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THE DEMAND THEORY OF THE WEAK AXIOM OF REVEALED PREFERENCE*

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ABSTRACT

In this paper we provide a statement of the relationship between the weak axiom of revealed preference (WA) and the negative semidefiniteness of the matrix of substitution terms (NSD). As a corollary we determine the relation between WA and the strong axiom of revealed preference (SA). The latter is equivalent to NSD and the symmetry of the matrix of substitution terms. The former, WA, implies NSD but is not implied by NSD. Also, WA is implied by the condition that the matrix of substitution terms is negative definite (ND), but it does not imply ND. Application of these results yield an infinity of demand functions which satisfy WA but not SA.

THE DEMAND THEORY OF THE WEAK AXIOM OF REVEALED PREFERENCE

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Introduction

The purpose of this paper is to provide a final step in the solution to a problem which has been central to the foundation of consumer demand theory since Samuelson's introduction of the weak axiom of revealed preference in 1938 [12]. A characterization, via classical restrictions, of the set of demand functions which satisfy the weak axiom is obtained.

Demand functions have as their arguments an ℓ dimensional price vector p and a wealth variable w.[1/] They assign to each p and w an ℓ dimensional vector of commodity amounts, called a demand. We assume that demands are non-negative and satisfy the balance condition that for each (p,w), the inner product of p with the demand at (p,w) is w. In addition we will assume that demand functions are homogeneous of degree zero in (p,w). It is a classical result (Johnson (1913 [10]) and Slutsky (1915 [16])), that demand functions which are generated by maximizing a quasi-concave (smooth) utility function satisfy the conditions of negative semidefiniteness NSD and symmetry S of a matrix of compensated price derivates (substitution terms).[2/] The converse theorem is also classical in background (Antonelli (1886 [1])) and both are central to the demand theory presented in Samuelson's Foundations of Economic Analysis [13].[3/]

In 1938 Samuelson [12] proposed a new foundation for the theory of consumer behavior. He defined the direct revealed preference relation R by xRy if

x is the demand in a situation where $y \neq x$ can be afforded, and took as his central axiom the asymmetry of R (called the weak axiom of revealed preference.) Samuelson left unanswered the question of whether his weak axiom was equivalent to the utility hypothesis; i.e., the assumption that demand is generated by maximizing a quasi-concave utility function. His efforts yielded the conclusion that the weak axiom implies NSD ; however, he seemed aware that S was probably not implied by his axiom, except for the special case of two commodities.

The question of a possible equivalence between the weak axiom and the utility hypothesis remained unanswered for over twenty years before an example due to Gale (1960 [5]) demonstrated that there exists a demand function satisfying the weak axiom which cannot be generated by utility maximizing behavior. In the meantime Houthakker (1950 [7]) showed that the acyclicity of R (called the strong axiom of revealed preference) is equivalent to the utility hypothesis. Even still, the question of the precise relationship between the weak axiom and the strong axiom (or equivalently the utility hypothesis) has remained open. Because of Gale's example they are not equivalent. Uzawa (1960 [18], p. 133) introduced a condition under which they are equivalent; however, it is marred by the fact that it looks very much like the strong axiom itself.

In this paper we provide a definitive statement of the relationship between the weak axiom and NSD . As a corollary we determine the relationship between the weak and strong axioms. The strong axiom is equivalent to NSD and S . The weak axiom implies NSD but is not implied by NSD ; however, the weak axiom is implied by the condition that the matrix of substitution terms is negative definite ND . Application of these results yield an infinity of Gale type examples and prove that an analog of ND for excess demand functions, since it implies the weak axiom, is sufficient for the stability of competitive equilibrium.

THE THEORY

Let $V = P \times (0,\infty)$ be an open convex cone of positive vectors in $R^{\ell+1}$. The first ℓ coordinates of points in V are interpreted as price variables and the remaining coordinates as an income variable. For this reason a generic element of V is denoted by (p,w) where $p \in P$ and $w > 0$. A demand function $h:V \to R^{\ell}$ is a continuously differentiable function which satisfies

(α) For all $(p,w) \in V$, $h(p,w) \geqq 0$,

(β) For all $(p,w) \in V$, for all $\lambda > 0$, $h(p,w) = h(\lambda p, \lambda w)$ (Homogeneity),

(γ) For all $(p,w) \in V$, $p \cdot h(p,w) = w$ (Balance).

Given a demand function h and $(p,w) \in V$, $A(h)(p,w)$ will denote the $\ell \times \ell$ matrix with generic entry

$$\frac{\partial h^i(p,w)}{\partial p_j} + h^j(p,w)\frac{\partial h^i(p,w)}{\partial w} = a_{ij}(h)(p,w).$$

It follows from (β) and (γ) that $p'A(h)(p,w)p = 0$ for every $(p,w) \in V$. Define also the relation R by xRy if $x = h(p^x,w^x)$, $x \neq y$, and $p^x \cdot y \leq w^x$. Some conditions which may be satisfied by a demand function are now stated. The first three involve conditions on derivatives, while the remaining three are finitistic.

(NSD) For every $(p,w) \in V$, $A(h)(p,w)$ is a negative semidefinite matrix; i.e., for all $v \in R^{\ell}$, $v'A(h)(p,w)v \leqq 0$.

(ND) For every $(p,w) \in V$, $A(h)(p,w)$ is a negative definite matrix; i.e., for all $v \in R^{\ell}$, $v \neq 0$, $v/\|v\| \neq p/\|p\|$, $v'A(h)(p,w)v < 0$. 4/

(S) For every $(p,w) \in V$, $A(h)(p,w)$ is a symmetric matrix.

(WWA) For every $(\tilde{p},\tilde{w})$, $(\bar{p},\bar{w}) \in V$, if $h(\tilde{p},\tilde{w}) \neq h(\bar{p},\bar{w})$ and $\tilde{p}\cdot h(\bar{p},\bar{w}) < \tilde{w}$, then $\bar{p}\cdot h(\tilde{p},\tilde{w}) > \bar{w}$.[5/]

(WA) R is asymmetric, or equivalently, for every $(\tilde{p},\tilde{w})$, $(\bar{p},\bar{w}) \in V$, if $h(\tilde{p},\tilde{w}) \neq h(\bar{p},\bar{w})$ and $\tilde{p}\cdot(\bar{p},\bar{w}) \leqq \tilde{w}$, then $\bar{p}\cdot h(\tilde{p},\tilde{w}) > \bar{w}$ (weak axiom).

(SA) R is acyclic (strong axiom).

Obviously ND implies NSD and WA implies WWA . We will show that WWA implies NSD [compare with Samuelson: WA implies NSD, Foundations, p. 111], ND implies WA , and NSD implies WWA. In addition we will show that WWA does not imply WA .

Lemma 1: Assume that h is a demand function, $(p^o,w^o) \in V$, $v \in R^\ell$, $p^o + v \in P$, and $x^o = h(p^o,w^o)$. Define $p(t) = p^o + tv$, and for all vectors $p \in P$ define $\xi(x^o,p) = h(p,p\cdot x^o)$. Then

(1) $\dfrac{\partial\xi^i(x^o,p^o)}{\partial p_j} = a_{ij}(h)(p^o,w^o)$, and

(2) $\lim\limits_{t\to 0} \left|(1/t^2)(p^o - p(t))\cdot(\xi(x^o,p^o) - \xi(x^o,p(t))) - \Sigma_j\Sigma_i a_{ij}(h)(p^o,w^o)v_i v_j\right| = 0.$

If in addition h satisfies WWA then

(3) $(p^o - p(t))\cdot(\xi(x^o,p^o) - \xi(x^o,p(t))) \leqq 0$ for all $t \in [0,1]$.

Proof: Statement (1) follows from a chain rule differentiation and the definition of $a_{ij}(h)(p^o,w^o)$. A variant of (2) is implicit in [11], p.110, and is attributed to L. Hurwicz. It is established as follows.

Since each ξ^i is continuously differentiable and $\|p(t) - p^o\| = t\|v\|$,

$$0 = \lim_{t\to 0} \left|\xi^i(x^o,p(t)) - \xi^i(x^o,p^o) - \sum_{j=1}^{j=\ell} \xi^i_j(x^o,p^o)(p_j(t) - p^o_j)\right| / t\|v\|$$

$$i = 1,2,\ldots,\ell$$

Since $tv_i = p_i(t) - p^o_i$, multiplying by $\|v\|(p_i(t)-p^o_i)/t$ yields

$$0 = \lim_{t\to 0} \left|(1/t^2)(p^o_i - p_i(t))(\xi^i(x^o,p^o) - \xi^i(x^o,p(t))) - \sum_{j=1}^{j=\ell} \xi^i_j(x^o,p^o)v_i v_j\right|,$$

and summing over i together with (1) complete the proof of (2).

Statement (3) combines $p(t)\cdot(\xi(x^o,p^o) - \xi(x^o,p(t))) = 0$ from the definition of ξ and $p^o\cdot(\xi(x^o,p^o) - \xi(x^o,p(t))) \leqq 0$ from WWA ∎

The following theorem is an immediate consequence of (2) and (3).

Theorem 1. If h is a demand function which satisfies WWA then h satisfies NSD .

Lemma 2: If h is a demand function which does not satisfy WA , then there exist vectors $p^o \in P$ and $0 \neq v \in T_{p^o}$ such that for all t between zero and one,

(3') $(p^o-p(t))\cdot(\xi(x^o,p^o) - \xi(x^o,p(t))) \geqq 0,$

where $p(t) = p^o + tv$.

Proof: We will consider three cases which together exhaust the ways in which (WA) can be violated.

If $\bar{p}\cdot h(\tilde{p},\tilde{w}) = \bar{w}$ and $\tilde{p}\cdot h(\bar{p},\bar{w}) < \tilde{w}$. let s^o be the smallest positive s such that $\tilde{p}\cdot \xi\ (h(\tilde{p},\tilde{w}),\ \bar{p} + s(\tilde{p} - \bar{p})) = \tilde{w}$. So defined, s^o is positive. If $p^o = \bar{p} + s^o(\tilde{p} - \bar{p})$, $x^o = \xi(h(\tilde{p},\tilde{w}),\ p^o)$, and v is the projection of $\bar{p} - p^o$ on T_{p^o} then $v \neq 0$ and we let $p(t) = p^o + tv$. Since $p^o \cdot h(\bar{p},\bar{w}) < p^o \cdot x^o$, the continuity of h and the definition of s^o imply that $p^o\cdot(\xi(x^o,p^o) - \xi(x^o,p(t))) > 0$. This inequality establishes the result.

If $\bar{p}\cdot h(\tilde{p},\tilde{w}) < \bar{w}$ and $\tilde{p}\cdot h(\bar{p},\bar{w}) < \tilde{w}$ let $\hat{x}\epsilon H = \{x: \bar{p}.x = \bar{w}$ and $\tilde{p}\cdot x = \tilde{w}\}$. The continuity of h guarantees that there exists $s\epsilon\ (0,1)$, such that $x' = \xi(\hat{x}, s\bar{p} + (1 - s)\tilde{p})\epsilon H$. Define $p' = s\bar{p} + (1-s)\tilde{p}$ and $w' = p'\cdot x'$. Since $\bar{p}\cdot x' = \bar{w}$ and $p'\cdot h(\bar{p},\bar{w}) < w'$, the previous case applies.

If $\bar{p}\cdot h(\tilde{p},\tilde{w}) = \bar{w}$ and $\tilde{p}\cdot h(\bar{p},\bar{w}) = \tilde{w}$ $(h(\tilde{p},\tilde{w}) \neq h(\bar{p},\bar{w}))$, then $x(s) = \xi(h(\tilde{p},\tilde{w}),\ s\bar{p} + (1-s)\tilde{p}) = \xi(h(\bar{p},\bar{w}), s\bar{p} + (1-s)\tilde{p})$. If $x(s)\epsilon H$ (defined above) for all $s\epsilon(0,1)$ define $x^o = \bar{x}$, $p^o = \bar{p}$, and $v = \tilde{p} - p^o$. If $x(\hat{s}) \notin H$ for some $\hat{s}\epsilon(0,1)$ define $\hat{p} = \hat{s}(\bar{p}) + (1 - \hat{s})\tilde{p}$, and $\hat{w} = \hat{p}\cdot x(\hat{s})$. By the definition of $x(\hat{s})$, $\hat{p}\cdot h(\tilde{p},\tilde{w}) = \hat{p}\cdot h(\bar{p},\bar{w}) = \hat{w}$. Also, since $x(\hat{s})\notin H$, either $\tilde{p}\cdot x(\hat{s}) < \tilde{w}$ or $\bar{p}\cdot x(\hat{s}) < \bar{w}$, in either event, the first case applies. ∎

The following theorem is an immediate consequence of (2) and Lemma 2. (Assume WA is not satisfied and prove that ND is violated.)

Theorem 2: If h is a demand function which satisfies ND then h satisfies WA.[6/]

Theorems 1 and 2 combine to yield the result that a demand function h satisfies ND if and only if there exists a neighborhood of h such that all demand functions in that neighborhood satisfy WA.[7/] Thus ND is an open infinitessimal analog of WA. This fact makes it very easy to exhibit demand functions which satisfy WA but cannot be generated by utility maximization. To achieve this take any demand function (for at least three commodities) which satisfy ND and S. All demand functions sufficiently close to h (and equal to h outside a compact set of prices) will satisfy ND but most will not satisfy S. Hence they must satisfy WA but cannot come from utility maximization. [8/]

Applying Theorem 2 in an approximation argument yields the following theorem, which together with Theorem 1 establishes the equivalence of NSD and WWA .

Theorem 3: If the demand function h satisfies NSD then it satisfies WWA .

Proof: Assume that h satisfies NSD but that WWA is violated. Then for some $(\tilde{p},\tilde{w}),(\overline{p},\overline{w}) \in V$ we have $h(\tilde{p},\tilde{w}) \neq h(\overline{p},\overline{w}), \tilde{p}\cdot h(\overline{p},\overline{w}) < \tilde{w}$, and $\overline{p}\cdot h(\tilde{p},\tilde{w}) \leqq \overline{w}$. Replacing $\overline{w}$ by $w' > \overline{w}$, close enough to $\overline{w}$, we have $h(\tilde{p},\tilde{w}) \neq h(\overline{p},\overline{w})$, $\tilde{p}\cdot h(\overline{p},w') < \tilde{w}$, and $\overline{p}\cdot h(\tilde{p},\tilde{w}) < w'$. Therefore, without loss of generality, we can assume

$h(\tilde{p},\tilde{w}) \neq h(\overline{p},\overline{w})$, $\tilde{p}\cdot h(\overline{p},\overline{w}) < \tilde{w}$, and $\overline{p}\cdot h(\tilde{p},\tilde{w}) < \overline{w}$.

Let $V' \subset V$ be an open convex cone such that $(\tilde{p},\tilde{w}),(\overline{p},\overline{w}) \in V'$ and any limit point of V' different from 0 is contained in V. Let $f: R^{\ell} \setminus \{0\} \to R$ be a continuously differentiable function which is homogeneous of degree one and such that for every $p \in R^{\ell} \setminus \{0\}$, $D^2 f(p)$, the Hessian matrix of f at p, satisfies the condition $v'D^2 f(p)v < 0$ for every $v \in R^{\ell}$, $v \neq 0$, $v/||v|| \neq p/||p||$. (For example, let $f(p) = -||p||$.)

For sufficiently small $\overline{\eta} > 0$, if $\eta < \overline{\eta}$ the function $h_{\eta}: V' \to R^{\ell}$ given by $h_{\eta}(p,w) = h(p,w - \eta f(p)) + \eta Df(p)$ is well defined. It is immediately seen to be a demand function; i.e., to satisfy (α), (β), and (γ). Moreover,

$A(h_\eta)(p,w) = A(h)(p,w - \eta f(p)) + \eta D^2 f(p)$, and so h_η satisfies ND and by Theorem 2, also WA. However, for η sufficiently small, $\tilde{p}\cdot h_\eta(\bar{p},\bar{w}) < \tilde{w}$ and $\bar{p}\cdot h_\eta(\tilde{p},\tilde{w}) < \bar{w}$, a contradiction ∎

If h satisfies S, then it is known that h can be locally integrated. 9/ Using this fact Hurwicz and Uzawa([9]) show that if h satisfies S in addition to NSD then h must satisfy WA. Since we have already demonstrated that WWA is equivalent to NSD, it follows that given S the conditions WWA, NSD, and WA are all equivalent. Since we are not concerned with integrability theory we will not pursue the above line; however, we will demonstrate by example that in the absence of S, WA and WWA are not equivalent. In particular the example shows that WWA cannot be replaced by WA in Theorem 3, and thus the theorem cannot be strengthened.

Example: Let $h: (0,\infty)^4 \to R^3$ be defined by

$$h(p^1, p^2, p^3, w) = \left(\frac{p_2}{p_3}, -\frac{p_1}{p_3}, \frac{w}{p_3}\right). \quad \underline{10/}$$

The function h is not positive valued, but this is inessential for our purpose. It satisfies NSD (hence WWA) since for every $(p,w) \in V$, $A(h)(p,w)$ is skew symmetric, and this implies $v'A(h)(p,w)v = 0$ for every $v \in R^\ell$. However, h does not satisfy WA: consider $(\tilde{p},\tilde{w}) = (1,1,1,1)$ and $(\bar{p},\bar{w}) = (2,1,1,2)$.

We conclude with a discussion of the corresponding theory for excess demand functions. An excess demand function $f\colon P \to R^{\ell}$ is a continuously differentiable function which is positive homogeneous of degree zero ($f(\lambda p) = f(p)$ for all $p \in P$ and $\lambda > 0$) and satisfies the balance condition that $p \cdot f(p) = 0$ for all $p \in P$. Note that if $h\colon V \to R^{\ell}$ is a demand function and ω is a positive vector in R^{ℓ}, then $h(p, p\cdot\omega) - \omega$ defines an excess demand function. The conditions analogous to ND and WA (for brevity we shall not be concerned here with NSD and WWA) are:

(ND^e) for every $p \in P$, $Df(p)$ is a negative definite matrix on $T_p \cap \{v \in R^{\ell}\colon\ v \cdot f(p) = 0\}$, i.e., if $v \neq 0$, $v \cdot f(p) = 0$, $v \in T_p$, then $v' D\, f(p) v < 0$.[11/]

(WA^e) for every $\tilde{p}, \bar{p} \in P$, if $f(\tilde{p}) \neq f(\bar{p})$ and $\bar{p} \cdot f(\tilde{p}) \leqq 0$, then $\tilde{p} \cdot f(\bar{p}) > 0$.

The following result is an analog of Theorem 2 and is proved in a similar way.

Theorem 4: If f is an excess demand function which satisfies ND^e then f satisfies WA^e .

As an application, let f be an excess demand function and consider the differential equation $\dot{p} = f(p)$. This defines a tâtonnement price dynamics. Arrow and Hurwicz [3] have shown that if f satisfies WA^e , then f is globally stable. Combining this fact with Theorem 4 yields the result that if f satisfies ND^e and $f(\bar{p}) = 0$ for some $\bar{p}$, then $\bar{p}$ is a globally stable equilibrium.[12/] This strengthens a result proved by Arrow and Hurwicz ([2] , Theorem 4) since our hypothesis requires negative definiteness of $Df(p)$ only on a proper subset of p.[13/]

A RELATED CONJECTURE
(with Wayne Shafer)

This concerns the theory of the "nontransitive consumer." Let W be a relation on Ω (the closed positive orthant of R^{ℓ}), T be defined by xWy and not yWx, $W(x) = \{x' \in \Omega : x'Wx\}$, $W^{-1}(x) = \{x' \in \Omega : xWx'\}$. The following three axioms provide the foundation for the theory of the nontransitive consumer.

(i) W is strongly connected

(ii) For all $x,y,z \in \Omega$, $x,y \in W(z)$ and $x \neq y$ imply $tx + (1-t)yTz$, $0 < t < 1$.

(iii) For all x, $W(x)$ and $W^{-1}(x)$ are closed.

It is known (Sonnenschein [17], and Shafer [15]) that under these axioms compact competitive budgets contain unique W-maximal elements and that demand functions exist and are continuous. It is also trivial to prove that nontransitive consumer demand functions satisfy WA. W. Shafer [15] has also shown that the nontransitive consumer demand functions need not satisfy S: consider $k(x,y) = y_1^{-\frac{1}{2}} x_2^{\frac{1}{2}} + \ln x_3 - x_1^{-\frac{1}{2}} y_2^{\frac{1}{2}} - \ln y_3$ and define xWy by $k(x,y) \geq 0$. The demand function for the first two commodities generated by W is $h_i(p,w) = w/[2p_i(1 + (p_2/p_1)^{\frac{1}{2}})]$, $i = 1,2$.

As we have stated

(4) $$\text{NSD and S} \Leftrightarrow \text{Strong Axiom} \Leftrightarrow \text{Utility Hypothesis.}$$

Since we have just shown

$$\text{ND, NSD} \Leftrightarrow \text{Weak Axiom,}$$ 14/

and since it is easily established

$$\text{Weak Axiom} \Leftarrow \text{Nontransitive Consumer,}$$

the following conjecture is natural.

Conjecture: Given a demand function h satisfying WA, there exists a nontransitive consumer who generates h: i.e.,

(5) ND, NSD $<=>$ Weak Axiom $<=>$ Nontransitive Consumer,

a counterpart of (4).

FOOTNOTES

1/ Static Demand Theory by D. Katzner [11] provides an excellent introduction to demand theory.

2/ Throughout this paper the terms semidefiniteness and definiteness do not presume symmetry.

3/ Surveys of this literature are found in references [8] and [11].

4/ In view of the fact that $p'A(h)(p,w) = 0$ and $A(h)(p,w)p = 0$ for all $(p,w) \in V$, NSD is equivalent to "$A(h)(p,w)$ is negative semidefinite on R^{ℓ}" and ND is equivalent to "$v'A(h)(p,w)v < 0$ for all $v \neq 0$, $v/\|v\| \neq p/\|p\|$". The latter is the definition of ND given in an earlier version of the paper; however, T. Rader pointed out to us that it leads to some difficulties in exposition which are avoided by the present formulation.

5/ The notation WWA is used to remind the reader that this condition is weaker than the weak axiom.

6/ M. K. Richter pointed out to us in private correspondence that the proof of Theorem 2 can be streamlined and the requirement that h be continuously differentiable replaced by differentiable. However, since (2) and Lemma 2 are needed for other purposes, the current treatment is more economical.

7/ Because of the openness of the domain, neighborhoods should be defined by the Whitney topology.

8/ The idea of peturbing a demand function which satisfies ND and S to obtain a demand function which satisfies WA but does not come from utility maximization was employed by Gale (and is attributed by him to Samuelson).

Since the question of the equivalence of the weak axiom and the utility hypothesis was classically carried out in the context of differentiable demand functions, we also note that the non-differentiability (on a set of measure zero) of the Gale example is inessential.

9/ Also, if there are only two commodities then h can be locally integrated.

10/ The function h can be altered to have positive values on an arbitrary compact subset of its domain. We do not know if there exists a function with the properties of h defined on $(0,\infty)^4$.

11/ Since $p'Df(p) = -f(p)$ and $Df(p)p = 0$ for all p, ND^e is equivalent to "if $vf(p) = 0$, $v \neq 0$, $v/\|v\| \neq p/\|p\|$ then $v'Df(p)v < 0$".

12/ Let $g:R^n \to R^n$ be a continuously differentiable function with $g(0) = 0$. Hartman and Olech([6], p. 548) have established that the following condition is sufficient for the asymptotic stability of $\dot{x} = g(x)$ at $x = 0$: for all $0 \neq v \in R^n$, $x \in R^n$, if $v \cdot g(x) = 0$, then $v'Dg(x)v < 0$. Notice the striking similarity between this condition and ND^e. This suggests that the weak axiom WA^e is the finitistic equivalent of the Hartman-Olech condition.

13/ Our analysis may shed some light on the question of whether every economy with sufficiently similar individuals will satisfy WA^e and thus generate a globally stable excess demand function. Since arbitrarily small perturbation of a given excess demand function which satisfies WA^e cannot be guaranteed to satisfy WA^e, there is some problem in obtaining an affirmative answer to the question. However, the fact that the individual excess demand functions which cause trouble are the ones with negative semi-definite but not negative definite Hessians (subject to constraint) suggests that the "bad cases" lie in a small closed set with empty interior (in the Whitney topology).

14/ This is an informal version of ND $\Rightarrow$ Weak Axiom $\Rightarrow$ NSD.

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