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TRADING GAMES WITH ASYMMETRIC INFORMATION

by

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Abstract

Two approaches to trading with asymmetric information are compared. The first one, which can be referred to as *institutional*, consists of delegating to a planner the organization of the transactions. The solutions are associated with (interim) individually rational, incentive compatible trading rules. The other approach is appropriate in a *decentralized* context. A game of contracts is modeled and solved by the communication equilibrium solution concept. Solutions in this sense are shown to correspond to *strongly* incentive compatible trading rules, which are in particular ex post individually rational. Further elements of comparison are provided.

Key words: communication equilibrium, incentive compatibility, individual rationality, trading rule.

1. Introduction

A traditional approach to trading with asymmetric information consists of delegating to a planner the organization of the transactions. The buyers and the sellers are asked to report their information to the planner who decides on the prices and the successful traders. In this approach, which will be referred to as institutional, the solutions are associated with strategies of the planner (called trading rules) which are incentive compatible and (interim) individually rational. Solving the problem in this way amounts to conceiving it as a Bayesian collective choice (see Myerson (1985)), in which the individual decision power is thus limited. A typical trading rule is the double auction--in particular, the first price and the second price auction when there is only one seller (see Myerson (1981), Wilson (1985)). But there are many other examples. If the informational context is not elementary (e.g., the traders are not symmetric), the optimal trading rule may not be a standard auction (see, again, Myerson (1981), Wilson (1985)).

The present paper is motivated by the following question: Is it always meaningful to look for an optimal mechanism in such a large set as all (interim) individually rational incentive compatible trading rules? Suppose no planner is available to organize the trade; is there any hope that the traders reach the previous solutions?

We start by formalizing a scenario of decentralized trading: the agents first exchange information, but they do not commit themselves at that time; then, they conclude (binding) contracts. A caricature of the last stage takes the form of a game, meaningless without pre-play communication. This leads us to use the "communication equilibrium" as a solution concept (see Forges (1986)). Here, trading is thus solved by means of a Bayesian game with communication (see Myerson (1985)).

The set of all communication equilibrium payoffs of the trading game is the largest conceivable one in a decentralized approach (we show in the sequel that under reasonable assumptions, this set is not "too large," because all solutions belonging to it may result from plain conversation between the traders: this follows from Forges (1990)). However the set of decentralized solutions may be strictly included in the set of institutional solutions, and this even in simple frameworks such as the independent private values. Without imposing any restriction on the information structure, we show that the decentralized solutions correspond to strongly incentive compatible trading rules, which involve ex post participation constraints from the traders. These rules are in particular ex post individually rational. The main message of this paper is that, in looking for an optimal trading rule, one should concentrate on the latter class because less commitment is then required from the traders. The corresponding solutions may thus emerge in less restrictive environments, e.g., decentralized ones.

Unlike most studies of trading with asymmetric information, this one will not be concerned by the design of an optimal procedure. Usually, after having described a set of "acceptable" rules (e.g., incentive compatible, individually rational), one is interested in finding the "best one," according to some criterion. Most often, this is achieved by maximizing an appropriate linear function over the set of acceptable rules. Here, we focus on the first part of that program, which consists of characterizing the feasible set. We argue that the latter should be restricted to strongly incentive compatible solutions.

Obviously, it may happen that the optimal (interim) individually rational incentive compatible trading rule is in fact strongly incentive compatible (we shall see that the optimal auction in the regular case, in Myerson (1981), enjoys such a property). But this phenomenon is far from being general (just relax the independence of the types of the buyers in the latter study).

Our goal here is also methodological. We want to establish a precise connection between communication equilibria and incentive compatible mechanisms in a context in which the latter are usually used. The communication equilibria make sense once an appropriate trading game is designed. The solutions obtained in this way are equivalent to the ones associated with a given class of incentive compatible trading rules, which reflects some decision power of the players.

The basic trading situation (described formally in Section 2) can be illustrated by the real estate market in a college town. All houses are essentially similar; each seller owns one house; each buyer wants to acquire one. Most sales take place around the same time (during the summer); buyers and sellers may identify each other because they all advertise in the university newspaper. They all have private information, which involves subjective and objective aspects. Subjective aspects are related with individual preferences (i.e., reservation price, possibilities of refusing or delaying the sale or the purchase). Objective aspects concern the evolution of the market which matters for delay of the sale as well as for resale. (This is analogous to "preference" and "quality" uncertainty; see Myerson (1981).) In such a context, informal discussions between buyers and sellers seem more plausible than a well-organized auction.

Section 3 is devoted to the institutional approach. We begin with the double auction and continue with incentive compatible trading rules. In Section 4, we introduce the game of contracts whose communication equilibria are interpreted as decentralized solutions. Their characterization in terms of trading rules is stated in Section 4.3.

Section 5 consists mainly of examples. Most of them are relative to the private values case. We first show that under this assumption, the double auctions are strongly incentive compatible. A similar result was already available for one seller and one buyer (Matthews and Postlewaite (1989)). In this case, all solutions associated with strongly incentive compatible trading rules can also be obtained in a double auction preceded by communication. We provide an example with more traders where this inclusion does not hold. Another example illustrates that double auctions may not be strongly incentive compatible if the informational structure is not the private values. Finally, we come back to a private values context, with one seller and two buyers; there, we exhibit an incentive compatible trading rule which is ex post individually rational but not strongly incentive compatible.

2. Model

We begin with the description of a basic situation G_0. This will be enriched into different games, according to the scenario of trading.

The agents consist of n sellers ($j = 1,\ldots,n$), each with a similar object for sale, and m buyers ($i = 1,\ldots,m$), each interested in acquiring such an object. The private information of seller j (resp. buyer i) is represented by a random variable ν_j (resp. μ_i) only observed by himself. To keep a simple framework, we assume that all these random variables take their values in a finite set L and that the traders share a common prior probability p on L^{m+n}. But the information of different agents need not be independent. The _value_ of the object to seller j (resp. buyer i) if he knew the full state of information $(\mu_1,\ldots,\mu_m,\nu_1,\ldots,\nu_n)$ would be $v_j(\mu_1,\ldots,\mu_m,\nu_1,\ldots,\nu_n)$ (resp. $u_i(\mu_1,\ldots,\mu_m,\nu_1,\ldots,\nu_m)$). The value of the object to a trader given his information can thus be evaluated as a conditional expectation. We shall adopt the short notations:

$$\mu = (\mu_1,\ldots,\mu_m)$$

$$\nu = (\nu_1,\ldots,\nu_n)$$

$$\lambda = (\mu,\nu)$$

$$\lambda_{-i} = (\mu_1,\ldots,\mu_{i-1},\mu_{i+1},\ldots,\mu_m,\nu)$$

and write $u_i(\lambda)$, $u_i(\mu,\nu)$, $u_i(\mu_i,\lambda_{-i})$, etc. All mappings u_i, v_j ($i = 1,\ldots,m$; $j = 1,\ldots,n$), defined on L^{m+n}, take finitely many (integer, say) values. The set of _prices_, Π, is the set of all integers between the minimum and the maximum value:

$$\underline{\pi} = \min\{u_i,\ i = 1,\ldots,m;\ v_j,\ j = 1,\ldots,n\}$$

$$\bar{\pi} = \max\{u_i,\ i = 1,\ldots,m;\ v_j,\ j = 1,\ldots,n\}$$

$$\Pi = \{\underline{\pi},\underline{\pi} + 1,\ldots,\bar{\pi} - 1,\bar{\pi}\}.$$

In other words, values (and thus prices) are expressed as integer amounts of money.

Observe that the value of the object to a trader may depend on information obtained by another. We do not make specific assumptions on the nature of the uncertainty faced by the agents. As pointed out in the Introduction, this may not only concern the preferences of the others but also the quality of the object. An extreme case, known as <u>private values</u> is the pure preference uncertainty:

$$u_i(\mu,\nu) = \mu_i$$
$$v_j(\mu,\nu) = \nu_j$$

Each trader knows his own reservation price and has only beliefs on the ones of the other agents. We shall particularize our approach in this case (in Section 5). The common value case is another extreme; then, the value of the object is the same for all traders but they only get to know an estimator of it (represented by the μ_i's and λ_j's). Many intermediate cases are conceivable (see, e.g., Milgrom and Weber (1982), Myerson (1981), and recall the real estate example of the Introduction).

The <u>basic situation</u> G_0 consists of m buyers with private information μ_i and valuation function u_i ($i = 1,\ldots m$), n sellers with private information ν_j and valuation function v_j ($j = 1,\ldots,n$), the prior probability distribution p and the set of prices Π. From this, various games will now be constructed; according to their rules, different sets of actions will be available to the players and different payoff functions will be defined.

3. <u>The Institutional Approach</u>

In this section, G_0 will be completed by an <u>institution</u> indicating precisely how trade can take place. In this approach, the exchanges are controlled by a planner. It is convenient to see a trading rule as a strategy of the planner. The latter is thus conceived as an extraneous player, indifferent between the possible outcomes of the trading rule and thus, reliable. We start with a simple, well-known trading rule (analyzed in detail in Wilson (1985)).

3.1 Double Auctions

From G_0, we define the game Γ_δ of δ-double auction (the parameter δ appears in the payoff function). The first stage is virtual and may be thought of as happening at time $-\infty$.

Stage 1: Buyers and sellers get their private information.

Stage 2: Each buyer i transmits a bid σ_i to the planner ($i = 1,...m$) and similarly, each seller j transmits an offer τ_j to the planner. The players may be thought of as moving simultaneously.

Stage 3: Given $\sigma = (\sigma_1,....\sigma_m)$ and $\tau = (\tau_1,....\tau_m)$, the planner determines a single price $\pi_\delta(\sigma,\tau)$, winning buyers (who get an object at that price) and winning sellers (who sell an object at that price).

The δ-double auction is a precise strategy of the planner for evaluating $\pi_\delta(\sigma,\tau)$ and the winning traders. Let $\sigma_{(i)}$ denote the bids in the decreasing order and set $\sigma_{(0)} = +\infty$, $\sigma_{(i)} = -\infty$ for $i > m$:

$$+\infty \geq \sigma_{(1)} \geq \dots \geq \sigma_{(k)} \geq \sigma_{(k+1)} \geq \dots \geq -\infty.$$

Proceed similarly with the offers, in the increasing order:

$$-\infty \le \tau_{(1)} \le \dots \le \tau_{(k)} \le \tau_{(k+1)} \le \dots \le +\infty.$$

Let then

$$k = \max\{\ell: \sigma_{(\ell)} \ge \tau_{(\ell)}\}$$

and, provided that $k \neq 0$,

$$\pi^1(\sigma,\tau) = \min\{\sigma_{(k)},\tau_{(k+1)}\}$$
$$\pi^2(\sigma,\tau) = \max\{\tau_{(k)},\sigma_{(k+1)}\}$$
$$\pi_\delta(\sigma,\tau) = \delta\pi^1(\sigma,\tau) + (1 - \delta)\pi^2(\sigma,\tau)$$

where δ is arbitrary in $[0,1]$. $\pi^1(\sigma,\tau)$ corresponds to the first highest price in the case of simple auctions (with only one seller) and $\pi^2(\sigma,\tau)$, to the second highest price.

If $k \neq 0$, the <u>successful traders</u> are selected among the buyers i with $\sigma_i \ge \pi_\delta(\sigma,\tau)$ and the sellers j with $\tau_j \le \pi_\delta(\sigma,\tau)$. In case of ties around (k), one proceeds to a random, uniform choice. Otherwise, pairs of winning traders are formed in the obvious way. Suppose, for instance, that $\sigma_{(k)} = \sigma_{(k+1)} = \dots = \sigma_{(k+\ell)}$ for some $2 \le \ell \le m - 1$. Then, $\pi_\delta(\sigma,\tau) = \sigma_{(k)}$, for all δ's. The buyer who will trade with the seller offering $\tau_{(k)}$ will be chosen with probability $1/(\ell + 1)$ among the $(\ell + 1)$ buyers bidding $\pi_\delta(\sigma,\tau)$. The same can be done for the sellers (if $k = 0$, there is no successful trader).

Observe that $\pi_\delta(\sigma,\tau)$ is chosen so as to clear the market, <u>with respect</u>

to the bids and offers: no unsuccessful trader could benefit from trading at that price. Observe, also, that the only action of each player in Γ_δ consists of submitting a bid or an offer. The exchange is otherwise organized by the planner. In particular, agents do not learn anything through the bids and offers of the others or the price, which is known after the sales take place.

To complete the description of Γ_δ, there remains to make the payoffs precise. Buyer i gets

$$u_i(\lambda) - \pi_\delta(\sigma,\tau) \text{ if he is successful; 0 otherwise;}$$

while seller j gets

$$\pi_\delta(\sigma,\tau) - v_j(\lambda) \text{ if he is successful; 0 otherwise.}$$

Let $\Phi_i(\sigma,\tau)$ (resp. $\Psi_j(\sigma,\tau)$) be the probability that buyer i (resp. seller j) be successful, given the bids σ and offers τ. These are 0 or 1 unless there are ties. Since the price $\pi_\delta(\sigma,\tau)$ is a deterministic function of (σ,τ), we can express the expected payoffs as follows:

$$\bar{x}_i(\lambda;\sigma,\tau) = (u_i(\lambda) - \pi_\rho(\sigma,\tau))\Phi_i(\sigma,\tau)$$

for buyer i ($i = 1,\ldots,n$) and

$$\bar{y}_j(\lambda,\sigma,\tau) = (\pi_\delta(\sigma,\tau) - v_j(\lambda))\Psi_j(\sigma,\tau)$$

for seller j ($j = 1,\ldots,m$).

Let $N(\Gamma_\delta)$ be the set of all (mixed) Nash equilibrium payoffs of Γ_δ. To be precise, we consider the vector payoffs of each player, constituted by his expected payoffs given each possible type (μ_i or ν_j); then we form the $(m + n)$-tuple made of the vector payoffs of the different players. $N(\Gamma_\delta)$ is thus a subset of $(\mathbb{R}^L)^{m+n}$. The set of solutions of the next sections will be defined in the same way.

The advantage of the procedure above is that it is simple and universal: it can be applied independently of the number of traders, their beliefs, and so on. To test the efficiency of the δ-double auction (or in the case of one seller, standard auction mechanisms, like the first-price or the second-price sealed-bid auction), one usually compares it to more sophisticated trading rules (see Wilson (1985)). We investigate such rules in the next section.

The main drawback of double auctions is that the equilibrium strategies of the players become complicated as soon as assumptions of symmetry and independence (of the information of the players) are relaxed.

3.2 Incentive Compatible, Interim Individually Rational Trading Rules

Unlike in the previous section, we shall not focus on a precise strategy of the planner but consider a family of solutions; each of them will be associated with a different trading rule. We consider thus a collection of games built on G_0, corresponding to different institutions for trading. In each game:

Stage 1: Exactly as in Section 3.1.

Stage 2: Each trader decides to participate in the trade or not and, if so, sends a message in L to the planner. The players may be thought of as moving simultaneously.

Stage 3: The planner selects successful buyers and sellers, associates them by pairs and chooses a price for each transaction.

To make the actions available to the planner more precise, let

$$M = \{0,1,\ldots,m\},\ N = \{0,1,\ldots,n\}.$$

The planner must decide on $t = (t_b, t_s)$ in an appropriate subset of $(N \times \Pi)^m \times (M \times \Pi)^n$, where Π is the set of prices introduced in Section 2. The component $t_b = (t_b(i))_{i \le i \le m} \in (N \times \Pi)^m$ concerns the buyers; $t_b(i) = (s_b(i), \pi_b(i))$ consists of a seller and a price for buyer i; $s_b(i) = 0$ means that buyer i is not successful; in that case, the price $\pi_b(i)$ is irrelevant; we can set $\pi_b(i) = \underline{\pi}$ for the sake of completeness. Otherwise, buyer i gets an object from seller $s_b(i)$ at the price $\pi_b(i)$. $t_s = (t_s(j))_{1 \le j \le n} \in (M \times \Pi)^n$ is defined similarly for the sellers with $t_s(j) = (b_s(j), \pi_s(j))$: $b_s(j)$ is the buyer to whom seller j sells the object at price $\pi_s(j)$, unless $b_s(j) = 0$, in which case seller j is not successful and $\pi_s(j) = \bar{\pi}$. Obviously, some feasibility constraints must be imposed on the actions of the planner, expressing that if buyer i buys from seller j then seller j sells to buyer i, at the same price. We define thus T as the set of all elements of $(N \times \Pi)^m \times (M \times \Pi)^n$ which satisfy

$$b_s(s_b(i)) = i \text{ if } s_b(i) \neq 0$$

$$\pi_b(i) = \pi_s(s_b(i))$$

$$s_b(b_s(j)) = j \text{ if } b_s(j) \neq 0$$

$\pi_s(j) = \pi_b(b_s(j))$.

Given the information λ and the action t of the planner, the payoff of buyer i can be evaluated as follows:

$$x_i(\lambda;t) = x_i(\lambda;t_b(i)) = [u_i(\lambda) - \pi_b(i)]I(s_b(i) \neq 0)$$

where I is the indicator function. Similarly, for buyer i,

$$y_j(\lambda;t) = y_j(\lambda;t_s(j)) = [\pi_s(j) - v_j(\lambda)]\ I(b_s(j) \neq 0)$$

In these expressions of the payoffs, λ denotes the true, full information, which might not be accessible to any trader. The relevant payoff to a trader is the conditional expected payoff, given his information (see below).

The scenario above indicates that the planner chooses his action t after having received a message in L from every trader. The interpretation of stage 2 is that each trader is invited to reveal his information to the planner. A strategy of the latter chooses thus t in T as a function of the report λ' (possibly different from the true information λ). A <u>trading rule</u> is a (possibly mixed) strategy q for the planner (q is a system of conditional probabilities $q(\bullet|\lambda')$ on T, for every λ' in L). A trading rule is said to be <u>incentive compatible</u> and (interim) <u>individually rational</u> if the three-stage game above, with the payoffs evaluated by means of the trading rule, admits the following Nash equilibrium: at stage 2, each trader decides to participate and reveals truthfully his information to the planner.

Let us write the incentive compatibility condition and the (interim) individual rationality condition explicitly. For this, let $p \times q$ be the following probability distribution on $L^{m+n-1} \times T$, induced by the prior p on L and the trading rule q.

$$(3.1) \qquad (p \times q)(\lambda_{-i}, t | \mu_i, \mu_i') = p(\lambda_{-i} | \mu_i) q(t | \mu_i', \lambda_{-i})$$

This is the probability that the opponents of buyer i hold information λ_{-i} and that the planner decides on t, if buyer i has information μ_i and reports μ_i' (possibly different from μ_i) to the planner.

If he participates and tells the truth, buyer i of type μ_i gets the expected payoff

$$(3.2) \qquad \alpha_i(\mu_i) = \Sigma_{\lambda_{-i}, t_b(i)} (p \times q)(\lambda_{-i}, t_b(i) | \mu_i, \mu_i) x_i(\mu_i, \lambda_{-i}; t_b(i))$$

Not participating would give him a payoff of zero, so that the <u>(interim) individual rationality condition</u> is

$$(3.3) \qquad \alpha_i(\mu_i) \geq 0, \ \forall \ \mu_i \in L$$

The <u>incentive compatibility condition</u> expresses that buyer i cannot gain in lying about his type:

$$(3.4) \qquad \alpha_i(\mu_i) \geq \Sigma_{\lambda_{-i}, t_b(i)} (p \times q)(\lambda_{-i}, t_b(i) | \mu_i, \mu_i') \times x_i(\mu_i, \lambda_{-i}; t_b(i)), \ \forall \ \mu_i, \mu_i' \in L.$$

The non-deviation for the sellers can be derived exactly in the same way.

We can now define $T(G_0)$ as the set of all solutions (understand $(m + n)$-tuples of equilibrium vector payoffs) which can be achieved by means of an incentive compatible, (interim) individually rational trading rule.

The approach followed in that subsection has become quite standard (see Myerson (1981), Wilson (1985)). It amounts to dealing with the trading problem G_0 as with a Bayesian collective choice problem (see Myerson (1985)). As usual, the focus on apparently specific, restrictive trading rules is justified by the revelation principle (see again Myerson (1985)).

Using the same arguments as for establishing this principle, one shows that $N(\Gamma_\delta)$ is included in $T(G_0)$. The inclusion may be strict, as we shall illustrate in Section 5. Observe that $N(\Gamma_\delta)$ is the set of solutions of one well-defined game Γ_δ while $T(G_0)$ is a collection of (canonical[1]) solutions, all associated with different games (namely, different trading rules).

The advantage of $T(G_0)$ is that it provides a tractable characterization of a large set of solutions, all attained with extremely simple strategies (the complexity of the trading mechanism is borne by the planner, not the agents). $T(G_0)$ is especially appropriate to isolate an optimal trading rule, maximizing a suitable objective function. This is usually linear in the players' payoffs, while $T(G_0)$ is a convex polyhedron. Classical linear programming can thus be used to find an optimal solution in $T(G_0)$. Obviously, the optimal trading rule for a given trading problem G_0 may depend on the parameters of G_0 (for instance, on the prior probability p). In general, unlike the double auction, it will not be universal. Such observations have led to the concept of uniform efficiency (see Wilson (1985)). As explained in the introduction, our study is not directly concerned with questions of efficiency. We will nevertheless return to this in Section 5.

3.3 Strongly Incentive Compatible Trading Rules

In the previous section, the agents delegate their trading power to the planner. Once he has accepted the trading rule (at stage 2), the trader has no way to escape. Buyers have signed blank checks to be filled out by the planner; sellers have given their object to the planner, and so on. An incentive compatible, interim individually rational trading rule may entail losses for some players in some circumstances, although interim individual rationality guarantees a positive expected payoff (see Section 5 for examples).

In this section we shall consider more restrictive scenarios of institutionalized trading, where the traders have some ex post control of the decision of the planner. Instead of deciding on participation at stage 2, the traders will now make that decision after stage 3,after having received information on the transaction that the planner has prepared for them. The motivation for limiting the freedom of the planner will be clear in Section 4.

Let us modify slightly the scenario of the previous section.[2] Stages 1 and 3 do not change. Stage 2 becomes:

> The players send, simultaneously, a message in L to the planner.

A Stage 4 is added:

> The planner transmits (secretly) $t_b(i)$ to each buyer i and $t_s(j)$ to each seller j. Each trader may then accept or refuse the transaction.

Let us fix a trading rule, i.e., a strategy of the planner, and let us investigate the conditions for truthful revelation and participation to be an equilibrium. The relevant probability distribution for buyer i of type μ_i is still $p \times q$ (see (3.1)) and his expected payoff if he follows the procedure is $\alpha_i(\mu_i)$ (see (3.2)). Suppose he sends μ_i' to the planner. At stage 4, he has two options: either accept the transaction $t_b(i)$ (selected on the basis of μ_i') or refuse it (and get 0). Conditions (3.4) are thus transformed into:

$$\begin{aligned}(3.5)\qquad \alpha_i(\mu_i) \geq \Sigma_{t_b(i)}\ & (p \times q)(t_b(i)|\mu_i, \mu_i') \\ & \times \max\{\Sigma_{\lambda_{-i}}\ (p \times q)(\lambda_{-i}|\mu_i, \mu_i', t_b(i))x_i(\mu_i, \lambda_{-i}; t_b(i)), 0\}, \\ & \forall\ \mu_i, \mu_i' \in L\end{aligned}$$

We shall refer to it as <u>strong incentive compatibility</u>. First, let us observe that it implies <u>ex post individual rationality</u>.

The right side of (3.5) is obviously greater than

$$(3.6)\qquad \Sigma_{\lambda_{-i}, t_b(i)}\ (p \times q)(\lambda_{-i}, t_b(i)|\mu_i, \mu_i')x_i(\mu_i, \lambda_{-i}; t_b(i))$$

which is exactly $\alpha_i(\mu_i)$ when $\mu_i' = \mu_i$. Hence,

$$\alpha_i(\mu_i) = \Sigma_{t_b(i)} (p \times q)(t_b(i)|\mu_i,\mu_i)$$
$$\times \max\{\Sigma_{\lambda_{-i}} (p \times q)(\lambda_{-i}|\mu_i,\mu_i,t_b(i))x_i(\mu_i,\lambda_{-i};t_b(i)),0\}$$

Since for any random variable z, $Ez = E \max\{z,0\}$ implies $z \geq 0$ a.s.,

$$(3.7) \qquad \Sigma_{\lambda_{-i}} (p \times q)(\lambda_{-i}|\mu_i,\mu_i,t_b(i))x_i(\mu_i,\lambda_{-i};t_b(i)) \geq 0,$$
$$\forall \mu_i, \forall t_b(i): (p \times q)(t_b(i)|\mu_i,\mu_i) > 0.$$

This last condition expresses <u>ex post individual rationality</u> for buyer i, namely, that after having reported his information correctly, buyer i has no chance to make a loss. This is obviously not guaranteed if buyer i lies.

It appears thus that the veto power introduced at stage 4 not only strengthens the individual rationality condition but also the incentive compatibility condition, which cannot be expressed without taking account of a possible ultimate refusal.

The difference between the two classes of incentive compatibility conditions ((3.3), (3.4) and (3.5)) is exactly the difference between the maximum of the expectations and the expectation of the maximum; the relevant order depends, obviously, on the moment at which the players are asked to agree on the procedure.

We have shown in passing (see (3.6)) that strong incentive compatibility implies incentive compatibility (3.4)). It is clear that it also implies interim individual rationality. Hence, if we define $T^*(G_0)$ as <u>the set of all solutions which can be achieved by means of a strongly</u>

incentive compatible trading rule (in an analogous way as for $T(G_0)$), it is apparent that $T^*(G_0)$ is included in $T(G_0)$. We will show in Section 5 that this inclusion may be strict. The relationship between $T^*(G_0)$ and $N(\Gamma_\delta)$ will also be investigated there.

4. A Decentralized Approach

In the previous section, we generate a family of games helpful to solve the basic problem G_0. Here, to the contrary, we start by constructing a single, somehow conceptional, game G from G_0, which models the contracts that the traders may eventually conclude. This game G is only meaningful if it is preceded by communication between the players.

4.1 Game of Contracts

The players of the game G are the m buyers and the n sellers of G_0. There is no extraneous planner. The information structure of G is the same as in G_0. Here, the transactions depend directly on the actions of the players. Each seller j must fill a contract mentioning the name $b_s(j)$ of the buyer and the price $\pi_s(j)$. As before, we adopt the convention that $b_s(j) = 0$ if seller j does not want to trade; in this case, we set also $\pi_s(j) = \bar{\pi}$. The set of actions of seller j is thus $M \times \Pi$, with the same notation as in Section 3.2. Sale takes place if the contract is signed by the buyer. The latter must thus decide which transactions to accept. Since the contract will typically be concluded after communication between the traders, each buyer i can as well pick a seller $s_b(i)$ and a price $\pi_b(i)$, thus an action in $N \times \Pi$ (with $(0,\underline{\pi})$ corresponding to no trade). With this formulation, we can adopt the following payoff functions, which express that sale requires that the actions of the seller and the buyer match:

$$
\begin{aligned}
x_i'(\lambda;t) &= [u_i(\lambda) - \pi_b(i)]\ I[s_b(i) \neq 0] \\
&\quad \times I[b_s(s_b(i)) = i, \pi_b(i) = \pi_s(s_b(i))] \\
y_j'(\lambda;t) &= [\pi_s(j) = v_j(\lambda)]\ I[b_s(j) \neq 0] \\
&\quad \times [s_b(b_s(j)) = j,\ \pi_s(j) = \pi_b(b_s(j))]
\end{aligned}
$$

for $\lambda \in L$, $t \in (N \times \Pi)^m \times (M \times \Pi)^n$, $i = 1,\ldots,m$, $j = 1,\ldots,n$. Observe that here, each component of t corresponds to the individual decision of a player.

4.2 Solutions

We observed that meaningful solutions of G should result from communication between the players before they make their decisions. Communication before G can obviously take various forms, but we will see that many specifications lead to the same set of solutions.

In the context of a decentralized approach, it seems natural to forbid that the players be helped by informational intermediaries (mediators). Let thus S(G) be the set of all solutions that can be achieved in G preceded by plain conversation between the players. We do not impose any other restriction on communication: traders can meet secretly but also make public statements; they can exchange their messages simultaneously; they can talk as long as they want, and so on.

Let us imagine that at the end of the communication process, some players decide not to trade while the others, organized in pairs (of a seller and a buyer) achieve informal, nonbinding, agreements. Seller j comes up to a project of contract specifying a buyer $b_s(j)$ and a price $\pi_s(j)$; if this is effective ($b_s(j) \neq 0$), seller j expects that buyer $b_s(j)$ will choose him (j) as a co-trader and $\pi_s(j)$ as a price. Thus, the relevant information to seller j after communication is an intended action $(b_s(j), \pi_s(j))$, consistent with an intended action of the associated buyer $b_s(j)$ (if $\neq 0$). And similarly for each buyer.

The players reach these informal agreements as a result of the messages that they have exchanged, which in turn were chosen as a function of their initial information. The effect of communication can thus be summarized by the probability distributions $q(\bullet|\lambda)$ induced on T, for each vector of information λ. The reader familiar with the revelation principle will have recognized standard arguments to establish it. By pursuing the reasoning along these lines, one will show that S(G) <u>is included in</u> D(G), <u>the set of (canonical) communication equilibrium payoffs of</u> G (see Forges (1986)).

In order to define D(G) precisely, let us recall that a <u>canonical communication device</u> for G receives an input in L from every player of G and selects accordingly a vector of outputs in T ($t_b(i)$--resp. $t_s(j)$--is transmitted secretly to buyer i--resp. seller j). Formally, a canonical communication device consists thus of a trading rule.[3] However, it is used here to <u>recommend an action</u> to each player. A <u>canonical communication equilibrium</u> is a specific Nash equilibrium of G extended by a canonical communication device ("Bayesian game with communication," to use the terminology of Myerson (1985)), where each player tells the truth and acts as suggested by the device (the equilibrium conditions are stated formally in the proof of Proposition 4.1).

If the inclusion of S(G) into D(G) is not surprising, one would a priori suspect that D(G) is a much larger set. This intuition may result from underestimating the possible complexity of the players' behavior during a plain conversation. If $(m + n) \geq 4$, all solutions in D(G) can be achieved without the help of a mediator, by means of plain conversation only (see Barany (1987) and Forges (1990)).[4] These studies show also that without specific assumptions on the underlying game G, the result is no longer true when $m + n \leq 3$. However, in a standard context for bargaining, namely, one seller and one buyer with private values, the coincidence of the corresponding sets S(G) and D(G) can be deduced from Matthews and Postlewaite (1989). The argument extends to an arbitrary number of players.[5] In view of this result and of the constructions of Barany (1987) and Forges (1990), it seems reasonable to conjecture that in trading games (perhaps with valuation functions satisfying further requirements), the power of plain conversation holds without any restriction on the number of participants.[6]

Hence, even if the description of D(G) uses canonical communication devices (which can be viewed as mediators), one can also argue that D(G) is a tractable representation of all solutions achievable in a decentralized context. In the next section, we compare this approach with the institutional one.

4.3 Relationship with Trading Rules

The relationship between the latter approach and the one of Section 3 is made precise by the following:

Proposition 4.1: $D(G) = T^*(G_0)$.

Proof: We start by showing that $D(G) \subseteq T^*(G_0)$. We thus fix a canonical communication device q, i.e., a probability distribution $q(\bullet|\lambda)$ on T for every $\lambda \in L$. The canonical communication equilibrium condition of buyer i is

$$(4.1) \qquad \alpha_i'(\mu_i) \geq \Sigma_{t_b(i)} (p \times q)(t_b(i)|\mu_i,\mu_i') \times \max_{t_b'(i)} \Sigma_{\lambda_{-i},t_{-i}} (p \times q)(\lambda_{-i},t_{-i}|\mu_i,\mu_i',t_b(i)) \times x_i'(\mu_i,\lambda_{-i};t_b'(i),t_{-i})$$

for all $\mu_i,\mu_i' \in L$, where

$$(4.2) \qquad \alpha_i'(\mu_i) = \Sigma_{\lambda_{-i},t} (p \times q)(\lambda_{-i},t|\mu_i,\mu_i)x_i'(\mu_i,\lambda_{-i};t)$$

and $p \times q$ is defined as in (3.1) (see Forges (1990), (2.2) and (2.3)). These equations express that buyer i of type μ_i cannot gain in telling $\mu_i' \neq \mu_i$ to the communication device, nor in taking an action $t_b'(i)$ different from the recommendation $t_b(i)$ of the device. Such a feasible deviation is $t_b'(i) = (0,\underline{\pi})$, i.e., no trade, which yields

$$(4.3) \qquad \alpha_i'(\mu_i) \geq \Sigma_{t_b(i)} (p \times q)(t_b(i)|\mu_i,\mu_i') \times \max\{\Sigma_{\lambda_{-i},t_{-i}} (p \times q)(\lambda_{-i},t_{-i}|\mu_i,\mu_i',t_b(i)) \times x_i'(\mu_i,\lambda_{-i};t_b(i),t_{-i}),0\}$$

for all $\mu_i,\mu_i' \in L$. This shows that q, viewed as a trading rule, is strongly incentive compatible (we used that q selects only outcomes in T so that x_i'

coincides with x_i in (4.1), (4.2) and (4.3)).

To establish that $T^*(G_0) \subseteq D(G)$, let q be a strongly incentive compatible trading rule. It can be used as a canonical communication device, to recommend an action $t_b(i)$ (resp. $t_s(j)$) to every buyer i (resp. seller j) of G. A priori, the players have more freedom in the context of D(G) than in the one of $T^*(G_0)$, where they can just accept or refuse the transaction. However, in D(G), each player can be controlled by the other ones. Assume thus that all opponents of buyer i report truthfully their information and make the decision chosen for them according to q. Let μ_i be buyer i's true type, μ_i' (possibly different from μ_i) be his reported type, and $t_b(i) = (s_b(i), \pi_b(i))$ be the action suggested to him. It is easy to show that any deviation $t_b'(i)$ will result in a null profit, so that conditions 3.5 imply conditions (4.1). Indeed, if $s_b(i) = 0$, there is no seller j who wants to trade with i (i.e., with $b_s(j) = i$); if $s_b(i) \neq 0$, $s_b(i)$ is the only seller who is ready to sell his object to buyer i. Playing $j \neq s_b(i)$ yields zero to buyer i, whatever the price he proposes. Finally, there is no way to buy from $s_b(i)$ at a price different from $\pi_b(i)$ $(= \pi_s(s_b(i)))$.

In the next section we shall present an example where $T^*(G_0)$ is strictly included in $T(G_0)$. The interpretation of Proposition 4.1 is thus that all solutions associated with (interim) individually rational, incentive compatible rules cannot be achieved in a decentralized context: only the ones which are <u>strongly incentive compatible</u> can. The good news is that this necessary condition is also sufficient.

5. <u>Private Values</u>

In this section, we investigate the consequences of a usual assumption, stating that the private information of each trader consists of his reservation price. Buyer i's payoff, introduced in Section 3.2, is now

$$x_i(\mu_i; t_b(i)) = [\mu_i - \pi_b(i)]\ I(s_b(i) \neq 0)$$

so that strong incentive compatibility (recall (3.5)) takes the following simple form

$$\alpha_i(\mu_i) \geq \Sigma_{t_b(i)}\ (p \times q)(t_b(i)|\mu_i,\mu_i')\ \max\{x_i(\mu_i; t_b(i)),0\}.$$

Similar expressions can be derived for every seller j.

We start by showing that under private values, a trading rule induced by a <u>double auction</u> (Section 3.1) is strongly incentive compatible. A similar result has been obtained by Matthews and Postlewaite (1989) for trading games with one seller and one buyer.

<u>Proposition 5.1</u>: Under private values, $N(\Gamma_\delta) \subseteq T^*(G_0)$.

<u>Proof</u>: We shall establish a more general result, namely, that all <u>communication</u> equilibrium payoffs of Γ_δ belong to $T^*(G_0)$. Let us fix a canonical communication device q' for Γ_δ: this selects a bid, σ_i, for each buyer i and an offer, τ_j, for each seller j, as a function of the reservation prices reported by the players. Γ_δ associates a single price $\pi_\delta(\sigma,\tau)$ and successful traders with (σ,τ). The two procedures combine in a trading rule, which determines the price and the successful traders directly

from the reports of the players (the scenario of the double auction suggests how to organize the successful traders in pairs). We shall show that this trading rule is strongly incentive compatible.

The communication equilibrium condition for buyer i in Γ_δ is

$$(5.1) \qquad \bar{\alpha}_i(\mu_i) \geq \Sigma_{\sigma_i} \; (p \times q')(\sigma_i|\mu_i,\mu_i') \\ \times \max_{\sigma_i'} \Sigma_{\sigma_{-i},\tau} \; (p \times q')(\sigma_{-i},\tau|\mu_i,\mu_i',\sigma_i) \; \bar{x}_i(\mu_i;\sigma_i',\sigma_{-i},\tau)$$

for every $\mu_i,\mu_i' \in L$, with $\bar{x}_i$ defined as in Section 3.1.

$$\bar{x}_i(\mu_i;\sigma,\tau) = (\mu_i - \pi_\delta(\sigma,\tau))\phi_i(\sigma,\tau)$$

and

$$\bar{\alpha}_i(\mu_i) = \Sigma_{(\sigma,\tau)} \; (p \times q')(\sigma,\tau|\mu_i,\mu_i)\bar{x}_i(\mu_i;\sigma,\tau).$$

Let $\sigma_i' = \min(\sigma_i,\mu_i)$ and set $\sigma' = (\sigma_i',\sigma_{-i})$. It is easily checked that

$$\bar{x}_i(\mu_i;\sigma',\tau) \geq \max\{\bar{x}_i(\mu_i,\sigma,\tau),0\}$$

so that (5.1) yields

$$\bar{\alpha}_i(\mu_i) \geq \Sigma_{(\sigma,\tau)} \; (p \times q')(\sigma,\tau|\mu_i,\mu_i') \; \max\{\bar{x}_i(\mu_i;\sigma,\tau),0\}.$$

Thus, if the planner selects the price $\pi_\delta(\sigma,\tau)$ and the probability of winning $\phi_i(\sigma,\tau)$ according to the trading rule induced by q' and Γ_δ, and submits them to buyer i, the latter reveals his type and accepts the transaction giving him the expected payoff $(\mu_i - \pi_\delta(\sigma,\tau))\phi_i(\sigma,\tau)$ if his reservation price is μ_i. Recall that $\Phi_i(\sigma,\tau)$ is different from 0 or 1 only if there are ties. Suppose that, in this case, the planner performs the ultimate lottery. Suppose further that the planner tells the price $\pi_\delta(\sigma,\tau)$ to buyer i only if he has won the auction. It is clear that buyer i will still reveal the truth and accept the transaction in the last scenario, which shows that the combined trading rule is strongly incentive compatible.

When there is only one seller and one buyer, $D(\Gamma_\delta)$ coincides with $T^*(G_0)$, as shown by Matthews and Postlewaite (1989). The next example illustrates that, in general, $D(\Gamma_\delta)$ may be <u>strictly included</u> in $T^*(G_0)$.

<u>Example 5.1</u>

$m = n = 2$.

The traders share the same information:

$\mu_1 = 2$, $\mu_2 = 4$

$\nu_1 = 1$, $\nu_2 = 3$

The set of prices is:

$\Pi = \{1,2,3,4\}$.

Consider the following trading rule: if each agent reports truthfully his reservation price (which is verifiable by the planner here, since information is complete), buyer 1 is associated with seller 1, and buyer 2 with seller 2. The price for the first (resp. second) transaction is 1 with probability 1/2, 2 with the same probability (resp. 3 or 4, again each price with probability 1/2).

If one agent cheats, no trade occurs. This trading rule is strongly incentive compatible and gives each player an expected payoff of 0.5.

Observe that no single price could yield a strictly positive gain to each trader. More generally, let (σ,τ) be bids and offers selected by a communication device for Γ_δ. Assume that it achieves a canonical communication equilibrium and that given (σ,τ), buyer 1 has a strictly positive probability of winning. Then $\pi_\delta(\sigma,\tau) \leq 2$ and seller 2's probability of winning must be zero. Hence, buyer 1 can only acquire the object from seller 1 (and the price $\pi_\delta(\sigma,\tau)$ must be in $[1,2]$). But, then buyer 2 gets the object for sure with a bid in $(2,4)$, which shows that (σ,τ) cannot make buyer 1 a winner with positive probability.

The total expected gain from trade associated with the trading rule described above is 2. For every δ, Γ_δ has an equilibrium where each trader bids or offers his true valuation, which yields a total expected gain from trade of 3 ($\pi_\delta \in [2,3]$, buyer 2 trades with seller 1). This reminds us that our discussion does not concern any form of efficiency. We shall come back to this at the end of this section.

Our next example illustrates that the assumption of private values is crucial in Proposition 5.1.

<u>Example 5.2</u>

$$m = 2,\ n = 1.$$

μ_1 and μ_2 are independent and may take two values $(\mu_L \leq \mu_H)$, each with probability 1/2. The assumption of private values is fulfilled on the buyers' side. The seller has no private information but his valuation of the object is influenced by the one of the buyers:

$$u_i(\mu_1,\mu_2) = \mu_i,\ i = 1,2,$$

$$v(\mu_1,\mu_2) = \begin{cases} v_L, & \text{if } (\mu_1,\mu_2) \neq (\mu_H,\mu_H) \\ v_H, & \text{otherwise} \end{cases}$$

where $v_L < \mu_L < \mu_H < v_H$ (these four values constitute the set Π). If both buyers have a high reservation price for the object, it becomes more valuable to the seller.

For appropriate price (e.g., 0,3,4,5), the following strategies describe an equilibrium of Γ_0 (δ = 0, second price auction): the buyers bid their true value (i.e., $\sigma_i = \mu_i$) and the seller offers v_L (or μ_L). The next table describes the price and the probabilities of winning for each buyer (i.e., (π,Φ_1,Φ_2)) as a function of the types of the buyers.

		buyer 2: μ_L	buyer 2: μ_H
buyer 1	μ_L	μ_L, 1/2, 1/2	μ_L,0,1
	μ_H	μ_L,1,0	μ_H,1/2,1/2

The equilibrium condition for the seller is

$$(3/4)(\mu_L - v_L) + (1/4)(\mu_H - v_H)$$
$$\geq \max\{(1/2)(\mu_H - v_L) + (1/4)(\mu_H - v_H),0\}$$

The corresponding trading rule is <u>not strongly incentive compatible</u>, because it is not ex post individually rational for the seller. He will not accept to sell the object at the price μ_H. Indeed, this reveals to him that both buyers value the object highly and thus increases his reservation price to $v_H > \mu_H$. If the seller can make informal contracts before committing himself, he will defer the sale if he learns that both potential buyers are ready to bid a high price.

The last example illustrates that even under private values, $T^*(G_0)$ may be strictly included in $T(G_0)$. Trading rules may be incentive compatible and <u>ex post</u> individually rational without being <u>strongly</u> incentive compatible. This phenomenon was already observed by Matthews and Postlewaite (1989) in the context of bargaining (one seller and one buyer). The framework of the next example is standard auction (as in Myerson 1981)); in particular, the seller's reservation price is common knowledge.

<u>Example 5.3</u>

$m = 2, n = 1.$

Buyer 1's reservation price, μ_1, takes two values (μ_L and μ_H), each with probability 1/2. Buyer 2's reservation price, μ_2, is independent of μ_1 and also takes two values (μ_L and μ_M), each with probability 1/2. The seller's reservation price is 0 and is known to everybody. We assume that $0 < \mu_L < \mu_M < \mu_H$; these four values constitute the set of prices.

Let us describe a trading rule. If we assume that the buyer can commit himself, we can even dispense with the planner here. If buyer 1 pretends that his reservation price is 0 or μ_M or if buyer 2 reports a type of 0 or μ_H, no trade occurs. In the other cases, the successful buyer and the expected price are determined as follows

		buyer 2	
		μ_L	μ_M
buyer 1	μ_L	$(1, \mu_L)$	$(2, \mu_M)$
	μ_H	$(1, \mu_L - \varepsilon)$	$(1, \mu_H)$

where $\varepsilon < \mu_H - \mu_L$. If buyer 1 reports μ_H and buyer 2 reports μ_L, we assume that the planner chooses the price 0 with probability ε/μ_L and the price μ_L with probability $1 - \varepsilon/\mu_L$.

This is certainly a strange trading rule. Nevertheless, it is <u>incentive compatible</u> and <u>ex post individually rational</u>: a buyer is never asked to pay more than the reservation price he reports and the seller never gets a negative payoff. The traditional approach would not reject such a rule a priori, i.e., would consider it as a candidate for an efficient solution.

Suppose now that buyer 1 must confirm his intention to buy at the last moment. Then type μ_L can gain by <u>pretending to be type μ_H</u> and <u>refusing</u> to trade if the price is greater than μ_L.

We conclude this section with some comments on Myerson (1981). First, let us observe that in the (regular) independent private values case, the <u>optimal</u> auction is <u>strongly</u> incentive compatible. This is thus a case where the optimal solution belongs to $T^*(G_0)$ although no solution of $T(G_0)$ is eliminated a priori. A similar phenomenon is pointed out in Zou (1990).

The framework of Myerson (1981) is a little different from the one developed above. The seller's reservation price ν is common knowledge. The buyers' reservation prices, μ_i, are independent and distributed over a bounded interval according to F_i. These distribution functions have a density f_i; regularity means that each g_i defined by

$$g_i(\mu_i) = \mu_i - (1 - F_i(\mu_i))/f_i(\mu_i)$$

is strictly increasing. To describe the optimal auction, let us set

$$\pi_b(i) = \inf\{\pi: g_i(\pi) \geq m_i(\mu_{-i})\}$$

where

$$m_i(\mu_{-i}) = \max\{\nu, \max_{\ell \neq i} g_\ell(\mu_\ell)\}$$

and μ_{-i} is interpreted as the truthful report of the opponents of buyer i. If he tells μ_i' to the planner, buyer i gets the object if $\mu_i' \geq \pi_b(i)$. An interesting feature is that $\pi_b(i)$ does not depend on buyer i's report. It is, however, a function of the probability assessments (i.e., F_ℓ, f_ℓ) and so, it is not universal.

Obviously,

$$(\mu_i - \pi_b(i))\ I(\mu_i \geq \pi_b(i)) \geq \max\{(\mu_i - \pi_b(i))\ I(\mu_i' \geq \pi_b(i)), 0\}$$

for every μ_i, μ_i', so that strong incentive compatibility is satisfied for each buyer. The procedure is also ex post individually rational for the seller: if a buyer is successful, he pays $\pi_b(i) \geq \nu$.

The property <u>does not necessarily extend to make general set ups</u>, e.g., when the types of the buyers are not independent. In this case, as is illustrated in Myerson (1981), the optimal trading rule may require that the seller makes losses in some circumstances. For instance, he may have to give the object for nothing to one buyer and to pay a compensation to the other.[7]

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Notes

1. "Canonical" in the sense that the solutions are adapted to the trading rule, by consisting of truthful strategies. The game induced by a trading rule may have non-canonical solutions. By the revelation principle, these are taken into account in $T(G_0)$.

2. Unless specified otherwise, the description of the previous section remains valid. In particular, the actions available to the planner are the same.

3. The definition used here is slightly different from the one of Forges (1986): we restrict to canonical communication devices selecting consistent actions for the players, i.e., in T, and not simply in $(N \times \Pi)^m \times (M \times \Pi)^n$. This seems meaningful in the present context. Notice that even without this restriction, D(G) is included in $T(G_0)$.

4. The structure of the game G simplifies the part of the plain conversation which follows Barany's construction. If a player does not send his message according to the right scheme, the others can react by refusing to trade with him (see also footnote 6).

5. In this context, it is true that every solution in D(G) can be achieved as a completely revealing equilibrium of G preceded by plain conversation. Example 3.2 in Forges (1990) shows that this result does not hold without the private values assumption.

6. Even without any further assumption on the payoff functions, the game G has two useful properties. First, each player has an action which guarantees him 0, for all states of information and all actions of the other players (indeed, a player of G can always refuse to trade). Hence, all payoffs in D(G) are positive. Second, for each player, there is a vector of actions of his opponents guaranteeing that this payoff will not exceed 0, whatever the state of information and his action. This enables us to show that every payoff in D(G) can be

achieved as a correlated equilibrium payoff of G preceded by plain conversation. More precisely, if G has the two properties above, Proposition 2 of Forges (1990) holds for any number of players. Indeed, if some player does not behave correctly during the preplay conversation, the other players can hold him down to 0. The important feature is that the effect of this reaction does not depend on the state of information of the players. In particular, an agent who is not honest during the conversation phase cannot make use of the information acquired then.

7. Here, we use exactly the definition of Forges (1986).