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INCREASING RETURNS, INDUSTRIALIZATION
AND INDETERMINACY OF EQUILIBRIUM*

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Abstract

This paper asks whether adjustment provesses over real time help to "select" the long run outcome in a model of industrialization, where multiple stationary states exist because of increasing returns in the manufacturing sector. "History" alone cannot in general determine where the economy will end up. Self-fulfilling expectations often make the escape from the state of pre-industrialization (the take-off) possible. The global bifurcation technique is used to determine when an underdevelopment trap exists and when a take-off path exists. The role of government policy and agricultural productivity in industrialization are then considered.

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I. INTRODUCTION

Recently there has been growing interest in the analysis of market economies in the presence of externalities. The traditional literature examined Marshallian external economies in the production process, while the recent studies are also concerned with the market size and the improved matching between potential buyers and sellers due to transaction externalities or the aggregate demand spillover. These studies show how there may exist multiple Pareto-ranked equilibria.[1] Facing the problem of equilibrium selection, numerous authors turn to historical factors. For example, Krugman and Obstfeld [1987, p. 130] discussed in the context of international trade,

> In interpreting the real world implications of the (indeterminacy) result, however, the right way to think of it is to say that initial advantages can cumulate over time, so that history and accidental factors—which we do not capture with our simple model—can have a persistent effect on the pattern of international trade.

Similarly, Blanchard and Summers [1988, p. 184] introduced the concept of "fragile equilibria", in the context of labor markets, to refer to situations "where outcomes are very sensitive to shocks and may be history dependent" and argued that,

> Research on multiple equilibria and on hysteresis suggest mechanisms that may generate unemployment rates that depend sensitively on the shocks an economy has experienced.

Some predict that the idea of history dictating the choice of equilibrium survives a formalization and that the explicit analysis of dynamic adjustment resolves the issue of multiplicity. Helpman [1984, p. 341] argued, for example,

> There remain, however, open questions which have to be answered before the relevance of this possibility can be evaluated. These have to do with dynamic adjustment processes which should help determine both autarky and trading equilibria.

because, for example, demonstrating the uniqueness of a perfect foresight path in a neighborhood of a stationary state does not necessarily rule out the existence of other perfect foresight paths in the large. One also needs to pay careful attention to the boundary conditions. Second, a model with multiple stationary states due to externalities generally needs to be highly nonlinear. It is well-known that the global analysis of nonlinear differential equations are still far from complete.

The goal of this paper is thus modest and limited. It addresses the above problem in the context of industrialization using a version of sectoral adjustment models developed in Matsuyama [1988a]. The economy has two (agriculture and manufacturing) sectors. The manufacturing sector is subject to increasing returns, producing multiple stationary states. One stationary state, with zero employment in manufacturing, can be considered as the state of pre-industrialization. This economy is inhabited by overlapping workers and each worker's career decision (choice of sector) is irreversible. Sectoral labor movement takes place due to the demographic change. This model provides a convenient framework in which to address the above question. First, the career decision by agents based on perfect foresight is treated explicitly. Second, the dynamics of employment is described by the relatively simple nonlinear differential equations on a plane, for which some mathematical results are available.

The history versus expectations distinction seems of particular importance in the context of development. The diversity of per capita income levels across countries suggests the presence of some sort of multiplicity. The idea of history determining the long run position of the economy then implies that many countries may be in underdevelopment traps.[4] A corollary transformation (or unbalanced growth), along which the economy traverses between two stationary states. Section III performs the global analysis. It first analyzes the case of zero rate of time preference. In this case, the dynamics can be described as a Hamiltonian system, whose global information is easily obtained. Then, by using a perturbation method, it is shown that, if the rate of time preference is sufficiently close to zero, there exist generally multiple perfect foresight paths leading to different stationary states. History, as captured by the initial manufacturing employment, cannot necessarily select the long run outcome. In particular, there is a case in which an industrialization path exists for the economy whose initial manufacturing employment is zero. A take-off is possible in such a case. However, there are also situations where history determines the outcomes. For example, there is a case in which, if the initial employment in the manufacturing sector is below some threshold level, the equilibrium is unique and the economy always converges to the zero level stationary state. The economy will be trapped into the state of pre-industrialization. The global bifurcation technique, which is new in economics, is used to find the exact condition under which the take-off path exists.

Section IV considers two applications. First, the role of the government policy is discussed. When the zero level stationary state is a trap under the laissez-faire, the government can make the escape from this stationary state (a take-off) possible, by subsidizing the production of the manufacturing good. The subsidy also eliminates the equilibrium path leading to the zero stationary state, and thus the possibility of de-industrialization when the initial manufacturing employment is large. In the second application, the effect of agricultural productivity in is positively related to the size of the sector: $h'(L^M) > 0$.[7] The aggregate production function is given by $X^M = h(L^M)L^M$. Take the agricultural good as a numeraire and let q denote the relative price of the manufacturing good, exogenously given in the world market. Then, perfect competition in the goods and labor markets ensures that $w^A = 1$ and $w^M = h(L^M)q$, where w^i is the wage rate in sector i, or

$$(1) \qquad W = h(L^M)q,$$

where W is the relative wage in manufacturing.

The economy is populated by a continuum of agents, whose measure is normalized to one. An agent of type τ can provide $g^i(\tau)$ efficiency units of labor service inelastically if she works in sector i (and she cannot work in both sectors at the same time). The index of type τ is numbered so that $g^A(\tau)/g^M(\tau)$ is a strictly increasing, differentiable function of τ; an agent with high τ has comparative advantage in agriculture relative to an agent with low τ. Let $\Phi(\tau)$ be the distribution function of τ with $\Phi'(\tau) > 0$ on the support of Φ, $[\tau^-,\tau^+]$. Let T denote the inverse function of $g^A(\tau)/g^M(\tau)$. Clearly $T' > 0$. Given the relative wage, W, all agents whose types are greater than T(W) work in agriculture and all agents whose types are smaller than T(W) work in manufacturing. That $T' > 0$ implies that, if $\tau^- < T(W) < \tau^+$, a higher relative wage in manufacturing attracts more agents to the sector. If $T(W) \geq \tau^+$, all agents work in manufacturing and, if $T(W) \leq \tau^-$, all agents work in agriculture. Thus, the labor supply schedule in agriculture is given by agriculture is simply given by (2) and does not impose any additional restriction. (Labor demand in agriculture is perfectly elastic.)

Facing the problem of equilibrium selection, one often appeals to a following story, which is sometimes referred to as the Marshallian tatonnement process. In the short run, the relative wage is determined by the demand condition. Suppose that the initial employment in manufacturing is somewhere between S_L and S_H. Then, the relative wage in manufacturing is higher than the level required to keep the employment constant. In responding to a higher wage rate, more agents switch sectors and this gradually increases the labor supply in manufacturing. This process continues until the economy converges to S_H. Similarly, if the initial labor supply is somewhere between S_0 and S_L, the economy converges to S_0, and if the economy is initially above S_H, it converges to S_H. Thus, S_H and S_0 are stable and S_L is unstable and history can help us to select the equilibrium. In particular, when the initial employment in the manufacturing sector is small, the sector will vanish and the economy will specialize in agriculture. The economy will be trapped into the state of pre-industrialization; the vicious circle of poverty results. In order to take off and industrialize, some sorts of government intervention would be necessary. This story may be also used to illustrate some unequalizing process, or the doctrine of "uneven development." Imagine that there are two economies: one's initial employment is slightly below S_L, while the other's is slightly above S_L.[8] The small difference at the beginning would magnify over time, and eventually, the two economies follow completely different courses. History matters in selecting the long run position of the economy.[9] In other words, "hysteresis exists."

The problem with this story is that it is not clear why sectoral model capable of describing industrialization as a continuous, self-sustained process of structural transformation (or unbalanced growth), along which the economy traverses between two stationary states. It should be emphasized that, in a dynamic model such as one developed below, an equilibrium is an entire path of the economy and, when the economy stays still, then it is in a stationary state.

The model is similar to Matsuyama [1988a]. Time is continuous and starts from zero (the initial period) and extends indefinitely into the future. The production structure is identical to the static economy. Although the size of the population is constant over time and equal to one, there are overlapping agents. Every agent throughout her lifetime faces a constant instantaneous probability of death p. The risk of death is independent and there exists no aggregate uncertainty. The constant population implies that a new cohort whose size is equal to p is born at each moment of time. Skill distribution within a cohort, and thus skill distribution of those alive, are as in the static economy. The relative price of the manufacturing good is constant over time and equal to q.

The labor allocation in this economy is sluggish because of the irreversibility of career decision. At the beginning of her life, every agent needs to decide in which sector to work and, once the career decision is made, she will be stuck to the sector for the rest of her life. [The idea is that, when you are young, you decide either to stay in the rural area and become a peasant, or to go to the urban area and be an industrial worker. Once you have acquired your life-style, it is very difficult (in the model, impossible) to change it.] The assumption of complete irreversibility is a strong one, but adopted to simplify the model. The cost of making this assumption seems cohort and the probability of death. Similarly, using the definition of function Z,

$$\dot{L}_t^M = p[Z(Q_t) - L_t^M]. \tag{4b}$$

Matsuyama [1988a] demonstrates that, for any given path of $\{Q_t\}_{t=0}^{\infty}$ and any initial conditions $(L_0^A, L_0^M) \in \Sigma \equiv \{(L^A, L^M) | \; 0 \le L^A \le Y(Q), \; 0 \le L^M \le Z(Q)$ for some Q satisfying $\tau^- \le T(Q) \le \tau^+\}$, a path of (L_t^A, L_t^M) stays inside Σ and they are on the frontier of Σ if and only if the economy is in a stationary state. Therefore, an equilibrium of this economy for given initial values, $(L_0^A, L_0^M) \in \Sigma$, is a path satisfying (4a)-(4b) and

$$Q_t = r\int_t^{\infty} h(L_s^M) q e^{-r(s-t)} ds \le \bar{Q} \tag{5}$$

for all $t \ge 0$, where $\bar{Q}$ is the upper bound of Q_t and equal to $h[\int_{\tau-}^{\tau+} g^M(\tau) d\Phi(\tau)]q$.

Two considerations simplify the problem of finding equilibria. First, the dynamics of $\{L^M, Q\}$ is independent of L^A, so that equation (4a) can be ignored for the purpose of this paper.[11] Second, differentiating equation (5) with respect to time shows that $\dot{Q}_t = r[Q_t - h(L_t^M)q]$ <u>and</u> $0 \le Q_t \le \bar{Q}$ for all $t \ge 0$ are equivalent to (5). Therefore, equilibria conditions now become: for a given $L_0^M \in [0, \int_{\tau-}^{\tau+} g^M(\tau) d\Phi(\tau)]$,

$$\dot{L}_t^M = p[Z(Q_t) - L_t^M], \tag{6a}$$

$$\dot{Q}_t = r[Q_t - h(L_t^M)q], \tag{6b}$$

and $Q_t \in [0, \bar{Q}]$ for all $t \ge 0$. Equations (6a)-(6b) jointly define a planar dynamical system in $\{L^M, Q\}$ on $[0, \int_{\tau-}^{\tau+} g^M(\tau) d\Phi(\tau)] \times [0, \bar{Q}]$, but the initial value for Q must be chosen to make a path consistent with these equilibrium which is the subject of the next section.

III. GLOBAL DYNAMICS: THE CASE OF A SMALL RATE OF TIME PREFERENCE

This section is more technical than the rest of the paper. Those who are more interested in the applications are advised to see just how to read the figures below, Figures IIIa-IIIb in particular, and skip the technical details on a first reading.

A. Some Important Results.

Proposition 1. Suppose that $r - p \equiv \theta \neq 0$. Then, no solution curve of the dynamical system, (6a)-(6b), is a Jordan curve.[12]

Proof of Proposition 1. Note that (6a)-(6b) implies that $r[Q - h(L)q]dL = p[Z(Q) - L]dQ$. Therefore, if a Jordan curve Γ solve (6a)-(6b),

$$\oint_{\Gamma}\{p[Z(Q) - L]dQ - r[Q - h(L)q]dL\} = 0,$$

where the integral sign is a line integral. From Green's Theorem,

$$\iint_{\Omega}(r - p)dLdQ = \iint_{\Omega}\theta dLdQ = 0,$$

where the integral is a surface integral and Ω is the region bounded by Γ, or $\partial\Omega = \Gamma$. This equality holds if and only if $r - p = \theta = 0$, which contradicts the assumption. Q.E.D.

Proposition 2. Suppose that $r - p = \theta = 0$. Define the Hamiltonian by

$$H(L,Q) \equiv QL - \int_0^Q Z(s)ds - \int_0^L h(z)qdz.$$

Then, all solutions of the planar dynamical system, (6a)-(6b), satisfy

Hamiltonian system.

Remark 1. Bifurcations occur when dynamics are not structurally stable. A slight change in parameters alters the topological dynamics. The points of parameters at which it occurs are called bifurcation points. The type of bifurcations discussed here is global in nature since the global information of the system is required to analyze them. They contrast with local bifurcations, whose analyses use only the local information. Saddle-node, transcritical, pitchfork and Hopf are examples of local bifurcations (Guckenheimer and Holmes [1986, Ch. 3]). The first three are concerned with changes of stationary states. Since loci $\dot{L} = 0$ and $\dot{Q} = 0$ and thus the stationary states are independent of r, p, and θ, their changes cannot produce these types of bifurcations. A saddle-node bifurcation occurs when parameters representing technology, and skill distribution change so as to shift the $\dot{Q} = 0$ locus down or to shift the $\dot{L} = 0$ locus up in Figure Ib, thereby eliminating the intersections. Hopf bifurcations are associated with the existence of closed orbits in the neighborhood of a stationary state, when a parameter is in a (one-side) neighborhood of the bifurcation point at which some roots of the stationary state are purely imaginary. One can conclude as a corollary of Proposition 1 that Hopf bifurcations cannot occur in the present model.

The next proposition is an immediate consequence of Proposition 2.

Proposition 3. Suppose $r - p = \theta = 0$ and let (L^*, Q^*) be a saddle point stationary state of (6a)-(6b).

(3.1) When the initial employment in the manufacturing sector is given by L_0, $H(L^*,Q^*) \notin H(L_0,[0,\bar{Q}])$ is a sufficient condition for the nonexistence of an equilibrium path converging to (L^*,Q^*). That $H(L^*,Q^*)$

Figure Ib. Proposition 3 suggests that, when $r - p = \theta = 0$, the qualitative nature of the dynamics crucially depends on whether $H(S_H) > 0$, called Case A below, or $H(S_H) < 0$, called Case B. The perturbation method below demonstrates that the distinction between these two cases remains important even when $r - p = \theta \neq 0$. Although the results obtained are general, I also work with the following example as a demonstration.

Example 1. Let $g^A(\tau) = \alpha k^2 \tau + \beta k$, $g^M(\tau) = k$, $\Phi'(\tau) = 1$ on $[\tau^-, \tau^+] = [0,1]$, $q = 1$, and $h(L) = \{\alpha + \beta(\lambda+\lambda^{-1})\}L - \beta L^2$, where $k > \lambda > 1$, $k > \sqrt{3}$, and $\alpha > \beta(k - k^{-1})$.[15] Then, $T(Q) = (Q - \beta)/(\alpha k)$, and (6a)-(6b) becomes, by omitting the time subscripts,

$$\dot{L} = p[Z(Q) - L], \tag{7a}$$

$$\dot{Q} = r[Q - \{\alpha + \beta(\lambda+\lambda^{-1})\}L + \beta L^2], \tag{7b}$$

with $0 \leq Q \leq \bar{Q} = h(k)$ and $0 \leq L \leq \int_{\tau-}^{\tau+} g^M(\tau)d\Phi(\tau) = k$ and $Z(Q)$ is given by; $Z(Q) = 0$ for $Q \leq \beta$; $Z(Q) = (Q - \beta)/\alpha$ for $\beta \leq Q \leq \alpha k + \beta$; $Z(Q) = k$ for $\alpha k + \beta \leq Q$. Three stationary states are; $S_H = (L, Q) = (\lambda, \alpha\lambda + \beta)$, $S_L = (\lambda^{-1}, \alpha\lambda^{-1}+\beta)$, and $S_0 = (0, 0)$. The Hamiltonian is given by

$$H(L,Q) = QL - \int_0^Q Z(s)ds - \{\alpha + \beta(\lambda+\lambda^{-1})\}L^2/2 + \beta L^3/3, \tag{8}$$

where $\int_0^Q Z(s)ds = 0$ for $Q \leq \beta$; $= (Q - \beta)^2/(2\alpha)$ for $\beta \leq Q \leq \alpha k + \beta$; $= k\{Q - \beta - (\alpha k)/2\}$ for $\alpha k + \beta \leq Q$. Some algebra shows that $H(S_H) = H(\lambda, \alpha\lambda + \beta) = \beta\lambda(3 - \lambda^2)/6$, $H(S_L) = H(\lambda^{-1}, \alpha\lambda^{-1} + \beta) = \beta\lambda^{-1}(3 - \lambda^{-2})/6 > 0$, and $H(S_0) = H(0,0) = 0$. Thus,

Case A:	$H(S_H) > H(S_0) = 0$	if $1 < \lambda < \sqrt{3}$,
The Borderline Case:	$H(S_H) = H(S_0) = 0$	if $\lambda = \sqrt{3}$,

from the right and the other leads to S_0. The zero level stationary state can be reached no matter where the economy starts. On the other hand, the high level stationary state can be reached only if the initial manufacturing employment is sufficiently large. The threshold level of employment can be determined by the intersection of the homoclinic orbit and the $\dot{L} = 0$ locus. For Example 1 ($1 < \lambda < \sqrt{3}$), these conditions, $H(L,Q) = H(\lambda, \alpha\lambda + \beta)$ and $Z(Q) = (Q - \beta)/\alpha = L$, can be reduced, from (8), to $(L-\lambda)^2\{2\lambda L - (3-\lambda^2)\} = 0$. Therefore, if

(9a) $$(3 - \lambda^2)/2\lambda \le L_0 \le k,$$

both stationary states, S_0 and S_H, can be reached. Note that this threshold employment level is smaller than λ^{-1}, the employment level of S_L. Thus, unlike what the Marshallian adjustment process suggests, the economy can take off even when the economy is initially located to the left of S_L, if expectations of agents are coordinated to the equilibrium path converging to S_H. It should be also noted that starting from the right of S_L does not guarantee the convergence to S_H. No matter how high the initial employment is, the economy may de-industrialize due to the self-fulfilling pessimism. If

(9b) $$0 \le L_0 < (3 - \lambda^2)/2\lambda,$$

the equilibrium is unique and the economy converges to S_0. Thus, the economy will be trapped into the zero level stationary state. History helps to select the long run position in this rather limited sense. The economy cannot take off under the laissez-faire; the active government intervention may be necessary to escape from the trap. (The role of government policy is considered in Section IV.A.) This unique equilibrium path (the stable equilibrium path converging to S_H. The economy may industrialize due to the self-fulfilling optimism. From Proposition 3, the unique value of Q_0 that is consistent with the convergence to S_H is given by $H(0,Q_0) = H(S_H)$, or, for Example 1, $Q_0 = \beta + [\alpha\beta\lambda(\lambda^2-3)/3]^{1/2}$. The economy can take off only if expectations of agents are coordinated. The active government intervention would not be necessary. What does matter is the confidence or the optimism among the private sector.

The Borderline Case: $H(S_H) = 0$. Before proceeding to the case of nonzero θ, brief mention should be made of the borderline case. See Figure IIc. In this case, there are a pair of heteroclinic orbits. One of them is the unstable manifold of S_H and the stable manifold of S_0. The other is the stable manifold of S_H and the unstable manifold of S_0. Along the first the economy moves from S_H to S_0 and along the second it moves from S_0 to S_H. There is also a continuous family of closed orbits in the area bounded by the heteroclinic orbits. (Those familiar with mechanics would notice the similarity of Figure IIc with the phase diagram of the simple pendulum.) These paths are all perfect foresight paths. In addition, there is a perfect foresight path to S_H from the right. Thus, for Example 1 ($\lambda = \sqrt{3}$), the equilibrium is unique and the economy converges to S_H if

(11a) $$\sqrt{3} \le L_0 \le k,$$

and both S_H and S_0 can be reached if

(11b) $$0 \le L_0 < \sqrt{3}.$$

[One may notice the discontinuities at $\lambda = \sqrt{3}$ by comparing the conditions from (9a) to (11b). This is due to the fact that the Hamiltonian dynamical or the initial value of Q consistent with the convergence to S_H, even one specifies functional forms.

One of the important differences made by the bifurcation is that all perfect foresight paths are isolated if $\theta > 0$. Although the equilibrium is globally indeterminate in general, it is locally determinate (in the sense of Woodford [1984]). Thus, despite multiple equilibria, one can still perform a local comparative static exercise using this model. (This exercise is one of the subjects of Matsuyama [1988a].) The effect of an infinitesimal shock to the economy can be analyzed if one is willing to accept the somewhat ad hoc assumption that the economy jumps to the nearby perfect foresight path.[17]

The Case of $r - p = \theta < 0$. Perfect foresight paths are no longer isolated if the rate of time preference is negative ($-p < \theta < 0$). See Figures IVa-IVb. The divergence is negative, thus flows direct inward compared to the case of the Hamiltonian system. Again, the change in θ causes a bifurcation by breaking the homoclinic orbits. The negative divergence implies that the low level stationary state, S_L, becomes a sink and thus there exists a continuous family of perfect foresight paths converging to it. It is locally indeterminate. (The possibility of a locally indeterminate stationary state is noted in the context of transaction externalities by Howitt and McAfee [1988].) Furthermore, because of the local indeterminacy, one may show that there are stationary sunspot equilibria in the neighborhood of S_L. With a negative rate of time preference, the indeterminacy of equilibria is more severe. In particular, one cannot hope to perform comparative static exercises in this case. However, the implication on the history versus expectations remains the same. The initial employment alone matters only when it is sufficiently small in Case A or when it is sufficiently large in Case B.

economy and efficient allocations may be unique. However, one cannot in general decentralize efficient allocations through simple linear tax and subsidy policies. This is because what one can best hope for by using these policy tools is to make the first-order conditions right. In a nonconvex economy, one also has to take care of some global conditions in order to implement efficient allocations. Another way of stating this difficulty is that the standard Euler equation and the transversality condition are only necessary for the optimality, but not sufficient, when the value function is not concave.[18]

A model with multiple equilibria also poses a serious problem concerning the validity of policy analyses based on comparative statics methodology. Of course, one may be able to establish the local uniqueness of equilibrium, as in the case of $\theta > 0$. Then, all equilibria are isolated from each other and small changes in policy parameters produce small and unique changes in each of these equilibria. Comparative statics exercise may be performed by limiting's one's attention to the neighborhood of the original equilibrium. However, the validity of such a restriction would crucially depend on the purpose of the analysis.

An alternative way of addressing the role of government policy in a model with multiple equilibria is to see how the government can affect the set of equilibria. One may argue, in the spirit of the mechanism design literature, that a certain policy is desirable if, by affecting the set of equilibria, it could either create a "good" equilibrium or eliminate a "bad" one. In this section, this approach will be adopted.

The case for government intervention in the process of industrialization would crucially depend on whether Case A or Case B prevails under the

Theorem, the values of L and Q at S_H can be expressed as a function of ν, $\{L(\nu), Q(\nu)\}$. Assume $r - p = \theta = 0$. The Hamiltonian is given by

$$H(L,Q;\ \nu) \equiv QL - \int_0^Q Z(s)ds - \nu[\int_0^L h(z)qdz].$$

Note that the subsidy affects not only the stationary states, but also the Hamiltonian. Define $\Psi(\nu) = H(L(\nu),Q(\nu);\nu)$. By differentiating it,

$$\Psi'(\nu) = H_L L'(\nu) + H_Q Q'(\nu) + H_\nu$$

$$= H_\nu = - \int_0^{L(\nu)} h(z)qdz < 0,$$

where use has been made of $H_L = H_Q = 0$ at a stationary state. Now, suppose that the initial employment is equal to zero. From Proposition 3, an equilibrium path converging to $\{L(\nu), Q(\nu)\}$ exists if and only if $\Psi(\nu) \leq 0$. Thus, $\Psi'(\nu) < 0$ suggests that, if there exists $\nu^* > 1$ such that $\Psi(\nu^*) = 0$, then the state of pre-industrialization is a trap under the laissez-faire, and a sufficiently high subsidy rate $\nu - 1 \geq \nu^* - 1$ can make industrialization possible. This critical level of the subsidy rate can be determined once the loci of $\dot{Q} = 0$ and $\dot{L} = 0$ or their underlying functions are specified. One can also show that the same critical level of the subsidy rate can be applied for he elimination of the de-industrialization equilibrium when the economy starts at S_H. [This is because the bifurcation occurs at $\nu = \nu^*$ by breaking the heteroclinic orbits connecting S_H and S_0.]

The subsidy policy has the similar effect even for the case of $r \neq p$. This can be shown by using the perturbation method, although, with $r \neq p$, the critical level of the subsidy rate necessary to make industrialization possible when the economy starts at S_0 is different from what is necessary to eliminate the possibility of de-industrialization when the economy starts at of Comparative Advantage, enhanced by the presence of external economies of scale in manufacturing, which is responsible for the sudden creation of a take-off path.

This result seems roughly consistent with recent experiences of successful industrialization in some countries, such as Hong Kong, Singapore, and South Korea, and much less satisfactory performances in India, Indonesia and Thailand.[21] It explains why Belgium was the first to become the leading industrial country in continental Europe, while the Netherlands lagged behind and did not take off until the last decades of the nineteenth century.[22] It also explained why New England became the manufacturing center of the United States during the antebellum period.[23]

On the other hand, this result is in striking contrast with the conventional wisdom in the development literature, which asserted that "[e]veryone knows that the spectacular industrial revolution would not have been possible without the agricultural revolution that preceded it [Nurkse, 1953, p. 52]," and that "revolutionary changes in agricultural productivity are an essential condition for successful take-off [Rostow, 1960, p.8]."

According to this conventional view, which partly comes from the experiences of the Industrial Revolution in England, there are positive links between agricultural productivity and industrialization. First, rising productivity in food production makes it possible to feed the growing population in the industrial sector. With more food being produced with less labor, it releases labor for manufacturing employment. Second, high incomes generated in agriculture provide domestic demand for industrial products. Third, it increases the supply of domestic savings required to finance industrialization.

high θ is beyond our capability. In this section we retreat to a (piece-wise) linear model to speculate the effect of a large rate of time preference. Assuming the linearity is not without cost. However, I believe that the following example is illuminating enough.

<u>Example 2</u>. Let $g^A(\tau)$, $g^M(\tau)$, $\Phi(\tau)$, q be as in Example 1, and $h(L) = (\alpha + \beta\delta^{-1})L$, where $0 < \delta < k$. A low δ represents strong externalities. As before, $Z(Q) = 0$ if $0 \le Q \le \beta$; $= (Q - \beta)/\alpha$ if $\beta \le Q \le \alpha k + \beta$; $= k$ if $\alpha k + \beta \le Q$. The three stationary states are; $S_H = (L, Q) = (k, (\alpha + \beta\delta^{-1})k)$, $S_L = (\delta, \alpha\delta + \beta)$ and $S_0 = (0, 0)$. The dynamics are given by

$$(13a) \qquad \begin{bmatrix} \dot{L} \\ \dot{Q} \end{bmatrix} = \begin{bmatrix} -p & 0 \\ -r(\alpha+\beta\delta^{-1}) & r \end{bmatrix} \begin{bmatrix} L \\ Q \end{bmatrix} \qquad \text{for } 0 \le Q \le \beta,$$

$$(13b) \qquad \begin{bmatrix} \dot{L} \\ \dot{Q} \end{bmatrix} = \begin{bmatrix} -p & p/\alpha \\ -r(\alpha+\beta\delta^{-1}) & r \end{bmatrix} \begin{bmatrix} L - \delta \\ Q - \alpha\delta - \beta \end{bmatrix} \qquad \text{for } \beta \le Q \le \alpha k + \beta,$$

$$(13c) \qquad \begin{bmatrix} \dot{L} \\ \dot{Q} \end{bmatrix} = \begin{bmatrix} -p & 0 \\ -r(\alpha+\beta\delta^{-1}) & r \end{bmatrix} \begin{bmatrix} L - k \\ Q - (\alpha+\beta\delta^{-1})k \end{bmatrix} \qquad \text{for } \alpha k + \beta \le Q.$$

Both S_0 and S_H are saddle points and the slopes of the stable manifolds of these stationary points are equal to $r(\alpha + \beta\delta^{-1})/(r + p)$, thus less than the slope of the $\dot{Q} = 0$ locus. The matrix given in (13b) has two distinct, real eigenvalues if $\alpha\delta(r - p)^2 > 4\beta rp$ and a pair of imaginary eigenvalues if $\alpha\delta(r - p)^2 < 4\beta rp$. Using these information, one can show that there are three generic cases. Figure Va depicts the case where perfect foresight paths consist of a pair of intertwining, noncrossing spirals around S_L. This occurs when system (13b) has a pair of imaginary roots, or

Likewise, the case of Figure Vc occurs if

$$(14c) \qquad \beta(r + p)/r(\alpha\delta + \beta) > \mu/(\mu + p).$$

Since $\mu = (1/2)[(r - p) + \{(r - p)^2 - 4rp\beta/(\alpha\delta)\}^{1/2}]$ and $r - p = \theta$, the left hand side approaches $\beta/(\alpha\delta + \beta)$ and the right hand side approach one as θ goes to infinity. Therefore, if the rate of time preference is sufficiently high, (14c) holds and, as shown in Figure Vc, there is a unique perfect foresight path for any initial employment. If the economy starts right of S_L, the economy will always reach S_H, while, if it starts left of S_L, then S_0 will be reached. History plays a decisive role in determining the long run position of the economy. Hysteresis exists. The intuition should be clear. If the future is heavily discounted, agents will not care much about the future actions of other agents, and this will eliminate the power of self-fulfilling expectations.

One can also see that, given any positive rate of time preference, a sufficiently large δ, or small increasing returns eliminate the power of self-fulfilling expectations. (As δ goes to infinity, the left hand side of (14c) approaches zero, while the right hand side approaches to $\theta/(\theta + p)$.) This is because if externalities are small there will not be enough interdependence among decisions. The similar results are demonstrated in Krugman's [1989] linear model. However, as seen in Case A of the previous sections, the result that small increasing returns make history more important seem crucially rest on the linearity of the model.[25]

VI. CONCLUDING REMARKS

The problems of poverty and stagnation among underdeveloped countries can be significant in the presence of three factors: technical interrelatedness, scale economies, and irreversibilities. Note that the dynamic economy discussed above includes all these elements in it. David [1985, p.335] argued, "[i]ntuition suggests that if choices were made in a forward-looking way, rather than myopically..., the final outcome could be influenced by expectations." Hopefully, the present analysis will stimulate further research interest in the issue of history versus expectations.

To some economists, a model with multiple equilibria may be unsettling in that the complete specification of the fundamentals cannot predict the unique outcome and that one need to rely on some extrinsic factors such as expectations.[26] In particular, it poses a serious problem concerning the validity of comparative statics. Of course, this does not justify making an assumption, such as weak externalities, in order to rule out multiple equilibria. The mere fact that certain parameter values ensure the uniqueness of equilibrium does not mean that they are more realistic than those implying multiplicity. Nor should one conclude that a model with multiple equilibria cannot yield useful predictions. First, the fact that the multiplicity results in certain cases and not in others itself allows one to make useful predictions. Second, one may argue, in the spirit of the mechanism design literature, that a certain policy is desirable if it could eliminate a "bad" equilibrium or generate "good" one by affecting the set of equilibria. Hopefully, the two applications discussed in Section IV have shown the usefulness of this approach.

Finally, from the technical point of view, the analysis here may have demonstrated the importance of global analysis in nonlinear models. By restricting one's attention to the local dynamics, one often fails to notice

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increase.

6. I doubt that external economies of scale of this type, usually attributed to Marshall [1920, Book IV, Chapters X-XI], are large enough to explain the huge diversity of economic performances across countries; other forms of externalities may be equally significant. And there are some conceptual problems about external economies; see Helpman and Krugman [1985, Chapter 2]. The result that the multiplicity of equilibria in the model is entirely due to the Marshallian externalities should not be taken literally. This formulation is adopted here because external economies of scale are a convenient way of making increasing returns consistent with perfect competition, and thus amenable to dynamic analysis. Implications of internal economies of scale in development are discussed in Murphy, Shleifer, and Vishny [1989a, 1989b].

7. For the sake of simplicity, we do not attempt to provide the sharpest results and instead assume that all functions are sufficiently "smooth."

8. Implicit here is the assumptions that labor is immobile across regions and that economies of scale are internal to each region.

9. See Buttrick [1958] and Nelson [1956] for this kind of argument in the context of neoclassical growth model and Krugman [1981] in the context of trade and development.

10. To justify this interpretation, consider the world economy in which agents alive as of time t maximize $E_t \int_t^{\infty}[C^A+v(C^M)]e^{\theta(t-s)}ds = \int_t^{\infty}[C^A+v(C^M)]e^{(p+\theta)(t-s)}ds$, where C^i is consumption of good i and $\theta > -p$ is the rate of time preference. Then, the equilibrium discount rate is given by $r = \theta + p$.

18. For the analysis of optimal control and regulation in dynamic nonconvex economies, see, for example, Skiba [1978], Dechert and Nishimura [1983] and Brock and Dechert [1985].

19. This point is the main difference between my argument and Graham's [1923] argument despite their apparent similarity.

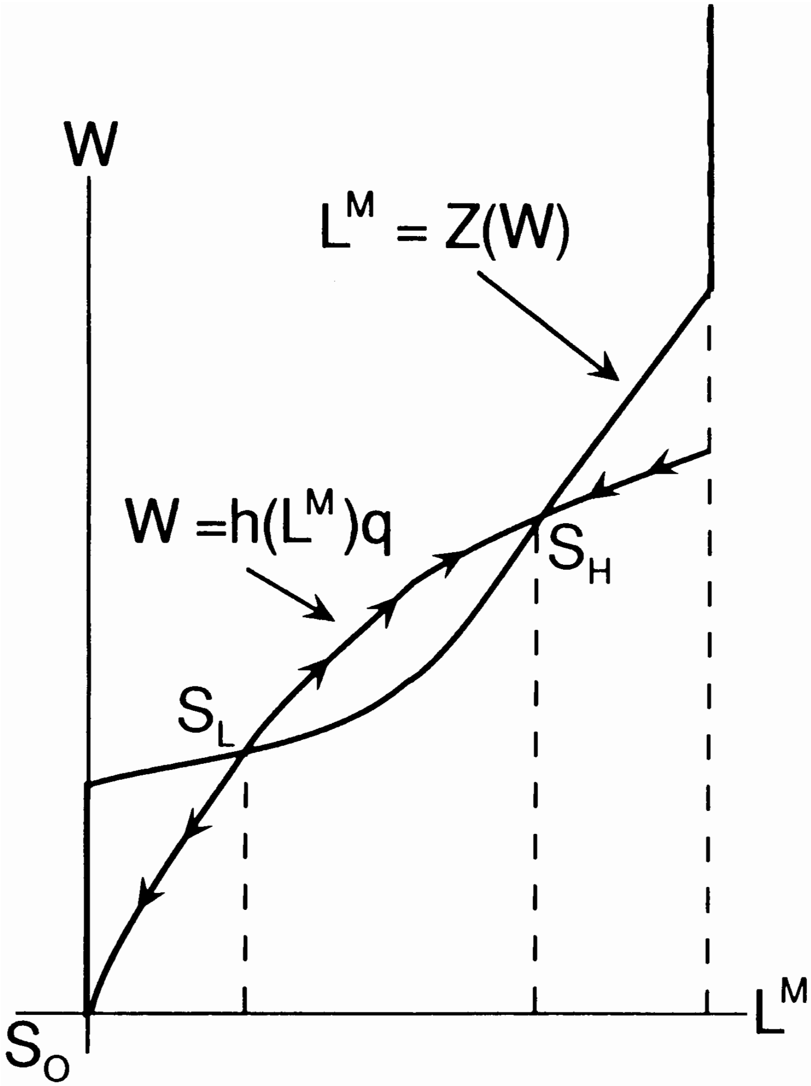
20. I would like to acknowledge that this implication of the model is first pointed out to me by Professor Uzawa.

21. One might think that these evidences are inconsistent with the model because larger economies are more likely to industrialize in the presence of increasing returns. This is not necessarily the case; If external economies arise due to some local informational exchanges or the specializied infrastructure, what matters is the density, not the absolute size. Increasing returns come from agglomeration and geographic concentration of activities. The model here has no implication about industrialization and the size of the economy.

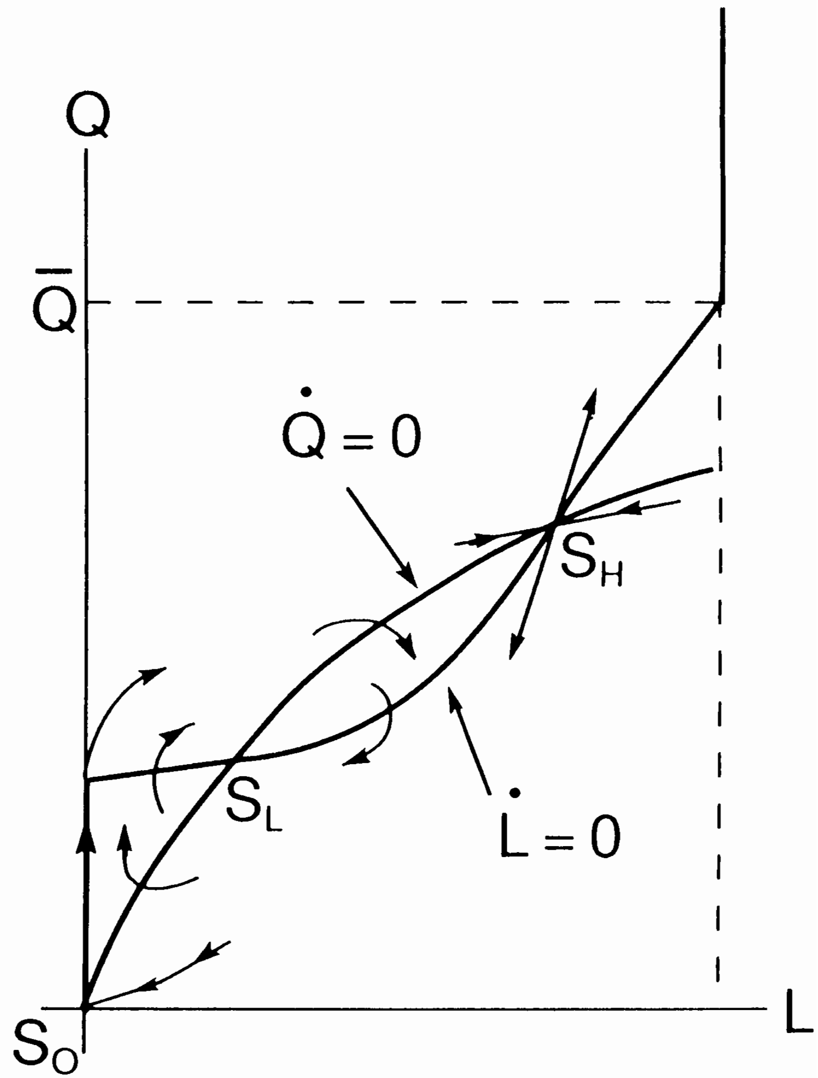
22. See Mokyr [1976] for a comparative study of industrialization in Belgium and the Netherlands.

23. Protection from British imports provided by Embargo of 1807, the subsequent war, and tariffs of 1816, 1824, and 1828 was probably important for industrialization in the United States, which is consistent with the result in Section IV.A. However, protection does not explain why industrialization, mostly in the cotton textile industry, started in New England, not in the South. See Field [1978] and Wright [1979].

24. The results of Murphy, Shleifer and Vishny [1989a, 1989b] crucially depend on the assumption that world trade is costly. They are careful enough to stress the importance of this assumption at length.

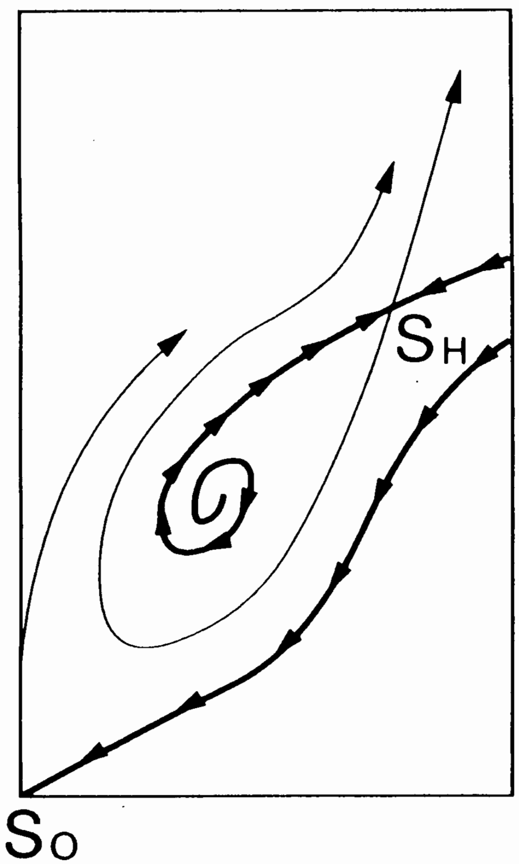


I a: The Static Economy

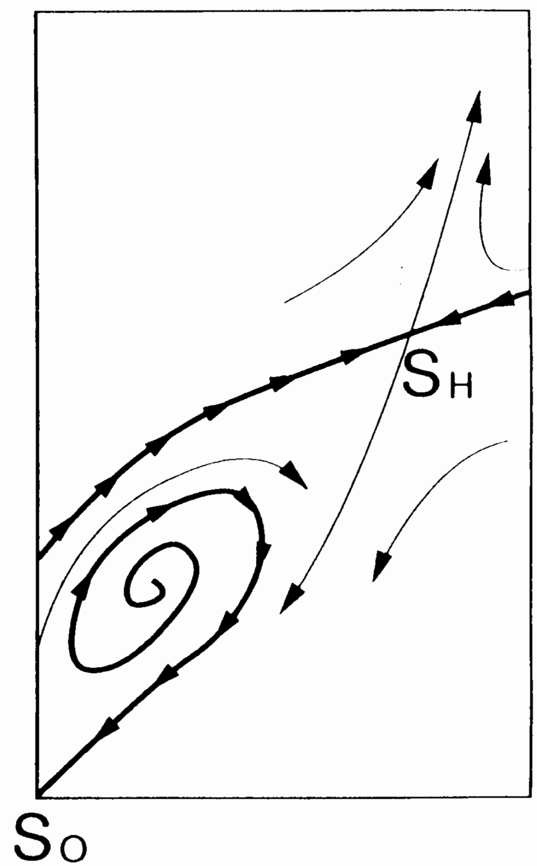


I b: The Dynamic Economy

Figures 1a–1b

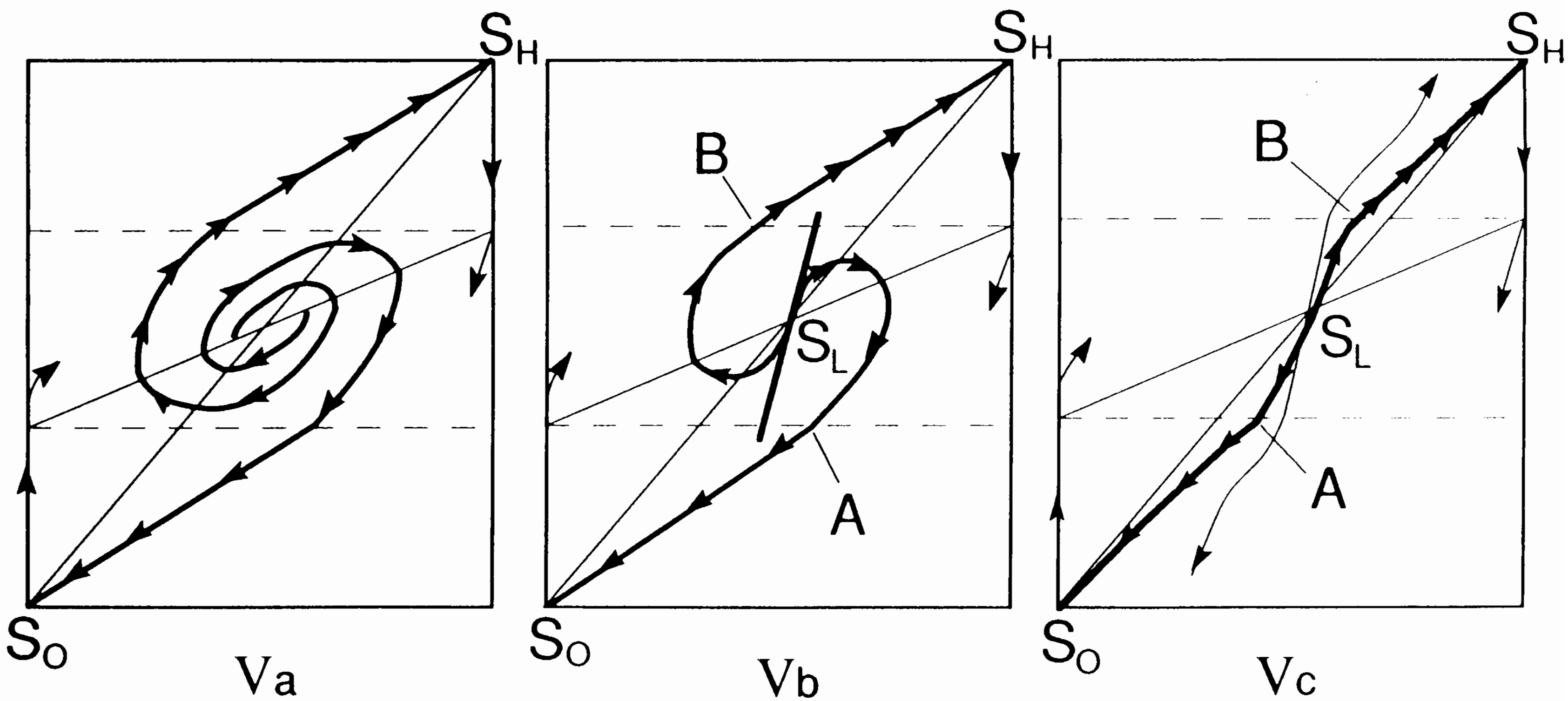


III a
Case A: $H(S_H) > O$



III b
Case B: $H(S_H) < O$

Figures IIIa–IIIb
Perturbations of the Hamiltonian Systems ($\theta > 0$).



Figures Va-Vc
The Case of a Large Rate of Time Preference