

Discussion Paper No. 876

**Managerial Incentives in an  
Entrepreneurial Stock Market Model\***

by

Richard E. Kihlstrom  
University of Pennsylvania

and

Steven A. Matthews  
Northwestern University

March 20, 1990

Forthcoming in the *Journal of Financial Intermediation* **1**, No. 1, 1990.

This is a revision of of the 1988 Discussion Paper No. 769 of the same title. This version has sharper results, but less discussion of examples.

---

\* Early versions of this paper have been presented under the title "Managerial Incentives in Publicly Traded Firms" in numerous seminars. We thank the participants at these seminars, two referees, and an editor for their comments. We are also grateful for the research support provided by the Economics Division and the Information Impact Program of the NSF.

*Running Head:* Managerial Incentives

*Proofs and correspondence should be sent to:* Steven A. Matthews  
Department of Economics  
Northwestern University  
Evanston, IL 60208  
(708) 491-3527

*Abstract:*

This paper addresses the First Theorem of Welfare Economics in a moral hazard environment. An entrepreneur sells equity in a firm which he supplies with an unobservable, costly input. How much equity he retains determines his incentives and is observed by investors. The investors have rational expectations which cause the equity price to increase in the amount of equity the entrepreneur retains. This gives the entrepreneur an incentive to retain equity and hence supply input. The entrepreneur may also be bound by an explicit incentive contract. In this framework, not all competitive equilibria are efficient, as defined relative to the moral hazard constraint. However, equilibria can be inefficient only if the entrepreneur's optimal input is nonunique or exhibits positive income effects.

*JEL Classification Numbers:* 021, 026, 512, 520





## 1. Introduction

A moral hazard problem arises when the shareholders of a firm cannot observe the conduct of its manager. Jensen and Meckling (1976) argue that this has implications for corporate capital structure, and informally discuss how it might affect financing choices and security prices. This paper formalizes the issue, shows how moral hazard affects stock market equilibria, and provides sufficient conditions for stock market equilibria to be efficient in an appropriate sense.

The agent in this paper is an entrepreneur who manages and, initially, owns a firm. A moral hazard problem arises because his managerial actions are unobservable to the outside investors in the firm. The entrepreneur's actions could, for example, correspond to how much of his physical or human capital that he invests in the firm, or the amount of effort that he supplies. For concreteness we take the latter interpretation, referring to the entrepreneur's "labor supply."

The outside investors purchase equity from the entrepreneur. Even though they do not observe his labor supply, they do observe the amount of equity he retains. The investors know that the entrepreneur's incentive to supply labor increases with his fractional share of the profit. Thus, the (rational expectations) value of the firm's equity increases in the entrepreneur's stake in the firm. The entrepreneur knows that by retaining more equity, he can realize a higher price for the equity that he sells, but at the cost of being induced to exert more effort and to bear more risk. In his portfolio decision he balances his reluctance to bear risk and exert effort against his desire to raise his firm's gross return and its market value.

An important issue is whether the entrepreneur has monopoly power in the equity market. Because investors infer that his labor supply is positively related to his share in the firm, the entrepreneur will be able to raise his firm's value by selling fewer shares. This way of affecting price resembles monopoly power, but in fact it is not. For example, if the investors are risk neutral, then the firm will be priced at its expected value and shares in riskless assets will be perfect substitutes for the shares of the firm. These perfect substitutes prevent the entrepreneur from having any

monopoly power, even though the amount of equity he decides to sell will still affect the equity price because it will affect the inferences of investors.

We define a Competitive Expectations Equilibrium (CEE) for two scenarios in which the entrepreneur arguably has no monopoly power. The first scenario, as described above, is that in which some investors are risk neutral. The second scenario is that in which the firm's technology satisfies a spanning condition, in which case the definition of a CEE relies on basically the same price-taking condition as is used in the shareholder unanimity literature (e.g., Ekern and Wilson (1974), Leland (1974), or Radner (1974)). A central result of that literature is that according to this price-taking definition, competitive stock market equilibria need not be (restricted) efficient unless spanning holds. Thus, spanning is a natural condition to assume in order to address the same efficiency question under the added complication of moral hazard. Nonetheless, as we shall discuss and emphasize, this standard price-taking assumption is less plausible in the presence of moral hazard.

An equilibrium allocation specifies the remunerations of all parties contingent on the firm's profit, as well as a labor supply choice for the entrepreneur. Our comparison set of feasible allocations is defined by two restrictions. First, since the entrepreneur's labor is not observable, a feasible allocation must specify a labor choice that is optimal for him given the way the allocation relates his compensation to profit. Second, a feasible allocation must specify that the entrepreneur's and the investors' incomes are linearly related to the firm's profit. This restriction, imposed originally in Diamond (1967), is appropriate here because the only assets traded are the firm's equity and a riskless asset. An equilibrium allocation must therefore specify linear risk-sharing rules, and the best that can be hoped for is efficiency in the set of linear allocations. We refer to equilibria that give rise to allocations which are Pareto optimal in this restricted set as "restricted efficient."

We show, by means of an example, that an economy can have many CEE which are Pareto-ranked and, hence, not restricted efficient. Whether an equilibrium is restricted efficient depends on how the firm is valued "off-the-equilibrium-path." One component of a CEE is a function which determines the value of the firm in terms of

the amount of equity retained by the entrepreneur. Theorems 5.1 and 5.2 show that a CEE is restricted efficient if this function is determined at each level of retained equity by maximizing the entrepreneur's proceeds from selling equity, subject to the constraint that the investors have rational expectations regarding his work incentives. For equity retentions less than one hundred percent, this condition requires that the value of the firm be the maximal value consistent with the investors' rational expectations.

Two kinds of conditions are shown to imply that every CEE is restricted efficient. First, every CEE is restricted efficient if, for each level of equity he might retain, the entrepreneur has a unique optimal labor choice which is independent of lump-sum transfers. This condition is not unduly restrictive. For example, the entrepreneur's labor supply will be unique if the production function is strictly concave in labor, and it will not exhibit income effects if the production function is additively separable in the labor and uncertainty terms, or if the entrepreneur has a constant absolute risk aversion utility function. Second, if the market does not allow short sales, so that the entrepreneur cannot retain more than one hundred percent of the firm's equity, then every CEE is restricted efficient if he works harder when his income is decreased, i.e., if his labor supply correspondence exhibits negative income effects.

It might appear that inefficiency occurs in this model because it does not allow the investors and entrepreneur to transfer income by trading securities other than equity, such as debt, or, more generally, by signing explicit incentive contracts which specify a payment to the entrepreneur for each realization of his firm's profit (e.g., Holmström, 1979, and Grossman and Hart, 1983). However, in the Appendix we show that a natural way of putting explicit contracts into the model does not affect the set of equilibrium allocations. Specifically, in the Appendix we allow the entrepreneur to specify an incentive contract in the firm's prospectus that is shown to the investors before the equity market opens. The investors can then base their inferences about the labor supply on the terms of this contract as well as on the amount of retained shares. Their expectations, as embodied in the value of the firm, can then counteract

the explicit incentives provided by the contract. Thus, adding incentive contracts in this way does not eliminate the restricted inefficient (or any other) equilibria.

A large related literature exists. Our formulation of competitive expectations equilibrium generalizes concepts employed in Marshall (1976) and Grossman and Hart (1982); it is related to the perfect information equilibrium concepts used in Diamond (1967) and Kihlstrom and Laffont (1982), and to the signaling equilibria in Leland and Pyle (1977). Early discussions of the efficiency of stock market equilibria in the presence of agency problems are in Stiglitz (1974), in a sharecropping setting, and in Jensen and Meckling (1976). Beck and Zorn (1982) is a formalization of the Jensen and Meckling discussion, distinguished from ours in part by not considering labor costs. Diamond and Verrechia (1982) and Ramakrishnan and Thakor (1984) contain financial models in which incentives are provided only by explicit contracts; if the managers in these models were allowed to also choose stock portfolios, the models would resemble that in our Appendix. Campbell and Kracaw (1985) and Marcus (1982) consider managerial incentives determined by portfolio choices and explicit contracts; the former paper does not address the efficiency issue, and the latter assumes exogenous contracts. Hughes (1988) contains a model similar to the example in Section 3. Finally, Prescott and Townsend (1984) consider the efficiency of competitive equilibria in the presence of moral hazard when agents can trade in incentive compatible lotteries.

The notion of a Competitive Expectations Equilibrium is formally presented in the next section. In Section 3 a simple CAPM illustrates the concepts. An example having a continuum of Pareto-ranked equilibria is in Section 4. The efficiency theorems are in Section 5. The Appendix considers explicit incentive contracts.

## **2. Competitive Expectations Equilibrium**

The model has two kinds of agents, one entrepreneur and  $n$  outside investors. The entrepreneur begins as the sole owner of a firm whose profit is  $g(L, \tilde{x})$ , where  $\tilde{x}$  is a random variable and  $L \geq 0$  is the amount of labor the entrepreneur supplies to the firm (other interpretations of  $L$  are possible, as mentioned in the introduction). The



function  $g$  increases in  $L$  for all  $x$  in the support of  $\tilde{x}$ . The investors do not observe  $L$ . The entrepreneur's opportunity cost of supplying an hour of labor to the firm is  $w > 0$ , the wage he earns in other employment.

We assume for most of the paper that the entrepreneur does not enter into an explicit incentive contract. Rather, the incentive for him to supply labor is determined entirely by the amount of the firm's equity that he retains, and by the proceeds from the equity that he sells. (Explicit incentive contracts are considered in the Appendix.) The entrepreneur's decision variables are  $L$ , his labor supply, and  $s$ , the fraction of the firm's equity that he retains.<sup>1</sup>

The investors observe  $s$  and use it to infer the level of  $L$ . The firm's market value,  $V(s)$ , is therefore a function of  $s$  and not of  $L$ . The entrepreneur's random income when he retains  $s$  and supplies  $L$  to the firm is

$$\tilde{I}_0 = sg(L, \tilde{x}) + (1-s)V(s) + (\bar{L} - L)w,$$

where  $\bar{L}$  is his stock of labor. We simplify notation by burying  $\bar{L}$  in the utility function. Hence, the entrepreneur chooses  $(s, L)$  to maximize his expected utility,

$$Eu_0(sg(L, \tilde{x}) - wL + (1-s)V(s)), \quad (2.1)$$

where  $u_0$  is strictly increasing and concave.

Investor  $i$  purchases a fraction  $t_i$  of the firm's equity. He chooses  $t_i$  to maximize his expected utility,

$$Eu_i(t_i g(L, \tilde{x}) - t_i V(s)), \quad (2.2)$$

where  $u_i$  is strictly increasing and concave.

An equilibrium consists not only of a specification of the entrepreneur's optimal  $(s, L)$  and the investors' optimal  $t_1, \dots, t_n$ , but also of the valuation function  $V(s)$ . This function embodies the beliefs of the investors about the entrepreneur's optimal labor supply in terms of the amount of equity that he retains. It also embodies the entrepreneur's beliefs about how the value of his firm depends on  $s$ ; he must have such beliefs in order for his maximization problem (2.1) to be well-defined. Implicit in our use of only one function  $V(\cdot)$  is the assumption that the entrepreneur and the

investors have the same out-of-equilibrium beliefs about how the value of the firm is related to  $s$ . Our first notion of an equilibrium embodies this assumption, the fact that equilibrium expectations should be correct on the equilibrium path (i.e. for  $s = s^*$ ), optimizing behavior, and balance.

DEFINITION 2.1: *An Expectations Equilibrium (EE) is an  $(n+3)$ -tuple*

$\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$  *such that*

- (i)  $(s^*, L^*)$  *maximizes (2.1) when  $V(\cdot) = V^*(\cdot)$ ;*
- (ii) *for each  $i$ ,  $t_i$  maximizes (2.2) when  $V(s) = V^*(s^*)$  and  $L = L^*$ ; and*
- (iii)  $\sum_{i=1}^n t_i^* = 1 - s^*$ . (2.3)

A Competitive Expectations Equilibrium is an EE in which the expectations embodied in  $V^*(\cdot)$  must be “competitive” and “rational” for both equilibrium and out-of-equilibrium values of  $s$ . To define these concepts, let

$$\lambda(s, y) \equiv \underset{L \geq 0}{\operatorname{argmax}} \operatorname{Eu}_0(\operatorname{sg}(L, \tilde{x}) - wL + y). \quad (2.4)$$

This correspondence gives the entrepreneur’s optimal labor supply when his share of the firm is  $s$  and his “lump-sum” income is  $y$ . In an EE his lump-sum income is  $(1-s^*)V^*(s^*)$ , so that

$$L^* \in \lambda(s^*, (1-s^*)V^*(s^*)). \quad (2.5)$$

Investors with rational expectations who observe the entrepreneur retain  $s^*$  will infer that the entrepreneur has an incentive to choose an  $L^*$  satisfying (2.5). If the investors observe a non-equilibrium  $s \neq s^*$ , they should (arguably) still believe that  $L$  is chosen optimally, i.e., they should expect the entrepreneur to choose some  $L$  in the set  $\lambda(s, (1-s)V^*(s))$ . This is what we mean by “rational” expectations.

It remains to define “competitive” rational expectations. As is clear from the unanimity literature (e.g., Ekern and Wilson (1974), Leland (1974), or Radner (1974)), a proper notion of competitive behavior in a stock market is problematic

unless investors are risk neutral or a spanning condition is satisfied. Thus, we define competitive rational expectations only for these scenarios.<sup>2</sup>

DEFINITION 2.2:

- (i) *In Scenario RN, at least one investor is risk neutral and able to buy and sell arbitrarily large amounts of shares in the firm.*
- (ii) *In Scenario SP, the following spanning condition holds: functions  $f(\cdot)$  and  $h(\cdot)$  exist such that for all  $(L,x)$ ,*

$$g(L,x) = f(L) + h(L)x. \quad (\text{SP})$$

In Scenario RN, competition causes the firm to be valued at its expected profit level; only this value is consistent with the risk neutral investors being in equilibrium. If they are rational, the risk neutral investors will expect the entrepreneur to choose an  $L$  in  $\lambda(s, (1-s)V^*(s))$ , causing them to bid the price of the firm up to  $Eg(L, \tilde{x})$ .

DEFINITION 2.3: *A Competitive Expectations Equilibrium (CEE) in Scenario RN is a vector  $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$  that is an EE and satisfies*

- (iv)  $V^*(s^*) = Eg(L^*, \tilde{x})$ , and
- $V^*(s) \in Eg(\lambda[s, (1-s)V^*(s)], \tilde{x})$  for all  $s$ .<sup>3</sup>

The following is a motivation for the CEE definition in Scenario SP. Given spanning (SP), owning the firm is equivalent to holding a portfolio composed of  $f(L)$  shares of a safe asset and  $h(L)$  shares of a risky asset. If profits were measured in dollars, a dollar invested in the safe asset would return one dollar with certainty, and a dollar invested in the risky asset would return  $\tilde{x}$  dollars. Competitive traders would take the prices of these assets as given. If the equilibrium price of the safe asset equaled one and the equilibrium price of the risky asset that pays  $\tilde{x}$  equaled  $p^*$ , then a standard arbitrage argument would imply that when rational investors inferred that  $L = L^*$  when they observed  $s = s^*$ , the firm would be valued at

$$V^*(s^*) = f(L^*) + p^*h(L^*). \quad (2.6)$$

Rational expectations require that if investors were to observe an  $s \neq s^*$ , they should believe that the entrepreneur is choosing an  $L$  in the set  $\lambda(s, (1-s)V^*(s))$ . Thus, competitive rational expectations should require that for  $s \neq s^*$ ,

$$V^*(s) \in f(\lambda[s, (1-s^*)V^*(s)]) + p^*h(\lambda[s, (1-s)V^*(s)]). \quad (2.7)$$

DEFINITION 2.4: A Competitive Expectations Equilibrium (CEE) in Scenario  $SP$  is a vector  $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$  that is an EE and for which a constant  $p^*$  exists such that

- (iv')  $V^*(s^*)$  satisfies (2.6), and  
 $V^*(s)$  satisfies (2.7) for  $s \neq s^*$ .

We note in passing that Definitions 2.3 and 2.4 are equivalent if both the spanning and the risk neutrality conditions hold, with  $p^* = E\tilde{x}$ .

The motivation for (iv') requires that traders take the price of a risky asset paying a return  $\tilde{x}$  as given. This is a standard price-taking assumption in securities market models in which spanning holds, see, e.g., Diamond (1967) or Radner (1974). The rationale for this assumption is most solid if such an asset is competitively traded. Unfortunately, in the present model two conceptual difficulties occur when there is a competitive market for any asset which has a return that is perfectly correlated with the firm's profit.

First, if an asset paying  $\tilde{x}$  exists and can be sold short, the entrepreneur can avoid the conflict between incentives and risk bearing by taking a short position in the asset and retaining all the shares of his firm. This strategy would induce him to provide the first-best labor supply, since he would be the sole owner of the firm. But being the sole owner would not subject him to excess risk, since he could optimally hedge by selling short the risky asset whose return is perfectly correlated with his firm's profit. Our model has little point if the moral hazard can be so circumvented. Thus, if a competitive market for a risky asset with return  $\tilde{x}$  does exist, we must assume that the entrepreneur is not able to take a short position in this asset.

(Similarly, the entrepreneur must not be able to take a long position in an asset whose return is negatively correlated with his firm's return.)

Second, if an asset paying  $\tilde{x}$  exists, then a joint observation of its return and the firm's return reveals the entrepreneur's labor choice. That is, the investors would know  $L$  if they could observe the realizations of both  $g(L, \tilde{x})$  and  $\tilde{x}$ . The entrepreneur could then be induced to supply the first-best labor level by entering into an explicit contract with the investors which relates his compensation to the realized returns of both his firm and the asset. With this contract in place, trade in the equity market would result in optimal risk sharing, and first-best optimality would be achieved. Thus, if such an asset exists, to prevent the moral hazard problem from being obviated we must assume that for some reason, the investors and the entrepreneur cannot sign a contract which relates his compensation to that asset's return.

Without these auxiliary assumptions, the moral hazard causes a problem only if the risk is idiosyncratic. Then, as an editor of this journal has remarked, Scenario RN might be the usual case. In a variety of competitive settings, investors can diversify away idiosyncratic risk (Connor, 1984). In these settings, if the risk that gives rise to the moral hazard problem is idiosyncratic, the firm's investors should treat this risk as though they were risk neutral. In this case Scenario RN should be assumed.

Nonetheless, in familiar models like the one in the next section, it would be restrictive to assume that investors appear risk neutral towards the firm. When they are risk averse, we know that even without moral hazard, stock market equilibria can be inefficient in the absence of spanning and price-taking. In order to have an appropriate benchmark from which to measure the extent to which moral hazard causes inefficiency in markets with risk averse investors, we must assume spanning and define a CEE as in Definition 2.4.

### **3. Moral Hazard in a Familiar Example**

In this section the set of special assumptions will ultimately include normally distributed risk and exponential utility. Thus, the example is a mean-variance model with one risky asset, chosen in order to help motivate the definition of a CEE in Scenario SP. We show that its unique CEE is restricted efficient.

The profit function in this example is additively separable:  $g(L, \tilde{x}) = f(L) + \tilde{x}$ , where  $f'(L) > 0$ ,  $f''(L) < 0$ , and  $E\tilde{x} = 0$ . Note that spanning (SP) holds, with  $h(L) \equiv 1$ . The entrepreneur's problem is to choose  $(s, L)$  to maximize

$$Eu_0(s[f(L) + \tilde{x}] - wL + (1-s)V(s)).$$

For a given  $s$ , his optimal labor choice is

$$L(s) \equiv \operatorname{argmax}_{L \geq 0} sf(L) - wL. \quad (3.1)$$

Lump-sum transfers do not affect the entrepreneur's labor supply.<sup>4</sup> Also, (3.1) implies that he is induced to work harder when he retains a greater share:  $L'(s) > 0$ .

An investor  $i$  with rational expectations who observes the entrepreneur retain share  $s$  will infer that the entrepreneur supplies  $L(s)$ . Thus,  $t_i$  maximizes

$$Eu_i(t_i[f(L(s)) + \tilde{x}] - t_i V(s)). \quad (3.2)$$

Now we assume that the risk  $\tilde{x}$  is normally distributed with variance  $\sigma^2$ , and that investor  $i$  has a constant absolute risk aversion measure  $c_i \geq 0$ , so that

$$u_i(I_i) = -e^{-c_i I_i}.$$

Hence, the expected utility (3.2) increases in the quantity  $t_i f(L(s)) - t_i V(s) - \frac{1}{2} c_i t_i^2 \sigma^2$ . The first order condition for choosing  $t_i$  to maximize this is

$$V(s) = f(L(s)) - c_i t_i \sigma^2. \quad (3.3)$$

Because  $V(s)$  does not directly depend on  $t_i$ ,  $c_i t_i$  must be the same for all  $i$ . Hence, because  $\sum t_i = 1-s$ ,

$$c_i t_i = c_m (1-s), \quad (3.4)$$

where

$$c_m \equiv \left( \sum_{i=1}^n (1/c_i) \right)^{-1} \quad (3.5)$$

is the market's aggregate risk aversion measure. From (3.3) and (3.4),

$$V(s) = f(L(s)) - c_m (1-s) \sigma^2. \quad (3.6)$$

Expression (3.6) is the inverse demand curve for the firm's equity. An increase in  $s$  raises the firm's price,  $V(s)$ , in two ways. First, it increases the firm's expected value,  $f(L(s))$ , through the incentive effect,  $f'(L(s))L'(s) > 0$ . Second, it decreases the supply of risk,  $(1-s)\sigma^2$ , and so causes the equilibrium price of risk,  $c_m(1-s)\sigma^2$ , to fall; this fall in the price of risk that must be paid to the investors in order for them to bear the risk raises the firm's price.

A monopolistic entrepreneur would realize that (3.6) is the true demand curve for equity. But a price-taking entrepreneur should take the price of risk as parametric, fixed at its equilibrium level  $c_m(1-s^*)\sigma^2$ . Accordingly, the CEE valuation function is

$$V^*(s) = f(L(s)) - c_m(1-s^*)\sigma^2. \quad (3.7)$$

This expression can be derived from the formal Definition 2.4 of a CEE by setting  $p^* = -c_m(1-s^*)\sigma^2$ , the negative equilibrium price of risk. An entrepreneur who believes his demand curve is given by (3.7) is generally "irrational" in the usual sense of being a price-taker and hence not accurately perceiving his demand curve.<sup>5</sup>

REMARK 3.1: Diamond (1967) and Kihlstrom and Laffont (1982) assume the price of risk is viewed parametrically, and show that then some equilibria are restricted efficient in the absence of moral hazard. But Stiglitz (1972) obtains inefficient equilibria because he does not assume managers take the price of risk as fixed. This strongly suggests that the price-taking assumption in (3.7) is necessary for restricted efficiency.

Now assume the entrepreneur has constant absolute risk aversion  $c_0$ . Then his expected utility is an increasing function of

$$\begin{aligned} sf(L(s)) + (1-s)V^*(s) - wL(s) - \frac{1}{2}c_0s^2\sigma^2 \\ = f(L(s)) - wL(s) - c_m(1-s)(1-s^*)\sigma^2 - \frac{1}{2}c_0s^2\sigma^2. \end{aligned}$$

The first order condition for maximizing this with respect to  $s$ , together with  $w = sf'(L(s))$  from (3.1), yields the following condition for the equilibrium  $s^*$ :

$$(1-s^*)f'(L(s^*))L'(s^*) = \sigma^2[c_0s^* - c_m(1-s^*)]. \quad (3.8)$$

Equations (3.4), (3.7), and (3.8) determine the CEE  $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ .

The CEE allocation is restricted efficient if it is Pareto optimal within the set of allocations in which (a) the labor supply is optimal for the entrepreneur given the way his income is determined, and (b) all agents' incomes are linearly related to the firm's profit. Restriction (a) reflects the moral hazard problem. Restriction (b) is imposed because simple stock market allocations must linearly relate incomes to profit.<sup>7</sup>

Rather than explicitly considering this class of feasible allocations, as we do in Section 5, we can take a simpler approach here. Because the investors have CARA utility functions, efficient risk-sharing among them requires that each  $t_i$  is determined from  $s$  by (3.4). Given this, the situation is as though there were just one investor, and that investor has CARA preferences with parameter  $c_m$ . A feasible allocation is then a triple  $(L, s, y)$ , where  $L = L(s)$  and  $y$  is a payment from the aggregate investor to the entrepreneur. The restricted efficient allocations are found by solving the following principal-agent problem:

$$\begin{aligned} \max_{s, y} \quad & (1-s)f(L(s)) - \frac{1}{2}c_m(1-s)^2\sigma^2 - y \\ \text{such that} \quad & sf(L(s)) - wL(s) - \frac{1}{2}c_0s^2\sigma^2 + y \geq \bar{u}_0. \end{aligned}$$

Using the constraint to eliminate  $y$ , and substituting  $w = sf'(L(s))$ , the first order condition for this problem is precisely the equation (3.8) that determines the CEE  $s^*$ . Thus, the CEE is restricted efficient, at least when  $f(L(s)) - wL(s)$  is concave. (By Corollary 5.2 below, this CEE is restricted efficient even without such concavity.)

Comparisons of the CEE to two benchmarks are of some interest.<sup>8</sup> First, if the entrepreneur acts monopolistically, i.e., correctly views (3.6) as the inverse demand function for the firm's shares, then he retains an inefficiently large share of the firm so as to raise its market value. He consequently supplies too much labor. Second, if  $L$  is observable, the value of the firm can depend directly on  $L$ . In this case the competitive equilibrium is efficient, with a lower  $s$  and a higher  $L$  than in the CEE.<sup>9</sup>



#### 4. An Example with Inefficient Equilibria

The investors in this section's example are risk neutral, so that it falls under Scenario RN. The example has a continuum of CEE which are Pareto-ranked, implying that the first welfare theorem fails. Restricted inefficient CEE exist because of income effects: the entrepreneur has little incentive to work if his income is low. In the CEE which are Pareto dominated, the entrepreneur's incentives are low because the proceeds from his equity sale are low. These proceeds are low because the value of the firm is low, which is consistent with rational expectations because the firm's expected profit actually is low because the entrepreneur supplies little labor.

The production function in this example is  $g(L, \tilde{x}) = f(L)\tilde{x}$ , where

$$f(L) = \begin{cases} L & \text{if } 0 \leq L \leq 1 \\ 1 & \text{if } 1 \leq L. \end{cases} \quad (4.1)$$

The random variable  $\tilde{x}$  is uniformly distributed on  $[0,2]$  and so has mean  $E\tilde{x} = 1$ . The wage rate is  $w = 1/4$ . The entrepreneur is risk averse with the utility function

$$u_0(I_0) = \begin{cases} \alpha I_0 & \text{if } I_0 < 0 \\ I_0 & \text{if } 0 \leq I_0, \end{cases} \quad (4.2)$$

where  $\alpha$ , his marginal utility at negative income levels, is much greater than one.<sup>10</sup>

Expected profit,  $Eg(L, \tilde{x}) - L/4$ , is maximized at  $L = 1$ . This is the first-best labor supply, since investors are risk neutral. But for  $y \geq 0$ , the entrepreneur's optimal  $L$  is

$$\lambda(s, y) = \begin{cases} 0 & \text{if } s < 1/4 \\ \min(1, 4y) & \text{if } s > 1/4. \end{cases} \quad (4.3)$$

To see this, note from (4.1) that the entrepreneur never sets  $L > 1$ . His income if  $L \leq 1$  is  $\tilde{I}_0 = y + L(s\tilde{x} - 1/4)$ , which is uniformly distributed on  $[y - L/4, y + L(2s - 1/4)]$  and has expected value  $y + L(s - 1/4)$ . If  $s < 1/4$ , then  $L = 0$  maximizes the entrepreneur's expected income without subjecting him to risk; this proves (4.3) if  $s < 1/4$ . If  $s > 1/4$ , the entrepreneur's expected income increases in  $L$  for  $L < 1$ . All income levels in the support of  $\tilde{I}_0$  are positive if  $L < 4y$ , so the entrepreneur is risk neutral with respect to  $L$

choices in this range. He therefore raises  $L$  to at least  $\min(1, 4y)$ . Raising  $L$  above  $4y$  lowers the left endpoint of the support of  $\tilde{I}_0$  below zero; this decreases expected utility because marginal utility is large at negative income levels. The optimal labor supply when  $s > 1/4$  is thus  $L = \min(1, 4y)$ , proving the bottom line of (4.3).

PROPOSITION: For any  $L_0 \in [0, 1]$ ,  $\langle L_0, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$  is a CEE in Scenario RN if  $s^* = 3/4$ ,  $\sum t_i^* = 1/4$ , and

$$V^*(s) = \begin{cases} L_0 & \text{if } s = 3/4 \\ 0 & \text{if otherwise.} \end{cases} \quad (4.4)$$

PROOF: Note that  $s^*, t_1^*, \dots, t_n^*$  add to one, and that the investors are indifferent as to the number of shares they hold because they are risk neutral and the price of the firm is equal to its expected profit:  $Eg(L_0, \tilde{x}) = L_0 = V^*(s^*)$ . This shows that (ii) and (iii) in Definition 2.1 are satisfied. It remains to show that the entrepreneur maximizes, which is condition (i) in Definition 2.1, and that the competitive expectations condition holds, which is (iv) in Definition 2.3. We do this in reverse order.

(iv) If the entrepreneur retains a fraction  $s \neq 3/4$ , the equity sale yields him only  $y = (1-s)V^*(s) = 0$ . In this case, from (4.3), he does not work and both expected profit and  $V^*(s)$  equal zero. This shows that (iv) is satisfied at  $s \neq 3/4$ . If  $s = 3/4 = s^*$ , the proceeds from his equity sale are  $y = (1-s^*)V^*(s^*) = L_0/4$ . From (4.3), he then works  $L = 4y = L_0$ . Expected profit is  $L_0 = V^*(3/4)$ , so that (iv) is also satisfied at  $s = s^*$ .

(i) As just noted, if  $s \neq 3/4$  the entrepreneur receives zero equity proceeds and supplies zero labor, which gives him zero expected utility. If  $s = 3/4$ , then  $L = L_0$  and  $y = L_0/4$ . In this case his expected utility is  $Eu_0(s^* \tilde{x} L_0 - L_0/4 + (1-s^*)L_0) = 3L_0/4$ , which is nonnegative. Thus,  $(L^*, s^*) = (L_0, 3/4)$  is optimal for the entrepreneur. ////

Each CEE in the class indexed by  $L_0 = V^*(3/4)$  gives the investors zero expected utility, and the entrepreneur prefers those with a higher  $L_0$ . The equilibrium with  $L_0 = 1$  is fully efficient. The firm's value is then  $V^*(s^*) = 1$ , which exceeds its value in any other CEE. This is not a coincidence, as is shown in the next section.

The inefficient equilibria arise because the entrepreneur's labor supply exhibits positive income effects. Suppose that in an inefficient CEE, the investors could give the entrepreneur an income transfer  $\Delta y > 0$ . This transfer, since it is positive, would necessarily make the entrepreneur better off. It would also induce him to work  $\Delta L = 4\Delta y$  additional hours and thereby increase expected profit by  $4\Delta y$ . This would make the investors as well off as before, since their expected portion of the additional profit is  $(1-s^*)(4\Delta y) = (1/4)(4\Delta y) = \Delta y$ .

The model could be modified to let the investors make such a transfer to the entrepreneur via an explicit incentive contract. However, the investors' expectations could then be indexed by the contract's terms so as to nullify its incentive properties; inefficient CEE would still exist. This is discussed in the Appendix.

## 5. Efficiency Results

The following definition gives the restrictions defining a restricted efficient allocation. An allocation is a vector  $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$ , where  $L$ ,  $s$ , and  $t_i$  are as before. The variables  $y$  and  $y_i$  are fixed payments to the entrepreneur and to investor  $i$ , respectively. An allocation specifies an income  $sg(L, \tilde{x}) - wL + y$  for the entrepreneur and  $t_i g(L, \tilde{x}) + y_i$  for investor  $i$ . This restricts the incomes to depend linearly on the firm's profit. Conditions (5.1) and (5.2) reflect the resource constraints, and condition (5.3) reflects the moral hazard problem.

DEFINITION 5.1: A feasible allocation is a vector  $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$  satisfying:

$$\sum_{i=1}^n t_i = 1 - s, \quad (5.1)$$

$$y + \sum_{i=1}^n y_i = 0, \quad (5.2)$$

$$L \in \lambda(s, y). \quad (5.3)$$

A restricted efficient allocation is Pareto optimal in the set of feasible allocations, and a CEE is restricted efficient if it gives rise to a restricted efficient allocation.

The following lemma indicates that restricted efficiency cannot require the entrepreneur to take a long position in his firm.

LEMMA 5.1: *For each feasible allocation in which the entrepreneur's share of the firm exceeds one, another feasible allocation in which his share equals one is weakly preferred by the entrepreneur and the investors.*<sup>12</sup>

PROOF: Let  $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$  be feasible, with  $s > 1$ . Define  $y^*$  by

$$\max_{\hat{L} \geq 0} Eu_0(g(\hat{L}, \tilde{x}) - w\hat{L} + y^*) = Eu_0(sg(L, \tilde{x}) - wL + y). \quad (5.4)$$

Define the certainty equivalent  $z_i$  for investor  $i$  by

$$u_i(z_i) = Eu_i(t_i g(L, \tilde{x}) - y_i). \quad (5.5)$$

We need to show  $\sum z_i \leq -y^*$ , for then the allocation  $\langle L^*, 1, 0, \dots, 0, y^*, y_1^*, \dots, y_n^* \rangle$  is preferred by all to  $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$ , where  $L^* \in \lambda(1, y^*)$  and  $y_i^* = z_i - (y^* + \sum z_i)/n$ .

Let  $\hat{y} = y - (1-s)Eg(L, \tilde{x})$ , and define

$$\alpha(\hat{s}) = Eu_0[\hat{s}g(L, \tilde{x}) - wL + (1-\hat{s})Eg(L, \tilde{x}) + \hat{y}].$$

For  $\hat{s} > 0$ , because  $u_0$  is concave,

$$\begin{aligned} \alpha'(\hat{s}) &= E\{u_0'(\cdot)[g(L, \tilde{x}) - Eg(L, \tilde{x})]\} \\ &= Cov(u_0'(\cdot), g(L, \tilde{x})) \leq 0. \end{aligned}$$

This shows that  $\alpha(1) \geq \alpha(s)$ . Hence,

$$\begin{aligned} \max_{\hat{L} \geq 0} Eu_0(g(\hat{L}, \tilde{x}) - w\hat{L} + \hat{y}) &\geq Eu_0(g(L, \tilde{x}) - wL + \hat{y}) \\ &= \alpha(1) \geq \alpha(s) = Eu_0(sg(L, \tilde{x}) - wL + y). \end{aligned}$$

This and (5.4) imply  $\hat{y} \geq y^*$ . As  $u_i$  is concave, (5.5) implies  $z_i \leq t_i Eg(L, \tilde{x}) - y_i$ . Sum this, using (5.1) and (5.2), to obtain  $\sum z_i \leq (1-s)Eg(L, \tilde{x}) - y = -\hat{y} \leq -y^*$ . ////

Our first theorem establishes a sufficient condition for a CEE in Scenario RN to be restricted efficient. It requires that if the entrepreneur sells a fraction  $s < 1$  of the

firm's equity, then his proceeds from the sale,  $(1-s)V^*(s)$ , must be the maximal amount consistent with the investors' rational expectations. The formal condition is

$$V^*(s) = \max\{v \mid v \leq \text{Eg}(L, \tilde{x}) \text{ for some } L \in \lambda(s, (1-s)v)\}. \quad (5.6)$$

From the competitive expectations condition, we know that

$$V^*(s) = \text{Eg}(L, \tilde{x}) \text{ for some } L \in \lambda(s, (1-s)V^*(s)). \quad (5.7)$$

Thus,  $V^*(s)$  is in the set on the right side of (5.6). Condition (5.6) strengthens (5.7) by requiring  $V^*(s)$  to be the maximal value satisfying (5.7). Furthermore, (5.6) requires that a greater value for the firm must exceed any rationally expected profit: if  $v > V^*(s)$ , then  $v > \text{Eg}(L, \tilde{x})$  for all  $L \in \lambda(s, (1-s)v)$ . This insures that a transfer from the investors to the entrepreneur cannot induce him to increase the firm's profits enough to compensate them for the transfer.

**THEOREM 5.1:** *In Scenario RN, a CEE is restricted efficient if its value function  $V^*(\cdot)$  satisfies (5.6) for all  $s < 1$ .*

**PROOF:** Suppose  $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$  is a feasible allocation which Pareto dominates the allocation generated by a CEE  $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$  satisfying (5.6). By Lemma 5.1, we can assume  $s \leq 1$ . By Definition 2.1(i),

$$\begin{aligned} \text{Eu}_0(\text{sg}(L, \tilde{x}) - wL + y) &\geq \text{Eu}_0[s^*g(L^*, \tilde{x}) - wL^* + (1-s^*)V^*(s^*)] \\ &\geq \text{Eu}_0[\text{sg}(L, \tilde{x}) - wL + (1-s)V^*(s)]. \end{aligned}$$

Consequently,

$$y \geq (1-s)V^*(s). \quad (5.8)$$

By Definition 2.1(ii), for any  $\bar{t}_i$ ,

$$\begin{aligned} \text{Eu}_i(t_i g(L, \tilde{x}) + y_i) &\geq \text{Eu}_i(t_i^* g(L^*, \tilde{x}) - t_i^* V^*(s^*)) \\ &\geq \text{Eu}_i(\bar{t}_i g(L^*, \tilde{x}) - \bar{t}_i V^*(s^*)). \end{aligned} \quad (5.9)$$

Choose  $\bar{t}_i = 0$  to obtain  $\text{Eu}_i(t_i g(L, \tilde{x}) + y_i) \geq u_i(0)$ . This and the concavity of  $u_i$  imply  $y_i \geq -t_i \text{Eg}(L, \tilde{x})$ . Sum this over  $i$  and use (5.1)–(5.3) to obtain

$$y \leq (1-s)\text{Eg}(L, \tilde{x}). \quad (5.10)$$

If  $s = 1$ , then (5.10) immediately implies

$$y \leq (1-s)V^*(s). \quad (5.11)$$

If  $s < 1$ , then (5.11) holds because of (5.6), (5.10), and the fact that  $L \in \lambda(s,y)$ .

Because the entrepreneur or an investor strictly prefers  $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$  to the CEE, either (5.8) or (5.11) is a strict inequality, a contradiction. ///

The next theorem is the analog of Theorem 5.1. It shows that a CEE in Scenario SP is restricted efficient if its value function satisfies, for all  $s < 1$ ,

$$V^*(s) = \max\{v \mid v \leq f(L) + p^*h(L) \text{ for some } L \in \lambda(s, (1-s)v)\}. \quad (5.12)$$

The interpretation of (5.12) is similar to that of (5.6), but weaker because the price of risk,  $p^*$ , is an equilibrium variable. For example, (5.12) requires  $V^*(s)$  to be larger than the value assigned to the firm for this  $s$  in any other CEE *with the same*  $p^*$ .

**THEOREM 5.2:** *In Scenario SP, a CEE is restricted efficient if its value function  $V^*(\cdot)$  and its risk price  $p^*$  satisfy (5.12) for all  $s < 1$ .*

**PROOF:** Suppose  $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$  is a feasible allocation which Pareto dominates a CEE  $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$  satisfying (5.12). By Lemma 5.1, we can assume  $s \leq 1$ . By the arguments used in Theorem 5.1, inequalities (5.8) and (5.9) hold. Using (SP) and setting  $\bar{t}_i = t_i h(L) / h(L^*)$  in (5.9), we obtain

$$Eu_i(t_i[f(L) + h(L)\tilde{x}] + y_i) \geq Eu_i([t_i h(L) / h(L^*)][f(L^*) + h(L^*)\tilde{x} - V^*(s^*)]). \quad (5.13)$$

Hence,

$$t_i f(L) + y_i \geq [t_i h(L) / h(L^*)][f(L^*) - V^*(s^*)].$$

Sum these inequalities over  $i$  and use (5.1) and (5.2) to obtain

$$(1-s)f(L) - y \geq (1-s)[h(L) / h(L^*)][f(L^*) - V^*(s^*)]. \quad (5.14)$$

By (2.6),  $f(L^*) - V^*(s^*) = -p^*h(L^*)$ . Substitute this into (5.14) to get

$$y \leq (1-s)[f(L) + p^*h(L)]. \quad (5.15)$$

This implies (5.11) if  $s = 1$ . If  $s < 1$ , then (5.11) follows from (5.12), (5.15), and the fact that  $L \in \lambda(s,y)$ . As before, either (5.8) or (5.11) is strict, a contradiction. ///

The following two corollaries deal with the case in which the entrepreneur's effort level is not influenced by income transfers, i.e., the case in which  $\lambda(s,y)$  does not depend on  $y$ . Suppose, in addition, that for each  $s$  the entrepreneur has a unique labor choice, so that  $\lambda(s,y)$  is a function and depends only on  $s$ . It is then immediate that any  $V^*(\cdot)$  satisfying (5.7) also satisfies (5.6). Since a CEE value function satisfies (5.7), this proves the following corollary for Scenario RN; the proof for Scenario SP is similar.

**COROLLARY 5.1:** *Under Scenario RN or SP, every CEE is restricted efficient if  $\lambda(s,y)$  is independent of  $y$  and single-valued.*

There are no income effects if the profit function satisfies a special case of spanning (SP) that we can refer to as *additive uncertainty*:

$$g(L,x) \text{ satisfies additive uncertainty if } g(L,x) = f(L) + x. \quad (\text{AU})$$

Under this assumption,  $\lambda(s,y)$  is independent of  $y$  because

$$\lambda(s,y) = \operatorname{argmax}_{L \geq 0} [sf(L) - wL]. \quad (5.16)$$

Since the profit function in the example of Section 4 satisfies (AU), income effects would be zero there even if the entrepreneur did not have a CARA utility function. The next corollary demonstrates that the restricted efficiency of the CEE in that example can be attributed either to the profit function exhibiting additive uncertainty, or to the entrepreneur's utility function exhibiting CARA.

**COROLLARY 5.2:** *Under Scenario RN or SP, any CEE is restricted efficient if either*

- (i) *the profit function satisfies (AU) and  $f$  is strictly concave, or*
- (ii) *the entrepreneur's utility function is CARA and  $g(L,\cdot)$  is strictly concave in  $L$ .*

**PROOF:** Part (i) follows from Corollary 5.1 and (5.16). Part (ii) follows from the well-known result that CARA utility functions exhibit no income effects. ///

Our final corollary deals with the case in which income effects are negative, so that the entrepreneur works less hard if income is given to him. In this case, transferring income from the investors to the entrepreneur is sure to hurt the investors, since the transfer will cause the profits of the firm to decrease. On the other hand, a transfer in the opposite direction must hurt the entrepreneur. This suggests that every CEE is restricted efficient if income effects are negative:

*COROLLARY 5.3: Under Scenario RN or SP, every CEE is restricted efficient if  $\lambda(s,y)$  decreases in  $y$ .*<sup>13</sup>

*PROOF:* We give the proof for Scenario RN; that for Scenario SP is similar. Let  $V^*(\cdot)$  be a CEE value function. Assume (5.6) fails at some  $s < 1$ . Since (5.7) must hold,  $V^*(s)$  is in the set on the right side of (5.6). Hence,  $v$  and  $L$  exist such that

$$V^*(s) < v \leq \text{Eg}(L, \tilde{x}) \quad \text{and} \quad L \in \lambda(s, (1-s)v). \quad (5.17)$$

From (5.7), we know that  $\hat{L} \in \lambda(s, (1-s)V^*(s))$  exists such that  $V^*(s) = \text{Eg}(\hat{L}, \tilde{x})$ . This and (5.17) imply  $\text{Eg}(\hat{L}, \tilde{x}) < \text{Eg}(L, \tilde{x})$ . Thus,  $\hat{L} < L$ , i.e., the entrepreneur works less hard when paid  $(1-s)V^*(s)$  than when paid  $(1-s)v$ , contrary to  $\lambda(s,y)$  decreasing in  $y$ . Hence, (5.6) holds for  $s < 1$ , and the CEE with  $V^*(\cdot)$  is restricted efficient by Theorem 5.1. ////



## Appendix: Explicit Incentive Contracts

We show here that each CEE allocation remains an equilibrium allocation even when the entrepreneur and investors are allowed to sign an explicit incentive contract. Thus, the inclusion of explicit contracts or, *a posteriori*, securities other than equity will not eliminate the restricted inefficient or any other CEE. We make the argument only for Scenario RN; that for Scenario SP is similar.

We start with a caveat. The model here allows the entrepreneur to specify an incentive contract in the firm's public prospectus. This fixes the contract for the remainder of time, and the investors base their expectations about the labor supply on its terms. However, as an editor has remarked, the results may be different if the contract can be renegotiated after the equity market closes. This is an interesting question for future study.

In order to make comparisons with the necessarily linear allocations achieved by the equity market, we restrict attention to linear incentive contracts. Such a contract gives the entrepreneur a compensation  $\tilde{y} = s_0\tilde{\pi} + y_0$  when the firm's profits are  $\tilde{\pi}$ ; it is characterized by a pair  $(s_0, y_0)$ . The entrepreneur chooses  $s_0$ ,  $y_0$ ,  $s$ , and  $L$ . The value of the firm depends on all observables:  $V(s, s_0, y_0)$ . The firm's profit net of the entrepreneur's compensation is  $(1-s_0)g(L, \tilde{x}) - y_0$ . The entrepreneur's income is

$$\begin{aligned}\tilde{I}_0 &= s[(1-s_0)g(L, \tilde{x}) - y_0] + s_0g(L, \tilde{x}) + y_0 + (1-s)V(s, s_0, y_0) - wL \\ &= [s + (1-s)s_0]g(L, \tilde{x}) + (1-s)[V(s, s_0, y_0) + y_0] - wL.\end{aligned}\tag{A.1}$$

The income of investor  $i$  when he purchases  $t_i$  shares of the firm is

$$\tilde{I}_i = t_i[(1-s_0)g(L, \tilde{x}) - y_0] - t_iV(s, s_0, y_0).\tag{A.2}$$

We now define an EE and a CEE when incentive contracts are allowed, which we refer to as an  $EE^*$  and a  $CEE^*$ . An  $EE^*$  is a vector  $\langle L^*, V^*(\cdot, \cdot, \cdot), s_0^*, y_0^*, s^*, t_1^*, \dots, t_n^* \rangle$  defined as in Definition 2.1, but with the added equilibrium variables  $s_0^*$  and  $y_0^*$ , with  $V^*(\cdot, \cdot, \cdot)$  replacing  $V^*(\cdot)$ , and with the entrepreneur choosing  $(s_0, y_0, s, L)$  and the investors choosing  $t_i$  to maximize their respective expected utilities from the incomes shown in (A.1) and (A.2). A  $CEE^*$  is defined for Scenario RN just as in Definition

2.3, but again with the added variables  $s_0^*$  and  $y_0^*$  and  $V^*(\cdot, \cdot, \cdot)$  replacing  $V^*(\cdot)$ , and, since only the net profit  $(1-s_0)g(L, \tilde{x}) - y_0$  is distributed, with (iv) replaced by

$$\begin{aligned} V^*(s^*, s_0^*, y_0^*) &= (1-s_0^*)Eg(L^*, \tilde{x}) - y_0^*, \text{ and} \\ V^*(s, s_0, y_0) &= (1-s_0)Eg(L, \tilde{x}) - y_0 \text{ for some} \\ L &\in \lambda([s + (1-s)s_0], (1-s)[V^*(s, s_0, y_0) + y_0]), \text{ for all } (s, s_0, y_0). \end{aligned} \quad (\text{A.3})$$

Now, let  $\langle \hat{L}, \hat{V}(\cdot), \hat{s}, \hat{t}_1, \dots, \hat{t}_n \rangle$  be a CEE for Scenario RN. Define

$$V^*(s, s_0, y_0) \equiv (1-s_0)\hat{V}(s+(1-s)s_0) - y_0. \quad (\text{A.4})$$

This function is the equilibrium expectations function in a CEE\* that gives the same allocation as the CEE  $\langle \hat{L}, \hat{V}(\cdot), \hat{s}, \hat{t}_1, \dots, \hat{t}_n \rangle$ . In this CEE\*, the incentives faced by the entrepreneur are the same regardless of how the explicit incentive contract is chosen, because the investor expectations embodied in (A.4) take into account and counteract the explicit incentives provided by  $(s_0, y_0)$ .

The CEE\* is defined as follows:  $L^* \equiv \hat{L}$ ,  $V^*$  as in (A.4),  $s_0^*$  arbitrary except that  $s_0^* \neq 1$ ,  $y_0^*$  arbitrary,  $s^* \equiv (\hat{s} - s_0^*) / (1 - s_0^*)$ , and  $t_i^* \equiv \hat{t}_i / (1 - s_0^*)$ . We show that this is a CEE\* in four steps.

(i) To show that the entrepreneur maximizes, substitute (A.4) into (A.1) to get

$$\tilde{I}_0 = \bar{s}g(L, \tilde{x}) + (1-\bar{s})\hat{V}(\bar{s}) - wL,$$

where  $\bar{s} = s + (1-s)s_0$ . The entrepreneur's expected utility from this income depends only on  $\bar{s}$  and  $L$ ; by the definition of a CEE, it is maximized at  $L = \hat{L}$  and  $\bar{s} = \hat{s}$ . Thus  $(s_0^*, y_0^*, s^*, L^*)$  maximizes the entrepreneur's expected utility, since then  $L = L^* = \hat{L}$  and  $\bar{s} = s^* + (1-s^*)s_0^* = \hat{s}$ .

(ii) To show that investor i maximizes, note that by substituting from (A.4) into (A.2), and evaluating the result at  $L = L^*$  and  $s = s^*$ , we obtain

$$\tilde{I}_i = t_i(1-s_0^*)(g(L^*, \tilde{x}) - \hat{V}(\hat{s})).$$

By (iv) in Definition 2.3,  $Eg(\hat{L}, \tilde{x}) = \hat{V}(\hat{s})$ . Hence, since  $L^* = \hat{L}$ , the expected income of investor i is zero regardless of how he chooses  $t_i$ . For the risk neutral investors, any

$t_i$  is optimal,  $t_i^*$  in particular. For the risk averse investors only  $t_i = 0$  is optimal; but then  $\hat{t}_i = 0$  by the definition of a CEE, so that  $t_i^*$  is optimal because  $t_i^* = \hat{t}_i / (1-s_0^*) = 0$ .

(iii) The supply and demand of equity are equal because:

$$\sum_{i=1}^n t_i^* = \sum_{i=1}^n \hat{t}_i (1-s_0^*)^{-1} = (1-\hat{s})(1-s_0^*)^{-1} = 1-s^*.$$

(iv) To verify the competitive expectations condition (A.3), note first that  $\hat{V}(\hat{s}) = \text{Eg}(\hat{L}, \tilde{x})$  and  $L^* = \hat{L}$  imply

$$\begin{aligned} V^*(s^*, s_0^*, y_0^*) &= (1-s_0^*) \hat{V}(s^* + (1-s^*)s_0^*) - y_0^* \\ &= (1-s_0^*) \hat{V}(\hat{s}) - y_0^* \\ &= (1-s_0^*) \text{Eg}(L^*, \tilde{x}) - y_0^*. \end{aligned}$$

This establishes the first line of (A.3). To establish the rest of (A.3), let  $(s, s_0, y_0)$  be given and let  $\bar{s} = s + (1-s)s_0$ . By (iv) in Definition 2.3, an  $L \in \lambda(\bar{s}, (1-\bar{s})\hat{V}(\bar{s}))$  exists such that  $\hat{V}(\bar{s}) = \text{Eg}(L, \tilde{x})$ . Hence, using (A.4),

$$\begin{aligned} V^*(s, s_0, y_0) &= (1-s_0) \hat{V}(\bar{s}) - y_0 \\ &= (1-s_0) \text{Eg}(L, \tilde{x}) - y_0. \end{aligned}$$

This finishes the proof of (A.3), since

$$\begin{aligned} L &\in \lambda(\bar{s}, (1-\bar{s})\hat{V}(\bar{s})) \\ &= \lambda(\bar{s}, (1-\bar{s}) \left( \frac{V^*(s, s_0, y_0) + y_0}{1-s_0} \right)) \\ &= \lambda([s+(1-s)s_0], (1-s)[V^*(s, s_0, y_0)+y_0]). \quad //// \end{aligned}$$

It is a simple exercise to verify that this CEE and CEE\* result in the same allocation  $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$ . (Recall the definition of feasible allocations in Definition 5.1.) This allocation is  $L = \hat{L}$ ,  $s = \hat{s}$ ,  $t_i = \hat{t}_i$ ,  $y = (1-\hat{s})\hat{V}(\hat{s})$ , and  $y_i = -\hat{t}_i \hat{V}(\hat{s})$ .

## References

- BECK, P., and ZORN, T. (1982). Managerial incentives in a stock market economy, *J. Finance* **37**, 1151-67.
- CAMPBELL, T. and KRACAW, W. (1985). The market for managerial labor services and capital market equilibrium, *J. Financial and Quantitative Analysis* **20**, 277-97.
- CONNOR, G. (1984). A unified beta pricing theory, *J. Econ. Theory* **34**, 13-31.
- DIAMOND, D. and VERRECHIA, R. (1982). Optimal managerial contracts and equilibrium security prices, *J. Finance* **37**, 275-88.
- DIAMOND, P. (1967). The role of a stock market in a general equilibrium model with technological uncertainty, *Amer. Econ. Rev.* **57**, 759-76.
- EKERN, S. and WILSON, R. (1974). On the theory of the firm in an economy with incomplete markets, *Bell J. Econ. and Mgmt. Sci.* **5**, 171-80.
- GROSSMAN, S. and HART, O. (1983). An analysis of the principal-agent problem, *Econometrica* **51**, 7-45.
- GROSSMAN, S. and HART, O. (1982). Corporate financial structure and managerial incentives, in J. McCall (Editor), "The Economics of Information and Uncertainty," Chicago: University of Chicago Press, 107-37.
- HOLMSTRÖM, B. (1979). Moral hazard and observability, *Bell J. Econ.* **10**, 74-91.
- HOLMSTRÖM, B., and MILGROM, P. (1987). Aggregation and linearity in the provision of intertemporal incentives, *Econometrica* **55**, 303-328.
- HUGHES, P. (1988). Risk sharing and valuation under moral hazard, in "Economic Analysis of Information and Contracts," (G.A. Feltham, A.H. Amershi, and W.T. Ziemba, Editors) Boston: Kluwer Academic Publishers, 247-268.
- JENSEN, M. and MECKLING, W. (1976). Theory of the firm: managerial behavior, agency costs and ownership structure. *J. Financial Econ.* **3**, 305-60.
- KIHLSTROM, R. and LAFFONT, J.-J. (1982). A competitive entrepreneurial model of a stock market, in "The Economics of Information and Uncertainty," (J. McCall, Editor) Chicago: University of Chicago Press, 141-98.

- KIHLSTROM, R. and MATTHEWS, S. (1988). Managerial incentives in an entrepreneurial stock market model, Northwestern, CMSEMS Paper 769.
- LELAND, H. (1974). Production theory and the stock market, *Bell J. Econ. and Mgmt. Sci.* **5**, 125-44.
- LELAND, H. and PYLE, D. (1977). Informational asymmetries, financial structure, and financial intermediation, *J. Finance* **32**, 371-87.
- MARCUS, A. (1982). Risk sharing and the theory of the firm, *Bell J. Econ.*, 369-78.
- MARSHALL, J. (1976). Moral hazard, *Amer. Econ. Rev.* **66**, 880-90.
- PRESCOTT, E. and TOWNSEND, R. (1984). Pareto optima and competitive equilibria with adverse selection and moral hazard, *Econometrica* **52**, 21-46.
- RADNER, R. (1974). A note on the unanimity of stockholders' preferences among alternative production plans: a reformulation of the Ekern-Wilson model, *Bell J. Econ. and Mgmt. Sci.* **5**, 181-86.
- RAMAKRISHNAN, R., and THAKOR, A. (1984). The valuation of assets under moral hazard, *J. Finance* **34**, 229-38.
- STIGLITZ, J. (1972). On the optimality of the stock market allocation of investment, *Quart. J. of Econ.* **86**, 25-60.
- STIGLITZ, J. (1974). Incentives and risk sharing in sharecropping, *Rev. Econ. Stud.* **41**, 219-55.

### Footnotes

- 1 Whether the entrepreneur decides on  $L$  before or after he decides on  $s$  is irrelevant; we view the two decisions as simultaneous under the assumption that nothing relevant happens between the times the two decisions are made.
- 2 In Kihlstrom and Matthews (1988), competitive rational expectations are defined for a scenario in which the entrepreneur is risk neutral. All CEE in this case are fully efficient. Their restricted efficiency is determined by an “invisible hand” welfare theorem analogous to Theorems 5.1 and 5.2 below. The fact that restricted and full efficiency coincide when the entrepreneur (agent) is risk neutral is well known from the principal-agent literature.
- 3 For out-of-equilibrium values of  $s$ , this condition (iv) does not require the investors to correctly infer the entrepreneur’s “true” labor choice, however that may be defined, unless  $\lambda(s, (1-s)V^*(s))$  is single-valued.
- 4 For every  $y$ ,  $\lambda(s,y) = \{L(s)\}$ .
- 5 If an investor is risk neutral, (3.7) is the true inverse demand function. For then  $c_m = 0$ , so that the price of risk,  $c_m(1-s^*)\sigma^2$ , is zero independent of the supply. In this case price-taking expectations are accurate.
- 6 This condition necessarily yields the unique maximum if  $c_0 > 0$  and  $f(L(s)) - wL(s)$  is concave in  $s$ ; the latter holds, for example, if  $f(L) = AL^\gamma$  and  $\gamma \in (0, 1/2]$ .
- 7 Because the entrepreneur has a CARA utility function, this linearity is not necessarily a restriction. In a continuous-time principal-agent model in which the agent has a CARA utility function and controls the drift term of a Brownian motion profit process, Holmström and Milgrom (1987) show that the optimal incentive contract is linear in the realized value of aggregate profit. Thus, in our model if  $L$  is the drift parameter and  $g(L, \tilde{x})$  is the integral of such a process (with  $L$  held constant), the optimal incentive contract would indeed be linear.
- 8 See Kihlstrom and Matthews (1988) for derivations of these comparisons.

- 9 When  $L$  is observable, its competitive equilibrium level is  $L(1)$ , which is greater than the CEE level  $L(s^*)$  because  $s^* < 1$ . The equilibrium level of  $s$  is determined by equating to zero the right side of (3.8), which yields  $s = c_0^{-1}/(c_0^{-1} + c_m^{-1})$ .
- 10 We have chosen a piecewise linear utility function for convenience only. At  $I_0 < 0$  we could have let  $u$  be any concave function with  $u'$  sufficiently large. Such a utility function could exhibit decreasing absolute risk aversion.
- 11 One might expect that work effort and hence  $V^*(\cdot)$  should increase in  $s$ ; this may seem what “sensible” beliefs off-the-equilibrium path should imply. Not so. For  $s > 3/4$ , rational expectations requires that  $L = V^*(s) = 0$  in every CEE; this follows from  $V^*(s) = g(\lambda(s, 4(1-s)V^*(s))) = \min(1, 4(1-s)V^*(s))$ . In no equilibrium can the entrepreneur sell less than a fourth of the firm’s equity and still obtain enough proceeds from the sale to induce him to work.
- 12 Although it would take us too far afield to do so here, it can similarly be shown, in either Scenario RN or SP, that if a CEE with  $s^* > 1$  exists, then another CEE with  $s^* = 1$  also exists. If  $u_0'' < 0$  and  $g(L^*, \tilde{x})$  has positive variance in every CEE, then  $s^* < 1$  in every CEE.
- 13 The correspondence  $\lambda(s, y)$  decreases in  $y$  if for each  $y < \hat{y}$ ,  $L \in \lambda(s, y)$ , and  $\hat{L} \in \lambda(s, \hat{y})$ , the inequality  $L > \hat{L}$  obtains.