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RISKY R&D IN OLIGOPOLISTIC PRODUCT MARKETS

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ABSTRACT: Recent work has established that firms select excessively risky R&D strategies. We argue that this conclusion is sensitive to the assumptions that the winner takes all and that consumers receive no surplus. We relax these assumptions and assume instead that firms compete as oligopolists after the completion of R&D. Our finding is that too litle risk-increasing R&D occurs. This is because investment which increases the riskiness of the distribution of a firm's production costs or product characteristics induces a positive externality to both consumers and rival firms. These results are robust across several standard models of product market competition.

I. Introduction

Recent work on the choice of R & D technology has established a presumption that firms opt for greater risk than is collectively optimal. This result follows from the negative externality associated with risk-increasing R & D. For example, in the patent race model considered by Klette and de Meza (1986), firms race for an innovation of known size with an R & D technology that yields risky discovery dates. There, a firm's R & D increases both the probability that it will discover the innovation very early and the probability that it will discover the innovation very late; because the innovation is more valuable at early dates, this in turn diminishes the expected profits of rival firms. Analogous results are obtained by Bhattacharya and Mookherjee (1986) and Dasgupta and Maskin (1987) in a setting where R & D can be interpreted either as inducing a distribution of discovery dates for an innovation of known size or, alternatively, as inducing a distribution over innovations with time playing no essential role.

The tendency towards excessive risk-taking identified by this literature stands in contrast to the popular notion that too little risk-taking occurs, a notion that has become especially prominent in the United States.[1] The purpose of this paper is to alter some of the assumptions maintained in the literature on the choice of R & D technology in order to identify possible sources of deficient risk-taking.

Two aspects of previous work appear fundamental. The first is the assumption that the rewards to R & D are of a "winner-take-all" nature.

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[1] Such an observation was made, for example, by the MIT Commission on Industrial Productivity (Dertouzos, Lester, and Solow, 1989).

This seems an appropriate idealization within the context of a patent race, if the random outcome induced by R & D is interpreted as the date of "discovery" of a commonly-sought prize, with the winner getting the legal right to the reward. But the winner-take-all setting is generally less compelling when the random outcome is instead interpreted as the quality of the innovation that comes out of the research; that is, as product quality in the case of product R & D or production cost in the case of process R & D.[2] Here, the winner-take-all setting may still apply if the product market is subject to sufficiently large scale economies so that only the lowest cost (or highest quality) firm enters at the production stage.[3] However, in the absence of such scale economies, the relative rewards to the "winner" and "loser" are likely to depend on how "drastic" the difference is in research outcomes and on the structure of the resulting product market; unless the difference in research outcomes is sufficiently drastic, the winner is unlikely to "take all". It thus seems important in the case where R & D leads to a distribution over innovations to extend the study of the choice of R & D technology to "smoother" settings, where the winner takes more but not all.

A second aspect of the previous literature which is potentially important is the assumption that the winner is able to appropriate all social surplus. We think it more likely that some consumer surplus associated with R & D is not appropriated. This implies that the

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[2] In reality, both the discovery date and the quality of the discovery are likely to be uncertain outcomes of a research project, a complication which we discuss in the concluding section.

[3] See Bagwell and Staiger (1989b) for a more formal development of this interpretation.

preferences which consumers hold about the effects of risky R & D must be explicitly accounted for.[4]

To explore these issues, we consider a duopoly game in which firms undertake R & D that affects the riskiness of the distribution of firm-specific product market variables. In particular, we consider investment in both process R & D, which affects the riskiness of the distribution of the investing firm's production costs, and product R & D, which alters the riskiness of the distribution of some characteristic of the firm's product. Such risk altering investments may be generated by a deterministic R & D technology which trades off efficiency with flexibility in the production process or product design, but which must be committed to prior to the realization of extrinsic uncertainty along a relevant dimension.[5] Alternatively, R & D may itself involve intrinsic risk, with the riskiness of the outcome increasing with the complexity of the project. In any event, we assume that acquiring risk is costly (globally for much of the paper but only locally in an extension section) to assure that interior R & D investment choices exist. After the effects of R & D are realized and observed, the firms compete in prices in a differentiated product setting. To relax the winner-take-all assumption, we assume explicitly that innovations are sufficiently non-drastic that equilibrium product market profits are always positive to both firms.

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[4] An alternative source of deficient risk-taking which we do not consider is managers who are risk averse with respect to their own human capital (Holmstrom and Ricarta i Costa, 1986). We also abstract from the possibility of spillovers of knowledge (Spence, 1984).

[5] An example would be process R & D which results in a production process requiring specialized imported inputs, with the price of such inputs denominated in foreign currency and subject to unhedgable exchange rate risk.

We consider first a quadratic utility model in which the representative consumer has variable demand. Solving for the equilibrium product market profits, we find that a firm's profit is convex in its own costs as well as the costs of its rival firm. A similar convexity is present in the product innovation characteristic. Thus, firms are intrinsically risk-loving with respect to their own risk-increasing R & D and the risk-increasing R & D of rival firms. The latter effect implies a cross-firm positive externality to risk-increasing investment. It follows that too little risk-increasing R & D occurs from the perspective of industry profit.

Consumers also benefit from added risk. This result follows, in part, from the convexity of the consumers' indirect utility function in prices. Since a firm's equilibrium price is positively related to its production costs and product innovation characteristic, it is not surprising that an investment in risk-increasing process or product R & D imparts a positive externality to consumers. Thus, consumers also agree that too little risk-increasing R & D occurs.

We consider next the standard Hotelling model of price competition. Here, consumers are heterogeneous and possess unitary demands. Individual consumers no longer have convex indirect utility functions. The results are nevertheless the same as in the quadratic utility model. Firms and consumers (in the aggregate) are intrinsically risk-loving with respect to all risk-increasing R & D.

Our results therefore suggest that, with respect to non-drastic innovations which maintain market structure, too little risk-increasing process and product R & D occurs. The optimal policy is to subsidize non-drastic risk-increasing R & D.

This paper complements our previous work (Bagwell and Staiger, 1989a,b), the emphasis of which has been on strategic and corrective process R & D policy in international markets. The present paper ignores strategic R & D policy and introduces consumer welfare analysis, product innovation, and the quadratic utility model.

The paper is organized in six sections. In section II, a general framework for assessing externalities and R & D policy is developed. Section III contains the quadratic utility model, while the Hotelling model is developed in section IV. A variety of extensions are discussed in section V. Concluding thoughts are offered in section VI.

II. General Framework

We begin with a general framework. For now there are two firms in the market (we extend our results to many firms in section V). The firms first choose R & D investment levels, with each firm's investment inducing a distribution for some product market variable. After the realized product market variables are commonly observed, the firms then compete in prices in the product market.

Let I^i denote the level of firm i's R & D investment, and let a^i be a product market variable determined as a random function of I^i. The investment technologies for the two firms are symmetric, and are summarized by the positive and continuously differentiable density function $f(a^i|I^i)$. The support for a^i is $[\underline{a},\overline{a}]$. As we illustrate in the next section, a^i may correspond to a process or a product innovation.

We are interested in R & D that affects the riskiness of the distribution of a^i. We thus assume that investment increases the riskiness of the distribution of a^i in a mean-preserving way, in the sense of second order stochastic dominance (Rothschild and Stiglitz, 1970). Defining $F(a^i|I^i) \equiv \int_{\underline{a}}^{a^i} f(z|I^i)dz$ and letting $F_{I^i}(a^i|I^i)$ denote the partial derivative of $F(a^i|I^i)$ with respect to I^i, we assume:

Assumption A: For every I^i, $\int_{\underline{a}}^{a^i} F_{I^i}(z|I^i)dz$ is zero if $a^i \in \{\underline{a},\bar{a}\}$ and is positive if $a^i \in (\underline{a},\bar{a})$.

It is also convenient at this point to state a second assumption, whose role is to ensure that the firm's investment problem is a concave program.

Assumption B: For every I^i, there exists an $\tilde{a} \in (\underline{a},\bar{a})$ such that $F_{I^iI^i}(\tilde{a}|I^i) = 0$, with $F_{I^iI^i}(a^i|I^i) \leq 0$ for $a^i \in [\underline{a},\tilde{a})$, $F_{I^iI^i}(a^i|I^i) \geq 0$ for $a^i \in (\tilde{a},\bar{a}]$, and $F_{I^iI^i}(a^i|I^I) \neq 0$ for some positive measure of $a^i \in (\underline{a},\bar{a})$.

Once investment choices are made and a^1 and a^2 realized, the firms compete in the product market. Let $\pi^i(a^i,a^j)$ denote firm i's equilibrium profit (gross of investment costs) in the product market. $\pi^1(a^1,a^2)$ and $\pi^2(a^2,a^1)$ are assumed to be symmetric. To avoid any winner-take-all influence, we assume $\pi^i(a^i,a^j) > 0$ for all a^i and a^j in $[\underline{a},\bar{a}]$. We also maintain the assumptions that $\pi^i_{a^i}(a^i,a^j)$ is nonzero and of invariant sign and that $\pi^i_{a^ia^i}(a^i,a^j) > 0$, with both conditions holding for all a^1 and a^2 in $[\underline{a},\bar{a}]$. These conditions will be met in the models of product market competition we consider below.

The firms choose investment levels simultaneously. They are forward looking and will anticipate the equilibrium product market profits. If it faces a unit cost of investment r, firm i's expected profit can be written as:

$$(1)\quad E\pi^i(I^i,I^j,r) \equiv \int_{\underline{a}}^{\overline{a}} \int_{\underline{a}}^{\overline{a}} f(a^i|I^i)f(a^j|I^j)\pi^i(a^i,a^j)da^i da^j - rI^i$$

The first order condition is:

$$(2)\quad E\pi^i_{I^i}(I^i,I^j,r) = \int_{\underline{a}}^{\overline{a}} \int_{\underline{a}}^{\overline{a}} f_{I^i}(a^i|I^i)f(a^j|I^j)\pi^i(a^i,a^j)da^i da^j - r = 0$$

In order to confirm that risk-increasing, rather than risk-reducing, investment is of interest to firms in this setting, we begin with the following Lemma, which states that a firm will always prefer more risk if more risk is costless to obtain.

<u>Lemma 1</u>: For all I^1 and I^2, $E\pi^i_{I^i}(I^i,I^j,r=0) > 0$.

<u>Proof</u>: Using (2):

$$E\pi^i_{I^i}(I^i,I^j,r=0) = \int_{\underline{a}}^{\overline{a}} \int_{\underline{a}}^{\overline{a}} f_{I^i}(a^i|I^i)f(a^j|I^j)\pi^i(a^i,a^j)da^i da^j$$

$$= \int_{\underline{a}}^{\overline{a}} f_{I^i}(a^i|I^i)K^i(a^i|I^j)da^i$$

where

$$(3)\quad K^i(a^i|I^j) \equiv \int_{\underline{a}}^{\overline{a}} f(a^j|I^j)\pi^i(a^i,a^j)da^j > 0$$

But successive integration by parts gives

$$E\pi^i_{I^i}(I^i,I^j,r=0) = -\int_{\underline{a}}^{\overline{a}} F_{I^i}(a^i|I^i)K^i_{a^i}(a^i|I^j)da^i$$

$$= \int_{\underline{a}}^{\overline{a}} [\int_{\underline{a}}^{a^i} F_{I^i}(z|I^i)dz]K^i_{a^ia^i}(a^i|I^j)da^i$$

which, under Assumption A, is positive since $\pi^i_{a^ia^i}(a^i,a^j)$ is globally positive and, by (3):

$$K^i_{a^ia^i}(a^i|I^j) = \int_{\underline{a}}^{\overline{a}} f(a^j|I^j)\pi^i_{a^ia^i}(a^i,a^j)da^j > 0. \qquad \text{QED}$$

Thus, provided the cost of investment (r) is not too high, firms will choose to undertake some risk-increasing investment. Intuitively, that firms like risk follows directly from our assumption that $\pi^i(a^i,a^j)$ is convex in a^i. We now assume that a solution to the first order condition (2) exists, so that a maximum obtains if the second order condition holds:

$$(4) \quad E\pi^i_{I^iI^i}(I^i,I^j,r) = \int_{\underline{a}}^{\overline{a}} \int_{\underline{a}}^{\overline{a}} f_{I^iI^i}(a^i|I^i)f(a^j|I^j)\pi^i(a^i,a^j)da^ida^j < 0$$

The solution to (2) then corresponds to an investment reaction curve. The next Lemma establishes that the second order condition must hold.

<u>Lemma 2</u>: For all I^1, I^2, and r, $E\pi^i_{I^iI^i}(I^i,I^j,r) < 0$.

<u>Proof</u>: Using (3) and (4) and integrating by parts yields:

$$E\pi^i_{I^iI^i}(I^i,I^j,r) = -\int_{\underline{a}}^{\overline{a}} F_{I^iI^i}(a^i|I^i)K^i_{a^i}(a^i|I^j)da^i$$

But, as noted above, $K^i_{a^i a^i}(a^i|I^j) > 0$. Thus, using Assumption B and the fact that $K_{a^i}(a^i|I^j)$'s sign is invariant:

$$E\pi^i_{I^iI^i}(I^i,I^j,r) = -[\int_{\underline{a}}^{\tilde{a}} F_{I^iI^i}(a^i|I^i)K_{a^i}(a^i|I^j)da^i + \int_{\tilde{a}}^{\bar{a}} F_{I^iI^i}(a^i|I^i)K_{a^i}(a^i|I^j)da^i]$$

$$< - K_{a^i}(\tilde{a}|I^j)\int_{\underline{a}}^{\bar{a}} F_{I^iI^i}(a^i|I^i)da^i = 0 \qquad \text{QED}$$

Investment reaction curves are thus well-defined.

We next assume that the investment reaction curves intersect, giving a symmetric Nash equilibrium investment level, $\hat{I}(r)$. The symmetric equilibrium is unique if the determinant of the Jacobian, J, associated with the first order conditions is globally positive:

$$(5) \quad |J| \equiv E\pi^1_{I^1I^1}(I^1,I^2,r)E\pi^2_{I^2I^2}(I^2,I^1,r) - E\pi^1_{I^1I^2}(I^1,I^2,r)E\pi^2_{I^2I^1}(I^2,I^1,r) > 0$$

We maintain this assumption. Total differentiation of the first order conditions now gives:

$$(6) \quad \hat{I}_r(r) = \frac{E\pi^i_{I^iI^i}(I^i,I^j,r)}{|J|} < 0$$

A subsidy on investment will thus raise investment for each firm.

We next characterize externalities across firms, that is, the sign of $E\pi^i_{I^j}(I^i,I^j,r)$.

Lemma 3: For all I^1, I^2 and r

$$\text{sign}\,\{E\pi^i_{I^j}(I^i,I^j,r)\} = \text{sign}\,\{\pi^i_{a^ja^j}(a^i,a^j)\}$$

provided that $\pi^i_{a^j a^j}(a^i,a^j)$ retains the same sign for all a^1 and a^2 in $[\underline{a},\overline{a}]$.

Proof: Using (1) and (3), we have that:

$$(7)\quad E\pi^i_{I^j}(I^i,I^j,r) = \int_{\underline{a}}^{\overline{a}} f(a^i|I^i)K^i_{I^j}(a^i|I^j)da^i$$

where

$$K^i_{I^j}(a^i|I^j) = \int_{\underline{a}}^{\overline{a}} f_{I^j}(a^j|I^j)\pi^i(a^i,a^j)da^j$$

But successive integration by parts gives:

$$(8)\quad K^i_{I^j}(a^i|I^j) = -\int_{\underline{a}}^{\overline{a}} F_{I^j}(a^j|I^j)\pi^i_{a^j}(a^i,a^j)da^j$$

$$= \int_{\underline{a}}^{\overline{a}} [\int_{\underline{a}}^{a^j} F_{I^j}(z|I^j)dz]\pi^i_{a^j a^j}(a^i,a^j)da^j$$

and the lemma thus follows. QED

The lemma has a familiar intuition. Since an increase in I^j acts to increase the riskiness of the distribution of a^j, the expected profit to firm i is increasing in I^j if its product market equilibrium profit is convex in a^j. Thus, if $\pi^i_{a^j a^j}(a^i,a^j) > 0$, then a positive externality accrues to a firm when its rival increases R & D investment.

Our final Lemma applies to consumer welfare. Let $U(a^1,a^2)$ represent the total consumer surplus in the product market equilibrium given a^1 and a^2. Assume $U(a^1,a^2)$ is symmetric in a^1 and a^2. Define:

$$EU(I^1, I^2) \equiv \int_{\underline{a}}^{\overline{a}} \int_{\underline{a}}^{\overline{a}} f(a^1|I^1) f(a^2|I^2) U(a^1,a^2) da^1 da^2$$

to be total expected consumer surplus as a function of investment.

We now have:

Lemma 4: For all I^1 and I^2,

$$\text{sign}\,\{EU_{I^i}(I^1,I^2)\} = \text{sign}\,\{U_{a^i a^i}(a^1,a^2)\}$$

provided that the sign of $U_{a^i a^i}(a^1,a^2)$ is the same for all a^1 and a^2 in $[\underline{a},\overline{a}]$.

Proof: The proof proceeds along the lines of the proof of the previous Lemmas, and is omitted.

Thus, for example, if $U_{a^i a^i}(a^1,a^2) > 0$, then consumers prefer greater riskiness in a^i and so an increase in I^i provides a positive externality to consumers.

We now represent the welfare of the economy and establish our major proposition. Let $W(r)$ be expected welfare when firms act noncooperatively:

$$W(r) \equiv E\pi^1(\hat{I}(r),\hat{I}(r),r) + E\pi^2(\hat{I}(r),\hat{I}(r),r) + EU(\hat{I}(r),\hat{I}(r)) - 2(\tilde{r}-r)\hat{I}(r)$$

where $\tilde{r}$ is the true social cost to a unit of investment. Thus, $W(r)$ is the sum of producer surplus, consumer surplus, and investment taxes. Alternatively, $W(r)$ is total producer and consumer surplus less subsidy costs.

We assume $W(r)$ has a maximizer, $\hat{r}$, which satisfies $W_r(\hat{r}) = 0$. Using the envelope theorem via (2), we write this as:

$$W_r(\hat{r}) = [2E\pi^i_{Ij}(\hat{I}(\hat{r}),\hat{I}(\hat{r}),\hat{r}) + 2EU_{Ii}(\hat{I}(\hat{r}),\hat{I}(\hat{r})) - 2(\tilde{r}-\hat{r})]\hat{I}_r(\hat{r}) = 0$$

Using (6), we have:

$$(9)\quad (\tilde{r}-\hat{r}) = E\pi^i_{Ij}(\hat{I}(\hat{r}),\hat{I}(\hat{r}),\hat{r}) + EU_{Ii}(\hat{I}(\hat{r}),\hat{I}(\hat{r}))$$

The relation of $\tilde{r}$ to $\hat{r}$ is thus purely a function of the nature of externalities. Absent any externalities to investment, $\hat{r}=\tilde{r}$: no investment subsidy or tax is desirable.

We conclude this section with the following proposition, which follows immediately from Lemma 3, Lemma 4, and (9):

<u>Proposition</u>: An investment subsidy $(\tilde{r}<\hat{r})$ is optimal if $\pi^i_{a^j a^j}(a^i,a^j)$ and $U_{a^i a^i}(a^1,a^2)$ are both positive for all a^1 and a^2 in $[\underline{a},\overline{a}]$.

Under the conditions of the proposition, R & D investment by any one firm imparts a positive externality to its rival firm as well as consumers. Since the firm is unable to appropriate these benefits, too little investment is undertaken and a subsidy is called for. We proceed in the next two sections to explore the plausibility of this conclusion in a variety of structural models.

III. Homogeneous Consumers with Variable Demands: The Quadratic Utility Model

A. General Setting

We begin with a simple quadratic utility model, similar to that developed by Shubik and Levitan (1980). The representative consumer has utility $M + f(q^1, q^2, \alpha^1, \alpha^2)$, where M is the quantity of a numeraire good that is consumed, q^i is the quantity of firm i's output consumed, and:

$$f(q^1, q^2, \alpha^1, \alpha^2) = \sum_{i=1}^{2} \frac{\alpha^i}{\beta} q^i - \frac{1}{2\beta} Q^2 - \frac{2\sigma^2}{\beta(1+\gamma)}$$

with

$$Q \equiv q^1 + q^2; \quad \sigma^2 \equiv \left(\frac{q^1 - q^2}{2}\right)^2.$$

The parameter $\gamma \in [0, \infty]$ can be interpreted as a measure of the degree of product differentiation, with $\gamma = \infty$ corresponding to perfect substitutes and $\gamma = 0$ corresponding to complete independence. The consumer maximizes utility subject to the budget constraint:

$$M + \sum_{i=1}^{2} P^i q^i = Y$$

where P^i is the price of firm i's product and Y is the consumer's income. Let $V(P^1, P^2, Y)$ be the consumer's indirect utility function, giving the maximized value of utility over quantities satisfying the budget constraint.

It is also convenient to define:

$$\tilde{U}(q^1,q^2,P^1,P^2,\alpha^1,\alpha^2) = f(q^1,q^2,\alpha^1,\alpha^2) - \sum_{i=1}^{2} P^i q^i$$

Thus, directly substituting for M, we have that:

$$V(P^1,P^2,Y) = \underset{q^1,q^2}{\text{maximum}} \quad Y + \tilde{U}(q^1,q^2,P^1,P^2,\alpha^1,\alpha^2)$$

With this structure, we see that the first order condition for utility maximization is:

$$\tilde{U}_{q^i}(q^1,q^2,P^1,P^2,\alpha^1,\alpha^2) = \frac{\alpha^i}{\beta} - \frac{Q}{\beta} - \frac{q^i - q^j}{\beta(1+\gamma)} - P^i = 0$$

Observe that α^i affects only the intercept of the marginal utility associated with firm i's product. Thus, it seems reasonable to interpret α^i as a product innovation variable.

It is easily verified that the second order conditions hold, and so the first order conditions can be solved for demand functions:

$$(10)\quad q^i(P^i,P^j,\alpha^i,\alpha^j) = \frac{1}{2}\{(\alpha^i - \beta P^i) + \frac{\gamma}{2}[(\alpha^i - \beta P^i) - (\alpha^j - \beta P^j)]\}$$

Notice that the products are substitutes provided that $\gamma > 0$, since a higher P^j leads to larger demands for firm i's product. We may also define a "choke price" for firm i, at which demand for firm i's product is zero. This price, denoted $P^{ic}(p^j,\alpha^i,\alpha^j)$, is:

$$P^{ic}(P^j,\alpha^i,\alpha^j) = \frac{\alpha^i}{\beta} - \frac{\gamma}{(2+\gamma)}[\frac{\alpha^j}{\beta} - P^j]$$

Observe that the demand for q^i and q^j is zero when $P^i = \alpha^i/\beta$ and $P^j = \alpha^j/\beta$; that is, these prices represent mutual choke prices.

We may now express the indirect utility function as:

$$V(P^1,P^2,Y) = Y + \tilde{U}(q^1(P^1,P^2,\alpha^1,\alpha^2),q^2(P^2,P^1,\alpha^2,\alpha^1),P^1,P^2,\alpha^1,\alpha^2)$$

The consumer surplus at the prices P^1 and P^2 for the products of firms 1 and 2 is the amount E such that:[6]

$$V(P^1,P^2,Y-E) = V(\alpha^1/\beta,\alpha^2/\beta,Y)$$

It is now direct to verify that:

$$E = \tilde{U}(q^1(P^1,P^2,\alpha^1,\alpha^2),q^2(P^2,P^1,\alpha^2,\alpha^1),P^1,P^2,\alpha^1,\alpha^2)$$

Thus, the maximized value of $\tilde{U}$ corresponds to consumer surplus.

Turning now to the firms, let firm i's profit be denoted as:

$$\tilde{\pi}^i(P^i,P^j,\alpha^i,\alpha^j,c^i) = (P^i-c^i)q^i(P^i,P^j,\alpha^i,\alpha^j)$$

with $q^i(P^i,P^j,\alpha^i,\alpha^j)$ as defined in (10) and c^i representing firm i's constant unit cost. The first order condition is:

[6] This is the "equivalent variation" definition of consumer surplus. Since V is linear in Y, the "compensating variation" definition of consumer surplus yields the same measure.

(11) $\tilde{\pi}^i_{P^i}(P^i, P^j, \alpha^i, \alpha^j, c^i) = (P^i - c^i) q^i_{P^i}(P^i, P^j, \alpha^i, \alpha^j) + q^i(P^i, P^j, \alpha^i, \alpha^j) = 0$

It is straightforward to verify using (10) that the second order condition holds. Solving (11) gives firm i's reaction curve, or profit-maximizing response:

(12) $P^{iR}(P^j, \alpha^i, \alpha^j, c^i) = (1/2)[P^{ic}(P^j, \alpha^i, \alpha^j) + c^i]$

The best response price is halfway between firm i's costs and firm i's choke price.

Let $\alpha^i \epsilon [\underline{\alpha}, \overline{\alpha}]$ and $c^i \epsilon [\underline{c}, \overline{c}]$. We want to ensure that the best response price is always strictly between the choke price and the unit cost; that is, we want conditions under which $P^{ic}(P^j, \alpha^i, \alpha^j) > c^i$. Such a condition would guarantee positive profits along the reaction curve. A sufficient condition for $P^{ic}(P^j, \alpha^i, \alpha^j) > c^i$ is easily demonstrated to be:

$$(\underline{\alpha}/\beta) - \overline{c} > \frac{\gamma}{(2+\gamma)}[(\overline{\alpha}/\beta) - \underline{c}]$$

Finally, we solve for the Nash prices, $\hat{P}^i(\alpha^i, \alpha^j, c^i, c^j)$, where reaction curves cross. After some calculation, we have:

$$\hat{P}^i(\alpha^i, \alpha^j, c^i, c^j) = \left[\frac{2(2+\gamma)^2 - \gamma^2}{(4+\gamma)(4+3\gamma)}\right]\frac{\alpha^i}{\beta} + \left[\frac{2(2+\gamma)^2}{(4+\gamma)(4+3\gamma)}\right]c^i - \left[\frac{\gamma(2+\gamma)}{(4+\gamma)(4+3\gamma)}\right]\left(\frac{\alpha^j}{\beta} - c^j\right)$$

It is now easy to see that $\hat{P}^i(\alpha^i, \alpha^j, c^i, c^j)$ is linear in all arguments, decreasing in α^j, and increasing in c^i, α^i and c^j.

The equilibrium mark-up, which we know is positive, is defined as:

$$\hat{m}^i(\alpha^i,\alpha^j,c^i,c^j) \equiv \hat{P}^i(\alpha^i,\alpha^j,c^i,c^j) - c^i$$

Explicit calculations reveal that the equilibrium mark-up is linear in all arguments, decreasing in c^i and α^j, and increasing in c^j and α^i.

Let $\hat{q}^i(\alpha^i,\alpha^j,c^i,c^j)$ denote firm i's equilibrium level of demand:

$$\hat{q}^i(\alpha^i,\alpha^j,c^i,c^j) = q^i(\hat{P}^i(\alpha^i,\alpha^j,c^i,c^j),\hat{P}^j(\alpha^j,\alpha^i,c^j,c^i),\alpha^i,\alpha^j)$$

This has explicit representation as:

$$\hat{q}^i(\alpha^i,\alpha^j,c^i,c^j) = \frac{(2+\gamma)}{4}\{[\frac{2(2+\gamma)^2-\gamma^2}{(4+\gamma)(4+3\gamma)}](\alpha^i-\beta c^i) - [\frac{\gamma(2+\gamma)}{(4+\gamma)(4+3\gamma)}](\alpha^j-\beta c^j)\}$$

Notice that $\hat{q}^i(\alpha^i,\alpha^j,c^i,c^j)$ is linear in all arguments, decreasing in c^i and α^j, and increasing in c^j and α^i. Also, $\hat{q}^i(\alpha^i,\alpha^j,c^i,c^j)$ is positive since the equilibrium price lies below the choke price.

We can now define firm i's equilibrium profit, $\hat{\pi}^i(\alpha^i,\alpha^j,c^i,c^j)$, as:

$$\hat{\pi}^i(\alpha^i,\alpha^j,c^i,c^j) = \tilde{\pi}^i(\hat{P}^i(\alpha^i,\alpha^j,c^i,c^j),\hat{P}^j(\alpha^j,\alpha^i,c^j,c^i),\alpha^i,\alpha^j,c^i)$$

or equivalently

$$\hat{\pi}^i(\alpha^i,\alpha^j,c^i,c^j) = \hat{m}^i(\alpha^i,\alpha^j,c^i,c^j)\hat{q}^i(\alpha^i,\alpha^j,c^i,c^j)$$

Equilibrium profit is thus positive under our assumption. Further, since

$\hat{m}^i(\alpha^i,\alpha^j,c^i,c^j)$ and $\hat{q}^i(\alpha^i,\alpha^j,c^i,c^j)$ are each positive, linear in their arguments, and share the same signs on all first partials, we have that equilibrium profit is decreasing in c^i and α^j and increasing in c^j and α^i, and quadratic in the various arguments.

B. Process Innovations

With the general framework now established, we consider first process innovations. Thus, assume $\alpha^1=\alpha^2=\overline{\alpha}=\underline{\alpha}$ so that firms are symmetric regarding the product innovation parameters. Investment affects only the riskiness of the possible production costs. In the notation of the previous section, $a^i=c^i$, $\underline{a}=\underline{c}$, and $\overline{a}=\overline{c}$. Similarly, let:

$$\pi^i(a^i,a^j) = \pi^i(c^i,c^j) \equiv \hat{\pi}^i(\overline{\alpha},\overline{\alpha},c^i,c^j)$$

Since equilibrium profit is globally decreasing in own cost, the maintained assumption on $\pi^i_{a^i}(a^i,a^j)$ is met. We are now ready to assess the direction of externalities associated with risk-increasing R & D.

Let us begin with externalities across firms. Using Lemma 3, we see that the sign of $\pi^i_{c^jc^j}(c^i,c^j)$ must be evaluated. Given the linearity of equilibrium mark-up and demand, it is straightforward to derive that:

$$\pi^i_{c^jc^j}(c^i,c^j) = 2\hat{m}^i_{c^j}(\overline{\alpha},\overline{\alpha},c^i,c^j)\hat{q}^i_{c^j}(\overline{\alpha},\overline{\alpha},c^i,c^j) > 0$$

Thus, firm i's equilibrium profits are convex in firm j's costs. We must also verify that firm i's profits are convex in its own costs, an assumption maintained in the previous section. We have:

$$\pi^i_{c^ic^i}(c^i,c^j) = 2\hat{m}^i_{c^i}(\overline{\alpha},\overline{\alpha},c^i,c^j)\hat{q}^i_{c^i}(\overline{\alpha},\overline{\alpha},c^i,c^j) > 0$$

Thus, each firm seeks risk-increasing R & D, and there is a positive externality across firms associated with such investments.

Consider next externalities received by consumers. The equilibrium consumer surplus is:

$$U(c^1,c^2) = \tilde{U}(\hat{q}^1(\overline{\alpha},\overline{\alpha},c^1,c^2),\hat{q}^2(\overline{\alpha},\overline{\alpha},c^2,c^1),\hat{P}^1(\overline{\alpha},\overline{\alpha},c^1,c^2),\hat{P}^2(\overline{\alpha},\overline{\alpha},c^2,c^1),\overline{\alpha},\overline{\alpha})$$

Now, since consumers are utility maximizing in the equilibrium, we know that partials of $\tilde{U}$ with respect to q^i are zero. Further, the partial of $\tilde{U}$ with respect to P^i is of course $-q^i$. We thus have:

$$U_{c^i}(c^1,c^2) = -[\hat{q}^i(\overline{\alpha},\overline{\alpha},c^i,c^j)\hat{P}^i_{c^i}(\overline{\alpha},\overline{\alpha},c^i,c^j) + \hat{q}^j(\overline{\alpha},\overline{\alpha},c^j,c^i)\hat{P}^j_{c^i}(\overline{\alpha},\overline{\alpha},c^j,c^i)]$$

which is negative. Next, we use the linearity of equilibrium prices to establish that:

$$U_{c^ic^i}(c^i,c^j) = -[\hat{P}^i_{c^i}(\overline{\alpha},\overline{\alpha},c^i,c^j)\hat{q}^i_{c^i}(\overline{\alpha},\overline{\alpha},c^i,c^j) + \hat{P}^j_{c^i}(\overline{\alpha},\overline{\alpha},c^j,c^i)\hat{q}^j_{c^i}(\overline{\alpha},\overline{\alpha},c^j,c^i)]$$

This expression cannot be signed by inspection, as the first term in brackets is negative while the second term is positive. Not surprisingly, however, one can show that own effects dominate cross effects in this model, whence the absolute value of the first term exceeds that of the second. Thus, it follows that:

$$U_{c^ic^i}(c^i, c^j) > 0$$

A positive externality accrues to consumers when risky investment is undertaken.

One way of understanding this result is to recall that the second derivative of $\tilde{U}$ with respect to P^i when a consumer is utility maximizing is $-q^i_{P^i}(P^i, P^j, \bar{\alpha}, \bar{\alpha}) > 0$. Given this convexity of the indirect utility function and the monotonic relation of price to costs, it seems reasonable that consumers enjoy risky process R & D.[7] This is not the whole story, however, since the relevant prices are equilibrium prices, and thus one must ensure that added riskiness in the distribution of c^i does not greatly harm consumers through the equilibrium price of firm j.

We are now ready to employ our Proposition. Since both rival firms and consumers receive positive externalities from any one firm's risk-increasing investment, there is too little risky process R & D. The optimal policy is to subsidize investment that adds risk to the distribution of production costs.

C. Product Innovations

Consider now product innovations. Let $c^1 = c^2 = \bar{c} = \underline{c}$. Investment now affects the riskiness of the marginal utility shift parameter. Thus, $a^i = \alpha^i$, $\underline{a} = \underline{\alpha}$, $\bar{a} = \bar{\alpha}$, and:

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[7] The observation that consumers can benefit from price instability was first made by Waugh (1944).

$$\pi^i(a^i,a^j) = \pi^i(\alpha^i,\alpha^j) \equiv \hat{\pi}^i(\alpha^i,\alpha^j,\overline{c},\overline{c})$$

Recall that $\pi^i(\alpha^i,\alpha^j)$ is globally increasing in α^i, corresponding to the maintained assumption placed on $\pi^i_{a^i}(a^i,a^j)$. As before, we begin with externalities across firms. Arguing as above, it is straightforward to show that:

$$\pi^i_{\alpha^j\alpha^j}(\alpha^j,\alpha^j) = 2\hat{m}^i_{\alpha^j}(\alpha^i,\alpha^j,\overline{c},\overline{c})\hat{q}^i_{\alpha^j}(\alpha^i,\alpha^j,\overline{c},\overline{c}) > 0$$

and

$$\pi^i_{\alpha^i\alpha^i}(\alpha^i,\alpha^j) = 2\hat{m}^i_{\alpha^i}(\alpha^i,\alpha^j,\overline{c},\overline{c})\hat{q}^i_{\alpha^i}(\alpha^i,\alpha^j,\overline{c},\overline{c}) > 0$$

Thus, firm i's equilibrium profits are convex in its rival's product innovation parameter as well as its own innovation parameter. This establishes that firms seek risk-increasing R & D and benefit from the risky investments selected by rival firms.

For consumers, define:

$$U(\alpha^1,\alpha^2) = \tilde{U}(\hat{q}^1(\alpha^1,\alpha^2,\overline{c},\overline{c}),\hat{q}^2(\alpha^2,\alpha^1,\overline{c},\overline{c}),\hat{P}^1(\alpha^1,\alpha^2,\overline{c},\overline{c}),\hat{P}^2(\alpha^2,\alpha^1,\overline{c},\overline{c}),\alpha^1,\alpha^2)$$

Since the partial of $\tilde{U}$ with respect to α^i is q^i/β, we may argue as before to establish that:

$$U_{\alpha^i}(\alpha^1,\alpha^2) = \hat{q}^i(\alpha^i,\alpha^j,\overline{c},\overline{c})[\frac{1}{\beta} - \hat{P}^i_{\alpha^i}(\alpha^i,\alpha^j,\overline{c},\overline{c})] - \hat{q}^j(\alpha^j,\alpha^i,\overline{c},\overline{c})\hat{P}^j_{\alpha^i}(\alpha^j,\alpha^i,\overline{c},\overline{c})$$

It is direct to establish that $1/\beta > \hat{P}^i_{\alpha^i}(\alpha^i,\alpha^j,\overline{c},\overline{c})$ and thus that

$U_{\alpha i}(\alpha^1,\alpha^2) > 0$. Next, again using the linearity of equilibrium prices, we have:

$$U_{\alpha i \alpha i}(\alpha^1,\alpha^2) = \hat{q}^i_{\alpha i}(a^i,\alpha^j,\overline{c},\overline{c})\left[\frac{1}{\beta} - \hat{P}^i_{\alpha i}(a^i,\alpha^j,\overline{c},\overline{c})\right] - \hat{q}^j_{\alpha i}(\alpha^j,\alpha^i,\overline{c},\overline{c})\hat{P}^j_{\alpha i}(\alpha^j,\alpha^i,\overline{c},\overline{c})$$

The expression cannot be signed by inspection, being the difference of two positive terms. Calculations reveal, however, that the first term is largest, whence:

$$U_{\alpha i \alpha i}(\alpha^1,\alpha^2) > 0$$

A positive externality goes to consumers when risky investment in product innovation is undertaken.

The intuition for this result is similar to that given for process innovations, though a bit more complex. The second derivative of $\tilde{U}$ with respect to α^i when a consumer is utility maximizing is $q^i_{\alpha i}(P^i,P^j,\alpha^i,\alpha^j)/\beta > 0$. Thus, there is a direct sense in which the consumers prefer a more risky distribution for α^i. But α^i also affects equilibrium prices, and this direction of influence causes a consumer to dislike riskiness, since the consumer's total expenditure is convex in α^i. As it turns out, the direct effect dominates, and consumers benefit from risky product innovation investment.

Employing our Proposition, we have that there is too little risk-increasing investment in product R & D. The optimal policy is to subsidize investment that adds risk to the distribution of the product innovation parameter.

IV. Heterogeneous Consumers with Inelastic Demands: The Hotelling Model

A. General Setting

We turn now to the traditional Hotelling spatial model of price competition. There is a unit length line, along which consumers are uniformly distributed with total mass one. Firm 1 is located at the left endpoint of the line, say point zero, and firm 2 is located at the right endpoint, point one.

A consumer can either buy from firm 1, firm 2, or not at all. Suppose a consumer is located at point $x \in [0,1]$. If he buys from firm 1, he earns utility s^1-tx-P^1, where s^1 gives the gross consumer surplus derived from firm 1's product, t is a transportation cost, and P^1 is the price of firm 1's product. Similarly, the utility from buying from firm 2 is $s^2-t(1-x)-P^2$.

The consumer who is indifferent between the two firms, known as the marginal consumer, is located at:

$$x^M(s^1,s^2,P^1,P^2) = \frac{(t+s^1-s^2+P^2-P^1)}{2t}$$

Assuming that s^1 and s^2 are sufficiently large that all consumers choose to buy and that s^1-P^1 and s^2-P^2 are sufficiently close that $x^M(s^1,s^2,P^1,P^2) \in (0,1)$, the quantity sold by firm 1 is $x^M(s^1,s^2,P^1,P^2)$ and the quantity sold by firm 2 is $(1-x^M(s^1,s^2,P^1,P^2))$. The profits to the respective firms are then:

$$\tilde{\pi}^1(P^1,P^2,s^1,s^2,c^1) = (P^1-c^1)x^M(s^1,s^2,P^1,P^2)$$

and

$$\tilde{\pi}^2(P^2, P^1, s^2, s^1, c^2) = (P^2 - c^2)(1 - x^M(s^1, s^2, P^1, P^2))$$

The firms select prices in a Nash fashion. The first order conditions are:

$$(13)\quad \tilde{\pi}^i_{P^i}(P^i, P^j, s^i, s^j, c^i) = \frac{(t + s^i - s^j + P^j - P^i) - (P^i - c^i)}{2t} = 0$$

Clearly, the second order condition holds. Solving (13) yields the reaction curve:

$$(14)\quad P^{iR}(P^j, s^i, s^j, c^i) = (c^i + t + s^i - s^j + P^j)/2$$

From (14), Nash prices may be found:

$$\hat{P}^i(s^i . s^j, c^i, c^j) = (3t + s^i - s^j + c^j + 2c^i)/3$$

The equilibrium mark-up, $\hat{m}^i(s^i, s^j, c^i, c^j)$, is defined to be $\hat{P}^i(s^i, s^j, c^i, c^j) - c^i$, and can be written:

$$\hat{m}^i(s^i, s^j, c^i, c^j) = (3t + s^i - s^j + c^j - c^i)/3$$

Note that the mark-up is linear in all arguments, increasing in c^j and s^i, and decreasing in c^i and s^j.

Define the marginal consumer in equilibrium to be:

$$\hat{x}^M(s^1, s^2, c^1, c^2) = x^M(s^1, s^2, \hat{P}^1(s^1, s^2, c^1, c^2), \hat{P}^2(s^2, s^1, c^2, c^1)).$$

Explicit calculation yields:

$$\hat{x}^M(s^1,s^2,c^1,c^2) = \frac{(3t+s^1-s^2+c^2-c^1)}{6t}$$

Note that the demand to firm i is linear in all arguments, increasing in s^i and c^j, and decreasing in c^i and s^j. The equilibrium profit to firm i can now be defined as:

$$\hat{\pi}^i(s^i,s^j,c^i,c^j) = \tilde{\pi}^i(\hat{P}^i(s^i,s^j,c^i,c^j),\hat{P}^j(s^j,s^i,c^j,c^i),s^i,s^j,c^i)$$

or more explicitly

$$(15)\quad \hat{\pi}^i(s^i,s^j,c^i,c^j) = \frac{(3t+s^i-s^j+c^j-c^i)^2}{18t}$$

Finally, we must provide assumptions under which profit is positive for each firm and every consumer purchases. Let $s^i \in [\underline{s},\overline{s}]$ and $c^i \in [\underline{c},\overline{c}]$. The marginal consumer is always willing to buy in equilibrium if:

$$(16)\quad \underline{s} - \overline{c} > 3t/2$$

Also, the equilibrium marginal consumer is always located strictly between zero and one, that is each firm always makes positive equilibrium profits, if:

$$(17)\quad 3t > \overline{s}-\underline{s} + \overline{c}-\underline{c}$$

The conditions (16) and (17) simply require total surplus to always be sufficiently high to warrant travel, and product-cost realizations to always be sufficiently close across firms that no one firm captures all buyers. We maintain these two assumptions and (15) is thus valid.

B. Process Innovations

Suppose now $s^1 = s^2 = \overline{s}$ and focus on process innovations. Thus, $a^i = c^i$, $\underline{a} = \underline{c}$, $\overline{a} = \overline{c}$, and:

$$\pi^i(a^i, a^j) = \pi^i(c^i, c^j) \equiv \hat{\pi}^i(\overline{s}, \overline{s}, c^i, c^j)$$

Since $\pi^i(c^i, c^j)$ is globally decreasing in c^i, the maintained assumption for $\pi^i_{a^i}(a^i, a^j)$ holds. From (15) we see that:

$$\pi^i_{c^i c^i}(c^i, c^j) = {}^1/_{9t} = \pi^i_{c^j c^j}(c^j, c^j) > 0$$

Firm i's equilibrium profits are convex in c^i and c^j. Each firm seeks risk-increasing R & D and benefits as rival firms seek such investments.

Consumer welfare is less obvious. Individual consumers no longer have convex indirect utility functions, as they now have unitary demands. The equilibrium consumer surplus in the market is:

$$U(c^1, c^2) = \int_0^{\hat{x}^M(\overline{s}, \overline{s}, c^1, c^2)} [\overline{s} - \hat{P}^1(\overline{s}, \overline{s}, c^1, c^2) - tx]dx +$$

$$\int_{\hat{x}^M(\overline{s}, \overline{s}, c^1, c^2)}^{1} [\overline{s} - \hat{P}^2(\overline{s}, \overline{s}, c^2, c^1) - t(1-x)]dx$$

Calculations reveal:

$$U_{c1c1}(c^1,c^2) = 2\hat{x}^M_{c1}(\bar{s},\bar{s},c^1,c^2)[\hat{P}^2_{c1}(\bar{s},\bar{s},c^2,c^1) - \hat{P}^1_{c1}(\bar{s},\bar{s},c^1,c^2) - t\hat{x}^M_{c1}(\bar{s},\bar{s},c^1,c^2)]$$

$$U_{c2c2}(c^1,c^2) = 2\hat{x}^M_{c2}(\bar{s},\bar{s},c^1,c^2)[\hat{P}^2_{c2}(\bar{s},\bar{s},c^2,c^1) - \hat{P}^1_{c2}(\bar{s},\bar{s},c^1,c^2) - t\hat{x}^M_{c2}(\bar{s},\bar{s},c^1,c^2)]$$

Substituting, we have:

$$U_{c1c1}(c^1,c^2) = 1/18t > 0$$

Thus, consumers receive a positive externality from risky process investments.

Once more, our Proposition indicates that too little risky R & D occurs. A subsidy on investment that increases the riskiness of the cost distribution is optimal.

C. Product Innovations

Finally, suppose $c^1=c^2=\bar{c}$, so that $a^i=s^i$, $\underline{a}=\underline{s}$, $\bar{a}=\bar{s}$, and:

$$\pi^i(a^i,a^j) = \pi^i(s^i,s^j) \equiv \hat{\pi}^i(s^i,s^j,\bar{c},\bar{c})$$

Again, $\pi^i(s^i,s^j)$ is globally increasing in s^i and the maintained assumption on $\pi^i_{ai}(a^i,a^j)$ is satisfied. Using (15), we have:

$$\pi^i_{sisi}(s^i,s^j) = {}^1/_{9t} = \pi^i_{sjsj}(s^i,s^j) > 0$$

Again, firms seek risky R & D and these investments impart positive externalities to rival firms.

Equilibrium consumer welfare is now:

$$U(s^1,s^2) = \int_0^{\hat{x}^M(s^1,s^2,\bar{c},\bar{c})} (s^1-\hat{P}^1(s^1;s^2,\bar{c},\bar{c})-tx)dx$$

$$+ \int_{\hat{x}^M(s^1,s^2,\bar{c},\bar{c})}^{1} (s^2-\hat{P}^2(s^2,s^1,\bar{c},\bar{c}) - t(1-x))dx$$

Calculations give:

$$U_{s1s1}(s^1,s^2) = 2\hat{x}^M_{s1}(s^1,s^2,\bar{c},\bar{c})[\hat{P}^2_{s1}(s^2,s^1,\bar{c},\bar{c}) - \hat{P}^1_{s1}(s^1,s^2,\bar{c},\bar{c}) - t\hat{x}^M_{s1}(s^1,s^2,\bar{c},\bar{c}) + 1]$$

$$U_{s2s2}(s^1,s^2) = 2\hat{x}^M_{s2}(s^1,s^2,\bar{c},\bar{c})[\hat{P}^2_{s2}(s^2,s^1,\bar{c},\bar{c}) - \hat{P}^1_{s2}(s^1,s^2,\bar{c},\bar{c}) - t\hat{x}^M_{s2}(s^1,s^2,\bar{c},\bar{c}) - 1]$$

Substitution yields:

$$U_{sisi}(s^1,s^2) = 1/18t > 0$$

Consumers again benefit from greater riskiness in the product innovation variable.

Appealing one last time to our Proposition, we conclude that a subsidy should be applied to investment in product innovation that adds risk to the distribution of the product innovation variable.

V. Extensions

We have shown that too little risk-increasing investment occurs in non-drastic process and product innovation. This section discusses a number of extensions of our model and the corresponding robustness of our conclusion.

Sequential Investments

In many models (e.g., Cournot competition), play becomes more aggressive when moves are made sequentially. This suggests that the "first mover" might invest heavily, hoping perhaps to reduce rival investment, and that our conclusion of deficient investment might require modification.

This suggestion is flawed in two respects. First, as we have seen, a firm prefers larger rival investment. Thus, a firm will only invest heavily if it expects that to raise rival investment. Second, there actually may be little response in a rival's investment to the first mover's selection. To see this, suppose firm 1 chooses I^1, and then firm 2 selects I^2. After all R & D choices are made, the realized product market variables are revealed. From (2), we see that firm 1's choice of I^1 will affect firm 2's optimal I^2 only if $E\pi^2_{I^2 I^1}(I^2, I^1, r)$ is nonzero, that is, only if the investment reaction curve is nonhorizontal. Using (7) and (8), we have:

$$E\pi^2_{I^2 I^1}(I^2, I^1, r) = \int_{\underline{a}}^{\overline{a}} \int_{\underline{a}}^{\overline{a}} f_{I^2}(a^2|I^2) \, [\int_{\underline{a}}^{a^1} F_{I^1}(z|I^1)dz] \, \pi^2_{a^1 a^1}(a^2, a^1) da^2 da^1$$

Integrating by parts successively gives:

$$E\pi^2_{I^2 I^1}(I^2, I^1, r) = - \int_{\underline{a}}^{\overline{a}} \int_{\underline{a}}^{\overline{a}} F_{I^2}(a^2|I^2) [\int_{\underline{a}}^{a^1} F_{I^1}(z|I^1)dz] \, \pi^2_{a^1 a^1 a^2}(a^2, a^1) da^2 da^1$$

$$= \int_{\underline{a}}^{\overline{a}} \int_{\underline{a}}^{\overline{a}} [\int_{\underline{a}}^{a^2} F_{I^2}(z|I^2)dz] [\int_{\underline{a}}^{a^1} F_{I^1}(z|I^1)dz] \pi^2_{a^1 a^1 a^2 a^2}(a^2, a^1) da^2 da^1$$

Thus, under Assumption A, $E\pi^2_{I^2 I^1}(I^2, I^1, r)$ takes the same sign as $\pi^2_{a^1 a^1 a^2 a^2}(a^2, a^1)$. As a general matter, there is no obvious sign for this fourth derivative to take, and so sequential investment choices should not disturb our conclusion in any predictable way. In particular, for both process and product innovation in the linear demand models considered above, $\pi^2_{a1a1a2a2}(a^2, a^1) = 0$. The second mover's investment is completely independent of the first mover's investment choice; the sequential game generates precisely the same behavior as the simultaneous move game.[8]

Quantity Competition

The examples developed above all assume firms compete in prices, where price reaction curves are positively sloped. By contrast, with quantity competition, quantity reaction curves are typically negatively sloped. Might our conclusion of underinvestment be sensitive to the slope of reaction curves in the product market choice variable?

Our basic results are robust to quantity competition. To see this, consider process innovations. Let $P(q^1+q^2) = 1-q^1-q^2$ be the market price when firm i produces output q^i. It is easy to show that equilibrium profit for the Cournot game is:

$$\pi^i(c^i, c^j) = (1-2c^i+c^j)^2/9$$

Assuming that $1+\underline{c} > 2\overline{c}$, it follows that equilibrium profits are always positive for each firm. Note, moreover, that $\pi^i_{ci}(c^i, c^j)$ is negative and that $\pi^i_{cici}(c^i, c^j)$ and $\pi^i_{cjcj}(c^i, c^j)$ are positive. Firms prefer risk-increasing investments, for themselves and their rivals. Finally, the given

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[8] We note that in this case our maintained assumption (5) must hold as well. The observation of flat investment reaction curves has a variety of strategic implications. For example, Bagwell and Staiger (1989a) argue that government subsidies of risky investments in international markets serve no strategic purpose in models such as those discussed in sections III and IV.

demand function can be modeled as deriving from a representative quadratic utility function. Arguments analogous to those developed above establish that consumers also benefit from risk-taking firms.

Alternative Costs of Investment

We have assumed throughout that an investment level I^i costs an amount rI^i. Thus, greater riskiness is always more costly. An alternative specification is that investment cost is a nonlinear function of I^i, say $rg(I^i)$. Lemma 2 continues to hold, provided $g_{I^iI^i}(I^i)$ is not too negative. Lemma 1, Lemma 3, and Lemma 4 are unaffected by the alternative specification. We continue to assume a unique symmetric Nash equilibrium in the investment game in which (6) is satisfied.

With the new specifications, the optimal policy as characterized in (9) becomes:

$$(\tilde{r}-\hat{r})g_{I^i}(\hat{I}(\hat{r})) = E\pi^i_{I^j}(\hat{I}(\hat{r}),\hat{I}(\hat{r}),\hat{r}) + EU_{I^i}(\hat{I}(\hat{r}),\hat{I}(\hat{r}))$$

The right hand side of this equation is positive in our examples. Thus, a subsidy remains attractive if $g_{I^i}(\hat{I}(\hat{r})) > 0$. To sign $g_{I^i}(\hat{I}(\hat{r}))$, note from (2) that firm i's first order condition is now:

$$E\pi^i_{I^i}(\hat{I}(\hat{r}),\hat{I}(\hat{r}),0) = \hat{r}g_{I^i}(\hat{I}(\hat{r})).$$

Lemma 1 continues to establish that the left hand side of this equation is positive, whence it must be that at the equilibrium level of investment, $g_{I^i}(I^i)$ is positive, that is, $g_{I^i}(\hat{I}(\hat{r})) > 0$. It follows that a subsidy remains optimal in our examples.[9]

Multiple Firms

The analysis above is carried out under the simplifying assumption that two firms operate in the market. Our basic results extend to the more general case of N firms.

The quadratic utility model must be generalized, with the consumer now maximizing:

$$\sum_{i=1}^{N} \frac{\alpha^i}{\beta} q^i - \frac{1}{2\beta} Q^2 - \frac{2}{\beta(1+\gamma)} \sum_{i=1}^{N} \sum_{j=i}^{N} \sigma^2_{ij} - \sum_{i=1}^{N} P^i q^i$$

where

$$Q \equiv \sum_{i=1}^{N} q^i ; \quad \sigma^2_{ij} \equiv \left(\frac{q^i - q^j}{2}\right)^2$$

This yields the demand function for firm i's product:

$$\frac{1}{N} \{(\alpha^i - \beta P^i) + \frac{\gamma}{N}[(\alpha^i - \beta P^i)(N-1) - (\sum_{j\neq i} \alpha^j - \beta \sum_{j\neq i} P^j)]\}$$

One can now verify that the best response price can be characterized as in (12), for a generalized definition of the choke price. Implicit

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[9] It is of course critical that $g_{I_i}(I^i) > 0$ at some I^i. For example, if $g_{I_i}(I^i) \equiv 0$ so that investment cost is independent of the investment level, then Lemma 1 establishes that firms will choose the maximal investment level possible. In this case, no interior solution exists and private and social optima agree. (Bhattacharya and Mookherjee, 1986; Klette and de Meza, 1986). It seems plausible to us, however, that $g(I^i)$ has increasing segments, an assumption the analogue of which is maintained in Dasgupta and Maskin (1987) and entertained by Klette and de Meza (1986).

differentiation of the best response system provides the constant derivatives of equilibrium prices with respect to the various parameters. With this approach, it is possible to confirm that the conclusions established above extend to the N-firm case.

Alternatively, the two-firm Hotelling model can be extended to many firms by placing firms around a circle as in Salop (1979). Under the assumption that firms locate symmetrically before undertaking R & D, it is straightforward to check that the properties of the equilibrium profit function established above for the two-firm Hotelling case are preserved with the introduction of more firms. For example, in the case of three firms labelled i, j, and k, the equilibrium profit function for firm i is given by:

$$\hat{\pi}^i(s^i, s^j, s^k, c^i, c^j, c^k) = (1/t)\left[\frac{(c^j+c^k-2c^i) - (s^j+s^k-2s^i)}{5} + \frac{t}{3}\right]^2$$

which, as readily checked, satisfies the properties established above.

First Order Stochastic Dominance

An alternative possibility is that investment exerts a first order stochastic shift in the distribution of a^i. For example, if a^i corresponds to production costs, $F_{I^i}(a^i|I^i) > 0$ for all $a^i \epsilon (\underline{a}, \overline{a})$ characterizes a first order shift (Hadar and Russell, 1969). Since $\pi^i_{a^j}(a^i, a^j) > 0$ in the process innovation models, we have using (7) and (8) that $E\pi^i_{I^j}(I^i, I^j, r) < 0$. When R & D shifts the mean cost in the sense of first order dominance, a negative externality is associated with investment across firms.[10] Of course, there remains a positive externality to consumers, who benefit from lower expected costs. A related tradeoff occurs when investment raises the expected value of the product innovation variable.

Since actual investment is likely to affect both the riskiness and the expected value of the relevant product market variable distribution,[11] one must exert caution in suggesting any broad R & D policy. Subsidies are unambiguously attractive, however, for R & D that primarily affects the riskiness of the relevant distribution, as there is then no trade off between firms and consumers, with both preferring greater investment.

VI Conclusion

Recent work on the choice of R & D technology has suggested that too much risk-taking occurs in R & D projects. We have argued that this result is sensitive to the winner-take-all assumption maintained in previous work. Much R & D occurs in smoother, winner-take-more environments. Using standard models of imperfect competition, we show that there is actually a positive externality associated with investment across firms, indicating that firms undertake R & D that is less risky than is collectively optimal. In addition, we show that consumers receive positive externalities from risk-increasing investments, which provides an additional basis from which to argue that too little risk-increasing R & D occurs. Our results apply to both process and product innovations. The key assumption in our work

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[10] This discussion is treated rigorously by Bagwell and Staiger, 1989a.

[11] That is, the most plausible class of investments may be those which induce mean-altering second order stochastic shifts.

appears to be that the innovation is "non-drastic," in the sense that the innovation is never sufficiently great to enable a firm to monopolize the market.[12] This suggests that risky R & D which maintains market structure should be subsidized, while, as in the patent race literature, an R & D tax may be attractive for risky and drastic R & D.

Finally, while the literature on the choice of R & D technology has been set in an environment in which either the "prize" or its "discovery date" is uncertain, it is probably more reasonable to think of both as uncertain outcomes of R & D investment. One way to capture this dual uncertainty would be to imagine that a firm's investment determines a distribution, from which it draws a (cost or product) parameter at each instant in time and then decides whether or not to terminate its R & D program and bring the product to market. In general, such an extension would require a careful treatment of the way in which innovation proceeds as well as the observability of rival progress. We view this as an important direction for future research.

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[12] Another possibly important assumption is that demand is linear. With nonlinear demands, the signs of terms such as $\hat{P}^i_{c^i c^i}(\bar{\alpha},\bar{\alpha},c^i,c^j)$ must be evaluated. Since this is an equilibrium price, its sign will depend on the sign and size of third order derivatives for $\tilde{\pi}^i(P^i,P^j,\bar{\alpha},\bar{\alpha},c^i)$. There are no obvious assumptions to place on such derivatives at a general level, and closed-form analysis of nonlinear models has proved quite difficult. However, since our purpose is only to establish a presumption that the market chooses too little risk in non-drastic R & D settings (or at the least that the opposite presumption depends heavily on the winner-take-all assumption), the restriction to linear demand settings does not seem inappropriate.

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