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LOCATION EQUILIBRIA UNDER ALTERNATIVE SOLUTION CONCEPTS*

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ABSTRACT

We provide a comparison of the location equilibria in a duopoly model under three alternative solution concepts. The first one is a simultaneous price and location game, the second is a two-stage location-then-price game. Third, we introduce a new solution concept, a two-stage price-then-location game. It is well known that no (pure strategy) equilibrium usually exists under the first two solution concepts when products are homogeneous. We show this is also true for the third concept. However, introducing sufficient product heterogeneity in a specific manner restores the existence of equilibrium in each case. We argue that, under certain "regularity" conditions, equilibrium locations are farther apart under the location-then-price game than in the simultaneous game. We provide simulation results for a specific functional form (the logit model) which illustrates this result, and show that locations in the price-then-location game may be either closer or farther from the center than the simultaneous game. Another contribution of the paper is to introduce a no-purchase option into the logit model of spatial competition.

I. Introduction

The objective of this paper is to compare equilibria for a duopoly competing over mill-prices and product locations. We consider three alternative equilibrium concepts. Under the first one, a one-stage game, prices and locations can be viewed as being simultaneously determined by firms. Under the second (a two-stage game), these are viewed as sequential decisions: locations are chosen bearing in mind the subsequent price equilibrium. The third equilibrium concept, which has not so far been considered in the context of spatial competition, reverses the order of the standard two-stage game. That is, <u>prices</u> are chosen first, and these are predicated on the known outcome of the ensuing <u>location</u> equilibrium.

The appropriateness of a particular game structure (simultaneous, location-then-price and price-then-location in the present case) is a matter of considerable debate. Different assumptions at this level yield different results. It has been suggested that the degree of flexibility of a given strategic variable (price and location here) should determine the "order of play".[1] From this viewpoint, if prices are less costly to adjust than locations, then a two-stage location-then-price equilibrium whereby locations are determined "prior" to prices is deemed the relevant equilibrium concept. If we use the relative flexibility criterion to determine the appropriate solution concept, then we can envisage situations where any one of our game structures is the pertinent one.

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[1] See Friedman (1983) for a similar discussion as to whether prices or quantities (i.e., Bertrand vs. Cournot competition) constitute the relevant decision variables.

For many location problems, such as store and product locations, prices are naturally viewed as more flexible than locations. Hence the preponderance of the location-then-price equilibrium in literature. On the other hand, when there are significant menu costs, or when prices are widely advertised, whereas locations are relatively easily changed, then the price-then-location equilibrium is perhaps the most apt. Somewhat ironically, the oft-cited example of ice-cream sellers on a beach may correspond more to the latter situation. Lastly, when both decisions entail broadly similar adjustment costs, then the simultaneous price-and-location equilibrium comes into play.

The above justification for the location-then-price equilibrium is often given. Yet we are aware of no explicit game-theoretical justification for such an argument. Perhaps indeed the relevant equilibrium concept can be determined endogenously from structural data.[2] Our purpose here is simply to compare the alternative solutions. That is, we do not necessarily condone the flexibility argument. Rather, at this stage, we wish to explore the implications of the alternatives.

In the next section of the paper we define the various equilibrium concepts and describe the model of duopoly competition under mill-pricing. Previous results on the (non-)existence of equilibrium are given for the simultaneous and location-then-price games when products sold are homogeneous. We then show there is no equilibrium (in pure strategies) in this context under the price-then-location game. In Section 3 we relax the assumption of product homogeneity. We prove that when a symmetric equilibrium exists it is

[2] See Thisse and Vives (1988) for a model of endogenous determination of spatial price policies in this vein.

necessarily at the center for both the simultaneous and price-then-location games. We then compute the equilibrium for the location-then-price game and find that firms may be dispersed. In order to broaden the basis of our comparison we extend the model in Section 4 to a situation where consumers can choose between the goods sold by the two firms and an outside option. Equilibrium locations and profits are computed and compared for the three games. Conclusions are discussed in Section 5.

II. (Non-)Existence of Equilibrium for the Three Games: The Case of Homogeneous Products.

To begin with, we cast our analysis (and illustrate the problem of existence of equilibrium) in the standard Hotelling (1929) model of horizontal product differentiation. As we shall show, there is no equilibrium (in pure strategies) for each of the three solution concepts considered under the original Hotelling specification of the primitives of the model.

The location space is a unit segment [0,1] over which consumers are located with unit density. There are two firms located at x_1, $x_2 \in [0,1]$. Each firm sets a mill price, p_1 and p_2 respectively, and each consumer will buy one unit of the product - which is homogeneous apart from its location - from one or the other firm. Consumers bear the cost of transporting the product and this transport cost is a linear function of the distance between consumer and firm. Hence the model can be thought of as one of shopping behavior. It can also be interpreted as a product selection problem where consumers have different preferences over the characteristics of products. We shall assume there are no production costs.

Given the assumptions above, consumers purchase from the firm quoting the lower full price, defined as mill price plus transport cost, i.e., a consumer at $x \in [0,1]$ will buy from Firm 1 if

$$p_1 + t|x - x_1| < p_2 + t|x - x_2| \tag{1}$$

where t is the transport cost rate. Firm 1's demand for $x_1 \leq x_2$ is then given by

$$D_1 = \begin{cases} 0 & \text{if } p_1 \geq p_2 + t(x_2 - x_1) \\ \dfrac{p_2 - p_1}{2t} + \dfrac{x_2 + x_1}{2} & \text{if } p_2 - t(x_2 - x_1) < p_1 < p_2 + t(x_2 - x_1) \\ 1 & \text{if } p_1 \leq p_2 - t(x_2 - x_1) \end{cases}$$

and $D_2 = 1 - D_1$.

Note the demand discontinuities at the critical prices where Firm 1 undercuts Firm 2's mill price and where Firm 1's mill price is undercut by Firm 2. Firm 1's profit is then

$$\Pi_1 = p_1 D_1 \tag{2}$$

and similarly for Firm 2.

a) Simultaneous Price and Location Equilibrium

Perhaps the most obvious equilibrium to consider is a standard Nash equilibrium whereby each firm simultaneously chooses its price and location.

Formally, a simultaneous (one-stage) price and location equilibrium is defined by a quadruple $(p_1^*, p_2^*, x_1^*, x_2^*)$ satisfying

$$\Pi_1(p_1^*, p_2^*, x_1^*, x_2^*) \geq \Pi_1 (p_1, p_2^*, x_1, x_2^*) \tag{3}$$

for all $x_1 \in [0,1]$ and for all $p_1 \geq 0$; and similarly for Firm 2.

A little reflection shows this game has no (pure strategy) equilibrium when products are homogeneous. The argument is as follows. Suppose we have a candidate equilibrium where the firms charge different prices and both have positive market share (note each firm can always ensure a positive market share). Then the firm charging the lower price necessarily can increase profit by locating next to its rival and capturing the whole market. Now, if both firms charge the same positive price, one can locate next to the other and undercut its price by an infinitesimal amount and again gain the whole market. Hence there can be no equilibrium. (This result applies to more general situations than the model considered here - see Novshek (1980)).

b) Sequential Location-Then-Price Equilibrium

The original solution concept envisioned by Hotelling was a two-stage process whereby locations are chosen in full anticipation of the ensuing price equilibrium. The solution is defined recursively. In the second stage, for given locations $(\dot{x}_1, \dot{x}_2)$, a price equilibrium is defined by a pair $(\bar{p}_1, \bar{p}_2)$ such that

$$\Pi_1(\bar{p}_1, \bar{p}_2; \dot{x}_1, \dot{x}_2) \geq \Pi_1(p_1, \bar{p}_2; \dot{x}_1, \dot{x}_2) \text{ for all } p_1 \geq 0. \tag{4}$$

and similarly for Firm 2. Given the solution to this problem, the first-stage profit functions are written as

$$\hat{\Pi}_1(\bar{x}_1, \bar{x}_2) = \Pi_1(\tilde{p}_1(\bar{x}_1, \bar{x}_2), \tilde{p}_2(\bar{x}_1, \bar{x}_2) ; \bar{x}_1, \bar{x}_2)$$

and likewise for Firm 2 so that equilibrium to the first-stage location game is characterized by a pair (x_1^*, x_2^*) which satisfies

$$\hat{\Pi}_1(x_1^*, x_2^*) \geq \hat{\Pi}_1(x_1, x_2^*) \quad \text{for all } x_1 \in [0,1] \tag{5}$$

and similarly for Firm 2. The full equilibrium to this game is therefore a quadruple $(p_1^*, p_2^*, x_1^*, x_2^*)$ with $p_1^* = \tilde{p}_1(x_1^*, x_2^*)$ and $p_2^* = \tilde{p}_2(x_1^*, x_2^*)$.

Under our present (Hotelling) assumptions there is no (pure strategy) equilibrium. The argument is given in d'Aspremont et al. (1979). The problem is that an equilibrium in the second stage of the game (the price stage) does not exist when firms are close to each other and, furthermore, when a second stage price equilibrium does exist, firms wish to move closer. Hence they are drawn exonerably closer to the region of non-existence of a second-stage solution and so do not know the payoffs for certain feasible locations. The profit functions therefore cannot be defined for all relevant locations so that no equilibrium can be said to exist.

We should note that equilibrium does exist for this game when transport costs are quadratic in distance.[3] However, it is still true that it fails to exist for the other two equilibrium concepts we consider.

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[3] See d'Aspremont et al (1979) for this result. Note however that non-existence prevails for the "linear-quadratic" class of transport cost functions $\tau(d) = ad + bd^2$ for a, b > 0 where d is the distance between firm and consumer - see Anderson (1988) for further details.

c) <u>Sequential Price-Then-Location Equilibrium</u>

Given the two solution concepts discussed above, it is a natural idea to look at the two stage game with the reverse ordering of stages. That is, we suppose now that prices are determined in the first stage and locations in the second - the first-stage price choices take into account the subsequent location equilibrium. In the second stage, for given prices $(\bar{p}_1, \bar{p}_2)$, a location equilibrium is defined by a pair $(\tilde{x}_1, \tilde{x}_2)$ such that

$$\Pi_1(\tilde{x}_1, \tilde{x}_2; \bar{p}_1, \bar{p}_2) \geq \Pi_1(x_1, \tilde{x}_2; \bar{p}_1, \bar{p}_2) \quad \text{for all } x_1 \in [0,1] \tag{6}$$

and similarly for Firm 2. Evaluating the first-stage profit functions at this solution gives reduced forms

$$\hat{\Pi}_1(\bar{p}_1, \bar{p}_2) = \Pi_1(\tilde{x}_1(\bar{p}_1, \bar{p}_2), \tilde{x}_2(\bar{p}_1, \bar{p}_2); \bar{p}_1, \bar{p}_2)$$

and likewise for Firm 2. The equilibrium to the first stage price game is a pair (p_1^*, p_2^*) such that

$$\hat{\Pi}_1(p_1^*, p_2^*) \geq \hat{\Pi}_1(p_1, p_2^*) \quad \text{for all } p_1 \geq 0 \tag{7}$$

and similarly for Firm 2. The full equilibrium to this game is a quadruple $(p_1^*, p_2^*, x_1^*, x_2^*)$ where $x_1^* = \tilde{x}_1(p_1^*, p_2^*)$ and $x_2^* = \tilde{x}_2(p_1^*, p_2^*)$.

We now show there is no pure strategy equilibrium to this game. Consider the second-stage location game. If $\bar{p}_1 = \bar{p}_2$, the only equilibrium at this stage is $\tilde{x}_1 = \tilde{x}_2 = 1/2$. For $\bar{p}_1 \geq \bar{p}_2 + t/2$, an equilibrium is $\tilde{x}_2 = 1/2$ with $\tilde{x}_1$ anywhere in $[0,1]$: Firm 1 can earn no profit wherever it locates. On the other hand, if $\bar{p}_1 \leq \bar{p}_2 - t/2$, an equilibrium is $\tilde{x}_1 = 1/2$ with $\tilde{x}_2$ anywhere in [0,1] and here Firm 2 can earn no profit. Finally, if $|p_1 - p_2| < t/2$ with $p_1 \neq p_2$ there can be no location equilibrium because the lower price firm can always usurp the whole market by locating adjacently to the higher price firm. The latter would then move to a position far away which guarantees a positive market share and the lower price firm would again gain by adjacent location.

This last case shows why there is no equilibrium to the full game. For any price set by Firm 2 say, there is always a set of prices Firm 1 could charge for which it cannot know its payoffs as there is no equilibrium to the second-stage location game. Again the full game cannot be well defined and no equilibrium can exist.

III. Candidate Equilibrium with Product Heterogeneity and No Outside Good.

One reason for non-existence of equilibrium in the spatial model is the assumption that products are homogeneous and hence perfect substitutes - the demonstrations of non-existence relied on undercutting arguments that one firm can act (either by price or location or both) so as to cut its rival completely from the market. If however the products sold by firms are heterogeneous, they are imperfect substitutes and this will tend to smooth out profit functions and complete undercutting will be more difficult if not impossible.

To model product demand in this section we use the approach introduced by Anderson and de Palma (1988). At each point in space, a fraction of the consumers patronize each firm; this fraction depends on the difference in full prices. At any point $x \in [0,1]$, the fraction of individuals purchasing from Firm i is $F_i = F(p_i + t|x - x_i| - p_j - t|x - x_j|)$, $i,j=1,2$, $i \neq j$, so that the fraction purchasing from Firm 2 is $F_2 = 1 - F_1(\cdot)$. We assume $F(\cdot) \in C^1$,

$F(\cdot) > 0$ and $F'(\cdot) < 0$. Note that we are assuming symmetric demand functions and that $F(0) = 1/2$.

For these individual demands, the total demand for Firm 1's product is

$$D_1 = \int_0^1 F(p_1 + t|x - x_1| - p_2 - t|x - x_2|)dx \qquad (8)$$

which is a continuous function of p_1.

We now show that the only possible symmetric equilibria for <u>both</u> the simultaneous and the price-then-location games involve $x_1 = x_2 = 1/2$. To do this it is sufficient to show that if $p_1 = p_2$, and $x_1 = 1 - x_2 < 1/2$, then 1's profit rises by moving toward the center. For the simultaneous game this property clearly implies there can be no symmetric equilibrium other than the center. For the price-then-location game it implies that if both firms set the same price then both cannot anticipate a second-stage location equilibrium other than at the center - again the center is the only possible candidate for the full game.

Given $\Pi_1 = p_1 D_1$, we need only look at the sign of $\frac{\partial D_1}{\partial x_1}$ evaluated at $x_1 = 1 - x_2$ and $p_1 = p_2$. We can write (8), with $p_1 = p_2$, as

$$D_1 = \int_0^{x_1} F(t[x_1 - x_2])dx + \int_{x_1}^{x_2} F(t[2x - x_1 - x_2])dx + \int_{x_2}^{1} F(t[x_2 - x_1])dx$$

or, integrating the first and last expressions and using symmetry of $F(\cdot)$,

$$D_1 = 1 - x_2 + [x_1 + x_2 - 1]F(t[x_1 - x_2]) + \int_{x_1}^{x_2} F(t[2x - x_1 - x_2])dx \qquad (9)$$

The derivative, with respect to x_1, is

$$\frac{\partial D_1}{\partial x_1} = [x_1 + x_2 - 1]F'(t[x_1 - x_2]) - \int_{x_1}^{x_2} F'(t[2x - x_1 - x_2])dx$$

If we evaluate this derivative at $x_1 = 1 - x_2$, we obtain

$$\left.\frac{\partial D_1}{\partial x_1}\right|_{x_1=1-x_2} = - \int_{x_1}^{x_2} F'(t[2x - 1])dx > 0 \tag{10}$$

which is the result we wished to prove.

The above argument shows the only candidate symmetric equilibrium for these two games is at the center. (For the other game, location-then-price, this argument does not hold and firms will not necessarily locate at the center, as we show below.) Let us now provide an example where this is indeed the full equilibrium to both these games. The example is the logit model taken from de Palma et al. (1985). For this model, for $i, j=1,2, i\neq j$,

$$F(p_i + t|x - x_i| - p_j - t|x - x_j|) = \frac{\exp([-p_i - t|x - x_i|]/\mu)}{\sum_{k=1,2} \exp([-p_k - t|x - x_k|]/\mu)} \tag{11}$$

where μ is interpreted as a measure of product heterogeneity. For $\mu \to 0$, the model reverts to the homogeneous products case of Section II. On the other hand, when μ becomes large, $F(\cdot)$ tends to one half. Note that (11) satisfies the properties of $F(\cdot)$ given at the start of the section. Under this demand specification, the profit function for Firm 1, Π_1, can be integrated explicitly to obtain (see de Palma et al. (1985))

$$\Pi_1 = \frac{x_1}{1 + He^{\gamma}} + 1 - x_1 - x_2 - \frac{\mu}{2c} \ln \left(\frac{1 + K}{1 + H}\right) + \frac{1 - x_2}{1 + Ke^{\gamma}} \qquad (12a)$$

where

$$\gamma = (p_1 - p_2)/\mu, \; H = \exp[-t(1 - x_1 - x_2)/\mu] \text{ and } K = 1/H \qquad (12b)$$

De Palma et al. (1985) consider the simultaneous equilibrium and show that there exists a symmetric equilibrium at the center providing $\mu \geq t$. The proof used there was to take the price equilibrium when both firms are centrally located with ($p_1 = p_2 = 2\mu$), and show that if Firm 1 locates at 1/2 and charges 2μ, then if 2 is located at any other point with any other price, its profit is increased if it moves toward the center (for $\mu \geq t$). Once at the center, its preferred price is 2μ since this is the price equilibrium for $x_1 = x_2 = 1/2$.

To derive a sufficient condition for existence in the price-then-location game we show that when μ is large enough the solution of the price game requires both firms to be at the center. Given that the candidate Nash equilibrium is $x_1 = x_2 = 1/2$, $p_1 = p_2 = 2\mu$, it suffices to show that $\frac{\partial \Pi_1}{\partial x_1}(p_1, p_2 = 2\mu, x_1, x_2 = 1/2) > 0$ for $x_1 < 1/2$ and for all $p_1 \in [0,\infty[$, and $\frac{\partial \Pi_1}{\partial x_1}(p_1 = 2\mu, p_2, x_1, x_2 = 1/2) > 0$ for $x_1 < 1/2$ and for all $p_2 \in [0,\infty[$. That is, we wish to show that the equilibrium to the location subgame where one firm chooses a price 2μ and the other any arbitrary price is at the center. The inequalities given imply the firm with the arbitrary price goes to the center if the other is at the center charging 2μ, and that the firm with price 2μ goes to the center if the firm with the arbitrary price is there. Hence any such subgame leads to a central equilibrium, and we know that the price equilibrium when both firms are at the center involves $p_1 = p_2 = 2\mu$. The first inequality is satisfied for $\mu \geq t$ (as discussed above, this condition was used in the proof of the simultaneous game by de Palma et al. (1985)). The second inequality is proved below. From (12) we obtain

$$\text{sign}\,\frac{\partial \Pi_1}{\partial x_1} = \text{sign}\left[K - H - \frac{2tx_1}{\mu}\,\frac{e^{\gamma} + H}{e^{\gamma}H + 1} + \frac{t(e^{\gamma} + K)}{\mu(e^{\gamma}K + 1)}\right] .$$

Replacing the term $2x_1$ by 1 in this expression we obtain the sufficient condition: $\frac{\partial \Pi_1}{\partial x_1} > 0$ if $\frac{\mu}{t}[1 + e^{\gamma}(H + K) + e^{2\gamma}] \geq [e^{2\gamma} - 1]$. Thus (given that $1 + e^{\gamma}(H + K) + e^{2\gamma} \geq (1 + e^{\gamma})^2$), $\frac{\partial \Pi_1}{\partial x_1} > 0$ if $\mu \geq t$. To sum up we have:

<u>Proposition</u>. For the spatially extended logit demand model (11), and for $\mu \geq t$, there is an equilibrium for both the simultaneous and the two-stage price-then-location games at the market center, $x_1 = x_2 = 1/2$, with mill prices $p_1 = p_2 = 2\mu$.

Analytic results are difficult for the location-then-price game. Indeed there are equilibria which are not at the center. The simulation of the equilibrium locations is given in Figure 1, which is taken from Anderson et al. (1989).

Insert Figure 1

For low values of μ/t there is no equilibrium for reasons similar to those given in Section II. When equilibrium does exist, initially firms move apart with increased μ and then start to move together. For intermediate values of μ/t there are two equilibria, one of which is the center, and for high parameter values there is only the central equilibrium.

The reason why dispersed equilibrium exists in this game but not for the others is explained by the game structure. In the location-then-price game, firms account for more intense price competition which they know will result as they locate closer together. In the simultaneous game, however, the price implications of a move together are not considered. This suggest that as long as closer locations entail more competition (lower equilibrium prices) - as is the case with the logit - the location-then-price game will involve greater equilibrium separation of firms than the simultaneous game. This intuition is borne out in the simulations of the next section.

For the price-then-location game, firms internalize the equilibrium locations conditional on price choices. Here however, equilibrium locations are totally unresponsive to price changes and remain at the center. This means prices cannot be used to affect one's rivals location behavior so that the equilibrium is exactly the same as the simultaneous game.

The result that two of the games give central locations as equilibria can be ascribed to the assumption that each consumer must buy one unit from one of the two firms. Introducing a non-purchase option will make firms less inclined to be at the same location because a move toward the outskirts of the market will reduce full price there and pick up consumers previously not buying. In such a context price will be expected to have some strategic impact in the sense that a price change will likely affect equilibrium locations. We should then expect the price-then-location equilibrium to differ from the simultaneous one. In the next section we simulate a model which allows for non-purchase by consumers.

IV. Equilibria with Outside Options.

In order to see how one might introduce non-purchase into the model it is useful to return to the underpinnings of the logit. The standard derivation of the logit model (11) is given from discrete choice theory. Specifically, let

$$u_1(x) = -p_1 - t\,|x - x_1| + \mu\epsilon_1 \tag{13}$$

and

$$u_2(x) = -p_2 - t|x - x_2| + \mu\epsilon_2 \tag{14}$$

be the utility of a consumer at $x \in [0,1]$ conditional on buying from Firms 1 and 2 respectively. The terms ϵ_1 and ϵ_2 are iid random variables distributed according to the double exponential, i.e.,

$$\text{Prob}(\epsilon_i < y) = \exp(-\exp - y) \quad . \tag{15}$$

The fraction of individuals at x buying from Firm 1 is then given by $\text{Prob}[u_1(x) \geq u_2(x)]$. McFadden (1973) has shown that under the specification (15), this fraction is given by (11).

This derivation of the logit model allows us to introduce an outside option in a straightforward way. We now assume there is a third option (a null option) for which the conditional utility is

$$u_0(x) = V_0 + \mu\epsilon_0. \tag{16}$$

where ϵ_0, ϵ_1 and ϵ_2 are ii double exponentially distributed according to (15). This constitutes a useful extension of previous work since now consumers are no longer obliged to buy from one of the two firms but may instead decide not to buy. As $\mu \rightarrow 0$, the model reduces to the case of a homogeneous good with reservation price equal to $-V_0$.

In this extended model, the fraction of consumers buying from Firm 1 is $\text{Prob}[u_1(x) \geq \max[u_2(x), u_0(x)]]$, and the corresponding multinomial logit expression is

$$F_1(x) = \frac{\exp([-p_1 - t|x - x_1|]/\mu)}{\exp(V_0/\mu) + \sum_{k=1,2} \exp(-p_k - t|x - x_k|]/\mu)} \tag{17}$$

We interpret V_0 as the attractiveness of the outside option. It is readily verified that the binomial logit of the previous section is given when $V_0 \rightarrow -\infty$ (see (11)) and all consumers buy. The higher is V_0, the more likely any given consumer is to not purchase from one of the two firms.

The model above is very complicated to deal with analytically because the profit functions are very cumbersome (even though they can be explicitly integrated). We thus resorted to numerical simulation to find equilibria for the three games for different values of μ and V_0. For the simultaneous game the program solved the first-order conditions evaluated at $p_1 = p_2$ and $x_1 = 1 - x_2$ using a Newton-Raphson procedure. Given this candidate solution, it was then checked by grid methods that one firm does not wish to deviate by choosing a different price and location. For low μ values the candidate was always rejected as firms wish to deviate (similar to the homogeneous case). For large μ values the candidate equilibrium could not be overturned.

For the two-stage games, the program started with a symmetric pair of first-stage variables and calculated the second-stage equilibrium. Then one firm's stage value was perturbed away from the symmetric value and a new second-stage equilibrium and the corresponding profit calculated. If this was higher than at the symmetric pair, a new pair of symmetric first-stage variables was selected, etc. Once a first-stage pair was found which was robust to small deviations of one firm, it was checked with respect to large deviations. Again, there is no equilibrium for low values of μ for both of these games with V_0 very small, since the homogeneous goods case is approached for μ small. However, for larger values of V_0 and $\mu \to 0$, we approach the homogeneous goods case with a reservation price, so equilibrium may exist. The equilibrium locations for the three games are illustrated in Figure 2.

Insert Figure 2

In panel (a) we see there are dispersed equilibria now as well as agglomerated ones for the simultaneous game, but for μ/t large enough the only equilibrium is at the center. For $V_0 \to -\infty$, as proved in Section III, firms locate together at the center. Also, for given μ as V_0 rises up to -0.25, firms move farther apart, as is expected because firms become more like local monopolies and tend toward the quartiles to avoid competition between each other. Indeed, it can easily be seen that for $\mu = 0$ and $V_0 = -0.25$ (that is, the reservation price is 1/4) both firms locate at the quartiles and are pure local monopolies - each firm serves exactly half the market. When V_0 rises further ($V_0 > -0.25$), firms begin to move back to the center because intra firm competition becomes a secondary issue and competition with the outside alternative becomes the most important factor.

In panel (b) for the location-then-price game we see that as V_0 rises there is a smaller range of μ values with dispersed equilibria. Moreover, firm locations tend to the quartiles for $V_0 > -1$ as μ goes to zero.

Panel (c) represents the price-then-location game. As V_0 increases, the solution is qualitatively similar in terms of firm locations to the simultaneous game in the sense that firms first move apart as V_0 rises and then back together. At approximately $V_0 \geq 0.5$, the two firms locate at the center for all values of μ where equilibrium occurs. Also, the range of non-existence of equilibria becomes larger as V_0 increases. Finally, as μ increases, the dispersed equilibria vanish and only clustered equilibria exist.

Insert Figure 3

Figure 3 shows a comparison of the dispersed equilibria for the three games for $V_0 = -0.25$, $V_0 = -0.5$ and $V_0 = -0.75$. When $V_0 = -0.25$ (panel (a)), the location of firms in dispersed equilibria are further apart for price-then-location, then simultaneous, then location-then-price games for any given value of μ. However, for $V_0 = -0.5$ (panel (b)), the order of the locations for dispersed equilibria changes to simultaneous, then price-then-location, then location-then-price. Panel (c) represents the case $V_0 = -0.75$. It is difficult to find a full intuition for these results, however, as we explained intuitively in Section 3, firms are farther apart in the location-then-price equilibrium locations game than in the simultaneous game. The price-then-location equilibrium locations may lie either inside or outside the simultaneous game ones.

Insert Figure 4

Firm profits are also different for the three games (Figure 4). When $V_0 = -0.25$ (panel (a)), they are almost identical for the three games. Equilibrium profit is an increasing (and convex) function of μ. We may explain this as follows: as μ increases, (A) competition between the two firms is weakened because products are now more differentiated, and (B) competition between firms is more intense because firms move toward the center (see Figure 3.a). Here (A) dominates (B).

When $V_0 = -0.5$ (panel (b)), the difference in profit under the three games becomes pronounced for the range $0.1 < \mu < 0.3$. The highest profit is obtained under the price-then-location game, followed by location-then-price, and then the simultaneous game. As μ becomes less than 0.1 or greater than 0.3, the profit differences vanish. These results are expected because, as shown in Figure 2, within the range $0.1 < \mu < 0.3$, the locations of dispersed equilibria are farthest apart. Beyond these ranges of μ values, the equilibrium locations tend to the same value and hence to same profit. Note that profit now decreases with μ, for $0.05 < \mu < 0.18$, so that (B) dominates (A) here. Interestingly, for $\mu > 0.25$, equilibrium locations for the simultaneous and the price-then-location games are at the center. This does not imply, as seen in Figure 4(b), that the price level is necessarily the same for these games. Indeed, the higher profit for the price-then-location game reflects higher mill prices there than for the simultaneous game. To explain the price differential, we must return to the structure of the games. In the price-then-location game each firm realizes that if it unilaterally raises its price above the simultaneous level of prices then its rival would move away from the center in the ensuing location game. This relocation is beneficial to it, so the equilibrium price level is higher. Note that this effect only operates for certain values of μ. When μ is large enough a small price change will not trigger a location change. Indeed, when μ is large enough both firms locate at the center and charge the same prices in all three games.

When $V_0 = -0.75$ (panel (c)), the highest profit is now obtained under the location-then-price game, followed by the simultaneous and the price-then-location games. But as μ increases, profit also goes up and the difference between the firms' profit in the three games vanishes. The price-then-location profit is much smaller than the two others because firms are at the center in this game (see Figure 3(c)). The difference between the three cases $V_0 = -0.25$, $V_0 = -0.5$ and $V_0 = -0.75$ can be (partially) explained as follows: for larger values of V_0, firms tend to compete less among themselves and more with the outside alternative. As a result, firms tend to occupy the same location (see Figure 3(a)), and behave as monopolists facing an elastic demand and therefore charge similar prices (see figure 4(a)).

V. Conclusions

Two equilibrium concepts have been the focus of analysis in location theory so far.[4] Either (following Hotelling (1929)) a two-stage location-then-price equilibrium is considered, or else (e.g., Eaton and Lipsey (1978), Novshek (1980)) a simultaneous equilibrium concept is used. We have considered a third possibility which reverses the stages of the usual two-stage game.

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[4] At least, in location theory where decision variables are locations and prices. Some authors have considered locations and quantities - see for example Salant (1986) and Hamilton et al. (1989), although here again the "sequence reversal" - quantity-then-locations - remains a topic for future research.

We argued that agglomeration is the only possibility both for the simultaneous game and for the new concept when consumers must purchase from one of two firms. However, other possibilities arise once consumers are allowed a no-purchase option. These results were shown by simulation - analytic results are very hard to find, even assuming specific functional forms. Using the simultaneous equilibrium as a benchmark, there are strong reasons to expect the equilibrium locations under the price-then-location game to be less central: under the latter equilibrium concept firms rationally anticipate a reduction in (deleterious) price competition as they move apart and this "strategic" effect induces them to move outside the equilibrium locations for the simultaneous game. However, no such argument is forthcoming for the new game we introduced, and indeed the simulations showed equilibrium locations could lie either side of the simultaneous game benchmark.

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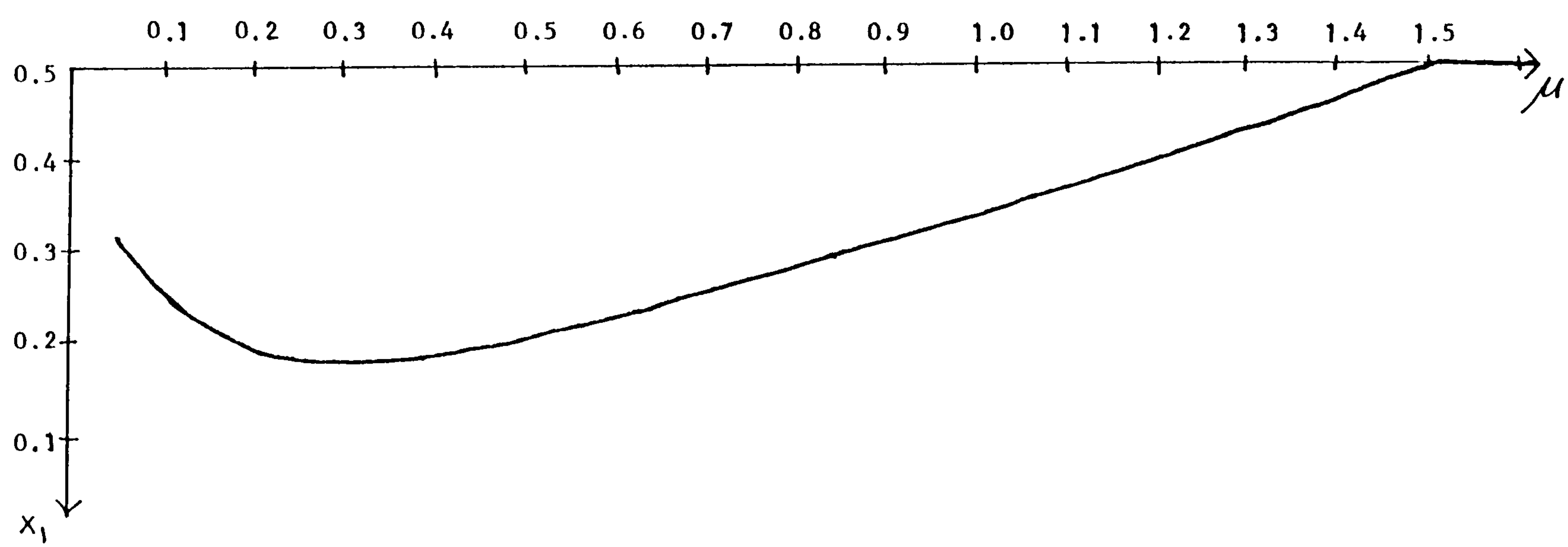

Figure 1: Equilibrium Solutions (Location-then-price)

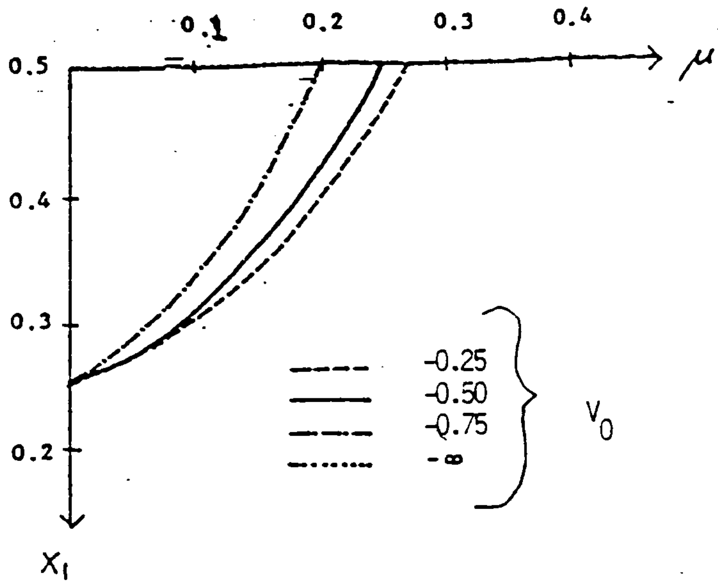

Figure 2 (a): Equilibrium Solutions (Simultaneous)

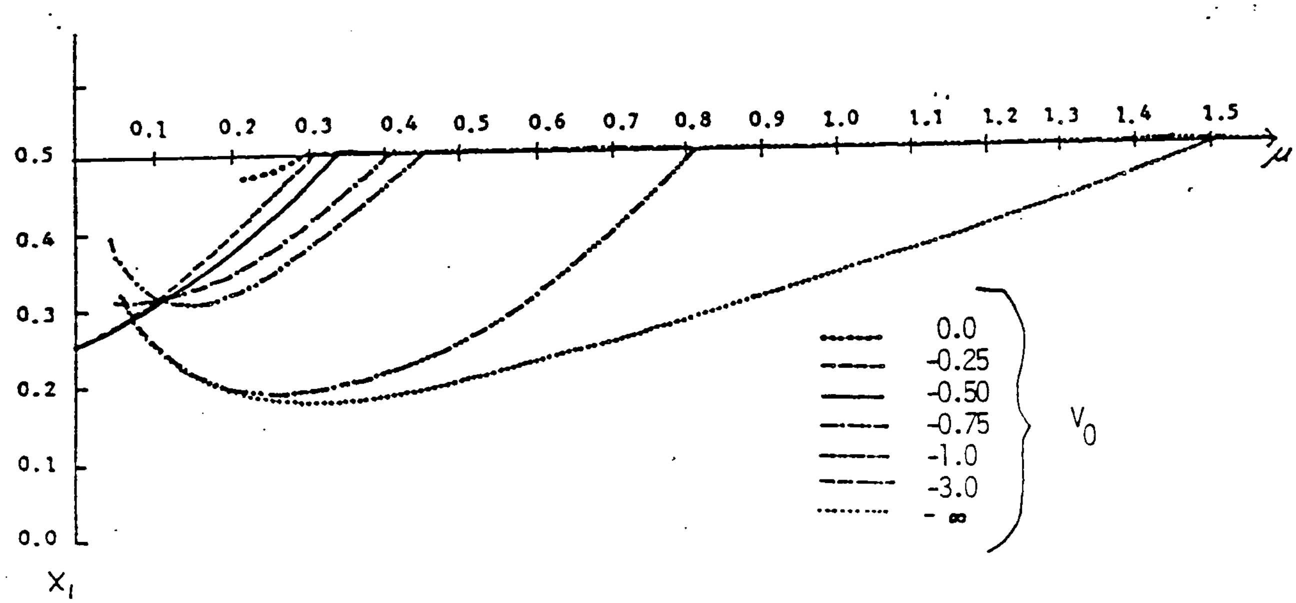

Figure 2(b): Equilibrium Solutions (Location-then-price)

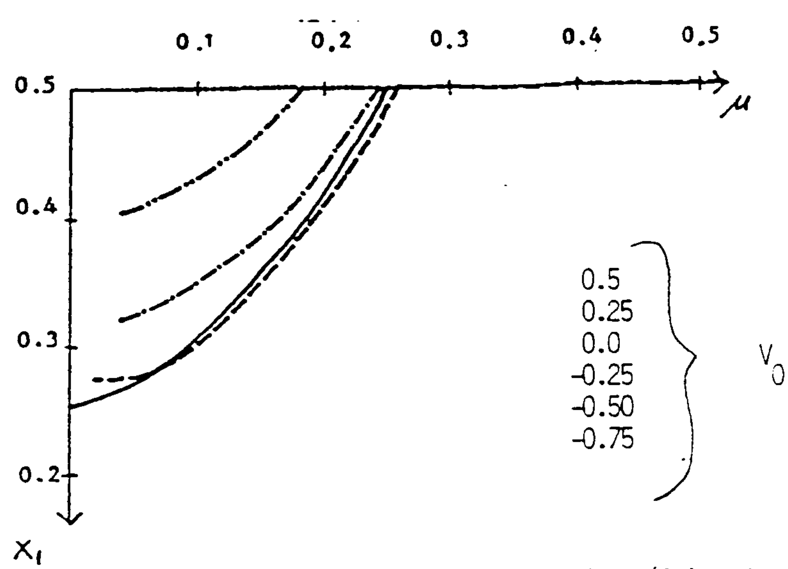

Figure 2 (c): Equilibrium Solutions (Price-then-location)

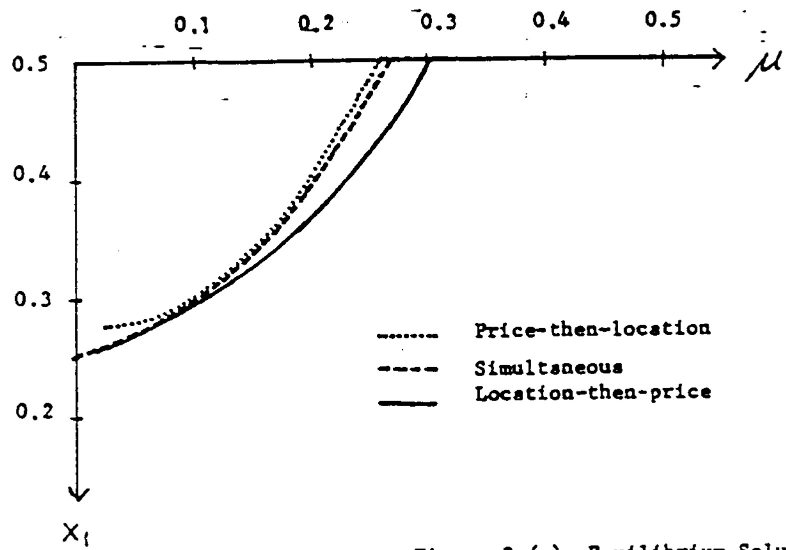

Figure 3 (a): Equilibrium Solutions (V_o = -0.25)

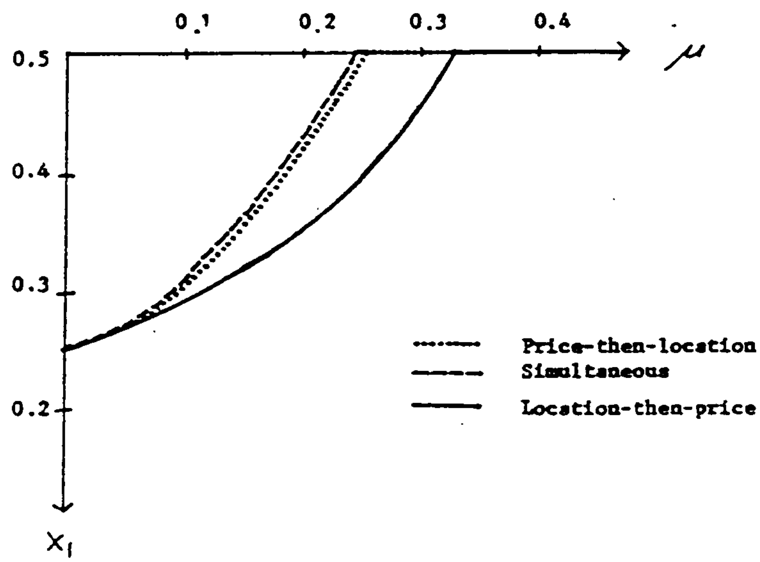

Figure 3 (b): Equilibrium Solutions (V_o = -0.5)

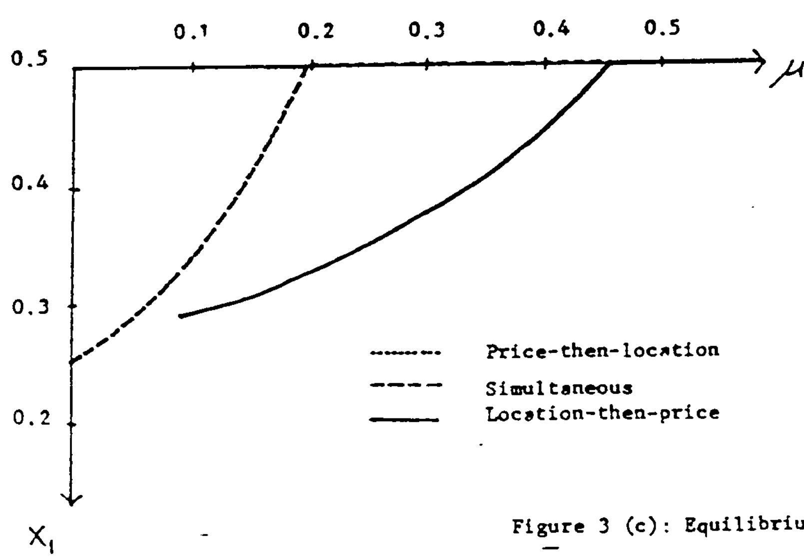

Figure 3 (c): Equilibrium Solutions (V_o = -0.75)

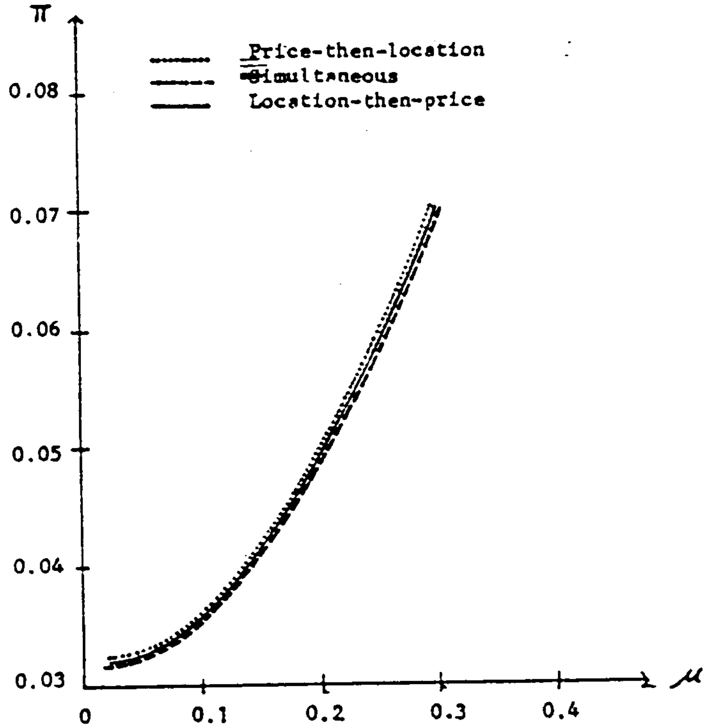

Figure 4 (a): Profit (V_o = −0.25)

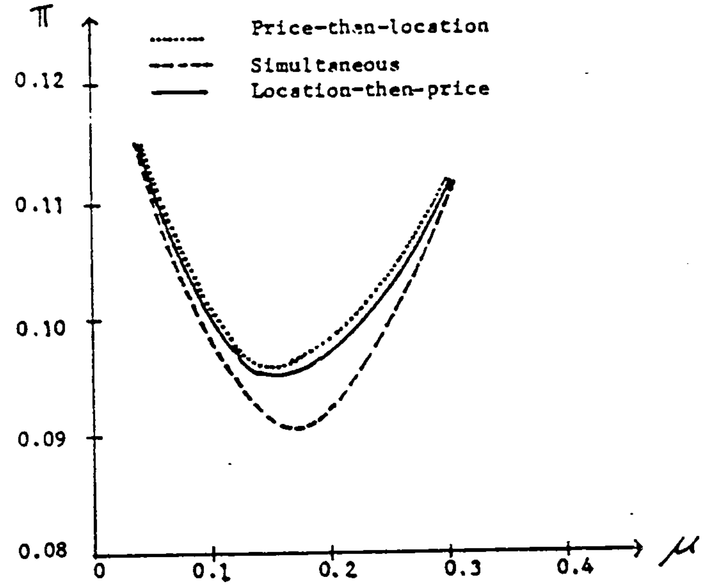

Figure 4 (b): Profit (V_o = −0.5)

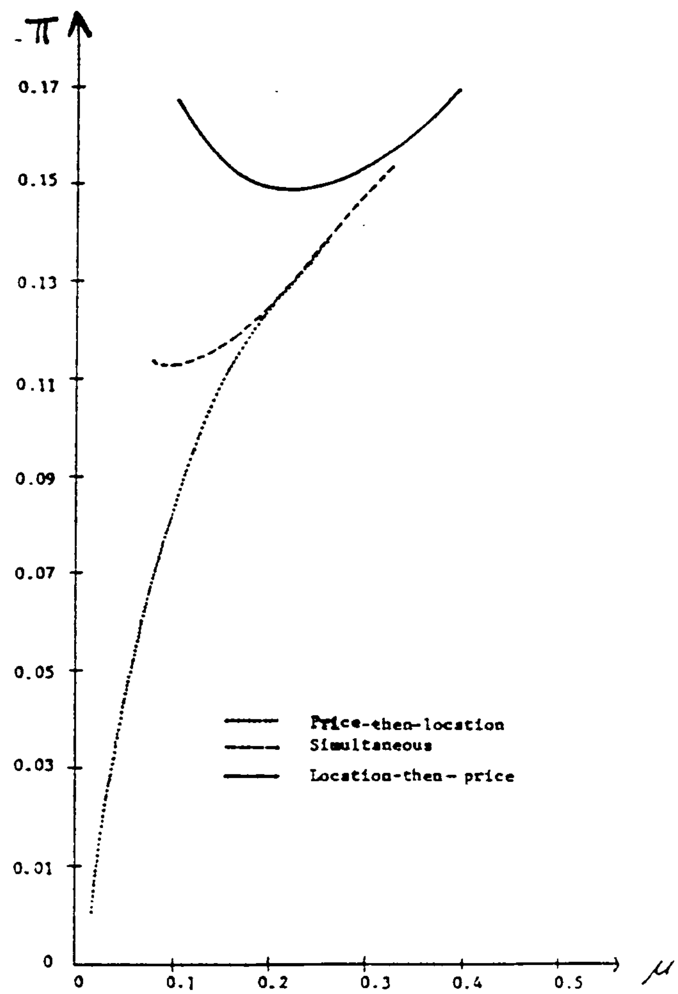

Figure 4 (c): Profit (V_o = −0.75)