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THE SENSITIVITY OF STRATEGIC AND CORRECTIVE R&D POLICY IN OLIGOPOLISTIC INDUSTRIES*

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Abstract

We evaluate the sensitivity of the case for an R & D subsidy in an export sector when the outcome of R & D is uncertain and when the resulting product market is oligopolistic. Investments in R & D are assumed to induce either first order or mean-preserving second order shifts in the distribution of a firm's costs, with firms then competing in either prices or quantities in the product market. When R & D reduces the mean of a firm's cost distribution in the particular sense of first order stochastic dominance, we find using standard models of product market competition that a national strategic basis for R & D subsidies exists, whether firms choose prices or quantities. This national strategic incentive to subsidize R & D must be balanced against the national corrective incentive to tax R & D that arises whenever the number of domestic firms exceeds one. However, when R & D preserves the mean but alters the riskiness of a firm's cost distribution in the sense of second order stochastic dominance, we find that the national strategic basis for R & D intervention completely disappears, while the national corrective incentive is now to subsidize R & D whenever the number of domestic firms exceeds one. We conclude that the crucial determinant of appropriate R & D policy is the nature of the R & D process itself.

I. Introduction

Recent work in international trade has established a strategic role for trade policy in oligopolistic industries. The essential logic is that an industrial policy can commit domestic firms to behavior that would be otherwise noncredible. This is potentially beneficial to the domestic country as a means of influencing the behavior of foreign firms. Using this logic, Spencer and Brander (1983) and Brander and Spencer (1985) have shown that domestic export and R & D subsidies can enable domestic firms to commit to "aggressive" output and investment strategies, which then induce a "soft" response from foreign firms, thereby shifting profits from the foreign to the domestic country.

A number of criticisms have been leveled against the position that exports should be subsidized.[1] The most damaging attack comes from Eaton and Grossman (1986). They note that an aggressive predisposition invites a soft response only when reaction curves are negatively sloped. In the Cournot quantity game that Brander and Spencer (1985) analyze, reaction curves are indeed negatively sloped, but in equally plausible price games, a positively sloped reaction curve emerges. Thus, whether a strategic export subsidy or tax should be used depends upon the rather subtle and difficult issue of whether firms choose prices or quantities. A clear strategic policy prescription can not be made.[2] A second important criticism, also

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[1] For a survey, see Grossman (1986).

[2] Some meaning can be given to the price-quantity distinction. Price competition is usually associated with short run competition, while quantity competition corresponds to rigid capacity choices followed by price competition (Kreps and Scheinkman (1983)). At a policy level, however, it may be difficult to assess which situation applies best.

made by Eaton and Grossman (1986) as well as by Dixit (1984) and Krishna and Thursby (1988), concerns the number of domestic firms. When there is more than one domestic firm, a negative externality arises within the domestic country as an increase in any one domestic firm's output decreases the profits of other domestic firms. In general, there will be too much production, providing the basis for a corrective export tax. Thus, while an export subsidy under quantity competition may offer a strategic benefit, it will exacerbate the social loss from excessive production whenever there is more than one domestic firm. Again, an export tax on balance may be preferred.

Though the limitations of an export subsidy policy are now understood, there has been comparatively little analysis of the possible limitations of an R & D subsidy policy.[3] Certainly, an understanding of the robustness and limitations of the R & D subsidy logic is important, as there are many countries which do subsidize the R & D of domestic firms, especially those involved in international markets.[4] Moreover, with the exception of certain primary products, developed countries are explicitly forbidden to subsidize

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[3] Exceptions include a companion paper, Bagwell and Staiger (1989), as well as Cheng (1987) and Dixit (1988). Bagwell and Staiger (1989) analyze strategic and corrective R&D policies in the context of "winner take all" product competition. Cheng (1987) attempts to generalize the results of Spencer and Brander (1983) to a dynamic setting by introducing continual technological innovation and allowing domestic consumption. Dixit (1988) adopts a free entry assumption and is concerned primarily with the effects of exit and entry on appropriate R & D policy in a patent race setting. Spencer (1988) is also concerned with capital or R & D subsidies rather than with export subsidies but her focus is on the effects of GATT-sanctioned countervailing duties in an oligopolistic setting rather than on the case for R & D subsidies per se.

[4] See, for example, the discussion in Hufbauer and Erb (1984), especially pp. 103-104, and also in Kominya, Okuno, and Suzumura (1988), especially chapters 7 and 8.

exports under the Subsidies Code of the General Agreement on Tariffs and Trade (GATT), a ban that does not extend to R & D subsidies. We analyze in this paper the extent to which the desirability of an R & D subsidy depends upon the nature of product market competition, the number of domestic and foreign firms, and the existence and form of uncertainty in the outcome of investments in R & D.

The R & D model developed by Spencer and Brander (1983) has two exporting countries, each with one firm, and a single importing country. Product market competition in quantities occurs after R & D leads in a deterministic fashion to lower production costs. With corrective policy issues (at the national level) ruled out by the assumption of a single firm per country, the appropriate strategic policy with quantity-setting firms is shown to be an R & D subsidy.

A potentially crucial element not captured by the Spencer and Brander (1983) analysis is the inherent uncertainty associated with R & D. We begin with the natural stochastic analogue to their deterministic model, and assume that R & D lowers the mean of a firm's cost distribution in the particular sense of a first order stochastic shift. With this characterization of R & D and with a single firm in each country, the key question is then whether a strategic R & D subsidy remains attractive independent of the nature of product market competition. The answer to this question hinges on the slope of investment reaction functions. Interestingly, employing standard models of product market competition, we find that these functions have a negative slope, whether firms compete in prices or quantities. Thus, in contrast to export subsidies, the positive strategic role of R & D subsidies is not particularly sensitive to the nature of product market competition.[5]

When we allow for greater numbers of firms, a negative externality arises among domestic firms engaged in R & D. Our results here are thus similar to those developed for the case of an export subsidy by Eaton and Grossman (1986), Dixit (1984) and Krishna and Thursby (1988): the appropriate R & D policy balances the strategic incentive to subsidize with the corrective incentive to tax, and a subsidy is relatively more attractive the smaller the number of domestic firms.

However, the effect of R & D on the distribution of costs could take on any of several plausible forms, of which a first order stochastic shift is just one. To explore the sensitivity of our results to the way R & D is modeled, we next consider a second order stochastic shift in which additional investment in R & D preserves the mean but alters the "risk" properties of the cost distribution. Such a shift in the cost distribution could itself represent a plausible outcome of R & D if increased R & D investment consists of committing to a potentially more efficient but increasingly inflexible production process prior to the resolution of uncertainty in the external environment. More generally, R & D investments are likely to contain elements of both mean-altering and risk-altering characteristics;[6] thus, a finding of policy sensitivity with respect to

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[5] Spencer (1986) conjectures that subsidies may be most useful when foreign capacity choices have yet to be made and domestic capacity choices are large and inflexible (see also note 2). In demonstrating that quantity competition is not essential to a useful R & D subsidy policy, we generalize the set of industries for which an R & D subsidy may have a positive strategic role.

[6] In particular, a mean altering second order shift may be the most reasonable scenario.

these two "pure" cases would indicate the need for detailed knowledge of the way uncertainty enters the R & D process when designing appropriate R & D intervention.

When R & D is risk altering, and provided firm profits are convex in own costs, firms are shown unwilling to invest if investment reduces risk, but will find risk increasing R & D investments attractive. With respect to such investments we find that, regardless of whether firms choose prices or quantities, the slope of investment reaction curves is now <u>zero</u> in the standard models of product market competition. Hence, there is now <u>no</u> strategic basis for R & D policy intervention. Moreover, for the standard models, a firm's profit is convex in its rival's costs. Investment which increases a firm's risk thus imparts a <u>positive</u> externality to rival firms.[7] It follows that firms undertake <u>too little</u> R & D, so that the corrective R & D policy in the presence of more than one domestic firm is now an R & D <u>subsidy</u>.

We conclude that the crucial determinant of appropriate R & D policy is the nature of the R & D process itself. If the dominant characteristics of R & D are captured by mean reducing shifts in the distribution of costs, then a strong strategic basis for R & D subsidies exists, but may be outweighed by the corrective incentive to tax R & D whenever there is more than one domestic firm. By contrast, if the effects of R & D serve largely to increase the riskiness of the cost distribution, then in general no

[7] This result is consistent with the popular notion that too little risk taking occurs and contrasts with the findings of the patent literature. There, a "winner take all" game is considered and risk increasing R & D by one firm exerts a negative externality to other firms, resulting in too much R & D. See, for example, Bhattacharya and Mookherjee (1986), Dasgupta and Maskin (1987), and Klette and de Meza (1986), as well as Bagwell and Staiger (1989).

strategic basis for intervention exists, but a corrective incentive to subsidize R & D arises whenever there is more than one domestic firm. Hence, the nature of the R & D process replaces the form of product market competition as the pivotal determinant of appropriate intervention.

The remainder of the paper is devoted to establishing these results. Section II presents the basic model and discusses at a general level the strategic and corrective issues that will provide the focus for the remainder of the paper. Section III considers appropriate R & D policy when investment in R & D leads to a first order shift in the cost distribution, while section IV considers the case where R & D alters the dispersion of a firm's cost distribution. Section V concludes.

II. The Basic Model

II.1. Basic Assumptions

We begin with a simple framework. There are two exporting countries and a third importing country. For now, we assume each exporting country has a single firm. The exporting countries are referred to as home and foreign countries, respectively, and asterisks (*) will be used to denote foreign country variables.

The game proceeds in three stages. First, the exporting governments simultaneously choose the unit costs of investment, r and r^*, for their respective firms. Let $\tilde{r}$ represent the social cost of investment, assumed to be constant across countries.[8] A home country subsidy (tax) on

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[8] The assumption of symmetric social costs of investment at home and abroad is made for simplicity. Our basic results go through in the more general case where $\tilde{r} \neq \tilde{r}^*$.

investment then occurs if $r < \tilde{r}$ $(r>\tilde{r})$.[9] Since all consumption takes place in the third country, each exporting country chooses its cost of investment with the goal of maximizing its firm's expected profit less subsidy costs.[10]

Next, in stage two, both firms have observed the policy choices of both exporting governments and then simultaneously choose nonnegative investment levels, I and I^*. We assume that production costs are randomly determined as a function of investment. Thus, $f(c|I)$ is the density of possible constant costs c, given the investment level I. $f^*(c^*|I^*)$ is defined analogously. We assume throughout that the two investment technologies are symmetric and well-behaved: $f(c|I) \equiv f^*(c|I)$, $f(c|I) > 0$, and $f(c|I)$ is continuously differentiable in c and I, for every $c \in [\underline{c},\overline{c}]$.[11] The goal of any one firm at this stage is to maximize its expected profit, given its cost of investment.

Finally, in the third stage, each firm has observed both its own realized production cost as well as that of its rival, and the firms then compete in the product market (in prices or quantities) with each firm

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[9] We restrict our analysis to a consideration of R & D policy alone. In part, this reflects the view, shared by Spencer and Brander (1983) and by Spencer (1988), that this is the most relevant case, since GATT codes effectively restrict direct export subsidies. Moreover, since the sensitivity of strategic export policy is now well established, it is natural to consider whether robust policy conclusions emerge from a model with R & D policy alone.

[10] We comment briefly on the effect of introducing domestic consumption in the concluding section.

[11] In the case of a new product, with no preexisting technology, we require only that $\underline{c}$ be positive. In the case of an existing product, we assume also that $\overline{c}$ lies below the cost of the existing technology. Thus, in this latter case, we assume that a new technology becomes available which is certain to be less costly than the existing production process, but by an uncertain amount.

having the goal of maximizing its profit given the realized pair of production costs. We look for subgame perfect equilibria (Selten (1975)) of this three stage game.

II.2. Strategic Issues

We now define national welfare and discuss at a general level the determinants of the appropriate R & D policy in a strategic international setting. Home welfare in this model is simply expected profits less subsidy costs, or

$$(1)\quad W(r,r^*) \equiv E\pi(\hat{I}(r,r^*),\hat{I}^*(r,r^*),r) - (\tilde{r}-r)\cdot\hat{I}(r,r^*)$$

Here, $E\pi(\cdot)$ is expected profits of the home firm (net of investment costs), while $\hat{I}(\cdot)$ and $\hat{I}^*(\cdot)$ denote equilibrium domestic and foreign investment levels, respectively. $W^*(r,r^*)$ is defined analogously for the symmetric foreign profit function $E\pi^*(\cdot)$. Differentiating (1) with respect to r and using the envelope theorem, we obtain

$$(2)\quad W_r(r,r^*) = E\pi_{I*}(\hat{I}(r,r^*),\hat{I}^*(r,r^*),r)\cdot\hat{I}^*_r(r,r^*) - (\tilde{r}-r)\cdot\hat{I}_r(r,r^*)$$

where subscripts denote partial derivatives.[12] Thus, the effect of a change in r on domestic welfare is composed of two terms. A change in r will affect foreign investment and thereby domestic profits, captured in the first term, and a change in r will also alter domestic investment and thus

[12] This calculation assumes $E\pi_r(\cdot) = -\hat{I}(\cdot)$; that is, r only directly enters into $E\pi(\cdot)$ through the total cost of investment $r\hat{I}(\cdot)$. In Section III, an explicit definition of $E\pi(\cdot)$ is provided.

domestic subsidy payments, as represented in the second term.

The sign of $W_r(r,r^*)$ therefore depends upon the signs of $E\pi_{I^*}(\cdot)$, $\hat{I}^*_r(\cdot)$, and $\hat{I}_r(\cdot)$. The effect of greater foreign investment on home profits is captured by $E\pi_{I^*}(\cdot)$. In the terminology of Fudenberg and Tirole (1984), a positive (negative) sign here means that investment makes a country "soft" ("tough"). The signs of $\hat{I}^*_r(\cdot)$ and $\hat{I}_r(\cdot)$ determine the directions of the equilibrium response of foreign and domestic investment, respectively, to a change in r. To characterize these signs, we begin with the first order conditions, $E\pi_I(I,I^*,r) = 0$ and $E\pi^*_{I^*}(I,I^*,r^*) = 0$, which respectively define investment reaction functions, $I(I^*,r)$ and $I^*(I,r^*)$. We maintain the assumption that the determinant of the Jacobian, J, associated with the first order conditions is globally positive,

$$(3) \quad |J| \equiv E\pi_{II}(I,I^*,r)\cdot E\pi^*_{I^*I^*}(I,I^*,r^*) - E\pi_{I^*I}(I,I^*,r)\cdot E\pi^*_{II^*}(I,I^*,r^*) > 0.$$

The reaction curves then have at most one intersection, which we assume exists. $\hat{I}(r,r^*)$ and $\hat{I}^*(r,r^*)$ denote this solution. Total differentiation of the first order conditions gives

$$(4) \quad \hat{I}_r(r,r^*) = \frac{E\pi^*_{I^*I^*}(I,I^*,r^*)}{|J|}$$

$$(5) \quad \hat{I}^*_r(r,r^*) = \frac{-E\pi^*_{II^*}(I,(^*,,r^*)}{|J|}$$

Thus, if the second order condition holds, a home subsidy will increase home investment. A home subsidy will decrease (increase) foreign investment if $E\pi^*_{II^*}(I,I^*,r) < 0$ ($E\pi^*_{II^*}(I,I^*,r) > 0$), or equivalently if investment reaction curves are negatively (positively) sloped.[13]

Next, we maintain the assumption that $W(r,r^*)$ is concave in r and set $W_r(r,r^*)$ in (2) equal to zero. Using (4) and (5) then yields an expression for the optimal domestic strategic R & D subsidy (if $\tilde{r}-r>0$) or tax (if $\tilde{r}-r<0$), for a fixed r^*:

$$(6)\quad \tilde{r}-r = \frac{-\,E\pi_{I^*}(\hat{I}(r,r^*),\hat{I}^*(r,r^*),r)\cdot E\pi^*_{II^*}(\hat{I}(r,r^*),\hat{I}^*(r,r^*),r^*)}{E\pi^*_{I^*I^*}(\hat{I}(r,r^*),\hat{I}^*(r,r^*),r^*)}$$

Thus, provided second order conditions hold ($E\pi^*_{I^*I^*}(I,I^*,r^*) < 0$), the decision to subsidize or tax R & D in a given environment can be deduced from the knowledge of whether the signs of $E\pi_{I^*}(I,I^*,r)$ and $E\pi^*_{II^*}(I,I^*,r^*)$ agree or disagree, respectively. In particular, one can use (6) to characterize the Nash equilibrium between governments. Such an equilibrium is defined by two policy choices, $\hat{r}$ and $\hat{r}^*$, satisfying $W_r(\hat{r},\hat{r}^*) = 0$ and $W^*_{r^*}(\hat{r},\hat{r}^*) = 0$. Assuming a Nash equilibrium exists and that second order conditions hold, we have from (6) that both countries subsidize (tax) in the Nash equilibrium if the signs of $E\pi_{I^*}(I,I^*,r)$ and $E\pi^*_{II^*}(I,I^*,r)$ agree (disagree).

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[13] The slope of the foreign investment reaction curve comes from totally differentiating the foreign first order condition to get $dI^*/dI = -E\pi^*_{II^*}(I,I^*,r)/E\pi^*_{I^*I^*}(I,I^*,r)$.

II. 3. Corrective Issues

It is also of interest to know if too little or too much R & D is undertaken relative to some socially optimal level. To explore this issue we limit our focus to the case in which $r=r^*$, so that symmetric firms undertake identical levels of investment in the noncooperative equilibrium. We then ask the following question: if common investment levels were selected to maximize total duopoly profit, would the investment selections be larger or smaller than those which would be selected by noncooperative firms? The answer to this question would indicate the appropriate symmetric corrective policy for R & D, if the two countries sought to maximize combined producer surplus, or if alternatively the two firms were located in a single country.

Any divergence between aggregate and private incentives requires an externality, which in our model is represented by the term $E\pi_{I^*}(\cdot)$. A negative (positive) externality occurs if investment makes a country tough (soft), and too much (too little) private investment will be shown to occur in this case.

To illustrate this, let $\tilde{I}(r,r^*)$ and $\tilde{I}^*(r,r^*)$ maximize total export profit, $E\pi(I,I^*,r) + E\pi^*(I,I^*,r^*)$. With $r=r^*$, we have $\tilde{I}(r,r) = \tilde{I}^*(r,r)$ and $\hat{I}(r,r) = \hat{I}^*(r,r)$. We must then have

$$E\pi(\tilde{I}(r,r),\tilde{I}^*(r,r),r) \geq E\pi(\hat{I}(r,r),\hat{I}^*(r,r),r) \geq E\pi(\tilde{I}(r,r),\hat{I}^*(r,r),r)$$

Now, maintaining the assumption that the joint profit maximization problem has an interior solution, and assuming $E\pi_{I^*}(\cdot) \neq 0$, it is straightforward to show $\tilde{I}^*(r,r) \neq \hat{I}^*(r,r)$, and thus that one of the above inequalities is strict. It follows that $\tilde{I}^*(r,r) < \hat{I}^*(r,r)$ if $E\pi_{I*}(\cdot) < 0$, and conversely if $E\pi_{I*}(\cdot) > 0$. We may thus conclude that noncooperative firms invest too much (too little) relative to joint profit maximizing levels for any given $r=r^*$ if $E\pi_{I*}(\cdot) < 0$ $(E\pi_{I*}(\cdot) > 0)$.

Next, let $r_c=r_c^*$ be the symmetric corrective R & D policy that maximizes $W(r,r^*) + W^*(r,r^*)$. Maintaining the assumption of an interior maximum, the solution must satisfy

$$W_r(r_c,r_c) + W_r^*(r_c,r_c) = 0$$

Direct calculation yields

$$(7)\quad (\tilde{r}-r_c) = E\pi_{I*}(\hat{I}(r_c,r_c),\hat{I}^*(r_c,r_c),r_c)$$

Hence, the sign of the symmetric corrective R & D policy will reflect the sign of $E\pi_{I*}(\cdot)$. Note in particular that free trade is not optimal when $E\pi_{I*}(\cdot) \neq 0$, as R & D policies are then set to counteract the excessive or deficient levels of noncooperative investment discussed above.

Having identified the critical role played by the signs of $E\pi^*_{I*I*}(\cdot)$, $E\pi_{I*}(\cdot)$, and $E\pi^*_{II*}(\cdot)$ in determining the appropriate strategic or corrective R & D policy, we now proceed to analyze the sign of these terms in a variety of settings.

III. R & D and First Order Shifts

In this section we explore the case where investment in R & D leads to a reduction in the mean of the firm's cost distribution as captured by the notion of first order stochastic dominance. For the present we limit our discussion to the case of one domestic and one foreign firm. Thus, we first consider the national incentives for strategic R & D policy, and postpone consideration of the interaction between strategic and corrective issues at the national level until later.

III.1 Strategic Issues

As discussed in the previous section, the sign of the appropriate strategic R & D policy intervention is determined by the signs of $E\pi^{*}_{I^{*}I^{*}}(\cdot)$, $E\pi_{I^{*}}(\cdot)$, and $E\pi^{*}_{II^{*}}(\cdot)$ or equivalently, given the symmetry of the expected profit functions, $E\pi_{II}(\cdot)$, $E\pi_{I^{*}}(\cdot)$ and $E\pi_{I^{*}I}(\cdot)$. Our approach is to define general foreign and domestic profit functions $\pi^{*}(c,c^{*})$ and $\pi(c,c^{*})$, respectively, and to characterize the link between the properties of these functions and the signs of $E\pi_{II}(\cdot)$, $E\pi_{I^{*}}(\cdot)$, and $E\pi_{I^{*}I}(\cdot)$ through a series of lemmas. We then evaluate the properties of these general profit functions under specific assumptions about market conditions, and assess the implications for appropriate strategic R & D policy.

1. Basic Assumptions

We let $\pi(c,c^{*})$ represent the home firm's profits (gross of investment costs) in the third stage if home costs are c and foreign costs are c^{*}, and define $\pi^{*}(c,c^{*})$ as the symmetric function for the foreign firm. We take these functions to be general profit functions, and assume only that they are continuously differentiable and positive functions of c and c^{*}. In particular, we have in mind that they correspond to either Cournot competition in quantities or Hotelling competition in prices with differentiated products.

We must also make a distributional assumption to convey the mean-cost -reducing nature of R & D investment. Defining $F(c|I) \equiv \int_{\underline{c}}^{c} f(s|I)ds$, and with $F_I(c|I)$ denoting the partial derivative of $F(c|I)$ with respect to I, we assume

<u>Assumption A</u>: For every I, $F_I(c|I) > 0$ for all $c \in (\underline{c},\bar{c})$.

Assumption A depicts the usual first order stochastic dominance condition (Hadar and Russell, 1969). Thus, an increase in investment shifts the density to lower costs. We also assume that this shifting process occurs at a decreasing rate as investment increases. The role of this assumption is to make the firm's investment problem a concave program.[14]

<u>Assumption B</u>: For every I and $c \in [\underline{c},\bar{c}]$, $F_{II}(c|I) \leq 0$, with a strict inequality holding over some positive measure of costs.

2. <u>The Investment Stage</u>

We now fix r and r^* and consider the choice of investment levels. The home country will want to choose its investment level to maximize its expected profit, which is given by

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[14] See Rogerson (1985) for a related assumption in the context of the principal-agent problem.

$$(8)\quad E\pi(I,I^*,r) \equiv \int_{\underline{c}}^{\overline{c}} \int_{\underline{c}}^{\overline{c}} f(c|I)f^*(c^*|I^*)\pi(c,c^*)dc^*dc - rI$$

The first order condition is then

$$(9)\quad E\pi_I(I,I^*,r) = \int_{\underline{c}}^{\overline{c}} \int_{\underline{c}}^{\overline{c}} f_I(c|I)f^*(c^*|I^*)\pi(c,c^*)dc^*dc - r = 0$$

We assume a solution to this equation, so that a maximum is obtained if the second order condition holds:

$$(10)\quad E\pi_{II}(I,I^*,r) = \int_{\underline{c}}^{\overline{c}} \int_{\underline{c}}^{\overline{c}} f_{II}(c|I)f^*(c^*|I^*)\pi(c,c^*)dc^*dc < 0$$

The solution to (9) corresponds to a reaction curve, $I = I(I^*,r)$. Exactly symmetric arguments apply for the foreign country.

We begin with the following lemma stating the condition on $\pi(c,c^*)$ which assures that the second order condition does indeed hold.

Lemma III.1: For all I, I^* and r, $E\pi_{II}(I,I^*,r) < 0$ provided that $\pi_c(c,c^*) < 0$ for every $c \in [\underline{c},\overline{c}]$ and $c^* \in [\underline{c},\overline{c}]$.

Proof: Observe that

$$E\pi_{II}(I,I^*,r) = \frac{d^2}{dI^2} \int_{\underline{c}}^{\overline{c}} f(c|I)K(c|I^*)dc$$

where

$$(11)\quad K(c|I^*) \equiv \int_{\underline{c}}^{\overline{c}} f^*(c^*|I^*)\pi(c,c^*)dc^* > 0$$

for all $c \in [\underline{c},\overline{c}]$. $K(c|I^*)$ is simply expected domestic profit given a domestic cost realization of c if the foreign investment level is I^*. We also have that, with $\pi_c(c,c^*) < 0$,

$$(12)\quad K_c(c|I^*) = \int_{\underline{c}}^{\overline{c}} f^*(c^*|I^*)\pi_c(c,c^*)\, dc^* < 0$$

for all $c \in [\underline{c},\overline{c}]$. Thus, integrating by parts, we obtain

$$(13)\quad E\pi_{II}(I,I^*,r) = -\int_{\underline{c}}^{\overline{c}} F_{II}(c|I)K_c(c|I^*)dc < 0$$

Q.E.D.

The reaction functions, $I = I(I^*,r)$ and $I^* = I^*(I,r)$ are thus well-defined provided that the condition of Lemma III.1 is satisfied. Moreover, by Lemma III.1, (3), and (4), we have that $\hat{I}_r(r,r^*) < 0$: a domestic R & D subsidy will raise domestic investment.

We next ask whether investment makes a country "tough" or "soft", i.e., whether greater investment by one country decreases or increases the expected profit of the other country. The following lemma establishes the property of $\pi(c,c^*)$ on which the answer depends.

<u>Lemma III.2</u>: For all I, I^* and r,

$$\text{sign}[E\pi_{I^*}(I,I^*,r)] = -\text{sign}[\pi_{c^*}(c,c^*)]$$

provided that the sign of $\pi_{c*}(c,c^*)$ is the same for all $c\epsilon[\underline{c},\bar{c}]$ and $c^*\epsilon[\underline{c},\bar{c}]$.

Proof: Observe that

$$(14)\quad E\pi_{I*}(I,I^*,r) = \int_{\underline{c}}^{\bar{c}}\int_{\underline{c}}^{\bar{c}} f(c|I)f^*_{I*}(c^*|I^*)\pi(c,c^*)dc^*dc$$

which using (11) can be rewritten as

$$(15)\quad E\pi_{I*}(I,I^*,r) = \int_{\underline{c}}^{\bar{c}} f(c|I)K_{I*}(c|I^*)dc$$

where

$$(16)\quad K_{I*}(c|I^*) = \int_{\underline{c}}^{\bar{c}} f^*_{I*}(c^*|I^*)\pi(c,c^*)dc^*$$

But integrating by parts yields

$$(17)\quad K_{I*}(c|I^*) = -\int_{\underline{c}}^{\bar{c}} F^*_{I*}(c^*|I^*)\pi_{c*}(c,c^*)dc^*$$

Using Assumption A, (17) implies that, for a fixed domestic cost realization, expected domestic profits are decreasing (increasing) in foreign investment when $\pi_{c*}(c,c^*)$ is positive (negative). With (15), the lemma is thus proved.[15] Q.E.D.

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[15] See also Spencer (1988) for a similar result in a deterministic setting under conjectural variations.

Intuitively, as I^* increases, the density on low c^*s also increases. Hence, expected home profits decline (rise) if $\pi_{c^*}(c,c^*)$ is positive (negative).

Finally, the next lemma identifies the slope of the reaction curves as a function of properties of $\pi(c,c^*)$.

<u>Lemma III.3</u>: For all I, I^* and r,

$$\mathrm{sign}[E\pi_{I^*I}(I,I^*r)] = \mathrm{sign}[\pi_{cc^*}(c,c^*)]$$

provided that the sign of $\pi_{cc^*}(c,c^*)$ is the same for all $c \in [\underline{c},\overline{c}]$ and $c^* \in [\underline{c},\overline{c}]$.

<u>Proof</u>: Observe that

$$(18)\ E\pi_{I^*I}(I,I^*,r) = \int_{\underline{c}}^{\overline{c}} \int_{\underline{c}}^{\overline{c}} f_I(c|I) f^*_{I^*}(c|I^*)\pi(c,c^*)dc^*dc$$

which with (16) simplifies to

$$(19)\ E\pi_{I^*I}(I,I^*,r) = \int_{\underline{c}}^{\overline{c}} f_I(c|I)K_{I^*}(c|I^*)dc$$

Integrating by parts yields

$$(20)\ E\pi_{I^*I}(I,I^*,r) = -\int_{\underline{c}}^{\overline{c}} F_I(c|I)K_{I^*c}(c|I^*)dc$$

But using (17)

$$E\pi_{I^*I}(I,I^*,r) = \int_{\underline{c}}^{\overline{c}} F_I(c|I) \int_{\underline{c}}^{\overline{c}} F^*_{I^*}(c^*|I^*)\pi_{c^*c}(c,c^*)dc^*dc$$

which, by Assumption A, yields the statement of the lemma.

Q.E.D.

To gain some intuition for this result, recall that by Lemma III.2, the effect of increasing I^* on $E\pi(I,I^*,r)$ is opposite in sign of $\pi_{c^*}(c,c^*)$, since increases in I^* raise the density on low c^*s. Analogously, Lemma III.3 implies that increasing I affects $E\pi_{I^*}(I,I^*,r)$ with the same sign that increases in c affect $\pi_{c^*}(c,c^*)$, since increasing I raises the density on low cs.

Thus, investment reaction curves are upward sloping if $\pi_{cc^*}(c,c^*) > 0$ for all c and c , and downward sloping if $\pi_{cc^*}(c,c^*) < 0$ for all c and c^*. With the properties of $\pi(c,c^*)$ that determine whether investment makes a country "soft" or "tough" (Lemma III.2), and whether second order conditions are met (Lemma III.1), we can now state the implications of the properties of $\pi(c,c^*)$ for the resulting R & D policies.

3. The Policy Stage

Lemmas III.1 through III.3, together with (6), allow us to state the following proposition.

Proposition III.1:

A) Facing a fixed foreign policy r^*, the home government's optimal strategic R & D policy as characterized in (6) will take the form of an R &

D subsidy if

I) $\pi_c(c,c^*) < 0$ and $\text{sign}(\pi_{c*}(c,c^*)) = -\text{sign}(\pi_{cc*}(c,c^*))$ for all $c \in [\underline{c},\overline{c}]$ and $c^* \in [\underline{c},\overline{c}]$

and will take the form of an R & D tax if

II) $\pi_c(c,c^*) < 0$ and $\text{sign}(\pi_{c*}(c,c^*)) = \text{sign}(\pi_{cc*}(c,c^*))$ for all $c \in [\underline{c},\overline{c}]$ and $c^* \in [\underline{c},\overline{c}]$

B) Any Nash equilibrium between the two governments will be characterized by strategic subsidies to R & D if I) holds and by strategic taxes on R & D if II) holds.

III.2 Corrective Issues

We now consider whether "excessive" R & D is undertaken. Our results are summarize in the next proposition, which relies on Lemmas III.1, III.2, III.3, (6) and (7).

Proposition III.2:

A). For a given symmetric home and foreign policy, $r=r^*$, the difference between the home firm's noncooperative investment level and its investment level when firms maximize total export profit is positive (negative) if $\pi_{c*}(c,c^*) > 0$ ($\pi_{c*}(c,c^*) < 0$) for all $c \in [\underline{c},\overline{c}]$ and $c^* \in [\underline{c},\overline{c}]$.

B). The symmetric joint welfare maximizing corrective R & D policy as characterized in (7) involves an R & D tax (subsidy) if $\pi_{c*}(c,c^*) > 0$

$(\pi_{c^*}(c,c^*) < 0)$ for all $c \epsilon [\underline{c},\overline{c}]$ and $c^* \epsilon [\underline{c},\overline{c}]$. Moreover, any Nash equilibrium between the two governments will be characterized by strategic R & D policies which conflict (agree) in sign with the symmetric joint welfare maximizing corrective R & D policy whenever $\pi_{cc^*}(c,c^*) < 0$ $(\pi_{cc^*}(c,c^*) > 0)$ and $\pi_c(c,c^*) < 0$ for all $c \epsilon [\underline{c},\overline{c}]$ and $c^* \epsilon [\underline{c},\overline{c}]$.

We now extend this line of inquiry to consider the potential national conflict between corrective and strategic incentives that arises when there are many domestic firms. To this end, we suppose that there are H home firms and F foreign firms. Defining domestic welfare as

$$W(r,r^*) \equiv \sum_{i=1}^{H} E\pi^i(\hat{I}(r,r^*),\hat{I}^*(r,r^*),r) - (\tilde{r}-r)\sum_{i=1}^{H} \hat{I}^i(r,r^*)$$

where $$\hat{I}(r,r^*) \equiv \hat{I}^1(r,r^*)\ldots\hat{I}^H(r,r^*)$$
$$\hat{I}^*(r,r^*) \equiv \hat{I}^{*1}(r,r^*)\ldots\hat{I}^{*F}(r,r^*)$$

we differentiate $W(r,r^*)$ with respect to r and impose symmetry among domestic and among foreign firms to obtain

$$W_r(r,r^*) = H\cdot\{F\cdot E\pi^i_{I^{*j}}(\cdot)\hat{I}^{*j}_r(r,r^*) + (H-1)E\pi^i_{I^j}(\cdot)\hat{I}^j_r(r,r^*) - (\tilde{r}-r)\hat{I}^i_r(r,r^*)\}$$

Maintaining the assumption that $W(r,r^*)$ is concave in r and setting $W_r(r,r^*) = 0$, we then derive explicit expressions for $\hat{I}^{*j}_r(r,r^*)$ and $\hat{I}^j_r(r,r^*)$ in analogy with our preceding analysis, and solve for the optimal domestic R & D policy for a fixed r^*:

$$(21)\quad \tilde{r}-r = \frac{-HF\ E\pi^{i}_{I^*j}(\cdot)\ E\pi^{*j}_{I_iI^*j}(\cdot)}{E\pi^{*j}_{I^*jI^*j}(\cdot) + (F-1)E\pi^{*j}_{I^*jI^*i}(\cdot)} + H(H-1)E\pi^{i}_{I_j}(\cdot)$$

The first term on the right hand side of (21) captures the national strategic R & D policy incentive. Provided second order and stability conditions are met, there will be a strategic incentive to subsidize (tax) R & D if the signs of $E\pi^{i}_{I^*j}(\cdot)$ and $E\pi^{*j}_{I_iI^*j}(\cdot)$ agree (disagree).[16] The three lemmas describe the associated properties of the underlying profit function that determine whether a strategic R & D tax or subsidy will be called for. But when the number of home firms H exceeds one, there is a second term on the right hand side of (21) which also enters into determining the sign of $W_r(\tilde{r},\tilde{r}^*)$, and this term captures the national corrective incentive to intervene. This term is the same sign as $E\pi^{i}_{I_j}(\cdot)$, and reflects the fact that domestic firm j's investment imposes a negative (positive) externality on domestic firm i's profits if $E\pi^{i}_{I_j}(\cdot) < 0$ ($E\pi^{i}_{I_j}(\cdot) > 0$) and should be taxed (subsidized) accordingly.

Since $E\pi^{i}_{I^*j}(\cdot)$ and $E\pi^{i}_{I_j}(\cdot)$ share the same sign, we conclude from (21) that whether or not the signs of the strategic and corrective incentives for a country to pursue R & D policy differ is determined by the sign of $E\pi^{*j}_{I_iI^*j}(\cdot)$, that is, by the slope of investment reaction curves. If $E\pi^{*j}_{I_iI^*j}(\cdot) < 0$, so that investment reaction curves slope downward, the two policy incentives will conflict in sign, and the sign of the optimal

[16] Formally, we derive the Jacobian for the multifirm model from two first order conditions, one for a representative domestic firm and one for a representative foreign firm. We then maintain the requirement that the Jacobian be negative definite. The denominator of the first term in (21) corresponds to a diagonal term in the Jacobian, and thus takes a negative sign.

national R & D policy will depend on the relative numbers of foreign and domestic firms. In contrast, if $E\pi^{*j}_{I^i I^{*j}}(\cdot) > 0$, so that investment reaction curves slope upward, the two policy incentives will agree in sign, and the sign of the optimal national R & D policy will correspond to the sign of $E\pi^{i}_{I^j}(\cdot)$.

III.3 Specific Market Conditions

Having established the general relationship between properties of $\pi(c,c^*)$ and both strategic and corrective incentives for R & D intervention, we now turn to a consideration of models reflecting specific assumptions concerning market conditions and evaluate the properties of $\pi(c,c^*)$ within these models, in an effort to assess the degree of sensitivity of strategic R & D policy to market conditions. We make the assumption of a single firm in each country, and then note what our results imply for the many firm case.

We begin with the Hotelling model of price competition. The standard model assumes consumers to have unitary demands and be uniformly distributed over the [0,1] interval, with a total mass of one. The two firms are located at opposite endpoints and simultaneously choose prices.[17] Letting t denote the consumers' transportation cost and assuming that a good is valued sufficiently, if $\overline{c}-\underline{c}$ is not too large it is straightforward to show that the equilibrium profit function is

$$\pi(c,c^*) = (t+(c^*-c)/3)^2/2t$$

[17] The model as developed below follows Tirole (1988), who also notes a negatively sloped investment reaction curve, though in a deterministic setting.

The foreign profit function is exactly symmetric. It is now direct to show that

(22) $\pi_c(c,c^*) < 0$; $\pi_{c*}(c,c^*) > 0$; $\pi_{cc*}(c,c^*) < 0$ for all c and c^*.

The first inequality is intuitive. As for the other inequalities, as c^* rises so too will the foreign price, which of course improves home profit. This improvement diminishes however as c rises, because the home demand increase generated by the rise in foreign price then becomes less valuable.[18]

With the relevant properties of $\pi(c,c^*)$ established in (22), Proposition III.1 can now be employed to characterize the incentives for R & D intervention in this setting. In particular, with $\pi_c(c,c^*) < 0$ and $\text{sign}(\pi_{c*}(c,c^*)) = -\text{sign}(\pi_{cc*}(c,c^*))$, condition I) of Proposition III.1 applies. Thus, despite the price-setting nature of product market competition between firms, each government has a strategic incentive to subsidize its firm's R & D investment, and any Nash equilibrium between the

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[18] The Hotelling model generates a linear demand and is otherwise not special; the conditions in (22) hold for any linear demand model of product differentiation, provided only that own effects aren't overwhelmed by cross effects. Specifically, if the demand for the home product is $a-bP+dP^*$, $(a,b,d) > 0$ with a large, and symmetrically for the foreign product, then the above inequalities hold if $2b^2 > d^2$. The additional properties attributed to the Hotelling model in the next section also hold if $2b^2 > d^2$. This seems to characterize the reasonable class of parameters. Moreover, these results are readily extended to the many-firm case. More generally, the linear focus we have adopted seems appropriate in evaluating a policy presumption, since with nonlinearities any policy presumption would require knowledge of third order effects in the underlying product market game.

two governments will involve R & D subsidies. The fact that $\pi_{c^*}(c,c^*) > 0$ indicates a negative externality to investment, so that Proposition III.2 implies that noncooperative firms "overinvest" for given r and r^*. Moreover, with $\pi_{cc^*}(c,c^*) < 0$, investment reaction curves are downward sloping. Hence, while the strategic incentive is for each government to subsidize R & D, the symmetric corrective R & D policy for both governments acting together would be to tax R & D. The analogous result implied by $\pi_{cc^*}(c,c^*) < 0$ when there are many domestic and foreign firms is that the national strategic incentive to subsidize R & D will run counter to the national corrective incentive to tax: the national R & D policy will thus depend on the balance of these two effects, and hence on the relative number of domestic and foreign firms.[19]

We now consider quantity competition. Following Kreps and Scheinkman (1983), it has become standard to think of quantity competition as corresponding to capacity choices followed by price competition. From this perspective, our quantity model is associated with a first stage in which unit costs of production or technologies are realized, followed by a stage of capacity choices, and then finally price choices. In other words, the quantity model allows for the additional possibility of a capacity choice after the cost realization.

The conditions listed in (22) are typically assumed to hold in quantity choice models, and hold in particular when demand is linear and costs are

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[19] Analogous properties for the equilibrium profit function can be derived in the many-firm case when firms are located around a circle as in Salop (1979).

constant.[20] To see this, let $P(q+q^*) = 1-q-q^*$ be the market price when domestic (foreign) output is $q(q^*)$. It follows directly that Cournot profit is

$$\pi(c,c^*) = (1-2c+c^*)^2/9$$

$\pi^*(c,c^*)$ is defined analogously. Assuming that $\bar{c}$ and $\bar{c}-\underline{c}$ are not too large, it is now straightforward to verify that the conditions listed in (22) hold. Our results for quantity competition are thus analogous to those established by Spencer and Brander (1983), although we do generalize their analysis to allow for uncertainty.

We have established that the case for strategic R & D subsidies is not particularly sensitive to whether firms choose prices or quantities, at least when R & D is modeled in the mean-reducing way we have modeled it here. However, the case for strategic R & D subsidies is sensitive to the number of domestic firms, due to the interaction of the strategic incentive to subsidize R & D with the corrective incentive to tax it. In the next section we explore the sensitivity of our results to the way in which R & D is modeled.

IV. R & D and Mean-Preserving Second Order Shifts

In the previous section we assumed that investment in R & D leads to a reduction in the mean of the firm's cost distribution in the sense of first order stochastic dominance. A plausible alternative, and one that we

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[20] It is also easily verified in this case that (22) holds for Cournot competition when products are differentiated.

consider in this section, is that investment in R & D affects the riskiness of the distribution of cost outcomes in a mean-preserving way, in the sense of second order stochastic dominance (Rothschild and Stiglitz, 1970). As before, we start by limiting our discussion to the case of one firm in each country, focusing on strategic incentives for R & D policy. We also maintain the assumption that $\pi_{cc}(c,c^*) > 0$ for all c and c^*, a condition that is easily verified to hold in the Hotelling and Cournot models above.

IV.1 Strategic Issues

1. Basic Assumptions

Our assumptions correspond to those of the previous section except that we replace Assumptions A and B with assumptions under which R & D alters the riskiness but not the mean of the cost distribution.

Assumption A': For every I, $\int_{\underline{c}}^{\bar{c}} F_I(s|I)ds = 0$ and either

1) (Risk-reducing investment)

$$\int_{\underline{c}}^{c} F_I(s|I)ds < 0, \quad \text{for all} \quad c \in (\underline{c},\bar{c}), \quad \text{or}$$

2) (Risk-increasing investment)

$$\int_{\underline{c}}^{c} F_I(s|I)ds > 0 \quad \text{for all} \quad c \in (\underline{c},\bar{c})$$

Assumption B′: For every I, there exists a $\tilde{c} \in [\underline{c},\bar{c}]$ such that $F_{II}(\tilde{c}|I) = 0$, with $F_{II}(c|I) \leq 0$ for $c \in [\underline{c},\tilde{c})$, $F_{II}(c|I) \geq 0$ for $c \in (\tilde{c},\bar{c}]$, and $F_{II}(c|I) \neq 0$ for some positive measure of $c \in (\underline{c},\bar{c})$.[21]

2. The Investment Stage

Risk-Reducing Investment

We begin with Assumption A′(1).

Lemma IV.0: For all I, I^*, and r, $E\pi_I(I,I^*,r) < 0$.

Proof: Using (8) and (11), first note that

$$E\pi_I(I,I^*,r) = \frac{d}{dI}\left[\int_{\underline{c}}^{\bar{c}} f(c|I)K(c|I^*)dc\right] - r$$

Integrating by parts twice yields

$$E\pi_I(I,I^*,r) = -\left[\int_{\underline{c}}^{\bar{c}} F_I(c|I)K_c(c|I^*)dc\right] - r$$

$$= \int_{\underline{c}}^{\bar{c}}\left[\int_{\underline{c}}^{c} F_I(s|I)ds\right]K_{cc}(c|I^*)dc - r$$

But from (12),

$$(23)\quad K_c(c|I^*) = \int_{\underline{c}}^{\bar{c}} f^*(c^*|I^*)\ \pi_c(c,c^*)dc^*$$

[21] This assumption ensures that the firm's investment problem for the risk-increasing case is a concave program, but plays no role in our results for the risk-reducing case.

and

$$(24)\ K_{cc}(c|I^*) = \int_{\underline{c}}^{\overline{c}} f^*(c^*|I^*)\ \pi_{cc}(c,c^*)dc^* > 0$$

Thus, application of Assumption A'(1) gives the desired result.

Q.E.D.

The intuition of the result is straightforward. Given the supposition that the firm's payoffs are convex in its own costs, the firm seeks investments that add risk. Thus, risk-reducing investments will not be undertaken. The relevant case given $\pi_{cc}(c,c^*) > 0$ is the case of risk-increasing investments.

Risk-Increasing Investment

We now suppose that Assumption A'(2) holds. An immediate implication from Lemma IV.0 is that firms will seek risk-increasing investment. It is therefore relevant to consider the appropriate R & D policy for such investments. As before, we must determine the signs of $E\pi_{II}(I,I^*,r)$, $E\pi_{I^*}(I,I^*,r)$ and $E\pi_{I^*I}(I,I^*,r)$ as functions of the properties of $\pi(c,c^*)$. We begin by showing conditions under which the second order condition holds.

Lemma IV.1: For all I, I^*, and r, $E\pi_{II}(I,I^*,r) < 0$, provided that $\pi_c(c,c^*) < 0$ for every $c \epsilon [\underline{c},\overline{c}]$ and $c^* \epsilon [\underline{c},\overline{c}]$.

Proof: Using (10), (11), and integrating by parts yields

$$E\pi_{II}(I,I^*,r) = \int_{\underline{c}}^{\bar{c}} f_{II}(c|I)\ K(c|I^*)dc$$

$$= - \int_{\underline{c}}^{\bar{c}} F_{II}(c|I)\ K_c(c|I^*)dc$$

But from (23) and (24), $K_c(c|I) < 0 < K_{cc}(c|I^*)$. Thus, using Assumption B',

$$E\pi_{II}(I,I^*,r) = - \left[\int_{\underline{c}}^{\tilde{c}} F_{II}(c|I)K_c(c|I^*)dc + \int_{\tilde{c}}^{\bar{c}} F_{II}(c|I)K_c(c|I^*)dc \right]$$

$$< - K_c(\tilde{c}|I^*) \int_{\underline{c}}^{\bar{c}} F_{II}(c|I)dc = 0$$

Q.E.D.

Thus, given our assumption that $\pi_{cc}(c,c^*) > 0$, reaction functions will be well-defined and $\hat{I}_r(r,r^*) < 0$ provided that, as in the previous section, $\pi_c(c,c^*) < 0$.

We next establish the properties of $\pi(c,c^*)$ which determines whether investment makes a country "tough" or "soft."

<u>Lemma IV.2</u>: For all I, I^*, and r,

$$\text{sign}[E\pi_{I^*}(I,I^*,r)] = \text{sign}[\pi_{c^*c^*}(c,c^*)]$$

provided that the sign of $\pi_{c^*c^*}(c,c^*)$ is the same for all $c\epsilon[\underline{c},\bar{c}]$ and $c^*\epsilon[\underline{c},\bar{c}]$.

$\underline{\text{Proof}}$: As in (15), we have that

$$E\pi_{I*}(I,I^*,r) = \int_{\underline{c}}^{\overline{c}} f(c|I)K_{I*}(c|I^*)dc$$

where

$$K_{I*}(c|I^*) = \int_{\underline{c}}^{\overline{c}} f^*_{I*}(c^*|I^*)\pi(c,c^*)dc^*$$

But integrating by parts twice and using Assumption A'(2) gives

$$(25)\quad K_{I*}(c|I^*) = -\int_{\underline{c}}^{\overline{c}} F^*_{I*}(c^*|I^*)\pi_{c*}(c,c^*)dc^*$$

$$= \int_{\underline{c}}^{\overline{c}} [\int_{\underline{c}}^{c^*} F^*_{I*}(s|I^*)ds]\pi_{c*c*}(c,c^*)dc^*$$

so that the lemma follows. Q.E.D.

Again, a simple intuition is available. The conditions of the lemma dictate whether home profits are convex or concave in foreign costs. Risky foreign investment technologies improve (worsen) expected home profit when home profits are convex (concave) in foreign costs.

We come finally to our last condition, which relates to the slope of investment reaction functions.

$\underline{\text{Lemma IV.3}}$: For all I, I^*, and r, $E\pi_{I*I}(I,I^*,r) = 0$ provided that $\pi_{c*c*cc}(c,c^*) = 0$ for all $c\epsilon[\underline{c},\overline{c}]$ and $c^*\epsilon[\underline{c},\overline{c}]$. If $\pi_{c*c*cc}(c,c^*) \neq 0$, then

$$\text{sign}[E\pi_{I^*I}(I,I^*,r)] = \text{sign}[\pi_{c^*c^*cc}(c,c^*)]$$

provided that the sign of $\pi_{c^*c^*cc}(c,c^*)$ is the same for all $c \in [\underline{c},\overline{c}]$ and $c^* \in [\underline{c},\overline{c}]$.

<u>Proof</u>: From (20), we have

$$E\pi_{I^*I}(I,I^*,r) = - \int_{\underline{c}}^{\overline{c}} F_I(c|I)K_{I^*c}(c|I^*)dc$$

But, using (25),

$$E\pi_{I^*I}(I,I^*,r) = - \int_{\underline{c}}^{\overline{c}} F_I(c|I) \int_{\underline{c}}^{\overline{c}} [\int_{\underline{c}}^{c^*} F^*_{I^*}(s|I^*)ds] \ \pi_{c^*c^*c}(c,c^*)dc^*dc$$

Integrating by parts and using Assumption A'(2), we have

$$E\pi_{I^*I}(I,I^*,r) = \int_{\underline{c}}^{\overline{c}} \int_{\underline{c}}^{\overline{c}} [\int_{\underline{c}}^{c} F_I(s|I)ds][\int_{\underline{c}}^{c^*} F^*_{I^*}(s|I^*)ds] \ \pi_{c^*c^*cc}(c,c^*)dc^*dc$$

Q.E.D.

The investment reaction functions therefore have no slope unless fourth order effects enter into the profit function. To gain some intuition, recall first that the sign of $E\pi_{I^*}(I,I^*,r)$ is determined by the sign of $\pi_{c^*c^*}(c,c^*)$. This is because an increase in I^* acts to increase the riskiness of the distribution of c^*, which increases expected home profit if home profit is convex in c^*, i.e., if $\pi_{c^*c^*}(c,c^*) > 0$. Similarly, the sign of $E\pi_{I^*I}(I,I^*,r)$ is determined by the sign of $\pi_{c^*c^*cc}(c,c^*)$. As I increases, the riskiness of the distribution of c increases, and this affects $E\pi_{I^*}(I,I^*,r)$ through the extent of convexity of $\pi_{c^*c^*}(c,c^*)$ in

c. In particular, if $\pi_{c^*c^*cc}(c,c^*) = 0$ so that $\pi_{c^*c^*}(c,c^*)$ is linear in c, then $E\pi_{I^*I}(I,I^*,r) = 0$. This implies by (5) that $\hat{I}^*_r(r,r^*) = 0$: a home subsidy has absolutely no affect on foreign investment.

3. <u>The Policy Stage</u>

With Lemmas IV.0 through IV.3, we can now characterize strategic R & D policy when R & D is risk-altering and profits are convex in own costs with the following proposition.

<u>Proposition IV.1</u>:

A) Facing a fixed foreign policy r^*, the home government's optimal strategic R & D policy as characterized in (6) will take the form of an R & D subsidy if

I) $\pi_c(c,c^*) < 0$ and $\text{sign}(\pi_{c^*c^*}(c,c^*)) = \text{sign}(\pi_{c^*c^*cc}(c,c^*))$ for all $c \in [\underline{c},\bar{c}]$ and $c^* \in [\underline{c},\bar{c}]$,

an R & D tax if

II) $\pi_c(c,c^*) < 0$ and $\text{sign}(\pi_{c^*c^*}(c,c^*)) = -\text{sign}(\pi_{c^*c^*cc}(c,c^*))$ for all $c \in [\underline{c},\bar{c}]$ and $c^* \in [\underline{c},\bar{c}]$,

and no strategic intervention if

III) $\pi_c(c,c^*) < 0$ and $\pi_{c^*c^*cc}(c,c^*) = 0$ for all $c \in [\underline{c},\bar{c}]$ and $c^* \in [\underline{c},\bar{c}]$.

B) Any Nash equilibrium between the two governments will be characterized by strategic subsidies to R & D if I) holds, by strategic taxes on R&D if II) holds, and by no strategic intervention if III) holds.

A simple calculation based on the linear Hotelling and Cournot models of the previous section establishes that $\pi_{cc}(c,c^*) > 0$ and $\pi_{c^*c^*cc}(c,c^*)$

= 0. Since (22) already established that $\pi_c(c,c^*) < 0$, Proposition IV.1 implies that the role for strategic R & D policy of any kind disappears in these models when the first order stochastic dominance assumption of the previous section is replaced by mean-preserving second order stochastic dominance. We are thus led to conclude that, while the case for strategic trade policy is not particularly sensitive to market conditions, the pivotal issue on which appropriate strategic R & D policy depends is the nature of the uncertainty associated with the R & D process itself.

IV.2 Corrective Issues

We turn now to corrective issues. The following proposition uses Lemma IV.2, and (7).

<u>Proposition IV.2</u>:

A). For a given symmetric home and foreign policy, $r=r^*$, the difference between the home firm's noncooperative investment level and its investment level when firms maximize total export profit is negative (positive) if $\pi_{c^*c^*}(c,c^*) > 0$ ($\pi_{c^*c^*}(c,c^*) < 0$) for all $c\epsilon[\underline{c},\overline{c}]$ and $c^*\epsilon[\underline{c},\overline{c}]$.

B). The symmetric joint welfare maximizing corrective R & D policy as characterized in (7) involves an R & D subsidy (tax) if $\pi_{c^*c^*}(c,c^*) > 0$ ($\pi_{c^*c^*}(c,c^*) < 0$) for all $c\epsilon[\underline{c},\overline{c}]$ and $c^*\epsilon[\underline{c},\overline{c}]$.

Thus, the critical term from a corrective point of view is the sign of $\pi_{c^*c^*}(c,c^*)$, which by Lemma IV.2 determines the sign of $E\pi_{I^*}(\cdot)$. While the previous subsection established the sensitivity of strategic R & D policy to the way in which the uncertainty in R & D outcomes is modeled, we note here that the corrective policies are just as sensitive. In particular, Lemma III.2 established that when R & D leads to a first order shift in the cost distribution, investment makes a country "tough" if $\pi_{c^*}(c,c^*) > 0$ for all c and c^*. This property of $\pi(c,c^*)$ was shown to be met in the linear Hotelling and Cournot models considered above. However, Lemma IV.2 establishes that, when R & D leads to a mean-preserving increase in the riskiness of the cost distribution, investment makes a country "soft" if $\pi_{c^*c^*}(c,c^*) > 0$ for all c and c^*. This property of $\pi(c,c^*)$ is also readily established for the Hotelling and Cournot models considered above.

Hence, when R & D leads to a more risky cost distribution, firms generally undertake too little investment in risk. In the case of a single firm per country, the strategic Nash equilibrium (with $\pi_{c^*c^*cc}(c,c^*) = 0$) of free trade will lead to too little R & D from a symmetric joint welfare perspective. Analogously, with more than one firm in a country, there is now a corrective incentive--and only a corrective incentive--to subsidize R & D.

V. Conclusion

We have analyzed the desirability of R & D subsidies for domestic firms involved in international markets. Our goal has been to examine the robustness of the conclusion drawn by Spencer and Brander (1983) that an R & D subsidy can play a positive strategic role. We have found that the strategic role played by an R & D subsidy remains attractive for a variety of forms of product market competition when R & D leads to a reduction in expected firm costs. In this respect, the case for a strategic R & D subsidy appears stronger than the case for a strategic export subsidy. The overall desirability of an R & D subsidy in this context does depend, however, on the balance between this strategic incentive to subsidize and the corrective incentive to tax which arises whenever there is more than one domestic firm. The subsidy is therefore most likely to improve national welfare in this setting when there are few domestic firms, though an R & D tax may be optimal whenever the number of domestic firms exceeds one.

However, the case for a strategic R & D subsidy is undone when R & D leads to a mean-preserving increase in the riskiness of the firm's cost distribution. In the particular models of product market competition that we considered, investment reaction curves are horizontal, and there is simply no strategic motive for intervention. More generally, to know the appropriate strategic policy, one must know the sign of a fourth derivative of the reduced form profit function. Since the sign of such a derivative is unlikely to ever be known by policy makers, there appears to be absolutely no basis for strategic policy intervention when R & D is modeled in this way. In this setting, the only rationale for policy intervention is corrective, as too little risk is undertaken when there are many domestic firms. Thus, a corrective subsidy is desirable whenever the number of domestic firms exceeds one.

Taking these results together, the pivotal issue for determining appropriate strategic and corrective R & D policy appears to be the way in which uncertainty in the R & D process is modeled. In particular, the nature and sign of appropriate R & D intervention corresponding to any given number of foreign and domestic firms depends critically on this issue. Definitive statements of the appropriate role for R & D intervention will therefore require a convincing treatment of the role that uncertainty plays in the R & D process.

We close with a brief discussion of two assumptions maintained throughout which if relaxed, would each tend to reduce the sensitivity of R & D policy to the form with which uncertainty enters the R & D process. The first concerns our assumption that all consumption occurs in a third (importing) country. The introduction of domestic consumers will clearly increase the desirability of an R & D subsidy in the mean-reducing case, since an added benefit of the R & D subsidy is now the lower expected price faced by domestic consumers. Moreover, the convexity of the indirect utility function in price suggests that the desirability of an R & D subsidy will be enhanced in the risk-increasing case as well. Hence, the introduction of domestic consumers is likely to strengthen the case for R & D subsidies regardless of the role that uncertainty plays in the R & D process.

A second assumption we have maintained throughout is the nonnegativity of profits for each firm when all firms enter in the final (production) stage of the market. This suggests that our model is most relevant in markets characterized by a limited degree of scale economies. In particular, this analysis must be modified in cases where scale economies, entering either on the supply side through large fixed costs or on the demand side through important network externalities, are sufficiently pronounced so that only one firm can profitably produce in the final stage. In a related paper (Bagwell and Staiger, 1989), we consider this alternative case in which firms battle for the monopoly position and show that strategic subsidies and corrective taxes are optimal for both first and mean-preserving second order stochastic shifts.[22] We summarize the combined implications of the two papers in Figure 1. Whether such "battles for monopoly" or the setting of the present paper is more representative of "typical" international R & D competition is not an issue we attempt to resolve here. Rather, we view these results simply as pointing to the key parameters on which appropriate R & D policy depends.

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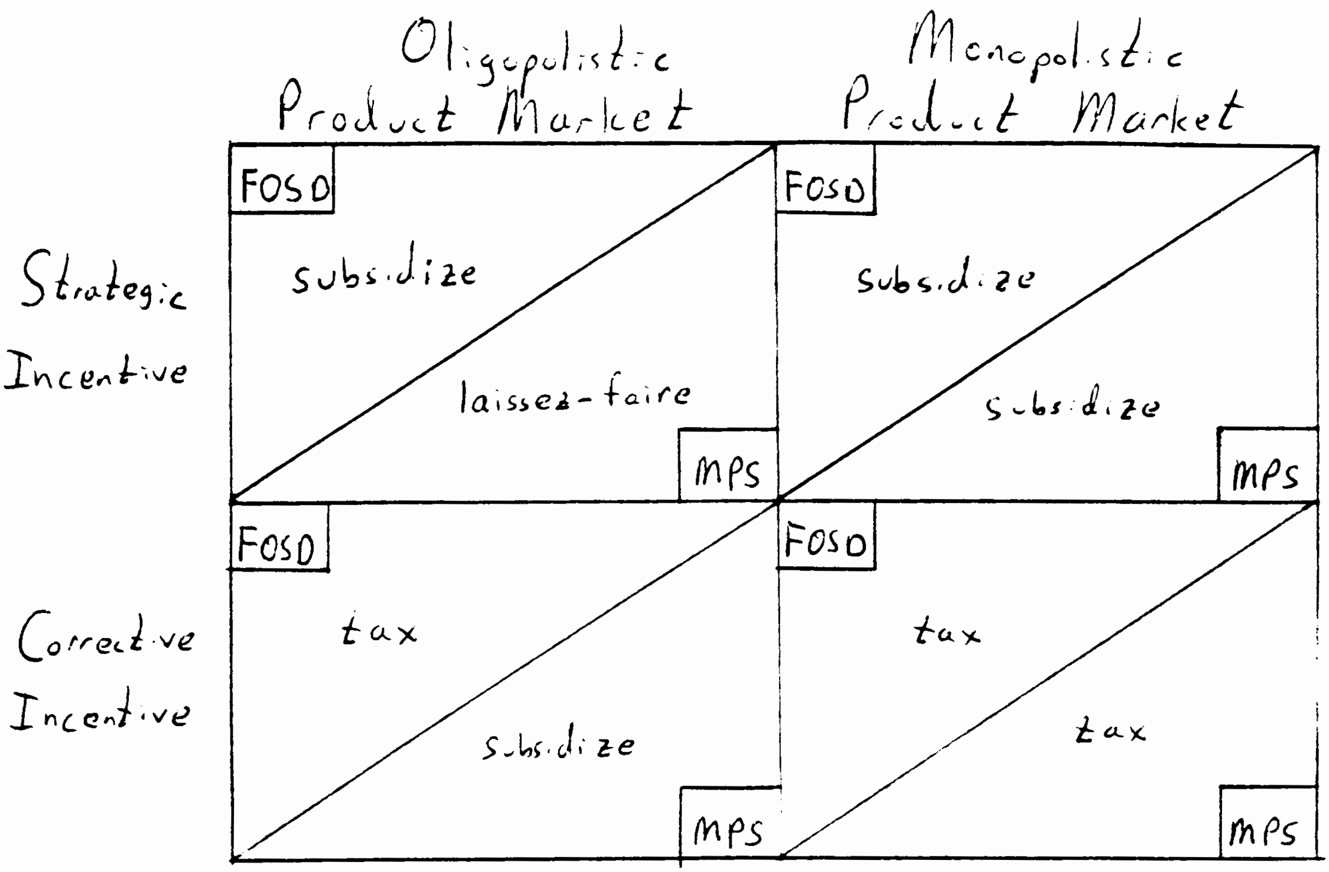
[22] The results of Bagwell and Staiger (1989) actually require assumptions slightly stronger than first and mean-preserving second order stochastic dominance, in that an additional monotonicity restriction is placed on the way investment effects the distribution of costs. See Bagwell and Staiger (1989) for details.

Figure 1



Note: FOSD denotes first order stochastic dominance; MPS denotes mean preserving spread.

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