Discussion Paper No. 871

Violence in incentives: Pain in a principal-agent model

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March 1990

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Abstract

One reason a person hurts another is to get that person to do something. This paper uses a model to show that threatening pain can be rational and that pain is inflicted upon people who are poor in the sense of having bad alternatives. The model corrects a confusion in previous models of slavery, gives an explanation why child and not adult laborers were beaten during the British industrial revolution, and prompts a discussion of the dangers of rational-choice modelling.

1. Introduction

Violence, which seems inherently irrational, and economics, which calls itself the study of rational behavior, seem to have little to do with each other. But violence is often used in incentives—one reason a person threatens to hurt another is to get that person to do something. This paper uses a model to show that threatening pain can be rational and that pain is inflicted upon people who are poor in the sense of having bad alternatives. The model corrects a confusion in previous models of slavery and gives an explanation why child and not adult laborers were beaten during the British industrial revolution.

2. Threats

Consider the following incentives (here the "principal" addresses the "agent"): (i) "If you take out the garbage, I'll give you five dollars," (ii) "If you don't take out the garbage, I'll fine you six dollars," (iii) "If you don't take out the garbage, I'll slap you," and (iv) "If you take out the garbage, I'll give you a hug." The first I call an "offer," since it involves an action which if carried out would make the agent better off (five dollars richer) and the principal worse off (five dollars poorer).[1] Both the second and third might be called "threats," since they involve actions which if carried out would make the agent worse off. They are different, however, in that carrying out the action of the second makes the principal better off while carrying out the action of the third makes him worse off, if slapping takes energy or is disagreeable. In this paper I call the third a "threat" and the second a "fine" (for lack of a more general term).[2] There is no ready name for the fourth incentive, in which carrying out the action makes both principal and agent better off.

Threats are not only socially wasteful, as is often acknowledged, but are wasteful for the principal. If the principal wants to create the possibility of the agent being worse off, she would prefer to use a fine because carrying out its action would make her better instead of worse off.[3] Threats are used when the principal cannot fine the agent, as in warfare. But even a principal who can fine agents cannot fine an agent who has nothing to give up.

This is an explanation why workers who receive threats are usually poor, in the sense of having little wealth. The most common, and perhaps cheapest, threat is to threaten to physically

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[1] "Better or worse off" refers to the action considered *by itself.* I call the first example an offer, even if having the garbage taken out is worth more than five dollars to the principal.

[2] Schelling (1980, p. 123, n. 5) makes this distinction, which is perhaps not so obvious: Osband (1987, p. 593), in considering Southern slavery, confuses them.

[3] Fines are more credible for this reason. If someone threatens you, there is the possibility of "calling her bluff." It would be foolish to respond similarly to a fine.

hurt. Hence, in this paper, pain is the name of what is "given" in a threat.[4] An explanation why workers who are whipped are poor, in the sense of having poor alternatives, is the result of a model.

3. The model

Consider Patrick Waldrum, an eight-year-old living in Wolverhampton in 1841, who

> goes a begging every day; it's hard work. Goes to it about eight o'clock in the morning, and begs till six or seven at night; gets 2*d*. sometimes in the day and some bread and cheese or meat....sometimes only gets a halfpenny all day; that's a very bad day's work, for working at it all day, and if it rains too. Has no father, has got a mother; she works sometimes a gardening for ******; and when she has no work she begs too....[She] beats him sometimes when he only earns a halfpenny a day; always beats him when he don't get nothing.[5]

If he works hard his chances of earning money improve, but there is always the possibility that he will get little or nothing, meaning that either he didn't work hard or that he just had bad luck. His mother cannot tell how hard he works. She only sees the earnings he brings home, and if they are low, she beats him.

In the basic principal-agent model, the agent chooses an effort level which determines a probability distribution over several possible outcomes which give different payoffs to the principal. The principal cannot observe the effort level, and so cannot give the agent an incentive which is based upon it. The principal can see the realized outcome, however, and hence gives the agent an incentive consisting of actions conditional on the outcomes. Facing an incentive, the agent chooses an effort level which maximizes her expected utility. Knowing this decision-making process, the principal picks an incentive which maximizes her own expected utility, given the constraint that the agent's expected utility must be at least his "reservation utility," the utility he could get in some alternative.

In my model, there are two outcomes: "good," which yields a payoff of one to the principal, and "bad," which yields a payoff of zero. The agent, who has a reservation utility of $\bar{U}$, chooses the effort level a, which is the probability that the good outcome occurs: the harder the agent works, the more likely the good outcome. In other models, the incentive is a set of conditional cash transfers, which are offers or fines. Here the principal cannot give the agent fines (since the agent has no wealth) but instead can give the agent threats. What is "given" in an offer I call

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[4]Scarry (1985, p. 16) reminds us that "the very word 'pain' has its etymological home in '*poena*' or 'punishment'."

[5]United Kingdom, Parliament, 1843b, p. q21. Quoted in Nardinelli 1982, p. 288.

"money" and what is "given" in a threat I call "pain." The incentive can thus be expressed as two ordered pairs: (m^g, p^g) and (m^b, p^b), the amounts of money and pain given if the outcome is good or bad, respectively.

The cost to the principal of giving an amount of money m and an amount of pain p is given by the cost function $C(m,p)$. The agent's utility from receiving money m, pain p, and exerting effort a is given by $U(m,p) - V(a)$.

We assume that $U: \mathbf{R}_+^2 \to \mathbf{R}$ is continuous and is differentiable over its interior. Also, $f: \mathbf{R}_+ \to \mathbf{R}$ and $g: \mathbf{R}_+ \to \mathbf{R}$ (where $f(m) = U(m,0)$ and $g(p) = U(0,p)$) are twice-differentiable over their interiors. Utility increases in money and decreases in pain: $\partial U(m,p)/\partial m > 0$, $\partial f(m)/\partial m > 0$, and $\partial U(m,p)/\partial p < 0$, $\partial g(p)/\partial p < 0$ for $m > 0$ and $p > 0$. The agent is risk averse with respect to lotteries over money alone (a standard assumption) and risk loving with respect to lotteries over pain alone (not standard, but one which has some support):[6] $\partial^2 f(m)/\partial m^2 < 0$ for $m > 0$ and $\partial^2 g(p)/\partial p^2 > 0$ for $p > 0$. The agent can be given arbitrarily large utility with money and arbitrarily large disutility with pain: $\lim_{m\to\infty} f(m) = \infty$ and $\lim_{p\to\infty} g(p) = -\infty$. We normalize $U(0,0) = 0$.

We assume that $V: [0,1) \to \mathbf{R}$ is three times differentiable over its interior. The marginal disutility of effort starts at zero (the right hand derivative of V at zero exists and is equal to zero) and increases as effort increases from zero: $V'(a) > 0$ and $V''(a) > 0$ for $a \in (0,1)$. The disutility of effort can get arbitrarily large: $\lim_{a\to 1} V(a) = \infty$, and we normalize $V(0) = 0$.

The principal is assumed to be risk neutral. His cost function C: $\mathbf{R}_+^2 \to \mathbf{R}$ is continuous and is differentiable over its interior, and $j: \mathbf{R}_+ \to \mathbf{R}$ and $k: \mathbf{R}_+ \to \mathbf{R}$ (where $j(m) = C(m,0)$ and $k(p) = C(0,p)$) are twice-differentiable over their interiors. The cost of providing a money-pain bundle increases in money and in pain: $\partial C(m,p)/\partial m > 0$, $\partial j(m)/\partial m > 0$ and $\partial C(m,p)/\partial p > 0$, $\partial k(p)/\partial p > 0$ for $m > 0$ and $p > 0$. There are constant or increasing marginal costs to providing money alone and pain alone: $\partial^2 j(m)/\partial m^2 \geq 0$ for $m > 0$ and $\partial^2 k(p)/\partial p^2 \geq 0$ for $p > 0$, and we normalize $C(0,0) = 0$.

These are the model's assumptions, except for two which are introduced later. Given the three functions U, V, and C, and the number $\bar{U}$, we try to find an optimal incentive (m^g, p^g) and (m^b, p^b) and the effort level a.

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[6]Tversky and Kahneman (1981, n. 6, p. 458) provide references, including Eraker and Sox (1981, p. 30), who found that in hypothetical scenarios "rather than preferring a certain and intermediate adverse drug effect, most patients were willing to risk a possible severe drug side effect in order to have a chance at experiencing no adverse reaction."

Given the incentive (m^g,p^g) and (m^b,p^b), what level of effort will the agent choose? Her expected utility is $aU(m^g,p^g) + (1-a)U(m^b,p^b) - V(a)$ and so her choice of a is determined by the first order condition $U(m^g,p^g) - U(m^b,p^b) - V'(a) = 0$. The principal knows this and takes this into account. He maximizes his expected profits given this condition (the "incentive compatibility" constraint) and the constraint that the agent's expected utility must be at least $\bar{U}$ (the "voluntary participation" constraint). His problem is

M. Find (m^g,p^g), (m^b,p^b), and a which maximize $a(1 - C(m^g,p^g)) + (1-a)(-C(m^b,p^b))$

such that $aU(m^g,p^g) + (1-a)U(m^b,p^b) - V(a) \geq \bar{U}$

and $U(m^g,p^g) - U(m^b,p^b) - V'(a) = 0.$

If $m^g > 0$ and $p^g > 0$, necessary Kuhn-Tucker conditions for a maximum imply that $(\partial C(m^g,p^g)/\partial m)/(\partial C(m^g,p^g)/\partial p) = (\partial U(m^g,p^g)/\partial m)/(\partial U(m^g,p^g)/\partial p))$, the familiar "tangency" condition in which the marginal rate of transformation equals the marginal rate of substitution. But by the assumptions, the left hand side of the equation is positive and the right hand side is negative. This is also the case for $m^b > 0$ and $p^b > 0$. Thus if (m^g,p^g), (m^b,p^b), and a is a solution of problem M, then either $m^g = 0$ or $p^g = 0$, and either $m^b = 0$ or $p^b = 0$. The principal would never use both money and pain after a given outcome, because he could use less of both and thereby give the agent the same utility at a lower cost. Since for a given outcome money and pain are not both given, we can represent the incentive (m^g,p^g), (m^b,p^b) with two numbers: the agent's utility if the good outcome occurs u^g, and her utility if the bad outcome occurs u^b. If one of these numbers is negative (positive) the corresponding bundle contains only pain (money).

Let's define the expenditure function $e(u)$, the principal's cost of giving the agent utility u.

$$e(u) = \begin{cases} C(f^{-1}(u),0) & \text{if } u > 0, \\ 0 & \text{if } u = 0, \\ C(0,g^{-1}(u)) & \text{if } u < 0, \end{cases}$$

where $f(m) = U(m,0)$ and $g(p) = U(0,p)$.

It is easy to show that $e(u)$ is twice-differentiable for nonzero u, is continuous, and is strictly convex. It is U-shaped, with minimum zero at zero. I assumed that the agent is risk loving with respect to pain to make the expenditure function convex over $(-\infty,0)$. Risk loving with respect to pain is analogous to risk aversion with respect to money: as the level of pain (money) increases, an additional unit of pain (money) becomes less effective in producing disutility (utility). For technical ease, we assume that e is twice-differentiable at zero. A reasonable "real world" utility

function and cost function might not satisfy this assumption, but could modified very close to zero so as to satisfy it.

So we get a simplified problem

S. Find u^g, u^b, and a which maximize $a(1 - e(u^g)) + (1-a)(-e(u^b))$

such that $au^g + (1-a)u^b - V(a) \geq \bar{U}$

and $u^g - u^b - V'(a) = 0$.

If u^g, u^b, and a solve problem S, then (m^g,p^g), (m^b,p^b), and a solve problem M, where, for $i \in \{g, b\}$: if $u^i > 0$, then $p^i = 0$ and $U(m^i,0) = u^i$; if $u^i < 0$, then $m^i = 0$ and $U(0,p^i) = u^i$; and if $u^i = 0$, then $m^i = 0$ and $p^i = 0$.

To solve problem S, we first consider the problem in which the participation constraint holds with equality. Call this problem ES, and instead of $\bar{U}$, use the dummy variable u. From the constraints we can solve for u^g and u^b:

$$u^g = u + V(a) + (1-a)V'(a), \quad u^b = u + V(a) - aV'(a).$$

Then $\pi(a,u) = a(1 - e(u^g)) + (1-a)(-e(u^b))$, where $\pi(a,u)$ is the expected profit of the principal given a and u, and u^g and u^b are understood as functions of a and u. Now ES is an unconstrained maximization problem in one variable: find a which maximizes $\pi(a,u)$. Since $\partial u^g/\partial a = (1-a)V''(a)$ and $\partial u^b/\partial a = -aV''(a)$, $\partial\pi/\partial a$ reduces to

$$\partial\pi/\partial a = 1 - e(u^g) + e(u^b) - a(1-a)V''(a)(e'(u^g) - e'(u^b)).$$

When $a = 0$, $u^g = u^b$ and thus $\partial\pi/\partial a = 1$. Since π is increasing at zero, if u^g, u^b, and a solves ES, then $a > 0$. Since $V'(a) > 0$ for nonzero a and $u^g - u^b = V'(a)$, we know that $u^g > u^b$. A solution to S must also solve ES for some $u \geq \bar{U}$, and so we know that if u^g, u^b, and a solve S, then $a > 0$ and $u^g > u^b$.

Necessary Kuhn-Tucker conditions for problem S are

$$0 = -ae'(u^g) + \lambda a + \mu,$$

$$0 = -(1-a)e'(u^b) + \lambda(1-a) - \mu,$$

$$0 = 1 - e(u^g) + e(u^b) - \mu V''(a),$$

where λ is nonnegative and μ is positive, negative, or zero. We can solve for λ and μ:

$$\lambda = ae'(u^g) + (1-a)e'(u^b), \quad \mu = a(1-a)(e'(u^g) - e'(u^b)).$$

Since $a > 0$, $u^g > u^b$, and e' is a strictly increasing function (e is strictly convex), μ is positive. Since λ is nonnegative and a is positive, $\lambda a + \mu$ is positive. From the first condition above, $ae'(u^g) = \lambda a + \mu$ and therefore $e'(u^g)$ is positive. But $e'(u) > 0$ only when $u > 0$, and so we know that if u^g, u^b, and

a solve S, then $u^g > 0$. Pain will not be given if the outcome is good. In any optimal incentive, the agent receives money with at least some probability.

So far our results are descriptions of possible solutions. Before considering existence and uniqueness, we need the main assumption of the model.

L. $(1-a)V'''(a) \geq 2V''(a)$ for $a \in (0,1)$.

Although it seems arbitrary, it is plausible and easily justified. The agent chooses the effort level according to the equation $u^g - u^b = V'(a)$. Given a utility difference between the two states, the agent supplies effort: the larger the difference, the higher the effort. This can be interpreted as a "production" function, with utility difference as input and effort as output. Now think of the level of effort measured not as a, but as proportional to $-\log(1-a)$. That is, a 0.9 probability of the good outcome corresponds to an effort level of 1, a 0.99 probability corresponds to an effort level of 2, a 0.999 probability corresponds to an effort level of 3, and so on. This measurement scale makes sense, as each unit of effort reduces the probability of the bad outcome by a multiplicative factor (for example, each unit of effort goes into laying down another layer of shingles, each with the same independent probability of leaking, and the bad outcome is a leaky roof). A sufficient condition for L is that the "production" of effort (measured in this "log-failure rate" scale) has constant or decreasing marginal returns with respect to utility difference.[7]

Back to problem ES. The second derivative of π is

$$\partial^2\pi/\partial a^2 = -V'''(a)(e'(u^g)(1-a) + e'(u^b)a) - ((1-2a)V''(a) + a(1-a)V'''(a))(e'(u^g) - e'(u^b)) - a(1-a)(V''(a))^2(e''(u^g)(1-a) + e''(u^b)a).$$

Since $e'' \geq 0$, the third term is nonpositive. To show that $\partial^2\pi/\partial a^2 < 0$ for all a, it suffices to show that

$$V''(a)(e'(u^g)(1-a) + e'(u^b)a) + ((1-2a)V''(a) + a(1-a)V'''(a))(e'(u^g) - e'(u^b)) > 0.$$

Since e' increases and $u^g > u^b$, $e'(u^g) - e'(u^b) > 0$. From assumption L, the above quantity is greater than or equal to

$$V''(a)(e'(u^g)(1-a) + e'(u^b)a) + ((1-2a)V''(a) + 2aV''(a))(e'(u^g) - e'(u^b))$$
$$= V''(a)((1-a)(e'(u^g) - e'(u^b)) + e'(u^g)),$$

which is positive. So π is strictly concave in a, and a solution to the first order condition $\partial\pi/\partial a = 0$ gives the unique optimum. But does a solution exist? At $a = 0$, $\partial\pi/\partial a = 1$. Since $\partial\pi/\partial a$ is continuous, it suffices to show that $\partial\pi/\partial a$ becomes negative as $a \to 1$.

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[7] In other words, if $V(a) = Y(r(a))$, where $r(a) = -\log(1-a)$, then $Y'''(r) \geq 0$ for $r \in [0,\infty)$ is sufficient for L.

Lemma 1. $\lim_{a\to 1} \partial\pi/\partial a = -\infty$.

Proof. In the Appendix.

So given u, a unique a maximizes $\pi(a,u)$. Since π is continuous in u, by the maximum theorem the function $a(u)$ is continuous. Since $a(u)$ is defined implicitly by $\pi_a(a(u),u) = 0$ (where $\pi_a = \partial\pi/\partial a$), π_a is differentiable in a and u, and $\partial^2\pi/\partial a^2 < 0$ for all a and u, by the implicit function theorem $a(u)$ is differentiable. Hence $u^g(u) = u + V(a(u)) + (1-a)V'(a(u))$, $u^b(u) = u + V(a(u)) - a(u)V'(a(u))$, and $\pi(u) = a(u)(1 - e(u^g(u))) + (1-a(u))(-e(u^b(u)))$ are also continuous and differentiable. It turns out that u^g and u^b both increase in u .

Lemma 2. $\partial u^g/\partial u > 0$ and $\partial u^b/\partial u > 0$.

Proof. In the Appendix.

As $u\to -\infty$, since $u^b < u$, we know that $u^b\to -\infty$. As $u\to\infty$, the first order condition $0 = \partial\pi/\partial a = 1 - e(u^g) + e(u^b) - a(1-a)V''(a)(e'(u^g) - e'(u^b))$ must hold. Since $e(u^g)\to\infty$ (since $u^g > u$, $u^g\to\infty$) and $a(1-a)V''(a)(e'(u^g) - e'(u^b)) > 0$, we know that $u^b\to\infty$. Since $u^b(u)$ is continuous and strictly increasing, there exists a unique u^* such that $u^b(u^*) = 0$.

Now we are finally ready to analyze problem S. From the envelope theorem, $\partial\pi/\partial u = -\lambda = -ae'(u^g) - (1-a)e'(u^b)$, which is negative if $u^b \geq 0$. So π decreases over $[u^*, \infty)$. Thus when $\bar{U} < u^*$, maximizing π over $[\bar{U},\infty)$ is equivalent to maximizing π over the compact set $[\bar{U},u^*]$. Since π is continuous, by the Weierstrass theorem at least one optimum exists. If $\bar{U} \geq u^*$, problem S has the unique solution $u^g(\bar{U})$, $u^b(\bar{U})$, and $a(\bar{U})$.

Remember that $u^g > 0$ always, and so money and not pain is given when the outcome is good. What happens when the outcome is bad? If $\bar{U} \geq u^*$, in the unique solution no pain is given. If $\bar{U} < u^*$, in a solution pain is given. All other things remaining the same, as the agent's reservation utility increases, the principal's optimal choice of incentive moves from one in which pain is used to one in which pain isn't used.

4. Discussion of the model

Figure 1 shows the optimum u^g, u^b , and a and expected profit π for $\bar{U}$ between –0.5 and 1, where $V(a)=(1/(1-a)-a-1)/4$ and $e(u)=u^2/4$. The a, u^g, u^b, and π curves are flat for low $\bar{U}$, where the voluntary participation constraint doesn't hold with equality.[8] In this region, the agent

[8]In models in which only offers and fines are available (as in the rest of the literature), and there are two outcomes, under weak assumptions it can be shown that the voluntary participation constraint will always bind. See Grossman and Hart 1983, p. 30.

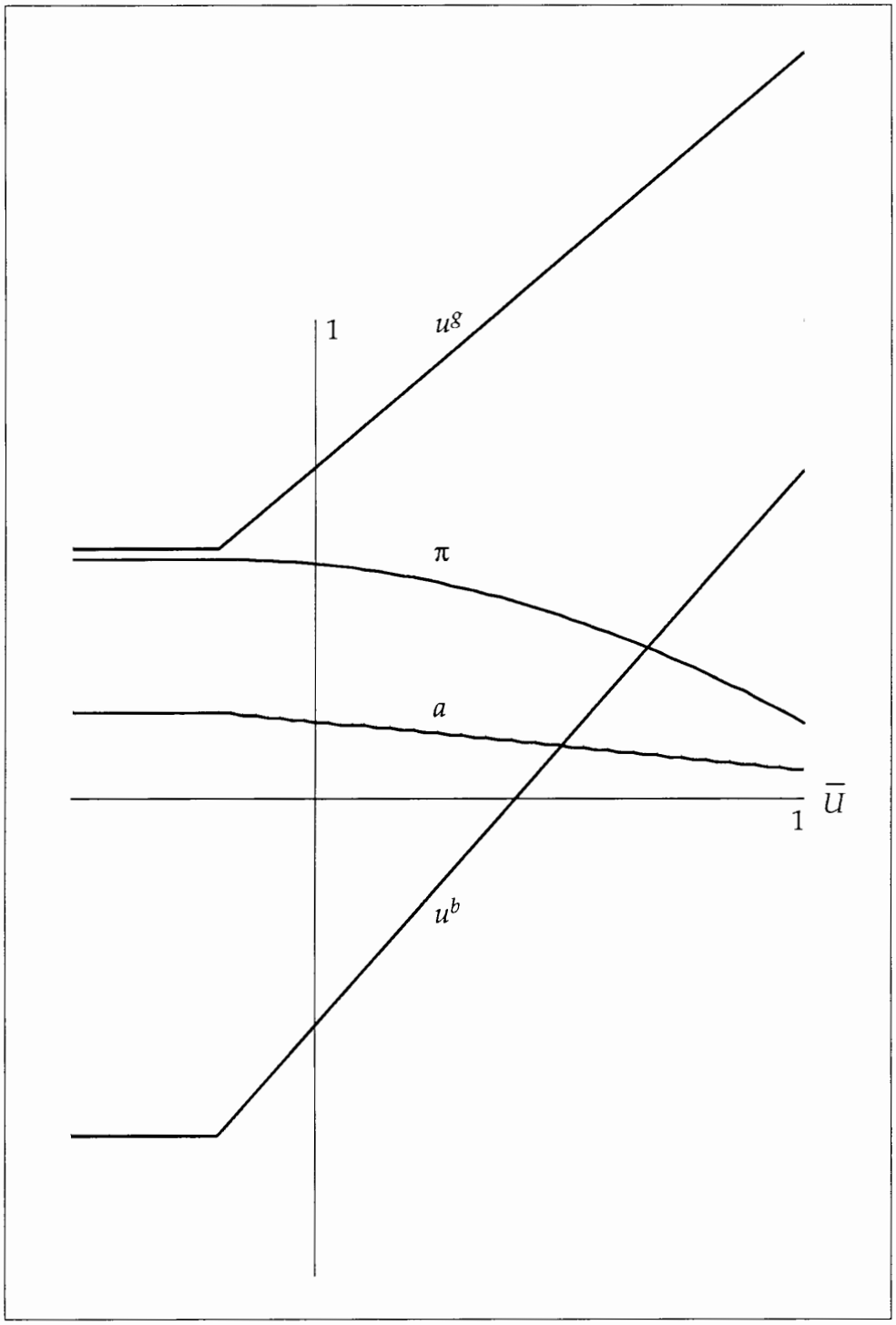

Figure 1.

receives a higher expected utility than her reservation utility because the principal would not profit by lowering it. When the voluntary participation constraint does hold with equality, u^g and u^b increase in $\bar{U}$. If the agent's $\bar{U}$ is below 0.4, pain is given to the agent with a probability of roughly 0.8. If it is above 0.4, only wages are given.

The model is interesting from the standpoint of principal-agent theory, since a "limitation of the present models is the restricted reward or penalty system used. It is virtually always stated in terms of monetary payments."[9] A related point is that in existing models there is no "wealth constraint": the agent is assumed to have enough wealth so that threats need not be given. Also, there are more general models, in which the agent's effort level determines a probability distribution over several or a continuum of outcomes and the optimal incentive is correspondingly more complex. However, "complex incentive schemes...stand in the way of serious extensions and applications. One can say little about comparative statics properties of the model and it is also hard to introduce additional variables into the analysis."[10] Although my model is simple in its probabilistic structure, at least comparative statics results are obtainable.

What makes the model interesting from a "welfare" point of view is an assumption that physically hurting someone is morally wrong in a way not expressed by the person's expected utility. That is, if I physically hurt another person with some probability, I am more culpable than if I never physically hurt him but give him the same or even a lower expected utility. This assumption, however, is not shared by standard present-day economics, in which a person's utility is not only the basis for her decisions but the only relevant consideration in ethically evaluating situations involving her.[11] Although expected utility might be useful in describing a person's preferences over money and pain (as in my model), money and pain cannot for the purposes of moral evaluation be reduced to a single number. I mention this to take care of the objection: "In the model, the agent always receives a expected utility of at least her reservation utility, and as economists, expected utility is all we need or are entitled to consider. Why care about whether or not the agent is whipped with some probability, for other than predictive purposes? After all, he *chooses* to take this risk."

This objection can also be met with an example. "Enough evidence exists now to show that woman-battering occurs to such a large extent in this and other industrialized countries that it

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[9]Arrow 1985, p. 50.

[10]Hart and Holmström 1987, p. 91.

[11]This conflation of utility as choice and as ethical criterion has been criticized by Amartya Sen. See Sen 1982.

cannot be viewed as a relatively rare problem of individual neuroticism or psychopathology."[12] Many women in violent relationships remain and are repeatedly beaten. When battered women were "asked directly, 'Why did you stay?' they usually stated that they felt they had few or undesirable or no alternatives."[13] I hope that an economist would not feel compelled to agree with a police officer who, since most women can find *some* sort of alternative in which they are not beaten, "describes battered women as a 'victimless' crime like rape and prostitution....'The husband "thumps" them once a week. It's a way of life. They must like it or they wouldn't stay.' "[14] That battered women choose to stay does not diminish the criminality of their male batterers. Nor does it make the social institution of marriage or cohabitation (like that of slavery to the extent that violence is legitimated) any more tolerable. What it does is make urgent the need to understand how social conditions (such as property relations and social norms) make inflicting and being a victim of violence rational choices.[15]

5. Slavery

None of the few formal economic models of slavery[16] explicitly models the monitoring or "policing" problem. My model might fill a need, especially if "the whole institution of slavery seems to have hinged on the question of policing costs."[17] Before discussing how the model is most helpful, I apply it to slavery in the American South.

Aside from the overwhelming objection that slaves and slaveowners just aren't rational in the economic sense, maybe the most immediate objection to my model's application to slavery concerns the notion of reservation utility. Usually it is interpreted as the utility or wage that the agent could get in another job or by simply not working. But slaves don't have these alternatives. There are, however, three ways to interpret a slave's reservation utility. The first is that there is a very large but not infinite disutility of death or suicide. Consider "Margaret Garner, fugitive

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[12]Pagelow 1981, p. 21.

[13]Pagelow 1981, p. 72.

[14]Waterbury 1976, p. 77, quoted in Pagelow 1981, p. 71. This opinion is held widely enough for the *Handbook for Domestic Violence Victims* (Illinois Coalition 1986, p. 4) to include: "*MYTH:* A woman who stays with her husband or boyfriend after being beaten must like to be beaten. *REALITY:* Being beaten hurts and no one likes it. There are many reasons why women remain with abusive men, including their fear of further violence, the financial hardships of leaving, their emotional attachment to their partners and their belief that families should stay together."

[15]See Gordon 1988, ch. 8.

[16]These include Barzel 1977, Bergstrom 1971, Canarella and Tomaske 1975, Findlay 1975, and Fogel and Engerman 1974.

[17]Barzel 1977, p. 109.

slave, who, when trapped near Cincinnati, killed her own daughter and tried to kill herself. She rejoiced that the girl was dead—'now she would never know what a woman suffers as a slave'—and pleaded to be tried for murder. 'I will go singing to the gallows rather than be returned to slavery.' "[18] The second interpretation is that some slaves had a very small chance to successfully escape: "During the 1850s about a thousand slaves a year ran away to the North, Canada, and Mexico."[19] The expected utility of an escape attempt (different for each person—most runaways were young men) could thus be considered as a slave's reservation utility. Thirdly, there were several "economic" reasons why slaveowners did not treat their slaves arbitrarily badly, such as the fact that health was necessary for hard work, and that they were literally "human capital" to be maintained for future productivity and resale value. Not uncommonly, a slaveowner's neighbors would intervene if he treated his slaves unusually badly.[20]

What are the results of the model? It tells us that slaveowners gain by lowering the reservation utilities of their slaves (to the extent they can), and that once they have done so they maximize profits by whipping their slaves. It also gives another reason for "paternalism" (why slaveowners did not make their slaves' expected utilities arbitrarily low) based on the cost of carrying out incentives: in cases such as the example in Section 4, the slave would receive a minimum expected utility, regardless of how low her reservation utility is.

I think the model's best use is not in application to specific data,[21] but in sharpening our intuitions. I show this by way of a critique of Robert Fogel and Stanley Engerman's model of a slaveowner's choice of how much "force" versus "wages" to use as incentives for his slaves.[22] Theirs is like mine in assuming an (economically) rational slaveowner:

> The shrewd capitalistic businessmen who ran the slave plantations were not usually psychological perverts who gloried in the exercise of unlimited force for its own sake.

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[18]Aptheker 1949, p. 12, quoted in Davis 1981, p. 21.

[19]Genovese 1974, p. 648.

[20]Genovese (1974, p. 41) explains why: " 'Harmony among neighbors is very important in the successful management of slaves,' wrote a planter in an article directed to his own class. A good manager among bad ones, he explained, faces a hopeless task, for the slaves easily perceive differences and become dissatisfied. It does no good, wrote another, to enforce discipline on your plantation if the next planter does not. These arguments cut in both directions. They called for strict discipline from those who tended to be lax and for restraint from those who tended to be harsh."

[21]One source for the frequency and severity of whipping is the diary of Bennet H. Barrow, presented in Davis 1943. Fogel and Engerman's treatment of it (1974, vol. 1, p. 144–148) is critiqued by Gutman and Sutch 1976.

[22]Fogel and Engerman 1974, vol. 2, p. 155–158. Canarella and Tomaske's model (1975) is exactly the same.

> They generally used force for exactly the same purpose as they used positive incentives—to achieve the largest product at the lowest cost. Like everything [*sic*] else, they strove to use force not cruelly, but optimally.[23]

It is best explained with their Figure B.7, reproduced below as Figure 2.

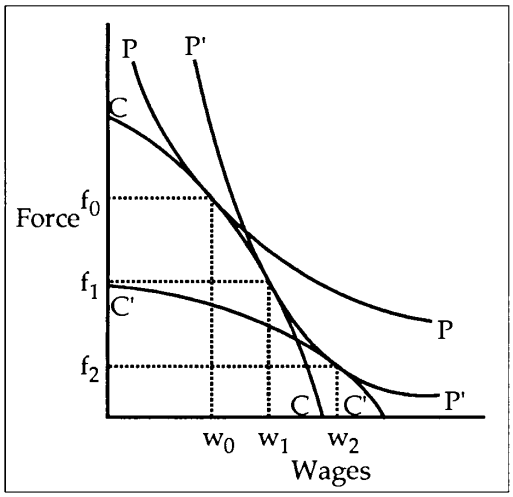

Figure 2.

This is the "usual production-isoquant mapping." The PP curve "is the unit isoquant for the 'production' of labor. It shows all of the various combinations of force and 'wages' (positive inducements) that will yield one unit of labor (measured, say, in man years)." The CC curve "is the isocost curve. It describes all of the different combinations of force and 'wages' which can be 'purchased' for a given total expenditure," and is concave because "the more force that is used, the greater the cost of an additional unit of force of a given 'quality.' "[24] The optimal choice of force versus wages is the point at which PP and CC are tangent.

The assumption that the "production" of labor isoquants are convex is crucial; they offer no argument that force and wages "produce" labor in the way that factor inputs produce output (as in the standard theory of the firm). But a bigger problem is the meaning of a "combination" of force and wages. Force and wages would not both be used *ex post*, after the slave has done something. The master would never efficiently use wages and a little force to reward the slave, or force and a little wages to punish the slave.[25] So a combination of force and wages means that both force and wages are used *ex ante*: force is used if one thing happens (like the unsatisfactory

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[23]Fogel and Engerman 1974, vol. 1, p. 232.

[24]Fogel and Engerman 1974, vol. 2, p. 155–156.

[25]Since a slave requires "positive" incentives like food to be a strong laborer, a slaveowner might whip the slave and give her food at the same time. But my argument still works; the point is that pain is given conditionally, and must be so modelled.

completion of a task), and wages are used if another happens (like satisfactory completion).

How should the cost of a force-wages combination be determined? It must depend on how often, or with what probability, force and wages are used.[26] If a certain amount of wages are given with complete certainty, the amount of force could be made arbitrarily large at no cost (since it is never used). This doesn't make much sense, because a slaveowner limits his threats at least partly because he might have to carry them out. Force would not be used with complete certainty, because this would mean that the threat never works.

So we are left with the case in which force and wages are both used with some likelihood. What determines when force versus wages are given? The slave's performance must be at least partly involved, for otherwise there would be no incentive. But then, how can the cost of the force-wages combination be determined independently of the labor "production" process, if the cost of implementing the combination depends on the slave's performance? Roughly speaking, the curve CC cannot be determined independently of the curve PP. This invalidates the idea of optimal choice (or "equilibrium") at a tangency.

I think that their confusion is widely shared. We forget that incentives are fundamentally *conditional*.[27] When incentives are narrowly conceived as offers it is easy to forget their conditional nature, for if they work they are always carried out. We commonly think of "exchanging" labor for a wage, which makes a wage seem more like an object than an incentive, in which money is given *if* an action is performed. But of course one doesn't "exchange" labor for whipping or pain. Because we almost always think of them as offers, we fetishize incentives.[28]

For if they do not correctly model the slaveowner's optimal force-wages choice, what do they model? Relabel the "force" axis in Figure 3 as "clothing" or some other good. Now if the slaveowner needs to give the slave a certain utility to motivate her to perform a task, he will give

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[26]"Quality" of force is puzzling. Are twenty lashes ten times as much force than two lashes, or force of a different quality? The proper distinction is between the severity of pain and how often or to what probability it is inflicted.

[27]Gutman and Sutch (1976, n. 3 on p. 58 and p. 73) grasp this ("As economists, Fogel and Engerman might be forgiven for their confusion of the frequency and incidence of punishment with its effectiveness, a point more within the expertise of sociologists and criminologists") only to let go in a different context: " 'It was the *universal custom* in Georgia,' explained Ralph Flanders [Flanders 1933, p. 146, italics are Gutman and Sutch's]... 'to allow slaves the privilege of raising small crops of their own for which the master paid cash, or which could be exchanged at the storeroom for anything they chose to buy.' There is a great difference between a 'universal custom' and a selective labor incentive. A custom is a habitual practice, an established way of doing things. If it happened this way, the garden patch hardly served as a positive labor incentive." Giving a teenager the keys to the car is an (almost universal) American custom, but is often used in an incentive.

[28]"The fetishism thesis can in fact be stated as follows: relations of interactions between men appear as relations of comparison between objects." (Elster 1985, p. 96).

her that utility at lowest cost. Say that level of utility is represented by PP, an indifference curve. Then the point of tangency of the isocost and indifference curves represents the clothing-wages combination which gives the slave that level of utility at lowest cost. The point of tangency thus legitimately represents the slaveowner's optimal choice.

Fogel and Engerman use a model of optimal choice between two offers to model the optimal choice between an offer and a threat. That they can do this and think it obvious, "usual," is evidence that they think of incentives as objects, "wages" and "force," which can be labels of axes like any good or commodity. This confusion is more literally displayed by Giorgio Canarella and John Tomaske: "For slaves, force is a discommodity."[29] Force, or violence, is undesirable, but is different from a pet skunk inherited from one's great-uncle: it is not an object.[30]

6. Child labor

Perhaps because in the United States today there is less apparent everyday violence, we forget how recently the threat of pain has been given to both "unfree" and "free" laborers.[31] Today, children are the main recipients. In this section I apply my model to child labor in the British industrial revolution.

One thing my model makes clearer is why people would receive pain in a labor relationship without being forced to. What is meant here by "being forced to" can be explained in terms of the model: one forces another person to be her laborer if one does so by (even indirectly) changing her reservation utility. "Voluntary" slavery can exist if the alternatives are bad enough and are the result of fate and not of the master.[32] But we can still call voluntary slavery "forced labor," since once a person becomes a slave his reservation utility is largely in the control of his master. Such a person would not be forced to enter the relationship but would be forced to continue in it. Apprentices in seventeenth and eighteenth century Britain were forced laborers whose masters had almost complete control over their working and non-working lives.

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[29]Canarella and Tomaske 1975, p. 626.

[30]In my model I did not use words like "wages" or "force," because of their implicit association with incentives. Rather, "money" and "pain," which have less of an association, are the "things" which are given in incentives (and not the incentives themselves). Still, one doesn't really "give pain" like one gives money—one whips, or one hurts (someone else).

[31]For example, whipping Asian contract laborers was standard practice on Hawaiian sugar plantations well into this century. See Takaki 1983.

[32]Patterson (1982, p. 130) explains: "Poverty was, of course, one of the main reasons for self-sale...in several advanced societies such as China and Japan it was at times a major source of slaves. In Russia between the seventeenth and nineteenth centuries self-sale as a result of poverty was the most important reason for enslavement among the mass of domestic slaves."

Children who worked for wages in nineteenth century British factories who were "free" laborers had reservation utilities which were not under the control of their employers. The job of lowering their reservation utilities was done by their parents, as told by John Bolling, a cotton master:

> The other day there were three children run away; the mother of one of them brought him back and asked us to beat him; that I could not permit; she asked us to take him again: at last I consented, and then she beat him.[33]

Parents sent their children off to work to supplement the family income, and sometimes received their children's wages directly. So the difference between an adult's reservation utility and a child's was not the difference between their market wages. A child laborer's low reservation utility was due not to her low average wage but to her basic dependence on her parents, as told by sixteen-year-old Frederick Hough:

> I left Mr. Okes's mill because of getting beat. I complained to my father, and told him that if ever the slubber beat me again I would come away....That day he beat us all round, soon after he came in; and I said that I would not stand it any longer; and next time I dropped an end, and he came out with the strap, and began to beat me again; as soon as he went into the bill-gate again I started, and came home, and never worked there again. That slubber was Joseph Addy. The next morning I got another place, and so my father said nothing about it. I do not think my father would have been angry with me for starting, if I had not got another place, because the man always beat me so.[34]

Since things got bad enough, Frederick Hough decided to leave. But, behind this, even for a sixteen-year-old, was the worry about how he would be treated by his father.

Another issue in interpreting the model is the extent to which children are rational. This concerns not just the model's applicability; one of the age-old justifications for physical violence toward children is that it is necessary to "discipline" them, where "discipline" is some sort of higher rationality which only adults have. The idea is that children are not rational enough to respond to anything less direct than physical pain: "I prefer fining to beating, if it answers.... Fining does not answer.... It does not keep the boys to their work."[35] I argue against this.

First, why should children who did not directly receive the wages they earned respond to fines (especially since, as they were paid little, only small fines were possible)? James McConnel used corporal punishment to motivate his apprentices because it wasn't enough to

> deprive the girls of the advantages which they enjoyed if good and industrious; for instance, they were denied the permission of going out occasionally to see their friends

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[33]United Kingdom, Parliament 1834, D. 1, p. 174, quoted in Pollard 1963, p. 261.

[34]United Kingdom, Parliament 1833, C. 1, p. 12–13.

[35]United Kingdom, Parliament 1833, C. 1, p. 44–45, quoted in Pollard 1963, p. 261.

> on Sundays, the use of books in the library...and part or the whole of the monthly gratuity [3*d.* to 9*d.*].[36]

It's worth asking whether an adult of the time would have responded with hard work to privileges of similar generosity. That children do not respond to fines and rewards to which adults would not respond isn't evidence that children are irrational.

Adults as much as children resisted the monotonous schedule of the factory:

> The personal inclinations and group *mores* of such old-established industrial workers as handloom weavers and framework knitters were opposed to factory discipline. "I found the utmost distaste," one hosier reported, "on the part of the men, to any regular hours or regular habits...The men themselves were considerably dissatisfied, because they could not go in and out as they pleased, and have what holidays they pleased, and go on just as they used to do"....
>
> ...[T]he acclimatization of new workers to factory discipline is a task different in kind, at once more subtle and more violent, from that of maintaining discipline among a proletarian population of long standing.[37]

For adult workers, this acclimatization was accomplished largely without corporal punishment.

Finally, before calling children "irrational," one should note that in factory owners' exhortations,

> such opprobrious terms as "idle" or "dissolute" should be taken to mean strictly that the worker was indifferent to the employer's deterrents and incentives. According to contemporaries, "it was the irrationality of the poor, quite as much as their irreligion, that was distressing. They took no thought for the morrow...The workers were by nature indolent, improvident, and self-indulgent."[38]

The reason child laborers were beaten is not that they were less rational than adults. The reason supplied by the model is that they had very bad alternatives. The model explains why an apprentice was more likely to be beaten than a "free" child laborer,[39] why young child laborers were more likely to be beaten than older ones,[40] and why free adults were rarely beaten.[41] Undoubtedly there were complementary reasons, such as macroeconomic ones concerning the demand and supply of labor. The historical context, including the institution of parish apprenticeship, also cannot be ignored.[42]

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[36]United Kingdom, Parliament 1843a, p. b65.

[37]Pollard 1963, p. 255 and p. 270.

[38]Pollard 1963, p. 269. The quotation is from Ashton 1955, p. 201.

[39]Nardinelli (1982, p. 287) finds that "in the metal trades, for example, the investigators found that 79% of the apprentices were beaten. Only 48% of the free child workers experienced corporal punishment."

[40]"We beat only the lesser, up to thirteen or fourteen. The boy of fifteen was considered too big to beat,' stated Samuel Miller in United Kingdom, Parliament 1833, C. 1, p. 45, quoted in Pollard 1963, p. 260. The reason isn't so much that older children had higher market wages, but that they could more easily live independently.

[41]Nardinelli (1982, p. 294) says that "in spite of the apparent effectiveness of corporal punishment in disciplining child workers, by the middle of the 19th century it was seldom used for older workers."

[42]See Nardinelli 1982, Fenoaltea 1984, Pinchbeck and Hewitt 1969, p. 223–259 and Dunlop 1912.

It should be emphasized that "the use of corporal punishment differed both among and between factories. Some firms used no corporal punishment, some firms used relatively mild punishment, and some firms used severe punishment."[43] It does seem, however, that corporal punishment of children is prevalent enough through history so that words for master-servant relationships are familial, such as "paternalistic."[44] Before the industrial revolution, children were beaten at home: "The experience of factory industry, far from being unique was in fact the experience of cottage and workshop industry writ large for all to see."[45] Today children throughout the world work under the threat of physical violence.[46] Closer to home, "over 97% of American children experience physical punishment....[F]or half of all American children, it does not end until the child leaves the paternal home."[47] The reader might object that the subject here is corporal punishment to obtain labor, not corporal punishment for misbehavior. But the model applies to the general situation in which the principal simply wants the agent to do (or refrain from doing) something. This switch of subject is represented in Boston child abuse records: "Disagreements about discipline often emerged from conflicts about children's work obligations. These conflicts were the most frequent origin of child abuse before World War II....After World War II, the more commonly cited provocation for child abuse was misbehavior."[48] The reader might object again that corporal punishment and child abuse are two different things. But the meaning of "acceptable" violence has of course changed through history, and today "the boundaries of family violence are openly in dispute among child-abuse experts, some of whom would declare all physical punishment to be abusive."[49]

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[43]Nardinelli 1982, p. 286.

[44]"The Latin *familia* had a wide spectrum of meanings: all the persons, free or unfree, under the authority of the *paterfamilias*, the head of the household; or all the descendants from a common ancestor, or all one's property; or simply all one's servants." (Finley 1973, p. 18). The Chinese character "slave" contains the character "woman." Lerner (1986, p. 9) finds that "men learned to institute dominance and hierarchy over other people by their earlier practice of dominance over the women of their own group. This found expression in the institutionalization of slavery, which began with the enslavement of women of conquered groups."

[45]Pinchbeck and Hewitt 1973, p. 406.

[46]See Mendelievich 1979.

[47]Straus 1983, p. 229.

[48]Gordon 1988, p. 181 and p. 185.

[49]Gordon 1988, p. 291.

7. Concluding remarks

Much is ignored in the model here, including the possibility of victims fighting back. Violence must be understood in many different ways—I hope that I have shown that rational choice modelling is one of these ways. But it has certain pitfalls, or dangers.

The first is that characterizing violence as the result of a rational choice has mixed effects. The confusion of "rational" in the instrumental sense and "rational" in the normative sense is the basis of Fogel and Engerman's intimation that a shrewd slaveowner is therefore not cruel, and a cruel slaveowner must be a psychological pervert who glories in power for its own sake.[50] In the debate over their *Time on the Cross*, this confusion was never completely exorcised.[51]

Calling violence a rational choice can be a justification bordering on an excuse, as in "rationalization." It is important to understand how some people hurt others because of their self-interests, and not because they are "evil." But a person who hurts others might be more likely to stop if she could think of herself only as evil. Calling his act a rational choice allows her to say to herself, "I may be cruel, but at least I'm rational," or "It may be cruel, but at least it's effective." In their periodicals southern slaveowners emphasized "efficient" slave management.[52] Another example is given by Elaine Scarry:

> Although the information sought in an interrogation is almost never credited with being a *just* motive for torture, it is repeatedly credited with being the motive for torture. But for every instance in which someone with critical information is interrogated, there are hundreds interrogated who could know nothing of remote importance to the stability or self-image of the regime. Just as within a precarious regime the motive for arrest is often

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[50]Here I am paraphrasing a quotation in Section 3 (Fogel and Engerman 1974, vol. 1, p. 232). Fogel and Engerman make this mistake in describing the slave as well as the slaveowner, as Barzel (1977, p. 104) notices: "It is puzzling why Fogel and Engerman so assiduously resist the notion of 'laziness' and try to impute to slaves the 'Protestant ethic.' Granted that owners had reason to encourage such an attitude, the slaves had at least as strong incentive to violate it. Fogel and Engerman's castigation of Stampp [Stampp 1956] for not recognizing the slave's 'superior' work attitude seems wholly misplaced. Idleness rather than adherence to the work ethic would have been the rational behavior."

[51]Fenoaltea (1981, p. 307) declares: "Here, then, is the central *curiosum* of the slavery debate. The best evidence that slavery was harsh and terror-driven is the superior 'efficiency' of slavery; the best evidence that slavery was mild and benevolent would be that slaves worked no harder than free men. *Time on the Cross* [Fogel and Engerman 1974] argues that slavery was benign, and advances the productivity argument that undermines that view; *Reckoning with Slavery* [David and others 1976] argues that slavery was harsh, and attacks the productivity argument that supports that view."

[52]In the words of "a committee of Georgia planters appointed to select the Southern Central Agricultural Society's prize essay for 1853...'the authors of these essays...generally concur...in the opinion that the pecuniary interests of the master are best consulted by a humane and liberal treatment of the slave.' " Breeden (1980, p. xx–xxi) offers "three possible explanations: first, cruel masters may have remained silent; second, the emphasis on slave welfare may have been feigned; and finally, these writers may have been genuinely influenced by concerns of humanity and duty." But another reason is that a master would not assuage any guilt by recommending that slaves be treated brutally.

> a fiction (the eggseller's eggs were too small—Greece), and just as the motive for punishing those imprisoned is often a fiction (the men, although locked in their cells, watched and applauded the television report that a military plane had crashed—Chile), so what masquerades as the motive for torture is a fiction.[53]

One can see how a rational choice model of torture would reproduce this fiction. To the extent that my model doesn't describe reality, it provides a fictional motive and hence a justification for hurting people. Bruno Bettelheim provides an illustration from Dachau and Buchenwald:

> Almost daily one guard or another, tinkering with his gun, would tell a prisoner that he would shoot him down except that a bullet cost three pfennig, and that was too much of Germany's money to spend on him....
>
> They repeated their statement so often because they had been told it so often in their indoctrination and must have been struck by its unusualness. Hard as it was to accept, it probably made such a deep impression on them that they believed it would impress prisoners as deeply; but prisoners by and large only found it silly. It was quite difficult for the average SS man to degrade human life to a thing of no value. He was impressed that his superiors could set the value of a human being below the trifling cost of a bullet. In his astonishment and disbelief he had to try it out again and again to convince himself.[54]

For Scarry, this device works because the guard's "movement toward a recognition of the internal experience of an exploding head and loss of life is interrupted and redirected toward a recognition of his own loss of three pfennig."[55] I would add that it works by allowing the guard to think of himself as making a rational choice (indeed, a cost-benefit choice). It is not simply a devaluation of human life: the guard doesn't say "you are worth three pfennig" but "shooting you would cost too much." Killing is made palatable by making it out to be something a rational person would choose to do.

Another effect is that once a person's act is described as rational, we can attach blame to her. This lies behind the child's excuse (after hitting a friend too hard and making her cry) of "I didn't *mean* it" as well as the insanity defense. This paper, by "establishing a motive," blames certain people for physically hurting other people. But the logic of rationality can twist into "blaming the victim." We have seen how a woman's remaining in a violent relationship is taken to mean that she consents and even desires to be beaten. Another example of how blame can be subtly shifted is that "there is not only among torturers but even among people appalled by acts of torture and sympathetic to those hurt, a covert disdain for confession."[56] But Yoram Barzel is fairly explicit:

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[53]Scarry 1985, p. 28.
[54]Bettelheim 1960, p. 240–241.
[55]Scarry 1985, p. 59.
[56]Scarry 1985, p. 29.

> Certain plantation activities involved the simultaneous work of large numbers of slaves, and frequently such work was assigned under a "task system" of uniform daily quotas....Had it been apparent that the quota would be set to the perceived ability of any particular slave, that individual would have had incentive to understate his capability. But where the quota was set at some "customary" level, no such incentive was present. ...The less able slave might be driven to exhaustion in a vain attempt to meet the quota, and the signal to abler workers would be clear.
>
> An implication of the model, then, is that occasionally slaves of wealth-maximizing owners would die of exhaustion...."It was the usual fate of such laggards [in slave caravans] to be killed by their disappointed masters, anxious to discourage any thought of feigning incapacity as a means of escape, just as in the Congo area it was usual to kill slaves who fell ill while carrying ivory."...
>
> ...As the notion of "shirking" does not apply to machines, we would not expect a machine to be driven to destruction. The maximizing behavior of slaves, in the above instance, is the source of their plight.[57]

The source of their plight is their masters; less immediately, the source is the system of unequal property relations. Barzel shifts blame away from where it belongs, where something can be done (change property relations and in the meantime get the masters to stop killing), to where nothing can be done (people cannot become machines). Not only can slaves be blamed because they are rational—their very rationality is to blame.

There is also the danger of sliding between description and prescription.[58] Here I have described a principal's optimal choice to better understand violence, but could be hired by an actual (not hypothetical) principal to help maximize him expected profit subject to constraints. The title of this paper would be "How to Whip Workers Efficiently" and would contain exactly the same model.[59] Consider the following by John Pratt and Richard Zeckhauser:

> The real world has provided substantial nourishment for the theory of agency. Now it is time to reciprocate. We hope our decision-maker readers, now acting as agents, will apply the concepts of agency theory and the lessons of this volume to the improvement of business practice and hence ultimately to the benefit of society.[60]

I might similarly declare my intentions: "The real world has provided nourishment for the theory of inflicting violence for profit and for punishing children. Now it is time to reciprocate."

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[57]Barzel 1977, p. 102–103. The quotation is from Fisher and Fisher 1970, p. 78, and the phrase in brackets is Barzel's.

[58]There are at least three meanings of "rationality": rationality in the normative sense, rationality as a theory of choice, and rationality in the instrumental sense. The confusion of the first and second appears in Section 4, and that of the first and third earlier in this section. Here appears the confusion between the second and the third.

[59]This is not a new subject for economics. Finley (1973, p. 17) tells us that "The book which became the model for the tradition still represented by Hutcheson [Adam Smith's teacher] was the *Oikonomikos* written by the Athenian Xenophon before the middle of the fourth century B. C....*Oikonomikos* is a guide for the gentleman landowner. It begins with a long introduction on the good life and the proper use of wealth, followed by a section on the virtues and leadership qualities necessary for the householder and on the training and management of his slaves, an even longer section on wifely virtues and the training of a wife, and the longest section of all, on agronomy."

[60]Pratt and Zeckhauser 1985, p. 35.

Finally, in modelling a social situation, one necessarily depersonalizes the people involved. This is more problematic in discussing violence than in discussing economics' more traditional subjects. There is the danger of diverting one's self from the necessity of concern and responsibility. But there is also the possibility of a more trenchant compassion.

Appendix

Lemma 1. $\lim_{a\to 1} \partial\pi/\partial a = -\infty$.

Proof. First consider u^g and u^b in the limit as $a\to 1$. Since $V(a)\to\infty$ as $a\to 1$, $V'(a)$ cannot be bounded. Since V' is increasing, $V'(a)\to\infty$. By assumption L, V''' is positive, and hence V'' increases and similarly $V''(a)\to\infty$. By L again, $V'''(a)\to\infty$. By l'Hopital's Rule, $\lim_{a\to 1} V'/V = \lim_{a\to 1} V''/V' = \lim_{a\to 1} V'''/V'' \geq \lim_{a\to 1} 2/(1-a) = \infty$, by L. Now $\lim_{a\to 1} u^b = \lim_{a\to 1} u + V(a)(1 - a(V'(a)/V(a))) = -\infty$ since $V(a)\to\infty$ and $V'(a)/V(a)\to\infty$ by the above. Clearly $u^g \to\infty$ since $V(a)\to\infty$ and $V'(a) > 0$.

Now $\partial\pi/\partial a = 1 - e(u^g) - a(1-a)V''(a)e'(u^g) + e(u^b) + a(1-a)V''(a)e'(u^b)$. Since $u^g \to\infty$ and e is convex and increasing over positive reals, $e(u^g)\to\infty$. Since $e'(u^g) > 0$ and $V'' > 0$, the third term is negative. Thus the sum of the first three terms goes to $-\infty$. So to derive the Lemma, it suffices to show that the sum of the remaining two terms becomes negative as $a\to 1$.

Now $\lim_{a\to 1} a(1-a)V''(a) + u^b = \lim_{a\to 1} a(1-a)V''(a) + u + V(a) - aV'(a) = \lim_{a\to 1} a((1-a)V''(a) - V'(a)) + u + V(a)$. Since $V(a)\to\infty$, this limit is ∞ unless $(1-a)V''(a) - V'(a)\to -\infty$, in which case its derivative $(1-a)V'''(a) - 2V''(a)$ must be negative for some a, contradicting L. Hence $a(1-a)V''(a) > -u^b$ for a close to 1. Since $u^b \to -\infty$, for a close to 1 we can assume $u^b < 0$. Then $e'(u^b) < 0$ and thus $a(1-a)V''(a)\, e'(u^b) < -u^b e'(u^b)$. Therefore $e(u^b) + a(1-a)V''(a)e'(u^b) < e(u^b) - u^b e'(u^b)$. But the right hand side of this inequality is negative since e is strictly convex and $e(0) = 0$. ∎

Lemma 2. $\partial u^g/\partial u > 0$ and $\partial u^b/\partial u > 0$.

Proof. Since $\pi_a(a,u) = 0$, we can take the derivative with respect to u of both sides to get $\pi_{aa}(a,u)(\partial a/\partial u) + \pi_{au}(a,u) = 0$. So $\partial a/\partial u = -\pi_{au}(a,u)/\pi_{aa}(a,u)$. Since $u^g = u + V(a) + (1-a)V'(a)$, by the chain rule $\partial u^g/\partial u = 1 + (1-a)V''(a)(\partial a/\partial u) = (1/\pi_{aa})(\pi_{aa} - (1-a)V''(a)\pi_{au})$. Now $\pi_{au} = -e'(u^g) + e'(u^b) - a(1-a)V''(a)(e''(u^g) - e''(u^b))$, and we know $\pi_{aa} < 0$. Therefore $\partial u^g/\partial u$ has the same sign as $-\pi_{aa} + (1-a)V''(a)\pi_{au}$,

$$
\begin{aligned}
&= V''(a)(e'(u^g)(1-a) + e'(u^b)a) + ((1-2a)V''(a) + a(1-a)V'''(a))(e'(u^g) - e'(u^b)) \\
&\quad + a(1-a)(V''(a))^2(e''(u^g)(1-a) - e''(u^b)a) \\
&\quad + (1-a)V''(a)(-e'(u^g) + e'(u^b) - a(1-a)V''(a)(e''(u^g) - e''(u^b))), \\
&= V''(a)e'(u^b) + ((1-2a)V''(a) + a(1-a)V'''(a))(e'(u^g) - e'(u^b)) + a(1-a)(V''(a))^2 e''(u^b).
\end{aligned}
$$

Since $e'(u^g) - e'(u^b) > 0$, by assumption L,

$$
\geq V''(a)e'(u^b) + V''(a)(e'(u^g) - e'(u^b)) + a(1-a)(V''(a))^2 e''(u^b),
$$

$= V''(a)e'(u^g) + a(1-a)(V''(a))^2 e''(u^b),$

which is positive.

Since $u^b = u + V(a) + aV'(a)$, by the chain rule $\partial u^b/\partial u = 1 - aV''(a)(\partial a/\partial u) =$ $(1/\pi_{aa})(\pi_{aa} - aV''(a)\pi_{au})$. Since $\pi_{aa} < 0$, $\partial u^b/\partial u$ has the same sign as $-\pi_{aa} - aV''(a)\pi_{au}$

$= V''(a)(e'(u^g)(1-a) + e'(u^b)a) + ((1-2a)V''(a) + a(1-a)V'''(a))(e'(u^g) - e'(u^b))$

$+ a(1-a)(V''(a))^2(e''(u^g)(1-a) + e''(u^b)a)$

$- aV''(a)(-e'(u^g) + e'(u^b) - a(1-a)V''(a)(e''(u^g) - e''(u^b)))$,

$= V''(a)e'(u^g) + ((1-2a)V''(a) + a(1-a)V'''(a))(e'(u^g) - e'(u^b)) + a(1-a)(V''(a))^2 e''(u^g).$

By assumption L,

$\geq V''(a)e'(u^g) + V''(a)(e'(u^g) - e'(u^b)) + a(1-a)(V''(a))^2 e''(u^g),$

which is positive. ∎

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