

Discussion Paper No. 868

THE SENSITIVITY OF STRATEGIC AND CORRECTIVE R&D POLICY  
IN BATTLES FOR MONOPOLY\*

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First Draft: August 1989  
Revised: December 1989

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\*We thank Avinash Dixit and Rob Porter for helpful comments. A preliminary version of this paper was written while Staiger was a National Fellow of the Hoover Institution.

## Abstract

We characterize the strategic and corrective role for R & D subsidies in an export market where R & D is an uncertain process and where the winner of the R & D competition monopolizes the market. Investments in R & D are assumed to induce either first order or mean-preserving second order shifts in the distribution of (i) a firm's costs, with the low cost firm then monopolizing the product market or, under a reinterpretation of the model, (ii) a firm's discovery dates, with the first firm to make the discovery enjoying patent protection of infinite duration. We show that, regardless of which form uncertainty takes in the R & D process, a national strategic incentive to subsidize R & D exists, but must be balanced against a national corrective incentive to tax R & D whenever a country has more than one firm involved in the R & D competition. We conclude that an R & D subsidy is likely to be attractive in markets where scale economies are sufficiently large that firms battle for the eventual monopoly position, provided only that the number of domestic firms involved in the R & D stage is low.





## I. Introduction

The effects of trade policy on the strategic interaction among firms has been an active area of research in international trade since the pioneering work of Spencer and Brander (1983) and Brander and Spencer (1985). The latter paper explores the strategic role of export subsidies, and its conclusions have been the subject of intense scrutiny. Perhaps the most central criticism comes from Eaton and Grossman (1986), who show that the sign of the appropriate strategic export policy depends on whether firms choose prices or quantities. An additional concern, pointed out by Dixit (1984), Eaton and Grossman (1986), and Krishna and Thursby (1988), is that a corrective incentive to tax exports arises when there is more than one domestic firm, clouding the original case for export subsidies still further.

Much less attention has been given to the sensitivity of the conclusions of Spencer and Brander (1983), who establish that R & D subsidies can also perform a strategic role.<sup>1</sup> Nevertheless, since the Subsidies Code of the General Agreement on Tariffs and Trade (GATT) explicitly prohibits developed countries from directly subsidizing exports, R & D subsidies may well be the more relevant case. In any event, their

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<sup>1</sup> Exceptions include a companion paper, Bagwell and Staiger (1989), as well as Cheng (1987) and Dixit (1988). Bagwell and Staiger (1989) analyze strategic and corrective R & D policies in the context of standard models of oligopoly behavior. Cheng (1987) attempts to generalize the results of Spencer and Brander (1983) to a dynamic setting by introducing continual technological innovation and allowing domestic consumption. Dixit (1988) adopts a free entry assumption and is concerned primarily with the effects of exit and entry on appropriate R & D policy in a patent race setting. Spencer (1988) is also concerned with capital or R & D subsidies rather than with export subsidies but her focus is on the effects of GATT-sanctioned countervailing duties in an oligopolistic setting rather than on the case for R & D subsidies per se.

widespread use is well-documented.<sup>2</sup>

The R & D model that Spencer and Brander (1983) consider has two exporting countries, each with a single firm, and one importing country. Firms engage in quantity competition in the product market after R & D leads in a deterministic fashion to lower production costs. With the assumption of a single firm per country ruling out corrective issues at the national level, the appropriate strategic policy is shown to be an R & D subsidy.

There are two fundamental assumptions in the Spencer and Brander (1983) analysis which we relax in this paper. First, R & D is an inherently uncertain process, and it is important to assess the robustness of any R & D policy prescription to the existence of uncertainty. The natural stochastic analogue to the deterministic model has R & D lowering the mean of the firm's cost distribution in the particular sense of a first order stochastic shift. This is not, however, the only plausible description of R & D activity. An additional possibility which we consider is that R & D preserves the mean but alters the "risk" properties of the cost distribution. Such a shift in the cost distribution could correspond to R & D that commits the firm to a potentially more efficient but increasingly inflexible production process, prior to the resolution of uncertainty in the external environment. More generally, it is likely that R & D investments contain both mean-altering and risk-altering characteristics, and it thus seems important to explore these two "pure" cases.<sup>3</sup>

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<sup>2</sup> See, for example, the discussion in Hufbauer and Erb (1984), and Kominya, Okuno, and Suzumura (1988).

<sup>3</sup> In particular, a mean-altering second order shift may be the most likely scenario.

Second, Spencer and Brander (1983) assume an oligopolistic product market. We consider an alternative assumption about product market behavior, namely, that the "winner" of the R & D competition monopolizes the market. Thus, we have in mind a situation in which firms undertake R & D to affect the distribution of their costs, knowing that only the lowest cost firm will actually operate in the final (production) stage. As we discuss in more detail in the next section, this "winner-take-all" feature is plausible when the product market is subject to sufficiently large scale economies, entering either on the supply side through large fixed costs of production or on the demand side through the existence of network externalities. Alternatively, under a reinterpretation of the model which we discuss in section VI, this set up may also be plausible in the context of an international patent race.

When firms of different countries are engaged in a battle for monopoly, we find that the strategic incentive to subsidize R & D remains whether R & D reduces the mean of a firm's cost distribution or increases its riskiness. Thus, in a "high stakes" R & D competition where the winner takes all, the case for strategic R & D subsidies is not particularly sensitive to the way in which uncertainty enters the R & D process. Moreover, when there is more than one domestic firm (at the R & D stage), we find a corrective incentive to tax R & D, again regardless of the particular way in which R & D is modeled. Hence, on balance, the appropriate R & D policy when firms battle for monopoly depends mainly on the number of firms engaged in R & D competition.

We thus conclude that, in battles for monopoly, there is a robust sense in which the optimal strategic policy is an R & D subsidy, while the optimal

corrective policy is an R & D tax. This result is of direct interest, but is also noteworthy in relation to our findings in a companion paper (Bagwell and Staiger, 1989). There, we assume that firms invest in a stochastic R & D process and then, given their cost realizations, compete in the product market according to standard models of oligopoly behavior. If the dominant features of R & D are captured by first order stochastic shifts in the cost distribution, we show that a case for strategic R & D subsidies in the home country can be made, although there is an opposing corrective incentive to tax R & D whenever the number of domestic firms exceeds one. However, if R & D serves largely to increase the riskiness of the cost distribution, then generally no basis for strategic intervention exists, and a corrective R & D subsidy is called for when there exists more than one domestic firm. We conclude in that paper that detailed knowledge of the R & D process is a crucial prerequisite to the design of appropriate R & D policy when the product market is oligopolistic. Taken together, the two papers highlight the important way in which the form of uncertainty in the R & D process and the nature of the resulting product market combine to determine appropriate R & D intervention.

The rest of the paper is organized as follows. The basic model is described in Section II. Mean-reducing and mean-preserving investments are considered in Sections III and IV, respectively. Section V considers the introduction of multiple domestic and foreign firms. Our model is re-interpreted as a patent race in Section VI. Section VII concludes.

## II. The Basic Model

We consider a simple model in which there are two exporting countries



and a third importing country. The export markets are imperfectly competitive; for now, we follow Spencer and Brander (1983) and assume each exporting country has a single exporting firm. One exporting country will be referred to as the home country, while the other is called the foreign country. Asterisks (\*) denote foreign country variables.

The basic game has three stages. In the initial stage, the two exporting governments simultaneously choose the effective unit costs of investment,  $r$  and  $r^*$ , to their respective firms. In both countries, the social cost of investment is  $\tilde{r}$ . Thus, for example,  $r < \tilde{r}$  indicates a home country subsidy on investment by the home firm. As all consumption takes place in a third country, the goal of each exporting country is to maximize its firm's expected profit less subsidy costs.

The second stage follows after both firms observe the policy choices of both exporting governments. The two firms then simultaneously choose nonnegative investment levels,  $I$  and  $I^*$ . We depart from the deterministic modeling approach of Spencer and Brander (1983) and model investment as a parameter in the random determination of production costs. Thus,  $f(c|I)$  is the density of possible constant costs  $c$ , given the investment level  $I$ .  $f^*(c^*|I^*)$  is then defined analogously. We maintain the assumptions that  $f(c|I) \equiv f^*(c|I)$ ,  $f(c|I) > 0$ , and  $f(c|I)$  is continuously differentiable in  $c$  and  $I$ , for every  $c \in [\underline{c}, \bar{c}]$ . The two investment technologies are thus symmetric and well-behaved. The goal of any one firm at this stage is to maximize its expected profit, given its cost of investment.

The third and final stage arrives when each firm has observed its

realized production cost as well as that of its rival.<sup>4</sup> The two firms then simultaneously chose whether or not to enter the production stage of the market. We assume that scale economies in the product market are sufficiently important that the market can only support the entry of one firm profitably. These scale economies could enter on the supply side and take the form of a sufficiently large fixed cost in production, or they could enter on the demand side in the form of network externalities. In either case, while multiple equilibria to this entry game exist, the efficient equilibrium will have the low cost firm enter and monopolize the market. We take this equilibrium to the entry game as focal, and concentrate on it throughout the paper.<sup>5</sup>

Throughout, we use the subgame perfect equilibrium concept (Selten (1975)). We thus solve the final stage first, the second stage second and the first stage last.

We close this section by defining national welfare and discussing at a

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<sup>4</sup> See Aoki and Reitman (1989) for a model in which a firm's investment, but not its realized cost, is known by the rival firm.

<sup>5</sup> When there is a large fixed cost to entry, the entry game admits three classes of equilibria. The most efficient firm may enter with probability one, the least efficient firm may enter with probability one, or the firms may play mixed entry strategies and earn zero expected gross profit. The latter case is uninteresting, since if firms play in this fashion no investment will occur. Of the two former cases, the efficient class of equilibria seems focal. A similar logic applies when there are network externalities and the firms offer new and incompatible products of similar, intrinsic value. Here, if both firms enter, consumers may coordinate on the efficient firm or the inefficient firm or worse, divide up asymmetrically between the two firms. The most efficient equilibrium for the subgame in which both firms enter entails consumers going to the lowest cost firm. (This equilibrium will also be unique, if consumers are sufficiently sensitive to price relative to network size.) If the efficient equilibrium is anticipated and if there is some slight fixed cost to entry, then the efficient firm will enter with probability one and monopolize the market.

general level the determinants of appropriate R & D policy. In the absence of domestic consumption, home welfare can be defined as simply expected profits less subsidy costs, or

$$(1) \quad W(r, r^*) \equiv E\pi(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r) - (\bar{r} - r) \cdot \hat{I}(r, r^*)$$

Here,  $E\pi(\cdot)$  is expected profits of the home firm (net of investment costs), while  $\hat{I}(\cdot)$  and  $\hat{I}^*(\cdot)$  denote equilibrium domestic and foreign investment levels, respectively.  $W^*(r, r^*)$  is defined analogously for the symmetric foreign expected profit function,  $E\pi^*(\cdot)$ . To see the domestic welfare effects of a policy-induced change in the domestic cost of R & D,  $r$ , we differentiate (1) with respect to  $r$  and use the envelope theorem to obtain

$$(2) \quad W_r(r, r^*) = E\pi_{I^*}(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r) \cdot \hat{I}_r^*(r, r^*) - (\bar{r} - r) \cdot \hat{I}_r(r, r^*)$$

where subscripts denote derivatives.<sup>6</sup> The domestic welfare effect of a change in  $r$  is composed of the difference between two terms. The first term captures the way in which a change in  $r$  alters equilibrium foreign investment, and through this channel, expected domestic profits. The second term simply gives the way in which a change in  $r$  alters equilibrium domestic investment, and through this, domestic subsidy payments.

In general, to determine the sign of  $W_r(r, r^*)$  requires knowledge of the signs of  $E\pi_{I^*}(\cdot)$ ,  $\hat{I}_r^*(\cdot)$ , and  $\hat{I}_r(\cdot)$ . The sign of  $E\pi_{I^*}(\cdot)$  tells how

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<sup>6</sup> This calculation assumes  $E\pi_r(\cdot) = -\hat{I}(\cdot)$ ; that is,  $r$  only directly enters into  $E\pi(\cdot)$  through the total cost of investment,  $r\hat{I}(\cdot)$ . In Section III, an explicit definition of  $E\pi(\cdot)$  is provided.

greater investment abroad affects the expected profit at home. In the language of Fudenberg and Tirole (1984),  $E\pi_{I^*}(\cdot) < 0$  would imply that investment (or a low cost of investment which leads to greater investment) makes a country "tough". The signs of  $\hat{I}_r^*(\cdot)$  and  $\hat{I}_r(\cdot)$  determine the direction of the equilibrium response of foreign and domestic investment, respectively, to a change in  $r$ .

To see what determines the signs of  $\hat{I}_r^*(\cdot)$  and  $\hat{I}_r(\cdot)$ , we write down each firm's first order condition, which implicitly defines investment reaction functions  $I(I^*, r)$  and  $I^*(I, r^*)$  provided that second order conditions are met:

$$E\pi_I(I, I^*, r) = 0$$

$$E\pi_{I^*}^*(I, I^*, r^*) = 0$$

We will assume throughout the paper that the determinant of the Jacobian,  $J$ , associated with the first order conditions is positive,

$$(3) \quad |J| \equiv E\pi_{II}(I, I^*, r) \cdot E\pi_{I^*I^*}^*(I, I^*, r^*) - E\pi_{I^*I}(I, I^*, r) \cdot E\pi_{II^*}^*(I, I^*, r^*) > 0,$$

and that this stability condition holds globally. With this condition, the reaction functions have at most one intersection, which we assume exists.  $\hat{I}(r, r^*)$  and  $\hat{I}^*(r, r^*)$  denote this solution. Totally differentiating the first order conditions then yields

$$(4) \quad \hat{I}_r(r, r^*) = \frac{E\pi_{I^*I^*}^*(I, I^*, r)}{|J|}$$

$$(5) \quad \hat{I}_r^*(r, r^*) = \frac{-E\pi_{II^*}^*(I, I^*, r)}{|J|}$$

Thus, a home subsidy will raise home investment provided that  $E\pi_{I^*I^*}^*(\cdot) < 0$ , which is simply the second order condition. And a home subsidy will reduce foreign investment if and only if  $E\pi_{II^*}^*(\cdot) < 0$ . Since, provided second order conditions hold, the sign of  $E\pi_{II^*}^*(\cdot)$  determines the sign of the investment reaction curve slopes, this last condition amounts to establishing whether investment reaction curves are negatively sloped.<sup>7</sup>

Drawing all this together and using (2), it is evident that, provided second order conditions hold ( $E\pi_{I^*I^*}^*(\cdot) < 0$ ) a small R & D subsidy by the home government to commit its firm to a larger investment will increase domestic welfare ( $W_r(r, r^*) < 0$ ) if investment makes a country "tough" ( $E\pi_{I^*}(\cdot) < 0$ ) and reaction curves are negatively sloped ( $E\pi_{II^*}^*(\cdot) < 0$ ). This is illustrated in Figure 1, where the depicted relationship between  $I$  and  $I^*$  is negative, and where a lower  $r$  (a subsidy) shifts  $I(I^*, r)$  out in a parallel fashion, thereby raising the equilibrium level of  $I$  and lowering that of  $I^*$ , with the latter effect benefiting the domestic country if it prefers lower foreign investment. The focus of the remaining sections is to explore the generality with which these conditions hold and, in so doing, to explore the generality of the case for R & D subsidies in the context of "winner-take-all" international R & D rivalry.

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<sup>7</sup> The slope of the foreign investment reaction curve comes from totally differentiating the foreign first order condition to get  $dI^*/dI = -E\pi_{II^*}^*(I, I^*, r)/E\pi_{I^*I^*}^*(I, I^*, r)$ .

### III. Mean-Reducing Investments

#### 1. Basic Assumptions

As noted above, we focus on the efficient equilibrium of the final stage entry game, so that only the lowest cost firm will choose to "open." Formally, let  $\pi(c, c^*)$  represent the home firm's profit (gross of investment costs) in the third stage if home costs are  $c$  and foreign costs are  $c^*$ .  $\pi^*(c, c^*)$  is the symmetric function for the foreign firm. Then, if  $\pi_m(c)$  is the profit from monopolizing the third country, we have that

$$\pi(c, c^*) \equiv \begin{cases} \pi_m(c), & c < c^* \\ 0, & c \geq c^* \end{cases} .^8$$

We assume only that  $\pi_m(c) > 0$  and  $\pi'_m(c) < 0$ , where the prime refers to a derivative, for every  $c \in [\underline{c}, \bar{c}]$ .<sup>9</sup>

We must also make a distributional assumption to convey the cost-reducing nature of investment. Let  $f_I(c|I)$  denote the partial derivative of  $f(c|I)$  with respect to  $I$ .

Assumption A: For every  $I$ , there exists  $\hat{c} \in (\underline{c}, \bar{c})$  such that  $f_I(\hat{c}|I) = 0$ ,  $f_I(c|I) < 0$  for  $c \in (\hat{c}, \bar{c}]$ , and  $f_I(c|I) > 0$  for  $c \in [\underline{c}, \hat{c})$ .

As shown in Figure 2, Assumption A simply amounts to the existence of a

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<sup>8</sup> Ties are actually irrelevant for our analysis, since they occur with zero probability.

<sup>9</sup> Our analysis in this section is thus more general than the constant cost assumption we employ. Note also that  $\pi_m(c)$  may embody a fixed cost component.

critical cost level such that a given raise in investment increases the density of lower costs and drops the density of higher costs. Defining  $F(c|I) \equiv \int_{\underline{c}}^c f(c|I)dc$ , it is straightforward to show that Assumption A implies  $F_I(c|I) > 0$  for all  $c \in (\underline{c}, \bar{c})$ , which is the usual first order stochastic dominance condition (Hadar and Russell, 1969). Assumption A also requires a "single crossing" of  $f_I(c|I)$  through the  $c$ -axis.<sup>10</sup>

Thus, an increase in investment shifts density to lower costs. We next assume that this shifting process occurs at a decreasing rate as investment increases. The role of this assumption is to make the firm's investment problem a concave program.<sup>11</sup>

Assumption B: For every  $I$  and  $c \in [\underline{c}, \bar{c}]$ ,  $F_{II}(c|I) \leq 0$ , with a strict inequality holding over some positive measure of costs.

## 2. The Investment Stage

We now fix  $r$  and  $r^*$  and consider the choice of investment levels. The home country will want to choose its investment level to maximize its expected profit, which is given by

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<sup>10</sup> Assumption A holds for the normal and exponential distributions, provided that greater investment lowers the mean, with  $\hat{c}$  equal to the mean. (Our analysis carries through where  $c$  is distributed on open intervals.) An example with finite support satisfying Assumption A, taken from Rogerson (1985), is  $F(c|I) = [(c-\underline{c})/(\bar{c}-\underline{c})]^{(\bar{I}-I)}$ , where  $\bar{I}$  is some maximal investment level.

<sup>11</sup> See Rogerson (1985) for a related assumption in the context of the principal-agent problem. The examples in note 10 are all consistent with Assumption B, provided in the normal and exponential cases that the mean's dependence on investment is not too concave.

$$(6) \quad E\pi(I, I^*, r) \equiv \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} f(c|I) f^*(c^*|I^*) \pi(c, c^*) dc^* dc - rI$$

The first order condition is then

$$(7) \quad E\pi_I(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} f_I(c|I) f^*(c^*|I^*) \pi(c, c^*) dc^* dc - r = 0$$

We assume a solution to this equation, so that a maximum is obtained if the second order condition holds:

$$(8) \quad E\pi_{II}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} f_{II}(c|I) f^*(c^*|I^*) \pi(c, c^*) dc^* dc < 0$$

The solution corresponds to a reaction curve,  $I = I(I^*, r)$ . Exactly symmetric arguments apply for the foreign country.

We begin with the following lemma stating that the second order condition does indeed hold.

Lemma III.1: For all  $I$ ,  $I^*$  and  $r$ ,  $E\pi_{II}(I, I^*, r) < 0$ .

Proof: Observe using (8) that

$$E\pi_{II}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} f_{II}(c|I) K(c|I^*) dc$$

where

$$(9) \quad K(c|I^*) \equiv \int_{\underline{c}}^{\bar{c}} f^*(c^*|I^*) \pi(c, c^*) dc^*$$



$$= \int_{\underline{c}}^{\bar{c}} f^*(c^* | I^*) \pi_m'(c) dc^* > 0$$

for all  $c \in [\underline{c}, \bar{c}]$ .  $K(c | I^*)$  is simply expected domestic profit given a domestic cost realization of  $c$  if the foreign investment level is  $I^*$ . We also have that, with  $\pi_m'(c) < 0$ ,

$$(10) K_c(c | I^*) = \int_{\underline{c}}^{\bar{c}} f^*(c^* | I^*) \pi_m'(c) dc^* - f^*(c | I^*) \pi_m(c) < 0$$

for all  $c \in [\underline{c}, \bar{c}]$ . Expression (10) says that, for fixed foreign investment  $I^*$ , expected domestic profits are decreasing in the domestic cost realization. Thus, integrating by parts, we obtain

$$(11) E\pi_{II}(I, I^*, r) = - \int_{\underline{c}}^{\bar{c}} F_{II}(c | I) K_c(c | I^*) dc < 0$$

Q.E.D.

The reaction functions,  $I = I(I^*, r)$  and  $I^* = I^*(I, r)$  are thus well-defined. Moreover, by Lemma III.1, (3), and (4),  $\hat{I}_r(r, r^*) < 0$ : a domestic R & D subsidy will raise domestic investment.

We next ask whether investment makes a country "tough", i.e., whether greater investment by one country decreases the expected profit of the other country. The following lemma establishes that investment does indeed correspond to a tough strategy.

Lemma III.2: For all  $I, I^*$  and  $r$ ,  $E\pi_{I^*}(I, I^*, r) < 0$ .

Proof: Observe that

$$(12) E\pi_{I^*}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} f(c|I) f_{I^*}^*(c^*|I^*) \pi(c, c^*) dc^* dc$$

which, using (9), can be rewritten as

$$(13) E\pi_{I^*}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} f(c|I) K_{I^*}(c|I^*) dc$$

But it is straightforward to show that

$$(14) K_{I^*}(c|I^*) = -\pi_m(c) F_{I^*}^*(c|I^*) < 0,$$

for all  $c \in (\underline{c}, \bar{c})$ . Expression (14) says that, for a fixed domestic cost realization, expected domestic profits are decreasing in foreign investment.

The lemma is thus proved.

Q.E.D.

Intuitively, as  $I^*$  increases, the density on low  $c^*$ s also increases. Thus, the home country is less likely to win and its expected profits decrease.

Finally, we must establish that reaction curves are negatively sloped. Our result is contained in the next lemma.

Lemma III.3: For all  $I, I^*$  and  $r$ , if  $I > I^*$ , then  $E\pi_{I^* I}(I, I^*, r) < 0$ .

Proof: Observe that

$$(15) E\pi_{I^* I}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} f_I(c|I) f_{I^*}^*(c^*|I^*) \pi(c, c^*) dc^* dc$$

which with (9) simplifies to

$$(16) \quad E\pi_{I^*I}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} f_I(c|I)K_{I^*}(c|I^*)dc$$

Unfortunately,  $K_{I^*}(c|I^*)$  is not of one global sign, so a simple application of Assumption A does not suffice to sign (16). Using (14), however, expression (16) can be rewritten as

$$(17) \quad E\pi_{I^*I}(I, I^*, r) = - \int_{\underline{c}}^{\bar{c}} f_I(c|I)F_{I^*}^*(c|I^*)\pi_m(c)dc.$$

Moreover, starting from symmetric initial investments ( $I=I^*$ ) implies that

$$\int_{\underline{c}}^{\bar{c}} f_I(c|I)F_{I^*}^*(c|I^*)dc = \int_{\underline{c}}^{\bar{c}} f_I(c|I)F_I(c|I)dc = \int_{F_I(\underline{c}|I)}^{F_I(\bar{c}|I)} F_I dF_I = 0,$$

since  $F_I(\bar{c}|I) = F_I(\underline{c}|I) = 0$ . Using this result, the single crossing property of  $f_I(c|I)$ , and the monotonicity of  $\pi_m(c)$ , we then have

$$\begin{aligned} & \int_{\underline{c}}^{\bar{c}} f_I(c|I)F_{I^*}^*(c|I^*)\pi_m(c)dc \\ &= \int_{\underline{c}}^{\hat{c}} f_I(c|I)F_{I^*}^*(c|I^*)\pi_m(c)dc + \int_{\hat{c}}^{\bar{c}} f_I(c|I)F_{I^*}^*(c|I^*)\pi_m(c)dc \\ &> \int_{\underline{c}}^{\hat{c}} f_I(c|I)F_{I^*}^*(c|I^*)\pi_m(\hat{c})dc + \int_{\hat{c}}^{\bar{c}} f_I(c|I)F_{I^*}^*(c|I^*)\pi_m(\hat{c})dc \end{aligned}$$

$$= \pi_m(\hat{c}) \int_{\underline{c}}^{\bar{c}} f_I(c|I) F_{I^*}^*(c|I^*) dc = 0,$$

where  $\hat{c}(\underline{c}, \bar{c})$  is as defined in Assumption A. The lemma now follows directly. Q.E.D.

The intuition for this result centers around the monotonicity of  $\pi_m(c)$ , and can be seen as follows. Suppose for the moment that  $\pi_m'(c) \equiv 0$ , so that monopoly profits conditional on winning,  $\pi_m$ , are independent of costs. Then using (13), (14), and (17) we could write

$$E\pi_{I^*}(I, I^*, r) = -\pi_m \int_{\underline{c}}^{\bar{c}} f(c|I) F_{I^*}^*(c|I^*) dc$$

$$E\pi_{I^*I}(I, I^*, r) = -\pi_m \int_{\underline{c}}^{\bar{c}} f_I(c|I) F_{I^*}^*(c|I^*) dc$$

The integral term in the first equation is simply the effect of an increase in foreign investment on the foreign firm's probability of winning. Thus,  $E\pi_{I^*}(I, I^*, r)$  would be proportional to this probability effect if  $\pi_m'(c) \equiv 0$ . The integral term in the second equation is then the effect of an increase in home investment on the foreign firm's probability of winning when there is also an independent increase in foreign investment. Intuitively, given the symmetry of the investment technologies, this effect must be zero if the home and foreign investment levels are initially identical. Thus, were  $\pi_m'(c) \equiv 0$ , we would have  $E\pi_{I^*I}(I, I^*, r) = 0$ . When  $\pi_m'(c)$  is monotonically decreasing in  $c$ , however, an increase in the probability with which the foreign firm wins is more costly in terms of lost profits to the home firm

the lower is the home firm's expected costs (the higher is  $I$ ). Thus, with  $\pi'_m(c) < 0$ , we have  $E\pi_{I^*I}(I, I^*, r) < 0$ .

Thus, about any symmetric equilibrium where  $I = I^*$ , reaction curves are negatively sloped. This implies by (5) that  $\hat{I}_r^*(r, r^*) > 0$  for a symmetric equilibrium. A home subsidy lowers foreign investment.

### 3. The Policy Stage

In the previous section, we showed that the illustration in Figure 1 is locally correct. About any symmetric equilibrium, a home subsidy will increase  $I$  and decrease  $I^*$ . Further, the decrease in  $I^*$  is desirable for the home country, since foreign investment hurts the domestic country. We develop in this section a number of implications that follow from this reasoning.

Proposition 1: From free trade ( $r=r^*=\tilde{r}$ ), a slight R & D subsidy improves that country's welfare while a slight R & D tax decreases that country's welfare. Further, the R & D subsidy has a "beggar thy neighbor" nature, as it decreases the welfare of the rival country.

Proof: Given the symmetric beginning at  $r=r^*=\tilde{r}$ ,  $\hat{I}(\tilde{r}, \tilde{r}) = \hat{I}^*(\tilde{r}, \tilde{r})$  follows from the symmetry of the countries. Then, Lemma III.2, Lemma III.3, and (2) give

$$W_r(\tilde{r}, \tilde{r}) = E\pi_{I^*}(\hat{I}(\tilde{r}, \tilde{r}), \hat{I}^*(\tilde{r}, \tilde{r}), \tilde{r}) \cdot \hat{I}_r^*(\tilde{r}, \tilde{r}) < 0$$

Hence, a slight R & D subsidy (tax) improves (decreases) home welfare.

Note also that

$$W_r^*(\tilde{r}, \tilde{r}) = E\pi_r^*(\hat{I}(\tilde{r}, \tilde{r}), \hat{I}^*(\tilde{r}, \tilde{r}), \tilde{r}) \cdot \hat{I}_r(\tilde{r}, \tilde{r}) > 0$$

Thus, a small R & D subsidy hurts the rival country.

Q.E.D.

Our results here are not as strong as those developed by Spencer and Brander (1983). In their model, a country's best response to the selection of free trade by the rival country is a subsidy. In contrast, we are only able to establish a negative slope for reaction curves about symmetric investments (Lemma III.3); thus, while it seems likely that the optimal response to free trade is a subsidy, we can only say that a slight subsidy (tax) is preferred (inferior) to a passive, free trade response and that the rival country experiences a welfare loss (gain) when this subsidy (tax) is imposed. The same local feature of our model also prevents us from characterizing the optimal subsidy/tax strategies when governments choose policies sequentially.<sup>12</sup>

Like Spencer and Brander (1983), however, we can argue that a subsidy is generally optimal when countries choose policies simultaneously. We say a symmetric, interior Nash equilibrium occurs at  $(\hat{r}, \hat{r}^*)$  if

$$(i) \quad \hat{r} = \hat{r}^*$$

$$(ii) \quad W_r(\hat{r}, \hat{r}^*) = 0 = W_{r^*}(\hat{r}, \hat{r}^*)$$

$$(iii) \quad W(\hat{r}, \hat{r}^*) \geq W(r, \hat{r}^*), \text{ for all } r \neq \hat{r}, \text{ and } W^*(\hat{r}, \hat{r}^*) \geq W^*(\hat{r}, r^*) \text{ for all } r^* \neq \hat{r}^*.$$

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<sup>12</sup> This limitation to local results is not present in Bagwell and Staiger (1989), where profits are continuous in  $c$  and  $c^*$ .

With this we have:

Proposition 2: In any symmetric, interior Nash equilibrium,  $\tilde{r} > \hat{r} = \hat{r}^*$ .

Both countries subsidize R & D.

Proof: Since  $\hat{r} = \hat{r}^*$ ,  $\hat{I}(\hat{r}, \hat{r}^*) = \hat{I}^*(\hat{r}, \hat{r}^*)$ . Then, as before,

$$E\pi_{I^*}(\hat{I}(\hat{r}, \hat{r}^*), \hat{I}^*(\hat{r}, \hat{r}^*), \hat{r}) \cdot \hat{I}_r^*(\hat{r}, \hat{r}^*) < 0$$

Further,  $\hat{I}_r(\hat{r}, \hat{r}^*) < 0$ . Thus, from (2),  $W_r(\hat{r}, \hat{r}^*) = 0$  requires  $\tilde{r} > \hat{r}$ , with symmetric arguments applying to the foreign country. Q.E.D.

The natural equilibrium to the three-stage game therefore involves subsidies on investments.

Two normative issues remain. First, is the welfare of the exporting countries higher or lower at the symmetric, interior equilibrium or at free trade? Second, for any given  $r=r^*$ , do the two firms invest more or less than they would if they chose symmetric investment levels to maximize total export profit?

We begin with the first question. Not surprisingly, welfare is lower when countries subsidize than when they do not.

Proposition 3:  $W(\tilde{r}, \tilde{r}) = W^*(\tilde{r}, \tilde{r}) > W(\hat{r}, \hat{r}^*) = W^*(\hat{r}, \hat{r}^*)$ . Welfare is higher at free trade than at the symmetric, interior Nash equilibrium.

Proof: Totally differentiating  $W(r, r^*)$  with respect to  $r$  and  $r^*$ , using the envelope theorem, and setting  $r=r^*$  and  $dr=dr^*$ , we find that

$$dW(r, r^*) = [E\pi_{1*}(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r) - (\tilde{r} - r)] [\hat{I}_r^*(r, r^*) + \hat{I}_{r^*}^*(r, r^*)] dr$$

Now consider beginning with  $r=r^* < \tilde{r}$  and then increasing  $r$  and  $r^*$  together until  $r=r^* = \tilde{r}$ . Then  $dr > 0$  along this path. The first term in the above expression is clearly negative. Finally, it can be shown that the second term is also negative, given that  $\hat{I}(r, r^*) = \hat{I}^*(r, r^*)$  along this path and  $|J| > 0$ . It follows that  $dW(r, r^*) > 0$  along this path when  $r=r^* < \tilde{r}$ . The proposition then follows from  $(\tilde{r}, \tilde{r}) > (\hat{r}, \hat{r}^*)$  and  $\hat{r} = \hat{r}^*$ , and symmetric arguments for the foreign country. Q.E.D.

The subsidy game has a definite prisoners-dilemma character. A unilateral subsidy can improve welfare over free trade, but bilateral subsidies are self-enforcing and inferior to free trade.

As a final matter, one might wish to know if the two firms invest too much or too little, from the viewpoint of total export profit. This is our second question. We show now that the firms invest too much, as they fail to internalize the negative externality that any one country's investment has on the other country's profit.

Proposition 4: For any  $r=r^*$ , the two exporting firms invest more than they would if their goal were to select investment levels to maximize joint export profit, assuming the joint problem has a symmetric interior solution.



Proof: Set  $r=r^*$ . Let  $\tilde{I}(r,r^*)$  and  $\tilde{I}^*(r,r^*)$ , with  $\tilde{I}^*(r,r^*) = \tilde{I}(r,r^*)$ , maximize total export profit,  $E\pi(I,I^*,r) + E\pi^*(I,I^*,r)$ . Then

$$E\pi(\hat{I}(r,r^*), \hat{I}^*(r,r^*), r) \geq E\pi(\tilde{I}(r,r^*), \tilde{I}^*(r,r^*), r)$$

and

$$E\pi(\tilde{I}(r,r^*), \tilde{I}^*(r,r^*), r) \geq E\pi(\hat{I}(r,r^*), \hat{I}^*(r,r^*), r)$$

Adding these inequalities together gives

$$E\pi(\tilde{I}(r,r^*), \tilde{I}^*(r,r^*), r) \geq E\pi(\hat{I}(r,r^*), \hat{I}^*(r,r^*), r).$$

But Lemma III.2 ensures  $E\pi_{I^*}(\cdot) < 0$  for all  $I$ , whence  $\tilde{I}^*(r,r^*) \leq \hat{I}^*(r,r^*)$ . Finally,  $\tilde{I}^*(r,r^*) = \hat{I}^*(r,r^*)$  is impossible, as a first order condition for joint profit maximization would then be violated, since

$$\begin{aligned} & E\pi_I(\hat{I}(r,r^*), \hat{I}^*(r,r^*), r) + E\pi_I^*(\hat{I}(r,r^*), \hat{I}^*(r,r^*), r^*) \\ &= E\pi_I^*(\hat{I}(r,r^*), \hat{I}^*(r,r^*), r^*) < 0 \end{aligned} \quad \text{Q.E.D.}$$

Thus, for any given  $r=r^*$ , the two firms overinvest. Too much investment even occurs at free trade, and of course a subsidy only exacerbates the problem, as Proposition 3 shows. If the two governments could cooperate, then the appropriate corrective policy would be to tax investment. Equivalently, if the two firms were located in a single country, an R & D tax acts to correct the negative externality between firms.

#### IV. Risk-Altering Investments

In the previous section we focused on investments which lowered expected cost. However, the effect of R & D on the distribution of costs could take on any of several plausible forms. To explore the sensitivity of our results to the way R & D is modeled, we now consider the case in which investment in R & D affects the riskiness of the distribution of cost outcomes in a mean-preserving way.

##### 1. Basic Assumptions

We maintain the "winner take all" assumption of the previous section, so that  $\pi(c, c^*)$  and  $\pi^*(c, c^*)$  continue to be defined as above. However, in addition to the property that  $\pi'_m(c) < 0$ , we now require that  $\pi''_m(c) > 0$  as well. This second derivative property will hold provided that the constant-cost monopoly problem is well-behaved.<sup>13</sup> Finally, we alter the way in which investment affects the distribution of costs to capture mean-preserving changes in risk.

Assumption D: For every  $I$ ,  $-\frac{d}{dI} \left[ \int_{\underline{c}}^{\bar{c}} cf(c|I)dc \right] = 0$ , and there exists  $\hat{c} \in (\underline{c}, \bar{c})$ ,  $c_1 \in (\underline{c}, \hat{c})$  and  $c_2 \in (\hat{c}, \bar{c})$  such that  $F_I(\hat{c}|I) = 0$ , and  $f_I(c_1|I) = 0 = f_I(c_2|I)$ , and either

(1) (risk-reducing investment)

$f_I(c|I) < 0$  for  $c \in (\underline{c}, c_1) \cup (c_2, \bar{c}]$ , and

$f_I(c|I) > 0$  for  $c \in (c_1, c_2)$ , or

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<sup>13</sup> For example, this condition is met in the case of constant costs if the demand curve slopes downward and profits are concave in price.

(2) (risk-increasing investment)

$f_I(c|I) > 0$  for  $c \in [\underline{c}, c_1) \cup (c_2, \bar{c}]$ , and

$f_I(c|I) < 0$  for  $c \in (c_1, c_2)$ .

Figure 3 illustrates Assumption D for the case of risk-increasing investment. As depicted, there is a critical cost level associated with any given investment level such that this investment raises the value of the distribution function evaluated at lower costs and reduces the value of the distribution function evaluated at higher costs. Assumption D(1) implies that higher levels of investment yield cost distributions which second-order stochastically dominate (Rothschild and Stiglitz, 1970) those associated with lower levels of investment. Assumption D(2) implies that higher levels of investment yield cost distributions which are second-order stochastically dominated by those associated with lower levels of investment. Both Assumptions D(1) and D(2) require the additional condition of a "single crossing" of  $F_I(c|I)$  through the  $c$ -axis and of  $f_I(c|I)$  on each side of this point.

As in the previous section, we will also require a regularity assumption on  $F_{II}$ . This ensures that the firm's investment problem for the risk-increasing case is a concave program, but plays no role in our results for the risk-reducing case.

Assumption E: For every  $I$ , there exists a  $\bar{c} \in (\underline{c}, \bar{c})$  such that  $F_{II}(\bar{c}|I) = 0$ , with  $F_{II}(c|I) \leq 0$  for  $c \in [\underline{c}, \bar{c})$ ,  $F_{II}(c|I) \geq 0$  for  $c \in (\bar{c}, \bar{c}]$ , and  $F_{II}(c|I) \neq 0$  for some positive measure of  $c \in (\underline{c}, \bar{c})$ .

Finally we assume that one of the following two conditions on the distribution of costs holds.

Assumption F: Either (1)  $c$  is distributed symmetrically about its mean for every  $I$  or (2)  $f(c|I)$  is nonincreasing in  $c$  for all  $c \in [\underline{c}, \bar{c}]$  and every  $I$ .

Assumption F is stronger than required, but is sufficient for the results below.<sup>14</sup>

## 2. The Investment Stage

### Risk-Reducing Investment

We begin with the case of investment which yields a mean-preserving reduction in the riskiness of the cost distribution. This is the case associated with Assumption D(1).

As before, we fix  $r$  and  $r^*$  and consider the investment decisions of each firm. With home firm expected profits still given by (6), we now establish that firms will not invest in risk-reducing R & D. This result is contained in the following lemma.

Lemma IV.0: For all  $I, I^*$ , and  $r$ ,  $E\pi_I(I, I^*, r) < 0$ .

Proof: Note first that

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<sup>14</sup> Examples of distributions that satisfy this assumption are the normal and the exponential.

$$E\pi_I(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} f_I(c|I)K(c|I^*)dc - r$$

with  $K(c|I^*)$  defined by (9),  $K_c(c|I^*) < 0$  defined by (10), and

$$K_{cc}(c|I^*) = \int_c^{\bar{c}} \pi_m''(c)f^*(c^*|I^*)dc^* - 2f^*(c|I^*)\pi_m'(c) - f_c^*(c|I^*)\pi_m(c).$$

Integrating by parts yields

$$E\pi_I(I, I^*, r) = - \left[ \int_{\underline{c}}^{\bar{c}} F_I(c|I)K_c(c|I^*)dc \right] - r$$

and, since the mean is fixed,

$$\int_{\underline{c}}^{\bar{c}} F_I(c|I)dc = 0.$$

Suppose first that Assumption F(1) holds so that the distribution of costs is symmetric. Then using the expression for  $K_c(c|I^*)$ ,  $E\pi_I(I, I^*, r)$  can be written as

$$\begin{aligned} E\pi_I(I, I^*, r) = & - \int_{\underline{c}}^{\bar{c}} F_I(c|I) \left[ \int_c^{\bar{c}} \pi_m'(c)f^*(c^*|I^*)dc^* \right] dc \\ & + \int_{\underline{c}}^{\hat{c}} F_I(c|I)f^*(c|I^*)\pi_m(c)dc + \int_{\hat{c}}^{\bar{c}} F_I(c|I)f^*(c|I^*)\pi_m(c)dc - r \end{aligned}$$

where  $\hat{c}$  is the mean of  $c$ . Now according to Assumptions D(1) and F(1),  $\bar{c}=\hat{c}$  and  $F_I(c|I)$  is negative for  $c$  below  $\hat{c}$  and positive for  $c$

above  $\hat{c}$ . Also we have established above that  $\int_{\underline{c}}^{\bar{c}} F_I(c|I)dc = 0$ . Thus, since  $[\int_{\underline{c}}^{\bar{c}} \pi'_m(c)f^*(c^*|I^*)dc^*]$  is negative and increasing (toward zero) in  $c$ , the first term in the expression above is negative. Moreover, with a symmetric distribution,  $f^*(c|I^*)$  is symmetric and  $F_I(c|I)$  is antisymmetric about  $\hat{c}$ , so that the second and third terms above would sum to zero absent  $\pi_m(c)$  which, since it is positive and decreasing in  $c$ , turns the sum of these two terms negative. Thus, under F(1) we have established that  $E\pi_I(I, I^*, r) < 0$ .

Finally, if F(2) holds instead, then  $K_{c_c}(c|I^*)$  is nonnegative. With  $K_c(c|I^*)$  negative and nondecreasing in  $c$  for all  $c \in [\underline{c}, \bar{c}]$ , the term  $[\int_{\underline{c}}^{\bar{c}} F_I(c|I)K_c(c|I^*)dc]$  will be positive, so that  $E\pi_I(I, I^*, r) < 0$ .

Q.E.D.

Lemma IV.0 establishes that firms will not invest in risk-reducing (mean preserving) R & D. This result comes from the fact that firm monopoly profits are a nonincreasing and convex function of costs. Thus, an investment which shifts probability weight from very low costs and from very high costs toward the mean will lower expected profits for two reasons: (i) since profits are nonincreasing in costs, it reduces the chance of winning when winning is the most profitable (low costs) and increases the chance of winning when winning is less profitable (higher costs), and (ii) since profits are convex in costs, it shifts probability weight to higher costs over a range (low costs) where profits fall steeply in costs and shifts probability weight to lower costs over a range (high costs) where profits are less sensitive to costs. Finally, while Lemma IV.0 suggests that firms will be uninterested in risk-reducing (mean-preserving) investments, it is also true that domestic and foreign governments, with their objectives

defined as in the previous section and acting individually or jointly, have no incentive to intervene to effect the no-investment choices of the firms. We thus leave the risk-reducing investment case and turn our attention for the remainder of this section to the case in which investment yields a mean-preserving increase in the riskiness of the cost distribution.

### Risk-Increasing Investment

According to the result above, firm expected profits are increasing in the riskiness of the cost distribution. Thus, we now consider the case in which investment preserves the mean of a firm's cost distribution but increases its risk. This corresponds to the case of Assumption D(2).

Once again we fix  $r$  and  $r^*$  and consider the investment decisions of each firm. With the home firm choosing its investment level to maximize its expected profit, integration by parts establishes that the first and second order conditions for a maximum are given by

$$E\pi_I(I, I^*, r) = - \left[ \int_{\underline{c}}^{\bar{c}} F_I(c|I)K_c(c|I^*)dc \right] - r = 0$$

and

$$E\pi_{II}(I, I^*, r) = - \left[ \int_{\underline{c}}^{\bar{c}} F_{II}(c|I)K_c(c|I^*)dc \right] < 0$$

We assume that a solution to the first order condition holds (the term in square brackets is non positive for every  $I$  according to D(2) and Lemma IV.0), so that a maximum is obtained if the second order condition is met. The next lemma verifies that the second order condition holds.

Lemma IV.1: For all  $I, I^*$ , and  $r$ ,  $E\pi_{II}(I, I^*, r) < 0$ .

Proof: We first note that integrating by parts yields

$$\int_{\underline{c}}^{\bar{c}} F_{II}(c|I)dc = 0$$

Also, according to Assumption E,  $F_{II}(c|I)$  is negative for  $c$  below some critical  $\tilde{c}$  and positive for  $c$  above this critical level, with strict inequality over some positive measure. Finally we have established above that  $K_c(c|I^*) < 0$ .

Suppose, then, that Assumption F(1) holds, so that the distribution of costs is symmetric. Then using (10),  $E\pi_{II}(I, I^*, r)$  can be written as

$$\begin{aligned} E\pi_{II}(I, I^*, r) = & - \int_{\underline{c}}^{\bar{c}} F_{II}(c|I) \left[ \int_c^{\bar{c}} \pi'_m(c) f^*(c^*|I) dc^* \right] dc \\ & + \int_{\underline{c}}^{\hat{c}} F_{II}(c|I) f^*(c|I^*) \pi_m(c) dc + \int_{\hat{c}}^{\bar{c}} F_{II}(c|I) f^*(c|I^*) \pi_m(c) dc \end{aligned}$$

where  $\hat{c}$  is again the mean of  $c$ . Arguments identical to those of Lemma IV.0 then establish that  $E\pi_{II}(I, I^*, r) < 0$  under Assumption F(1). Suppose, next, that Assumption F(2) holds. Identical arguments to those of Lemma IV.0 then establish that  $E\pi_{II}(I, I^*, r) < 0$  in this case as well. Q.E.D.

Thus, reaction functions are well-defined and a domestic R & D subsidy will raise domestic investment ( $\hat{I}_r(r, r^*) < 0$ ). The next lemma shows that expected domestic profits are decreasing in foreign investment.



Lemma IV.2: For all  $I, I^*$ , and  $r$ ,  $E\pi_{I^*}(I, I^*, r) < 0$ .

Proof: Using (12), (13) and (14), we have

$$\begin{aligned} E\pi_{I^*}(I, I^*, r) &= - \int_{\underline{c}}^{\bar{c}} F_{I^*}^*(c|I^*)f(c|I)\pi_m(c)dc \\ &= - \left[ \int_{\underline{c}}^{\hat{c}} F_{I^*}^*(c|I^*)f(c|I)\pi_m(c)dc + \int_{\hat{c}}^{\bar{c}} F_{I^*}^*(c|I^*)f(c|I)\pi_m(c)dc \right] \end{aligned}$$

Suppose first that Assumption F(1) holds so that the distribution of costs is symmetric. Then  $\hat{c}$  is the mean of  $c$ , and absent  $\pi_m(c)$  the two terms above would sum to zero. Since  $F_{I^*}^*(c|I^*)$  is positive below  $\hat{c}$  and negative above it by Assumption D(2), and since  $\pi_m(c)$  is nonincreasing in  $c$ ,  $E\pi_{I^*}(I, I^*, r) < 0$  in this case. Alternatively, if Assumption F(2) holds, then  $E\pi_{I^*}(I, I^*, r) < 0$ , since  $f(c|I)\pi_m(c)$  is a positive and decreasing function of  $c$ . Q.E.D.

Thus, the home country earns greater expected profits the lower is foreign investment. This is because, with lower foreign investment, the distribution of foreign costs is less risky. As a result, the domestic firm wins less often when winning doesn't mean much (when its own costs are high) and wins more often when winning means a great deal (when its own costs are low). As such, expected domestic profits rise as foreign investment falls.

Finally, we turn to the question of whether or not reaction curves are negatively sloped. The next lemma establishes that they are, at least starting from symmetric investment levels.

Lemma IV.3: For all  $I, I^*$ , and  $r$ , if  $I=I^*$ , then  $E\pi_{I^*I}(I, I^*, r) < 0$ .

Proof: We start by observing that, as established in (17) and imposing the condition that  $I=I^*$ ,

$$E\pi_{I^*I}(I, I^*, r) = - \int_{\underline{c}}^{\bar{c}} F_I(c|I)f_I(c|I)\pi_m(c)dc$$

Note also that

$$\int_{\underline{c}}^{\hat{c}} F_I(c|I)f_I(c|I)dc = \int_{F_I(\underline{c}|I)}^{F_I(\hat{c}|I)} F_I dF_I = 0$$

and

$$\int_{\hat{c}}^{\bar{c}} F_I(c|I)f_I(c|I)dc = \int_{F_I(\hat{c}|I)}^{F_I(\bar{c}|I)} F_I dF_I = 0$$

Finally, we rewrite  $E\pi_{I^*I}(I, I^*, r)$  in four parts as

$$E\pi_{I^*I}(I, I^*, r) = - \left\{ \left[ \int_{\underline{c}}^{c_1} F_I(c|I)f_I(c|I)\pi_m(c)dc + \int_{c_1}^{\hat{c}} F_I(c|I)f_I(c|I)\pi_m(c)dc \right] \right. \\ \left. + \left[ \int_{\hat{c}}^{c_2} F_I(c|I)f_I(c|I)\pi_m(c)dc + \int_{c_2}^{\bar{c}} F_I(c|I)f_I(c|I)\pi_m(c)dc \right] \right\}$$

To sign this expression, note that Assumption D(2) implies (see Figure 3)

$$F_I(c|I)f_I(c|I) \begin{cases} \geq 0 & \text{for } c \in [\underline{c}, c_1] \cup [\hat{c}, c_2] \\ \leq 0 & \text{for } c \in [c_1, \hat{c}] \cup [c_2, \bar{c}] \end{cases}$$

Using this and the fact that  $\pi_m(c)$  is nonincreasing in  $c$  implies

$$E\pi_{I^*I}(I, I^*, r) < -[\pi_m(c_1) \int_{\underline{c}}^{\hat{c}} F_I(c|I)f_I(c|I)dc + \pi_m(c_2) \int_{\hat{c}}^{\bar{c}} F_I(c|I)f_I(c|I)dc] = 0$$

Q.E.D.

The intuition for this result is analogous to that for Lemma III.3, applied over the regions  $c \in [\underline{c}, \hat{c}]$  and  $c \in [\hat{c}, \bar{c}]$ . Thus, around any symmetric equilibrium with  $I = I^*$ , reaction curves are negatively sloped. Consequently, starting from a position of symmetric equilibrium investment levels,  $\hat{I}_r^*(r, r^*) > 0$  and a home subsidy lowers foreign investment.

### 3. The Policy Stage

With the lemmas of the previous subsection, each proposition in Section III.3 holds. Thus, when investment yields a mean preserving increase in the riskiness of the distribution of costs as depicted above, (i) a slight subsidy (starting at free trade) increases that country's welfare and reduces welfare in the rival country, (ii) in any symmetric interior Nash equilibrium, both countries subsidize, (iii) welfare of each exporting country is higher under free trade than in the symmetric interior Nash equilibrium, and (iv) for any  $r=r^*$ , combined investment at home and abroad is higher than the symmetric investment choice which maximizes joint profits.

## V. Greater Number of Firms: Strategic and Corrective Issues

In the previous sections, a corrective policy for R & D arises only if governments cooperate and maximize total welfare. When governments interact

noncooperatively, there is no incentive for corrective policy, since each country has but a single firm and thus no externalities within the country. All that remains is the possibility of a strategic policy, which results in a subsidy as shown in Proposition 2.

Suppose now that a country has more than one firm entering the R & D competition. Certainly, there remains a role for strategic subsidies. As before, by committing its firms to greater investment, a country's welfare is improved by the associated reduction of investment in the rival country. On the other hand, there is now a role for a corrective policy as well. A negative externality to R & D arises among the firms of a country, as each firm's marginal investment lowers the probability that all other firms - including those in the same country - will win the R & D competition. This negative externality within a country will tend to make the level of R & D in any one country excessive. A corrective role for an R & D tax is thus provided.

To explore the tradeoff between strategic and corrective incentives, we suppose that there are  $H$  home firms and  $F$  foreign firms, and allow R & D investment to affect either the mean or the riskiness of the cost distribution. Defining domestic welfare as

$$W(r, r^*) = \sum_{i=1}^H E\pi^i(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r) - (\tilde{r} - r) \sum_{i=1}^H \hat{I}^i(r, r^*)$$

where

$$\hat{I}(r, r^*) \equiv \hat{I}^1(r, r^*) \dots \hat{I}^H(r, r^*)$$

$$\hat{I}^*(r, r^*) \equiv \hat{I}^{*1}(r, r^*) \dots \hat{I}^{*F}(r, r^*)$$

we differentiate  $W(r, r^*)$  with respect to  $r$  and impose symmetry among

domestic and among foreign firms to obtain

$$W_r(r, r^*) = H \cdot \{F \cdot E\pi_{I^*j}^i(\cdot) \hat{I}_r^{*j}(r, r^*) + (H-1)E\pi_{Ij}^i(\cdot) \hat{I}_r^j(r, r^*) - (\bar{r}-r)\hat{I}_r^i(r, r^*)\}$$

Restricting our analysis to small subsidies starting from free trade, so that  $r=r^*=\bar{r}$ , we then have

$$W_r(r, r^*) = H \cdot E\pi_{Ij}^i(\cdot) \cdot \{F \cdot \hat{I}_r^{*j}(r, r^*) + (H-1) \cdot \hat{I}_r^j(r, r^*)\}$$

In analogy with our preceding analysis, it can be shown that  $E\pi_{Ij}^i(\cdot) < 0$  and that  $\hat{I}_r^j(r, r^*) < 0$  and  $\hat{I}_r^{*j}(r, r^*) > 0$  provided that the second order and stability conditions are met. Thus, starting from free trade,

$$(18) \text{ sign } (W_r(r, r^*)) = -\text{sign } (F \cdot \hat{I}_r^{*j}(r, r^*) + (H-1) \cdot \hat{I}_r^j(r, r^*))$$

Hence, in the presence of more than one domestic firm, the sign of  $W_r(r, r^*)$  is in general indeterminate. The term  $F \cdot \hat{I}_r^{*j}(r, r^*)$  is positive, and captures the strategic rent-shifting effect of a change in  $r$ ; a higher domestic interest rate increases foreign investment ( $\hat{I}_r^{*j}(r, r^*) > 0$ ) thereby shifting expected rents toward the foreign country and lowering domestic welfare. However, the term  $(H-1)\hat{I}_r^j(r, r^*)$  is negative, and captures the impact of a change in  $r$  on the national externality problem; with more than one domestic firm ( $H > 1$ ), a higher domestic interest rate reduces domestic investment ( $\hat{I}_r^j(r, r^*) < 0$ ) thereby mitigating against excessive investment and thus increasing domestic welfare. Hence, whether a country's welfare improves with a small R & D tax or a small R & D subsidy

depends in general on the relative importance of these two effects.<sup>15</sup>

To establish conditions under which  $W_r(r, r^*)$  is negative and thus a small R & D subsidy starting from free trade improves national welfare, we use (18) and note that  $W_r(r, r^*) < 0$  implies

$$(19) - \left( \frac{\hat{I}_r^{*j}(r, r^*)}{\hat{I}_r^j(r, r^*)} \right) > \frac{H-1}{F}$$

Using explicit expressions for  $\hat{I}_r^{*j}(r, r^*)$  and  $\hat{I}_r^j(r, r^*)$ , we find that (19) reduces to

$$(20) R > \frac{1}{1+F/(H-1)}$$

where  $R \equiv \frac{E\pi_{I_i I_j}^i(\cdot)}{E\pi_{I_i I_i}^i}$  denotes the negative of the slope of the investment reaction curve of a representative firm, starting from free trade. Since all (foreign and domestic) firms are symmetric starting from free trade,  $R$  is invariant to the ratio of foreign to domestic firms. Thus, holding the total number  $(H+F)$  of firms fixed, condition (20) is more likely to be satisfied, and thus a small R & D subsidy is more likely to improve national welfare starting at free trade, the greater the number of foreign relative to home firms. Moreover, for any fixed ratio of foreign to domestic firms, condition (20) is more likely to be met the steeper are investment reaction curves, which suggests that a smaller total (foreign plus domestic) number of firms will also increase the likelihood of national welfare improvements

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<sup>15</sup> This indeterminacy is reminiscent of Dixit (1988) who raises similar concerns in a competitive (free entry) setting in the context of a patent race.

from small R & D subsidies.<sup>16</sup>

## VI. Comparison with the Patent Literature

Before concluding, we note that the model developed above can be re-interpreted as a patent race. To see this, note that (6) can be written as

$$E\pi(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} f(c|I) \int_c^{\bar{c}} f^*(c^*|I^*) \pi_m(c) dc^* dc - rI$$

A simple change of variables gives

$$E\pi(I, I^*, r) = \int_0^{\bar{t}} f(t|I) \int_t^{\bar{t}} f^*(t^*|I^*) \pi_m(t) dt^* dt - rI$$

In this latter formulation, an investment outlay at time zero induces a distribution over dates for the discovery of an innovation. A firm wishes to discover the innovation first, because the first discovery can be patented. Finally,  $\pi_m(t)$  is the monopoly profit associated with the discovery. Assuming the market demand is invariant through time,  $\pi_m(t) = e^{-rt} \pi_m$  where  $\pi_m$  is the flow profit from the innovation. To see that the two formulations are equivalent, recall that the only assumptions we placed on  $\pi_m(c)$  were  $\pi_m(c) > 0$ ,  $\pi'_m(c) < 0$ , and  $\pi''_m(c) > 0$ . Clearly, these assumptions are also satisfied by  $\pi_m(t)$ .

Patent race models in which investment reduces the mean time to discovery were first studied by Loury (1979) and Dasgupta and Stiglitz (1980). These authors were primarily interested in whether the optimal

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<sup>16</sup> We have been unable to verify this last result at a general level. However, the result does hold for the exponential case if the hazard function is not too concave and  $\pi_m(c)$  is exponential in  $c$ .

amount of investment occurs. Consequently, while the negative externality between firms and the corresponding corrective issues that we discuss have also been identified in this literature, our analysis of investment reaction functions and the associated strategic issues is novel.<sup>17</sup>

Bhattacharya and Mookherjee (1986), Dasgupta and Maskin (1987), Dasgupta and Stiglitz (1980), and Klette and De Meza (1986) have considered the possibility that investment affects the riskiness of discovery dates in patent race models. Under distributional assumptions similar to our own, this literature has identified a tendency for firms to invest more in risk than is collectively optimal, due to the negative externality that one firm's investment has on the expected profit of other firms. Previous work does not, however, analyze investment reaction functions and the corresponding strategic issues.

Thus, our basic results can be interpreted in terms of a patent race. There are some important qualifications to this interpretation, however. First, as shown by Lee and Wilde (1980), if the costs of investment are not completely borne (or committed to) at time zero, then reaction curves may slope upward, in which case in our setting strategic taxes would occur.<sup>18</sup> In the context of patent races, our results thus are certain to apply only if investment costs are invariant with respect to the realized date of discovery. Second, patent races quite plausibly involve a variety of feedback phenomena, in which the intensity of effort at a date by one firm

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<sup>17</sup> We note that this previous work used an exponential (constant hazard rate) density function. The exponential function is a particular distribution that satisfies our assumptions.

<sup>18</sup> See Dixit (1988) for a recent model that also makes this point and which attempts to synthesize the strategic patent literature.



affects the probability of discovery by that firm, as well as the intensity of effort by other firms, at immediately following dates. Neither our model nor those discussed above admit these effects.

## VII. Conclusion

We have analyzed the desirability of R & D subsidies for domestic firms involved in "winner-take-all" international rivalries. Our goal has been to examine the robustness of the conclusion drawn by Spencer and Brander (1983) that an R & D subsidy can play a positive strategic role. We have found that the strategic role played by an R & D subsidy in this setting remains attractive under several alternative specifications of the way uncertainty enters the R & D process.

This result contrasts sharply with our findings in Bagwell and Staiger (1989). There, scale economies are sufficiently small that all competitors in the R & D stage also choose to enter and compete as oligopolists at the production stage. The way in which uncertainty enters the R & D process then turns out to be a crucial determinant of the nature of appropriate R & D intervention.<sup>19</sup> Together, as illustrated in Figure 4, the two papers provide a structural basis from which to determine optimal R & D policy. A strategic R & D subsidy is likely to be attractive in markets in which scale economies are sufficiently large that firms battle for the eventual monopoly

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<sup>19</sup> The difference between the two papers arises from the fact that, in the setting here where firms battle for the monopoly position, firms care only about rival costs in so far as these costs determine whether or not they "win" (have lowest costs) and not by how much they win. In contrast, when all firms engaged in R & D will also choose to produce as in Bagwell and Staiger (1989), the amount by which rival costs differ from own costs becomes important. In the latter setting, the importance of "winning" is replaced by the curvature properties of the profit function, and the way in which R & D affects the distribution of costs then becomes pivotal.

position, whether R & D affects the riskiness or the mean of the cost distribution. Similarly, in oligopolistic product markets with associated smaller scale economies in which R & D is thought to lower expected costs without substantially increasing the riskiness of the distribution, a strategic R & D subsidy is likely to be appropriate. In either of these cases, a corrective incentive to tax R & D emerges if the number of domestic firms undertaking R & D exceeds one. On balance, an R & D subsidy is more likely to be preferred the smaller the number of domestic firms. However, if the product market is oligopolistic and R & D primarily affects the riskiness of the cost distribution, then an R & D subsidy offers no strategic advantage, but does constitute an attractive corrective policy when there are multiple domestic firms.

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Figure 1

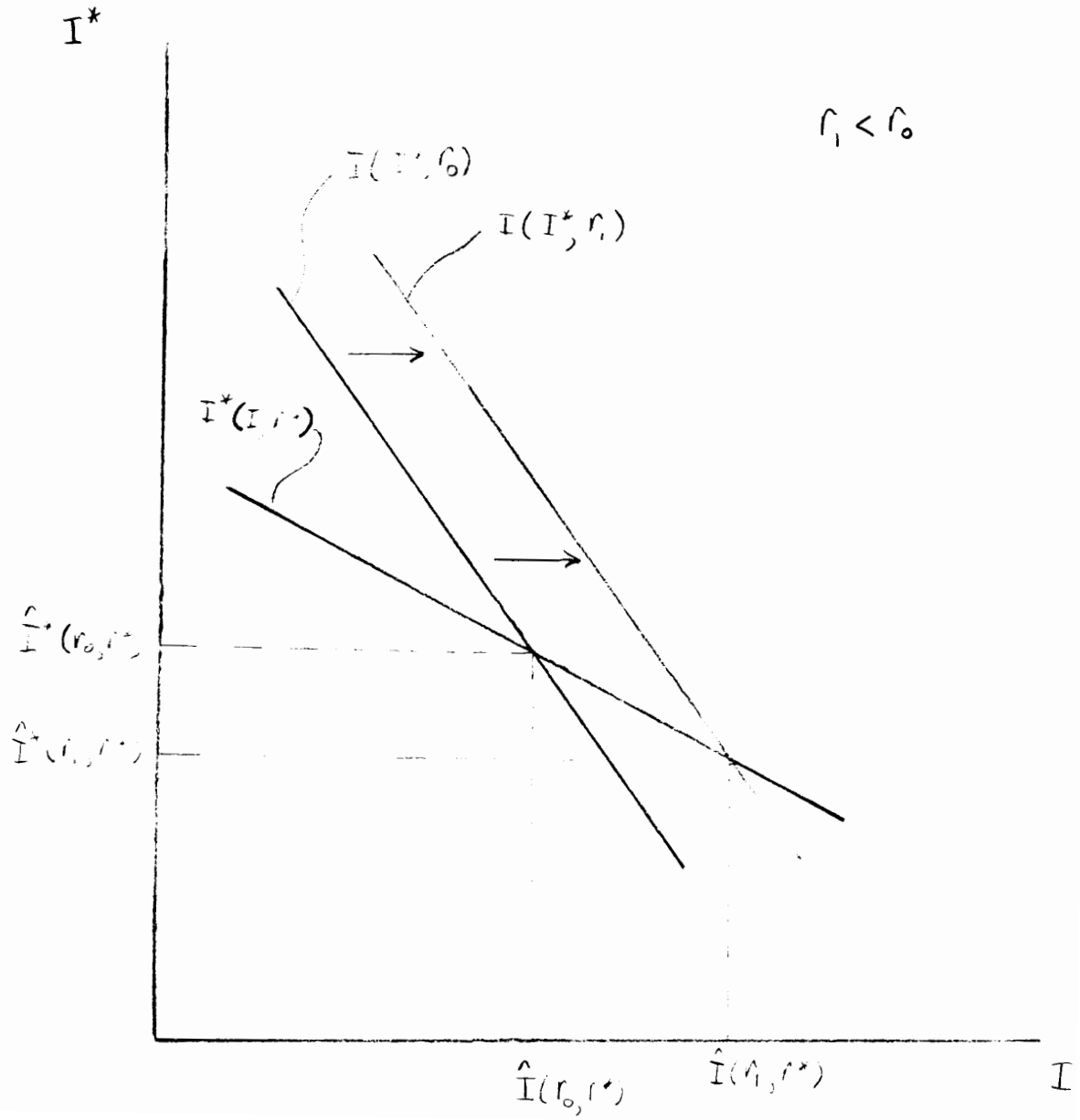


Figure 2

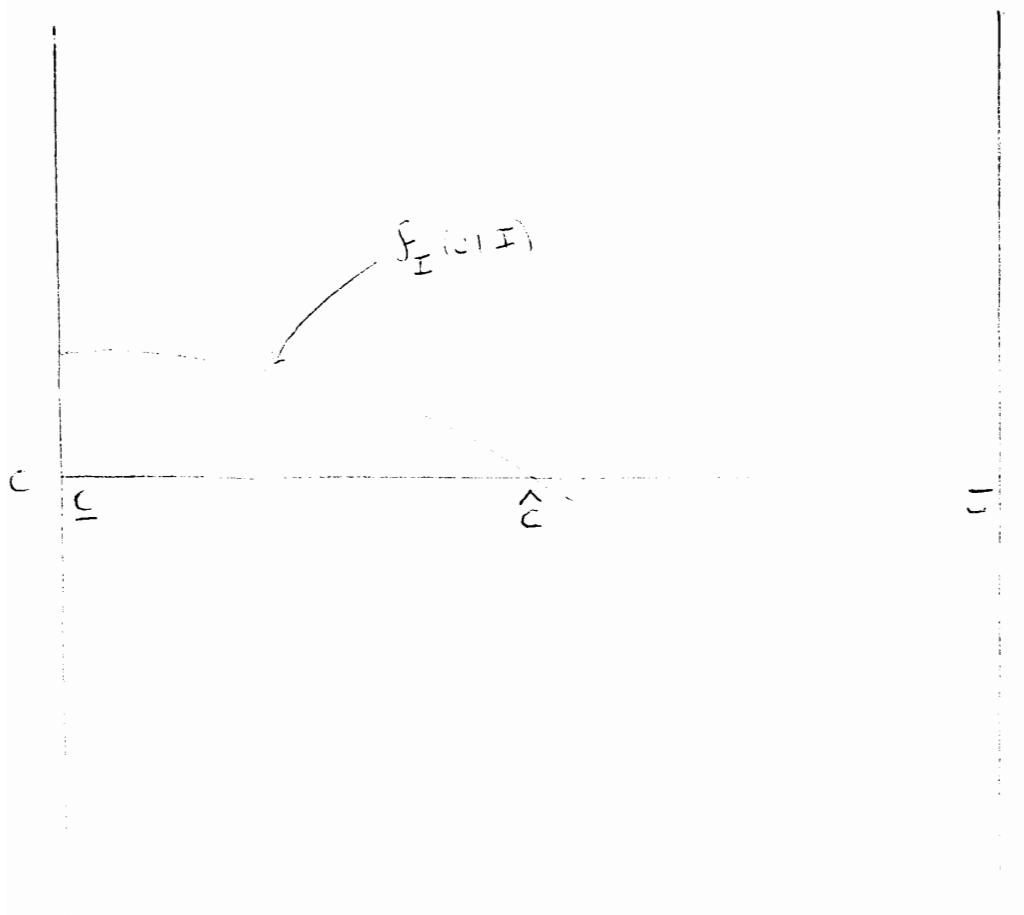


Figure 3

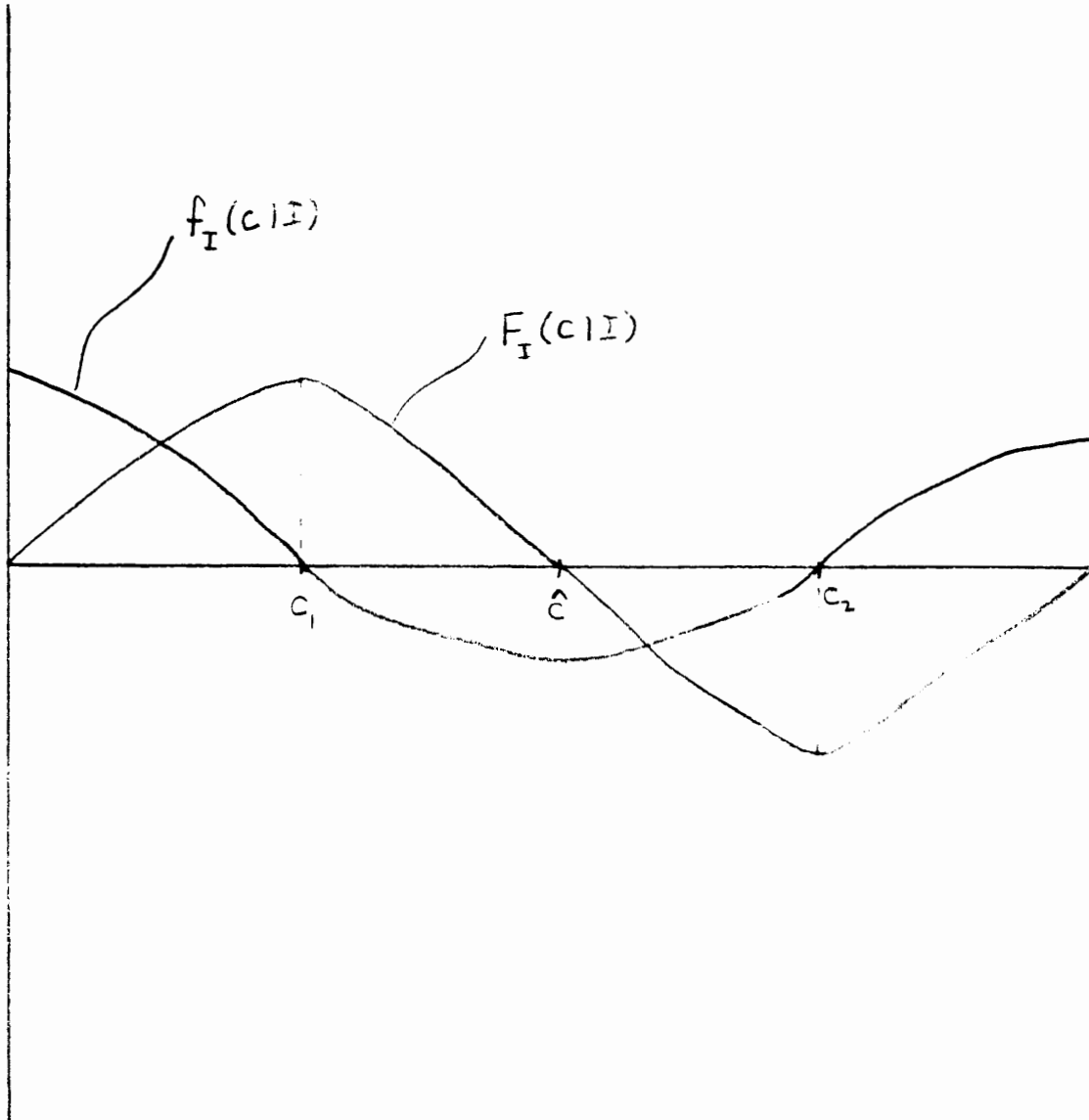
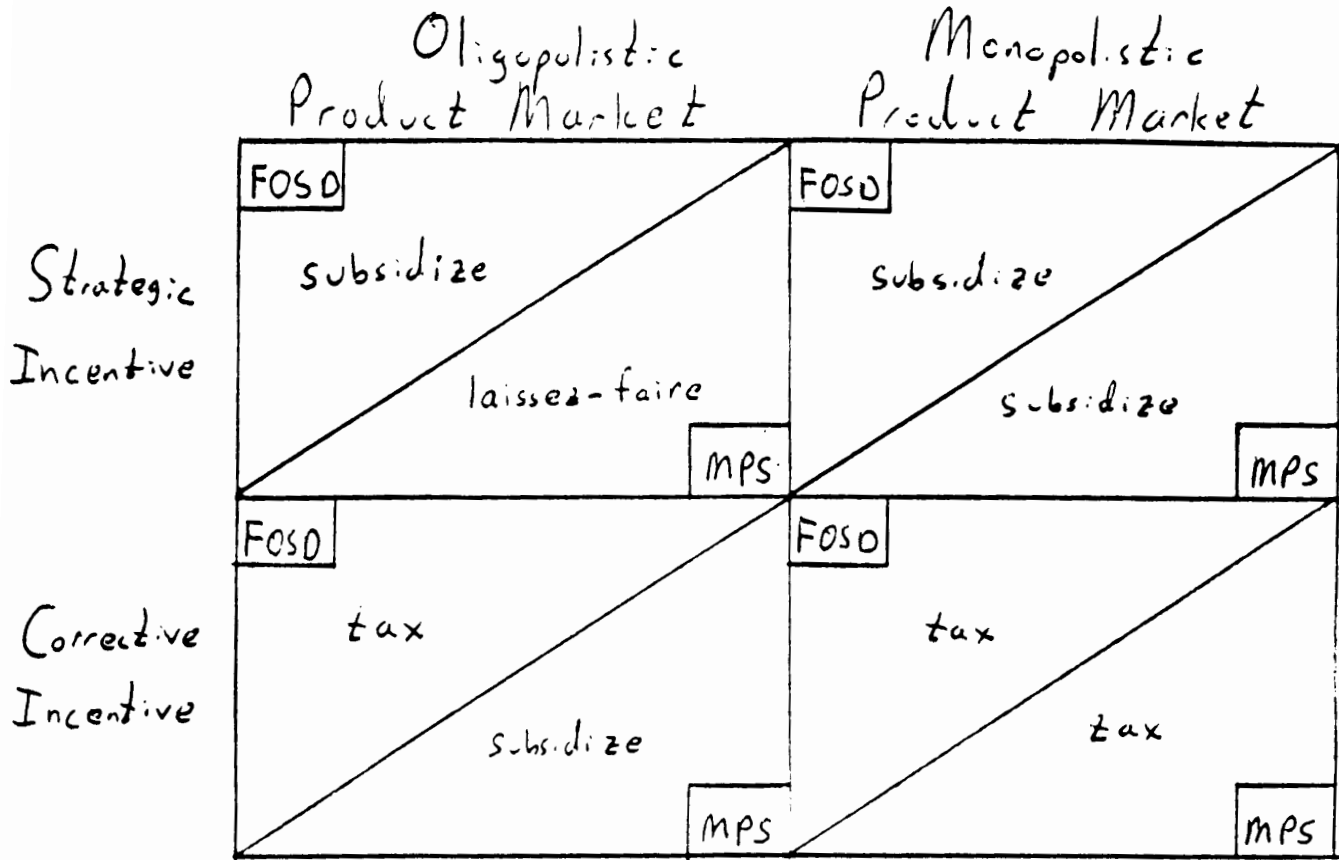


Figure 4



Note: FOSD denotes first order stochastic dominance; MPS denotes mean preserving spread.