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LEARNING FROM MISTAKES: A NOTE ON JUST-IN-TIME SYSTEMS

by

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Abstract

We consider an inventory model in which the production technology is not given but, instead, admits improvement through a sequential learning process controlled by the firm. It is shown that in this framework some salient characteristics of Just-in-Time (JIT) systems (sequential inventory reduction, emphasis in variability reduction, and suboptimal inventory levels relative to the static optimum) emerge as profit maximizing policies. Thus, the model permits rationalization of JIT systems within the standard optimization paradigm.

Central to our approach is the learning process considered; it is characterized as an investment of optional intensity and random yield. The probability of a successful investment is larger after a poor performance of the production system so that the process can be seen as one of learning from mistakes.

1. Introduction

The purpose of this article is to develop a framework in which the new inventory management techniques known as "just in time" (JIT) can be reconciled with the standard inventory literature. The conventional wisdom on inventory management is seriously challenged in several ways by the widespread success of JIT implementations. In the first place, the "optimization" approach to inventory management suggests that the optimal level of inventory be determined by balancing the costs of having too much and too little inventory. Typically, the optimal balance is achieved at some positive level of inventory.[1] JIT, on the other hand, strives for the systematic elimination of the costs associated with having too little inventory. Thus the long term goal set by JIT is "zero inventories." To study this process of sequential technological improvement and inventory reductions, an element of learning has to be introduced into the inventory model. We refer to the particular learning process considered in this paper as "learning from mistakes." In rough outline, the learning process is initiated by observing a "mistake" or a quality "problem" occurring at the production system; tracing the origins of the problem and eliminating them improves the the overall quality of the system. Under some reasonable conditions, the result of these improvements is a gradual reduction in the "optimal" level of inventory. But the JIT principles carry the relationship between inventory reduction and learning one step further. Not only is the "optimal" amount of inventory decreased as a result of learning; at each point in time the system is operated deliberately with "suboptimal" levels of inventory (relative to the static optimum) in order to accelerate the learning process. The popular "rocks hidden by water"[2] analogy is a vivid expression of this idea. Examining this phenomenon in the context of our model shows that it can be the consequence of some "observability" constraint. If the latter can be eliminated, then a system in which learning and inventory decisions are decoupled (so that inventories are adapted to the current technology) will perform better than JIT.

The organization of the paper is as follows. In the next section we cover some of the related literature. In Section 3 we consider the basic model without learning. This model serves as a convenient benchmark for the results of the JIT model developed in the subsequent sections. In Section 4 the notion of improvement is introduced. We demonstrate the well known JIT principle that a reduction in the variability of a production system can be as useful in increasing profits as an increase in its "average" yield. In Section 5 we develop the model with learning. Section 6 examines the relation between technology improvements and inventory reduction. In Section 7 we introduce the observability constraint and characterize its effects on the production policy and the results are summarized in Section 8. We also argue in this concluding section that the ideas of JIT are not limited to systems involving physical inventory and briefly discuss how the same principles can be applied to the service sector. We also note the futility of implementing JIT within a static framework, i.e., reducing inventory below optimal levels without harnessing the learning process which is an integral part of JIT.

2. A Review of The Literature

This section briefly discusses some of the previous literature on learning and JIT. Our focus is on learning in a production environment which results in increased cost efficiency.[3]

The basic premise in the concept of learning is that technologies improve with time and experience. This seems to be an empirical fact of general validity. Muth (1986) gives an account of the empirical evidence, summarized in the progress or experience curve relating accumulated production to the per-unit production costs. He also develops a model of search among unknown technological possibilities which replicates the empirical relationship.

Learning processes can be separated into two broad categories. On the one hand, autonomous learning (or learning-by-doing) relates to the idea that "practice makes perfect," i.e., that repetition of an acitivity tends to reduce the cost of performing it. Fine (1986, 1988) considered the management of quality in a learning environment and showed that quality control criteria should change over time as a consequence of learning. Suri and de Treville (1986) analyzed the optimal management of inventories under the assumption that learning takes time, so that the gains of inventory reduction must be balanced against the temporary loss of efficiency that results when the operators of a productive system are confronted with a new production environment. As a consequence, they argued that inventory reduction must occur gradually.

Arrow (1986), Spence (1981), Fudenberg and Tirole (1982), and Bhattacharya (1984), among others, have studied the consequences of learning for the dynamic evolution of markets. Learning effects, by reducing production costs, result in prices decreasing over time and industry output consequently increasing. Bhattacharya (1984) argued that an early entrant in a new industry can obtain a cost advantage by exploiting learning effects so that experience acts as an entry barrier in the more mature phases of the industry.

The other broad category of learning processes is labelled as induced. Such learning occurs as a consequence of deliberate actions taken by the firm. Induced learning is thus the consequence of an investment. There seems to be general agreement that in practice learning is mostly induced, so that models of autonomous learning are justified only by their relative simplicity.

A framework for the analysis of manufacturing systems in an induced learning environment is provided in Fine and Porteus (1988). Their approach to the design of production systems emphasized gradual process improvement as the source of cost efficiency. They provided conditions under which an investment in process improvement is an optimal policy for the firm and showed how this concept applies to several aspects of the firm's production policy (inventories, set-up times, and so forth).

A related approach was taken in Jaikumar (1988) and Bohn and Jaikumar (1989). The first paper provides optimal rules for the management of inventories and searches-for-problems in a production line subject to random occurrences. The second paper develops a detailed analysis of the different elements involved in sequential process improvement and illustrates them in the context of an example. Induced learning in a competitive environment is the subject of Flaherty's (1980) study of investments in cost reduction and Spence's (1982) analysis of R&D policies.

The optimal design of JIT systems in a static (non-learning environment) was considered, among others, by Porteus (1985, 1986), Karmakar (1986), Bitran and Chang (1987), and Zangwill (1987). This literature provides decision models characterizing the optimal inventory policy in complex production systems.

Finally, the practice of JIT management is disussed in numerous books and articles: for instance, Hall (1983) and Schonberger (1982) provide detailed comparisons of JIT and standard inventory management, as well as some intuitive explanations on the rationale underlying JIT.

3. The Basic Inventory Model

The purpose of this section is to develop an inventory model, without learning, in which the production and inventory policies can be explicitly characterized. These policies provide a benchmark against which the performance of the learning version of the model can be compared.

<Insert Figure 1 about here>

We consider a simple production system (see Figure 1) consisting of two activity centers (machines, work stations, etc.), A and B, and an inventory C, serving as a buffer between them. At each period t, the rate of operation of the two machines can be changed. If the rate of A at time t is a_t then the output of A, in that period, is given by $m_t = a_t x_t$, where $0 \le x_t \le 1$ is the random yield of machine A. Similarly, if the production rate of center B at time t is set to b_t, and the inventory at C at the beginning of this period is I_{t-1} then the quantity out of C into B in period t is given by $q_t = \min\{I_{t-1} + m_t, b_t\}$. Finally, the output of B in period t is given by $r_t = q_t y_t$, where $0 \le y_t \le 1$ is the random yield of machine B. We thus have:

(1) $m_t = a_t x_t$ (Output of A)

(2) $q_t = \min\{I_{t-1} + m_t, b_t\}$ (Output of C)

(3) $r_t = q_t y_t$ (Output of B)

(4) $I_t = I_{t-1} + m_t - q_t = \max\{I_{t-1} + m_t - b_t, 0\}$ (Inventory)

(5) $S_t = \max\{-I_{t-1} - m_t + b_t, 0\}$ (Shortage)

We assume that the rates a_t and b_t can be independently chosen each period within some bounds $A_1 \le a_t \le A_2$, $B_1 \le b_t \le B_2$. The yields $x_t, y_t \in [0,1]$ are independent random variables with distributions $x_t \sim H_\alpha(\cdot), y_t \sim G(\cdot)$. The cumulative distribution function $H_\alpha(\cdot)$ is assumed to be continuous with a density $h_\alpha(x) > 0$ on $(0,1)$ so that the inverse, $H_\alpha^{-1}(\cdot)$, is well defined. The parameter α represents the "state" of the system and its precise nature will be discussed in the next section where the learning process is introduced.

Let us denote the average values of y_t and x_t by μ_y and $\mu_x(\alpha)$, respectively. Let p be the value (price) of every final unit; let c_a and c_b be the per-unit costs of the rates of centers A and B, respectively; and let c_I and c_s be the per-unit (per-period) costs of inventories and shortages. We assume that p, c_a, c_b, c_I, and c_s are all nonnegative. The profits generated in each period are then given by:

(6) $$G(I_{t-1}, a_t, b_t, x_t, y_t) = pr_t - c_b b_t - c_a a_t - c_I I_t - c_s S_t$$

where r_t, I_t and S_t can be expressed in terms of the decision variables, a_t, b_t, the random yields x_t, and y_t, and the inventory level I_{t-1}, using (1)-(5). Let $P(I_{t-1}, a_t, b_t | \alpha)$ be the expectation of the profit in period t with respect to x_t, y_t, and conditional on the values I_{t-1}, a_t, b_t. It follows from (6) that

$$(7) \qquad P(I_{t-1}, a_t, b_t|\alpha) = p\mu_y E(q_t) - c_a a_t - c_b b_t - C_I E(I_t) - c_s E(S_t)$$

where all the expectations are taken with respect to x_t. We assume that profits are discounted at a rate $\rho, 0 < \rho < 1$, over the (infinite) time horizon. Specifically, let $A = \{a_1, a_2, \ldots\}$ and $B = \{b_1, b_2, \ldots\}$ be sequences of production decisions. Then, total expected discounted profits are given by:

$$(8) \qquad \Pi(I_0, A, B|\alpha) = \Sigma_{t=1,2,\ldots} \rho^{t-1} P(I_{t-1}, a_t, b_t|\alpha).$$

where I_0 is the initial inventory. Let

$$V(I_0|\alpha) = \max_{A,B} \Pi(I_0, A, B|\alpha)$$

be the optimal expected profits conditional on the initial level of inventory, I_0. As a benchmark for the results that follow, we compute $V(I_0|\alpha)$ and the associated optimal production strategies A* and B*. Clearly, the function $V(I_0|\alpha)$ must satisfy the optimality equation of dynamic programming:

$$(9) \qquad V(I_0|\alpha) = \max_{A_1 \le a_1 \le A_2; B_1 \le b_1 \le B_2} \{p\mu_y E(q_1) - c_a a_1 - c_b b_1 -$$

$$- C_I E(I_1) - c_s E(S_1) + \rho E(V(I_1|\alpha))\}.$$

The solution of this equation is characterized as follows:

<u>Proposition 1</u>: Assume that (a) $B_1 = 0$; (b) $B_2 \geq 2A_2$, and (c) $I_0 \leq A_2$. Then the optimal production policy (A^*, B^*) is given by:

(i) $$a_t^* = \begin{cases} A_1, & \text{if } F_\alpha(z^*) \leq 0 \\ A_2, & \text{otherwise.} \end{cases}$$

where

$$z^* = H_\alpha^{-1}\left(\frac{(1 - \rho)(p\mu_y - c_b) + c_I}{(1 - \rho)p\mu_y + \rho c_b + c_I + c_s}\right)$$

and

$$F_\alpha(z^*) = (p\mu_y + c_s) \int_0^1 x dH_\alpha + \rho(P\mu_y - c_b) \int_{z^*}^1 x dH_\alpha - c_a$$

(ii) $$b_t^* = I_{t-1} + a_t^* z^*.$$

(iii) $$v(I_0|\alpha) = a^*(F_\alpha(z^*)/(1 - \rho)) + (p\mu_y - c_b)I_0.$$

<u>Proof</u>: See Appendix 1. []

According to part (i) of the proposition, center A operates at full capacity as long as the system is generating profits for any initial condition. Otherwise, if $F_\alpha(z^*) \leq 0$, shooting down the operation would be optimal. The reader may recognize part (ii) of the proposition as the solution of a dynamic version of the newsboy problem. Also, note that the last term in (iii) gives the value of one unit of inventory. This value is simply the expected revenue generated by transforming one unit of inventory into one of the final product, $p\mu_y$, minus its cost, c_b.

4. The Model with Learning: Sources of Technology Improvement

JIT systems are characterized by sequential learning. We are interested in examining the interaction between learning and production decisions and specifically the relation between quality improvements and inventory levels. To that end we introduce a notion of learning which we call "learning from mistakes." Specifically, learning in our model means modifying the (stochastic) performance of center A.[4] Naturally, we are seeking modifications which improve the performance of the system as measured by expected profits. We will not be concerned here with modeling the technological and managerial details which are the essence of sequential learning. Rather, we abstract from these details and summarize the entire relevant state of the system at any given time by a unique (and highly multi-dimensional) parameter α. Thus, α captures the entire set of procedures and policies with respect to maintenance, training, quality management, purchasing, etc., which collectively determine the stochastic behavior of the output of center A. In general, each value of the parameter α will be associated with a different distribution of the yield, $H_\alpha(\cdot)$. The static solution of the inventory problem introduced in the previous section characterizes the optimal inventory given the state α. In contrast, JIT approaches are concerned with improvements resulting from the utilization of "better" choices of the parameter α. We refer to such improvements as <u>learning</u>. The learning process examined here is sequential since, in practice, the universe of possible parameter values α is so large, complex and multi-dimensional that the only effective way to search over it is incrementally, by a costly and slow process of trial and error. In contrast to the standard "learning-by-doing" processes in which learning is autonomous, the one considered here is a <u>managed</u> process; its rate and intensity can be controlled by management. We call this process "learning from mistakes" since the occurrence of bad instances (low yields) is assumed to contain information which, if discovered, can expedite the learning rate.

We will now formally characaterize the concept of learning. We begin by explicitly spelling out the natural notion of "improvement." We use the short cut "system α" to refer to the inventory system in which the yield x_t is distributed according to $H_\alpha(\bullet)$.

<u>Definition 1</u>: System α is <u>better</u> than system β if and only if for every level of initial inventory, $I \leq A_2$,

$$V(I|\alpha) > V(I|\beta).$$

That is, for any level of initial inventory, system α yields higher optimal expected profits. In light of Proposition 1 we can omit the dependence on the inventory level from this definition and use instead a direct criterion: system α is better than system β iff

$$F_\alpha(z^*) > F_\beta(z^*).$$

Although this definition is sufficient for the analysis that follows, it is interesting to examine some specific situations which give rise to an improvement. As it turns out, the concept of stochastic dominance plays a critical role here:

<u>Proposition 2</u>: Assume that $\rho(\mu_y p - c_b) \geq c_I$. Then system α is better than system β if $H_\alpha(\bullet)$ dominates $H_\beta(\bullet)$ in the sense of first order or second order stochastic dominance ((FOSD) and (SOSD), respectively).

<u>Proof</u>: See Appendix 1.

The assumption in the proposition simply requires the value of one unit of inventory be available "tomorrow" to be at least as big as the cost of holding this unit in inventory for one period. If this condition does not hold, it would be optimal to discard leftover inventory at each period.

The reader may recall that $H_\alpha(\bullet)$ dominates $H_\beta(\bullet)$ in the sense of FOSD if $H_\alpha(x) \leq H_\beta(x)$ for every $x \in [0,1]$. In our context this basically means that the (typical) yield of center A is higher in the α system or, in other words, that the productivity of system α is higher. Similarly, $H_\alpha(\bullet)$ dominates $H_\beta(\bullet)$ in the sense of SOSD if $\int_0^x H_\alpha(y)dy \leq \int_0^x H_\beta(y)dy$ for every $x \in [0,1]$. Roughly speaking, this means that the typical <u>variability</u> of the α system is smaller than that of the β system. We will refer to these changes in the technology as quality improvements since they result in a more reliable and predictable system. It is well known that both productivity and quality improvements are major objectives of JIT systems. In the sequel we examine in detail the interaction between such improvements and inventory reductions.

5. Learning From Mistakes: The Dynamics of Learning

As stated earlier, we model learning as the process of moving from a given system α_1 to a better system α_2. We assume that the set of parameters α is discrete and can be indexed, $\alpha_0, \alpha_1, \alpha_2, \ldots$, with higher indices corresponding to better systems. Let $N(\alpha)$ be the system which follows α in this sequence (i.e., if $\alpha = \alpha_i$ for some $i = 0, 1, \ldots$, then $N(\alpha) = \alpha_{i+1}$). Then our convention implies that for every α, the system $N(\alpha)$ is better than the system α.

Learning occurs by sequentially moving from the current system α to the improved subsequent system $N(\alpha)$. The process of learning is modeled as an investment problem with a random yield. Specifically, let $0 \le k_t \le K$ be the effort or investment in learning made in period t. Then, with a certain probability f the search is successful and we move in period $t + 1$ to the improved system $N(\alpha)$. With probability $(1 - f)$ the search fails, and the system stays in state α. The probability f is assumed to depend on the current state, α, on the effort k, and on the most recent random yield, x. Formally, $f = f(k, x|\alpha)$. The probability function $f(\bullet,\bullet|\alpha)$ is assumed to be twice continuously differentiable in its two first arguments, (weakly) increasing in the first, and (weakly) decreasing in the second. The requirement that f be increasing with k represents the fact that more effort increases the probability of identifying the source of a given problem and thus of improvement. In practice, one also would expect f to decrease with x representing the "learning from mistakes" feature, namely, that the occurrence of a bigger "problem" is more informative about its underlying causes and thus is more useful for the learning process. We believe that this feature is an important part of JIT. However, it is not required for the analysis presented here. In order to emphasize the induced nature of the learning process we add the requirement $f(0,x|\alpha) = 0$. The cost of one unit of effort is denoted by w.

The timing of events is as follows: at the beginning of each period, a production decision is made which determines a_t and b_t. Depending on the realized random yield, x_t, an investment decision, k_t, which determines the intensity of the search, is made. If the search is successful, the system moves to state $N(\alpha)$. The shift to the new state materializes in time for the production decisions due in period $t + 1$.

How does the learning process interact with the production policy a_t, b_t? Let $V_1(I,x|\alpha)$ be the value of being in state α, with inventory I, and the last realization of the yield being x, just before making an investment decision. Let $V_2(I|\alpha)$ be the value of being in state α, with inventory I, just before making a production decision. The optimality principle of dynamic programming requires the optimal policy and the value functions to satisfy:

(10) $$V_1(I,x|\alpha) = \max_{0 \le k \le K} \{-wk + \rho f(k,x|\alpha)V_2(I|N(\alpha))$$

$$+ \rho(1 - f(k,x|\alpha))V_2(I|\alpha)\}$$

and

(11) $$V_2(I|\alpha) = \max_{A_1 \le a \le A_2 ; B_1 \le b \le B_2} \{P(I,a,b|\alpha) +$$

$$+ \int_0^1 V_1(\max\{0, I + ax - b\}, x|\alpha)dH_\alpha(x)\}.$$

The optimal production and investment policies in the learning scenario are characterized as follows:

Proposition 3: Under the assumptions of Proposition 1:

(i) The optimal production policy, for any given α, is the same as in the no-learning scenario.

(ii) The optimal investment policy does not depend on the current inventory level.

(iii) Supose that $[V(0,N(\alpha)) - V(0,\alpha)](\partial f(0,x|\alpha)/\partial k) > w$ for some x. Then $k^*(x^1) > 0$ for some x^1 in $[0,1]$.

Proof: Consider (11). Arguing as in Proposition 1, it follows that V_2 must be linear in I, as long as the bounds on b are not effective; and in this case, the coefficient of I in V_2 does not depend on α. But then, from (10),

$$V_1(I,x|\alpha) = (p\mu_y - c_b)I + \max_{0 \le k \le K} \{-wk + \rho f(k,x|\alpha)V_2(0|N(\alpha))$$
$$+ \rho(1 - f(k,x|\alpha))V_1(0|\alpha)\}.$$

This proves (ii). Part (i) now follows as in Proposition 1.

To prove (iii), suppose that $k^*(x) \equiv 0$. Then $V_1(I,x|\alpha) = \rho V_2(I|\alpha)$ for every x and $V_2(I|\alpha) = V(I|\alpha)$. Now, since $V_2(I|N(\alpha)) \ge V(I|N(\alpha))$, it follows that $V_2(I|N(\alpha)) - V_2(I|\alpha) \ge V(I|N(\alpha)) - V(I|\alpha) = V(0|N(\alpha)) - V(0|\alpha)$. Hence, for some x, $[V_2(I|N(\alpha)) - V_2(I|\alpha)](\partial f(0,x|\alpha)/\partial k > w$ which implies $k^*(x) > 0$.

But this contradicts $k^*(x) \equiv 0$. We conclude that there exists x^1 such that $k^*(x^1) > 0$. □

The first and second parts of the proposition state that in the absence of other constraints, it is optimal to <u>separate learning and production</u> decisions. Production is made at the profit-maximizing level given the current state of the technology, and changes in the production policy are made only after improvements have materialized. This separability is not commonly observed in a JIT system. In fact, JIT principles seem to advocate for inventory levels below the profit-maximizing level. Later on it will be seen how the separation of learning and production decisions is affected by the introduction of observability constraints.

The last part of the proposition shows that the optimal investment is strictly positive under some reasonable circumstances. Specifically, the inequality stipulated in part (iii) is satisfied if $N(\alpha)$ is strictly better than α and $\lim_{k\to 0} \partial f(kx|\alpha)/\partial k = \infty$. The latter condition will hold in practice if there are some learning options capable of generating improvements with a "very small investment." As an example, consider the suggestion box and other similar methods that allow worker participation in the adjustment of technology. These methods are utilized extensively by JIT systems.

Since the investment policy is unaffected by the inventory level, Fine and Porteus' (1988) characterizations of optimal investment policies are also valid in our model. Thus, we will not pursue a detailed study of the dynamic pattern of the investment policy. The reader is referred to Fine and Porteus (1988) for this analysis.

6. Inventory Reductions

A common characteristic of JIT systems is the gradual reduction of inventories as the learning process evolves. In this section we examine the conditions under which these reductions are predicted by the model. The result, roughly stated, is that the implication holds true as long as the improvements are relatively more effective in "bad" states of nature than in "good" ones. In order to derive this result in a way which is readily interpretable, it is convenient to separate the factors determining x into an underlying random effect, denoted y, and the deterministic response x of system α to the shock y. Specifically, we define the primitive random shock of the system, y, as the random variable satisfying $x = g_\alpha(y)$, for some increasing continuous function $g_\alpha(\bullet)$ with the property $y \sim F(y)$ (independent of α).[5] Thus y can be interpreted as the randomness built into the system, which does not change with α. The function $g_\alpha(\bullet)$ is the production function, relating, for a given state of the system, each value of the primitive random shock to the effective rate of production. With this notation, the function $g_\alpha(\bullet)$ captures the current state of the system.

A technological improvement from β to α will be said to be a uniform loss reduction if the increase in productivity is higher in bad states of nature than in better ones. Formally, if

$$y \geq x \text{ and } \alpha \text{ better than } \beta \text{ imply } g_\alpha(y) - g_\beta(y) \leq g_\alpha(x) - g_\beta(x).$$

When this property holds, the model replicates JIT behavior as shown in the following:

Proposition 4: Suppose α is better than β and $a^*(\alpha) = a^*(\beta)$. If α is a uniform loss reduction with respect to β, then $E[I|\alpha] < E[I|\beta]$.

Proof: Denote by y^* the solution of $F(y) = H_\alpha(z^*)$.[6] Notice that y^* does not depend on the state α. Also, $E(I|\alpha) = a^*\int_{y^*}^{\infty} (g_\alpha(y) - g_\alpha(y^*))dF(y)$. Therefore:

$$\begin{aligned} E[I|\alpha] - E[I|\beta] &= a^*\int_{y^*}^{\infty} \{[g_\alpha(y) - g_\alpha(y^*)] - [g_\beta(y) - g_\beta(y^*)]\}dF(y) = \\ &= a^*\int_{y^*}^{\infty} \{[g_\alpha(y) - g_\beta(y)] - [g_\alpha(y^*) - g_\beta(y^*)]\}dF(y) \\ &\leq 0 \qquad \square \end{aligned}$$

A final remark. The reader may note that the properties of stochastic dominance and uniform loss reduction are independent statements about the nature of the learning process. As an example, notice that first order stochastic dominance is equivalent to the property $g_\alpha(y) \geq g_\beta(y)$ for every y, and this last relation is independent of the uniform loss property $g_\alpha(y) - g_\beta(y) \leq g_\alpha(x) - g_\beta(x)$.

6. Observability Constraints

Up to this point we have shown that the inventory reduction obtained by JIT systems can be the consequence of an ongoing learning process. We address now the question of why, in practice, inventory reductions seem to precede, rather than follow, process improvement. Our answer rests on the observation that some minor alterations on the performance of a production unit are hardly observable while major failures are more likely to be noticed and investigated.

Specifically, we consider a system in which only those disturbances generating a shortage, and therefore forcing the system to a halt, would trigger a search for the causes of the problem. Thus, as long as the system keeps going on, no investment in learning will occur. This we label the observability constraint. The zero inventory level, at which the constraint becomes binding, has been chosen arbitrarily and our results would apply similarly to any other arbitrarily chosen level as long as, in some states of nature, learning is not possible due to an exogenous constraint. However, the zero inventory level is particularly significant since, in practice, shortages are typically highly visible and are also a standard trigger activating the search and review procedures characteristic of JIT.

When the constraint is binding, improvement opportunities may be lost. A way around this is to decrease the average inventory level so that learning opportunities become more frequent. Hence, to some degree, shortages are artificial. This explanation rationalizes the parable about the water hiding the rocks cited in the introduction: smaller inventories permit more "problems" to be identified. We will formalize this agrument. We define

$$\sigma(S_t) = \begin{cases} 1 \text{ if } S_t > 0 \\ 0 \text{ otherwise} \end{cases}$$

and let us denote the value of being in state α, with inventory I, shortage S, and the last value of the yield equal to x, just before making an investment decision, by $W_1(I,\sigma(S),x|\alpha)$. Similarly, $W_2(I|\alpha)$ is the value, given the state α, of being just before a production decision. The optimality conditions require in this case:

$$(12) \qquad W_1(I,0,x|\alpha) = \rho W_2(I|\alpha)$$

since $S = 0$ implies $k = 0$.

$$(13) \qquad W_1(0,1,x|\alpha) = \max_{0\le k\le K} \{-wk + \rho f(k,x|\alpha)W_2(0|N(\alpha)) + \rho(1 - f(k,x|\alpha))W_2(0,\alpha)\}$$

$$(14) \qquad W_2(I|\alpha) = \max_{A_1\le a\le A_2;\, 0\le b\le B_2} \{P(I,a,b|\alpha) + \int_{[0,(b-I)/a]} W_1(0,1,x|\alpha)dH_\alpha(x) + \int_{[(b-I)/a,1]} W_1(I + ax - b,0,x|\alpha)dH_\alpha(x)\}.$$

Let (a_t^+,b_t^+) be the optimal production rates in the informationally constrained problem. The change in inventory policy is characterized as follows:

<u>Proposition 5</u>: Under the assumptions of Proposition 1, if b^+ is an interior solution then

(i) $a^+ \le a^*$ and $a^+ \in \{A_1,A_2\}$

(ii) $(b^+ - I)/a^+ \ge (b^* - I)/a^*$

(iii) $E[I|a^+,b^+,\alpha] \le E[I|a^*,b^*,\alpha]$

(iv) $V(I|\alpha) \leq W_2(I|\alpha) \leq V_2(I|\alpha)$.

<u>Proof</u>: Let $k^+(x)$ be optimal in the constrained problem. Then

$$W_2(I|\alpha) = \max_{A_1 \leq a \leq A_2, 0 \leq b \leq B_2} \{P(I,a,b|\alpha)$$

$$+ \rho\int_{[0,1]} W_2(\max\{0, I + ax - b\}|\alpha)dH_\alpha$$

$$+ \int_{[0,(b-I)/a]} \{\rho[W_2(0|N(\alpha)) - W_2(0|\alpha)]f(k^+(x),x|\alpha) - wk^+(x)\}dH_\alpha$$

if $b^+ \leq B_2$. We can write as in Proposition 1, $W_2(I|\alpha) = (p\mu_y - c_b)I + G(\alpha)$, and defining $b' = (b - I)/a$, it follows that

$$0 = a^+F_\alpha'(b'^+) + \{\rho[W_2(0|N(\alpha)) - W_2(0|\alpha)]f(k^+(b'^+),b'^+|\alpha)$$
$$- wk^+(b'^+)\}h_\alpha(b'^+)$$

The second term on the right side is nonnegative since k^+ is optimal. Therefore $F_\alpha'(b'^+) \leq 0, F_\alpha'(b'^*) = 0$, and $F_\alpha(\bullet)$ is concave; this implies $b'^+ \geq b'^*$. Also notice that $F_\alpha(b'^+) \leq F_\alpha(b'^*)$ implies (i).

Consider $E[I|a,b',\alpha] = a\int_0^1 \max\{0,x - b'\}dH_\alpha$. This is an increasing function of a and it is decreasing on b'; thus (i) and (ii) jointly imply (iii). The inequalities in (iv) are trivial. []

Intuitively, the proposition states that it may be optimal to increase the probability of a shortage so that learning opportunities arise more often. It remains to be seen that the proposition is not trivial, i.e., that the inequalities are strict under some reasonable assumptions.

Corollary: If $\lim_{k\to 0} \partial f(k, x(\alpha)/\partial k = \infty$, then (ii) and (iii) hold as strict inequalities.

Proof: Otherwise, it would be the case that $w_2(0|N(\alpha)) = w_2(0|\alpha)$, which implies $k(x) \equiv 0$. Hence, $w_2(0|\alpha) = V(0(\alpha)$ but $V(0|\alpha) < V(0|N(\alpha))$ $\leq w_2(0|N(\alpha))$, contradicting $w_2(0|N(\alpha)) = w_2(0|\alpha)$. []

The condition given in the corollary was discussed in Section 5.

7. Concluding Remarks

Finally, we discuss some limitations inherent to our model as well as some policy implications. The analysis has emphasized the technological aspects in the management of inventories and process improvement. Two other aspects of the problem are the motivational properties often attributed to JIT, and the relationship between production policy and the competitive position of the firm.

Incentives and information enter in our model without explicit modeling, subsumed in the observability constraint. A possible extension of the model would derive the constraint from the maximizing behavior of individuals, given the impossibility of complete monitoring of their actions.

Another extension concerns competition. The choice of production policy is an important determinant of the firm's market share and profits. It is possible that competition among firms is one of the causes in the adoption of aggressive cost reduction policies beyond what is justified on the grounds of efficiency (De Groote (1989) modeled this possibility in the context of the firm's inventory policy).

It is important to remark that our results regarding process improvement apply whether or not the firm carries inventories. Thus, some of our findings could be applied to the management of operations in the service sector as well. First, the model suggests that performance criteria should change with time. Second, failures in providing adequate service may be the source of improvements if the adequate review procedures are in place.

As an example demonstrating these two points in a non-inventory setting, consider the problem of missing price tags on items in a department store. Such "defects" represent an obvious loss in productivity for the cash register operators and support personnel. They are also a major source of irritation to customers "stuck" in a line waiting for the price of an item to be determined. The standard approach is to "buffer" against the second problem by providing a support system that can react fast whenever a price is missing. The "optimal" investment in such a support system is determined by balancing its costs and benefits much as the optimal inventory in the static model of Section 2 is obtained by balancing the costs of inventory and of shortages. The sequential improvement approach would use each occasion in which a price tag is discovered as missing to find the causes of such an event, then try to eliminate them in the future. Such a policy would result in a gradual reduction in both the frequency of missing tags and the required "buffer." In analogy to the effects of the "observability" constraints of JIT, we may wish to operate the support system at "suboptimal" levels. In the short run this will increase the irritation and costs caused by missing tags as the system will not be as efficient at handling them as they occur. However, this increased irritation and cost will increase the visibility of such problems. If used to trigger improvements, the overall effects over the long run may be far superior service at lower overall cost.

Finally, if the causal relationship between process improvement and inventory reduction is accepted as central to understanding JIT, three important policy implications follow. First, there is some evidence that firms sometimes adopt inventory policies mimicking JIT without, at the same time, developing the required programs to support sequential improvement. Our analysis suggests that such imitation strategies may be even less effective than standard policies. In our model, inventory reduction is justified in the context of learning and should not be adopted in isolation.

Second, there are some claims in the literature about the intrinsic superiority of JIT techniques over conventional inventory management. We have found, however, that the JIT pattern in which inventory reduction anticipates learning is a second-best phenomenon. Specifically, if the informational constraint could be relaxed (using better supervision methods, an improved incentive policy, etc.), then a more classical pattern in which inventories adjust to the current available technology, will obtain.

Finally, it is important to emphasize the premium placed by our model on variability reduction. It justifies the frequent claim that improved predictability can be a major factor in allowing for inventory reduction. Furthermore, it suggests that, in evaluating investments in technology, variability measures and not only average yields, should be considered. It seems to be a common practice, however, to ignore the former.

References

Alpern, S. (1988), "High-Low Search in Product and Labor Markets," A.E.R., 78, No. 2, 356-62.

Alpern, S. and D. Snower (1987), "Inventories as an Information-Gathering Device," ICERD Discussion Paper No. 87/151, L.S.E.

Arrow, K. (1962), "The Economic Implications of Learning by Doing," Review of Economic Studies, 29, 155-173.

Bhattacharya, G. (1984), "Learning and the Behavior of Potential Entrants," Rand Journal of Economics, 15, 281-289.

Bielecki, T. and P. R. Kumar (1988), "Optimality of Zero-inventory Policies for Unreliable Manufacturing Systems," Operations Research, 36, 532-541.

Bitran, G. R. and Li Chang (1987), "A Mathematical Programming Approach to a Deterministic Kanban System," Management Science, 33, No. 4, 427-41.

Bohn, R. and R. Jaikumar (1989), "The Dynamic Approach: An Alternative Paradigm for Operations Management," working paper, Harvard Business School.

De Groote, X. (1989), "The Strategic Design of Production Processes," unpublished manuscript, the Wharton School, University of Pennsylvania.

Fine, C. H. (1988), "A Quality Control Model with Learning Effects," Operations Research, 36, 437-44.

Fine, C. H. and E. L. Porteus (1988), "Dynamic Process Improvement," Research Paper No. 967, Sloan School of Management (MIT).

Fine, C. H. (1986), "Quality Improvement and Learning in Productive Systems," Management Science, 32, 1301-1315.

Flaherty, M. T. (1980), "Industry Structure and Cost Reducing Investment," Econometrica, 48, 1167-1209.

Hall, R. W. (1983), Zero Inventories, Dow Jones-Irwin Press, Homewood, Illinois.

Jaikumar, R. (1988), "Contingent Control of Synchronous Lines: A Theory of JIT," Working Paper No. 88-061, Harvard Business School.

Karmarkar, U. (1986), "Push, Pull and Hybrid Control Systems," Working Paper Series No. QM8614, The Graduate School of Management, University of Rochester.

Milgrom, P. and J. Roberts (1988), "Communication and Inventories as Substitutes in Organizing Production," mimeo, Stanford University.

Muth, J. F. (1986), "Search Theory and the Manufacturing Progress Function," Management Science, 32, 948-62.

Porteus, E. L. (1985), "Investing in Reduced Setups in EOQ Model," Management Science, 31, 998-1010.

Porteus, E. L. (1986), "Optimal Lot Sizing, Process Quality Improvement, and Setup Cost Reduction," Operations Research, 34, 137-144.

Reyniers, D. (1987), "Active Learning About Demand in the Newsboy Problem," mimeo.

Rothschild, M. and J. Stiglitz (1970), "Increasing Risk I: A Definition," J.E.T., 2, 225-43.

Schonberger, R. J. (1982), Japanese Manufacturing Techniques: Nine Hidden Lessons in Simplicity, New York: The Free Press.

Spence, M. (1984), "Cost Reduction, Competition, and Industry Performance," Econometrica, 52, 101-122.

Spence, M. (1981), "The Learning Curve and Competition," Bell Journal of

Economics, 12, 49-70.

Suri, R. and S. De Treville (1986), "Getting from 'Just-in-case' to 'Just-in-time'," Journal of Operations Management, 6, 295-304.

Zangwill, W. (1987), "Eliminating Inventory in a Series Facility Production System," Management Science, 33, No. 9, pp. 1150-1164.

Appendix 1

Proof of Proposition 1: The functional equation (9) can be written as:

$$V(I_0|\alpha) = \max_{A_1 \leq a \leq A_2;\, 0 \leq b \leq B_2} \{p\mu_y E_x[\min\{I_0 + ax, b\}]$$

$$- c_I E_x[\max\{0, I_0 + ax - b\}] - c_s E_x[\max\{0, b - I_0 - ax\}]$$

$$- c_a a - c_b b + \rho E_x[V(\max\{0, I_0 + ax - b\}|\alpha)]\}.$$

Define $b' = (b - I_0)/a$. Then

$$V(I_0|a) = \max_{A_1 \leq a \leq A_2;\, -I_0/a \leq b \leq (B_2 + I_0)/a} \{ap\mu_y E_x[\min\{x, b'\}]$$

$$- ac_I E_x[\max\{0, x - b'\}] - ac_s E_x[\max\{0, b' - x\}]$$

$$- a(c_a + c_b b') + (p\mu_y - c_b)I_0 + \rho E_x[V(a \max\{0, x - b'\}|\alpha)]\}.$$

Suppose that the bounds on b' are not effective. Then the choice of (a,b') does not depend on I_0 and the value function can be written as

$$V(I_o|\alpha) = (p\mu_y - c_b)I_0 + G,$$

where G is a constant to be determined and b' must be chosen to

$$\max_{b'} F_\alpha(b') = \max_{b'} \{p\mu_y E_x[\min\{x, b'\}] +$$

$$+ (\rho p\mu_y - \rho c_b - c_i)E_x[\max\{0, x - b'\}]$$

$$- c_s E_x[\max\{0, b' - x\}] - c_a - c_b b'\}$$

a^*, which appears linearly in the functional equation, with coefficient $F(b'^*)$, must satisfy

$$a^* = \begin{cases} A_1 & \text{if } F(b'^*) \quad 0 \\ A_2 & \text{otherwise.} \end{cases}$$

The optimal rate z^* must satisfy $F'_\alpha(z^*) = 0$. (Notice that $F''_\alpha(b') \leq 0$); hence, $0 = (1 - H_\alpha(b'))((1 - \rho)p\mu_y + \rho c_b + c_i) - c_s H_\alpha(b') - c_b$. The solution of this equation yields part (ii) in the proposition, and

$$F_\alpha(z^*) = (p\mu_y + c_s) \int_0^1 x dH_\alpha + \rho(p\mu_y - c_b) \int_{z^*}^1 x dH_\alpha - c_a.$$

Solving the functional equation for these values of b and a, it follows that

$$V(I|\alpha) = a^* F_\alpha(z^*)/(1 - \rho) + (p\mu_y - c_b)I_0.$$

It only rests to show that the bounds on b are indeed not effective. Consider

$$0 < b^* = I + a^* z^* \leq I + a^* \qquad \text{(since } 0 \leq z \leq 1)$$

$$\le I + A_2 \qquad \text{(since } a^* \le A_2\text{)}$$

$$\le 2A_2 \qquad \text{(since } I \le A_2\text{)}$$

$$\le B_2 \qquad \text{(by assumption).}$$

So, b^* satisfies the conditions. Furthermore,

$$I' = a^* \max\{0, x - z^*\} \le A_2 \max\{0, x - z^*\} \le A_2 \max\{0, 1 - z^*\}$$

$$= A_2(1 - z^*) < A_2.$$

Hence, $b_t = b^*$ when $I_t \le A_2$, and this implies $I_{t+1} \le A_2$, which in turn implies $b_{t+1} = b^*$. []

Proof of Proposition 2: Consider $F_\alpha(b')$ for any arbitrarily chosen b'. Let us define $u(x|b)$ as

$$u(x|b') = p\mu_y \min\{x, b'\} + (\rho p\mu_y - \rho c_b - c_I) \max\{0, x - b'\}$$

$$- c_s \max\{0, b' - x\} - c_b b' - c_a.$$

Then, $F_\alpha(b') = E_{x|\alpha}[u(x|b')]$. Clearly, $u(x|b)$ is an increasing function of x. Furthermore

$$du(x|b)/dx = \begin{cases} p\mu_y + c_s & \text{if } x < b' \\ \rho(p\mu_y - c_b) - c_I & \text{if } x > b' \end{cases}$$

and since $p\mu_y + c_s > \rho(p\mu_y - c_b) - c_I$, we conclude that $u(x|b')$ is a concave function of x. Both definitions of stochastic dominance imply,

$$E_{x|\alpha}\,(u(x|b')) \geq E_{x|\beta}\,(u(x|b'))$$

(since $u(x|b')$ is increasing and concave) whenever H_α dominates H_β. Hence

$$F_\beta(b'^*(\beta)) = E_{x|\beta}\,(u(x|b'^*(\beta))) \leq E_{x|\alpha}\,(u(x|b'^*(\beta)))$$

$$\leq E_{x|\alpha}\,(u(x|b'^*(\alpha))) = F_\alpha(b'^*(\alpha)). \qquad []$$

Notes

1. For an interesting exception, see Bielecki and Kumar (1988).

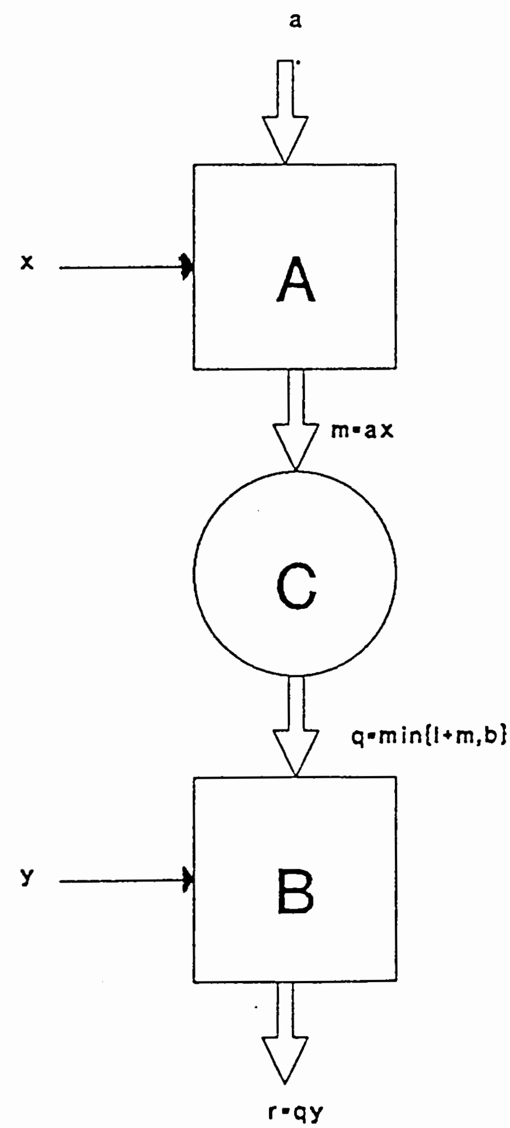
2. JIT practitioners often claim that inventories hide production problems and compare this to sea water hiding the rocks at the bottom of the sea.

3. The term learning is also applied to processes which reveal information about the demand faced by a firm. Alpern and Snower (1987, 1988), Reyniers (1987), and Milgrom and Roberts (1988) have characterized inventory policies and search procedures in the presence of an uncertain demand.

4. Which is the one setting the limit to the overall performance of the system.

5. The pair (g_α, F) is not unique since, for every strictly increasing function $r(\cdot)$, the pair $(g_\alpha r, Fr)$ also satisfies the definition. The property discussed below, however, is not affected by the particular representation chosen.

6. Notice that, by construction, $H_\alpha(g_\alpha(y)) = F(y)$.



<u>FIGURE 1</u>: The Production System