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CONTRACTUAL SOLUTIONS TO THE HOLD-UP PROBLEM*

by

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1. Introduction

A. Motivation

The hold-up problem as first described by Klein, Crawford and Alchian [1978] and Williamson [1975,1977] has come to be accepted by economists as a fundamental determinant of contractual and organizational structure. A hold-up problem occurs when two factors are present. First, parties to a future transaction must make non-contractible specific investments prior to the transaction in order to prepare for it. Second, the exact form of the optimal transaction (e.g. -- how many units if any, what quality level, the time of delivery) cannot be specified with certainty ex-ante. It depends on the resolution of uncertain parameters and these parameters cannot be objectively measured and contracted upon. The problem is that these two factors create what at least superficially appear to be conflicting requirements for the nature of the contractual structure governing the transaction. The first factor suggests that the contract ought to be relatively rigid and inflexible so that investing parties do not fear that subsequent bargaining will "rob" them of the value of their specific investments. However, the second factor suggests that the contract ought to be relatively flexible in order to allow the parties to tailor the exact form of the transaction to be optimal given the realization of the uncertain parameters. An important task for economic theory is therefore to explore the extent to which contractual arrangements can simultaneously deal with both of these apparently conflicting requirements.

The literature investigating this issue is in its infancy.[1] Basically, one type of model has been investigated. It is assumed that there are only two parties, a buyer and a seller, and the transaction decision is simply the

number of units to exchange (this is often a 0-1 decision). Contracts are assumed to be very simple, usually only specifying a price if exchange occurs and a price if no exchange occurs. Finally, a fixed bargaining game is assumed to be played after the resolution of uncertainty. The contractual provisions thus influence the final outcome by affecting the threat points of the bargaining game. Three points have emerged. First, in the absence of any contract, investment is likely to be inefficiently low under most plausible bargaining games. Second, rather simple written contracts can achieve surprisingly complicated final outcomes when they are viewed as affecting subsequent bargaining rather than as completely describing the outcome. Third, even in the simple environments being considered, simple contracts used in the presence of an existing ex-post bargaining game will not generally fully resolve the problem -- i.e. -- the optimal simple contract still generally results in inefficient investment.

From the standpoint of mechanism design, the existing literature restricts itself to a very small subset of the set of all possible incentive compatible mechanisms. Given the economic plausibility of their restrictions, the results are of great interest. Nonetheless, it is also clearly of great interest to be able to identify the optimal mechanism from the set of all possible incentive compatible mechanisms for a variety of reasons. Can the hold-up problem be solved by complex enough contracts? Are the fully optimal contracts too complex to implement? How close do simple contracts come to the fully optimal solution? All of these questions (and many more) require one to calculate the fully optimal mechanism.

This paper shows that within the class of all possible incentive compatible mechanisms, that first-best solutions to the hold-up problem generally exist even in extremely complex environments involving n agents with arbitrarily complex transaction decisions and utility functions. The basic approach is very simple. A large literature exists which analyzes the mechanism design question for the model where each of n agents has a utility function which depends on a collective choice and his own type parameter. Although the type parameter is determined probabilistically, it is not affected by any ex-ante investment decisions. This paper generalizes this model by allowing each individual to make an investment decision which affects the probabilistic realization of his own type. The remarkable result which is shown is that the efficient mechanisms identified in the existing literature for the no-investment case also create first-best investment incentives when an investment decision is added. Section B will explain this result in more detail.

B. Method of Analysis

Three different information scenarios are considered. It is always assumed that investment levels and type realizations cannot be specified in an objectively verifiable manner in contracts. However, whether the agents can observe one another's investment levels and type realizations is allowed to vary. This yields three cases of interest.[2]

Completely Private Information (CPI): Each agent's investment choice and the realization of each agent's type are his own private information.

Partially Private Information **(PPI)**: Each agent's investment choice is public information. However, the realization of each agent's type is his own private information.

Non-Private Information **(NPI)**: Each agent's investment choice and the realization of each agent's type are public information.

In the existing mechanism design literature which does not allow an investment choice there are of course only two information scenarios -- private types and public types. For the private types case D'Aspermont and Gerard-Varet [1979a,b] have shown that a mechanism exists which yields the efficient outcome as a Bayesian-Nash equilibrium. More recently Cremer and Riordan [1985] have created an "improved" mechanism where n-1 of the n agents have a dominant strategy. The public-types case was first extensively explored by Maskin [1977].[3] Recently, Moore and Rupello [1988] have demonstrated that almost any choice rule (including the efficient one) can be implemented in the type of model considered in this paper where there is transferable utility.

The major result of this paper is to show that by a remarkable coincidence, the D'Aspermont-Gerard-Varet mechanism and the Cremer-Riordan mechanism continue to provide first-best solutions even when an <u>ex-ante</u> investment choice exists for the CPI case. The reason is very straightforward. Both of the above mechanisms create incentives for truthful revelation of types because they exhibit a property which D'Aspermont and Gerard-Varet term being "subjectively discretionary." However, this property also automatically guarantees that each agent will have the correct investment decisions.

This result turns out to depend critically on the assumption that each agent's investment choice is private. Intuitively, the reason for this is as follows. Suppose one agent contemplates deviating from his equilibrium investment choice. Then, because the other agents do not observe this deviation, they will continue to truthfully report their types as part of a Bayesian-Nash equilibrium at the second stage of the game. Thus an individual deviation from the equilibrium investment level does not disrupt the subsequent operation of the mechanism. However the Cremer-Riordan mechanism gives dominant strategies to n-1 of the agents to truthfully report their types. Thus it follows immediately that the Cremer-Riordan mechanism will continue to provide first-best outcomes when an investment choice exists for the PPI case so long as only one of the agents makes an investment choice. (One chooses the Cremer-Riordan mechanism which gives the other n-1 agents dominant strategies.) Thus a weaker result is obtained for the PPI case.

Finally, consider the NPI case. The key insight is that any mechanism which implements the D'Aspermont-Gerard-Varet or Cremer-Riordan mechanisms as social choice functions will automatically create first-best investment incentives at the earlier stage because they are subjectively discretionary. Therefore the achievability of the first-best simply depends on the implementability of a particular social choice function. However, as Moore and Rupello [1988] show, any social choice function can be implemented. Thus the first-best is achievable under the NPI case by using the mechanism of Moore and Rupello [1988] to implement the outcome choice functions and transfer rules of the D'Aspermont-Gerard-Varet or Cremer-Riordan mechanisms.

Finally, note that this paper requires that the sum of transfers be zero. However if this assumption is relaxed then Groves mechanisms can be considered. The above results imply that Groves mechanisms will implement efficient investment and exchange decisions for all three information cases.

C. Existing Literature

I am aware of only one other paper which solves for the fully optimal incentive compatible mechanism in a collective choice model with non-contractible investment. This is by Konakayama, Mitsui, and Watanabe [1986]. For the case of CPI, they consider a very simple problem where the buyer and seller must decide whether to exchange one unit of a good with fixed characteristics. By investing in advance the seller can stochastically reduce his cost of production. The buyer makes no investment decision. In this environment they prove the rather surprising result that the buyer and seller can sign a contract which produces the first-best outcome. (Under the optimal contract trade occurs if and only if the buyer's value exceeds the seller's cost and the seller chooses a level of investment which maximizes the expected gains from trade.)

Although the analysis of the paper is elegant and clear it seems to rely heavily on the special structure of the particularly simple exchange problem being assumed. Thus it is not clear from their analysis whether the authors have identified a special case of an important general idea or have simply identified a very special case with an anomalous sort of solution. It will be shown below that the authors have in fact simply identified the Cremer-Riordan mechanism for their particular problem.

D. Organization of the Paper

Section 2 describes the model for the CPI case. Then Section 3 analyzes this case. Section 4 considers the PPI case and then Section 5 considers the NPI case. Section 6 briefly considers Groves mechanisms. Finally, Section 7 draws conclusions.

2. The Model; The CPI Case

The model will be described for the CPI case. Subsequent sections which analyze other cases will introduce appropriate alterations. Since the model is so well known it will be described as briefly as possible. See Green and Laffont [1977] and D'Aspermont and Gerard-Varet [1979a] for more detailed discussions.

There are n agents indexed by $i \in \{1, \ldots, n\}$. Agent i is of some type a_i chosen from the set A_i. The collective choice to be made is to choose an element x from the set X. If agent i is of type a_i, the collective choice is x, and he receives I dollars, then agent i's utility is given by

$$(2.1) \qquad v_i(x,a_i) + I.$$

Finally, agent i also makes an investment choice y_i from the set Y_i. Agent i's type is then determined by the density function $f(a_i/y_i)$.[4] It will be assumed that the a_i's are distributed independently of one another.

Some mathematical notation will be useful. Let A denote $\pi_{i=1}^{n} A_i$ and let a denote an element of A. Let a_{-i} denote the vector $(a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$ and let (a_{-1}, a_i^*) denote the vector $(a_1, \ldots, a_{i-1}, a_{i+1}^*, \ldots, a\ )$. Define Y, y, y_{-1}, and (y_{-i}, y_i^*) similarly. Finally let $E_{-1}\ (\ /y_{-i})$ denote the expectation operator over a given the investment vector y and let $E_{-i}(\ /y_{-i})$ denote the expectation operator over a_{-i} taking y_{-i} as given.

The sequence of decisions and distribution of information is as follows. The relationship can be thought of as evolving over 4 periods. In period 1 the agents' types are not yet drawn and investment levels have not yet been chosen. The density functions, f_i, and utility functions, v_i, are public information. The agents can sign a contract[5] in period 1. In period 2 agents choose an investment level. Their choice is private information. In period 3 the agents' types are drawn according to the densities f_i. Their types are private information. Finally in period 4 the contract is executed.

It will be assumed that unique maxima to various maximization problems exist. Let $\phi^*(a)$ be the unique maximum over the set X to

(2.2) $$\sum_{i=1}^{n} v_i(x, a_i\ )\ .$$

Let y^* be the unique maximum over the set Y to

$$(2.3) \qquad E\{\sum_{i=1}^{n} v_i (\phi^{*}(a), a_i)/y\} .$$

An outcome is defined to be a vector $y = (y_1, \ldots, y_n)$ and a function $\phi: A \to X$ where y gives the investment choices of agents and ϕ denotes the rule for determining the collective choice given the types. An outcome will be said to be exchange-efficient if $\phi = \phi^{*}$, investment efficient if $y = y^{*}$, and efficient if it is both exchange and investment efficient. The efficient outcome of course maximizes the expected sum of utilities and would be the rule the agents would choose if y and a were public information and could be contracted upon.

Following the familiar revelation mechanism approach, a contract is a function (d,t) from A to X x R^n where d is called the decision rule and t is called the transfer rule. It is interpreted as follows. In period 1 the agents agree on the contract. Then in period 4 the agents announce their types. If $\hat{a}$ is the vector of announcements, then $d(\hat{a})$ is the collective choice and $(t_1(\hat{a}), \ldots, t_n(\hat{a}))$ is the vector of payments received by the individuals - i.e. - agent i receives $t_i(\hat{a})$ dollars. If $t_i(\hat{a})$ is negative agent i pays money. As part of the definition of a contract it will be assumed that the transfer rule must balance. That is, it will be assumed that the contract must satisfy

$$(2.4 \qquad \sum_{i=1}^{n} t_i(a) = 0$$

for every $a \in A$. This means that side payments between the individuals must sum to zero.

A strategy for agent i is a vector $(y_i, \alpha_i(a_i))$ where y_i is an investment level and $\alpha_i(a_i)$ is agent i's announced type given his real type is a_i. Let (y,α) denote a vector of strategies for all n individuals. The vector (y,α) will be called an equilibrium strategy under the contract (d,t) if (y,α) is a Bayesian-Nash equilibrium given the contract (d,t).

Definition:

(y,α) is an equilibrium strategy given the contract (d,t) if (2.5) and (2.6), below, hold.[6]

$$(2.5) \qquad \alpha(a_i) \ \varepsilon \ \underset{\hat{a}_i}{\text{argmax}} \ E_{-1} \left[v_i(d(\alpha_{-i}, \hat{a}_i), a_i) + t_i(\alpha_{-i}, \hat{a}_i)/y_{-i}\right]$$

$$(2.6) \qquad y_i \ \varepsilon \ \underset{\hat{y}_i}{\text{argmax}} \ E\left[v_i(d(\alpha), a_i) + t_i(\alpha)/(y_{-i}, \hat{y}_i)\right]$$

A contract will be said to implement an outcome if equilibrium strategies will yield that outcome.

Definition:

The contract (d,t) implements the outcome (y,ϕ) if there exists an equilibrium strategy $(\hat{y},\hat{\alpha})$ to (d,t) such that

$$(2.7)\qquad d(\hat{\alpha}(a)) = \phi(a) \quad \text{for every } a \in A$$

and

$$(2.8)\qquad y = \hat{y}\ .$$

A contract will be called exchange efficient, investment efficient or efficient if, respectively, it implements an exchange efficient, an investment efficient or the efficient outcome. The goal of the paper is to investigate whether an efficient contract exists.

A few more definitions regarding the structure of contracts will be useful. Consider the contract (d,t). It will be called naively exchange efficient (NEE) if it selects the optimal collective choice given the announcements - i.e. - if $d = \phi^*$. The transfer rule for agent i can always be written in the form

$$(2.9)\qquad t_i(a) = \sum_{j \neq i} v_j(d(a), a_j) + g(a)$$

for some function g(a). The contract (d,t) will be called discretionary for agent i if g(a) does not depend on a_i - i.e. - if

$$(2.10)\qquad g(a_{-i}, \hat{a}_i) = g(a_{-i}, \hat{\hat{a}}_i)$$

for every $a_{-i} \varepsilon A_{-i}$ and $\hat{a}_i$, $\hat{\hat{a}}_i \varepsilon A_i$. It will be called subjectively discretionary for agent i given y_{-i} if agent i's expectation of g (calculated when he knows his type) does not depend on a_i - i.e. - if

$$(2.11) \qquad E_{-i}\left[g(a_{-i},\hat{a}_i)/y_{-i}\right] = E_{-i}\left[g(a_{-i},\hat{\hat{a}}_i)/y_{-i}\right]$$

for every $\hat{a}_i$, $\hat{\hat{a}}_i \varepsilon A_i$. Obviously, the first property implies than the second. That is, if (d,t) is discretionary for agent i, then it is subjectively discretionary for agent i for every $y_{-i} \in Y_{-i}$.

3. Analysis: The CPI Case

A. A Fixed Investment Level

In Section A it will be assumed that there is only a trivial investment choice. Each Y_i is the one element set $\{y_i\}$. This produces the standard collective choice model considered by the public goods literature. The results of these papers necessary for this paper's analysis will be reported in this section. Then Section B will show how these results generalize to cases where the investment choice is non-trivial.

Early work on dominant strategy mechanisms relied on the observation that if a contract (d,t) is NEE and discretionary for agent i, then it is a dominant strategy for agent i to truthfully reveal his type. In particular, then, if (d,t) is NEE and discretionary for every i, then it implements the efficient outcome by an equilibrium strategy where it is a dominant strategy for each individual to truthfully reveal his type. Unfortunately, except for very special cases, it is not possible to create such contracts which balance.[7]

D'Aspermont and Gerard-Varet [1979a] observed that the weaker property of being subjectively discretionary is sufficient to induce a Bayesian-Nash equilibrium where each individual truthfully reveals his type. This is stated as Proposition 1, below.

Proposition 1: (D'Aspermont and Gerard-Varet [1979a], Theorem 5)

Suppose $Y_i = \{y_i\}$ for every i. Suppose that (d,t) is NEE and is subjectively discretionary given y_{-i} for every i. Then:

(i) It is an equilibrium strategy for each individual to truthfully reveal his type.

(ii) Therefore (d,t) implements the efficient outcome.

proof:

See D'Aspermont and Gerard-Varet [1979a].

QED

They then constructed a (balanced) contract which was NEE and subjectively discretionary given y_{-i} for every i. This is described in the Appendix. More recently Cremer and Riordan [1985] have constructed another efficient contract. Like the original mechanism of D'Aspermont and Gerard

Varet, the Cremer-Riordan mechanism is NEE and subjectively discretionary given y_{-i} for every i and this is why it is efficient. However, it is "improved" in the sense that (n-1) of the n transfer rules are discretionary (not merely subjectively discretionary). Thus n-1 of the n individuals have a dominant strategy and only the n^{th} individual has merely a Bayesian-Nash optimal strategy. This is also described in the Appendix.

B. Variable Investment Levels

In Section B it will be assumed that the sets Y_i may have more than one element. Proposition 2 shows that Proposition 1 generalizes to this environment. Recall that (ϕ^*, y^*) is the efficient outcome.

Proposition 2:

Consider the contract (d,t). Suppose that (d,t) is NEE and is subjectively discretionary for y^*_{-i} for every i. Then (d,t) implements (ϕ^*, y^*).

proof:

Consider agent i's choice of a strategy. Suppose that for every $j \neq i$, that agent j chooses y^*_j and truthfully reveals his type. It will be shown that it is optimal for agent i to do the same. Suppose agent i chooses $\hat{y}_i$. By Proposition 1 agent i will find it optimal to report his type truthfully. Therefore if agent i chooses $\hat{y}_i$ his expected utility will be

$$(3.1) \qquad E\left\{v_i(d(a),a_i) + t_i(a)/(y^*_{-i},\hat{y}_i)\right\}.$$

Substitute (2.9) into (3.1) to yield

$$(3.2) \qquad E\left\{\sum_{j=1}^{n} v_j(d(a),\ a_j)/(y^*_{-i},\ \hat{y}_i)\right\}$$

$$+\ E\left\{g(a)/(y^*_{-i},\ \hat{y}_i)\right\}$$

It is obviously sufficient to show that the second term of (3.2) does not depend on $\hat{y}_i$. Rewrite it as

$$(3.3) \qquad E_i\left\{E_{-i}\ \{g(a)/y^*_{-i}\}/\hat{y}_i\right\}$$

where $E_i\{\ /\hat{y}_i\}$ denotes the expectation over a_i given $\hat{y}_i$. Since t_i is subjectively discretionary given y^*_{-i} as defined by (2.11), the inner expectation does not depend on a_i. Therefore (3.3) can be rewritten as

$$(3.4) \qquad E_i\ \{h(y^*_{-i})/\hat{y}_i\}$$

where h is a function of y^*_{-i}. Expression (3.4) equals $h(y^*_{-i})$ which does not depend on $\hat{y}_i$.

QED

An immediate Corollary of this is that the D'Aspermont-Gerard-Varet and Cremer-Riordan mechanisms (constructed for $y = y^*$) will induce the efficient outcome. Thus efficient contracts exist.

4. The PPI Case

This section will now consider the case where investment levels are publicly observed but agents' types are private. It is instructive to investigate why Proposition 2 is no longer true for this case. In the proof, agent i could assume that all other individuals would continue to tell the truth no matter what his choice of investment, $\hat{y}_i$. This was because they could not observe $\hat{y}_i$ and were assuming that agent i chose y_i^*. It is straightforward to show that if investment is observable that the argument will still work for agent i if the transfer rules for all other individuals are discretionary. This is because all other agents then have a dominant strategy to truthfully reveal their types. Of course a balanced contract cannot be created when all n transfer rules are discretionary. The Cremer-Riordan [1985] mechanism allows one to choose n-1 of the transfer rules to be discretionary. Thus if investment is observable, the Cremer-Riordan [1985] mechanism will implement the efficient outcome if only one of the n individuals makes an investment choice. One simply chooses the Cremer-Riordan mechanism which gives the other n-1 individuals the discretionary transfer rule. This result is summarized in Proposition 3.

Proposition 3:

For the case of PPI, suppose that only individual 1 makes a non-trivial investment choice. (i.e. - Y_i is a singleton set for every $i \neq 1$). Then the Cremer-Riordan mechanism (constructed for $y = y^*$) which gives individuals 2 through n dominant strategies implements the efficient outcome.

proof:

As above.

QED

In case of an exchange between a buyer and seller where only one of the two parties must invest in advance, Proposition 3 suggests that the Cremer-Riordan mechanism (constructed for $y = y^*$) which gives the non-investing party a dominant strategy is the best mechanism to use. This mechanism is efficient whether investment is observable or not.[8]

5. The NPI Case

This section will consider the case where agents' investment levels and types are public information. First consider the following simple problem. Suppose that the agents have signed a contract governing how x and a vector of transfer payments $(I_1, \ldots, I_n)$ would be determined. Furthermore suppose that the agents can predict that the equilibrium outcome of the operation of this contract will result in the choices

$$x = d(a_1, \ldots, a_n) \tag{5.1}$$

and

$$I_i = t_i(a_1, \ldots, a_n) \tag{5.2}$$

for some functions d and $\{t_i\}_{i=1}^n$. Note that the equilibrium outcomes will in general depend on the realization of the agents' types.

The set of functions (d,t) was called a contract in the previous sections. It will now be convenient to view (d,t) as the outcome of a possibly more complicated contract where strategies may not simply consist of reporting one's own type. For the purposes of this section (d,t) will be called a social choice function. As in the previous sections it will be required that the transfers sum to zero as part of the definition.

<u>**Definition**</u>:

A social choice function is a pair of functions (d,t) where $t = (t_1, \ldots, t_n)$ and

(5.3) $$d: A \to X$$

(5.4) $$t_i: A \to R$$

(5.5) $$\sum_{i=1}^{n} t_i = 0 .$$

A Bayesian-Nash equilibrium in investment levels can now be defined given the agents' expectations that the operation of the contract will produce the social choice function (d,t). This is formally defined below.

<u>**Definition**</u>:

Let $y = (y_1, \ldots, y_n)$ denote a vector of investments. Then y is an equilibrium given (d,t) if

$$(5.6) \qquad y_i \in \underset{\hat{y}_i}{\operatorname{argmax}}\ E\left[v_i(d(a),a_i) + t_i(a)/(y_{-i},\hat{y}_i)\right]$$

for every i.

The definitions of being naively exchange efficient (NEE) subjectively discretionary and discretionary were given for the ordered pair of functions (d,t) and did not depend on the interpretation of (d,t) as a contract. Therefore these definitions can also be applied when (d,t) is viewed as a social choice function. The key insight is that y^* is an equilibrium given (d,t) if the social choice function is NEE and subjectively discretionary for every agent given y^*. This is stated as Proposition 4. Since the proof is simply a somewhat simplified version of the proof of Proposition 2 it will not be presented.

<u>Proposition 4</u>:

Suppose that (d,t) is naively exchange efficient and subjectively discretionary given y^*_{-i} for every i. Then y^* is an equilibrium given (d,t).

In previous sections the D'Aspermont-Gerard-Varet and Cremer-Riordan mechanisms (constructed for $y = y^*$) were viewed as contracts. Now instead view them as social choice functions. As stated in previous sections, both are NEE and subjectively discretionary given y^*. Therefore if a contract could be found which yielded either of these two social choice functions as a unique equilibrium, then this contract would be efficient. It would yield an efficient choice of x because the social choice function is NEE. It would yield an efficient choice of y by Proposition 4.

All of the above analysis has therefore reduced the question to one considered in the standard implementation literature. Namely, does a contract exist which implements a given social choice rule. The relevant result for the purposes of this paper is contained in Moore and Rupello [1988]. They show that when utility functions are additively separable in a transfer payment as in this paper (and given some very weak additional assumptions) that almost any social choice function can be implemented as the unique sub-game perfect equilibrium to an appropriately designed stage game. In particular, the social choice functions determined by the D'Aspermont-Gerard-Varet and Cremer-Riordan mechanisms can be so implemented.

The mechanism of Moore and Rupello will not be described. Readers should refer to Moore and Rupello [1988] for a description of the mechanism as well as an illuminating discussion of how their result relates to the implementation literature in general. Basically the mechanism involves n separate stages, one for each agent. At stage i, agent i is induced to truthfully reveal his type. This is accomplished by designing a game where some other agent has the incentive to correct agent i's report if he lies.

Two points should be noted about this mechanism. First, it is a full order of magnitude more complicated than mechanisms considered in the previous sections because now agents must make reports not only about their own types, but about other agents' types.

Second, the fact that the Moore-Rupello mechanism has n entirely separate stages means that there is a particularly simple method to use it to implement the Cremer-Riordan mechanism as a social choice function. Suppose that the Cremer-Riordan mechanism which gives agents 2 through n a dominant strategy to tell the truth is used. Then a two step mechanism of the following sort would implement the Cremer-Riordan mechanism.

Step 1: Agent 1 is induced to truthfully reveal his type, through the Moore-Rupello mechanism.

Step 2: Agents 2 through n play the Cremer-Riordan mechanism as a revelation game where agent 1's type from stage 1 is used for agent 1's report.

The point being made here is that the Moore-Rupello device does not have to be used n times, one for each agent. Rather, it only has to be used once for the agent who does not have a dominant strategy. This considerably simplifies the nature of the mechanism.

6. Groves Mechanisms

A contract (d,t) is called a Groves mechanism if it is NEE and discretionary for every agent. Such contracts were not considered by this paper because balanced Groves mechanism generally do not exist. The purpose of this brief section is to note a simple fact that is probably obvious to most readers given the previous analysis. This is that a Groves mechanism will implement the efficient outcomes under all three information cases. Truthful revelation is a dominant strategy for every agent and will thus occur for every information case. Then the technique used in proving Proposition 2 establishes that y^* is a Bayesian-Nash equilibrium.

Therefore if the requirement that contracts balance is not important, perhaps because of the existence of an extra agent who can act as a residual claimant, then Groves mechanisms provide not only a first-best solution to the simple collective choice problem (as has been established in the existing literature) but also a solution to the collective choice problem when ex-ante investments must be made.

7. The Contracting Problem of Konakayama, Mitsui, and Watanabe [1986]

Konakayama, Mitsui, and Watanabe [1986] consider a special case of the general contracting problem considered by this paper for the CPI case. In their example n = 2. Let agent 1 be the seller and agent 2 be the buyer. Let X = {0,1} be the decision set where 0 denotes no trade and 1 denotes trade. Agent 1's utility is

$$(7.1) \qquad v_1(x, a_1) = \begin{cases} -a_1, & x = 1 \\ 0\ , & x = 0 \end{cases}$$

where $A_1 = [0,1]$. That is, a_1 is the seller's cost if trade occurs. If no trade occurs he does not produce the good and his cost is zero. Agent 2's utility is

$$(7.2) \qquad v_2(x, a_2) = \begin{cases} a_2, & x = 1 \\ 0\ , & x = 0 \end{cases}$$

where $A_2 = [0,1]$. That is, a_2 is the buyer's value if trade occurs. If no trade occurs the buyer does not consume the good and his value is zero.

Finally the authors assume that only agent 2 (the buyer) makes a non-trivial investment decision. For agent 2, $Y_2 = R$ and agent 2's type is determined by $F_2(a_2/y_2)$. Agent 1's type is determined by $F_1(a_1)$.

The authors construct a contract which implements the first-best outcome. Furthermore, agent 2 has a dominant strategy to truthfully reveal his type. It is straightforward to show that the authors' contract is the Cremer-Riordan mechanism for their special environment.

8. Conclusions

A hold-up problem is said to occur when parties to an exchange must make non-contractible specific investments ex-ante and the nature of the optimal ex-post transaction should vary depending on the resolution of non-contractible uncertainties. The major conclusion of this paper is that first-best contractual solutions to the hold-up problem exist if certain environmental properties are satisfied and if "powerful" contracts can be written.

Three important environmental properties must be satisfied:

1) Risk Neutrality

2) No Externalities

   Each agent's investment directly affects only his own type. Thus a situation, for example, where a seller's investment affects the quality of a product he will sell to a buyer is not allowed.

3) Only one investor under PPI

   If the PPI case holds, then only one agent makes an investment decision.

The contracts which can be written are assumed to be "powerful" in three senses:

1) Complexity

Complex contracts can be written

2) Commitment to Participate

Agents cannot simply decide to renege on the contract if it turns out that they expect to make losses at some point.

3) No Renegotiation

The contract can prevent agents from attempting to renegotiate the outcomes of the contract.

Therefore the most important conclusion of this paper may be that it simply clarifies the nature of the hold-up problem. In particular, the hold-up problem does <u>not</u> necessarily cause inefficiencies. Rather if one believes that the hold-up problem generally causes inefficiencies, then it must be because one believes that one or more of the above requirements is generally not satisfied.

One general comment will be made about these requirements before discussing a number of them in more detail. The general comment is that this paper has not, strictly speaking, demonstrated that any of these six requirements are <u>necessary</u> for a first-best solution. Rather, it is simply the case that the contractual solution based on subjectively discretionary transfers does not appear to generalize in any obvious fashion when these requirements are not met. Therefore an important topic for future research concerns investigating to what extent these requirements can be relaxed.

Now the requirements will be briefly discussed in more detail. First consider the environmental properties. Two remarks should be noted. First, although one can imagine many cases where the properties are not satisfied, there are many contracting situations where they will be. The result holds for remarkably general environments in many respects. Arbitrarily general type spaces, investment spaces, and utility functions are allowed. Furthermore, the collective decision itself may be arbitrarily complex and arbitrarily large numbers of agents can be involved.[9] The second remark concerns the requirement that there only be one investor under the PPI case. This requirement probably reflects a more general economic idea. Namely, the hold-up problem will generally be easier to solve if investment levels are private information. This is because the actual levels of investment will not affect exchange and bargaining at the second stage. Intuitively, one might think that "opportunism" is less severe when specific investments are private information.[10]

Now consider the issue of complexity. Two remarks should be noted. First, the complexity requirement is clearly most severe for the NPI case where exchange occurs under symmetric information. This is because mechanisms where agents report not only their own type but other agents' types must be used. Second, however, it is not at all clear that the required contracts are too complex to be used, especially for the CPI and PPI cases. The Cremer-Riordan mechanism is actually quite simple in the standard sorts of two-person exchange environments. Both Riordan [1984] and Konakayama, Mitsui, and Watanabe [1986] have considered simple exchange problems where the Cremer-Riordan mechanism could easily be written in a simple contract. Even for the NPI case, it was shown that one could limit use of the more complex Moore-

Rupello mechanism to eliciting a single agent's type. Furthermore, even the Moore-Rupello mechanism does not appear to be particularly complex in simple exchange problems. Therefore in the simple types of two agent exchange problems considered in the existing literature, it is far from clear that the first-best contracts are too complex to write.

Now consider the issue of commitment to participate. The contractual solutions identified in this paper satisfy ex-ante individual rationality. That is, at the time of signing, all agents want to sign.[11] However, as uncertainty resolves itself over the course of the transaction, individual agents may well end up in a position where they expect to, or actually definitely will, make losses. This paper has assumed that in such cases the affected agents cannot simply renege on the contract. They must continue to participate. Three remarks should be noted about this. First, the existing literature on the hold-up problem generally assumes that binding commitments are possible as well. Second, it seems clear that contracts will generally be unable to provide first-best solutions without the possibility of making binding commitments. For example, Myerson and Satterthwaite [1983] have solved for the optimal contract in a simple two person model with interim individual rationality constraints[12] when there is no investment problem. Even with no investment problem an efficient outcome cannot be achieved. Therefore the optimal contract with an investment problem would clearly not be efficient either.[13] The third remark is that this requirement may bind most severely for the NPI case. This is because the Moore-Rupello mechanism requires potentially very large penalties to be levied as part of the device which elicits truthful revelation of types. The large penalties are levied only in the event that an agent lies. Thus they never occur on the equilibrium path. Nonetheless, the mechanism only works if agents believe very large penalties can be levied.

Finally, consider the issue of renegotiation. Note that the contracts in this paper yield ex-post efficient outcomes. Thus no renegotiation is possible in equilibrium. However, the equilibrium outcomes are supported by threats of inefficient outcomes off the equilibrium path. For these threats to be credible requires that these outcomes will not be renegotiated. The defense of this assumption must be that renegotiation can be slow and/or inefficient due to bluffs, mistaken intentions, etc. In this case, particularly if decisions must be made quickly, the assumption that no renegotiation occurs may not be unreasonable. Note that this criticism applies to almost all of the mechanism design literature and not merely this paper.

Therefore, in this paper's environment, where it is assumed that arbitrarily complex contracts can be signed, renegotiation is a negative. Any mechanism, including the "state of nature" mechanism assumed to govern renegotiation, can be created through a contract. Thus the inability to prevent renegotiation simply limits the set of mechanisms which can be chosen. It is interesting to contrast this situation to that in the existing literature on the hold-up problem which assumes that only extremely simple contracts can be signed. Far from creating a problem, renegotiation is then viewed as being extremely desirable. It is the existence of renegotiation which allows rather simple contracts to in fact implement rather complicated mechanisms. Therefore in the existing literature, the question of whether renegotiation can be prevented or not generally does not arise. This is because the parties want renegotiation to occur given the restriction that they must write extremely simple contracts. Obviously an interesting question for future research concerns investigating the nature of the optimal renegotiation-proof contract in the general setting of this paper.[14]

Appendix

The purpose of this brief Appendix is to explicitly define the D'Aspermont-Gerard-Varet and Cremer-Riordan mechanisms for those who are unfamiliar with them.

D'Aspermont and Gerard-Varet [1979a] prove that their mechanism has subjectively discretionary transfer rules in Theorem 6 of their paper and this result will not be reproven here. Cremer and Riordan [1985] do not explicitly prove (or even claim) that their transfer rules are subjectively discretionary.[15] Therefore, although the proof is simple, it will be given here.[16]

Fix the vector of investments at some levels given by y. Then the D'Aspermont/Gerard-Varet Mechanism given y is calculated as follows. Define $\gamma_i(a_i)$ by

$$\text{(A.1)} \qquad \gamma_i(a_i) = E_{-i}\left\{\sum_{j \neq i} v_j(\phi^*(a), a_j)/y_{-i}\right\}$$

for every i. Then the D'Aspermont-Gerard-Varet contract is given by

$$\text{(A.2)} \qquad d(a) = \phi^*(a)$$

$$\text{(A.3)} \qquad t_i(a) = \gamma_i(a_i) - \frac{1}{n-1} \sum_{j \neq i} \gamma_j(a_j) + z_i$$

where z_i are any n constants satisfying

$$(A.4)\qquad \sum_{i=1}^{n} z_i = 0.$$

Now consider the Cremer-Riordan solution. Define $\delta_i(a_i)$ by

$$(A.5)\qquad \delta_i(a_i) = E_{-i}\left\{\sum_{j=1}^{n} v_j(\phi^*(a),a_j)/y_{-i}\right\}.$$

Recall that Cremer and Riordan construct a contract where n-1 of the n agents have dominant strategies. The contract where all individuals except agent i have a dominant strategy is defined as follows. The decision rule is

$$(A.6)\qquad d(a) = \phi^*(a)$$

The transfer rules for all individuals except agent i are defined by

$$(A.7)\qquad t_j(a) = \sum_{k\neq j} v_k(\phi^*(a),a_k) - \delta_i(a_i) + z_j$$

where the z_j are (n-1) constants. Finally, agent i's transfer rule is

$$(A.8)\qquad t_i(a) = -\sum_{j\neq i} t_j(a) .$$

The transfer rules for all individuals other than agent i are obviously discretionary (and thus also subjectively discretionary). It will now be shown that t_i is also subjectively discretionary.

Proposition:

The transfer rule t_i defined by (A.8) is subjectively discretionary given y_{-i}.

proof:

The transfer rule t_i can be rewritten as

$$(A.9) \qquad t_1(a) = \sum_{j=2}^{n} v_j(\phi^*(a), a_j) + \Gamma(a)$$

where

$$(A.10) \qquad \Gamma(a) = (n-1)\delta_i(a_i) - (n-1) \sum_{j=1}^{n} v_i(\phi^*(a), a_i) - \sum_{j \neq i} k_i .$$

By taking expectations and using the definition of $\delta_i(a_i)$ in (A.5), we have

$$(A.11) \qquad E_{-i}\left\{\Gamma(a)/y_{-i}\right\} = - \sum_{j \neq i} k_i .$$

In particular, the expectation does not depend on a_i. Therefore t_i is subjectively discretionary given y_{-i} .

QED

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Notes

1. See Chung [1989], Green and Laffont [1987], Grossman and Hart [1986], Hart and Moore [1988], Huberman and Kahn [1988], Rogerson [1984], Shavell [1984], Teresawa [1984], Tirole [1986], Wiggins [1988], and Williamson [1983].

2. It will become clear that the fourth possible case (where investment choice is private but type is public) is in fact equivalent to the NPI case. Once type is known the issue of whether investment is known or not is irrelevant. Thus only three cases need to be considered.

3. See Maskin [1985] for a general survey and further references.

4. It is no more general to assume that the utility functions are of the form

$$u_i(x, a_i, y_i) + I$$

with type determined by $g_i(a_i/y_i)$. In this case define a "new" type space A_i^* to be $A_i \times Y_i$. Now a density function $f_i(a_i^*/y_i)$ and utility function $v_i(x, a_i^*) + I$ can be defined.

5. A contract will be defined below.

6. The vector $(\alpha_{-1}, \hat{\alpha}_i)$ denotes $(\alpha_1(a_1), \ldots, \alpha_{i-1}(a_{i-1}), \hat{a}_i, \alpha_{i+1}(a_{i+1}), \ldots, \alpha_n(a_n))$. The vector α denotes $(\alpha_1(a_1), \ldots, \alpha_n(a_n))$.

7. See Green and Laffont [1977].

8. Note that Konakayama, Mitsui and Watanabe [1986] chose the reverse mechanism. In their example only the buyer invests. However, they chose the Cremer-Riordan mechanism which gives the buyer a dominant strategy. See the discussion in Section 7.

9. It may be that the chief problem created by complex environments is that the required contractual solutions also become more complex. This will be discussed further below.

10. This intuition is also illustrated by Tirole's [1986] analysis.

11. Given that the contracts maximize the expected sum of utilities, lump sum transfer payments can always be chosen so that all agents will want to participate, <u>ex-ante</u>.

12. They consider a private-types model. Interim individual rationality means that agents still wish to participate after they learn their type but before they learn other agents' types.

13. An interesting research topic would be to see how the need to provide investment incentives affects the Myerson-Satterthwaite solution.

14. See Maskin and Moore [1987] for a discussion of the concept of "renegotiation-proof" and for other references.

15. In the Cremer-Riordan mechanism the n-1 individuals with dominant strategies clearly have discretionary transfer rules. The question is whether the transfer rule of the n^{th} individual is subjectively discretionary.

16. Cremer and Riordan directly prove that their contract is efficient in their model with no investment choice. A somewhat simpler proof would be to show that their transfer rules are subjectively discretionary. Efficiency of the contract then follows from D'Aspermont and Gerard-Varet's result reported as Proposition 1 in this paper.