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A TEMPORAL AND SPATIAL EQUILIBRIUM ANALYSIS OF COMMUTER PARKING

by

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Abstract

In major cities parking costs typically exceed automobile running costs, while the time to find a parking spot and walk to work can be comparable to driving time. Yet models of urban commuting have ignored parking completely. The purpose of this paper is to examine the effects of parking on morning rush hour congestion and to assess the relative merits of road tolls and parking fees as tools for congestion relief.

The paper extends Vickrey's (1969) bottleneck road congestion model by assuming on-street parking is located along commuting routes radiating from the CBD. Absent pricing, drivers occupy parking spots in order of increasing distance from the CBD.

Three pricing schemes are considered: 1) an optimal time-varying road toll, 2) competitively set parking fees, and 3) optimal location-dependent parking fees. The optimal road toll is shown to eliminate queueing, but does not affect the order in which parking spots are occupied. In contrast, competitive parking fees do nothing to reduce queueing, but induce drivers to park in order of decreasing distance from the CBD, so that in the aggregate commuters arrive at work closer to their preferred time. Optimal parking fees reduce queueing in addition to supporting the efficient order of parking.

For reasonable parameter values competitively set parking fees are found to be relatively inefficient -- indeed potentially welfare-reducing. Optimal parking fees, however, are generally superior to a road toll. In light of the logistical drawbacks of tolls and political opposition that road pricing has met, this suggests that parking fees deserve more attention than they have received in the literature.

Notational Glossary

(in alphabetic order)

Greek characters

α	Unit cost of in-vehicle travel time
β	Unit cost of arriving at work early
γ	Unit cost of arriving at work late
δ	$\beta\gamma/(\beta+\gamma)$
λ	Unit cost of walking time
$\tau(t)$	Road toll at time t
$\phi(n)$	Parking fee at parking location n
$\bar{\phi}$	Parking fee at most distant parking location under competitive pricing

English characters

$C(t)$	Driver's trip cost when traversing bottleneck at time t
$D(t)$	Length of queue at time t
E^i	Efficiency of pricing regime i
N	Number of commuters
n	Parking location
n'	Parking location at and beyond which drivers experience no queue under optimal location-dependent parking fee.
$\hat{n}$	Parking location of individual who arrives at work on time
r	Departure rate from residential area
s	Capacity of bottleneck
SDC	Aggregate schedule delay cost
t	Time
t_d	Departure time
t^*	Desired arrival time at work
t_0^i	Earliest departure time in pricing regime i
t_1^i	Latest departure time in pricing regime i
$\hat{t}^i$	Departure time for which individual arrives on time in pricing regime i
T^0	Travel time in absence of queue.
TC	Aggregate trip costs
TTC	Aggregate travel time cost
w	Time to walk past one parking spot
WTC	Aggregate walking time cost

Superscripts for Pricing Regimes

f	Free: No toll or parking fee
r	Optimal road toll
o	Optimal road toll and parking fees
p	Optimal parking fees
c	Competitive parking fees

1. Introduction

In recent years significant progress has been made in modeling the dynamics of the morning rush hour. Commuters are assumed to choose departure times for work that minimize the sum of their individual costs of in-vehicle travel time and schedule delay (the cost of arriving either before or after their preferred or official starting time at work). In equilibrium no individual can reduce trip costs by departing at a different time. Those traveling at the peak of the rush hour arrive close to their preferred time, but suffer the greatest congestion delays. Those traveling on the tails have quicker trips, but incur substantial schedule delays.

This work is a substantial improvement over earlier static analyses, in that individuals' travel time and route choice decisions, and the evolution of congestion are endogenous. However, it neglects an important aspect of the urban commute: parking. In major urban areas, the time to find a parking spot and walk to work can be an appreciable fraction of total travel time. Parking fees, moreover, often exceed vehicle operating costs (Gillen [1977b]). In addition to when they travel, commuters may thus have a preference where they park. This additional margin of adjustment can affect the efficiency gains from road-pricing and other policies for congestion relief.

Various aspects of parking have been considered in the literature. Descriptions of parking patterns, the effects of on-street parking on traffic circulation, and the technology of off-street parking are found (e.g. Institute of Traffic Engineers [1976]) as well as discussions of parking policy (e.g. Adiv and Wang [1987], Miller and Everett [1982], Shoup [1982], U.S. DOT [1982]). Some empirical work has been done identifying the determinants of modal choice and parking location (e.g. Gillen [1977a,b, 1978], Hunt [1988]). But no model has been developed that incorporates parking as a determinant of commuters' travel time decisions, or considers analytically the interaction of parking and traffic flow congestion.

Such a model could be used to address several important issues. First, what is the impact of the search time for parking and subsequent walking time to work on drivers' departure time decisions and the level of traffic congestion? Second, what is the effect of the monetary cost of parking, and how effective can parking fees be in alleviating congestion?[1] Third, how well could a comprehensive parking fee policy serve as a supplement to or substitute for road pricing?

The purpose of this paper is to take a first step toward an analytical treatment of parking and congestion by adding a simple parking module to the generic rush-hour traffic model. Parking either on-street or in off-street parking lots is assumed available along commuting routes radiating out from the CBD, which is treated as a point in space. All commuters are employed in the CBD, to which they must walk from where they park. Travel modes other than automobile are ignored. The model is thus very simple, and ignores some important features of parking in the real world, such as the large amount of employer-provided parking in many cities. Search time for parking is also not considered. But the model does incorporate in a tractable manner the time and money costs of parking in commuters' travel decisions.

The model is specified in Section 2. Derivation of the equilibrium departure time distribution and parking location choice of commuters absent road tolls or parking fees is carried out in Section 3. Equilibrium with an optimal time-varying toll is considered in Section 4, and with an optimal combined time-varying road toll and location-dependent parking fee in Section 5. In section 6 parking fees are considered in isolation. Competitive pricing of parking is analyzed in Section 7. Aggregate trip costs in the five road toll/parking fee regimes are compared in Section 8. Section 9 concludes with a brief summary and discussion of possible extensions.

2. The Model

The rush-hour congestion model on which this paper builds was developed by Vickrey (1969), and extended by Hendrickson and Kocur (1981), Fargier (1983), Cohen (1987) and Arnott *et al.* (1985) *inter alios*. The precise assumptions and notation employed here follow Arnott *et al.* N identical commuters travel each morning, one per car, from their homes in the suburbs to work downtown. Travel is uncongested except at a single bottleneck with a maximum flow capacity of s cars per unit time.[2] If the arrival rate at the bottleneck exceeds s, a queue develops.

The bottleneck is taken to be far enough from the CBD that all drivers park beyond it.[3] Since employer-provided parking is assumed unavailable, commuters must use either on-street parking or off-street parking lots. It is assumed that the number of parking spaces as a function of distance from the CBD is predetermined; the most straightforward interpretation of this assumption is that the government determines the amount of on- and off-street parking. Furthermore, to simplify the algebra, it is assumed that the number of parking spaces per unit distance from the CBD is constant. Parking spots, which are treated as a continuous variable, are indexed by n in order of increasing distance from the CBD. Walking time to the centre from location n is taken to be wn, where w depends on the spatial concentration of parking, walking speed, delays at intersections *etc*.

In-vehicle travel time consists of free-flow travel time to the parking spot, T^0, plus queueing time at the bottleneck. Driving speed is assumed to be much greater than walking speed, so that in-vehicle travel time within the parking area can be ignored. Without loss of generality we set $T^0 = 0$; drivers thus reach the tail of the queue at the bottleneck as soon as they leave home, and reach the parking area immediately after exiting the bottleneck. A driver traversing the bottleneck at time t experiences a travel time $D(t)/s$, where $D(t)$ is the length of the queue.

Individuals are assumed to have a common preferred arrival time at work (*e.g.* their official starting time), t^*. The cost of arriving early is taken to be β per unit of time early, and the cost of arriving late γ per unit of time late. The unit cost of in-vehicle travel time (including vehicle operating costs and the opportunity cost of time) is α, and the unit cost of walking time λ. The trip cost of a commuter traversing the bottleneck at t and parking at n is

$$C(t) = \alpha \frac{D(t)}{s} + \lambda wn + \beta(\text{time early}) + \gamma(\text{time late}),$$

where for individuals arriving at work before t^*, time late $= 0$, and for those arriving after t^*, time early $= 0$. To assure existence of a deterministic equilibrium we assume $\alpha > \beta$[4], and $\lambda > \beta$. Empirical evidence, considered in Section 8, supports these assumptions. The latter guarantees that commuters destined to arrive at work early do not dawdle or choose a roundabout walking route from where they park.

3. Equilibrium with no Tolls or Parking Fees (regime f)

To derive the equilibrium without pricing (road use and parking are both free) we first consider the trip cost of an individual traversing the bottleneck at t who arrives at work early:

$$(1)\quad C^f(t) = \alpha \frac{D(t)}{s} + \lambda wn + \beta(t^*-t-wn), \qquad t_0^f \le t \le \hat{t}^f,$$

where the superscript f denotes equilibrium with roads and parking free. t_0^f is the time at which the first driver departs (and traverses the bottleneck) and $\hat{t}^f$ is the traversal time at which an individual arrives on time at t^*. Since $dC^f(t)/dn = (\lambda-\beta)w > 0$, parking spots are occupied in strict order of increasing n. On the assumption that the bottleneck operates at capacity throughout the rush hour the parking location of an individual who travels at t is:

$$(2)\quad n(t) = s(t - t_0^f).$$

In equilibrium all drivers incur the same trip cost, C^f. Substituting (2) into (1) and rearranging, queue length is found to be

$$D(t) = \frac{s}{\alpha}\,[C^f - \beta(t^*-t) - (\lambda-\beta)ws(t-t_0^f)], \qquad t_0^f \le t \le \hat{t}^f.$$

Queue length changes at a rate

$$\dot{D}(t) = \frac{s}{\alpha}\,[\beta - (\lambda-\beta)ws], \qquad t_0^f \le t \le \hat{t}^f,$$

which is positive provided:

$$\beta(1+ws) > \lambda ws,$$

a condition which is hereafter assumed to hold. To interpret this condition note that, given parking in order of increasing distance, ws is the rate at which walking time increases with t. $\beta(1+ws)$ is the rate at which the cost of arriving early decreases with t, and λws the rate at which walking time cost increases. Delaying departure is desirable (thus leading to a growing queue) if the former exceeds the latter.

A further condition for equilibrium is that the departure rate from home be finite. With t_d as departure time we have

$$t_d = t - D(t)/s,$$

$$dt_d/dt = 1 - \dot{D}/s.$$

The departure rate is

$$r(t_d) = s + \dot{D}(t)\, dt/dt_d$$

$$= \frac{\alpha s}{\alpha - \beta + (\lambda - \beta)ws},$$

which is indeed finite and positive given the assumptions $\alpha > \beta$, $\lambda > \beta$. The departure rate is greater the larger is β, *i.e.* the more desirable on-time arrival, and the smaller is α, *i.e.* the lower the time cost of queueing. The departure rate is lower the greater λ and w, *i.e.* the greater the penalty in increased walking time from delaying departure.

Turning to individuals who arrive late we have

$$(3)\quad C^f(t) = \alpha \frac{D(t)}{s} + \lambda wn + \gamma(t + wn - t^*), \qquad \hat{t}^f \le t \le t_1^f,$$

where t_1^f is the time at which the last driver traverses the bottleneck. Since $dC^f(t)/dn = (\lambda+\gamma)w > 0$, equation (2) continues to hold. Substituting (2) into (3) and differentiating with respect to t one obtains

$$\dot{D}(t) = -\frac{s}{\alpha}[\gamma + (\lambda+\gamma)ws] < 0, \qquad \hat{t}^f \le t \le t_1^f.$$

The queue thus dissipates over time. The departure rate, which can be derived as earlier, is

$$r(t) = \frac{\alpha}{\alpha + \gamma + (\lambda+\gamma)ws}, \qquad \hat{t}^f \le t \le t_1^f,$$

which is still positive. Drivers are willing to delay departure despite their increasing lateness because of decreasing travel time. The last individual departs just as the queue reaches zero. He thus departs at t_1^f and immediately traverses the bottleneck. Were the last individual to depart before the queue reaches zero he would be delayed at the bottleneck without arriving at work any earlier. Were he to depart later he would arrive later without any reduction in travel time. The last driver thus escapes queueing, as of course does the first because there is no earlier traffic.

The timing of the rush hour is determined as follows. Since the bottleneck operates at capacity throughout the interval $[t_0^f, t_1^f]$,

$$(4) \quad t_1^f - t_0^f = N/s.$$

Trip costs of the first and last individuals are respectively

$$(5) \quad C^f(t_0^f) = \beta(t^* - t_0^f),$$

$$(6) \quad C^f(t_1^f) = \lambda wN + \gamma(t_1^f + wN - t^*).$$

Equating (5) and (6) and using (4) one obtains

$$(7) \quad t_0^f = t^* - \frac{\gamma + (\lambda+\gamma)ws}{\beta+\gamma}\frac{N}{s},$$

$$(8) \quad t_1^f = t^* + \frac{\beta - (\lambda+\gamma)ws}{\beta+\gamma}\frac{N}{s}.$$

Departures begin and end earlier the larger is w. Since in equilibrium trip costs are the same for all drivers, aggregate trip costs are simply $TC^f = N \cdot C^f(t_0^f)$, or given (5) and (7)

$$(9) \quad TC^f = \lambda \frac{\beta w}{\beta+\gamma} N^2 + \delta(1+ws)\frac{N^2}{s},$$

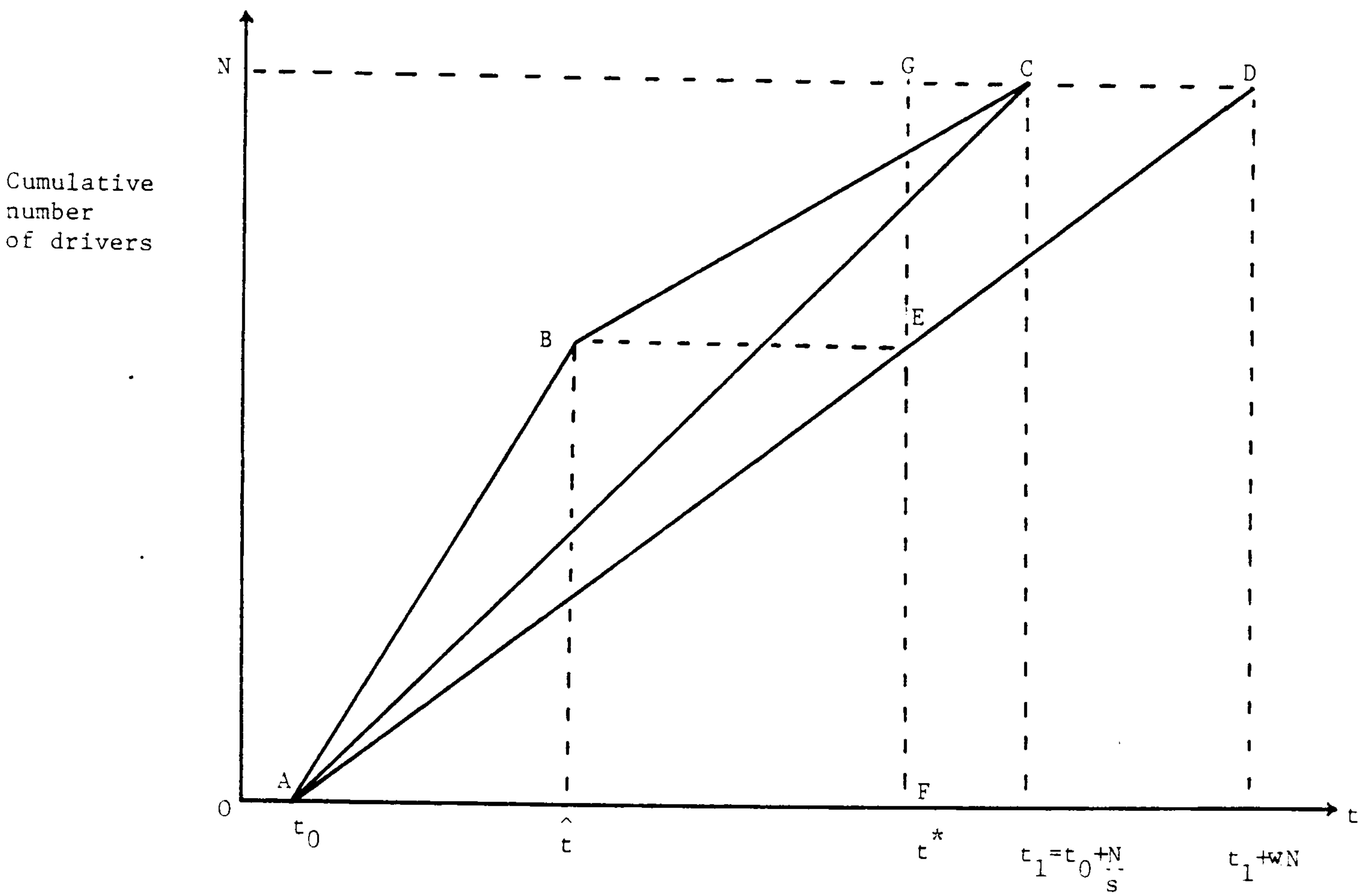
where $\delta = \beta\gamma/(\beta+\gamma)$.

Equilibrium is depicted in Figure 1.[5] Arrivals at the CBD are spread over a period of length $N/s + wN$.

Figure 1

Equilibrium with No Tolls

or Parking Fees



Cumulative departures from home :	ABC
Cumulative number through bottleneck :	AC (slope s)
Cumulative arrivals at the CBD:	AED (slope $s/(1+ws)$)
Aggregate queueing time	ABCA
Aggregate walking time	ACDA
Aggregate time early	AEFA
Aggregate time late	EGDE

4. A Time-varying Road Tolls (regime r)

Suppose that a time-varying road toll can be charged to drivers while they are in transit.[6] If a toll $\tau(t)$ is levied at the bottleneck the cost of a trip becomes

$$(10) \quad C^r(t) = \alpha \frac{D(t)}{s} + \tau(t) + \lambda wn + \beta(\text{time early}) + \gamma(\text{time late}),$$

where the superscript r indicates equilibrium with a road toll.

Since the toll varies with time, but not parking location, drivers continue to park in order of increasing n. However, the toll can be adjusted to eliminate queueing, which is pure deadweight loss. Setting $D(t) = 0$ in (10) and differentiating with respect to t one obtains as conditions for equilibrium without queueing

$$(11) \quad \dot{\tau}(t) = \beta - (\lambda-\beta)ws, \qquad t_0^r < t < \hat{t}^r,$$

$$(12) \quad \dot{\tau}(t) = -\gamma - (\lambda+\gamma)ws, \qquad \hat{t}^r < t < t_1^r,$$

with t_0^r, t_1^r and $\hat{t}^r$ defined analogously to their counterparts in Section 3. The toll increases while drivers are arriving early, and decreases while they are arriving late.

Now aggregate walking time is independent of when the rush hour begins and ends. With zero queueing time the optimal timing of the rush hour is determined by minimizing aggregate schedule delay costs. Since commuters pass through the bottleneck at rate s and park in order of increasing n they reach the CBD at the rate $s/(1+ws)$ over the period $[t_0^r, t_1^r+wN]$, which is of length $N/s + wN$ as is the case without tolling. Given the uniform arrival rate, schedule delay costs are minimized by equating the schedule delay costs of the first and last commuters:

$$(13) \quad \beta(t^* - t_0^r) = \gamma(t_1^r + wN - t^*).$$

Combining (13) with the condition $t_1^r - t_0^r = N/s$ one obtains

(14) $t_0^r = t^* - \frac{\gamma(1+ws)}{\beta+\gamma}\frac{N}{s}$,

(15) $t_1^r = t^* + \frac{\beta-\gamma ws}{\beta+\gamma}\frac{N}{s}$.

Comparing (14) and (15) with (7) and (8) it is clear that with $w>0$ the rush hour begins later than without tolling. Competition between drivers for convenient parking spaces in the no-toll regime induces them to leave too early, just as the desire to arrive at work close to t^* induces drivers to bunch departure times.

The departure (= bottleneck transit) time of the driver who arrives at work on time, $\hat{t}^r$, is determined by the condition

$$\hat{t}^r + ws(\hat{t}^r - t_0^r) = t^*.$$

Substituting for t_0^r with (14)

(16) $\hat{t}^r = t^* - \frac{\gamma}{\beta+\gamma} wN$.

Since the level of the toll does not affect equilibrium we can set $\tau(t_1^r) = 0$. Given (11) and (12) the optimal time-varying toll is

$$\tau(t) = \begin{cases} \lambda wN + [\beta-(\lambda-\beta)ws](t - t_0^r), & t_0^r \le t \le \hat{t}^r \\ \lambda wN + [\beta-(\lambda-\beta)ws](\hat{t}^r - t_0^r) - [\gamma+(\lambda+\gamma)ws](t-\hat{t}^r), & \hat{t}^r \le t \le t_1^r \\ 0 & t = t_1^r \end{cases}$$

with t_0^r, t_1^r and $\hat{t}^r$ given by (14), (15) and (16) respectively. The toll outside $[t_0^r, t_1^r]$ should be big enough to dissuade drivers from departing then.

Since the toll eliminates queueing time and is a transfer with no social cost, aggregate trip costs are the sum of walking time and schedule delay costs. Walking time costs are simply

(17) $WTC^r = \frac{\lambda}{2} wN^2$.

Since drivers arrive at work at a uniform rate throughout the rush hour, and since the first and last drivers incur the same schedule delay costs, average schedule delay cost is half that of the first driver, and aggregate schedule delay cost N times this:

(18) $SDC^r = \frac{1}{2}\beta(t^* - t_0^r)N = \frac{\delta}{2}(1+ws)\frac{N^2}{s}$.

Summing (17) and (18) we have

(19) $TC^r = \frac{\lambda}{2}wN^2 + \frac{\delta}{2}(1+ws)\frac{N^2}{2}$.

The optimal time-varying toll yields a cost saving (neglecting collection costs) equal to the difference between (19) and (9). These benefits are considered along with those of other tolling schemes in Section 8.

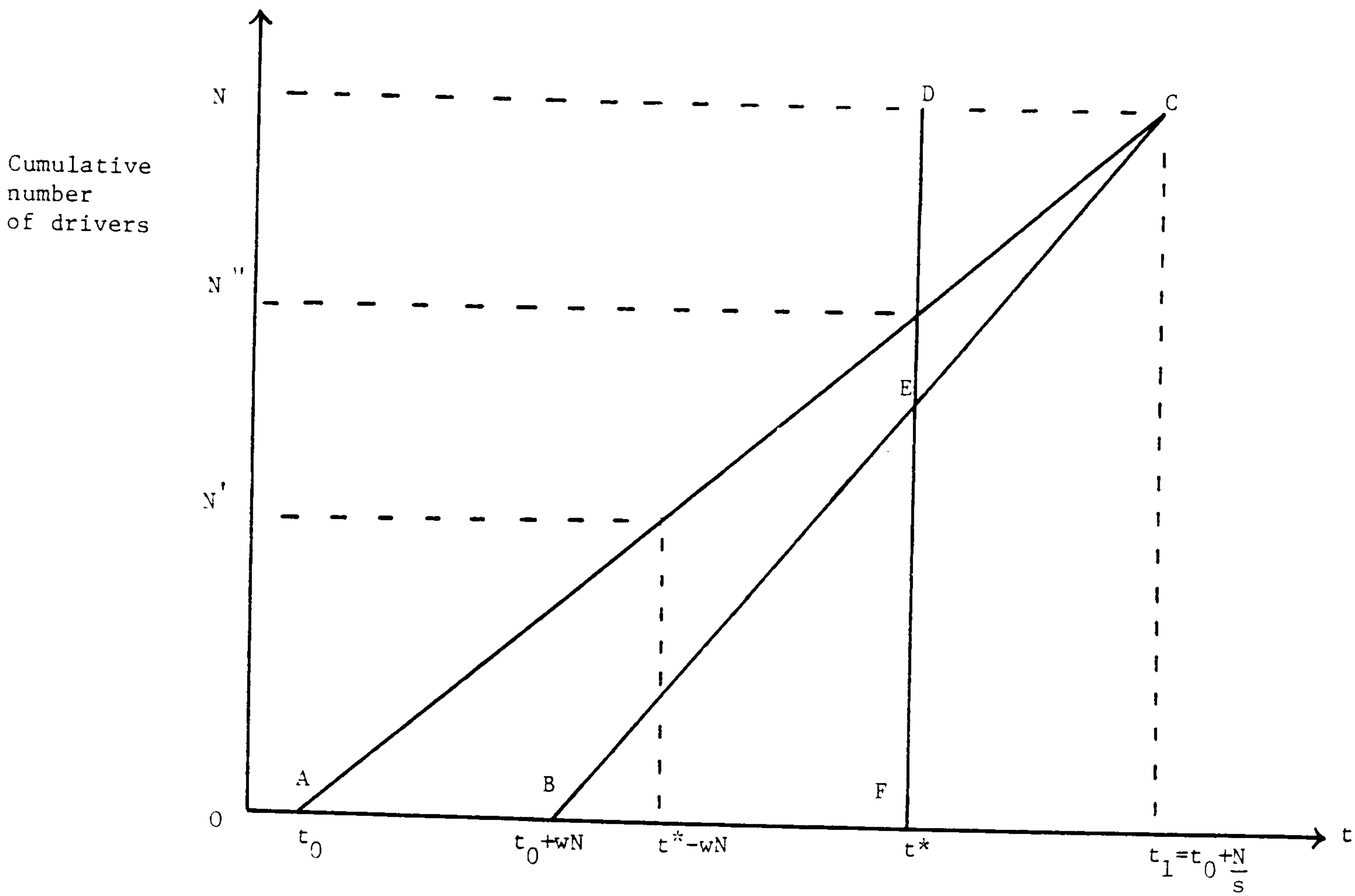
5. A Time-Varying Road Toll and Parking Fees (regime o)

Road tolls affect the travel time decisions of commuters, but not where they park. Early and late drivers alike prefer to park as close to the centre as possible. Parking spaces are occupied in strict order of increasing distance, which is inefficient because arrival times are spread over a period of duration $N/s+wN = (1+ws)N/s$. Were drivers to park in order of decreasing n, the arrival period could be compressed to $N/s-wN = (1-ws)N/s$, with a corresponding reduction in aggregate schedule delay, as shown in Figure 2.[7] (The arrows in the figure are considered below.)

Parking in strict sequence of decreasing n is in fact sufficient but not necessary to minimize schedule delay costs. This is shown in Figure 3, where the cumulative number of drivers is graphed against parking location rather than time. Drivers parking to the left of the 'on-time locus' arrive early. To the right they are late. Drivers departing before N' in the order are early no matter where they park. After N" they are invariably late.

Figure 2

Optimal Departure and Arrival Rates



Cumulative departures from home (= cumulative transits through bottleneck)	AC
Cumulative arrivals at the CBD	BC
Aggregate walking time	ACBA
Aggregate time early	BEFB
Aggregate time late	EDCE

If two early drivers switch parking places, and both still arrive early, schedule delay is unaffected. This is shown by the arrows labelled '1' in Figure 3. The earlier individual is assumed to park closer to the CBD than with parking in strict sequence. The previous occupant moves to the earlier driver's former spot. Figure 2 shows the effect of the switch on arrival time.

If two late drivers switch places and both still arrive late, as shown by the arrows labelled '2' in Figures 2 and 3, schedule delay is again unaffected. But if, say, two early drivers switch and one now arrives late ('3'), or if an early and a late driver switch ('4'), schedule delay increases. Schedule delay costs are minimized if and only if no early driver parks closer to the centre than any late driver.

The optimal timing of the rush hour is determined, as in Section 4, by equalizing the schedule delay costs of the first and last drivers:

(20) $\beta(t^* - t_0^o - wN) = \gamma(t_1^o - t^*)$,

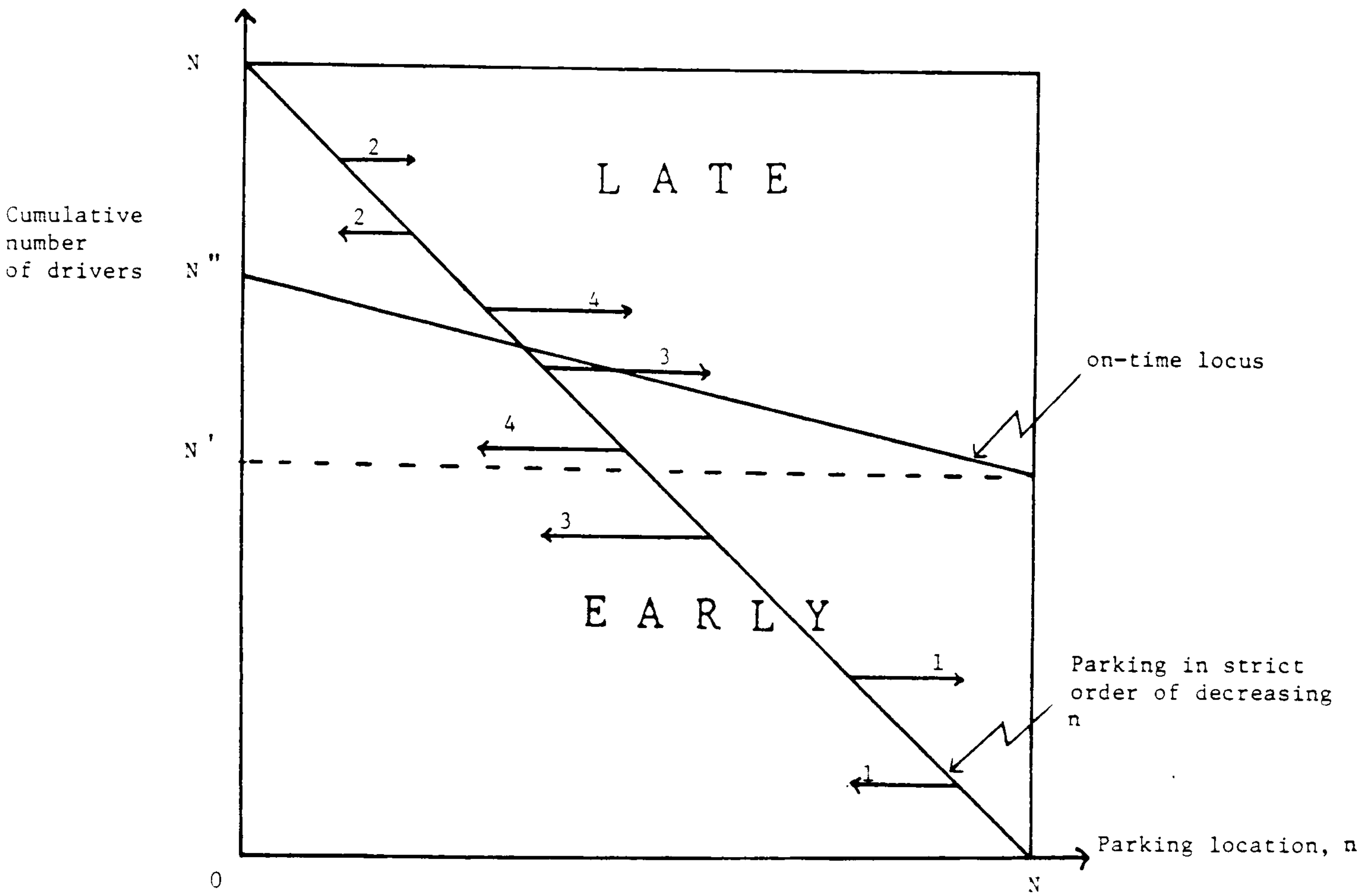
where the superscript ^{o} denotes the fully optimal equilibrium. Given the condition $t_1^o - t_0^o = N/s$, (20) yields:

(21) $t_0^o = t^* - \frac{\gamma+\beta ws}{\beta+\gamma}\frac{N}{s}$,

(22) $t_1^o = t^* + \frac{\beta(1-ws)}{\beta+\gamma}\frac{N}{s}$,

The rush hour begins later than if there is only a road toll if $t_0^o <$ t_0^r, or (given (21) and (14)) if $\gamma > \beta$. Since late arrival at work is generally more costly than early arrival this is probable. The rush hour begins later than in the no-toll equilibrium if $t_0^o > t_0^f$, or (given (21) and (7)) if $\lambda+\gamma > \beta$, which is true since $\lambda > \beta$ by assumption.

Figure 3



1: Both drivers early: Schedule delay unchanged

2: Both drivers late: Schedule delay unchanged

3: One early driver becomes late: Schedule delay increases

4: One driver early and one late: Schedule dalay increases

The departure (= bottleneck transit) time for arrival at t^* is defined by the condition

$$\hat{t}^o + ws(t_1^o - \hat{t}^o) = t^*,$$

or using (22)

$$(23)\quad \hat{t}^o = t^* - \frac{\beta}{\beta+\gamma} wN.$$

In addition

$$SDC^o = \frac{1}{2}\beta(t^* - t_0^o + wN) = \frac{\delta}{2}(1-ws)\frac{N^2}{s},$$

$$WTC^o = \lambda w \frac{N^2}{2},$$

$$(24)\quad TC^o = \frac{\lambda}{2}wN^2 + \frac{\delta}{2}(1-ws)\frac{N^2}{s}.$$

Given (24) and (19), $TC^o < TC^r$ if and only if $w>0$: confirmation that compressing work arrival times reduces aggregate costs (the arrival rate of individuals at work is $s/(1-ws) > s$).

Decentralization

To support both the optimal departure rate and parking location choice a location-dependent parking fee is required as well as a time-varying road toll. Let $\phi(n)$ be the parking fee at location n. Assuming as in Section 4 that queueing is eliminated the trip cost for an early driver is

$$C^o(t) = \tau^o(t) + \phi^o(n) + \lambda wn + \beta(t^* - t - wn), \qquad t_0^o \le t \le \hat{t}^o.$$

Differentiating with respect to t and imposing the condition

$$n(t) = N - s(t - t_0^o), \qquad t_0^o \le t \le \hat{t}^o,$$

we have

$$(25)\quad \dot{\tau}^o(t) - \phi_n^o s = \beta + (\lambda-\beta)ws, \qquad t_0^o \le t \le \hat{t}^o,$$

where $\phi_n^o \equiv d\phi^o(n)/dn$.

Now schedule delay + walking costs increase with n at rate $(\lambda-\beta)w$.

To induce early drivers to park in reverse order the parking fee gradient must satisfy:

$$(26)\quad \phi^{o}_{n} \le -(\lambda-\beta)w, \qquad \hat{n} < n < N,$$

where $\hat{n}$ is the parking location of the individual who arrives on time. For late drivers

$$C^{o}(t) = \tau^{o}(t) + \phi^{o}(n) + \lambda wn + \gamma(t + wn - t^{*}), \qquad \hat{t}^{o} \le t \le t^{o}_{1}.$$

Differentiating with respect to t one obtains

$$(27)\quad \dot{\tau}^{o}(t) - \varphi^{o}_{n}s = -\gamma + (\lambda+\gamma)ws, \qquad \hat{t}^{o} \le t \le t^{o}_{1}.$$

To induce drivers to park in reverse order

$$(28)\quad \phi^{o}_{n} \le -(\lambda+\gamma)w, \qquad 0 < n < \hat{n}.$$

If the parking fee gradient is chosen to satisfy (26) and (28) with equality,[8] (25) and (27) dictate that the toll satisfy

$$(29)\quad \dot{\tau}^{o}(t) = \begin{cases} \beta & t^{o}_{0} < t < \hat{t}^{o} \\ -\gamma & \hat{t}^{o} < t < t^{o}_{1}. \end{cases}$$

Using (29), the formulae for t^{o}_{0}, t^{o}_{1} and $\hat{t}^{o}$ given by (21), (22) and (23), and imposing the (arbitrary) boundary conditions $\tau(t^{o}_{0})=0$ and $\phi(N)=0$ the road toll and parking fee are found to be those shown in Figure 4. Toll revenue is

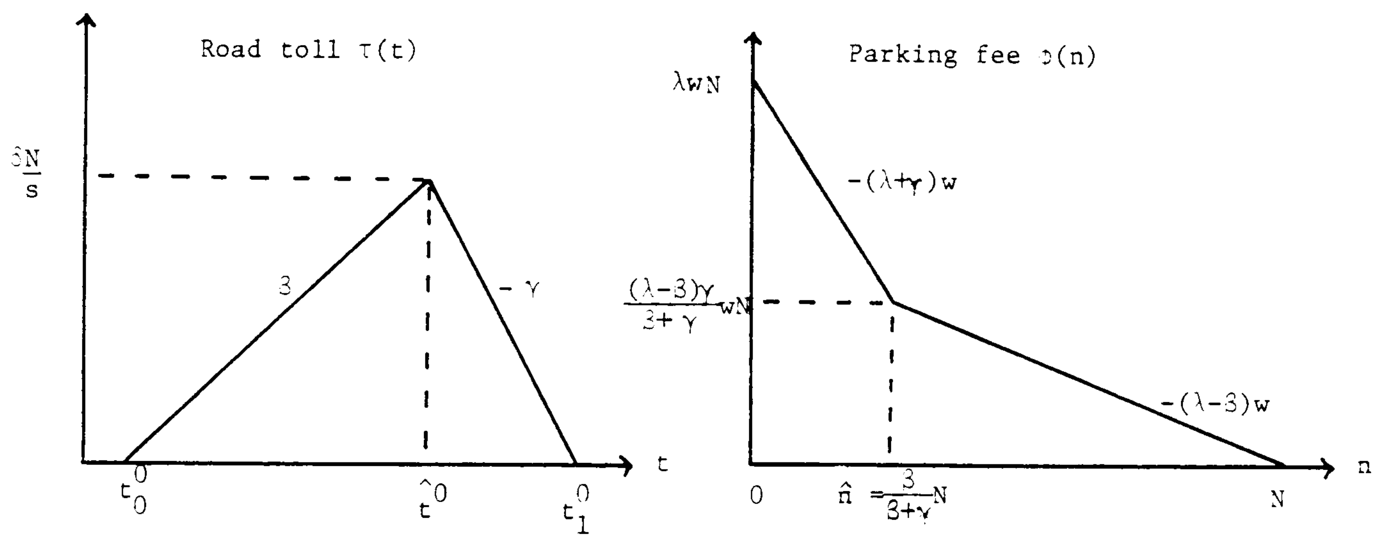
$$(30)\quad TR^{o} = \frac{1}{2}\beta s(\hat{t}^{o} - t^{o}_{0})^{2} + \frac{1}{2}\gamma s(t^{o}_{1} - \hat{t}^{o})^{2} = \frac{\delta}{2}\frac{N^{2}}{s}.$$

The parking fee (weakly) induces drivers to park in order of decreasing n, thereby minimizing aggregate schedule delay costs. The road toll eliminates queueing.[9] The two pricing instruments are targetted independently on the two margins of adjustment: the toll on departure time, the parking fee on parking location.

The toll and fee shown in Figure 4 are not unique in supporting the optimum; any combination satisfying (25)-(28) works. Moreover, if parking

Figure 4

Optimal Combined Road Toll
and Parking Fee Schedules



fees could be made time- as well as location-dependent it would be possible in principle to support both the efficient departure rate and parking location without a road toll.

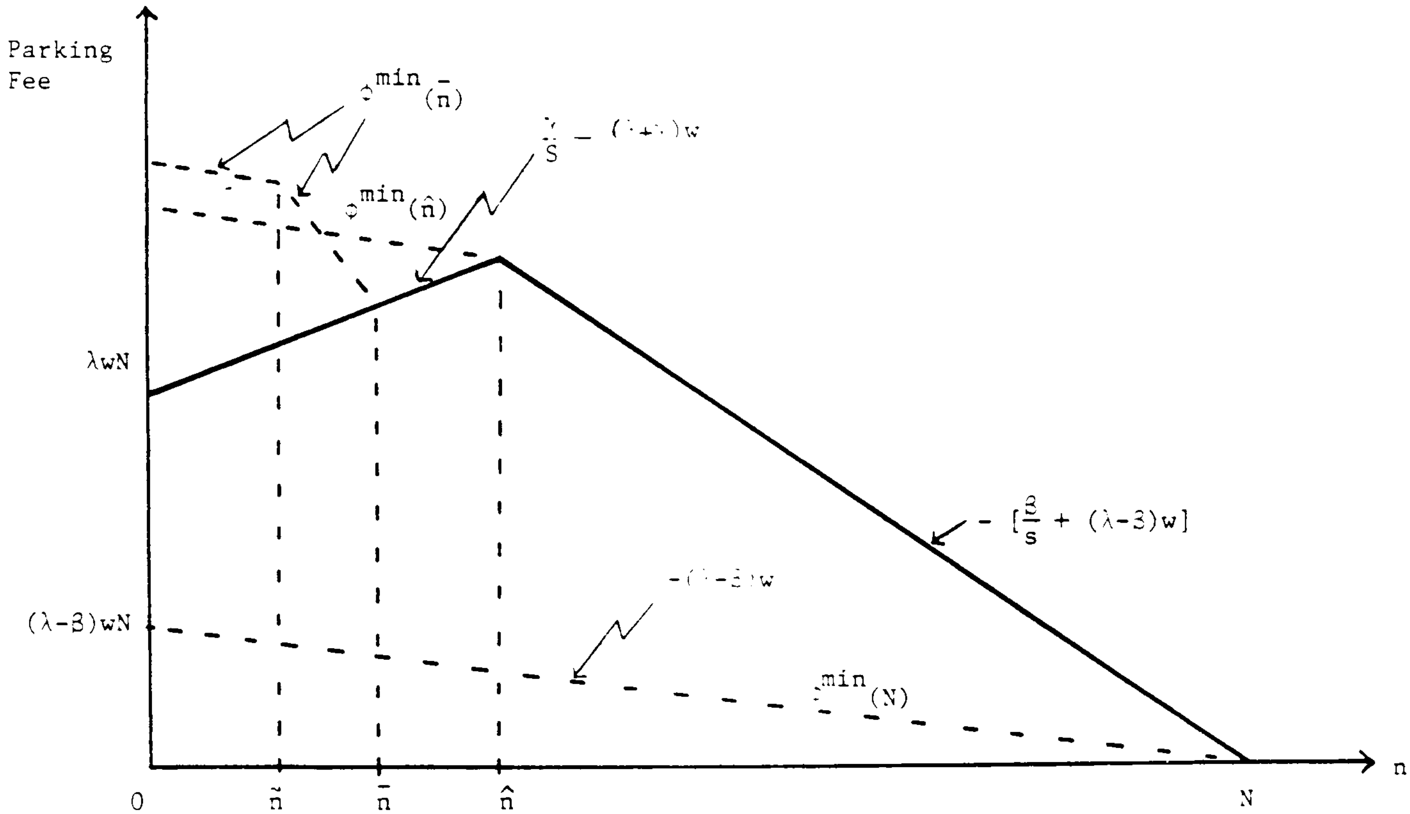
While time-varying parking fees are not a practical policy it is instructive to see how they would operate. The parking fee *actually paid* by drivers would have to follow the solid curve in Figure 5, with a location gradient determined by setting $\dot{r}^{o}(t)=0$ in (25) and (27). However, the parking fee *schedule* in effect when a driver arrived could not remain fixed throughout the rush hour. The first driver would park at $n=N$ as long as the fee schedule lay everywhere on or above the locus labelled $\phi^{min}(N)$ with slope $-(\lambda-\beta)w$, which it does. But the driver supposed to park at $\hat{n}$ would prefer to park closer unless the fee schedule lay on or above $\phi^{min}(\hat{n})$. Similarly, the driver supposed to park at $\bar{n}$ would pay up to $(\lambda+\gamma)w(\bar{n}-\tilde{n})$ to move from $\bar{n}$ to $\tilde{n}$ (the location from where he would arrive on time) and up to $(\lambda-\beta)w$ per unit distance between $n=\tilde{n}$ and $n=0$, as shown by the locus $\phi^{min}(\bar{n})$. The parking fee at closer locations would thus have to be steadily reduced during the later stages of the rush hour. This would appear administratively very difficult, and is unlikely to be implemented in the foreseeable future.

6. <u>Location-dependent Parking Fees but no Road Toll</u> (regime p)

The two previous sections concerned road tolls, either as the only pricing instrument, or in conjunction with parking fees. Tolls, however, have disadvantages. Toll booths force drivers to slow down or stop, with expenditure of time and fuel, and may create the very queues they are supposed to alleviate.[10] Tolls may divert traffic to untolled routes,

Figure 5

Optimal Time-Varying and Location
Dependent Parking Fee Schedule



thereby relocating rather than reducing congestion, and possibly increasing travel times. Road pricing has also met political opposition, which raises the question whether it would be implemented even if it passed a cost-benefit test on economic criteria.

Parking fees suffer neither of the technological disadvantages of road pricing mentioned above. Moreover, parking fees are almost ubiquitous in large cities, suggesting that a comprehensive system of fees varying with location would be more likely to be acceptable politically than road pricing. The purpose of this section is to consider parking fees as the sole pricing instrument available and examine how they fare *vis à vis* tolls on efficiency grounds. It is assumed that fees can be location- but not time-dependent.

Before deriving the optimal location-dependent parking fee schedule it is helpful to consider the two (nonoptimal) schedules in Figure 6. The one in panel (a) is that used in combination with the road toll in Section 5 (see Figure 4). Aggregate schedule delay costs are minimized because the fee supports both the optimal departure time interval and parking in order of decreasing n. The drawback is that queueing is not prevented; indeed aggregate travel time is the same as in the no-toll equilibrium without parking.

The schedule in panel (b) of Figure 6 does eliminate queueing, and also induces drivers to park in the correct order, but at the cost of having everyone arrive early. To see this, note that the trip cost of an early driver is

(31) $C^p(t) = \alpha\frac{D(t)}{s} + \phi(n) + \lambda wn + \beta(t^* - t - wn), \qquad t_0^p < t < \hat{t}^p,$

where the superscript p denotes the parking fee regime. With parking in

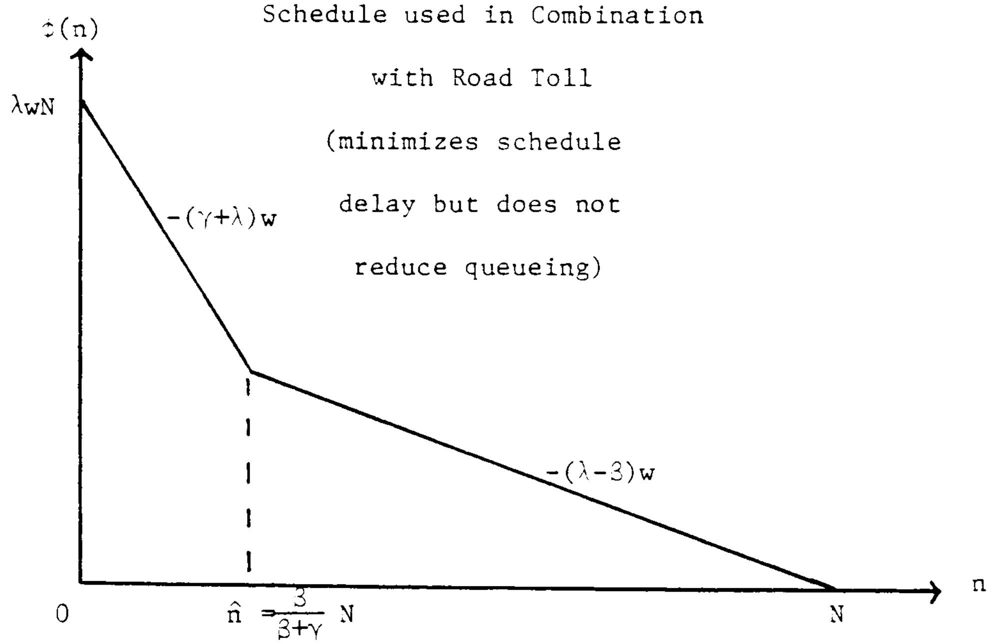
Figure 6

Two Location Dependent

Parking Fee Schedules

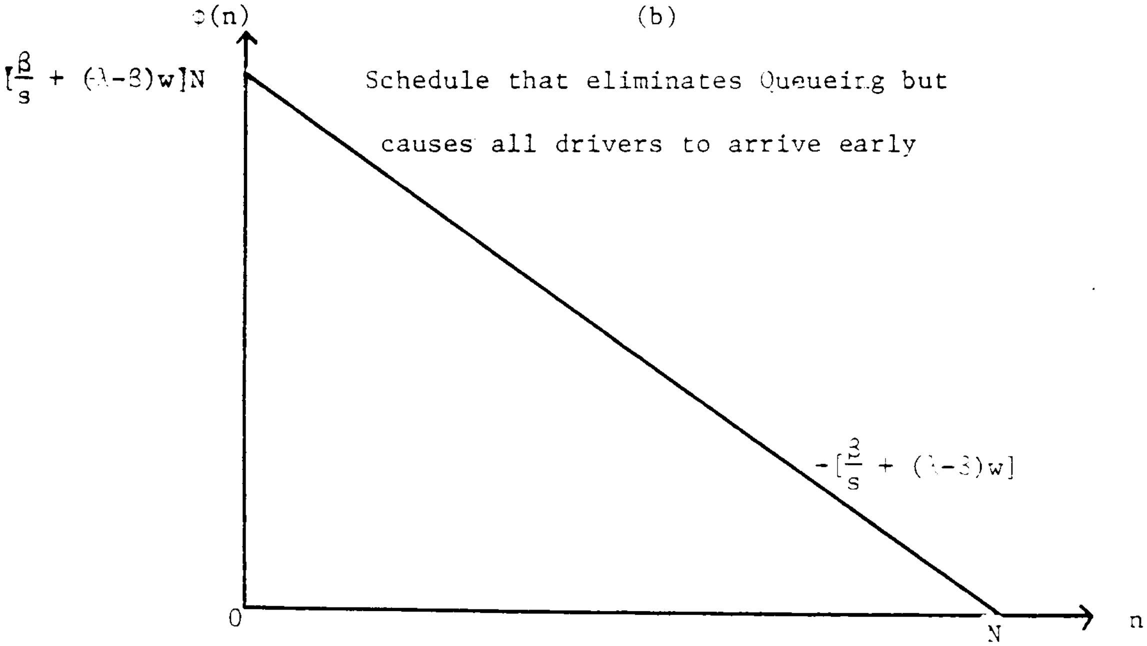
(a)

Schedule used in Combination with Road Toll (minimizes schedule delay but does not reduce queueing)



(b)

Schedule that eliminates Queueing but causes all drivers to arrive early



reverse order

(32) $n(t) = N - s(t - t_0^P)$.

Substituting (32) into (31), differentiating with respect to t and setting $\dot{C}^P(t) = 0$, one obtains

$$\dot{D}(t) = \frac{s}{\alpha}\,[\phi_n s + \beta + (\lambda-\beta)ws],$$

which is zero if

(33) $\phi_n = -[\frac{\beta}{s} + (\lambda-\beta)w]$

as it is in Figure 6. No driver is willing to arrive late because it would only increase his schedule delay with no offsetting reduction in either walking or queueing time.

To sum up: The first parking fee schedule in Figure 6 minimizes schedule delay costs, but does nothing about queueing. The second schedule eliminates queueing but induces drivers to travel too early. As we now show, the optimal location-dependent schedule is a hybrid of these two schedules.[11]

Early drivers

Consider first the trip cost of an early driver, given in equation (31). To induce drivers to park in order of decreasing n it is necessary to have

(34) $\phi_n \le -(\lambda-\beta)w, \qquad 0 \le n \le \hat{n},$

where $\hat{n}$ is the parking spot of the driver who arrives on time.[12] In equilibrium

(35) $C^P(t) = C^P(t_0^P) = \phi(N) + \lambda wN + \beta(t^* - t_0^P - wN)$.

Without loss of generality let $\phi(N)=0$. Equating the right-hand sides of (35) and (31) and rearranging:

(36) $D(t) = \frac{s}{\alpha}\,[\beta(t-t_0^P) - \phi(n) + (\lambda-\beta)w(N-n)], \qquad t_0^P \le t \le \hat{t}^P.$

Now setting the parking fee above the level that would drive the queue to zero would interrupt flow through the bottleneck, which is obviously inefficient.[13] We thus impose the condition $D(t) \geq 0$ on (36), which yields

$$\phi(n) \leq \beta(t - t_0^p) + (\lambda-\beta)w(N-n), \qquad t_0^p \leq t \leq \hat{t}^p.$$

Substituting out for t using (32) this becomes

(37) $\phi(n) \leq \beta\frac{N-n}{s} + (\lambda-\beta)w(N-n), \qquad \hat{n} \leq n \leq N.$

Late drivers

For late drivers, trip costs are

(38) $C^p(t) = \alpha\frac{D(t)}{s} + \phi(n) + \lambda wn + \gamma(t + wn - t^*).$

To induce drivers to park in reverse order

(39) $\phi_n \leq -(\lambda+\gamma)w, \qquad 0 \leq n \leq \hat{n}.$

In equilibrium,

(40) $C^p(t) = C^p(t_1^p) = \phi(0) + \gamma(t_1^p - t^*).$

Equating the right-hand sides of (40) and (38), rearranging and using the relation

$$t_1^p = t_0^p + \frac{N}{s},$$

one obtains

(41) $D(t) = \frac{s}{\alpha}\,[\gamma(t_0^p + \frac{N}{s} - t) - (\lambda+\gamma)wn + \phi(0) - \phi(n)],$

which yields a condition analogous to (37):

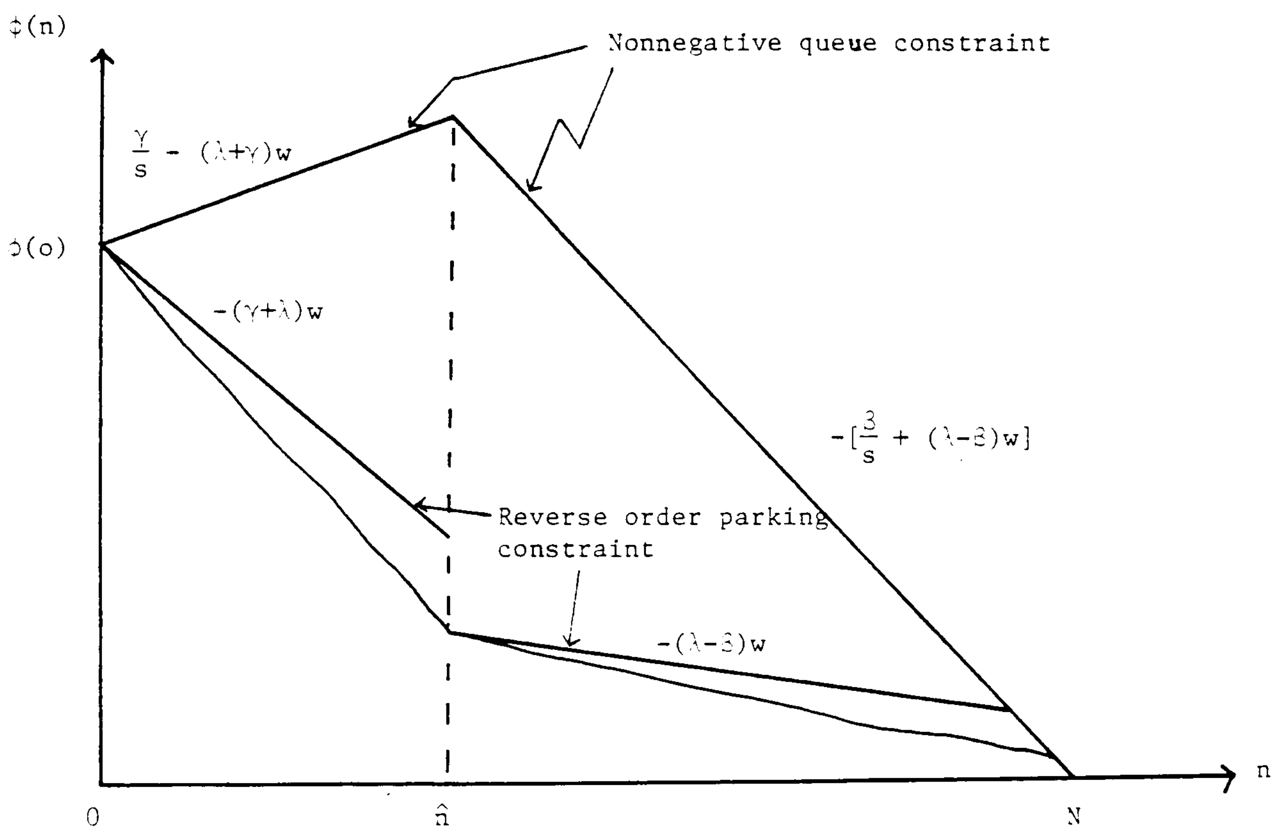
(42) $\phi(n) \leq \gamma\,\frac{n}{s} - (\lambda+\gamma)wn + \phi(0), \qquad 0 \leq n \leq \hat{n}.$

Conditions (34), (37), (39) and (42) restrict the optimal parking fee as shown in Figure 7. (The general level of the toll is fixed by the condition $\phi(N)=0$. $\phi(0)$ is treated as a given for the moment.) Conditions (42) and (37) impose upper bounds on the toll over the ranges $0 \leq n \leq \hat{n}$ and $\hat{n} \leq n \leq N$ respectively. Conditions (39) and (34) impose an upper bound on the parking fee gradient over the same respective ranges (the

Figure 7

Restrictions on Optimal Location Dependent

Parking Fee Schedule



particular loci shown in Figure 7 are representative only). It is clear from the slopes of the constraints that the nonnegative queue constraint will never be binding over the range $0 \leq n \leq \hat{n}$. A feasible parking fee schedule is shown by the wiggly line.

The value of $\hat{n}$ can be derived from (32) and the definition of $\hat{t}$:

$$\hat{t} + w[N - s(\hat{t} - t_0^p)] = t^*,$$

which yields

$$\hat{t} = \frac{t^* - w[N + st_0^p]}{1-ws}, \text{ and}$$

$$(43) \quad \hat{n} = N - s\,\frac{t^* - wN - t_0^p}{1-ws}.$$

t_0^p is solved by equating the trip costs of the first and last drivers:

$$\beta(t^* - t_0^p - wN) + \lambda wN = \gamma(t_0^p + \frac{N}{s} - t^*) + \phi(0), \text{ whence}$$

$$(44) \quad t_0^p = t^* + \frac{(\lambda-\beta)wN}{\beta+\gamma} - \frac{\gamma}{\beta+\gamma}\,\frac{N}{s} - \frac{\phi(0)}{\beta+\gamma}, \text{ and}$$

$$(45) \quad \hat{n} = \frac{[\beta + (\lambda-\beta)ws]N - s\phi(0)}{(1-ws)(\beta+\gamma)}.$$

The timing of the rush hour, and schedule delay costs, are thus determined by choice of $\phi(0)$.

Now aggregate travel time costs are

$$TTC = \alpha s \int_{t_0^p}^{t_1^p} \frac{D(t)}{s}\,dt,$$

which can be written

$$(46) \quad TTC = \int_{n=0}^{\hat{n}} [\gamma\,\frac{n}{s} - (\lambda+\gamma)wn + \phi(0) - \phi(n)]dn$$

$$+ \int_{n=\hat{n}}^{N} [\beta \frac{N-n}{s} + (\lambda-\beta)w(N-n) - \varphi(n)]dn.$$

In light of (37) and (42) TTC is thus the area in Figure 7 between $\phi(n)$ and the nonnegative queueing constraint.

For given $\phi(0)$, which determines schedule delay, it is thus optimal to choose $\phi(n)$ as large as possible, as shown by the bold curve in Figure 8. Queueing is eliminated for drivers parking in the most distant spots $n \in [n', N]$. Over this range the fee schedule matches that in panel (b) of Figure 6.[14] The fee for the remaining spots falls with distance, just quickly enough to induce parking in order of decreasing n. Over this range it has the same shape as the fee schedule in panel (a) of Figure 6. It follows immediately from Figure 8 and Figure 6 that

(47) $\phi(0) = [\frac{\beta}{s} + (\lambda-\beta)w](N - n') + \lambda wn' = [\frac{\beta}{s} + (\lambda-\beta)w]N - \frac{\beta}{s} (1-ws)n'.$

Substituting (47) into (44) and (45) one has

(48) $t_0^p = t^* - \frac{N}{s} + \frac{\beta}{\beta+\gamma} (1-ws) \frac{n'}{s},$

(49) $\hat{n} = \frac{\beta}{\beta+\gamma} n'.$

Aggregate costs are now easily determined. As always,

(50) $WTC = \frac{\lambda}{2} wN^2.$

Aggregate travel time costs are from Figure 8 and (49)

(51) $TTC = \frac{1}{2} \frac{\gamma}{s} \hat{n}^2 + \frac{1}{2} \frac{\beta}{s} (n' - \hat{n})^2 = \frac{\delta}{2} \frac{(\hat{n})^2}{s}.$

Schedule delay costs are (counting drivers as they arrive at work)

$$SDC = \frac{s}{1-ws} [\frac{\beta}{2} (t^* - t_0^p - wN)^2 + \frac{\gamma}{2} (t_0^p + \frac{N}{s} - t^*)^2],$$

or, given (48),

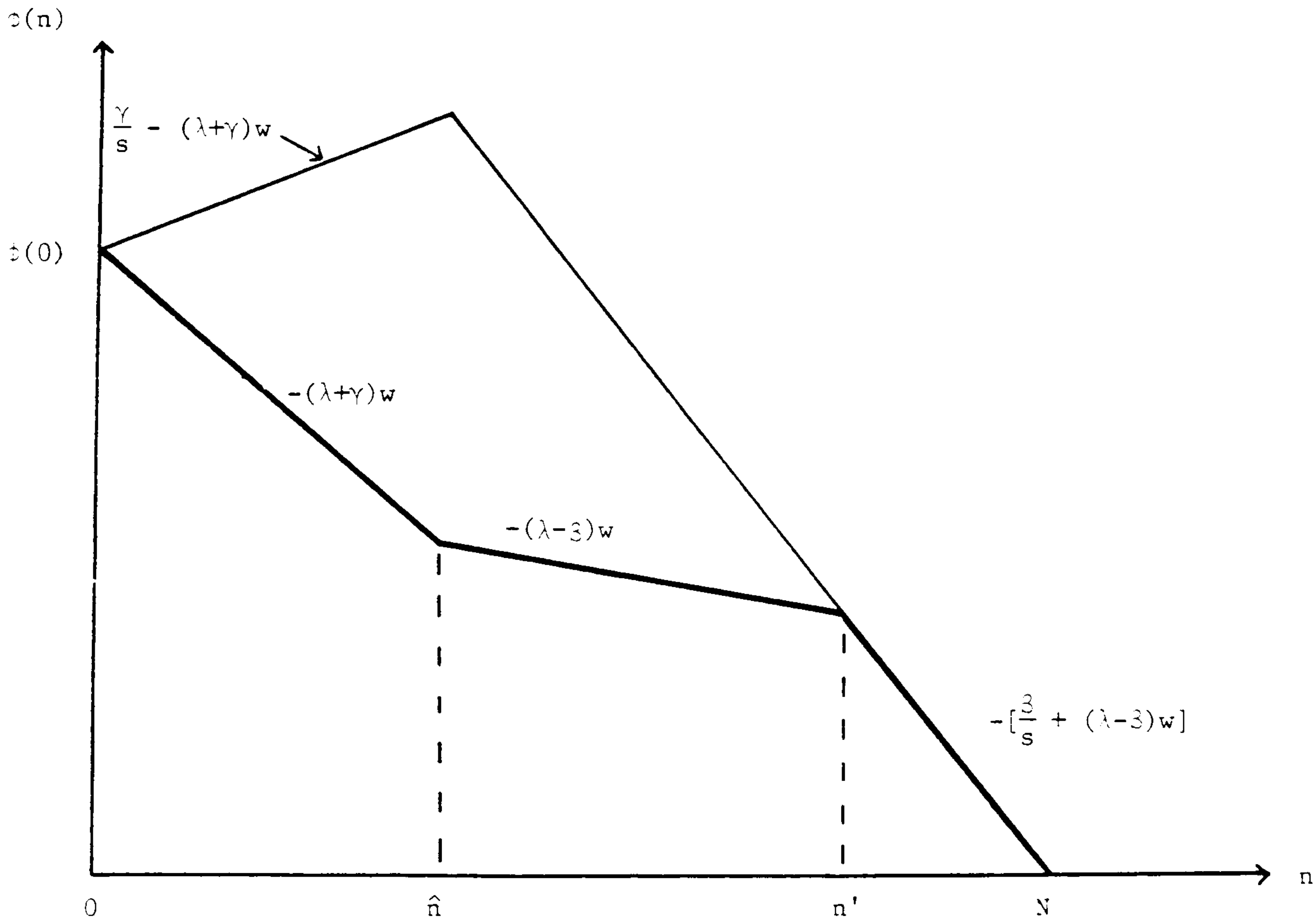
(52) $SDC = \frac{1-ws}{2s} [\beta(N - \frac{\beta}{\beta+\gamma} n')^2 + \gamma \frac{\beta^2}{(\beta+\gamma)^2}(n')^2].$

Adding (50), (51) and (52)

Figure 8

The Optimal Location Dependent

Parking Fee Schedule



$$(53)\quad TC^P = \frac{\beta + (\lambda-\beta)ws}{2}\frac{N^2}{s} - \frac{\beta^2}{\beta+\gamma}(1-ws)\frac{Nn'}{s}$$
$$+ \left[\frac{1-ws}{2}\frac{\beta^2}{\beta+\gamma} + \frac{\beta\gamma}{2(\beta+\gamma)}\right]\frac{(n')^2}{s}.$$

Finally, the optimal fee schedule is obtained by choosing n' to minimize TC. The solution can be readily shown to be

$$(54)\quad n' = \frac{\beta(1-ws)}{\gamma + \beta(1-ws)} N,$$

$$(55)\quad TC^P = \frac{\lambda}{2}wN^2 + \frac{\beta(1-ws)}{2}\left[1 - \frac{\beta^2(1-ws)^2}{(\beta+\gamma)(\gamma+\beta(1-ws))}\right]\frac{N^2}{s}.$$

Savings in costs from the optimal location-dependent toll are the difference between (55) and (9). The savings will be compared with those of the other toll regimes in Section 8.

7. Competitive Pricing of Parking (regime c)

In many cities parking is operated by the private rather than public sector. In this section we investigate how parking will be priced in such an environment. For simplicity the market is assumed to be perfectly competitive in that each parking spot is owned by a different operator. Because locations close to the CBD are most convenient, however, they command a differential Ricardian rent over locations further away.

To derive the competitive equilibrium we begin with the observation that, since the demand for a given parking location is all-or-nothing (either some driver will occupy it or it will remain vacant) the profit-maximizing parking fee is the reservation price of drivers. In equilibrium the parking fee schedule is such that no owner can raise his parking fee without losing custom. Owners are assumed to take the fees set by other owners as given, but to perceive correctly drivers' parking decisions.[15]

The main features of the equilibrium are given in the following two propositions.

Proposition 1 All drivers arriving strictly early at work park in locations $n \in (\hat{n}^c, N]$, for some $\hat{n}^c$ to be determined. All drivers arriving strictly late park in the interval $[0, \hat{n}^c)$.

Proof See Appendix 1.

Proposition 2 In equilibrium the parking fee schedule is of the form

$$(56)\quad \phi(n) = \begin{cases} \bar{\phi} + (\lambda-\beta)w(N-\hat{n}^c) + (\lambda+\gamma)w(\hat{n}^c-n), & 0 \le n \le \hat{n}^c \\ \bar{\phi} + (\lambda-\beta)w(N-n), & \hat{n}^c \le n \le N, \end{cases}$$

where $\bar{\phi}$ is some value, determined outside the model, which does not affect the results. If $\hat{t}^c$ is the time after which drivers occupy parking spots $n \in [0, \hat{n}^c)$, then

$$(57)\quad \hat{t}^c + w\hat{n}^c = t^*.$$

Proof See Appendix 2.

To solve for $\hat{n}^c$ observe that since locations $n \in [\hat{n}^c, N]$ are occupied during the period $[t_0^c, \hat{t}^c]$,

$$(58)\quad N-\hat{n}^c = s(\hat{t}^c - t_0^c).$$

Combining (58) with (57)

$$(59)\quad \hat{n}^c = \frac{N - s(t^* - t_0^c)}{1-ws}.$$

Now since the first and the last drivers, who escape queueing, are as well off parking at n=N and n=0 respectively, equality of trip costs dictates

(60) $\bar{\phi} + \lambda wN + \beta(t^* - t_0^c - wN) = \phi(0) + \gamma(t_0^c + \frac{N}{s} - t^*)$.

From (56) we have

(61) $\phi(0) = \bar{\phi} + (\lambda-\beta)wN + (\beta+\gamma)w\hat{n}^c$.

Solving (59), (60) and (61) simultaneously one has

(62) $t_0^c = t^* - \frac{\gamma+\beta ws}{\beta+\gamma}\frac{N}{s}$,

(63) $\hat{t}^c = t^* - \frac{\beta}{\beta+\gamma} wN$,

(64) $\hat{n}^c = \frac{\beta}{\beta+\gamma} N$.

Since (62) matches (21), (63) matches (23) and the parking fee schedule (56) matches that in Figure 4, the timing of the rush hour and schedule delay costs are identical to that in the full optimum. In the absence of a road toll, however, departure time slots must be rationed by queueing. Travel time costs in the competitive pricing equilibrium thus equal toll revenue in the full optimum, given by equation (30):

(65) $TTC^c = TR^o = \frac{\delta}{2}\frac{N^2}{s}$.

Total social costs in the competitive equilibrium are

(66) $TC^c = TC^o + TTC^c = \frac{\lambda}{2} wN^2 + \frac{\delta}{2} [2-ws] \frac{N^2}{s}$

with TC^o given by (24).

The parking fee schedules under competition and social management are congruent because in both cases location rents are expropriated from commuters: under competition because of profit maximization, and under management to delay start of the rush hour.

Congruence of the competitive and optimal parking fee schedules might lead one to expect that the competitive pricing regime is unambiguously superior to the equilibrium with no pricing considered in Section 3. But subtracting (66) from (9) one obtains:

Proposition 3

$TC^c < TC^f$ if and only if either $\gamma < \beta$ or $\gamma > \beta$ and $\lambda < \frac{3\beta\gamma}{\gamma-\beta}$.

We thus have the perhaps surprising finding that competitive pricing can be welfare-reducing relative to no pricing at all. (This happens if travel time costs rise by more than schedule delay costs fall.) This result provides a motivation beyond the logistical and political drawbacks of road tolls for investigating optimal parking fee schemes as practical measures for urban congestion relief.

As a final observation, note that the full optimum can be supported by the time-varying toll of Figure 4, since the competitive parking fee is invariant to whether departure time slots are rationed by queueing or by a toll.

8. Comparison of the Various Road Toll and Parking Fee Regimes

Total social costs in the regimes considered in Sections 3-7 are listed in Table 1. It is straightforward to establish

Proposition 4

$TC^o < \mathrm{Min}(TC^r, TC^p)$, and $\mathrm{Max}(TC^r, TC^p) < \mathrm{Min}(TC^f, TC^c)$.

However, the rankings of TC^r and TC^p, of TC^f and TC^c and the relative efficiency of the regimes are parameter-dependent. A natural measure of efficiency of a regime is the reduction in aggregate trip costs from the no-toll equilibrium achieved as a proportion of the savings in the full social optimum. For regime i:

$$E^i = \frac{TC^f - TC^i}{TC^f - TC^o}, \quad i = r, p, c.$$

TABLE 1

Aggregate Social Costs for Various Toll and Parking Fee Regimes

Regime	Total Costs
(f) Free: No toll or parking fee	$\lambda \frac{\beta}{\beta+\gamma} wN^2 + \delta(1+ws)\frac{N^2}{s}$
(r) Optimal road toll	$\frac{\lambda}{2} wN^2 + \frac{\delta}{2}(1+ws)\frac{N^2}{s}$
(o) Optimal road toll and parking fees	$\frac{\lambda}{2} wN^2 + \frac{\delta}{2}(1-ws)\frac{N^2}{s}$
(p) Optimal parking fees	$\frac{\lambda}{2} wN^2 + \frac{\beta(1-ws)}{2}[1 - \frac{\beta^2(1-ws)^2}{(\beta+\gamma)(\gamma+\beta(1-ws))}]\frac{N^2}{s}$
(c) Competitive parking fees	$\frac{\lambda}{2} wN^2 + \frac{\delta}{2}(2-ws)\frac{N^2}{s}$

To compute the E^i, parameter values must be chosen. Since total costs in all regimes are proportional to N^2/s no value for N need be specified. The parameters w and s enter only as the product ws. Now $ws = wN/(N/s)$ is the ratio of maximum walking time to the duration of vehicular flow through the bottleneck. In a city with a 2 hour rush hour flow a reasonable value for wN might be 12 minutes. As a benchmark value we thus choose $ws = 0.1$, and use 0.25 as a high value and 0 as a low value.[16]

Empirical evidence (Quarmby (1967), Lee and Dalvi (1969), Beesley (1973), Domencich and McFadden (1975), *inter alios*) indicates that walking and waiting time savings are valued between 1 and 3 times as much as in-vehicle travel time savings. We thus take 2α as a benchmark value for λ, and α and 3α as low and high values. For α, β and γ we use Small's (1982) estimates: $\alpha = \$6.40/\text{hr.}$, $\beta = \$3.90/\text{hr.}$, $\gamma = \$15.21/\text{hr.}$ (λ thus takes values 6.40, 12.80 and 19.20.) The resulting values of the E^i are given in Table 2, part (a).

With $ws = 0$, competitive parking fees confer no cost savings because differential Ricardian location rents are zero. A road toll is fully efficient since the order of parking is irrelevant. Perhaps surprisingly, the optimal location-dependent parking fee schedule achieves nearly 80% efficiency. With $w = 0$ this would entail charging different rates at different parking spots, despite them being equally attractive, *a priori*.

With $ws = 0.1$, optimal parking fees are marginally superior to a road toll. Competitive parking fees result in a small efficiency gain with $\lambda = \alpha$, but a loss with $\lambda = 3\alpha$. Finally, with $ws = 0.25$ parking fees are much superior to a road toll.

TABLE 2

Efficiency of Toll and Parking Fee Regimes relative to Social Optimum

(a) α=\$6.40/hr., β=\$3.90/hr., γ=\$15.21/hr.

ws	λ/α	Optimal Road Toll (r)	Optimal Parking Fees (p)	Competitive Parking Fees (c)
0	Irrelevant	1.0000	0.7959	0
0.1	1	0.8302	0.8408	0.1511
* 0.1	2	0.8106	0.8224	0.0530
0.1	3	0.7859	0.7992	-0.0707
0.25	1	0.6540	0.8884	0.3079
0.25	2	0.5614	0.8585	0.1227
0.25	3	0.4011	0.8068	-0.1979

(b) β=\$3.90/hr., ws = 0.1, λ=2α.

α	γ	Optimal Road Toll (r)	Optimal Parking Fees (p)	Competitive Parking Fees (c)
3.9	15.21	0.8263	0.8371	0.1314
* 6.4	15.21	0.8106	0.8224	0.0530
10.0	15.21	0.7823	0.7959	-0.0885
6.4	3.80	0.8471	0.6331	0.2357
6.4	7.61	0.8246	0.7230	0.1229
* 6.4	15.21	0.8106	0.8224	0.0530
6.4	30.42	0.8027	0.8980	0.0137
6.4	$+\infty$	0.7942	1.0000	-0.0290

* Benchmark parameters

As a further test of sensitivity, different values of α and γ are considered in part (b) of Table 2. (β is fixed, since only the ratios α/β and γ/β affect efficiency.) Neither the efficiency of the road toll or parking fees varies much with α, but the latter is sensitive to γ. In the limiting case $\gamma \to +\infty$ (no lates at work allowed) optimal parking fees are fully efficient. Competitive fees, in contrast, are welfare-reducing.

9. Concluding Remarks

Previous theoretical work on the dynamics of rush hour traffic congestion ignores parking as a facet of the urban commute. In this paper we take a first step at rectifying this oversight by examining the impact of time and money costs of parking on morning commuters' departure time and parking location decisions. We show that when road use and parking are free, drivers occupy parking spots in order of decreasing accessibility. This prolongs the period during which individuals arrive at work and increases aggregate schedule delay costs. Competitive pricing of parking leads to minimization of schedule delay costs, but fails to alleviate congestion. The welfare gain relative to no pricing is modest, and for some parameter values negative.

Two types of optimal pricing schemes are considered: time-varying road tolls and location-dependent parking fees. The road toll can be designed to eliminate congestion (at least in the pure queueing congestion model assumed) but does not alter the order in which parking spots are occupied. The optimal location-dependent parking fee on the other hand does not eliminate queueing, but does induce commuters to park at the most remote spots first, thereby considerably reducing schedule delay costs.

For most reasonable parameter values the parking fee is more efficient than the road toll. In light of the logistical drawbacks of tolls, and political opposition that road pricing has met, the results of the paper suggest that parking fees deserve more attention than they have received so far.

Neglected from this paper are several important features of parking in the real world that deserve investigation. First, we have assumed that all parking spots are located along radial commuting routes and do not require search. Parking in residential and commercial areas, however, is often not arranged systematically by accessibility; nor are the majority of drivers commuters who park all day. Search may be required for vacant spots. Search adds a stochastic element to commuters' arrival times; indeed, some commuters may not know whether they will arrive early or late for work.

Second, there are differences between off-street and on-street parking that appear to matter to commuters, such as entry and exit delays, awkward ramp geometry and poor visibility for some off-street stalls (Hunt (1988)). Users of on-street parking delay traffic while they are entering or exiting spots, or double-parking. Off-street parkers create congestion when queues develop outside parking garages. Users of both types of parking contribute to flow congestion while cruising for parking.

Third, commercial traffic, shoppers and through commuters add to road congestion and the demand for parking space in urban areas. To discourage commuters from using on-street parking, and to encourage shopping and business trips during off-peak hours, time-of-day or length-of-stay dependent parking fees may be employed.

Fourth, a large proportion of commuters in many cities use employer-provided and/or subsidized parking (Shoup (1982)), a practice believed to contribute significantly to overall congestion. This could be examined in an extended version of the present model, as well as (second-best) pricing of publicly-managed parking.

Fifth, we have treated the number of automobile commuters as fixed. In practice, some commuters may have access to public transit or a carpool. Besides affecting travel time and parking location decisions, road tolls and parking fees would alleviate congestion by reducing the total amount of traffic.

Finally, the CBD has been treated as a point. More realistic would be to model it as an area within which employment and parking are distributed. Depending on where they live, drivers travel varying distances on downtown streets on their way to work. Within limits, road tolls can be used to charge drivers on the basis of distance travelled, while parking fees cannot. This reveals a drawback of parking fees *vis a vis* road pricing as an instrument for alleviating congestion.

FOOTNOTES

[1]Gillen (1978) found empirically that parking fees are not very effective in reducing auto usage or relocating parking. But he did not consider the changes they may induce in the departure time distribution.

[2]A bottleneck can be created by a bridge, tunnel, intersection *etc*. More than one bottleneck may exist, but with pure queueing congestion only the smallest bottleneck capacity is consequential. In the absence of any bottleneck, s is the flow capacity at any point on the road.

In a typical city, several roads link the suburbs with the CBD. N may be interpreted as the number of commuters using a representative road, and s the capacity of the road.

[3]Pedestrian congestion is assumed away.

[4]The case $\alpha < \beta$ is treated for the model without parking in Arnott *et. al.* (1985).

[5]The equilibrium is a <u>weak</u> equilibrium with respect to departure time (but not parking location) in that individual drivers are indifferent as to when they travel in the interval $[t_0^f, t_1^f]$. If the daily departure rate were to deviate from that shown, however, trip costs would no longer be equal and departure times would adjust. The dynamics of adjustment are not considered here.

[6]Tolls could be imposed at booths, or by a system of electronic road pricing whereby vehicles fitted with electronic number plates would be identified by sensor loops under the roadway and charged on, say, a monthly basis by mail. Electronic road pricing was first proposed by Vickrey (1963) and the Ministry of Transport (1964) in the U.K., and experimented with in Hong Kong; see Dawson and Catling (1986) and Pretty (1988).

[7]To avoid complications it is assumed that $ws < 1$. This means that if drivers park in order of decreasing n those who depart later will not overtake drivers who departed earlier.

[8]The resulting equilibrium is then a <u>weak</u> equilibrium with respect to both departure time and parking location. See footnote 5 above. If (26) and (28) hold as strict inequalities for all n then the equilibrium is weak with respect to time but strong with respect to location.

[9]The road toll in Figure 4 decentralizes the optimal departure rate for the bottleneck model without parking ($w=0$), as shown by Arnott *et. al.* (1985) *inter alios*.

[10]Electronic road pricing would not suffer this disadvantage.

[11]The parking fee schedule derived below relies critically on two assertions that we are reasonably confident are correct, but so far have been unable to prove. The two assertions are noted with footnotes below.

[12]That the optimum entails parking in reverse order (for early and late drivers) is the first assertion that remains unproven.

[13]This is the second unproven assertion.

[14]For the parking pattern to be sustainable, drivers must be prevented from parking within a certain distance beyond $n=N$.

[15]It is necessary to assume that parking fees are fixed in advance, since otherwise owners with vacant spaces near the end of the rush hour will be able to raise their rates and gouge the last drivers.

[16]With $w = 0$ all parking is located next to the workplace, which might be true of a small town.

APPENDIX 1

Proof of Proposition 1

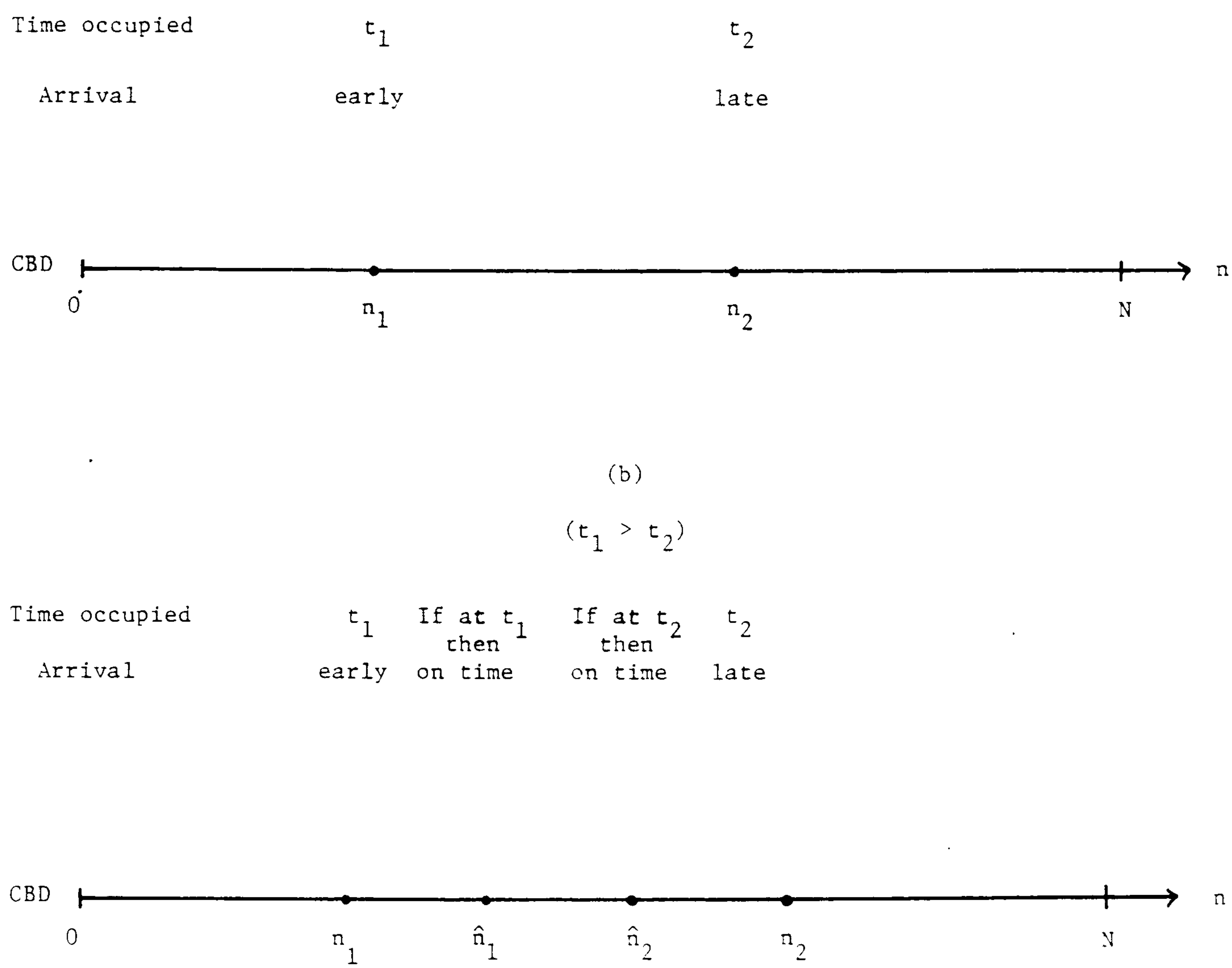
(By contradiction) Suppose some driver passes the bottleneck at time t_1, parks at location n_1, and arrives early at work, while another passes the bottleneck at t_2, parks at $n_2 > n_1$ and arrives late, as shown in panel (a) of Figure A1. On this assumption we have

Lemma There exists at least one driver who arrives early and one who arrives late, with the early driver passing the bottleneck sooner and parking closer than the late driver.

Proof of Lemma Given the situation depicted in Figure A1 (a), the proof is trivial if $t_1 < t_2$, so assume $t_1 > t_2$. Then there exist locations $\hat{n}_1$ and $\hat{n}_2$, with $n_1 < \hat{n}_1 < \hat{n}_2 < n_2$ as shown in Figure A1 (b), such that a driver passing the bottleneck at time t_1 and parking at $\hat{n}_1$, or passing the bottleneck at t_2 and parking at $\hat{n}_2$ arrives on time. If the lemma were false then all locations between $\hat{n}_1$ and $\hat{n}_2$ would have to be occupied between t_2 and t_1. (If such a spot were occupied before t_2, the driver, who would be early, would park sooner and closer than the driver at (t_2, n_2). If such a spot were occupied after t_1, the driver, who would be late, would park later and further than the driver at (t_1, n_1).) There are $(t_1-t_2)/w$ such locations. But the maximum number of drivers who can park between t_2 and t_1 is $s(t_1-t_2)$. Since $ws < 1$ by assumption, $s(t_1-t_2) < (t_1-t_2)/w$, a contradiction. □

Figure A1

(a)


Time occupied
t_1
t_2
Arrival
early
late
CBD
0
n_1
n_2
N
n
(b)
$(t_1 > t_2)$
Time occupied
t_1
If at t_1 then
If at t_2 then
t_2
Arrival
early
on time
on time
late
CBD
0
n_1
$\hat{n}_1$
$\hat{n}_2$
n_2
N
n

Proof of Proposition 1, concluded

Without loss of generality assume the situation in Figure A1 (a), obtains, with $t_2 > t_1$. In equilibrium, $C(t_1,n_1) = C(t_2,n_2)$ and the driver at t_1 must be unable to reduce his trip cost by parking at n_2 instead of n_1. There are two cases to consider.

Case 1: The driver at t_1 would still be early parking at n_2.

For equilibrium,

$$C(t_1,n_2) - C(t_1,n_1) = \phi(n_2) - \phi(n_1) + (\lambda-\beta)w(n_2-n_1) \geq 0.$$

Since the driver at t_2 would eliminate some or all of his late time by parking at n_1, we have

$$C(t_2,n_1) - C(t_2,n_2) < -[C(t_1,n_2) - C(t_1,n_1)] \leq 0.$$

The driver at t_2 would thus prefer to park at n_1 rather than n_2. If the owner at n_1 raised $\phi(n_1)$ slightly, the driver at t_1 would no longer park there (otherwise the original fee would not be profit-maximizing). But the driver at t_2 would still park at n_1, so the original fee couldn't be profit-maximizing.

Case 2: The driver at t_1 would be late parking at n_2

There exists a location $\hat{n}_1 \in (n_1,n_2)$ at which the driver at t_1 would arrive on time. For equilibrium:

$$C(t_1,n_2) - C(t_1,n_1) = \phi(n_2)-\phi(n_1) + (\lambda-\beta)w(\hat{n}_1-n_1) + (\lambda+\gamma)w(n_2-\hat{n}_1) \geq 0.$$

Since the driver at t_2 would be late parking at $\hat{n}_1$ we have again

$$C(t_2,n_1) - C(t_2,n_2) < -[C(t_1,n_2) - C(t_1,n_1)] \leq 0.$$

The proof concludes as for Case 1. □

APPENDIX 2

Proof of Proposition 2

The proof entails 3 lemmas. The first two establish the slope of the fee schedule for $n \geq \hat{n}$ and for $n \leq \hat{n}$. The third establishes equation (57).

Lemma 1 $\phi(n) = \bar{\phi} - (\lambda-\beta)w(n-N)$, $\hat{n} \leq n \leq N$.

Proof In equilibrium all parking spots $n \in [0,N]$ are occupied. Suppose

$$\phi(n') > \phi(n) - (\lambda-\beta)w(n'-n), \text{ for some } n, n'; \qquad \hat{n} \leq n < n' \leq N.$$

The driver parking at n' would then prefer to park at n. If n were as yet unoccupied the driver at n' would relocate, even if $\phi(n)$ were raised slightly. If n were occupied, the earlier driver would vacate if $\phi(n)$ were increased (otherwise the original fee would not have been profit-maximizing). This would free up n for the driver at n'. In either case, the original $\phi(n)$ would not have been profit-maximizing, in contradiction to equilibrium.

Suppose now that

$$\phi(n') < \phi(n) - (\lambda-\beta)w(n'-n), \text{ for some } n, n'; \qquad \hat{n} \leq n < n' \leq N.$$

Then at least one of the following two cases must occur.

Case 1 There exists $\tilde{n} \in (n,n')$ and a neighbourhood $\nu(\tilde{n})$ within which $d\phi/dn$ exists and is less than $-(\lambda-\beta)w$. Within any interval inside $\nu(\tilde{n})$, however small, some driver must arrive strictly early (since $ws < 1$). Such a driver would prefer to park slightly further away, even if the fee there were raised slightly, contradicting equilibrium.

Case 2 There exists $\tilde{n} \in [n,n')$ at which the fee takes a downward jump. A driver parking slightly closer than $\tilde{n}$ would prefer to park slightly beyond $\tilde{n}$, even if the fee were raised slightly, again a contradiction. This concludes the proof of Lemma 1. □

<u>Lemma</u> <u>2</u> $\phi(n) = \phi(\hat{n}) - (\lambda+\gamma)w(n-\hat{n}), \qquad 0 \le n \le \hat{n}.$

<u>Proof</u> The proof is analogous to that of Lemma 1. Suppose

$$\phi(n') < \phi(n) - (\lambda+\gamma)w(n'-n), \text{ for some } n,\ n'; \qquad 0 \le n < n' \le \hat{n}.$$

Then the driver parking at n would prefer to park at n'. The same argument as in the first part of the proof of Lemma 1 establishes a contradiction. Suppose now that

$$\phi(n') > \phi(n) - (\lambda+\gamma)w(n'-n), \text{ for some } n,\ n'; \qquad 0 \le n < n' \le \hat{n}.$$

Then at least one of the following two cases must occur.

<u>Case</u> <u>1</u> There exists $\tilde{n} \in (n,n')$ and a neighbourhood $\nu(\tilde{n})$ within which $d\phi/dn$ exists and is greater than $-(\lambda+\gamma)w$. Within any interval inside $\nu(\tilde{n})$, however small, some driver must arrive strictly late. Such a driver would prefer to park slightly closer, even if the fee there were raised slightly, contradicting equilibrium.

<u>Case</u> <u>2</u> There exists $\tilde{n} \in (n,n']$ at which the fee takes an upward jump. The driver parking at n' would prefer to park slightly closer, even if the fee were raised slightly, a contradiction. This proves Lemma 2. □

The final step is to prove

<u>Lemma</u> <u>3</u> $\hat{t} + w\hat{n} = t^*$.

<u>Proof</u> The proof derives from the observation that for the group of early drivers

$$\lim_{t \uparrow \hat{t}} n(t) = \hat{n}.$$

If the limit were $\bar{n} > \hat{n}$ then a driver parking just before $\hat{t}$ at $\bar{n}$ would be on time or early, and a driver parking just after $\hat{t}$ at $n \le \hat{n} < \bar{n}$ on time or late, which is impossible. Hence

$$\lim_{t \uparrow \hat{t}} t + wn(t) = \hat{t} + w\hat{n} = t^*. \quad □$$

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