before that date. If i has innovated already, j innovates iff* θ^* *has elapsed since i's innovation.* Hence, the equilibrium outcome is: firm i (j) adopts at x^I with probability p (1-p) and the remaining firm adopts at $x^I + \theta^*$.

ii) *In a maturation market, the only pure strategy equilibrium is one in which firm i tries to innovate at all dates after* x^M *while j tries to innovate at all dates after* x^I, *if neither has innovated till that date. If i has innovated already, j innovates iff* θ^* *has elapsed since i's innovation.* The equilibrium outcome is i adopting at x^M and j at $x^M + \theta^*$. There are two asymmetric equilibria for i=1,2.

Proof: Since the proof of i) is contained in that for ii), we only demonstrate the latter. It is clear that if the game was ever at $x \geq x^I$, a dominant strategy for either firm is to move immediately (recall (A2): if both try to move, nature selects the actual entrant). It is now straightforward to check that the exhibited strategies in fact form a subgame perfect equilibrium. j anticipating the possibility of being a follower only after x^I, is best off leading at x^M or at any time x thareafter if the game ever gets there. This self-enforces i's decision to hold out till x^I.

That this is the only pure strategy equilibrium follows by a simple argument. Consider any such equilibrium. Clearly, for any $x < x^M$, it is a dominant strategy to not adopt. We first show that it cannot be the case that there is $x_i < x_j$, $x_i, x_j \in [x^M, x^I)$, with i adopting at x_i (respectively j adopting at x_j). For some $x_1 < x_j$, $F(x_j) \geq L(x)$, $x \in [x_1, x_j)$. So if i anticipates following at x_j, it is a dominant strategy for him to not adopt for $x \in [x_1, x_j)$. But then, it is a dominant strategy for j to adopt immediately at any such date, rather than wait till x_j. So in fact j moves at x_1. In turn, there is $x_j < x_2 < x_1$ such that for any $x \in [x_2, x_1)$, i's dominant strategy is to wait and consequently j's is to immediately lead. This argument works back to x_j. It is clear that a finite iteration of this logic in fact works back to x_i.[18] But then, i should not be moving at x_j. In other words, there can only be one leader in equilibrium. That implies of course the given strategies. •[19]

Remark: From the arguments above it should be clear how small a role the "no simultaneous move" assumption, (A2), plays. Beyond x^I, we have a "pure pre-emption game." If simultaneous moves are possible, all we require is that there are equilibria in such subgames. Clearly, such equilibria result in payoffs which are no more than $L(x^I) = F(x^I)$. The rest of the logic works exactly as stated. Possible equilibria may be in mixed strategies or the class of correlated equilibria Fudenberg-Tirole (1985) consider.

We shall call the equilibrium of Fig. 1, pre-emption equilibrium and that of Fig. 2, maturation equilibrium. Further, (see Tirole, 1989), we shall say that rents are dissipated if the full returns of a monopoly situation are not realized, i.e. if an equilibrium outcome is $x \neq x^M$, for the first entrant.

Corollary 2 : In a pre-emption equilibrium, rents are equalized and this is achieved through a dissipation of rent. In a maturation equilibrium a follower makes strictly higher profits, although a leader realises the full monopoly rent.

Finally, it should be remarked that although a leader makes less relative to the follower, he makes more of course relative to his alternative option as a follower, for which

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[18] Else, there is an accumulation point $\bar{x} < x^I$, s.t. $F(\bar{x}) = L(\bar{x})$.

[19]The remaining two possible configurations are: $L \geq F$ always. The equilibrium then is trivial: each firm attempts to move at every instant. The outcome is probabilistic entry by i at 0 and an optimal follow by j at θ^*. Conversely, $F \geq L$ throughout. The equilibrium strategies are: i moves at all $x \geq x_M$, j never moves if i has not moved before. The outcome is the rent preserving one of x^M, $x^M + \theta^*$. These two trivial equilibria are of course special versions of pre-emption and maturation equilibria respectively. From hereon we supress discussion of these trivial cases.

he has to wait till x^I. In a symmetric game (like Battle of the Sexes, whose equilibria are like the asymmetric eqilibria of the maturation market), equilibrium behavior does not of course allow us to say which of two asymmetric equilibria will actually get played, i.e. whether firm 1 or 2 will actually lead. In the next section, we show that even a little bit of asymmetry will resolve this issue.

4. Entrants Versus Incumbents

A repeated theme in both the popular and academic literature is that incumbents frequently have insufficient incentives to innovate. The argument rests on the fact of the "cannibalisation effect", that an incumbent's returns to innovation are only the net increase in returns, net of returns foregone on older products.[20] In this section we examine this hypothesis in our model. It will be very useful to make a distinction between a direct incumbent and an indirect incumbent. A direct incumbent is one who is (at period 0) in the relevant market (and is making some instantaneous profits $\pi \geq 0$). The size of the current profits π is then a measure of incumbent inertia. Our principal finding (Proposition 3) is that there is a critical level of profits, say ,$\hat{\pi}$ above which the cannibalisation effect directly leads to the non-incumbent (entrant) adopting first (and making lower lifetime profits). However, for $\pi < \hat{\pi}$ either firm could lead in equilibrium, and a forward induction argument suggests that in fact, an incumbent adopting first is the more reasonable outcome. The intuition for this result is the following: if an entrant were to be the low-tech player in the market it would introduce a technology earlier than that which a low-tech incumbent would introduce (this is precisely because of the "cannibalisation factor"). Hence, an entrant, but not an incumbent, can credibly signal early on a desire to be a high technology firm by passing up on the best low-tech entry point for itself.

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[20]One counter argument is clearly the "lock-in effect", whose extreme implication would be that a market lost is lost forever.

An indirect incumbent is one who may not be in the precise market under consideration but has better information about it, perhaps by virtue of selling similar products. A domestic firm facing competition from a foreign enterprise may be said to be in such a situation. As an index of indirect incumbency advantage, we shall maintain as a hypothesis, that the incumbent is better able to maintain a monopoly position. We shall translate this to mean that an incumbent makes mR, $m \geq 1$, in a monopoly situation, whereas an entrant only makes R (as before). m is of course an index of incumbency advantage. We show (Proposition 4) that for <u>any</u> advantage (i.e. for any $m > 1$), the <u>unique</u> equilibrium is one in which the <u>incumbent</u> necessarily adopts first. As long as m is not very large (i.e. the monopoly advantage is not unilaterally big), the incumbent, despite his advantage, makes strictly <u>less</u> in equilibrium.

4.1 <u>Direct Incumbents</u>

From hereon, let firm 1 refer to the incumbent and firm 2 to the entrant. Then, between period 0 and the first adoption x, firm 1 makes a flow profit $\delta\pi$ (and firm 2, the entrant, makes nothing). So,

$$F_1(x) = (1 - e^{-\delta x})\pi + F(x) \qquad (11)$$

$$L_1(x) = (1 - e^{-\delta x})\pi + L(x)$$

Of course, $F_2 = F$ and $L_2 = L$. It is immediate that x_i^I, the intersection of F_i and L_i are identical for both firms. Hence, beyond x^I both firms will try to adopt if neither has adopted till that point. Further, precisely because adopting a new product means foregoing current profits π, if firm 1 <u>had</u> to lead it would lead later than firm 2 in a similar situation, i.e. $x_1^M > x_2^M$.

(please insert Figure 3 here)

In Fig. 3, there are clearly two equilibria: the first has the entrant moving at $x \geq x_2^M$, if the other has not moved yet. This is done in anticipation of the incumbent (firm 1) only adopting at $x \geq x^I$, if neither has adopted till such point. Of course, as a follower each follows after the optimal gap of θ^*. The second equilibria has the roles reversed with firm 1 (the incumbent) leading at x_1^M, and firm 2 only leading after x^I. Since $L(x_2^M) < F(x_1^M)$, the entrant would rather follow at x_1^M, than lead at x_2^M. So, the cannibalisation effect, if small, does not rule out the incumbent leading. In fact, since an entrant would rather follow than lead, one could appeal to the forward induction ideas of Kohlberg-Mertens (1986) to argue that an entrant can credibly signal to an incumbent that he is not going to lead. For instance, suppose an incumbent observes at x_1^M that firm 2 does not adopt (and passes up adoption also for $x \in [x_2^M, x_1^M)$. The only credible inference that the incumbent can draw from this, is that the two firms should coordinate on the second equilibria and he should adopt at x_1^M. The idea is very simple. Both players know that before x_1^M, firm 1 has a dominant incentive to not innovate. However, if 2 passes up the opportunity to lead at x_2^M, then a reasonable (and forward induction says the only reasonable) inference is " 2 passed up the maximum it could have guaranteed as a leader. It would not have done so if it was waiting to be a leader at some other point. Hence, it follows that it must be playing to be the high tech firm in the industry, i.e. the relevant equilibrium is the one with firm 1 innovating first." Note that firm 1 cannot credibly signal before 2's adoption date, precisely because the cannibalisation factor means it is strictly better off not innovating early.

When does such a signal cease to be credible for the entrant? Note that as π increases, x_2^M and x^I remain unchanged (i.e. the entrant's absolute situation is unchanged). But x_1^M increases to the right (the incumbent becomes more and more inert). Let x^* denote the date at which $L(x_2^M) = F(x^*)$. Clearly, if $x_1^M > x^*$, the entrant would do strictly better by moving as a leader at x_2^M. Put differently, there is a critical profit level $\hat{\pi}$, below which the entrant can credibly signal his unwillingness to lead and force the incumbent, despite the cannibalisation effect, into a leadership position. For $\pi \geq \hat{\pi}$, the cannibalisation effect dominates. So we have,

Proposition 3: *Suppose the symmetric game had a maturation equilibrium.* Then, in the asymmetric equilibrium,under direct incumbency, for $\pi \leq \hat{\pi}$, the unique forward induction proof outcome is: incumbent adopts at x_1^M, entrant follows at $x_1^M + \theta^*$. For $\pi > \hat{\pi}$, the unique subgame perfect equilibrium outcome is: entrant adopts at x_2^M, incumbent follows at $x_2^M + \theta^*$.

If the symmetric game had a pre-emption equilibrium, then so does the asymmetric game with an outcome: probabilistic move by firm i at x^I, j follows at $x^I + \theta^*$.

4.2 Indirect Incumbent

An indirect incumbent, firm 1, is better able to exploit a monopoly position, and hence makes mR(x) as a monopolist, where $m \geq 1$. Hence,

$$L_1(x) = e^{-\delta x}\left\{(1 - e^{-\delta\theta^*})mR(x) + e^{-\delta\theta^*}r(-\theta^*)\right\} \tag{12}$$

$$F_1(x) = F(x)$$

For expositional purposes, in this sub-section we assume $r(-\theta^*) = 0$. The reader is invited to check that none of the results are predicated on this; it merely makes the presentation a lot clearer. Then,

$L_1(x) = mL(x)$. Of course, $L_2 = L$, $F_2 = F$. Clearly, it follows that (starting from a maturation equilibrium in the symmetric game), we have figure 4.

(please insert Figure 4 here)

Clearly, in any equilibrium, a dominant strategy for firm 1 is to adopt beyond x_1^I if neither has adopted before. But then, there is $x^1 < x_1^I$ such that it is a dominant strategy for 2 to not adopt at any $x \in [x^1, x_1^I)$. x^1 is formally defined through

$$x^1 = \max \left\{ z: L(x) \leqq F(x_1^I) \quad x \geq z \right\} \tag{13}$$

In the figure, $x^1 = 0$. More generally, there is some left neighborhood of x_1^I, in which firm 2 does better by waiting to follow, than by leading. Given this, firm 1's dominant strategy is to lead on $[x^1, x_1^I)$. An identical argument as in Proposition 1 now leads through an iterated elimination of dominated strategies to: firm 1 adopts at x^M (and any time thereafter). The entrant, firm 2 follows at $x^M + \theta^*$. Note, despite the incumbency advantage, the entrant makes strictly more than the incumbent.

As $m\uparrow$, x_1^I decreases and hence at some critical advantage $\hat{m}$, $x_1^I = x^M$. Clearly, for any $m \geq \hat{m}$, the equilibrium outcome is incumbent moves at x^M and makes more than the entrant.

Proposition 4: *Suppose the symmetric game has a maturation equilibrium.* Then, there is some critical incumbency advantage $\hat{m}$ s.t. i) $m < \hat{m}$: the unique equilibrium has incumbent adopt at x^M, the entrant at $x^M + \theta^*$. No rent is dissipated but the entrant makes strictly more in equilibrium.

ii) $m \geq \hat{m}$: the unique equilibrium has the same outcome as above, but the incumbency advantage is sufficiently big to overwhelm the first mover disadvantage. The incumbent makes more.

Finally, if the symmetric equilibrium was a pre-emption equilibrium, incumbent adopting at x^I and entrant following at $x^I + \theta^*$ is an equilibrium outcome.

5. Consumer Diversity, Learning and Lock-In

The question we now turn to is: what underlying market conditions results in pre-emption or maturation outcomes? We focus first on diversity in consumer preferences, which for a fixed "differentiation technology", is the exclusive determinant of payoffs to differentiation. Secondly, we investigate the very nature of the differentiation technology, in particular emphasizing the elements of learning and market lock-in. The conjecture that "as maturation becomes more profitable, maturation equilibria are more likely", may or may not be true. The problem is that pre-emption may simultaneously also become more profitable. This could arise, for instance, from an increase in consumer preference for diversity. On the other hand, if the returns to being a first innovator decline substantially, a firm may be willing to lead only for sufficiently large monopoly rents, i.e. for a sufficiently delayed first innovation. However, following after such a long lag may be less attractive than earlier pre-emption. In what follows we isolate robust sufficient conditions under which such anomalous behavior does not arise and in fact the conjecture is exactly true. In particular, we establish critical levels of the "primitives", consumer diversity, learning possibilities and lack of market lock-in, such that there are maturation equilibria above these levels and pre-emption equilibria below them.

5.1 Consumer Diversity

By consumer preferences over diversity, we mean the distribution of consumer preferences for quality. In Example 2.2, consumer preference for quality (which scales the utility function) is uniformly distributed on [a,b], $0 < a < b$. An increase in diversity could be modelled by a decrease in $\frac{a}{b}$ or a-b. Notice that if b also changes, there is a level effect as well. To concentrate all attention on diversity effects in a duopoly, consider (equivalently) decreases in $\frac{a}{b}$ or a-b, with b fixed.

A change in consumer diversity has in general several effects. It will typically affect the optimal product diversity θ^*. This in turn affects the relative attractiveness of being a leader by both affecting monopoly rents and the returns as a technological laggard in a duopoly. The two effects might conflict. Even if they do not, the change in the relative attractiveness of leaders to followers is ambiguous. To facilitate analysis, in this sub-section we consequently restrict attention to Example 2.2. It is easy to infer that the optimal product diversity θ^* is unchanged, when consumer diversity changes. Further,

$$L(x) = e^{-\delta x}\left\{\frac{(1 - e^{-1})b^2}{4}x + (\delta e)^{-1}\left(\frac{b - 2a}{3}\right)^2\right\}$$

$$\equiv e^{-\delta x}\{\lambda_1 x + \lambda_2(b - a)\}$$

$$F(x) = e^{-\delta x}(\delta e)^{-1}\left(\frac{2b - a}{3}\right)^2$$

$$\equiv e^{-\delta x}\phi(b - a) \qquad (14)$$

An increase in consumer diversity increases a follower's lifetime returns, ($\phi(b - a)$ is increasing), since the payoffs to differentiation increase. On the other hand, it also increases a technological laggard's returns ($\lambda_2(b - a)$ is increasing). It is easy to see that we have a

Maturation equilibrium $(x^M < x^I) \Leftrightarrow \frac{a}{b} < \xi$, and a

Pre-emption equilibrium $(x^M > x^I) \Leftrightarrow \frac{a}{b} > \xi$, where $\xi = 2 - \frac{3}{2}\left[e\left(1 - \frac{1}{e}\right)\right]^{\frac{1}{2}}$.

It is also clear from (14) that as diversity increases, a potential leader would like to innovate earlier, i.e. $x^M(b\text{-}a)$ is a decreasing function. This follows directly from the fact that not adopting has a higher waiting cost, the postponement of greater duopoly returns $\lambda_2(b - a)$.

Proposition 5: In Example 2.2, there is a critical consumer diversity ξ s.t. the equilibria are maturation (pre-emption) for diversity more (less) than ξ. In maturation equilibrium, increasing consumer diversity decreases the level of innovation, i.e. x^M decreases, and leaves the product diversity θ^* unchanged. The profits of both firms increase.

(As the algebra demonstrates, in the simple version of this example all equilibria are in fact pre-emption equilibria. However, the example is illustrative of the more general principle that if the relative attractiveness of a follower increases and the first introduction time is faster, then increases in preference diversity make maturation equilibria more likely. Note also that with more general parametric specifications, even in this example both pre-emption and maturation equilibria emerge.)

Obviously the analysis is unchanged if duopoly returns are more generally of the form, $l(b-a)r(\theta)$ and $f(b-a)r(-\theta)$. $f' \gtreqless 0$ is equivalent to the level of innovation increasing (decreasing) in a maturation equilibrium.

5.2 Learning and Market Lock-In

Suppose that after the first adoption at x by firm i, firm j's payoff relevant state evolves at a rate γ^{-1}. γ is the growth rate net of the conflicting effects of learning/imitation and market lock in. For any γ, there is an optimal entry gap θ_γ determined from the follower's problem (see 3.1). In particular, firm j enters after a time period $\gamma\theta_\gamma$, during which the state has grown by the amount θ_γ. So,

$$\begin{aligned} F_\gamma(x) &= e^{-\delta x}\left\{ e^{-\delta\gamma\theta_\gamma} r(\theta_\gamma) \right\} \equiv e^{-\delta x}\phi(\gamma) \\ L_\gamma(x) &= e^{-\delta x}\left\{(1 - e^{-\delta\gamma\theta_\gamma}) R(x) + e^{-\delta\gamma\theta_\gamma} r(-\theta\gamma)\right\} \\ &\equiv e^{-\delta x}\left\{\lambda_1(\gamma)R(x) + \lambda_2(\gamma)\right\} \end{aligned} \tag{15}$$

As before, the implication of changes in the differentiation technology, γ, for a potential leader could be either through monopoly returns ($\lambda_1(\gamma)$) or duopoly returns ($\lambda_2(\gamma)$). To simplify the analysis, we consider only the first effect, and assume for this sub-section $r(-\theta) \equiv 0$ (and hence, $\lambda_2(\gamma) = 0$).

So, $L_\gamma(x) = e^{-\delta x}R(x)\lambda_1(\gamma)$, and we have the situation of Figure 5 (where L' and F' refer to $e^{-\delta x}R(x)\lambda_1(\gamma')$ and $e^{-\delta x}\phi(\gamma')$ respectively).

(please insert Figure 5 here)

Fig. 5 shows a case where an increase in learning possibilities (γ' rather than γ), leads to a shorter time-lag of adoption, thereby lowering leader's returns. Of course, the follower must necessarily make more as a faster learner. Since the monopoly returns, $\lambda_1(\gamma)$, depend precisely on the adoption time-lag, as long as the follower takes less time imitating, when the imitation or differentiation technology improves, decreasing γ makes maturation more likely. Suppose $\gamma\theta_\gamma$ is in fact increasing in γ.

Proposition 6: There is a critical γ^* s.t. for all $\gamma < \gamma^*$ the equilibrium is a maturation equilibrium with outcomes x^M (independent of γ) and $x^M + \theta_\gamma$. For all , $\gamma > \gamma^*$ the equilibrium is a pre-emption equilibrium, with outcomes $x^I(\gamma)$ and $x^I(\gamma) + \theta_\gamma$. $x^I(\gamma)$ is decreasing in γ, i.e. the less productive is the differentiation technology, the lower is the level of innovation.

5.3 Productivity of Basic Technology

A last comparative static exercise of interest is to ask what happens if the underlying technology grows at a rate ρ^{-1} till the first adoption (and then, at 1 thereafter). The question is: what is the effect of the technological environment on market performance? This case is not equivalent to changes in imitation technology (5.2) as we will see.

$$F_\rho(x) = e^{-\rho\delta x}\phi \tag{17}$$

$$L_\rho(x) = e^{-\rho\delta x}\left\{\lambda_1 R(x) + \lambda_2\right\}$$

Arguments as above establish.

Proposition 7: Suppose R is concave. Then, there is a critical level of technological growth, say $\bar{\rho}$, such that

$\rho > \bar{\rho} \Rightarrow$ maturation equilibrium. Entry times are decreasing and profits are decreasing in

$\underline{\rho}$.

$\rho < \bar{\rho}$ $\Rightarrow$ pre-emption equilibrium, all of whose outcomes (independently of ρ) are x^I and $x^I + \theta^*$. Firm's profits are decreasing in ρ.

6. Does the Market Under Innovate?

One motivation for this paper was to examine the validity of the conjecture: in imperfectly competitive markets there is a tendency to generate too little of innovation, as a consequence of threats of pre-emption. The bench-mark for such a comparison could be taken to be either a cartel's or a social planner's choices. Since neither is faced with the possibility of a rival pre-empting, the argument goes that the level of innovation, i.e. the quality or time of first entry should be higher in such contexts. We examine the validity of this conjecture and show that product differentiation possibilities fundamentally affect the market's innovation incentives. In general, there are two conflicting determinants of the comparison between the two classes of outcomes. Consider a cartel. On the one hand, the cartel internalises the returns to differentiated products and hence may innovate earlier since the waiting costs are consequently greater. On the other hand, the cartel is better able to control the flow of innovations and may be able to generate monopoly returns for a longer period. This would lead to a later first innovation, since it is important to have a better quality in the first innovation. The fact that there are these conflicting incentives suggests of course that the comparative statics are ambiguous. As an illustration we give sufficient conditions under which in any maturation equilibrium, the low technology leader in fact innovates too late relative to both the cartel and social optimum. For a social planner the trade-offs are similar although the returns are different. We restrict ourselves to Example 2.2 and derive the "over innovation of duopoly" result.

Product diversity, i.e. θ, will differ in the three contexts. Cartels and social planners have to trade-off "cannibalisation effects" against "efficiency effects" and the outcome is hence, ambiguous.

6.1 The Cartel Solution

A cartel picks two adoption dates x and x + θ. Let P(θ) denote the cartel's optimum profits when two products, of diversity θ, are in the market and suppose that P(θ) > r(θ) for all θ. To maintain consistency with the duopoly problem, we assume that this optimum profit only depends on relative quality. Also to keep the two structures comparable, we assume that a cartel will in fact innovate twice. The second adoption problem is

$$\max_{\theta \geq 0} (1 - e^{-\delta\theta})R(x) + e^{-\delta\theta}P(\theta) \tag{18}$$

Let us suppose that a solution to this problem exists, denote a solution by θ_x and consider the first adoption problem.

$$\max_{x \geq 0} \; e^{-\delta x} \left\{ (1 - e^{-\delta\theta_x})R(x) + e^{-\delta\theta_x} P(\theta_x) \right\} \tag{19}$$

A first order condition for this problem is:

$$[R'(x_c) - \delta R(x_c)] (1 - e^{-\delta\theta_x}) = \delta e^{-\delta\theta_x} P(\theta_x) \tag{20}$$

In contrast a leader in a potential duopoly maximizes rents by selecting a level of innovation x^M such that

$$[R'(x^M) - \delta R(x^M)] (1 - e^{-\delta\theta^*}) = \delta e^{-\delta\theta^*} r(-\theta^*) \tag{21}$$

Comparing the trade-offs in (20) - (21) yields

Proposition 9: *Suppose that monopoly returns are concave in quality. Suppose further that either* i) $r(-\theta^*) = 0$ or ii) $\theta_x \leq \theta^*$. Then, $x_c \leq x_M$, i.e. a cartel innovates earlier than a duopoly in a maturation equilibrium.

Remark The optimal product diversity may, in general, be more or less than the amount of product differentiation in a duopoly equilibrium, θ^*. Further information on joint profits $P(\theta)$ would be required to answer that question.

6.2 The Social Optimum

Consider the social planner's problem of maximizing some index of social welfare by the choice of an innovation and differentiation decision. The obvious indices would be consumer surplus (if the primitives are demand functions) or aggregate utility (if the underlying consumer preferences are specified). We illustrate with Example 2.2, using aggregate utility.

It is easy to see that the social welfare problem is

$$\text{Max } e^{-\delta x} \left\{ x(1 - e^{-\delta\theta}) + (x + \theta)e^{-\delta\theta} \right\} \tag{22}$$

It is straightforward to see that the social diversity $\theta_S = \theta^*$. Comparing the social planner's first adoption problem with that of a potential leader in a duopoly, some straightforward algebra yields

Proposition 10: In Example 2.2, a social planner adopts at a date sooner than that in a duopoly maturation equilibrium. The amount of product diversity is identical in the two cases.

7. Extensions

Let us briefly discuss some extensions of the current model. The principal assumption which facilitated analysis is (A1), that duopoly returns depend only on relative qualities. Dropping this assumption changes some details but not the main substance of the results. It is clear that in a general setting the optimal amount of product differentiation engaged in by a follower will depend on the level of the first innovation. Denote this dependence $\theta(x)$. The principal complication arises from not knowing, in a general formulation, any qualitative features about this optimal reaction function. For example, if $\theta(x)$ is increasing in x, the above analysis and results remain completely unchanged. Else, it would still be the case that there are only two kinds of equilibria. There may however be several possible maturation or pre-emption equilibria in a single game.

The assumption of deterministic technological change chosen in this paper is virtually inessential to the general analysis. In fact in many interesting applications, like the timing of asset sales or learning about market characteristics, an uncertainty formulation is natural. A similar comment pertains to the requirement that the maturation process be a non-decreasing process as in this paper. For quality of information that is a natural restriction, but not for content of information. Dutta-Rustichini (1990b) develop a general theory for games of entry which can handle such issues.

As argued in the introduction repeat innovations are an impotant stylised fact of the innovative process. In a sense this is the classic repeated oligopoly problem but not quite. Quality or product attributes are costly and less reversible choices than prices. Hence the issue here is much closer to that of a dynamic game with historical product choices affecting current decisions. Such a dynamic analysis is contained in Dutta-Rustichini (1990a). Finally, it should be pointed out that our analysis remains unchanged if we drop the irreversibility of quality assumption. Similar issues as in the case of level-dependent returns arise.

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Figure 1

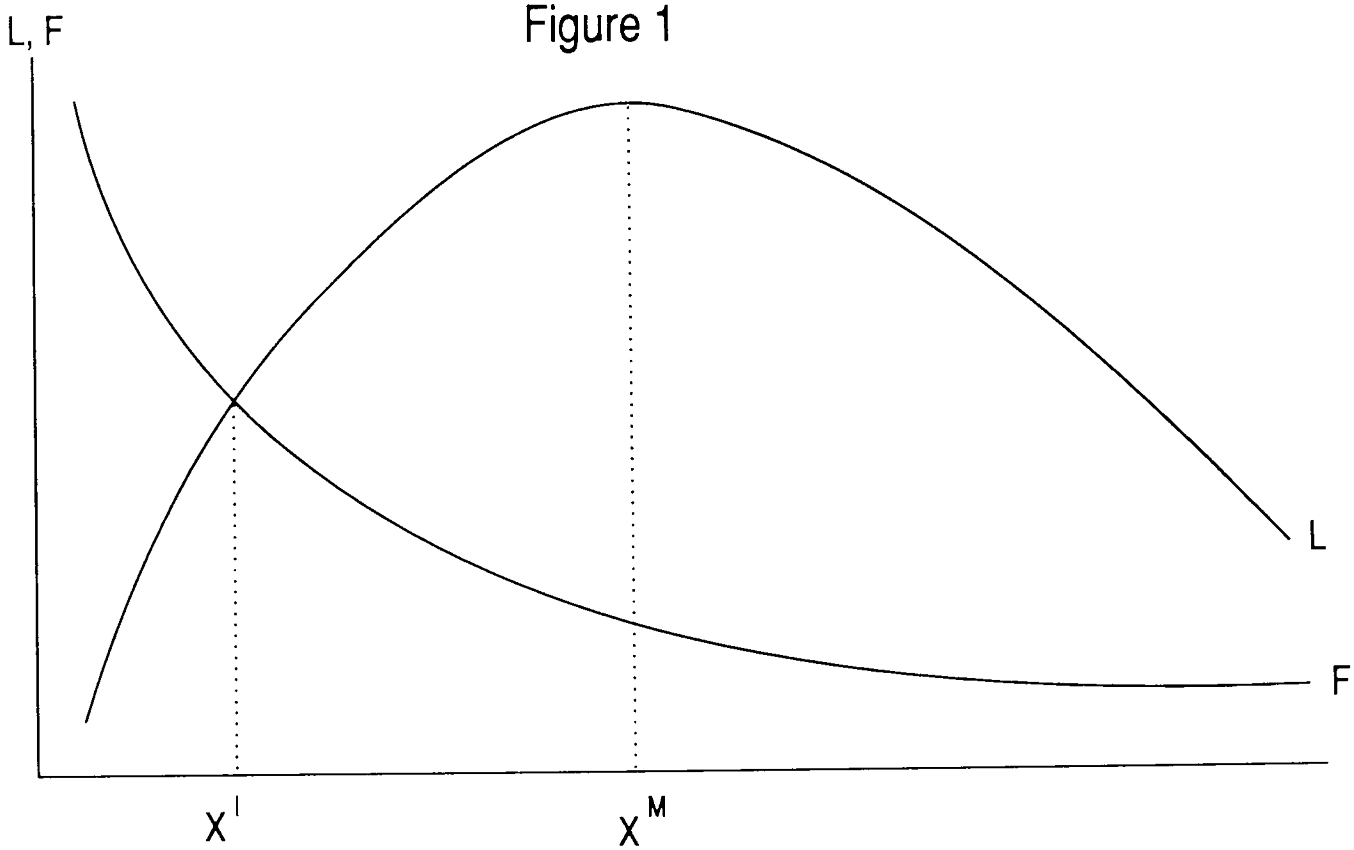
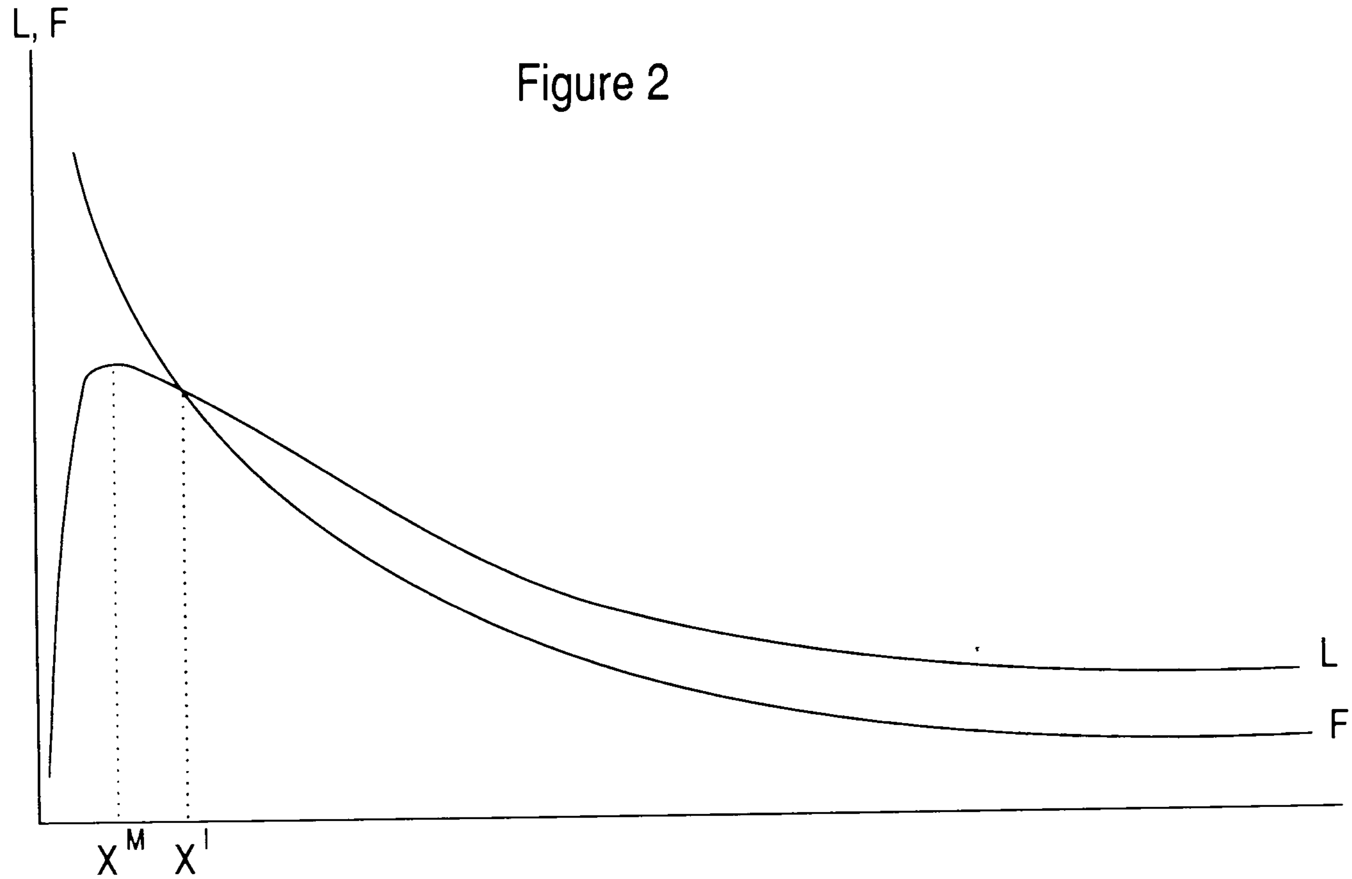


Figure 2



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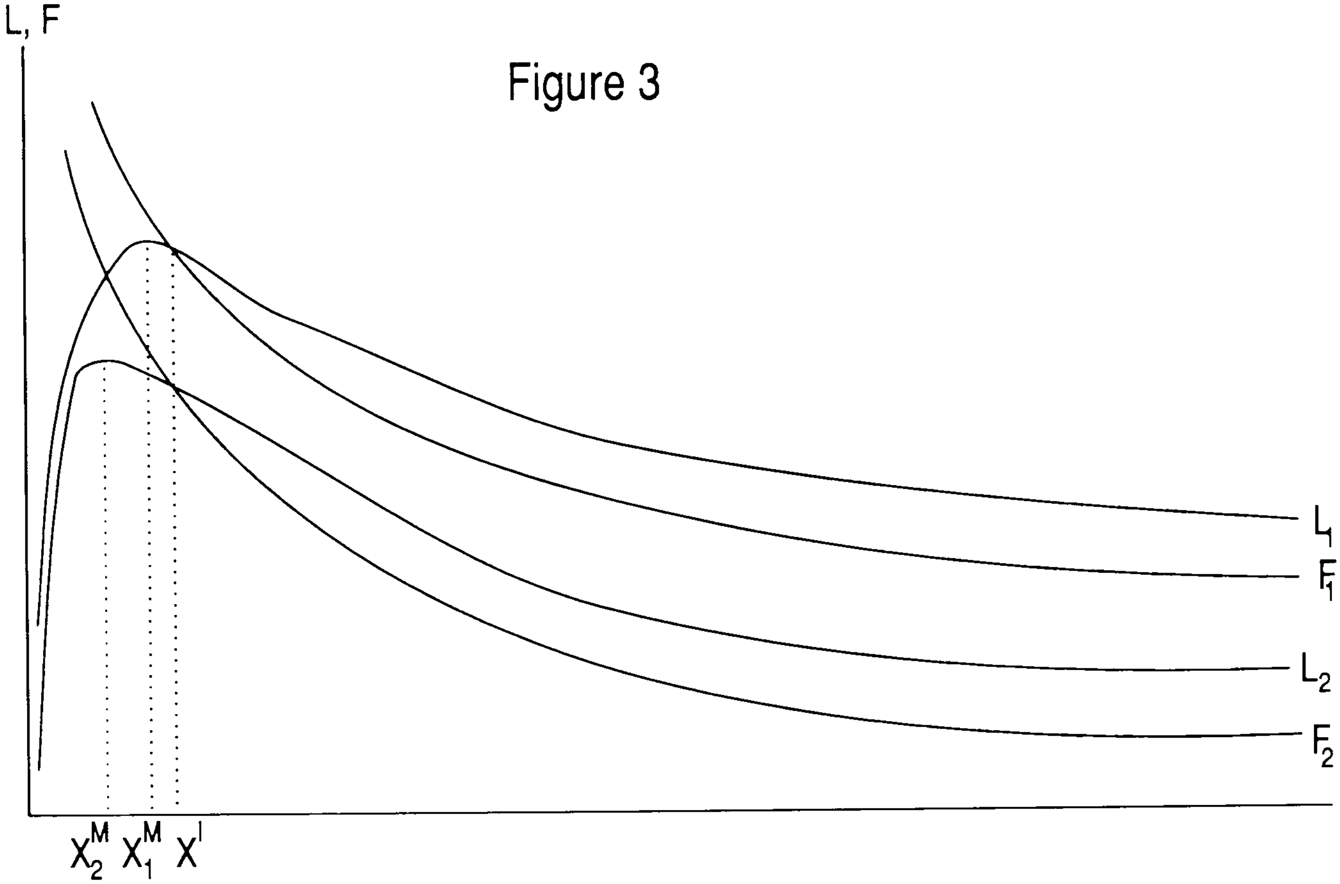

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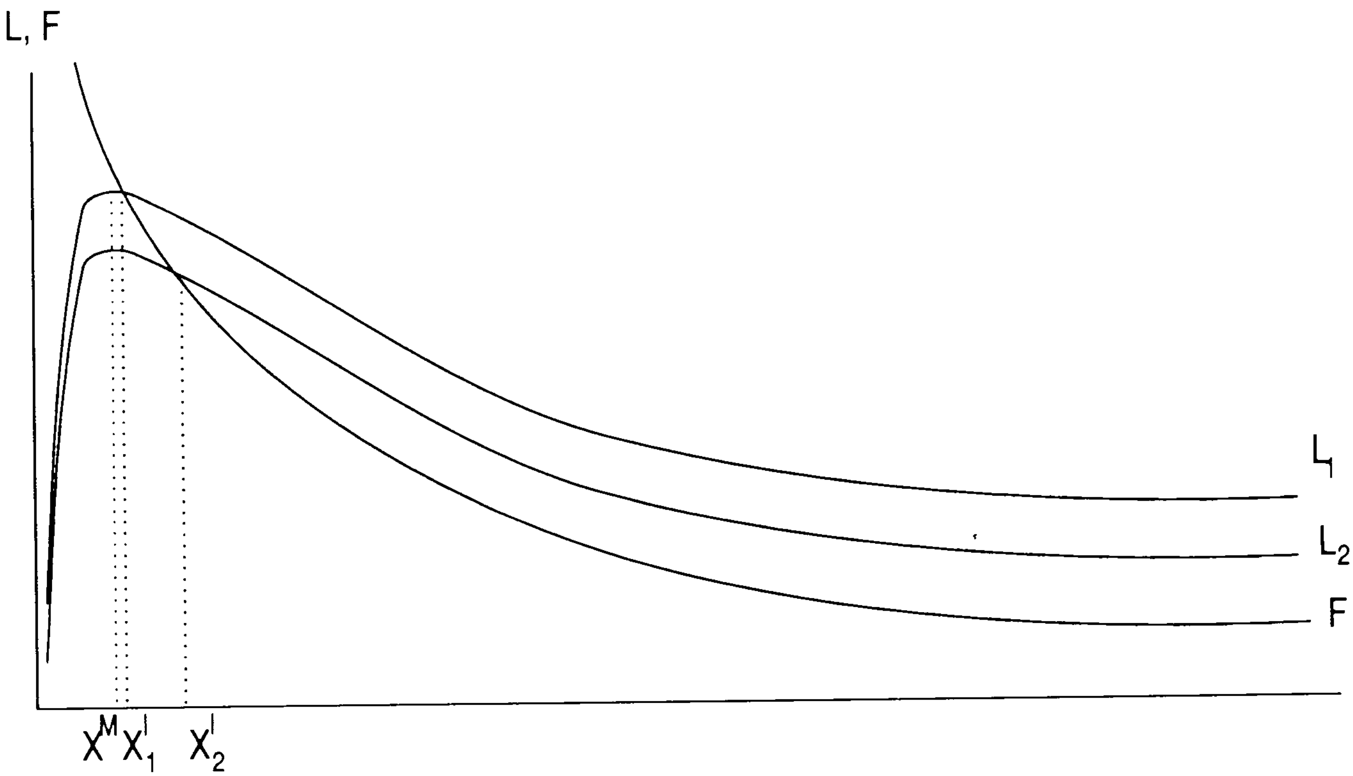

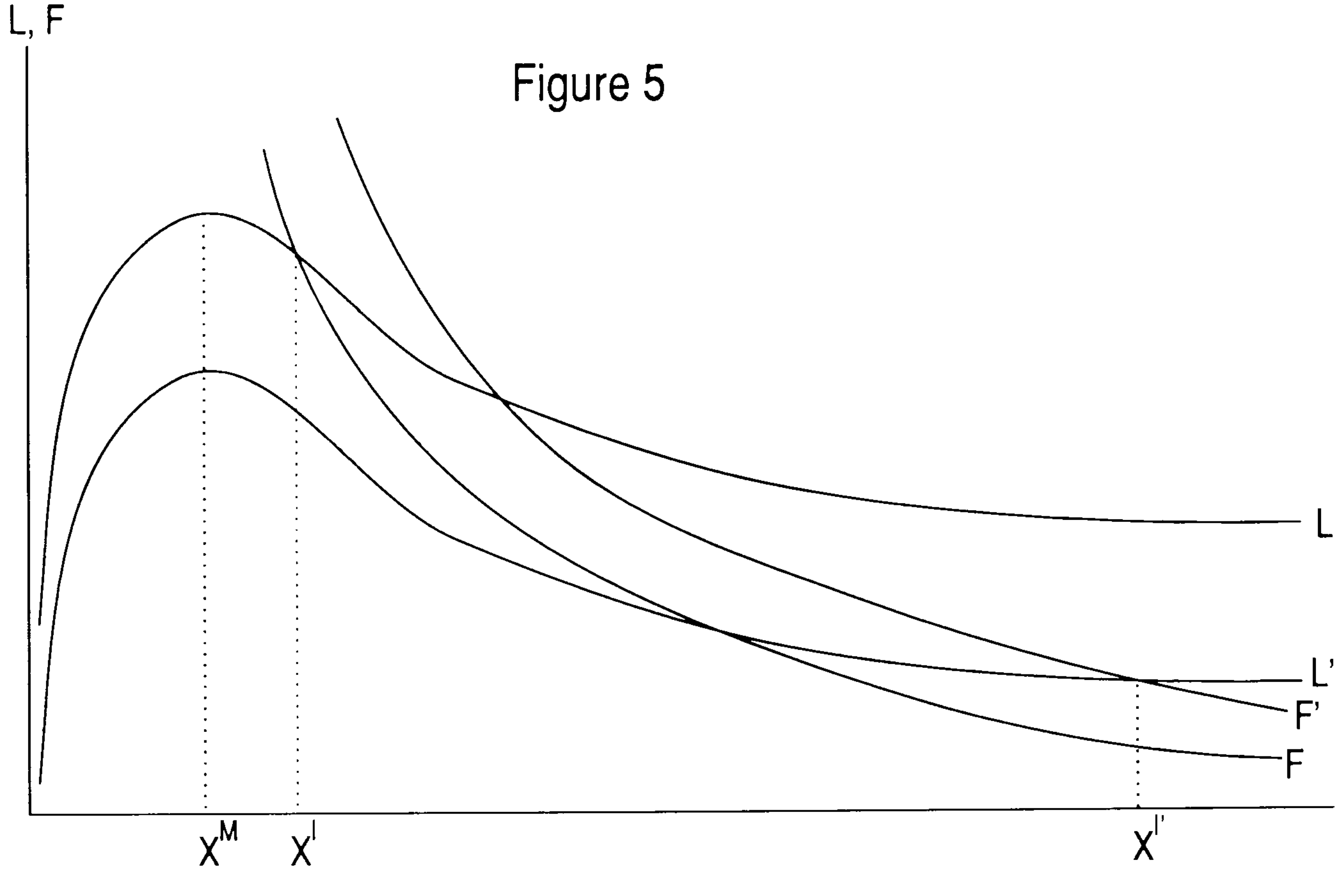
L, F
Figure 5
L
L'
F'
F
X^M
X^I
$X^{I'}$