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IMPLEMENTING A PUBLIC PROJECT AND DISTRIBUTING ITS COSTS

by

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Abstract

We provide a game form which undertakes a public project exactly when the total benefit of the project to individuals in a society outweighs its cost. The game form is simple, as well as balanced and individually rational. The game form can be adjusted to distribute cost according to a wide class of rules. For example, it can distribute costs so that each individual pays a share of the cost which is proportional to his or her benefit. We discuss the informational limitations of our work (at least two individuals need to know the average value of the project), and the relation of this work to the literature on mechanism design and public goods.

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1) INTRODUCTION

Several agents must decide whether or not to undertake a project that will benefit them all: it is efficient to do so if and only if the sum of individual benefits outweighs its total cost. The project is called "public" because it is consumed without rivalry by all agents.

We propose a family of mechanisms achieving an efficient and equitable outcome. The social planner (who designs the mechanism) tries to achieve two goals, a normative goal of equity and a positive goal of inducing the agents to achieve the desired outcome even though the planner chooses a mechanism in ignorance of agents' personal characteristics. The positive goal is usually referred to as the implementation issue.

In this paper we follow one stream of the implementation literature which looks at "universal" mechanisms: that is, the mechanism uses no statistical information about the distribution of agents' characteristics. It places the burden of acquiring information about the preference profile upon the agents themselves: once agents know enough about each other's preferences, the unique "reasonable" non-cooperative equilibrium implements the desired outcome. In other words, the implementation property requires the agents to know at least some summary of the overall preference profile. This is admittedly a strong assumption, but it appears necessary if we wish to use simple, intuitive mechanisms such as the Divide and Choose method. The alternative route, relying on the existence of Bayesian beliefs about mutual preferences, has a stronger claim to realism as a model of individual behavior, but its mechanisms are pegged to the Bayesian characteristics of a particular group of agents (see e.g. the bilateral trade mechanisms of Myerson and Satterthwaite [1983]).

The literature on non-cooperative implementation under complete

information has followed two directions; it has produced general characterization theorems and designed simple mechanisms for specific problems. The general results are technically impressive, but generally impractical for producing plausible mechanisms. The early results on implementation in Nash equilibrium (Maskin [1977] and [1985]) and in strong equilibrium (Peleg [1978] Moulin and Peleg [1982]) demonstrated that implementability imposes severe restrictions, in particular if one wishes to implement a single valued solution. In sharp contrast with those fairly negative conclusions, stand the recent characterization results on implementation by subgame perfect equilibrium (Moore and Repullo [1988], Abreu and Sen [1987]), undominated Nash equilibrium (Palfrey and Srivastava [1986]), and undominated strategies (Jackson [1989]). They reach the striking conclusion that virtually anything (any social choice function) is implementable. However, the mechanisms used to prove those broad possibility results are distastefully complex, in part because the theorems cover an enormous array of collective decision problems (applications include anything from voting rules to the exchange of goods). In fact, if we restrict our attention to "reasonable" mechanisms, then the striking results are mitigated (Jackson [1989]). Only strategy proof social choice functions are implementable in undominated strategies, while for undominated Nash implementation some social choice functions are ruled out - but the extent of this restriction is not yet known. (For subgame perfect implementation the issue is more subtle. The definition of implementation does not account for mixed strategy equilibria which may exist in "reasonable" mechanisms. This issue is largely unexplored, and is discussed with reference to Nash implementation in Jackson [1989].)

The inapplicability of the general results to simple collective decision problems is compensated in part by several papers dealing with the implementation issue in specific contexts. Examples include voting (McKelvey and Niemi [1978], Moulin [1979], Herrero and Srivastava [1989]), fair division (Crawford [1979], Demange [1984], Glazer and Ma [1989]), bargaining over lotteries (Moulin [1984a], Binmore, Rubinstein and Wolinsky [1986], Howard [1988]) and public decision with monetary transfers (Moulin [1981] [1984b]).

In this paper, we focus on the very simple (and often studied) model of provision of an indivisible public good. The normative goal is captured by a cost sharing rule (e.g. costs are proportional to benefits) and the positive goal is taken to mean implementation in undominated Nash equilibrium (or subgame perfect equilibrium: See footnote 1).

The mechanisms which we propose i) undertake the project in equilibrium when its collective benefit outweighs its total cost, and only then; ii) collect costs from individual agents that exactly balance the cost of the project; iii) do not force any individual to participate in the decision making process against his will and iv) accomodate a large class of cost sharing rules.

The more formal definitions of these properties are as follows. A mechanism translates agents' valuations of the project into a level of the project (0 or 1) and a cost to be paid by each agent. Transfers among the agents may be incorporated in the specification of the costs. A mechanism is successful if it always chooses the first best level of the public good. It is feasible if the sum of the costs of the agents is at least c when the project is undertaken, and at least 0 otherwise. It is balanced if the sum of the costs of the agents is c when the project is undertaken, and 0 otherwise. A mechanism is individually rational if the benefit each agent obtains from the project (times 0 or 1) outweighs the cost that agent pays.

Our mechanisms allow for a family of cost sharing-rules for which each agent's cost strictly decreases in the other agent's valuations for the project (and which must also satisfy another monotonicity property: see (4) below). For each such cost sharing rule we construct a simple two stage mechanism in which the agents report an estimate of the collective benefit accruing from the project in the first stage, whereas in the second stage they report their own benefit for the project. It does not rely on "doomsday" threats, is budget balanced (for all strategies) and individually rational (in equilibrium and in the sense that every agent has a strategy which guarantees a "no loss" outcome). It is reminiscent of the auctioning the leadership mechanism originally proposed by Crawford [1979] for the division of resources problem, and later applied to public decision with money by Moulin [1981] [1984b].

To put our mechanism into perspective we recall the classical results on direct revelation mechanisms (whereby an agent's message is a report of his or her valuation for the project). It is well known (see Green and Laffont [1979]) that a mechanism which is successful and balanced will not be dominant strategy incentive compatible. There is a non-trivial trade-off between efficiency and dominant strategy incentive compatibility. Groves mechanisms (Groves [1973], see also Vickrey [1961] and Clarke [1971]) are dominant strategy incentive compatible, but fail to achieve balance. The difficulty is more acute if individual rationality is considered. Every successful, individually rational, dominant strategy incentive compatible mechanism will fail to be feasible (see Green and Laffont [1979]).

One way out of the efficiency/incentive compatibility trade-off (for direct revelation mechanisms) is to allow the social planner to use statistical information about agents' valuations, and to replace dominant strategy equilibrium by Bayesian incentive compatibility. Of course those mechanisms are not very meaningful among a few agents, and are much more demanding on the social planner. D' Aspremont and Gerard-Varet [1979] demonstrate a mechanism which is Bayesian incentive compatible, successful and balanced. However, individual rationality is not satisfied (an ex-ante individual rationality constraint is satisfied, but the appropriate interim individual rationality constraints are not). Mailath and Postlewaite [1988] show that if individual rationality is required along with Bayesian incentive compatibility, then it is generally impossible to achieve efficiency (see also Myerson and Satterthwaite [1983] and Myerson [1985]). Moreover, Mailath and Postlewaite show that as the number of agents involved grows, the probability of ever undertaking the project goes to zero.

The restrictiveness of our analysis, and of the non-Bayesian implementation literature in general, is in terms of information held (or acquired) by the agents. As mentioned earlier, this is the price to pay for dealing with "universal" mechanisms. The same remark applies whether we deal with voting, division of private goods, etc. In our problem, we must assume that individuals know their own valuations, and at least two agents know the average of all the agents valuations. It is interesting to note that this is the minimal informational requirement which needs to be satisfied in order to achieve ex-post efficiency and individual rationality with any mechanism! Essentially, for any coarser information partitions, a Bayesian incentive compatibility condition needs to be satisfied. Our information structure satisfies NEI (non-exclusivity of information as defined by Postlewaite and Schmeidler [1986]). That is, any agent's type can be figured out by pooling the information of all the other agents. The NEI condition identifies the situations in which Bayesian incentive compatibility will fail to bite (see Blume and Easley [1988] and Jackson [1990]).

The rest of the paper is organized as follows. Section 2 presents our mechanism in the simple context with two agents and the proportional cost sharing rule (cost shares are proportional to benefits). In Section 3 we define our family of cost-sharing rules and discuss a few examples. Section 4 defines our mechanism and states our main result, the proof of which is given in Section 5.

2) AN EXAMPLE WITH TWO AGENTS

In this section we provide an example which illustrates the general structure of the game forms we consider. The game form described below implements the correct public project decision and distributes costs among two individuals in proportion to the benefits they receive from the project.

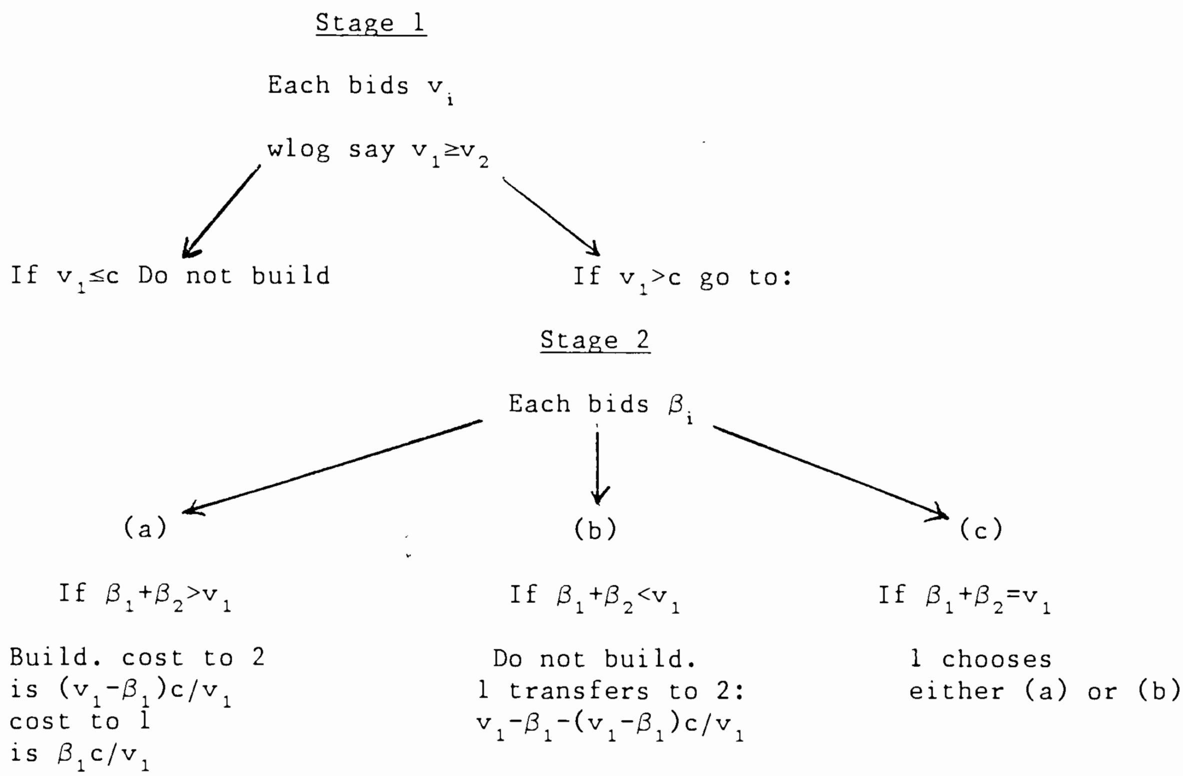
Let c be the cost of the project (which is common knowledge), and let b_1 and b_2 be the benefits the agents receive from the project if it is built.

In the first of two stages, agents simultaneously submit bids. These bids v_1,v_2 are interpreted as the agents' estimates of the joint benefit from the project. If the highest bid is less than or equal to c, the project is not undertaken. Otherwise a second set of (simultaneous) bids are solicited. These bids β_1, β_2 are interpreted as the agents' reports of their own valuations for the project.

Without loss of generality, say that agent 1 had the highest first stage bid v_1 (ties can be broken according to any rule). If the sum of the second stage bids is greater than the winning first stage bid, then the project is undertaken and agent 1 pays $(\beta_1/v_1).c$, while agent 2 pays $((v_1-\beta_1)/v_1)c$. If the sum of the second stage bid is less than the winning first stage bid, then the project is not undertaken and agent 1 transfers $v_1-\beta_1-((v_1-\beta_1)/v_1)c$ to 2.

If the sum of the second stage bids is equal to the winning first stage bids, then agent 1 can decide to follow either of the above prescriptions.

This game form is represented in the figure below.



In the unique Nash equilibrium in undominated strategies, each agent bids the correct total valuation of the project in the first round and their own valuation in the second round. The formal proof of this is presented in the next section. The analysis of the game form is made easy by the fact that agent 2 has a dominant strategy in the second stage which is to bid his or her true valuation. (The second stage is essentially a pivotal mechanism for agent 2.) This determines a best response for agent 1 in the second stage, which depends on the winning first stage bid. Namely, $\beta_1=(v_1-b_2)^+$ and build (where $z^+=\max(z,0)$ and b_2 is agent 2's true valuation). It then remains to verify that the first stage equilibrium bids are the true total valuation (if it is greater than c, and any bids not higher than c otherwise.) This follows since agents prefer not to win with a bid higher than the true total valuation, but do prefer to win with a bid lower than the true valuation. To see this, first notice that the winning agent's cost $c(v_1-b_2)^+/v_1$ is increasing for $v_1>b_2$ and non-decreasing otherwise. Thus the "winner" would like to win with as low a bid as possible. However, also notice that if (and only if) $v<b_1+b_2$, then an agent is better off as the "winner (1 pays $c(v-b_2)^+/v$ as opposed to $c(v-(v-b_1)^+)/v$ if 1 loses). Thus, no agent will wish to "lose" when $v<b_1+b_2$. In sum the only equilibrium bids are $v_1=v_2=b_1+b_2$.

The cost (or transfers) are balanced by design. The unique equilibrium results in the first best level of the public good, and shares the cost proportionally. It is also clear that the particular cost function is not critical in the analysis of the equilibrium. Thus, the game form will work for a whole class of cost sharing rules which are non-decreasing in own benefit and non-increasing in other's benefit. The equilibrium outcomes are clearly individually rational (in fact bidding the true total valuation in the first round and the true benefit in the second round is individually rational, provided that no agent plays a weakly dominated strategy), but we can go beyond that to say that each agent has a strategy which guarantees him or her a utility of at least 0 even ignoring the other agent's actions. That strategy is to bid 0 in the first round and to bid the true benefit in the second round.

3) A FAMILY OF COST SHARING RULES

Denote by c the positive cost of the project and assume that n agents share this cost. Denote by b_i agent i's benefit for the project. The domain of b_i contains all non-negative numbers.

If the profile $(b_1,..,b_n)$ is such that $\sum_{i=1}^{n} b_i < c$, the project will not be undertaken and (by individual rationality) no transfer of money among players occurs. Thus we need only to define a cost sharing rule over the domain B:

$$B=\{(b_1,..,b_n) : b_i\geq 0 \text{ all } i, \text{ and } \sum_{i=1}^{n} b_i\geq c\}$$

All rules discussed in the literature (see e.g. O'Neill [1982], Aumann and Maschler [1985], Young [1987]) can be written as a mapping γ associating to every profile $(b_1,..,b_n)$ in B a vector of cost shares $\gamma_i(b_1,..,b_n)$, $i=1,..,n$, and satisfying:

budget balance: $\sum_{i=1}^{n} \gamma_i(b_1,..,b_n)=c$, and

core bounds: $0\leq\gamma_i(b_1,..,b_n)\leq b_i$.

The "core" here refers to the possibility for any coalition to build the project at its own cost.

Another primitive assumption is <u>anonymity</u>: the rule discriminates among agents only on the basis of their benefit level. Formally, this says that by switching b_i and b_j in the profile, we exchange γ_i and γ_j and leave every other γ_k unchanged. This in turn allows us to represent the cost-sharing rule by a <u>single</u> real valued function $\gamma(b_1;b_2,...,b_n)$.

Definition

A cost-sharing rule is a mapping γ defined over B and with range $[0,c]$ and satisfying:

anonymity: $\gamma(b_1;b_2,..,b_n)$ is a symmetrical function of $b_2..,b_n$,

budget balance: $\sum_{i=1}^{n} \gamma(b_i;b_{-i})=c$, and

core bounds: $0\leq\gamma(b_i;b_{-i})\leq b_i$.

In addition to these three primitive properties, the two following monotonicity properties are typically true:

$\gamma(b_1;b_2,..,b_n)$ is non-increasing in b_i, $i\geq 2$. (1)

Note that in view of budget balance and anonymity, property (1) implies that $\gamma(b_1;b_{-1})$ is non-decreasing in b_1. The second monotonicity property is as follows:

$$\gamma(b_1-\lambda;b_2+\lambda,b_3,..,b_n)\geq\gamma(b_1;b_2,..,b_n)-\lambda, \text{ for all } \lambda>0 \text{ and all } b\epsilon B \text{ s.t.} b_1\geq\lambda. \quad (2)$$

Property (2) says that when a unit of agent 1's benefit is shifted to agent 2, agent 1's share does not reduce by more than the amount transferred.

These two properties are satisfied by the proportional cost sharing rule:

$$\gamma(b_1;b_2,..,b_n)=c\ b_1/(b_1+...+b_n).$$

They also hold true for the two methods: equal cost under the core bounds (agent i pays either a common cost-share αb_i, or whichever is less), and equal benefits under the core bounds (agent i pays either zero or $b_i-\beta$, whichever is more). They can also be checked for the talmudic solution of Aumann and Maschler [1985], and for all the examples of parametric methods discussed by Young [1987].

In the theorem stated in the next section, we will in fact require a <u>strict</u> monotonicity version of the properties (1) and (2), thereby eliminating some popular methods, such as equal cost and equal benefits (under core bounds). However, a straightforward approximation argument shows that every cost-sharing rule satisfying (1) and (2) is arbitrarily close to a rule satisfying the following properties (3) and (4):

$$\gamma(b_1;b_{-1}) \text{ is decreasing in } b_2 \text{ if } b_1>0, \text{ and} \quad (3)$$

$$\gamma(b_1-\lambda;b_2+\lambda,b_{-1,2})>\gamma(b_1;b_{-1})-\lambda \text{ for all } \lambda>0 \text{ and all b s.t.} b_1\geq\lambda \text{ and } \sum_{i=1}^{n} b_i>c. \quad (4)$$

Notice that (3) implies that $\gamma(b_1;b_{-1})$ is increasing in b_1 if $\sum_{i\geq2} b_i>0$. Hence

$$\gamma(b_1-\lambda;b_2+\lambda,b_{-1,2})<\gamma(b_1;b_{-1}) \text{ for all } \lambda>0 \text{ and all b in B s.t. } b_1\geq\lambda. \quad (5)$$

We remark that the proportional cost-sharing rule satisfies (3) and (4), indeed.

4) THE MECHANISM AND THE THEOREM

We are given a cost sharing rule γ satisfying properties (3) and (4). In order to construct a mechanism to implement the efficient project decision and distribute costs according to γ, we define an auxiliary function θ over R^n_+. We use the notation

$$x_{N/i} = \sum_{j\neq i} x_j .$$

We now define θ for all $v\geq c$, $b_i\geq 0$, $i \geq 2$:

$$\theta(v;b_2,\ldots,b_n)=\gamma(v-b_{N/1};b_2,..,b_n) \text{ if } b_{N/1}\leq v, \text{ and}$$

$$\theta(v;b_2,\ldots,b_n) = (v-b_{N/1}) \frac{c}{v} \qquad \text{if } v \leq b_{N/1}.$$

Note that $\gamma(c;b_{-1}) = 0$ by the core bounds so θ is well defined at $b_{N/1}=v$. Also, check the following properties:

$$\theta(c;b_{-1}) = c-b_{N/1}, \text{ for all } b_{-1}, \ \theta(v;0)=c \text{ for all } v\geq c, \text{ and}$$

$$\sum_{i=1}^{n} \theta(b_N;b_{-i})=c \text{ for all } b \text{ such that } b_N\geq c. \tag{6}$$

In view of the monotonicity properties (3), (4), and (5) for γ, we get the following properties of θ:

$$\theta \text{ is increasing in } \sigma \text{ if } b_{N/1}>0, \tag{7}$$

$$\theta(v;b_{-1})>\theta(v;b_2+\lambda,b_3,..,b_n)>\theta(v;b_{-1})-\lambda \text{ for all } \lambda>0 \text{ and all } v>0, \text{ and} \tag{8}$$

$$\theta(v+\lambda;b_2+\lambda,b_3,..,b_n)<\theta(v;b_{-1}) \text{ if } b_{N/1}<v. \tag{9}$$

We are ready to define the mechanism.

<u>THE MECHANISM</u>

<u>Stage 1</u>: Agents simultaneously submit $v_i\geq 0$

If all $v_i\leq c$ then do not build. Otherwise let i^* be one of the agents with the highest bid. Denote $v_{i^*}=v$ and go to stage 2.

Stage 2, <u>given</u> i^* <u>and</u> v

Agents simultaneously submit $\beta_i\geq 0$.

If $\beta_N > v$ the project is built. Agent i, $i \neq i^*$, pays $\theta(v,\beta_{-i})$ and agent i^* pays the balance (namely $c - \sum_{i \neq i^*} \theta(v,\beta_{-i})$). Note that agent i actually receives money if $v < \beta_{N/i}$.

If $\beta_N < v$ the project is not built. Agent i, $i \neq i^*$ receives

$t_i = v - \beta_{N/i} - \theta(\sigma,\beta_{-i})$ from agent i^* (so agent i^* pays t_{N/i^*}).

If $\beta_N = v$, agent i^* chooses either one of the above two outcomes.

THEOREM

Suppose the cost-sharing rule γ satisfies properties (3) and (4). Then for every profile $(b_1,..,b_n)$ in R^n_+ consider the game induced by the above mechanism at every Nash equilibrium in undominated strategies[1]*, the correct decision is taken (the project is built if $b_N > c$ and not built if $b_N < c$) and the cost-sharing rule γ is implemented (nobody pays anything if the project is not built; agent i pays $\gamma(b_i;b_{-i})$ if it is built).*

Moreover, if at least two agents derive positive benefits from the project ($b_i > 0$), then in equilibrium the highest first stage bid is equal the joint surplus, and the second stage bids reveal the agents' true benefits.

Finally every agent can guarantee a non-negative net utility by bidding zero in the first stage and reporting truthfully in the second stage.

We remark that to sustain the equilibrium it is required that each agent knows his or her own benefit, and that at least two agents who derive positive benefit, and that at least two agents who derive positive benefits know the total valuation.

[1]The mechanism can be altered so that the theorem is true for subgame perfect equilibrium. The change would be to have agents announce β_i one at a time in stage 2, with i^* moving first.

4) PROOF OF THE THEOREM

a) We analyze Stage 2 first.

Say that agent i* "won" stage 1 and consider the mechanism from the point of view of a different agent $i \neq i^*$.

Set $\alpha = v - \beta_{N/i}$.

If $\beta_i > \alpha$, then agent i's utility is $b_i - \theta(v, \beta_{-i})$.

If $\beta_i < \alpha$, then agent i's utility is $\alpha - \theta(v, \beta_{-i})$.

If $\beta_i = \alpha$, then agent i's utility is one of these two.

Thus the truthful report $\beta_i = b_i$ is a dominating strategy (and the unique undominated strategy).

Next, consider agent i*. Given that the other agents report truthfully, agent i*'s best reply β_{i^*} is as follows:

If $b_{N/i^*} \leq v$, then send $\beta_{i^*} = v - b_{N/i^*}$ and build if $b_N \geq c$, or don't build if $b_n < c$, and

If $b_{N/i^*} > v$, then send $\beta_{i^*} = 0$ and build. (10)

Notice that agent i* needs only to know the joint benefit (not each and every benefit of the other agents) to compute the best reply. We prove (10) by checking agent i*'s payoff. We distinguish two cases.

Case 1: $b_{N/i^*} \leq v$.

Set $a = v - b_{N/i^*}$

If $\beta_i^* > a$ then agent i* gets: $b_{i^*} - c + \sum_{j \neq i^*} \theta(v; \beta_{i^*}, b_{-i^*}, j)$. (11)

If $\beta_{i^*} < a$, then agent i* gets: $(n-1)(\beta_{i^*} - v) + (n-2) b_{N/i^*} + \sum_{j \neq i} \theta(v; \beta_{i^*}, b_{-i^*, j})$.

If $\beta_{i^*} = a$, then agent i* gets the best of the two above numbers.

In view of (8) (applied to the variable β_{i^*}) the best strategy for agent i* is to set $\beta_{i^*} = a$. Notice that property (6) implies that:

$$\sum_{j \neq i} \theta(v; v - b_{N/i^*}, b_{-i^*, j}) = c - \theta(v; b_{-i^*}).$$

Thus agent i* decides whether to build or not by comparing the payoffs:

If the project is built: $b_{i*}-\theta(v;b_{-i*})$, and

If the project is not built: $c-b_{N/i*}-\theta(v,b_{-i*})$.

Clearly agent i* will build if $b_N>c$ and will not build if $b_N<c$, and so the efficient decision will be implemented.

Case 2: $b_{N/i*}>v$

As a<0, we have $\beta_{i*}>a$ no matter what, so agent i*'s utility is (2) which decreases in β_{i*}. The best reply is $\beta_{i*}=0$ and to build. Agent i*s final payoff is then

$$b_i*-c+\sum_{j\neq i*}\theta(v;\tilde{b}_{-j}),\text{ where }\tilde{b}_j=b_j\text{ and }\tilde{b}_{i*}=0.$$

b) We analyze Stage 1

To analyze the Nash equilibrium bids in stage 1 we take care first of the easy case in which $b_N\leq c$.. Since every agent can guarantee a no loss and the joint utility is non-positive (there is no surplus from building the project) the game is inessential (see Moulin [1986] chapter 1) and its unique Nash equilibrium outcome is zero utility for all.

Now we assume, until the end of this proof, that $b_N>c$. Let us first take care of the case in which all b_i are zero except for b_1; and $b_1>c$. Then in any equilibrium, agent 1 "wins" stage 1 with a bid $v,b_1\geq v>c$ and in the second stage the project is built and agent 1 pays its full cost, as the core bounds command.

From now on we assume that at least two agents have a positive valuation for the project (and know the total valuation.) We know from the analysis of stage 2 that in any equilibrium the winning bid v is greater than c and the good will be produced. Note that the outcome where all bids are less than or equal to c in stage 1 cannot result from an undominated Nash equilibrium because any agent would gain by bidding $v_i>c$ given the strategies which must be played in any undominated Nash equilibrium in stage 2.

We now prove the following claim by contradiction.

Claim a: There is no undominated Nash equilibrium for which the winning bid v is such that $v<b_N$.

Suppose there is one, and call agent 1 the winner of stage 1. We can pick another agent, say agent 2, such that $b_2>0$. Agent 2's final utility at the resulting equilibrium in stage 2 is:

$$b_2-\theta(v;\beta_1^*,b_{-1,2}) \text{ where } \beta_1^*=(v-b_{N/1})^+. \tag{12}$$

Case i: $v\geq b_{N/1}$.

By sending the bid $v'=b_N$, agent 2 wins and sends $\beta_2^*=b_2$ in stage 2 and obtains finally utility:

$$b_2-\theta(v',b_{-2}). \tag{13}$$

To see that (13)>(12), we check that:

$$\theta(b_N;b_1,b_3..) > \theta(v;\beta_1^*,b_3..).$$

Set $\delta=b_N-v$ so that $b_1=\beta_1^*+\delta$, and the above inequality reads:

$$\theta(v+\delta,\beta_1^*+\delta,..) < \theta(v;\beta_1^*,...),$$

which follows from (9) and $b_2>0$(since $b_1+b_3+..+b_N<(b_N-c)+c$).

Case ii: $v<b_{N/1}$.

By sending the bid $v'=b_{N/1}$, agent 1 still wins and still sends the bid $\beta_1=0$ in stage 2 to get

$$b_1-\theta(v';b_2,b_3,...).$$

By contrast, with the bid v and $\beta_1=0$ in stage 2 he gets

$$b_1-c+\sum_{i\neq 1}\theta(v,\bar{b}_{-i}) \text{ where } \bar{b}=0,\bar{b}_i=b_i \;\; i\neq 1.$$

Check that: $\theta(v';\bar{b}_{-1})+\sum_{i\neq 1}\theta(v;\bar{b}_{-i})<c$.

This follows from $v'=\bar{b}_N$, formula (6), property (7), and $v<v'$.

The next claim is also proven by contradiction.

Claim b: There is no undominated Nash equilibrium for which the winning bid is v, $v>b_N$.

<u>Proof</u>: Suppose that agent 1 wins stage 1 with $v>b_N$ and assume that $b_{N/1}>0$. His utility will be

$$b_1-\theta(v;b_{-1}).$$

If he is sole winner of stage 1 he would like to lower σ a bit because θ increases in v ((7)). So somebody else (agent 2) must have bid v as well. Suppose that agent 1 lets agent 2 win stage 1. He gets:

$$b_1-\theta(v;\beta_2,b_3..),\text{ where } \beta_2=v-b_{N/2}.$$

This is better, since θ decreases in b_2 ((8)) and $\beta_2>b_2$.
The case $b_{N/1}=0$ is an exception: here $\theta(v,0)=c$ so any message $b_1\geq v_1>c$ yields an equilibrium (as discussed previously). But they all have the right payoff. Conclusion: All equilibria have $v=b_N$. Hence in stage 2, <u>everybody</u> reports truthfully and agent i's final utility is

$$b_i-\theta(v;b_{-i})=b_i-\gamma(b_i;b_{-i}),\text{ as required.}$$

Finally, to complete the proof of the theorem, we need to show that the (myopic) strategy $v_i=0$ in stage 1 and $\beta_i=b_i$ in stage 2 <u>guarantees</u> a non-negative utility to agent i, no matter what other players do.
This is easily checked. For instance, if the project is built, agent i's final utility is $b_i-\theta(v,\beta_{-i})$ (14)
If $v\leq\beta_{N/i}$ the "cost share" θ is negative so that agent i's final utility is positive. If $v\geq\beta_{N/i}$ then the cost share θ is bounded above (in view of the core bounds):

$$\theta(v,\beta_{-i})=\gamma(v-\beta_{N/i};\beta_{-i})\leq v-\beta_{N/i}$$

thus the utility level (14) is worth $b_i+\beta_{N/i}-v$ which must be non-negative when the project is built.

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